DOKUZ EYLÜL UNIVERSITY

GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

AN INTEGRATED MULTI-CRITERIA DECISION MAKING METHODOLOGY FOR RISKY INVESTMENT PROJECTS EVALUATION

by

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March, 2009

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A Thesis Submitted to the
Graduate School of Natural and Applied Sciences of Dokuz Eylül University
In Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Industrial Engineering, Industrial Engineering Program

by
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March, 2009
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Ph.D. THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"AN INTEGRATED MULTI-CRITERIA DECISION MAKING METHODOLOGY FOR RISKY INVESTMENT PROJECTS EVALUATION"** completed by **ÖZGÜR ARMANERİ** under supervision of **PROF.DR. HASAN ESKİ** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy.

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ACKNOWLEDGMENTS

Although only my name appears on this thesis, this work would not be possible without the help of many people. First, I would like to express my gratitude to my supervisor, Prof. Dr. Hasan ESKİ, whose expertise, understanding, and patience, added considerably to my graduate experience. I am deeply thankful to him for the endless support, guidance and encouragement that made this study possible. It has been great pleasure and honor to work with him and learn from him.

I would like to thank the other members of my thesis committee, Prof. Dr. G. Miraç BAYHAN and Prof. Dr. Serdar KURT, for the assistance they provided at all levels of the study.

My very special thanks go to my friends, Dr. Hasan SELİM, Dr. Ceyhun ARAZ, Özgür YALÇINKAYA, and Emrah EDİS for their encouragement, help and friendship throughout my academic and private life. I would also like to thank my friends Dr. Özcan KILINÇCI, Dr. Özgür ESKİ, Dr. Gonca TUNÇEL, Dr. Özlem Uzun ARAZ and all my colleagues for their encouragement and friendship.

I would also like to give special thanks to my parents, Ümran and Yüksel ARMANERİ, and my brothers, Barış and İ. Hakkı ARMANERİ, for the support they provided me through my entire life, their forever love, encouragement, and confidence in me. In particular, I would like to express my special gratitude to my wife and best friend, Sergül ARMANERİ, without whose love, understanding and encouragement, I would not have finished this thesis. Lastly, I am grateful to my little daughter, Yağmur, for adding happiness to my life.

Özgür ARMANERİ

AN INTEGRATED MULTI-CRITERIA DECISION MAKING METHODOLOGY FOR RISKY INVESTMENT PROJECTS EVALUATION

ABSTRACT

The aim of this research is to propose a novel methodology for risky investment projects evaluation. The proposed methodology consists of three main stages. The first stage of the methodology includes opportunity and pre-feasibility studies. The aim of this stage is to give prominence to project ideas which have the highest chance of attaining the goals planned by entrepreneurs and investors. Therefore, in the first stage, the investment projects are classified by using a multi-criteria sorting (MCS) method which does not require a training sample, and takes into account the inherent risk and uncertainty associated with the values of evaluation criteria. This MCS method named as PROMSORT was proposed for financial classification problems. In the scope of this dissertation, this method has been adapted to the investment project evaluation and selection problems.

After assigning of the project alternatives to the groups, the second stage of the proposed methodology begins. In this stage, a new net present value (NPV) formulation that eliminates the weakness of using the traditional formulation of NPV has been developed. In uncertain and risky environments, the risky project parameters are determined by probability distributions by using simulation models. For that reason, in the second stage, a computer simulation model for new NPV formulation has been developed by using computer simulation software. Also, the second simulation model has been developed in order to calculate the expected cash flows for each project in each period.

The budgets of the enterprises are generally not enough to implement all of the investment proposals which have high expected utility level at the same time. In these cases, the enterprises prefer to implement the investment project proposals at the number allowed by the size of their budgets. Besides the lack of budget, the other reasons of this complexity may be some technical limitations such as earliest and latest start dates and precedence relations.

However, in today's high competitive environments, enterprises have to act well-planned. The first step of acting well-planned is to determine a planning horizon and to predict how much budget to allocate for carrying out investment projects each period over that planning horizon. In this new case, the main objective of enterprises is to maximize the expected utility of all investment projects which are carried out over the planning horizon. In the third stage of the proposed methodology, this type of problem is called as optimal project selection and scheduling problem. The last original contribution of this dissertation is to construct multi-objective mathematical models such as multi-objective linear programming model and fuzzy multi-objective linear programming models in order to solve this problem.

Keywords: Evaluation of Investment Projects, Multi-Criteria Decision Making, Decision Making Under Risk, Optimal Project Selection and Scheduling

RİSKLİ YATIRIM PROJELERİNİN DEĞERLENDİRİLMESİ İÇİN BÜTÜNLEŞİK ÇOK KRİTERLİ KARAR VERME METODOLOJİSİ

ÖZ

Bu araştırmanın amacı, riskli yatırım projelerinin değerlendirilmesi için özgün bir metodoloji önermektir. Önerilen metodoloji üç ana aşamadan oluşmaktadır. Metodolojinin birinci aşaması fırsat ve ön-yapılabilirlik çalışmalarını içermektedir. Bu aşamanın amacı, girişimciler ve yatırımcılar tarafından planlanan amaçlara ulaşmada en yüksek şansa sahip proje fikirlerini öne çıkarmaktır. Bu nedenle, birinci aşamada, yatırım projeleri bir referans kümeye ihtiyaç duymayan ve değerlendirme kriterlerinin değerleriyle ilişkili doğal risk ve belirsizliği dikkate alan çok kriterli sınıflandırma yöntemi kullanılarak sınıflandırılır. PROMSORT olarak adlandırılan bu çok kriterli sınıflandırma yöntemi, finansal sınıflandırma problemleri için önerilmiştir. Bu tez kapsamında, bu yöntem yatırım projesi değerlendirme ve seçme problemlerine uyarlanmıştır.

Proje alternatiflerini gruplara atadıktan sonra önerilen metodolojinin ikinci aşaması başlar. Bu aşamada, geleneksel net bugünkü değer (NBD) formülasyonu kullanmanın zayıflığını yok eden yeni bir NBD formülasyonu geliştirilmiştir. Belirsiz ve riskli ortamlarda, riskli proje parametreleri simülasyon modelleri kullanılarak olasılık dağılımları ile belirlenir. Bu sebepten dolayı, ikinci aşamada, bir bilgisayar simülasyonu yazılımı kullanılarak yeni NBD formülasyonu için bir bilgisayar simülasyonu modeli geliştirilmiştir. Ayrıca, her dönemde her proje için beklenen nakit akışlarını hesaplamak için ikinci bir simülasyon modeli geliştirilmiştir.

İşletmelerin sahip olduğu bütçe, genelde, beklenen fayda düzeyi yüksek olan bu yatırım önerilerinin hepsini aynı anda gerçekleştirmeye yetecek kadar çok değildir. Böyle durumda işletmeler, sahip oldukları bütçenin izin verdiği sayıda yatırım projesi önerisini gerçekleştirme yoluna gider. Bütçe yetersizliğinin yanında, bu karmaşıklığın diğer sebepleri en erken başlama ve tamamlanma zamanları ve öncelik ilişkileri gibi teknik sınırlamalar olabilir.

Ancak, günümüzün yoğun rekabet ortamında, işletmeler, planlı hareket etmek zorundadır. Planlı hareket etmenin ilk adımı, bir planlama ufkunun belirlenmesi ve bu planlama ufku boyunca her dönem yatırım projelerinin gerçekleştirilmesi için ne kadar bütçe ayrılacağının tahmin edilmesidir. Bu yeni durumda işletmelerin temel amacı, planlama ufku boyunca gerçekleştirilecek olan tüm yatırım projelerinin sağlayacağı faydayı maksimize etmektir. Önerilen metodolojinin üçüncü aşamasında, bu tip problem, optimal proje seçimi ve çizelgelemesi problemi olarak adlandırılır. Bu tezin son özgün katkısı, bu problemi çözmek için çok amaçlı doğrusal programlama modeli ve bulanık çok amaçlı doğrusal programlama modelleri gibi çok amaçlı matematiksel modeller oluşturmaktır.

Anahtar sözcükler: Yatırım Projelerinin Değerlendirilmesi, Çok Kriterli Karar Verme, Risk Altında Karar verme, Optimal Proje Seçimi ve Çizelgelemesi

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CHAPTER ONE
INTRODUCTION

In this chapter, the motivation, research objectives and original contributions of this work are stated, and the organization of this dissertation is outlined.

1.1 Motivation

The main purpose of economic activities and economic science is to overcome the shortage between the requirements and the economic resources. In other words, the main purpose is to provide a balance between the requirements and the economic resources. From this point of view, it can be said that the focus of the economic activities is the management of limited resources.

Enterprises are units in which two important economic functions - investment and production - are carried out. The enterprises, which are one of the most important elements of the economic life, are defined as the economic units that manufacture the products for the aim of getting maximum profit. Enterprises face various investment alternatives during their operating periods. However, the fact that economic resources are limited and there exist alternative usage areas forces them to make a decision and choice between how and where to use these resources. Therefore, enterprises which are faced with the problem of using economic resources for many different investment alternatives have to make a ranking and choice in order to ensure the topmost benefit among these different investment alternatives. In order to be able to make this ranking and choice, investment alternatives need to be assessed in accordance with certain criteria (Eski & Armaneri, 2006).

Investments are the basic factors which lead notable changes in economy of any country from a macro perspective and enable enterprises to attain their goals and maintain their existence from a micro perspective. It is apparent that investments are of great importance for economic growth. Increasing the amount of economic resources and improving their qualities are very important during the war against scarcity. All these efforts lead to an increase in production of total goods and services and raising welfare.

While the growth is required for all countries, it is of vital importance for especially developing and underdeveloped countries. In order to provide a success in economic growth, analyzing investment proposals in accordance with scientific fundamentals and taking investment decisions on the basis of results obtained from these analyses is very important.

From a micro perspective, fixed capital investments are the basic reason for enterprises to be able to carry on their existence. Because, in today's high competitive environments, enterprises have to make investment so as to increase their market value, to make a profit in the future and to accommodate to changing socio-economic and technologic conditions. Thus, enterprises should analyze and assess the available investment areas and alternatives in a rational way. This obligation is inevitable not only for attaining their goals in the future but also for maintaining their existence. The reason for this is that fixed capital investments affect all functions of enterprises such as supply, production, personnel, marketing and financing and naturally require great expenditures.

Despite all these points mentioned, in our country, one cannot say that both public and private sector investment projects are prepared and assessed in a rational manner. It can be said that preparation and assessment studies of investment projects are generally regarded as a formality to be performed. However, problems concerning the preparation and assessment of investment projects are of importance for both enterprise management and economy of a nation. Therefore, investment projects should absolutely be prepared and assessed on the basis of scientific facts.

The development of an industrial investment project from the stage of the initial idea until the plant is in operation can be shown in the form of a cycle comprising three distinct phases. These phases are called as the pre-investment phase, the investment phase, and the operational phase. The detail analyses of these phases are discussed in the following sections.

Pre-investment phase cover the time period between the birth of a project idea and decision-making to invest. In other words, it is pre-investment phase in which an investment project is prepared and assessed and the decision regarding the investment is taken. As an example, decisions about foundation of a new production facility in any sector or forming new production lines in existing facility are within the scope of pre-investment phase. Some studies which affect decision-making in pre-investment phase should be conducted. These studies are opportunity studies, pre-feasibility studies, feasibility studies and investment project evaluation and decision, respectively.

The first step of the pre-investment phase is the birth of idea of investment. In enterprises, many project ideas are put forward over a certain time period by entrepreneurs or managers. However, some of them can be rejected even without a need to a detailed analysis. The idea of producing black and white television can be held as an example of this type of ideas. Therefore, many project proposals should be eliminated if they do not have a good chance of ensuring the lowest cost and highest advantage. This case is the focus of opportunity studies.

The aim of the opportunity studies is to carry out preliminary election of project ideas and to give prominence to ideas which are promising among other ones and which have the highest chance of attaining the goals planned by entrepreneurs and investors.

During the investment project evaluation process, after completing the opportunity studies, the pre-feasibility studies should be carried out. As it will be explained in Section 2.3.1.2, the main aim of pre-feasibility studies is to determine whether it is necessary to conduct a detailed and comprehensive feasibility study for project ideas qualified as a result of opportunity studies and, if found necessary, which subjects require a more careful and detailed study.

While superficially assessing the project ideas with the opportunity and pre-feasibility studies, certain criteria should be taken into account. It is not a desirable situation that any project idea is assessed by considering only one assessment criterion. Therefore, more than one criterion should be taken into account while determining whether project ideas will be suitable for the goals of entrepreneurs.

Assessing certain number of project ideas in accordance with several criteria with opportunity and pre-feasibility studies and determining which of them will be exposed to feasibility studies is a type of discrete decision making problems. Discrete problems involve the examination of a discrete set of alternatives. Each alternative is described along some attributes. Within the decision making context these attributes have the form of evaluation criteria. When considering a discrete decision making problem, there are four different kinds of analyses (decision making problematics) that can be performed in order to provide meaningful support to decision makers; ranking, choice, description and classification/sorting.

The problem, that is tried to be solved by the opportunity and pre-feasibility studies, is better addressed through the classification/sorting problematic. Sorting problematic involves the assignment of a set of alternatives in homogenous groups defined in a preference order. There are many statistical and econometric classification methods, which constitute the traditional approach to develop classification models. However, they are several shortcomings due to the restrictive statistical assumptions. A significant drawback of all these methods is the exclusion of qualitative criteria such as quality of management, market position, etc (Zopounidis & Doumpos, 1999; Araz & Özkarahan, 2005). In order to overcome these shortcomings, alternative sorting approaches have been developed by researchers. A significant part is devoted on the development of multicriteria sorting (MCS) methods. MCS problem consists in assigning a set of alternatives evaluated on multiple criteria to one of the predefined classes. Many of the approaches proposed assume that a set of training sample exists.

As known, the sample of observations used to develop the classification/sorting model is referred to as the training sample. Here, the observations are referred to as alternatives. If the developed model performs satisfactorily in the training sample, it can be used to decide upon the classification of any new alternative that becomes under consideration. However, in many cases, especially the case of project evaluation and selection, the training sample may not be possible, but the literature indicates that the investment projects are mostly classified by using MCS methods that require a training sample. This kind of behavior is not realistic. For that reason, the investment project alternatives should be classified into the predefined ordered classes by using one of the MCS methods that does not require a training sample. This fact is the first motivator of this dissertation.

On the other hand, it is quite clear that investment decision-making never takes place under conditions of certainty, but only under those of uncertainty or risk. Therefore, it is necessary to define and locate the investment decision-making problem in its real conditions, and possibly find suitable and appropriate solutions (Jovanovic, 1999). For that reason, when the project alternatives are classified in the opportunity and pre-feasibility studies, the inherent risk and uncertainty associated with the values of evaluation criteria should be handled carefully. As a consequence, besides the necessity to classify the investment projects by using a MCS method which does not require a training sample, this MCS method should be taken into account the inherent risk and uncertainty associated with the values of evaluation criteria. This fact is the second motivator of this dissertation.

As a summary, after determining promising project ideas among other ones in the opportunity and pre-feasibility studies, the next step is to make feasibility studies for these investment projects. A feasibility study should provide all data necessary for an investment decision. As mentioned, enterprises have to make a ranking and choice in order to ensure the topmost benefit among different investment alternatives. In order to be able to make this ranking and choice, firstly, the feasibility studies should be conducted for these alternatives and then they need to be assessed in accordance with certain criteria. At this point, the evaluation process of the project alternatives requires some data for non-realized investments, i.e., total amount of investment, cash flows during the economical life of the project, discount rates, and salvage value. However, it is nearly impossible to know the values of these parameters with a complete certainty before the project is realized.

Because of the uncertainty and risk of the future, the values of the project parameters can not be estimated with complete certainty. Any wrong value that is estimated by the decision maker will directly affect the return and the profitability of the project. Therefore, it is necessary to consider uncertainty and risk phenomena while evaluating projects. Several methods have been presented in the literature to handle the analysis of the investment projects under uncertainty or risk. One of the methods for analyzing complex, real-world decision making situations involving risk is simulation.

Simulation is a statistics based behavioral approach that applies predetermined probability distributions and random numbers to estimate risky outcomes. Recently, the usage of simulation in investment project evaluation under uncertain and/or risky environments has been increasing. Because, simulation based project evaluation approaches enable to make more reliable investment decisions since they permit including future uncertainty and risk in analysis process.

In simulation based project evaluation approaches, the risky project parameters are defined as probability distributions. The expected profitability of the project is calculated via simulation approach. It is well known that, project profitability is generally determined by checking net present value (NPV) of the project. In literature, much of the studies that use simulation approach to calculate the expected NPV of the project are used the traditional formulation of NPV which will be expressed in Section 2.4.2.1. In these studies, it is often assumed that the effect of inflation is same both on project inflows and outflows, so the effect of inflation on project inflows and outflows is not taken into account. But it is obvious that inflation effect will be different for cost and revenue components, and it should be considered in project evaluation process. The other important point is; in most of these studies, only the net cash flows or gross cash inflows and cash outflows are simulated in order to provide a sufficient number of NPVs and to develop the NPV distribution.

However, defining only the net cash flows or gross cash inflows and cash outflows as probability distributions can make the calculation process easy, but it is not realistic. Instead of it, individual inflow and outflow components, such as sales volume, sale price, revenues, material cost, labor cost, depreciation, taxes, and any other risky parameters should be defined as probability distributions. In such a situation, it is necessary to develop a new NPV formulation that eliminates the weakness of using the traditional formulation of NPV while evaluating the projects. This fact is the third motivator of this dissertation.

In this new situation, the number of parameters, which are defined as probability distributions, will increase. Therefore, using Monte Carlo simulation approach for modeling the new developed NPV formulation will cause some complexities. Because, if the numbers of the random variables in the mathematical model increases, providing a sufficient number of NPVs to define the NPV distribution would be more difficult by using Monte Carlo simulation. Hence, it is a necessity to develop a computer simulation model for new NPV formulation by using computer simulation software. By the help of this model, all parameters affecting the NPV of the project can be defined as discrete and continuous probability distributions if required. Therefore, this fact is the fourth motivator of this dissertation.

During any time period, most enterprises, especially public enterprises, have to make a ranking and selection among a number of investment project proposals. Some of these investment proposals may be promising and can allow the enterprises and entrepreneurs to realize their objectives. However, the budgets of the enterprises are generally not enough to implement all of these investment proposals which have high expected utility level at the same time. In these cases, the enterprises prefer to implement the investment project proposals at the number allowed by the size of their budgets.

For example, when public enterprises allocate funds for investment project proposals, they make a ranking among investment proposals on the basis of effectiveness measures such as the expected profitability and social utility. At a certain time period, these enterprises determine which project proposals to carry out in accordance with certain effectiveness measures by taking into account the budget possibilities, and they start operations in order to perform these project proposals. This process is repeated in the next time period. That is, the project proposals are redetermined and they are reevaluated and reranked by taking certain effectiveness measures into account. The budget possibilities are recontrolled for that time period. A decision is taken for carrying out the project or projects which are best compatible with the objectives of enterprises or entrepreneurs and these projects start to be out into practice. This process continues throughout subsequent time periods, similarly.

As a result of this process, some of the investment project proposals with high expected utility level are selected and performed and the others are not. Besides the lack of budget, the other reasons of this complexity may be some technical limitations such as earliest and latest start dates and precedence relations between specific projects.

If the amount of budget planned by the enterprises to be allocated for investments is estimated only for the current time period, then it is inevitable to implement the process described above. However, in today’s high competitive environments, enterprises, especially public enterprises, have to act well-planned. The first step of acting well-planned is to determine a planning horizon and to predict how much budget to allocate for carrying out investment projects each period over that planning horizon. In this way, there will not be the cases in which some of the project proposals, evaluated at the beginning of each period and predicted to have high expected utility level, cannot carried out due to the lack of budget allocated for investments for that period. If some of the project proposals cannot be put into practice owing to lack of budget, these project proposals will have the chance of being carried out at other periods in the planning horizon. The reason for this is that the budget estimate regarding each period in the planning horizon is certain. In this new case, the main objective of the enterprises is to maximize the expected utility of all investment projects which are carried out over the planning horizon.

At the same time, some projects whose economic evaluation in the current time period is unfavorable may, as a result of expected population and income growth, fare much better at a later date (Medaglia et al., 2008). Therefore, the planning process of the enterprises relies on optimal project selection and scheduling and the efficient allocation of scarce resources. This process is complicated due in part to the fact that investment project should be considered according to multiple objectives, project cash flows are uncertain, the estimated budget for each time period can be flexible, and there are several limitations.

According to this perspective, a feasibility study should be made for all project proposals at the beginning of each planning horizon and these proposals should be evaluated in accordance with obtained results. Then, it should be determined which of these investment proposals will be suitable for implementing. At this point, one should not consider the budget estimate for investments concerning only the first period of planning horizon. The budget amounts planned to be allocated each year over the horizon should also be taken into account. In this way, if some projects have not been implemented during the first period due to the lack of budget, it can be decided to carry out the projects in the subsequent periods. We should note here that the utility level of a project proposal carried out in the first periods of the horizon may be different the utility level of this project proposal carried out in the subsequent periods. So, while conducting feasibility studies for project proposals, the utility levels of them should be determined for each period in the planning horizon.

Since the project parameters are uncertain, the expected values of effectiveness measures such as a NPV of a project proposal include risk. Therefore, before determining the periods in which the projects will be implemented, it should be taken into account the possibility that expected values of effectiveness measures regarding the selected projects may deviate.

It should be noticed to some operational business and technical constraints while one tries to determine which of the project proposals that have high expected utility level will be implemented in which period of planning horizon. As an example, there may be earliest and latest start dates of the projects, and there may be precedence relations between specific projects, or there may be mutually exclusive projects that have the highest chance of attaining the goals planned by entrepreneurs and investors.

The optimal project selection and scheduling problem that was explained in details can be solved by developing multi-objective mathematical models. This fact is the fifth motivator of this dissertation.

As a consequence, the literature contains a number of studies in which risky investment projects are evaluated. However, we have not been able to encounter a methodology covered all the facts defined above. In the studies contained in literature, several methodologies have been proposed which discuss the facts mentioned above separately. On the other hand, some facts have never been studied by researchers before. Therefore, it is believed that for risky investment projects evaluation, there is a need for an integrated multi-criteria making methodology which covers all the facts expressed above. This fact is the last and *major motivator* of this dissertation.

1.2 Research Objectives

As mentioned in the previous section, it has not been able to encounter a methodology covered all defined facts in the literature. Therefore, the aim of this research is to propose a novel methodology for risky investment projects evaluation. Preliminary election of project ideas and to give prominence to ideas which are promising among other ones is inherently a multi-criteria decision making problem. Therefore, the proposed methodology should be based on the multi-criteria evaluation of the investment project alternatives. Considering the facts described in the previous section, the main objectives of this research are as follows;

(1) Motivated by the fact that the investment project alternatives should be classified into the predefined ordered classes by using one of the MCS methods that does not require a training sample, the first objective of this research is to classify the investment projects by using a MCS method which does not require a training sample.

(2) Motivated by the fact that investment decision-making never takes place under conditions of certainty, but only under those of uncertainty or risk, the second objective of this research is to classify the investment projects by using a MCS method which does not require a training sample, and takes into account the inherent risk and uncertainty associated with the values of evaluation criteria.

(3) Motivated by the fact that the inflation effects on individual inflow and outflow components should be considered in project evaluation process, and all risky inflow and outflow components, such as sales volume, sale price, revenues, material cost, labor cost, depreciation, taxes, and any other risky parameters should be determined by probability distributions, the third objective of this research is to develop a new NPV formulation that eliminates the weakness of using the traditional formulation of NPV while evaluating the projects.

(4) Motivated by the fact that if the numbers of the random variables in the mathematical model increases, providing a sufficient number of NPVs to define the NPV distribution would be more difficult by using Monte Carlo simulation, the fourth objective of this research is to develop a computer simulation model for new NPV formulation by using computer simulation software. By the help of this model, all risky parameters can be defined as discrete and continuous probability distributions if required.

(5) Motivated by the fact that the optimal project selection and scheduling problem explained in details in the previous section can be solved by constructing multi-objective mathematical models, the fifth objective of this research is to construct multi-objective mathematical models such as multi-objective linear programming model and fuzzy multi-objective linear programming models in order to solve this problem.

(6) Motivated by the fact that it has not been able to encounter a methodology covered all defined facts in the literature, the last and major objective of this

research is to propose an integrated multi-criteria decision making methodology for risky investment projects evaluation which includes all the facts described in the previous section.

1.3 Original Contributions

The following list summarizes the original contributions to be achieved with this dissertation to the investment project evaluation and selection literature.

(1) The major contribution of this dissertation is to propose an integrated multi-criteria decision making methodology for risky investment projects evaluation which includes all the facts described in the previous sections. This integrated methodology will be explained in details in Chapter Six.

(2) The second original contribution of this dissertation is to classify the investment projects by using a MCS method which does not require a training sample, and takes into account the inherent risk and uncertainty associated with the values of evaluation criteria. This MCS method named as PROMSORT that assigns alternatives to predefined ordered categories was proposed by Araz & Ozkarahan (2005) for financial classification problems. It was also used to solve the strategic supplier selection problem by Araz et al. (2007). This new MCS procedure has been adapted to the investment project evaluation and selection problems. In the scope of this dissertation, this method has been used in order to assign project alternatives to predefined ordered categories in the first stage of the proposed methodology.

(3) The third original contribution of this dissertation is to develop a new NPV formulation that eliminates the weakness of using the traditional formulation of NPV, which will be expressed in Section 2.4.2.1, while evaluating the projects. The developed NPV formulation will be explained in details in Section 6.2.2.

(4) The fourth original contribution of this dissertation is to develop a computer simulation model for new NPV formulation by using computer simulation software. As mentioned, in uncertain and risky environments, the values of the project parameters can not be estimated with complete certainty, and it is necessary to consider uncertainty and risk phenomena while evaluating projects. The risky project parameters are defined as probability distributions by using simulation models. Also, the expected profitability of the project is calculated via simulation. In the developed NPV formulation, the numbers of the random variables have been increased. So, providing a sufficient number of NPVs to define the NPV distribution would be more difficult by using Monte Carlo simulation. In this dissertation, the second simulation model is developed in order to calculate the expected cash flows for each project in each period. The developed computer simulation models will be explained in details in Section 6.2.2.

(5) The fifth original contribution of this dissertation is to construct multi-objective mathematical models such as multi-objective linear programming model and fuzzy multi-objective linear programming models in order to solve the optimal project selection and scheduling problem that was explained in details in the Section 1.1. The constructed multi-objective mathematical models will be explained in details in Section 6.2.3.

1.4 Organization of the Thesis

This dissertation is divided into seven chapters. The organization of this dissertation is as follows;

An overview of investment project evaluation and selection is presented in Chapter 2. In this chapter, basic concepts and definitions related with investment project evaluation and selection process are explained, and investment project cycle is explained. Investment project evaluation and selection methods under certainty are covered in this concern. Then, investment project evaluation and selection process under risky and uncertain environments is presented, separately. The review of the related literature and an overview of approaches and methods used for solving project evaluation and selection problem are also provided in this chapter.

In Chapter 3, taxonomy of the multi-criteria decision making problems is described and some methods used for solving these problems are reviewed. Chapter 3 also provides a comprehensive overview of multi-criteria classification problem and reviews some methods to solve these problems. At the end of this chapter, one of the MCS procedures, called PROMSORT, which will be used to assign the project alternatives to predefined ordered categories in the first stage of the proposed methodology, is presented in details.

Chapter 4 is devoted to describe the usage of simulation in risky investment project evaluation and selection process. In this chapter, after giving all information about Monte Carlo simulation of a risky investment project, computer simulation modeling of a risky investment project is explained in details. This section covers computer simulation model building, simulation software packages types, and output analysis of simulation. At the end of this chapter, advantages of using simulation in risky investment projects evaluation and the key points that should be taken into account in investment project evaluation via the simulation method are presented.

An overview of fuzzy mathematical programming is presented in Chapter 5. Basic concepts and definitions of fuzzy set theory, fuzzy linear programming, fuzzy multi-objective and fuzzy multi-attribute decision making are explained in this concern. Fuzzy multi-objective linear programming and fuzzy multi-objective modeling approaches which are employed in the computational experiments performed in this dissertation are also explained in this chapter.

Chapter 6 proposes an integrated multi-criteria decision making methodology for risky investment projects evaluation that consists of three main stages. The names of these stages are opportunity and pre-feasibility studies, feasibility study, and investment project evaluation and decision. In this chapter, computational experiments are presented in order to explore the application of the proposed methodology. In our experiments, several different well known multi-objective modeling approaches are employed in the third stage of the proposed methodology.

Finally, Chapter 7 concludes the dissertation that covers summary, concluding remarks and contributions of this dissertation and also suggestions for future research.

CHAPTER TWO
INVESTMENT PROJECT EVALUATION AND SELECTION: AN OVERVIEW AND LITERATURE REVIEW

In this chapter, an overview of investment project evaluation and selection will be presented. At first, basic concepts and definitions related with investment project evaluation and selection process will be explained, and investment project cycle will be discussed. This chapter covers investment project evaluation and selection methods under certainty, and also investment project evaluation and selection process under risky and uncertain environments will be presented, separately. The review of the related literature, an overview of approaches and methods used for solving project evaluation and selection problem, and gaps in the existing literature are also provided in this chapter.

2.1 Introduction

It is of great importance to understand what the concept investment means in order to make investment decisions and compare investment alternatives with each other more comprehensible. There exist several definitions in literature regarding the investment concept. Therefore, investment can be defined in the following ways (Eski & Armaneri, 2006, pp.317-319);

Investment, from the point of economics, is defined as net additions made by an individual, an enterprise or a country to existing capital assets and technical equipment stock over a certain period of time. In this case, allocation of resources by individuals, enterprises or countries in attempt to found new production places or renew old and worn-out machines and equipment is named as investment. A newly-founded factory or a newly-built power plant, a new road, new machines and improvements in production capacity are called as investment in terms of economy. Investment is a concept that is very closely related to capital accumulation (industrialization). Thus, this concept bears strategic importance to especially countries which are on the path towards economic development and growth. As can be inferred from the definition, economically net increases in inventory over a certain period of time are regarded as investment, too.

In terms of business science, the investment concept is specified as net additions to fixed assets or raw materials, semi-finished and finished stocks over a certain period of time. Fixed capital goods, machines, buildings, transport vehicles can be listed as examples of fixed assets. In literature, investments in fixed assets are also called as fixed capital investments. Fixed capital investments can be generally defined as expenses made for all kinds of durable goods which are used constantly or recurrently during the production process over more than one year by production units, generally, to produce goods or services.

The investment concept, from an entrepreneur point of view, is defined as investing the existing monetary resources in fixed assets such as machinery and equipment, production facility and transport vehicles. By transporting these existing monetary resources into fixed capital investment, planned operations will be performed with these fixed assets bought.

As for financing science, the investment concept is specified as transforming cash assets, which do not provide income if they are not used, into less liquid assets with a view to obtain income.

As can be seen, the investment concept has different definitions according to several points of view. However, investments, regardless of the way they are defined, enjoy a significant role in enabling enterprises to attain their goals, to maintain their competitive power, to be able to adapt to all kind of changes under the related environmental and competitive conditions and to reach their targets. Naturally, investments are of utmost importance for not only enterprises, but also for economic development of countries and raising their development level.

In the light of definitions presented concerning the investment concept, it is possible to classify investments into two categories as those made in current assets and those made in fixed assets. The use of funds at disposal for provision of current assets or fund expenses on current assets can be defined as investment in current assets. This type of investments is also known as working capital investments. As previously specified, expenses on assets such as factory building, land, machine and equipment, which serve the enterprises for a long time, are characterized as investments in fixed assets. For enterprises, there are differences between investments in fixed and current assets due to following reasons (Akgüç, 1998);

(1) Investments in fixed assets generally require higher cost when compared to investments in current assets.

(2) Investments in fixed assets are as a whole and indivisible. Investments in current assets, on the other hand, are divisible. For example, it is possible to make or not to make sales on credit to a customer or keep less or more stock of a certain raw material or finished goods. This dissimilarity also affects financing resources of fixed and current assets.

(3) In investments in fixed assets, the funds owned by the firm become dependent for a long time. Thus, predictions during investments in fixed assets extend over a long time and are of great importance. In this kind of investments, since the possibility of improving or amending the decisions regarding the capital expenditures at short intervals is limited, deviations in expectations about future lead to notable consequences on the part of firms. Since investments in current assets, on the other hand, are relatively shorter-term when compared to investments in fixed assets, it is possible to revise and amend the decisions taken and expectations put forward within a few months' period.

(4) One of the most significant elements that separate investments in fixed assets from those in current assets is liquidity. Investments in current assets have a high chance of being liquidated quickly and without experiencing a value loss. However, this is not the case for investments in fixed assets.

(5) Investments in fixed assets affect the risk degree of a firm directly and considerably. Investments in current assets, on the other hand, affect risk degree of a firm in a limited manner.

Considering the dissimilarities specified above, investments in fixed assets, that is; fixed capital investments, are much more important for the enterprises when compared to investments in current assets, and precise assessment of investments in fixed assets is of great importance for the success of enterprises in the future. The reason for this is that when a decision is made to invest in fixed assets, this will mean that most of the limited capital of firms will be allocated for fixed assets for a very long time, and if this investment decision is taken without a proper analysis, this may cause notable, irreversible, negative consequences on the part of enterprises (Eski & Armaneri, 2006).

Economic resources at disposal are not enough for satisfying all needs and realizing all targets of enterprises or countries. Naturally, limited resources constitute an obstacle to finance all investment alternatives possible and perform all these investment alternatives at the same time. For this reason, both enterprises and countries are supposed to carry out appropriate investments so as to use their resources in a proper and rational manner. Otherwise, already limited resources would be wasted. Therefore, it is necessary to make a choice between competing investment proposals, to list them in accordance with certain evaluation criteria and give up some of the investment proposals at least for a while should the problem of lack of resources arise (Eski & Armaneri, 2006).

It is apparent that investments are of great importance for economic growth. Increasing the amount of economic resources and improving their qualities are very important. All these efforts lead to an increase in production of total goods and services and raising welfare. Naturally, during the process of realizing economic growth with already limited economic resources, analyzing investment proposals, which will require the use of existing limited resources, and investment expenditures to be made for these proposals in accordance with scientific fundamentals and taking investment decisions on the basis of results obtained from these analyses will enable to get expected results from the investments (Eski & Armaneri, 2006).

2.2 Investment Project Concept

The concept “project” is one which we frequently encounter in our daily lives and often implies a work we are involved in. Project is the activity of planning how and in which manner available resources will be used. It is also possible to define the concept “project” as set of activities which are related to each other and will be performed over a certain period of time and within a framework of a schedule. Project Management Institute (2004) defines a project as a temporary endeavor undertaken to create a unique product or service. Here, temporary means that every project has a definite end. Unique means that the product or service is different in some distinguishing way from all similar products or services. Turner (1999) defines a project as an endeavor in which human, (or machine), material and financial resources are organized in a novel way, to undertake a unique scope of work, of given specification, within constraints of cost and time, so as to deliver beneficial change defined by quantitative and qualitative objectives. Archibald (2003) defines a project as the entire process required to produce a new product, new plant, new system, or other specified results. Burke (2003) states that the main difference between project management and general management (or any other form of management for that matter) relates to the definition of a project and what the project intends to deliver to the client and stakeholders.

Regardless of the way it is defined, it is very important to carry out a project and carry on the activities in accordance with this project in order to use available resources effectively during operations which have many sub-phases and require high expenditures of resources. The reason for this is that attaining the goal will be easier if, before starting an operation, all details concerning the any operation is presented and planned, potential problems are determined and their solutions are specified before starting the project (Eski & Armaneri, 2006).

All projects have some characteristics in common notwithstanding their size and coverage. These can be listed as the following (Eski & Armaneri, 2006);

(1) Projects are temporary efforts; that is, all projects have a starting and ending date. In other words, a project has a certain due time to be completed. Studies which will last forever or have unclear starting dates cannot be described as project.

(2) It is accepted beforehand that when the project is completed, it will yield a product which has not been made before and uniqueness of which will not be controversial. For instance, construction of a new factory building will be a unique work for the person who performed the construction. The reason for this is that, although there may be many similar buildings, a product is developed which is absolutely different from them based on one or several characteristics such as its location, architecture, area or the material used.

(3) Each project has a scope and budget. Projects should be carried out considering available budget facilities and budget shares which can be allocated for the related project.

The concepts “project” and “investment project” are closely inter-related by definition. Investment projects also have all characteristics of projects specified above. But in order for a project to be described as an investment project, it has to cover one of the investment types discussed previously. In this sense, an investment project can be defined as an investment proposal for providing new opportunities to increase the production of goods and services and ensure utmost benefit with the least usage of resource over a certain time period or expanding or improving existing opportunities. It is also possible to describe investment projects as directly-related set of activities towards production of goods and services by using limited economic resources over a certain time period within the framework of an outline (Eski & Armaneri, 2006).

The most important elements which separate investment projects from other project types (e.g. design projects, research and development projects) can be listed as the following (Eski & Armaneri, 2006);

(1) With an investment project, a certain and long-lasting new production capacity is created or existing capacity is renewed or increased.

(2) Investment projects require production factors such as labor force, capital goods, raw material and so on.

(3) As a result of investment projects, goods and services are produced by applying certain production technologies.

In general, investment projects can be classified into the following categories (Salvatore, 1996, p.590);

(1) *Replacement:* Investments to replace equipment that is worn out in the production process.

(2) *Cost Reduction:* Investments to replace working but obsolete equipment with new and more efficient equipment, expenditures for training programs aimed at reducing labor costs, and expenditures to move production facilities to areas where labor and other inputs are cheaper.

(3) *Output expansion of traditional products and markets:* Investments to expand production facilities in response to increased demand fort he firm's traditional products in traditional or existing markets.

(4) *Expansion into new products and/or markets:* Investments to develop, produce, and sell new products and/or enter new markets.

(5) *Government regulation:* Investments made to comply with government regulations. These include investment projects required to meet government

health and safety regulations, pollution control, and to satisfy other legal requirements.

In general, investment decisions to replace worn-out equipment are the easiest to make since management is familiar with the specifications, productivity, and operating and maintenance costs of existing equipment and with the time when it needs to be replaced.

Investment projects to reduce costs and expand output in traditional products and markets are generally more complex and usually require more detailed analysis and approval by higher-level management. Familiarity with the product and the market, however, does not usually make these projects among the most challenging that management is likely to face.

Investment projects to produce new products and move into new markets, on the other hand, are likely to be very complex because of the much greater risk involved. They are also likely to be the most essential and financially rewarding in the long run since a firm’s product line tends to become obsolete over time and its traditional market may shrink or even disappear (witness the market for slide rules which have been practically replaced entirely by hand-held calculators during the past decade).

Finally, investment projects to meet government regulations often give rise to special legal, evaluation, and monitoring problems requiring outside expert assistance.

It is clear that the generation of ideas and proposals for new investment projects is crucial for the future profitability of the firm. In well-managed and dynamic firms, all employees are encouraged to come up with new investment ideas. Most large firms, however, are likely to have a research and development division especially entrusted with the responsibility of coming up with proposals for new investment projects. Such a division is likely to be staffed by experts in product development, marketing research, industrial engineering and so on, and they may regularly meet with the heads of other divisions in brainstorming sessions to examine new products, markets, and strategies (Salvatore, 1996, p.591).

Investment decisions may be tactical or strategic. A tactical investment decision generally involves a relatively small amount of funds and does not constitute a major departure from what the firms has been doing in the past. Strategic investment decisions involve large sums of money and may also result in a major departure from what the company has been doing in the past. Acceptance of a strategic investment will involve a significant change in the company's expected profits and in the risks to which these profits will be subject (Bierman & Smidt, 1990).

Investment projects represent sizable outlays of funds that commit a firm to some course of action. Consequently, the firm needs procedures to analyze and properly select its investment projects.

2.3 Investment Project Cycle

Morris & Hough (1987) describe the activity sequence of project cycle as follows: "every project, no matter of what kind or for what duration, essentially follows the activity sequence of pre-feasibility/feasibility, design and contract negotiation, implementation, handover and in-service support"(Morris & Hough, 1987, p.74).

Just like Morris & Hough, Sell (1991) also divides any project into several phases. First phase, naturally, is project conception. Then opportunity and pre-feasibility studies follow. After that, feasibility study consisting of techno-economic, financial and economic analyses is performed. The next phases are investment and operating activities. Finally, ex-post evaluation is performed.

As a consequence, the development of an industrial investment project from the stage of the initial idea until the plant is in operation can be shown in the form of a cycle comprising three distinct phases;

(1) The pre-investment phase,
(2) The investment phase,
(3) The operational phase.

Each of these three phases is divisible into stages, some of which constitute important consultancy, engineering and industrial activities.

Several parallel activities take place within the pre-investment phase and even overlap into the succeeding investment phase. Thus, once an opportunity study has produced fairly dependable indications of a viable project, investment promotion and implementation planning are initiated, leaving the main effort; however, to the final investment appraisal and the investment phase (Figure 2.1). To reduce wastage of scarce resources, a clear comprehension of the sequence of events is required when developing an investment proposal from the conceptual stage by way of active promotional efforts to the operational stage (Behrens & Hawranek, 1991, p.9).

All phases of the project cycle lend themselves to important consultancy and engineering work to be carried out. Increasing importance should, however, be attached to the pre-investment phase as a central point of attention, because the success or failure of an industrial project ultimately depends on the marketing, technical, financial and economic findings and their interpretation, especially in the feasibility study. The costs involved should not constitute an obstacle to an adequate examination and appraisal of a project in the pre-investment phase; as such a process might save considerable costs, including those relating to misdirected investment, after start-up of the enterprise (Behrens, 1989, p.1002).

2.3.1 The Pre-investment Phase

As can be seen in Figure 2.1, the first phase included in life cycle of investment projects is the pre-investment phase which means the time period between the birth of investment idea and decision-making to invest. Pre-investment phase consists of several sub-studies.

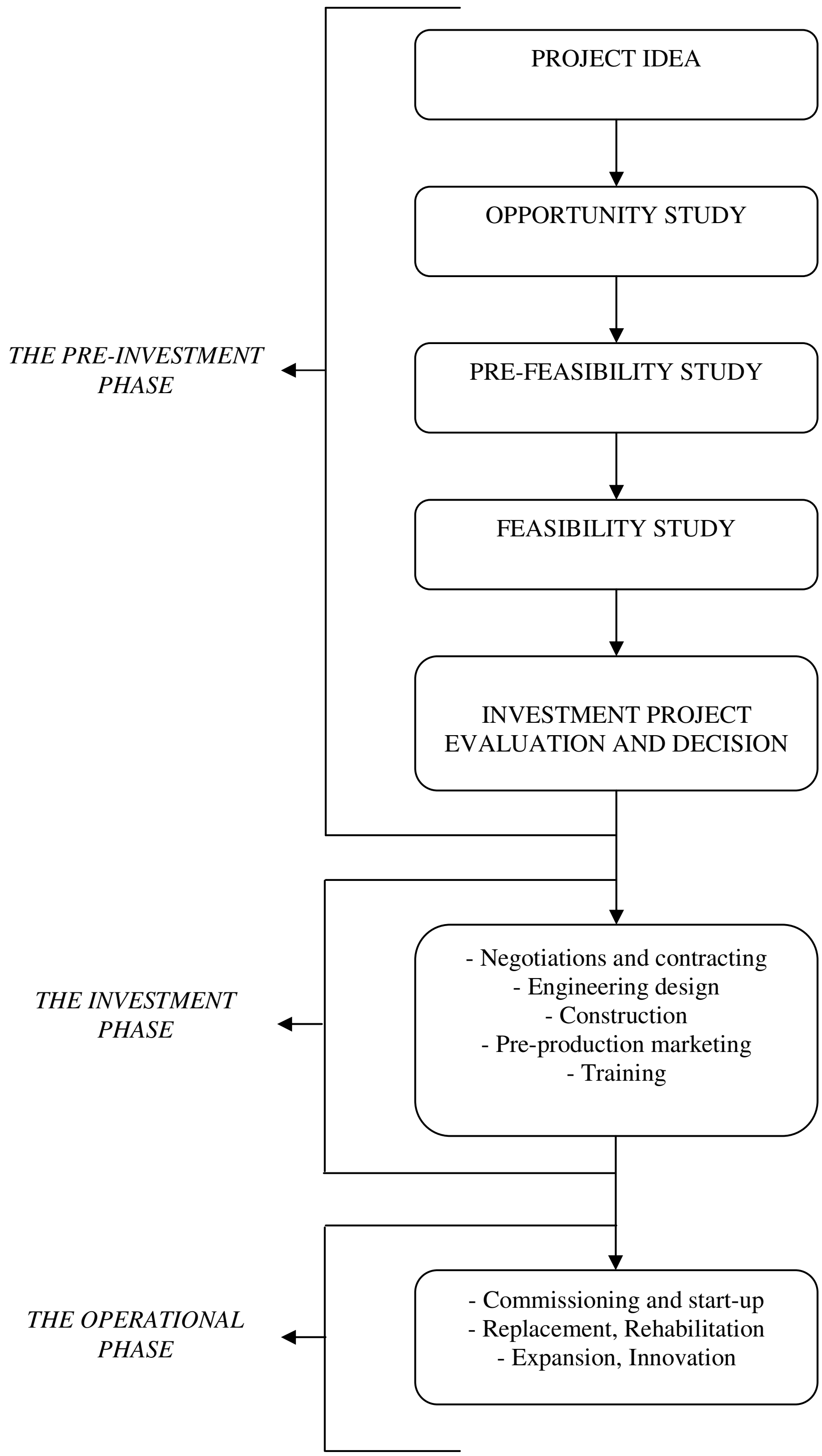


Figure 2.1 Investment Project Cycle

The first step of pre-investment phase is the birth of an investment idea. Opinions about fixed capital investment proposal can be put forward by entrepreneurs. For example, foundation of a new production factory of battery can be approached as a new idea of a project. It is natural that this idea which includes only the general scope of the project can be made into a detailed one and different ideas of project can be composed. For instance, foundation of a new production factory of battery within the borders of the city Izmir or Manisa should be handled with as separate project ideas. Again, project ideas about building new production facilities which will operate in different fields and sectors can be put forward. It is highly important that in order for a project idea to be a good one and suitable for targets, it should be put forward by entrepreneurs who are experienced, learned, constructive and who have strong intuition about future (Eski & Armaneri, 2006).

In enterprises, many project ideas are put forward over a certain time period by entrepreneurs or managers. However, some of them can be rejected even without a need to a detailed analysis. The idea of producing black and white television can be held as an example of this type of ideas. Therefore, many project proposals should be eliminated if they do not have a good chance of ensuring the lowest cost and highest advantage. At this point, this case is the focus of opportunity studies (Eski & Armaneri, 2006).

As a consequence, after creating project ideas, the pre-investment phase (Figure 2.1) comprises several stages;

(1) Identification of investment opportunities (opportunity studies),
(2) Pre-feasibility studies,
(3) Feasibility studies,
(4) Investment project evaluation and decision.

2.3.1.1 Opportunity Studies

Conducting detailed analyses and assessments for all of many different project ideas is not an applicable way in practice since this requires a very long time and high cost. Furthermore, when all project ideas are analyzed in detail, some investment opportunities which firstly seem applicable may be missed. Thus, it is of great importance to determine the idea or ideas quickly which will satisfy the goals and expectations of the entrepreneur in the best way among many different ones, and to conduct detailed analyses and assessments for these determined project alternatives.

The opportunity study is very rough study. This very rough study is performed to make clear the possibilities of technically realizing the project in the region. Furthermore, first signals of the chances of the project to be an economic success are monitored. Opportunity studies are based on predictions rather than detailed analyses.

The main instrument used to quantify the parameters, information and data required to develop a project idea into a proposal is the opportunity study, which should analyze the following:

(1) Natural resources with potential for processing and manufacture, such as timber for wood-based industries.
(2) The existing agricultural pattern that serves as a basis for agro-based industries.
(3) Future demand for certain consumer goods that have growth potential as a result of increased population or purchasing power or for newly developed goods such as synthetic fabrics or domestic electrical products.
(4) Imports, in order to identify areas for import substitution.
(5) Environmental impact.
(6) Manufacturing sectors successful in other countries with similar economic background and levels of development, capital, labor and natural resources
(7) Possible interlinkage with other industries, indigenous or transnational.
(8) Possible extension of existing lines of manufacture by backward or forward integration, linking, for example, a downstream petrochemical industry with a refinery, or an electric-arc steel plant with a steel rolling-mill.
(9) Possibilities for diversification, for example, from a petrochemical complex into the pharmaceutical industry

(10) The general investment climate
(11) Industrial policies
(12) Availability and cost of production factors.
(13) Export possibilities

Opportunity studies are rather sketchy in nature and rely more on aggregate estimates than on detailed analysis. Cost data are usually taken from comparable existing projects and not from quotations of sources such as equipment suppliers. Depending on the prevailing conditions under investigation, either general opportunity studies (sector approach) or specific project opportunity studies (enterprise approach), or both, have to be undertaken (Behrens, 1989, p.1008).

General Opportunity Studies are realized generally by the public with the aim of determining the location of specific investment suggestions. The companies can also make these studies. General opportunity studies may be divided into the following three categories;

(1) Area studies designed to identify opportunities in a given area such as an administrative province, a backward region or the hinterland of a port.

(2) Industrial studies designed to identify opportunities in a delimited industrial branch such as building materials or food processing

(3) Resource-based studies designed to reveal opportunities based on the utilization of natural, agricultural or industrial products such as forest-based industries, downstream petrochemical industries and metal-working industries.

Specific project opportunity studies should follow the initial identification of general investment opportunities in the form of products with potential for domestic manufacture, and an investment profile should be circulated to potential investors. Although in many developing countries a governmental investment promotion agency or a chamber of commerce and industry may perform such work, it is most often undertaken by the prospective investor or an entrepreneurial group.

A specific project opportunity study, which is more common than a general opportunity study, may be defined as the transformation of a project idea into a broad investment proposition. As the objective is to stimulate investor response, a specific project opportunity study must include certain basic information; the mere listing of products that may have potential for domestic manufacture is not sufficient. While such list – derived from general economic indicators such as past imports, growing consumer demand or from one of the general opportunity studies relating to areas, sectors or resources – can serve as a starting-point, it is necessary, first, to be selective as to the products so identified, and secondly, to incorporate data relating to each product so that a potential investor, either domestic or foreign, can consider whether the possibilities are attractive enough to proceed to the next stage of project preparation. Such data can be supplemented with information on basic policies and procedures that may be relevant to the production of the particular product. A broad investment profile would then emerge that would be adequate for the purpose of stimulating investor response (Behrens & Hawranek, 1991, p.12).

As a summary, it should be stated that the aim of opportunity studies is to carry out preliminary election of project ideas and to give prominence to ideas which are promising among other ones and which have the highest chance of attaining the goals planned by entrepreneurs and investors.

2.3.1.2 Pre-feasibility Studies

The project idea must be elaborated in a more detailed study. However, formulation of a feasibility study that enables a definite decision to be made on the project is a costly and time-consuming task. Therefore, before assigning larger funds for such a study, a further assessment of the project idea might be made in a pre-feasibility study, the principal objectives of which are to determine whether;

(1) All possible project alternatives have been examined,

(2) The project concept justifies a detailed analysis by a feasibility study,

(3) Any aspects of the project are critical to its feasibility and necessitate in-depth investigation through functional or support studies such as market surveys, laboratory tests or pilot-plants tests,

(4) The project idea, on the basis of the available information, should be considered either non-viable or attractive enough for a particular investor or investor group,

(5) The environmental situation at the planned site and the potential impact of the projected production process are in line with national standards.

A pre-feasibility study should be viewed as an intermediate stage between a project opportunity study and a detailed feasibility study. The structure of a pre-feasibility study should be the same as that of a detailed feasibility study.

A detailed review of available alternatives must take place at the stage of the pre-feasibility study, since it would be too costly and time-consuming to have this done at the feasibility study stage. In particular, the review should cover the various alternatives identified in the following main fields (components) of the study:

(1) Project or corporate strategies and scope of project,
(2) Market and marketing concept,
(3) Raw materials and factory supplies,
(4) Location, site and environment,
(5) Engineering and technology,
(6) Organization and overhead costs,
(7) Human resources, in particular managerial (entrepreneurial) staff, labor costs and training requirements and costs,
(8) Project implementation schedule and budgeting.

Support or functional studies cover specific aspects of an investment project, and are required as prerequisites for, or in support of, pre-feasibility and feasibility studies, particularly large-scale investment proposals. Examples of such studies are as follows (Behrens & Hawranek, 1991, p.14);

(1) Market studies of the products to be manufactured, including demand projections in the market to be served together with anticipated market penetration,

(2) Raw material and factory supply studies, covering current and projected availability of raw materials and inputs basic to the project, and the current and projected price trends of such materials and inputs,

(3) Laboratory and pilot-plant tests, which are carried out to the extent necessary to determine the suitability of particular raw materials or products,

(4) Location studies, particularly for potential projects where transport costs would constitute a major determinant,

(5) Environmental impact assessment, which covers current environmental conditions in the area surrounding the envisaged site, possible low emission technologies or environmental protection technologies, alternative sites, the use of alternative raw materials and auxiliary materials,

(6) Economies-of-scale studies that are generally, conducted as a part of technology selection studies. The principal task of such studies is to assess the size of plant that would be most economic after considering alternative technologies, investment costs, production costs and prices. Several plant capacities are analyzed and the broad characteristics of the project developed, including a computation of results for each capacity,

(7) Equipment selection studies, which are required when large plants with numerous divisions are involved and the sources of suppliers and the costs are widely divergent. The ordering of equipment, including preparation of and invitations for bids, their evaluation, contracting and deliveries, is usually carried out during the investment or implementation phase. When

very large investments are involved, the structure and economics of the project depend heavily on the type of equipment, its price and production costs; even the operational efficiency of the project is a direct function of the selected equipment.

The contents of a support study vary, depending on its type and the nature of the projects. However, as it relates to a vital aspect of the project, the conclusions should be clear enough to give direction to the subsequent stage of project preparation. In most cases, the results of a support study, when undertaken either before or together with a feasibility study, form an integral part of the latter and lessen its burden and cost.

The main aim of pre-feasibility studies is to determine whether it is necessary to conduct a detailed and comprehensive feasibility study for project ideas qualified as a result of opportunity studies and, if found necessary, which subjects require a more careful and detailed study. For instance, as a result of this study, some of the project ideas qualified through opportunity studies will not be found necessary to undergo a feasibility study and these ideas will be eliminated by means of pre-feasibility study. If it is agreed that any project idea requires a detailed feasibility study as a result of the pre-feasibility study, in this case, it should be determined that which research bases in feasibility study requires a more careful study. For example, as a result of the pre-feasibility study, it can be recognized that a project idea will be feasible if the data obtained through market research proves positive. Therefore, feasibility study will be conducted only with the aim of introducing market circumstances. This will provide both cost and time saving.

2.3.1.3 Feasibility Studies

A feasibility study should provide all data necessary for an investment decision. The commercial, technical, financial, economic and environmental prerequisites for an investment project should therefore be defined and critically examined on the basis of alternative solutions already reviewed in the pre-feasibility study.

The result of these efforts is then a project whose background conditions and aims have been clearly defined in terms of its central objective and possible marketing strategies, the possible market shares that can be achieved, the corresponding production capacities, the plant location, existing raw materials, appropriate technology and mechanical equipment and, if required, an environmental impact assessment.

The financial part of the study covers the scope of the investment, including the net working capital, the production and marketing costs, sales revenues and the return on capital invested.

Final estimates on investment and production costs and the subsequent calculations of financial and economic profitability are only meaningful if the scope of the project is defined unequivocally in order not to omit any essential part and its related cost. The scope should be defined in drawings and schedules that should then serve as a supporting structure during further project work.

Although feasibility studies are similar in content to pre-feasibility studies, the industrial investment project must be worked out with the greatest accuracy in an iterative optimization process, with feedback and interlinkages, including the identification of all commercial, technical and entrepreneurial risks.

"… Should weak points be revealed initially and the profitability of the project prove inadequate, then sensitive parameters such as the size of the market, the production program or the mechanical equipment selected should be examined more closely, and better alternatives should be looked for, in order to improve the feasibility of the project. All of the assumptions made, data used and solutions selected in a feasibility study should be described and justified in order to make the project more comprehensible to the promoter or investor in his evaluation of the study. If a project is not viable despite a review of all alternatives, that fact should be stated and the reasons given. In other words, even a feasibility study that does not lead to an investment recommendation is of great value as it prevents the misallocation of scarce capital" (Behrens & Hawranek, 1991, p.16).

A feasibility study must be related to available production factors and local market and production conditions, and this requires an analysis that has to be translated into costs, income and net profits.

A feasibility study should be carried out only if the necessary financing facilities can be identified with a fair degree of accuracy. For that reason, possible project financing must be considered as early as the feasibility study stage, because financing conditions have a direct effect on total costs and thus on the financial feasibility of the project.

Feasibility studies are generally prepared for four main aims. Ayanoğlu et al. (1996) lists these aims as the following;

(1) Those who are to make an investment decision on both a macro and micro basis with the aim of using their own resources effectively need feasibility studies.

(2) Governments request feasibility studies for the projects of private firms that want to take advantage of incentives and loans provided by the state.

(3) If corporations demand any external financing (loan) from a financing institution in order to carry out its investments, financing institutions request feasibility study in order to make sure that the demanded debt or interest can be paid in due time.

(4) Feasibility studies are needed in order to predict any potential difficulties during practice phase of the project and take necessary precautions.

2.3.1.4 Investment Project Evaluation and Decision

During this stage, project alternatives are assessed and analyzed by considering the data obtained from feasibility study. The profitability and payback periods of project proposals and the added-value to be achieved are tried to be determined. During these analyses, tables for each project presenting cash flows are taken into account. Several key assumptions are tested via sensitivity analyses and an attempt is made to determine prospects of the project. Methods used in assessment and analysis of projects are going to be discussed in detail in the following sections.

2.3.2 The Investment Phase

The investment or implementation phase of a project provides wide scope for consultancy and engineering work, first and foremost in the field of project management. The investment phase can be divided into the following stages:

(1) Establishing the legal, financial and organizational basis for the implementation of the project
(2) Technology acquisition and transfer, including basis engineering
(3) Detailed engineering design and contracting, including tendering, evaluation of bids and negotiations
(4) Acquisition of land, construction work and installation
(5) Pre-production marketing, including the securing of supplies and setting up the administration of the firm
(6) Recruitment and training of personnel
(7) Plant commissioning and start up

Detailed engineering design comprises preparatory work for site preparation, the final selection of technology and equipment, the whole range of construction planning and time-scheduling of factory construction, as well as the preparation of flow charts, scale drawings and a wide variety of layouts (Behrens & Hawranek, 1991, p.20).

The construction stage involves site preparation, construction of buildings and other civil works, together with the erection and installation of equipment in accordance with proper programming and scheduling.

The personnel recruitment and training stage, which should proceed simultaneously with the construction stage, may prove very crucial for the expected growth of productivity and efficiency in plant operations.

Plant commissioning and start-up is usually a brief but technically critical span in project implementation. It links the preceding construction phase and the following operational (production) phase. The success achieved at this point demonstrates the effectiveness of implementation planning and the execution of the project and is a portent of the future performance of the project.

Good project planning and efficient project management must ensure that the necessary action for setting up a factory, such as construction, delivery and assembly of the equipment, recruitment and training of the operating personnel and the delivery of all production inputs, is taken in good time before the projected start-up.

Any delay or gaps in the planning of one of the above-mentioned stages would have a negative effect on the successful implementation of the project, especially during the start-up phase. In order to avoid this, effective, balanced organization of the various activities is necessary, and can be achieved only by careful scheduling.

In summary, it is to be noted that in pre-investment phase, the quality and dependability of the project are more important than the time factor, while in the investment phase, the time factor is more critical in order to keep the project within the forecasts made in the feasibility study. It is therefore conceptually wrong when investors, complaining about the costly and time-consuming project preparation process, try to short-circuit the stages of project preparation and analysis, moving directly from project identification to the application for a loan. Industrial investment usually involves long-term financial commitments and the time used to study all strategic market, locational, technical, managerial, organizational and financial project alternatives, so as to find the optimal solution, usually pays for itself many times (Baum, 1978, p.46).

2.3.3 The Operational Phase

Operational phase of investment projects is the one in which investment projects analyzed, assessed and agreed to be performed in pre-investment phase start to be realized and normal production activities following the test production are commenced. Operating period, on the other hand, is the one in which normal production facilities are carried on. During the operational phase of investment projects, both production facilities are performed, and obtained results are monitored and assessed.

The problems of the operational phase need to be considered from both a short and a long-term viewpoint. The short-term view relates to the initial period after commencement of production when a number of problems may arise concerning such matters as the application of production techniques, operation of equipment or inadequate labor productivity owing to a lack of qualified staff and labor. Most of these problems have their origin in the implementation phase.

The long-term view relates to chosen strategies and the associated production and marketing costs as well as sales revenues. These have a direct relationship with the projections made at the pre-investment phase. If such strategies and projections prove faulty, any remedial measures will not only be difficult but prove highly expensive (Behrens & Hawranek, 1991, p.21).

2.4 Investment Project Evaluation and Selection Under Certainty

Certainty concept in project evaluation means that the values of all project parameters are assumed to be known with complete certainty; the project analysis is concerned with measuring the economic worth of projects and selecting the best investment projects.

The investment project evaluation methods under certainty assumption consist of two groups. These are as follows;

(1) Static Methods,
(2) Dynamic Methods.

2.4.1 Static Methods

Methods to be discussed in this section are those which are based on the assumption that money does not have a time value, that is, today's monetary value will remain the same in the future in terms of real purchasing power. Static methods are studied under two categories. The detailed information about advantages and disadvantages of the application of methods and practice samples can be found in Eski & Armaneri (2006).

2.4.1.1 Profitability Ratios Method

One of the most important goals for any business is to earn a profit. The ratios examined thus far provide useful clues as to the effectiveness of a firm's operations, but the profitability ratios show the combined effects of liquidity, asset management, and debt on operating results. Therefore, ratios that measure profitability play a large role in decision making (Park, 2002).

Ratios that indicate how profitably the capital introduced in order to carry out investments is used are called as *profitability ratios*. In the simplest way, profitability ratios can be defined as proportion of annual net profit expected from an investment proposal to total amount of capital introduced for an investment proposal. This can be shown as the following;

$$\text{Profitability Ratio} = \frac{\text{Annual Net Profit}}{\text{Total Capital Amount}} \qquad (2.1)$$

The notion "net profit" means the profit obtained after subtracting taxes from annual profits. Considering that an investment proposal will provide profit in more than one year over its economic life, it is of great importance to determine to which year's value will replace the concept "annual net profit" included in the equation above. For this reason, it is recommended in literature that one should consider the net profit of a year which reflects the process of operating period over the economic life of investment proposal and in which capacity utilization ratio is high. This kind of year is called "normal year".

Total capital amount also reflects the total of equity capitals introduced for financing of total investment cost of investment proposal and foreign capital used. In this case, one can also study how profitably equity capitals are used and the profitability ratio can be arranged as follows;

$$\text{Profitability Ratio} = \frac{\text{Annual Net Profit}}{\text{Equity Capital Amount}} \tag{2.2}$$

If profits of an investment experience considerable differences and fluctuations from one year to another over its economic life, it is difficult to determine which year will be the normal year. In these cases, one will have to resort to average profitability ratios and to assess the investments by taking these ratios into account. Therefore, the average profitability ratio for an investment alternative is determined as follows (Eski & Armaneri, 2006, pp. 359-360);

$$\text{Average Profitability Ratio} = \frac{\text{Average Net Profit}}{\text{Average Investment Amount}} \tag{2.3}$$

Average net profit is calculated by dividing the total profits expected over the economic life of an investment proposal into economic life period of investment. Average investment amount is determined as follows;

Average Investment Amount = Net working capital + Salvage value of investment +1/2 [Fixed capital investment amount - Salvage value of investment] (2.4)

As a result, if investment alternatives are evaluated on the basis of their profitability ratios, the one with the highest ratio will be the one to be preferred first. If it is wanted to rank the alternatives, this ranking is made with the highest profitability ratio on the top and the lowest ratio at the bottom.

2.4.1.2 Payback Period Method

Expressions such as "This investment will pay for itself in less than three years" are common in business and industry and emphasize the tendency to evaluate assets in terms of a payback period. The payback period is commonly defined as the length of time required to recover the initial cost of an investment from the net cash flow produced by that investment (Thuesen & Fabrycky, 2001; Eski & Armaneri, 2006).

That is, if A_t = the net cash flow in period t; I_t = investment cost in period t and m = investment establishment period length, then the payback period is defined as the value of n that satisfies the equation (Eski & Armaneri, 2006);

$$\sum_{t=0}^{m-1} I_t = \sum_{t=m}^{n} A_t \qquad (2.5)$$

If the payback period is less than the maximum acceptable payback period, the project is acceptable. However, if the payback period is greater than the maximum acceptable payback period, the project should be rejected. The length of the maximum acceptable payback period is determined by management. This value is set subjectively on the basis of a number of factors, including the type of project and the perceived risk of the project (Gitman, 2003).

When comparing the payback period for investment alternatives it is usually more desirable to have a short payback period than a longer one. A short payback period indicates that the investment provides revenues early in its life sufficient to cover the initial outlay. Thus, an investment with a short payback period can be viewed as having a higher degree of liquidity than one with a longer payback period. This quicker return of the capital invested also shortens the time span over which the investment is susceptible to possible economic loss.

In general, the most serious deficiencies of the payback period are that it fails to consider;

(1) The time value of money,

(2) The consequences of the investment following the payback period, including the magnitude and timing of the cash flows and the expected life of the investment.

Because of the limitations just mentioned, the payback period tends to favor shorter-lived investments. Experience has generally indicated that this bias is unjustifiable and in many cases economically unsound. Nevertheless, it must be said that the payback period does give some measure of the rate at which an investment will recover its initial outlay. For situations where there is a high degree of uncertainty concerning the future and a firm is interested in its cash position and borrowing commitments, the payback period can supply useful information about investments under consideration (Thuesen & Fabrycky, 2001).

2.4.2 Dynamic Methods

Methods to be discussed in this chapter are those which are based on the assumption that money has a time value, that is, today's monetary value will not remain the same in the future in terms of real purchasing power.

Dynamic methods are analyzed under seven titles. One should read Eski & Armaneri (2006) in order to get detailed information about all seven methods and to comprehend how they are practiced. In the following chapter, four commonly-used ones of the methods contained in the referred study will be discussed.

2.4.2.1 Net Present Value Method

The net present value (NPV) of a project is calculated by discounting all flows to the present and subtracting the present value of all outflows from the present value of all inflows. In simple mathematical terms (Eski & Armaneri, 2006);

$$NPV = \underbrace{\sum_{t=1}^{n} \frac{A_t}{(1+i)^t} + \frac{H}{(1+i)^n}}_{\text{The present value of all inflows}} - \underbrace{\sum_{t=0}^{n} \frac{I_t}{(1+i)^t}}_{\text{The present value of all outflows}} \tag{2.6}$$

In this equation, A_t represents cash inflow in period *t; H* represents the salvage value of the investment; *n* represents the life cycle of the project; *i* represents the accepted discount rate, and I_t represents cash outflow in period *t*. The investment establishment period length can be more than one periods (Eski & Armaneri, 2006).

Some of these terms must be explained further. Inflows are shown from period 1 to period *n*; however, inflows may not occur in all periods. Should the project under consideration be the construction of a plant, the time elapsed before the first shipment of product, and thus the first inflow, may not occur until period 3, for example. Outflows are shown starting in period 0. The discount rate, *i*, is the interest rate used to evaluate the project. This rate represents the cost of the funds employed and is often called the cost of capital (Keat & Young, 2000).

As a result of the feasibility studies, the cash inflows and outflows are estimated for each period over the economical life of the project. Therefore, it can be easily calculated the net cash flow for each period by subtracting the cash outflows from the cash inflows. That is, if A_t represents the net cash flow at the end of period *t*, the NPV of the project is calculated by adding up the present worth of each net cash flow at the accepted discount rate. Sum of the present worth of each net cash flow is defined as the project's NPV;

$$NPV = \sum_{t=0}^{n} \frac{A_t}{(1+i)^t} \tag{2.7}$$

A_t will be positive if the corresponding period has a net cash inflow, or negative if there is a net cash outflow. Notice that, the net cash flow at the end of period *n* contains the salvage value of the project.

In this context, a positive NPV means the equivalent worth of the inflows is greater than the equivalent worth of outflows, so, the project makes a profit. Therefore, the decision rule for NPV methods is;

If $NPV > 0$, accept the project.
If $NPV = 0$, remain indifferent.
If $NPV < 0$, reject the project.

In other words; a project is considered acceptable if its NPV is positive; it is not acceptable if its NPV is negative. If projects with positive NPVs are purchased, the value of the firm will increase; purchasing projects with negative NPVs will lower the value of the firm. In general, higher NPV projects are better than lower NPV projects. So if two projects are mutually exclusive, and both have positive NPVs, the one with the higher NPV should be chosen (Weston et al., 1996, p.496).

Economic lives of alternative investment proposals can be varied. NPV of a project alternative with a long economic life may be larger than a project alternative with a short economic life. However, this does not necessarily mean that one project is more profitable than another. A high NPV may result from a project alternative having long economic life and naturally more cash flow. Therefore, in order to be able to compare projects through NPV method, the length of time periods during which NPV of projects are calculated have to be same. Anyway, there will not arise any problem if economic lives of projects are the same. However, it is important how to determine the length of calculation period for projects with different economic lives. For such a calculation, it is generally recommended that one should calculate the least common multiple of economic lives of projects and determine the NPVs of projects by considering cash flows estimated to take place during that period (Eski & Armaneri, 2006).

Investment projects are not always made with the aim of maximizing the profitability. In some cases, investments are expected to bring about not high profitability but the least cost. In public investments, for example, it is evident that annual net cash flows and naturally NPVs of investment alternatives are generally negative. In such cases, choice among alternative investments is made on the basis of the principle cost minimization. In this case, the project with the lowest absolute NPV-the closest one to zero- among negative NPVs will be one to be chosen (Eski & Armaneri, 2006).

2.4.2.2 Internal Rate of Return Method

Internal rate of return (*IRR*) is the break-even discount rate which equates the present value of a project's cash outflows to the present value of its cash inflows, or

$$\underbrace{\sum_{t=1}^{n}\frac{A_t}{(1+IRR)^t}+\frac{H}{(1+IRR)^n}}_{\text{The present value of all inflows}}=\underbrace{\sum_{t=0}^{n}\frac{I_t}{(1+IRR)^t}}_{\text{The present value of all outflows}} \qquad (2.8)$$

The *IRR* is the unknown variable for which we solve. Actually, the *IRR* solution is only a special case of the NPV method; the *IRR* of a project is the discount rate that causes NPV to equal zero. Note that the NPV expression is equivalent to

$$NPV=\sum_{t=0}^{n}\frac{A_t}{(1+i)^t}=0 \qquad (2.9)$$

The accept/reject criterion for the *IRR* is based on a comparison of the *IRR* with the minimum acceptable discount rate (i_{min}). i_{min} is indicated by company policy, management, or the project decision maker and it represents the cost of capital of the project.

In most project cash flows, it would be able to find a unique positive *IRR* that causes NPV to equal zero. However, it may be encountered some cash flows that cannot be solved for a single rate of return. By the nature of the NPV function, it is certainly possible to have more than one rate of return for certain types of cash flows. For some cash flows, it may not be found a specific rate of return at all (Park, 2002).

Once the type of an investment cash flow is identified, several ways to determine its rate of return are available. The calculation of the *IRR* can be easily accomplished with a hand-held business calculator or a computer (using, for instance, an Excel function).

One of the most practical methods to determine the *IRR* is Trial-and-error method. The first step in the trial-and-error method is to make an estimated guess at the value of *IRR*. For a project, we compute the present value of net cash flows using the "guessed" discount rate and observe whether it is positive, negative, or zero. Suppose the NPV is negative. Since we are aiming for a value of i that makes NPV=0, we must raise the present value of the cash flow. To do this, we lower the discount rate and repeat the process. If NPV is positive, however, we raise the discount rate in order to lower NPV. The process is continued until NPV is approximately equal to zero. Whenever we reach the point where NPV is bounded by one negative and one positive value, we use linear interpolation to approximate the *IRR* (Park, 2002; Eski & Armaneri, 2006).

Let us assume that positive NPV is observed by using discount rate i_1 and negative NPV is observed by using discount rate i_2.

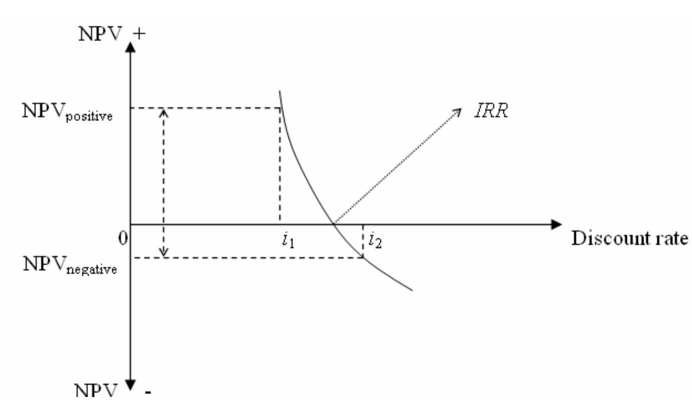


Figure 2.2 Calculation of *IRR* with Linear Interpolation
(Eski & Armaneri, 2006)

As seen in Figure 2.2, the *IRR* that makes NPV=0 should be between i_1 and i_2. If the positive and negative NPVs are close to zero, a good approximation of the IRR value can be obtained, using the following linear interpolation formulas (Eski & Armaneri, 2006);

$$IRR = i_1 + \frac{NPV_{positive}(i_2 - i_1)}{NPV_{positive} + \left|NPV_{negative}\right|} \tag{2.10}$$

$$IRR = i_2 - \frac{\left|NPV_{negative}\right|(i_2 - i_1)}{NPV_{positive} + \left|NPV_{negative}\right|} \tag{2.11}$$

It should be noted that i_1 and i_2 should not differ by more than one or two percentage points. The above formula will not yield realistic results if the difference is too large, since the discount rate and the NPV are not related linearly.

According to this method, the decision rule for a simple project is as follows (Eski & Armaneri, 2006);

If $IRR > i_{min}$, accept the project.
If $IRR = i_{min}$, remain indifferent.
If $IRR < i_{min}$, reject the project.

In case of comparing more than one investment projects with each other, the project with the highest *IRR* among the projects that have *IRR* higher than the value i_{min} will be chosen.

NPV and *IRR* methods have just described that conform to the criteria specified for a valid capital budgeting decision. In a large majority of cases, either NPV or *IRR* can be used with confidence. These two tests of investment worth give consistent accept/reject indicators. In some cases, however, problems may arise. When independent projects are being analyzed, both *IRR* and NPV criteria give consistent results. “Independent” implies that if a company is considering several projects at the same time, they can all be implemented simultaneously as long as they pass NPV or *IRR* tests, and as long as funds are not limited. The adoption of one independent project will have no effect on the cash flows of another. However, proposals may be mutually exclusive. This occurs when two solutions for a particular proposal are offered, only one of which can be accepted.

In those cases, the reason for the differences between *IRR* and NPV results is the implicit reinvestment assumption. In the NPV calculation, as inflows occur, they are automatically assumed to be reinvested at the minimum accepted discount rate (cost of capital). The *IRR* solution assumes reinvestment at the *IRR*. Conflicting accept/reject signals may not occur frequently in investment project analysis, but they do occur and they may cause the analyst some anxious moments (Keat & Young, 2000).

2.4.2.3 Benefit-Cost Ratio Method

Benefit-Cost Study is a method for assessing the range of benefits and costs associated with several alternative means for solving a problem. The costs and benefits are usually quantified in money terms and expressed as a ratio, although this procedure permits the presentation of intangible and qualitative benefits and costs as well (Wedley et al., 2001, pp.342-347).

Benefit-cost ratio is an economic indicator of efficiency, computed by dividing costs by benefits. The results of this technique determine which alternative returns the most benefits for each units of money spent. Here, it should be expressed the values of benefits and costs in present value equivalents.

Mathematical expression of benefit-cost ratio is as follows;

$$\text{Benefit} - \text{Cost Ratio} = \frac{\text{Sum of the present values of cash inflows}}{\text{Sum of the present values of cash outflows}} \qquad (2.12)$$

Since we are using a ratio, all inflows and outflows are expressed in positive units. Naturally, for accepting a project, its benefit-cost ratio must be greater than 1. According to this method, the decision rule for a simple project is as follows (Eski & Armaneri, 2006);

If Benefit-Cost Ratio > 1, accept the project.
If Benefit-Cost Ratio $= 1$, remain indifferent.
If Benefit-Cost Ratio < 1, reject the project.

Benefit-cost ratio yields the same decision for a project as does the NPV criterion. This implies that it could be used the benefit-cost ratio in evaluating investment projects instead of using NPV criterion.

If it is necessary to compare many project alternatives with one another, the project with the greatest benefit-cost ratio will be the chosen one.

2.4.2.4 Discounted Payback Period Method

As mentioned before, one of the most deficiencies of the classical payback period is that it fails to consider the time value of money. To include consideration of the time value of money when calculating the payback period, a method known as the discounted payback period may be used.

That is, if A_t = the net cash flow in period t; I_t = investment cost in period t and m = investment establishment period length, then the discounted payback period is the value of n that satisfies the expression (Eski & Armaneri, 2006);

$$\sum_{t=0}^{m-1} \frac{I_t}{(1+i)^t} = \sum_{t=m}^{n} \frac{A_t}{(1+i)^t} \qquad (2.13)$$

In this method, the discounted payback period is calculated by finding the present value of each cash flow at the accepted discount rate. Therefore, if the discounted payback period is less than the maximum acceptable payback period determined by management, the project is acceptable. When comparing the payback period for investment alternatives it is usually more desirable to have a short payback period than a longer one (Eski & Armaneri, 2006).

2.5 Investment Project Evaluation and Selection Under Risk and Uncertainty

In previous section, cash flows of the projects were assumed to be known with complete certainty; our analysis was concerned with measuring the economic worth of projects and selecting the best investment projects. Although these types of analyses can provide a reasonable decision basis in many investment situations, it should be certainly considered the more usual uncertainty. In this type of situation, management rarely has precise expectations about the future cash flows to be derived from a particular project. In fact, the best that a firm can reasonably expect to do is to estimate the range of possible future costs and benefits and the relative chances of achieving a reasonable return on investment.

In project management, it is common to refer to very high levels of uncertainty as sources of risk. Risk is present in most investment projects. Principal sources of uncertainty include random variations, inaccurate or inadequate data, and the inability to forecast satisfactorily as a result of lack of experience. Specifically, there may be uncertainty in scheduling, uncertainty in cost, and technological uncertainty. There are other sources of uncertainty, including those of an organizational and political nature (Shtub et al., 2005).

The term *risk* is used to describe an investment project whose cash flow is not known in advance with absolute certainty, but for which an array of alternative outcomes and their probabilities are known. Although no future events are known with certainty, some events can be assigned probabilities, and others cannot. Where future events can be defined and probabilities assigned, we have a case of risk. If there is no way to assign any probabilities to future random events, pure uncertainty is addressed.

In other words, risk represents the probability distribution of the consequences of each alternative. A probability distribution implies an ability to quantify the consequences of an alternative. On the other hand, uncertainty is addressed when the consequences of each alternative belong to some subset of all possible consequences, but that the decision maker cannot assign definite probabilities to the occurrence of particular outcomes.

These definitions suggest that quantifiable factors surrounding a project represent risks, whereas qualitative factors that affect decision maker's confidence in project estimates represent uncertainties. Levels of risk and uncertainty can affect a decision maker's choice of models, techniques, and processes used for making the investment decisions (Alessandri et al., 2004).

In project evaluation and selection literature, the term of project risk is widely used. Moreover, this term will be used in the next sections. In the light of the information given above, it can be easily defined. The term *project risk* is used to refer to variability in a project's NPV. A greater project risk means a greater variability in a project's NPV, or simply that the risk is the potential for loss (Park, 2002).

2.5.1 Origins of Project Risk

The decision to make a major capital investment such as introducing a new product requires cash flow information over the life of a project. The profitability estimate of an investment depends on cash flow estimations, which are generally uncertain. The factors to be estimated include the total market for the product; the market share that the firm can attain; the growth in the market; the cost of producing the product, including labor and materials; the selling price; the life of the product; the cost and life of the equipment needed; and the effective tax rates. Many of these factors are subject to substantial uncertainty. A common approach is to make single-number "best estimates" for each of the uncertain factors and then to calculate measures of profitability, such as NPV or rate of return for the project. This approach has two drawbacks (Park, 2002);

(1) No guarantee can ever ensure that the "best estimates" will ever match actual values.

(2) No provision is made to measure the risk associated with an investment or the project risk. In particular, managers have no way of determining either the probability that a project will lose money or the probability that it will generate large profits.

Because cash flows can be so difficult to estimate accurately, project managers frequently consider a range of possible values for cash flow elements. If a range of values for individual cash flows is possible, it follows that a range of values for the NPV of a given project is also possible. Clearly, the analyst will want to try to gauge the probability and reliability of individual cash flows occurring and, consequently, the level of certainty about overall project worth (Park, 2002).

It is possible to summarize the reasons for the inability to make correct and precise estimations during assessment and analysis process of projects as the following (Eski & Armaneri, 2006);

(1) The prices of the many materials and equipment, especially production inputs, which are directly or indirectly related to the project increase in the course of time. Also, prices of products and services produced may increase or decrease at previously unanticipated levels.

(2) During the project evaluation process, expected capacity utilization ratio for the investment project may not occur. In this case, predictions regarding project inflows and outflows will reflect deviations.

(3) There may be changes in prices of production cost as well as amount and quality of them. Technological developments may occur.

(4) If the amount of fixed capital investment and working capital for foundation of investment project is determined less than required, this will cause the deviation of all estimated values.

(5) In addition, unpredictable changes in natural, economic and social factors may cause deviations in estimations regarding project parameters.

2.5.2 Methods of Describing Project Risk

Analyzing project risk can be done in a number of ways, which range from making informal judgments to calculating complex economic and statistical analyses. In this section, three methods of describing project risk will be explained briefly. For detailed information, the reader can refer to see Park (2002) and Eski & Armaneri (2006).

2.5.2.1 Sensitivity Analysis

One way to glean a sense of the possible outcomes of an investment is to perform a sensitivity analysis. Sensitivity analysis determines the effect on the NPV of variations in the input variables (such as revenues, operating cost, and salvage value) used to estimate after-tax cash flows. A sensitivity analysis reveals how much the NPV will change in response to a given change in an input variable. In calculating cash flows, some items have a greater influence on the final result than others. In some problems, the most significant item may be easily identified. For example, the estimate of sales volume is often a major factor in a problem in which the quantity sold varies among the alternatives. In other problems, the items that have an important influence on the final results can be wanted to locate so that they can be subjected to special scrutiny (Park, 2002).

Sensitivity analysis is sometimes called “what-if” analysis. It begins with a base-case situation, which is developed using the most-likely values for each input. Then the specific variable of interest is changed by several specified percentages above and below the most-likely value, while holding other variables constant. Next, a new NPV is calculated for each of these values. A convenient and useful way to present the results of a sensitivity analysis is to plot sensitivity graphs. The slopes of the lines show how sensitive the NPV is to changes in each of the inputs: The steeper the slope, the more sensitive the NPV is to a change in a particular variable. Sensitivity graphs identify the crucial variables that affect the final outcome most (Park, 2002).

2.5.2.2 Break-even Analysis

When a sensitivity analysis of a project is performed, it is asked how serious the effect of lower revenues or higher costs will be on project's profitability. Managers sometimes prefer to ask how much sales can decrease below forecasts before the project begins to lose money. This type of analysis is known as break-even analysis. In other words, break-even analysis is a technique for studying the effect of variations in output on a firm's NPV or other measures.

To illustrate the procedure of break-even analysis based on NPV, the generalized cash flow approach can be used. At first, the present value of cash inflows [$f(x)_1$] as a function of an unknown variable (X). Next, the present value of cash outflows [$f(x)_2$] as a function of X. NPV is of course the difference between these two numbers. Then the break-even value of x that makes $f(x)_1 = f(x)_2$ is searched (Park, 2002).

2.5.2.3 Scenario Analysis

Although both sensitivity and break-even analyses are useful, they have limitations. It is difficult to specify precisely the relationship between a particular variable and the NPV; the relationship is further complicated by interdependencies among the variables. Holding operating costs constant while varying unit sales may ease the analysis, but in reality, operating costs do not behave in this manner. However, it may complicate the analysis too much to permit movement in more than one variable at a time.

A scenario analysis is a technique that does consider the sensitivity of NPV to both changes in key variables and to the range of likely variable values. For example, the decision maker may consider two extreme cases, a "worst-case" scenario (low unit sales, low unit price, high variable cost per unit, high fixed cost, and so on) and a "best-case" scenario. The NPVs under the worst and the best conditions are then calculated and compared to the expected, or "base-case", NPV (Park, 2002).

The interpretation of the scenario analysis results is not easy. For example, it could be said that there is a chance of losing money on the project, but there is not a specific probability for this possibility on hand, yet. Clearly, the estimates of the probabilities of occurrences of the worst-case, the best-case, the base-case (most likely), and all the other possibilities are needed. The need to estimate probabilities leads us directly to develop a probability distribution.

It will be possible to comment on the risk of project through determining variation range of each parameter affecting NPV and probability distribution in this range as well as determining how probably each scenario will occur. At this point, probability analyses are carried out during assessment process of risky investment projects in order to determine potential variation range of a parameter which involves risk and affects NPV of projects. These analyses are also conducted with the aim of determining how probably this parameter will have each value within this range; that is, determining probability distribution.

2.5.3 Probability Analysis in Describing Project Risk

When evaluating projects under risk and uncertainty, one considers the fact that all parameters that affect the profitability of a project may have different values in the future from the ones calculated today. During the process of determining project risk, discussed in detail in previous section, it is studied how potential changes in estimated values of project parameters have an impact on project profitability, and thus, an attempt is made to determine the parameters to which project profitability is sensitive. However, the fact that project profitability is too sensitive to potential changes in any parameter does not necessarily mean this project has high risk.

Even if project profitability is sensitive to changes in any parameter, it is not possible to reach a decisive conclusion about the risk of a project before determining the variation range of that project parameter and the probabilities of taking the values within this range. Probability analysis is an analysis technique aiming to determine potential variation range of a parameter involving uncertainty and risk as well as the probability distribution within this range (Eski & Armaneri, 2006).

In the light of this information, probability analysis can be characterized as a supplementary analysis technique which ensures a proper interpretation of results obtained through project risk identification methods. It should be noted here that there are many parameters which affect project profitability and it will be very time-consuming to conduct probability analysis for all these parameters. Therefore, it is more appropriate to determine probability distribution for parameters to which project profitability has been found sensitive through sensitivity analysis and break-even analysis.

In probability analyses, parameters affecting project profitability are not taken as a constant value. They are defined as *random variables* which have a variation range and a probability distribution within this range. Probability analysis and probability theory are included among fundamentals of statistics and there exist a lot of papers on this issue (Montgomery & Runger, 2003; Walpole et al., 2002; Devore, 2000).

Random variables are classified as either discrete or continuous. Any random variables that take on only isolated (countable) values are discrete random variables. Continuous random variables may have any value in a certain interval. Potential values of a parameter in the future are taken into account while determining what kind of random variable a project parameter will be defined as. After defining a project parameter as discrete or continuous random variable, one makes an attempt to determine probability distribution of this random variable. While determining probability distribution of the random variable, its expected value and variance are also considered.

The expected value is a weighted average value of the random variable where the weighting factors are the probabilities of occurrence. All distributions (discrete and continuous) have an expected value. $E(X)$ is used to denote the expected value of random variable X. For a random variable X that has discrete values, an expected value is determined as follows;

$$E(X) = \sum_{i=1}^{n} x_i f\left(x_i\right) \tag{2.14}$$

where n is the number of discrete events and $f(x_i)$ is the probability of occurrence of the x_ith value of the discrete random variable. The variance tells us the degree of spread, or dispersion, of the distribution on either side of the mean value. As the variance increases, the spread of the distribution increases; the smaller the variance, the narrower the spread about the expected value. For a random variable that has only discrete values, the equation to compute variance is as follows;

$$Var(X) = \sum_{i=1}^{n} [x_i - E(X)]^2 f(x_i) \tag{2.15}$$

In practice, the actual calculation of the variance is somewhat easier if the following formula is used;

$$Var(X) = E(X^2) - [E(X)]^2 \tag{2.16}$$

To calculate the standard deviation (σ_x), the positive square root of $Var(X)$ is taken which is measured in the same units as is X.

$$\sigma_x = \sqrt{Var(X)} \tag{2.17}$$

Suppose X is a continuous random variable with probability density function $f(x)$. Then, an expected value of X is determined as follows;

$$E(X) = \int_{L}^{U} x f(x) dx \tag{2.18}$$

where L and U are the lower and upper bounds of the continuous probability distribution. Then, some background information and data must be available in order to be able to determine the probability distribution of a random variable - whether it is defined as discrete or continuous.

The most widely used approach in determining probability distribution of a random variable is to benefit from special probability distributions. In statistics, there are several standard probability distributions for discrete and continuous random variables such as uniform distribution, binominal distribution, Poisson distribution, normal distribution and exponential distribution. Information about such probability distributions and all other ones can be easily found in statistics books (Montgomery & Runger, 2003).

Probability assessments may be based on past observations or historical data, if the same trends or characteristics of the past are expected to prevail in the future. Any probability assessments based on objective data are called objective probabilities. However, in many real investment situations, no objective data are available to consider. In these situations, subjective probabilities are assigned.

2.5.4 Project Risk Measures

The most commonly used criterion in measuring risk of a project proposal is variability of the NPV of a project. Variability of the NPV of a project is determined through calculating the variance and standard deviation. Thus, one can measure a project risk statistically by calculating variance and standard deviation of the NPV of a project proposal.

As can be predicted, it is necessary to have calculations of more than one NPV concerning the project proposal so as to be able to calculate variance and standard deviation of this project proposal. Under highly uncertain and risky conditions, risky project parameters are not predicted as one single value but set of values with certain probability distributions. In this way, different NPVs are obtained for all possible combinations of project parameters.

It is more accurate to calculate expected value of the NPV of a project proposal before measuring project risk by calculating variance and standard deviation of the NPV of a project proposal. Because, expected NPV of a project proposal is an important criterion used in project risk measurement. It is evident that a project proposal with an expected negative NPV is too risky. In the following sections, the criteria that are used to measure the project risk will be described.

2.5.4.1 Expected Value

Expected value criterion aims to find out expected value of the NPV of a project proposal determined through its probability distribution and thus to measure project risk. If the NPV of the project is defined as a discrete random variable, *NPV,* then the expected NPV [$E(NPV)$] of the project is calculated as follows;

$$E(NPV) = \sum_{i=1}^{n} NPV_i f(NPV_i) \tag{2.19}$$

where n is the number of calculated NPVs and $f(NPV_i)$ is the probability of occurrence of the NPV_ith value of the discrete random variable.

When expected NPV of a project proposal has been calculated through the use of this equation and if the value found is negative, it can be said that this is highly risky project without a need to calculate variance or standard deviation. However, if expected NPV of project proposal is found positive, this does not necessarily mean that the project has low risk. The reason for this is that even if expected NPV of a project proposal is positive, it is not possible to reach a certain conclusion about project risk without calculating variance and standard deviation of NPV.

When standard deviations of the NPVs of two different project proposals are equal, expected NPVs of projects can be used for determining which project has higher risk. Naturally, if the standard deviations of the NPVs of two different project proposals are equal, the less risky project will be the one with higher expected NPV.

2.5.4.2 Standard Deviation

As mentioned before, the most commonly used criterion in measuring risk of a project proposal is variability of the NPV of a project. The variability of project's

NPV is determined by calculating variance and standard deviation. As mentioned, the variance tells us the degree of spread, or dispersion, of the distribution on either side of the mean value. To be most useful, any measure of risk should have a definite value (unit). One such measure is the standard deviation.

Although the standard deviation of the NPV of a project is the main criterion for determining project risk, it is not possible to reach a certain conclusion about project risk without calculating the expected NPV of a project. The standard deviation of project's NPV is considered to be the dispersion of possible NPVs around the expected value of project's NPV. Therefore, the greater the potential differences from the average, the greater risk. However, in order to measure the project risk exactly, it is necessary to determine the expected NPV of a project.

The standard deviation is the square root of the weighted average of the squared deviations of all possible outcomes from the expected value. The standard deviation of project's NPV (σ_{NPV}) can be calculated by using the equation below;

$$\sigma_{NPV} = \sqrt{\sum_{i=1}^{n} [NPV_i - E(NPV)]^2 f(NPV_i)} \qquad (2.20)$$

In order to be able to make a correct decision by considering only standard deviations of NPVs while trying to find out which of the two project proposals are more risky, one should know that expected NPVs of project proposals are equal. If it is known that expected NPVs of project proposals are equal, the less risky project will be the one with lower standard deviation of NPV.

2.5.4.3 Coefficient of Variation

When the expected values of two projects are equal, or at least close to one another, the standard deviation is a proper measure of risk. But since the standard deviation is an absolute measure, it may not serve our purposes if the two projects being compared have divergent expected values (Keat & Young, 2000).

Since the expected values of the two projects are so dissimilar, an absolute measure of risk may not give an adequate answer. In such cases, another concept is introduced, the coefficient of variation, which measures risk relative to expected value. The simple formula for the coefficient of variation is as follows;

$$CV = \frac{\sigma_{NPV}}{E(NPV)} \tag{2.21}$$

The smaller the coefficient of variation of a project is, the less risky the project proposal is. Then, as the coefficient of variation increases, so does the risk of project proposal.

2.5.5 The Main Risky Investment Project Evaluation Methods

Under highly uncertain and risky environments, it is not rationale behavior to assume that predictions about project proposal for the future will be certain and to assess the projects in this way. In these cases, a need arises to evaluate project proposals under uncertainty and risk. The two terms risk and uncertainty have somewhat different meanings, even though they are often used interchangeably. Therefore, evaluation of project proposals under uncertainty and risk is approached in different ways. During the evaluation process of projects under risk, it is aimed to consider project risk in the analyses. Because there is a possibility that one or several project parameters may deviate from expected values.

The main risky investment project evaluation methods will be explained in the following sections. The detail information about all methods and consider application examples can be found in Eski & Armaneri (2006).

2.5.5.1 The Risk-Adjusted Discount Rate Method

The risk-adjusted discount rate (RADR) is probably the most practical risk adjustment method and is the one most frequently used in business. This method makes the risk adjustment within the NPV calculation without the use of the standard deviation. Here, risk adjustment is made in the denominator of the NPV calculation.

The discount rate at which project cash flows are discounted to the present comprises two components, the risk-free discount rate, i_{rf}, and the risk premium, e. The risk-free discount rate is, in the ideal sense, the pure time value of money. The risk premium represents a judgment as to the additional return necessary to compensate for additional risk. Therefore, the RADR is equal to the risk-free discount rate plus the risk premium. The magnitude of RADR depends on the risk of the project; the higher the risk, the higher the RADR (Keat & Young, 2000).

Therefore, the NPV calculation can be adjusted subject to risk by using a RADR, as follows;

$$NPV = \sum_{t=0}^{n} \frac{A_t}{(1+RADR)^t} \tag{2.22}$$

where A_t is the *risky* net cash flow from the investment project. This method, however, has the serious shortcomings that RADRs are subjectively assigned by managers and decision makers, and variations in net cash flows or returns are not explicitly considered. The risk premiums for each project are subjectively determined. This method is most useful for the evaluation of relatively small and repetitive investment projects (Salvatore, 1996; McGuigan et al., 2002).

2.5.5.2 Certainty Equivalent Method

The RADR modifies the discount rate in the denominator of the NPV calculation to incorporate risk. The certainty equivalent method, on the other hand, uses a risk-free discount rate in the denominator and incorporates risk by modifying the numerator of the NPV calculation, as follows;

$$NPV = \sum_{t=0}^{n} \frac{b_t A_t}{(1+i_{rf})^t} \tag{2.23}$$

where A_t is the *risky* net cash flow or return from the investment project, i_{rf} is the *risk-free* discount rate, and b_t is the certainty equivalent coefficient for period t. Assigning a size to the certainty equivalent coefficient is fraught with at least as many problems as estimating the risk premium. The size of the certainty equivalent coefficient depends on the decision maker's attitude toward risk. The value of b_t ranges from 0 to 1.

For each risky net cash flow, A_t, a certainty equivalent coefficient is assigned. If risk increases as a function of time, the certainty equivalent coefficients will decrease as we move into the future.

The risk-free net cash flows, b_tA_t, are obviously smaller than the risky net cash flows, A_t, as would be expected for a risk-averse investor. These risk-free cash flows are then discounted at the risk-free discount rate to obtain the present value of the net cash flows (Keat & Young, 2000).

2.5.5.3 Expected Net Present Value Method

The expected NPV method is one of the commonly used methods in the risky investment projects evaluation. As known, the values of the project parameters may represent deviations from expected values under highly uncertain and risky conditions. In the event that one or more of these parameters deviate from their expected values, the net cash flows regarding this period will also deviate. Because, there have been deviations from expected project inflows or outflows.

At this point, the expected NPV method proposes to define net cash flows regarding all periods as probability distributions rather than to define each risky parameter which constitutes the net cash flows of project proposals as probability distributions.

The process of evaluating risky investment projects with the expected NPV method is summarized as follows (Eski & Armaneri, 2006);

(1) At first, the net cash flows for each period of the project's economical life are forecasted with probability distributions. It should be noted that, in the expected NPV method, the forecasted net cash flows for each period are assumed to be independent.

(2) Then, the expected net cash flows are determined for each period. Accordingly; if A_{it} represents the probable *i.* net cash flow of an investment project in period *t*, and $f(A_{it})$ represents the probability of occurrence of the A_{it}, the expected net cash flow for period *t*. [$E_t(A)$] is determined as follows;

$$E_t(A) = \sum_{i=1}^{m} A_{it} f(A_{it}) \qquad t=0,\ 1,\ \ldots,\ n \qquad i=1,\ 2,\ \ldots,\ m \tag{2.24}$$

(3) In order to determine the expected NPV of an investment project, the expected net cash flows are discounted at the risk-free discount rate (i_{rf}) and then they are added up. As a consequence, the expected NPV of an investment project [$E(NPV)$] is calculated by using the equation below;

$$E(NPV) = \sum_{t=0}^{n} \frac{E_t(A)}{(1+i_{rf})^t} \tag{2.25}$$

It should be noted that the expected net cash flows are discounted at the risk-free discount rate (i_{rf}).

If the expected NPV of the project is negative, it can be said that the project is proposal is too risky. However, if expected NPV of project proposal is found positive, this does not necessarily mean that the project has low risk. As mentioned in Section 2.5.4.1, the reason for this is that even if expected NPV of a project proposal is positive, it is not possible to reach a certain conclusion about project risk without calculating variance and standard deviation of NPV.

(4) After determining the $E(NPV)$ of the project proposal, the standard deviation and coefficient of variation of the project's NPV are calculated in order to measure the project risk. For calculating the standard deviation of the NPV of project proposal, firstly, the standard deviations of each period's net cash flow should be calculated. In that case, the standard deviations of net cash flow in period t [S_t] is determined as follows;

$$S_t = \sqrt{\sum_{i=1}^{m} \left[A_{it} - E(A_t)\right]^2 f(A_{it})} \qquad t=0, 1, \ldots, n \qquad (2.26)$$

As a consequence, the standard deviation of the NPV [σ_{NPV}] is calculated by using the equation below;

$$\sigma_{NPV} = \sqrt{\sum_{t=0}^{n} \frac{S_t^2}{(1+i_{rf})^{2t}}} \qquad (2.27)$$

The coefficient of variation of the project proposal is determined as follows;

$$CV = \frac{\sigma_{NPV}}{E(NPV)} \qquad (2.28)$$

As mentioned in Section 2.5.4.3, the smaller the coefficient of variation of a project is, the less risky the project proposal is. In other words, as the coefficient of variation increases, so does the risk of project proposal.

While one can easily determine the project proposal with the highest risk among other ones thanks to coefficient of variation, other data is also required in order to determine the risk of a single project proposal. For measuring risk of a project proposal, firstly, the NPV of the project proposal is assumed to have a normal distribution with mean $E(NPV)$ and variance σ_{NPV}^2. If the NPV of the project is denoted as random variable X, the distribution of this variable is shown as follows;

$$X \sim N\left[E(NPV),\ \sigma_{NPV}^{2}\right] \tag{2.29}$$

Then, in order to be able to find out how likely the random variable X exists below or above a certain value, the distribution of the random variable X is transformed to standard normal distribution. The transformation from the normal distribution to standard normal distribution is made like that;

$$Z = \frac{X - E(NPV)}{\sigma_{NPV}} \tag{2.30}$$

A normal random variable with a mean 0 and variance 1 is called a standard normal random variable and is denoted as $Z \sim N[0,\ 1]$. Thus, random variable Z can be used for probability analyses related with the random variable X.

Accordingly, in order to find the probability that the random variable X is less than or equal to any x value, both the random variable X and the value of x should be standardized. Then, this equation is obtained;

$$P(X \leq x) = P\left(\frac{X - E(NPV)}{\sigma_{NPV}} \leq \frac{x - E(NPV)}{\sigma_{NPV}}\right) = P(Z \leq z) \tag{2.31}$$

The value of the probability $P(Z \leq z)$ is determined via cumulative standard normal distribution tables. The probability value determined through these statistical tables also represents the probability that the NPV of a project is less than any certain x value. Similarly, one can also easily find the probability of NPV of a project to be over a certain value or between two values.

For detailed information about the expected NPV method, the reader can refer to see Eski & Armaneri (2006).

2.5.5.4 Decision Tree Method

As mentioned in the previous section, in the expected NPV method, the forecasted net cash flows for each period are assumed to be independent. However, the policy maintained by an enterprise over a certain period may affect not only monetary values in that period but also the values of following periods. For that reason, the assumption that the forecasted net cash flows for each period are independent can not be valid for every situation.

On the other hand, most investment problems involves only a single decision at the time of investment (accept or reject), or this single decision could entail a different decision option such as, make a product in house or farm out, and so on. Once the decision is made, there are no later contingencies or decision options to follow up. However, certain classes of investment decisions cannot be analyzed in a single step.

Decision tree method is especially suitable when decisions have to be made sequentially, for instance, if a decision two years hence depends on the outcome of an action undertaken today. In other words, the actions taken at one stage depend on actions taken in earlier stages. Such decision making can be extremely complex, and the use of a tree diagram facilitates the process because it illustrates the sequence in which decisions must be made (Keat & Young, 2000; Park, 2002).

The decision tree is a method commonly used in a wide range of areas and with different purposes. For instance, it is successfully applied while taking decisions about issues such as modernization of existing production facilities, determining the capacity of a new production facility, purchasing or hiring a machine or equipment.

In this chapter, we deal with the issue about how investment projects are assessed under risky conditions. Therefore, it will be focused on how to apply the decision tree method in the event that net cash flows estimated for each period are not dependent from each other.

For example, let's suppose that an initial investment cost of a project is 30000 TL and its economic life is two years. Then it is foreseen that this investment project provides 15000 TL net cash flow in the first year with a probability of 0.40 and 18000 TL with a probability of 0.60. Also, it is presumed that the net cash flows regarding the second year are estimated as follows;

(1) If the investment project provides 15000 TL net cash flow in the first year, it is foreseen for the second year that the project provides 22000 TL with a probability of 0.40, 26000 TL with a probability of 0.30 and 30000 TL with a probability of 0.30.

(2) If the investment project provides 18000 TL net cash flow in the first year, it is foreseen for the second year that the project provides 26000 TL with a probability of 0.30, 32000 TL with a probability of 0.40 and 22000 TL with a probability of 0.30.

In that case, the all probable net cash flow series for a sample investment project is shown in Figure 2.3.

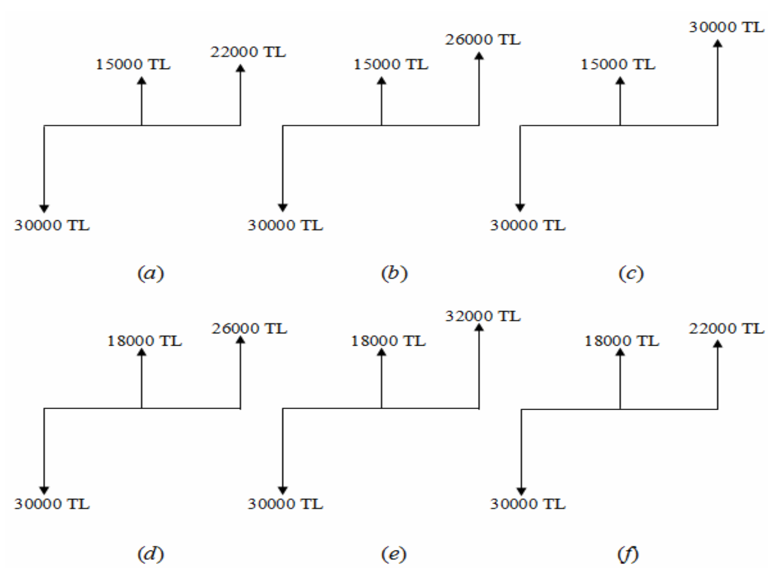


Figure 2.3 The Probable Net Cash Flow Series for a Sample Investment Project

As seen, there are six different probable net cash flow series for a sample investment project and all of them can be represented graphically as the branches of a tree. This kind of presentation is called as decision tree. Therefore, the decision tree for a sample investment project proposal is shown in Figure 2.4.

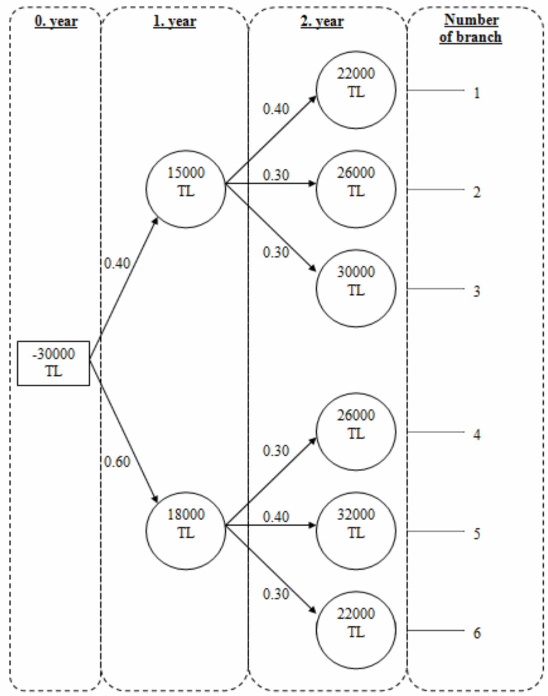


Figure 2.4 Decision tree for a sample investment project proposal

The nodes of the decision tree from which the tree's branches emanate are either decision nodes, (□), or chance, (O). The branches that emanate from a decision node represent alternative courses of actions about which the decision maker must make a choice; those leaving the chance nodes represent chance events. The occurrence of a chance event is considered to be a random variable over which the decision maker has no control.

As can be seen in Figure 2.4, there are six probable cash flow series for the sample project proposal and, naturally, the NPV of each series is different from one another. Then, six different NPVs will be calculated for the project proposal and the real NPV of the project may be equal to one of these values. As discussed in detail in previous sections, if more than one NPV can be calculated for a project proposal, it is easy to calculate the expected NPV of the project, standard deviation of the NPV and coefficient of variation by determining the possible values for the NPV and the probability of the occurrence of these values.

Therefore, during the evaluation of risky investment projects via the decision tree method, the next step upon forming the decision tree is to find out the probability of the occurrence of each branch of the decision tree. The probability of a net cash flow in any branch of the decision tree over any time period is expressed in accordance with the cash flows realized in the previous year or years. Because of this reason, all probabilities within the decision tree are called as *conditional probability*. For detailed information about the conditional probability concept, the reader can refer to see Montgomery & Runger (2003).

The probability of the occurrence of the cash flow series in any branch of the decision tree (joint probability) is calculated by multiplying all probability values on that branch (Eski & Armaneri, 2006).

Therefore, the joint probabilities for each branch of the decision tree can be calculated as follows. It should be noted that, the sum of the joint probabilities should be equal to 1.

Number of Branch	Joint Probability
1	0.40*0.40 = 0.16
2	0.40*0.30 = 0.12
3	0.40*0.30 = 0.12
4	0.60*0.30 = 0.18
5	0.60*0.40 = 0.24
6	0.60*0.30 = 0.18

Each branch of the decision tree represents the probable net cash flow series of the sample project. Thus, six different NPV will be calculated for this project proposal. It should be noted that the net cash flows on each branch are discounted at the risk-free discount rate (i_{rf}). Discounting the net cash flows at a rate that includes a risk premium could result in double-counting risk. On the other hand, the probability of the occurrence of calculated NPV for each branch is equal to the joint probability of its branch. Therefore, if the net cash flows on each branch are discounted at i_{rf}=0.10, the NPV of the sample project proposal is defined as a probability distribution as seen in the table below.

Number of Branch (*i*)	**NPV of Branch *i* (*NPV*$_i$)**	**Joint Probability of Branch *i*** $f(NPV_i)$
1	1818 TL	0.16
2	5124 TL	0.12
3	8430 TL	0.12
4	7851 TL	0.18
5	12810 TL	0.24
6	4545 TL	0.18

It is discussed in detail in Section 2.5.4 how to find out the expected NPV of a project proposal when the NPV of a project proposal is defined as a probability distribution. As is the case for the expected NPV method, in the decision tree method, one should calculate the standard deviation of the NPV of the project and coefficient of variation with the aim of determining the project risk after finding out the expected NPV of the project proposal. It is also explained in detail in Section 2.5.4 how to carry out these calculations.

The detail information about decision tree analysis can be found in Salvatore (1996), Keat & Young (2000), Thuesen & Fabrycky (2001), Park (2002), Shtub et al. (2005) and Eski & Armaneri (2006).

2.5.5.5 Simulation Method

Another method for analyzing complex, real-world decision making situations involving risk is simulation. Simulation is a statistics based behavioral approach that applies predetermined probability distributions and random numbers to estimate risky outcomes. The first step in simulation is the construction of a mathematical model of the managerial decision making situation that we seek to simulate. For example, the firm might construct a model for the strategy of expanding the output of a product. The model would specify in mathematical (i.e., equational) form the relationship between the output of the product and its price; output, input prices, and costs of production; output and depreciation; output, selling costs, and revenue; output, revenues, and taxes; and so on. The manager could then substitute likely values or best estimates for each variable into the model and estimate the firm's profit. By then varying the value of each variable substituted into the model, the firm can get an estimate of the effect of the change in the variable on the output of the model or profit of the firm. This simplest type of simulation is often referred to as sensitivity analysis (Salvatore, 1996; Gitman, 2003).

As an another example; by trying the various cash flow components together in a mathematical model and repeating the process numerous times, the decision maker can develop a probability distribution of project returns. Figure 2.5 presents a flowchart of the simulation of the NPV of a project.

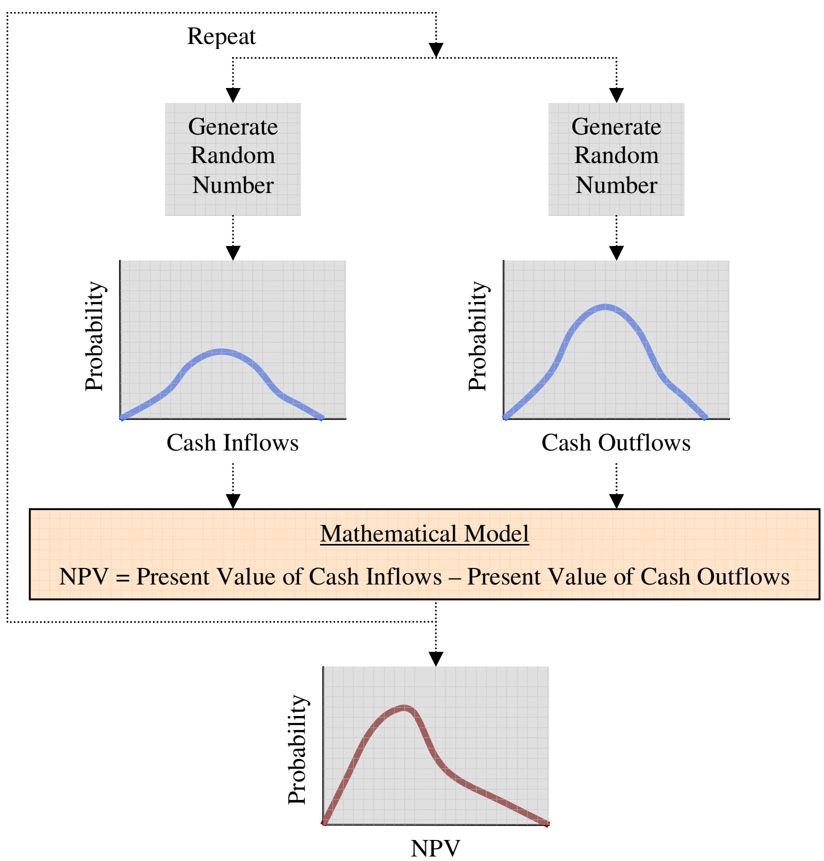


Figure 2.5 Flowchart of a NPV Simulation (Gitman, 2003)

The process of generating random numbers and using the probability distributions for cash inflows and cash outflows enables the decision maker to determine values for each of these variables. Substituting these values into the mathematical model results in an NPV. By repeating this process perhaps a thousand times, decision maker can create a probability distribution of NPVs. Here, only gross cash inflows and cash outflows are simulated.

In more sophisticated simulation models, the model builder needs to estimate or specify the probability distribution of each variable in the model. For example, in order to simulate the NPV formulation, the analyst needs the probability distribution of individual inflow and outflow components, such as sales volume, sale price, revenues, material cost, labor cost, depreciation, taxes, and so on. Randomly selected values of each variable of the model are then fed into the simulation model to generate the NPV. This process is then repeated a large number of times. Each time (i.e., for each computer run), a new randomly selected value for each variable is fed into the model, and the NPV is recorded. A large number of such trials, or iterations, are conducted, so as to generate the probability distribution of the NPV.

The probability distribution of the NPV so generated can then be used to calculate the expected NPV, the standard deviation of the distribution of NPV and the coefficient of variation. The output of simulation provides an excellent basis for decision making, because it enables the decision maker to view a continuum of risk-return tradeoffs rather than a single point estimate.

Simulation can be a good tool for decision making. However, in obtaining solution, it is made at least two assumptions that may have omitted some important relationships among the variables. First, it is assumed that the deviations obtained with the use of random numbers remain the same in each year for which estimated cash flows were calculated. This need not be the case. A set of different random number calculations for each year may have been more appropriate. With the use of a computer, such calculations could have been taken care of quite efficiently. Second, it has been assumed here that the variables are statistically independent. It is much more likely tat the various factors are interrelated. If interdependencies actually exist, they must be included in the simulation model. Such a model would, of course, be considerably more complex. A large number of estimates relating to these relationships would have to be made (Keat & Young, 2000; Eski & Armaneri, 2006).

In Chapter Four, simulation in risky investment project evaluation and selection process will be explained in details.

2.5.6 Investment Decisions Under Uncertainty

It may be inappropriate or impossible to assign probabilities to the several futures identified for a given decision situation. Often, no meaningful data are available from which probabilities may be developed. In other instances the decision maker may be unwilling to assign a subjective probability, as is often the case when the future could prove to be unpleasant. When probabilities are not available for assignment to future events, the situation is classified as decision making under uncertainty (Thuesen & Fabrycky, 2001)

Several criteria are available for making the actual decision under uncertainty. These decision criteria, which will be presented in this section, include Laplace, maximax, maximin, Hurwicz and minimax regret.

2.5.6.1 Laplace Criterion

The Laplace criterion postulates that if no information is available about the probabilities of the various outcomes, it is reasonable to assume that they are equally likely. Therefore, if there are n outcomes, the probability of each is $1/n$. This approach also suggests that the decision maker calculate the expected payoff for each alternative and select the alternative with the largest value. The use of expected values distinguishes this approach from the criteria that use only extreme payoffs. This characteristic makes the approach similar to decision making under risk.

2.5.6.2 Maximax Criterion

With the maximax criterion, the decision maker selects the decision that will result in the maximum of the maximum payoffs. The maximax criterion is very optimistic. The decision maker assumes that the most favorable state of nature for each decision alternative will occur. Thus, for example, using this criterion, the investor would optimistically assume that good economic conditions will prevail in the future.

It should be pointed out that the maximax decision rule as presented deals with profit. However, if the payoff table consisted of costs, the opposite selection would be indicated: the minimum of the minimum costs, or a *minimin criterion*. For the subsequent decision criteria we encounter, the same logic in the case of costs can be used (Taylor, 1999).

2.5.6.3 Maximin Criterion

In contrast to the maximax criterion, which is very optimistic, the maximin criterion is pessimistic. With the maximin criterion, the decision maker selects the decision that will reflect the maximum of the minimum payoffs. For each decision alternative, the decision maker assumes that the minimum payoff will occur. Of these minimum payoffs, the maximum is selected. However, if the payoff table consisted of costs, the conservative approach would be to select the maximum cost for each decision. Then the decision that resulted in the minimum of these costs would be selected (Taylor, 1999).

2.5.6.4 Hurwicz Criterion

The Hurwicz criterion strikes a compromise between the maximax and maximin criteria. The principle underlying this decision criterion is that the decision maker is neither totally optimistic nor totally pessimistic. With the Hurwicz criterion, the decision payoffs are weighted by a coefficient of optimism, a measure of the decision maker's optimism. The coefficient of optimism, which we will define as α, is between zero and one (i.e., $0 \leq \alpha \leq 1.0$). If $\alpha=1.0$, then the decision maker is said to be completely optimistic; if $\alpha=0$, then the decision maker is completely pessimistic. Given this information, if α is the coefficient of optimism, $1-\alpha$ is the coefficient of pessimism.

The Hurwicz criterion requires that, for each decision alternative, the maximum payoff be multiplied by α and the minimum payoff be multiplied by $1-\alpha$. Then, it specifies selection of the decision alternative corresponding to the maximum weighted value.

It should be pointed out that when $\alpha=0$, the Hurwicz criterion is actually the maximin criterion; when $\alpha=1.0$, it is the maximax criterion. A limitation of the Hurwicz criterion is the fact that α must be determined by the decision maker. Therefore, the Hurwicz criterion is a completely subjective decision making criterion (Taylor, 1999).

2.5.6.5 Minimax Regret Criterion

The minimax regret criterion examines the regret, opportunity cost or loss resulting when a particular situation occurs and the payoff of the selected alternative is smaller than the payoff that could have been attained with that particular situation. With this decision criterion, the decision maker attempts to avoid regret by selecting the decision alternative that minimizes the maximum regret. To use the minimax regret criterion, a decision maker first selects the maximum payoff under each state of nature. All other payoffs under the respective states of nature are subtracted from these amounts. These values represent the regret that would be experienced by the decision maker if a decision were made that resulted in less than the maximum payoff. The values are summarized in a modified version of the payoff table known as a *regret table*.

In order to make the decision according to the minimax regret criterion, the maximum regret for each decision must be determined. The decision corresponding to the minimum of these regret values is then selected (Taylor, 1999).

2.6 Literature Review

In the scope of this dissertation, investment project evaluation and selection literature has been reviewed. During the review of literature, firstly, the studies on evaluation and selection of investment projects have been analyzed. In this way, an attempt was made to determine the trend of studies in this field from past to present and which issues have attracted attention mostly over recent years. The purpose of this survey is to review the studies on evaluation and selection of investment projects, to determine the subjects which have not been analyzed or analyzed insufficiently in the literature and to focus on these specified issues.

Naturally, evaluation and selection of investment projects is an extremely significant field in which many scientific studies have been conducted. For that reason, nearly all studies which have been conducted especially for the past 30 years have been analyzed; however, only the ones which seem important have been mentioned in this section of the thesis. The following section includes information about important studies on evaluation and selection of projects conducted from past to present for the last 30 years.

Gaber et al. (1992) have proposed integrating artificial neural networks with Knowledge Based Systems to improve risk assessment for new project evaluation. This research examines the use of artificial neural networks for classifying new industrial projects according to their risk. Then, they have compared the performance of the artificial neural networks to expert classification, and the network correctly classified most of the projects considered. The results show a great potential for artificial neural networks to improve decision making in that field.

Chiu & Park (1994) have proposed a discounted cash flow model. They represent the cash flows and discount rates by triangular fuzzy numbers and use present worth as the figure of merit, and first derive an exact present worth formulation, and later an approximate form with much less computational effort. They further state that when the future estimated discount rates are within an absolute range of 4%, the approximate present worth is a close estimate of the exact present worth.

Badiru & Sieger (1998) have presented an artificial neural network model for economic analysis of risky projects. Outputs of conventional simulation models have been used as neural network inputs. The neural network model has been then used to predict the potential returns from an investment project having stochastic parameters. The nondeterministic aspects of the project include the initial investment, the magnitude of the rate of return, and the investment period. Analysis of the outputs of the neural model has indicated that more predictive capability could be achieved by coupling conventional simulation with neural network approaches. The trained network was able to predict simulation output based on the input values with a high level of accuracy for conditions not in its training set.

Van Groenendaal (1998) has considered the variability in the NPV of an investment project. He said that this variability is an indication of the project's risk and risk analysis is one way to estimate this variability. He suggested that the analysis of the variability is usually restricted to deterministic sensitivity analysis, such as "one-factor-at-a-time" and scenario analysis. He believes that these deterministic analyses, however, do not account for the total variability in the NPV. For this reason, he has presented a three-step procedure to improve traditional sensitivity analysis through the application of experimental design theory in combination with regression metamodelling. He has shown that the use of experimental designs leads to more adequate information about the variability in the NPV, and that regression metamodelling can be used evaluate the variability.

Chiadamrong (1999) has proposed an integration of financial and strategic justification approaches based on the fuzzy logic. He has developed a fuzzy multi-criteria decision model. Because he believes that the new manufacturing systems can offer not only the financial benefits but also longer-term strategic benefits. Developed model in this study allows not only consideration of the strategic aspects of the investment but also determination of the quantifiable aspects of the investment. Then, both results are integrated by considering the financial and strategic importance of each and obtaining the overall aggregate index.

Jovanovic (1999) has studied the investment project evaluation and the decision-making problems under uncertainty and risk. He has presented some of the methods used for investment decision-making under uncertainty. These methods are break-even analysis, sensitivity analysis, theory of games and decision making theory. He has shown one of the procedures of sensitivity analysis application in investment decision making under uncertainty and risk in his study. He has used the NPV, *IRR* and payback period criteria for application of sensitivity analysis.

Jinlou & Lean (2000) have created optimal model of project investment with exposure preference and benefit pursuing. Their optimal model successfully solves the paradox relationship of exposure and benefit in the project investment decision. At the same time, they have verified sensibility and feasibility of the process by calculating and analyzing of true examples.

Beuthe et al. (2000) have proposed methodology of multi-criteria decision aid for the assessment of public investment projects which takes into account the uncertainty of projects' measures.

Karsak & Tolga (2001) have proposed a fuzzy decision algorithm to select the most suitable advanced manufacturing system (AMS) alternative from a set of mutually exclusive alternatives. In their study, they consider both economic evaluation criterion and strategic criteria such as flexibility, quality improvement, which are not quantitative in nature, for selection. The economic aspects of the AMS selection process are addressed using the fuzzy discounted cash flow analysis. The decision algorithm aggregates the experts' preference ratings for the economic and strategic criteria weights, and the suitability of AMS investment alternatives versus the selection criteria to calculate fuzzy suitability indices. The fuzzy indices are then used to rank the AMS investment alternatives. Triangular fuzzy numbers are used throughout the analysis to quantify the vagueness inherent in the financial estimates such as periodic cash flows, interest rate and inflation rates, experts' linguistic assessments for strategic justification criteria, and importance weight of each criterion.

Lee & Kim (2001) have suggested an integrated approach for interdependent IS project selection problems using Delphi, analytic network process concept and zero-one goal programming. Although goal programming incorporates multiple objectives and arrives at an optimal solution, its major drawback is that the decision maker(s) must specify goals and priorities a priori. In order to overcome this problem, they have used Delphi method. Another shortcoming of goal programming is the lack of a systematic approach to set priorities and trade-off among objectives and criteria. In order to overcome this problem, they have used analytic network process developed by Saaty (1980).

Mohamed & McCowan (2001) have proposed a method capable of modeling the effects of both monetary and non-monetary aspects of an investment options, using interval mathematics and possibility theory to handle the inherent uncertainty associated with such aspects. They have presented two numerical examples to demonstrate its application in the assessment and ranking of available investment options. Their proposed methodology has provided an accurate method for comparing different project alternatives.

Chen (2002) has proposed a linguistic multi criteria decision making method based on fuzzy measures and integrals. In this model, decision maker's opinions have been described by linguistic terms expressed in trapezoidal fuzzy numbers. Then, the proposed model has applied to solve a selection of information system (IS) project problem for high-technology software company. This proposed method is practical and useful and provides more flexible and objective information in dealing with IS project selection problems in a fuzzy environment.

Karsak & Kuzgunkaya (2002) have presented a fuzzy multiple objective programming approach to facilitate decision making in the selection of a flexible manufacturing system (FMS) in their paper. They introduce fuzzy set theory in the model to incorporate the vague nature of future investments and the uncertainty of the production environment. They use linguistic variables and triangular fuzzy numbers to quantify the vagueness inherent in decision parameters, e.g., increase in market response, improvement in quality, reduction in setup cost, and so forth. The model proposed in their paper determines the most appropriate FMS alternative through maximization of objectives such as reduction in labor cost, reduction in setup cost, reduction in work-in-process (WIP), increase in market response and improvement in quality, and minimization of capital and maintenance cost and floor space used. They assign priorities to these objectives indicating their importance levels using linguistic variables.

Naumov & Khodusov (2002) have analyzed investment projects' effectiveness evaluation methods in their paper and they have proposed the method namely as modified NPV-method, or NPV↑. The basis of their method is the assumption that project's investments will be invested from the outgoing flows of it or other projects. Also their method takes into the account the fact that temporary free money got from the project can be invested to have more profits.

Powers et al. (2002) have purposed the model that shows a special purpose simulation tool for project selection based on influences that govern the project selection process. Then they have adopted a graphical and hierarchical approach for the non- simulation experts to use the model to derive expected results for project selection process and decision making under uncertain conditions.

Borgonovo & Peccati (2004) have discussed the sensitivity analysis of valuation equations used in investment project valuation. Since financial decisions are often based on the nominal value of the project economics chosen as valuation criterion (NPV, *IRR* etc.), they have focused on the use of local sensitivity analysis techniques. In particular, they have presented the differential importance measure and general results of the NPV and *IRR* sensitivity on changes in the cash flows.

Hwang (2004) has written the paper concerned with development of web-based multi-attribute evaluation model for engineering project. Major technologies used for this study were; analytic hierarchy process (AHP), multi-attribute utility with multi echelon structure and integrating the prioritized set of evaluated results. He has developed a 3-step approach, three sub-modules were: (1) web-based brainstorming module for idea and alternative generation, (2) web-based AHP and Fuzzy AHP module for evaluation of alternatives, and (3) priority integration module to aggregate the multiple rank-ordered sets based on fuzzy set priority. Finally, he has developed a systematic and practical computer program, he has validated his proposed model by comparative computations for various multi-structured project evaluation examples.

Ammar & Khalifa (2005) have characterized the optimal profit (best return) of an investment problem with trapezoidal fuzzy numbers. They have given a solution algorithm to determine the optimal profit for investing of F money in the n investment polices and also they have considered the alternative approach for people who need to be more precise in their requirements. In their study, a fixed sum of money, F, is to be spread among n possible investment polices; each of which has a history of fuzzy returns. The investment problem is to determine how much money should be allocated to each possible investment policy so that the total expected of fuzzy return is maximized. They have used the dynamic programming approach to characterize the set of solutions to such problem.

Duarte & Reis (2006) have presented the development process of an evaluation system to help the Portuguese Public Administration to choose a portfolio of projects for financing. They have used multiple attribute value theory, which focuses on the prescription of decisions in non-structured multiple objective decision scenarios. Problem structuring involved defining objectives in agreement with national development program and European Community policies, and attributes to measure the achievement of projects with respect to them. The approach required the assessment of value functions for each of the attributes, validation of independence conditions of decision makers and, finally, the aggregation of single attribute value functions into an overall multiple attribute value function. All these structuring steps have been carried out based on the preferences of a panel of decision makers with wide experience in managing and selecting projects within similar programs.

Coldrick et al. (2005) reviews the development of a project selection and evaluation tool that can be applied to a wide range of research, technology and investment decisions. Firstly, they have given background on project selection models and then they have given the introduction of the model and its application to a sample group of projects. Finally, they have discussed some conclusions as to the applicability of such models.

Dimova et al. (2006) have considered the problem of investment project estimation as the multiple criteria hierarchical task of choosing the optimal alternative (project). They have used the mathematical tools of fuzzy sets theory for representation of uncertainty and building of local criteria, based on the quantitative and qualitative parameters characterizing the considered projects. They have analyzed the problems of ranked local criteria aggregation and presented some new theoretical results, which can be useful for proper choice of aggregation method. They present the generalized method for multiple criteria hierarchical estimation of investment projects in fuzzy setting. The key issue is the analysis of the familiar approaches to aggregation of local criteria.

Büyüközkan et al. (2005) have proposed an integrated multi criteria decision making model in fuzzy environment with four main stages to evaluate wastewater treatment investment for Siemens, İstanbul. In the first stage, they want to made the decision “to invest for the waste treatment” or “not to invest” based on two main evaluation criteria: economic and environmental. Each main criterion consists of several under-criteria. They use Fuzzy AHP methodology to weight all evaluation criteria. After identifying the evaluation criteria weights, they evaluate two alternatives by using fuzzy PROMETHEE approach. The purpose of the second stage is to choose “the most suitable technology for water treatment” between two alternatives. They evaluate these two alternatives based on four evaluation criteria. At the third stage, they decide the system type for the technology chosen in the second stage by using fuzzy AHP. Finally, the last stage consists in choosing the most suitable company which will give the service of waste water system construction. In this stage they evaluate three companies by using fuzzy TOPSIS approach.

Çekyay et al. (2005) have introduced a method that integrates fuzzy analytic network process (FANP) with zero-one goal programming (ZOGP) to solve IS project selection problem. They use analytic network process to determine the effect of each criterion to the overall performance of each project. Because the criteria used at the evaluation process of IS projects have interdependencies. Since the human perception and judgments are vague, they prefer FANP to improve the quality of responses of decision makers. They use the weights calculated at FANP stage as input for project selection to obtain the coefficient of the objective function of ZOGP model.

Rabbani et al. (2005) have presented a decision-making methodology based on Data Envelopment Analysis (DEA) and AHP for projects evaluation and selection. At first, they have identified the important criteria in investment projects selection and grouped into quantitative and qualitative criteria categories. Then, they generate feasible alternatives of projects. They have weighted the qualitative criteria by AHP. They have used DEA methodology to solve the project selection problem by simultaneously considering both quantitative and qualitative data.

Rebiasz (2007) have proposed a method for quantification of project-specific risk. When assessing investment project risk it is very common to apply two analytical methods for describing parameter uncertainty: probability distribution and possibility distribution. This study discusses methods for integrating the above-mentioned approaches into a description of the uncertainty of parameters in calculations of effectiveness and investment project risk.

Literature review indicates that investment decision-making never takes place under conditions of certainty, but only under those of uncertainty or risk. Therefore, it is necessary to define and locate the investment decision-making problem in its real conditions, and possibly find suitable and appropriate solutions (Jovanovic, 1999). The estimation of the investment efficiency is rather an uncertain problem and so the proper methods for operating in uncertain must be used (Dimova et al., 2006). Several methods have been presented in the literature to handle the analysis of the investment projects under uncertainty or risk (Choobineh & Behrens, 1992; Badiru & Sieger, 1998; Jovanovic, 1999; Karsak & Tolga, 2001; Mohamed & McCowan, 2001; Borgonovo & Peccati, 2006; Huang, 2007; Medaglia et al., 2007; Rebiasz, 2007).

According to the results of the literature survey, it can be said that there are several approaches in project evaluation and selection under uncertain and/or risky environment. The first approach in investment project evaluation is multi-criteria or multi-objective investment evaluation and selection under uncertainty. The process of investment project selection among different project alternatives is a complex problem due to the vagueness of the available information related with each alternative. Moreover, there are several criteria such as; market conditions, availability of raw materials, management desire, and flexibility which are involved in investment project evaluation and selection process. The selection process that takes into account several criteria for decision making is multi-criteria decision making (MCDM) process. There are several MCDM approaches used in the literature, which seek to take explicit account of more than one criterion in supporting the decision process. Main advantage of the MCDM tools is that; they allow incorporating uncertainty of the future and the multi-objectivity.

Effective project evaluation necessitates incorporating the many conflicting objectives of decision maker(s) into decision models. Many models and methodologies for MCDM have been developed (Bellman & Zadeh, 1970; Cochrane & Zeleny, 1973; Ignizio, 1982a; Zeleney, 1982; Saaty, 1986; Keeney & Raiffa, 1993; Tamiz et al., 1998; Teng & Tzeng, 1998; Saaty & Vargas, 2000; Karsak & Tolga, 2001; Aouni & Kettani, 2001; Lee & Kim, 2001; Al-harbi, 2001; Oral et al., 2001; Enea & Piazza, 2004). For decisions with multiple objectives, Keeney & Raiffa (1993) propose a method to determine “the utility function” of the decision maker in mathematical form. This utility function then represents a decision maker’s level of satisfaction with different alternatives. Utility theory is a branch of decision analysis that involves the building of mathematical models to describe the behavior of a decision maker when faced with making a choice among alternatives in the presence of risk (Badiru & Pulat, 1995). Among other models are statistical methods such as; Bayesian theory, fuzzy set theory, and mathematical programming.

Goal programming (GP) is perhaps the oldest methodology in the field of multi-criteria decision making (Romero, 1986). Linear goal programming (LGP) formulation was first introduced by Charnes & Cooper in 1952 (Khorramshahgol et al., 1988). The procedure to formulate a LGP model starts with specifying a target or aspiration value for each objective, thus transforming all objectives into goals. Some survey papers that include GP methods and applications can be seen in literature (Romero, 1986; Tamiz et al., 1998; Aouni & Kettani, 2001).

Steuer & Na (2003) presented the widest review related with MCDM for economical and financial problems. However, according to the literature survey, there are not many papers related with multi-criteria or multi-objective project evaluation.

The second approach in project evaluation is simulation and post-simulation analysis based investment evaluation and selection approach. Some projects have high uncertainty, and simulation based investment project selection analysis could evaluate the projects with a greater confidence. Although, in economic analyses, it is often assumed that all factors are deterministic in nature, but, in reality, some factors have stochastic properties. In some cases where the stochastic nature of a factor is recognized, some flaws may still exist because of the simplifying approach that is used in the analysis. Two of common, but wrongful, practices in analytical approaches involve representing a distribution simply by its mean and using the wrong probability distribution. Simulation approach avoids these pitfalls by allowing both parametric and nonparametric factors over the range of factor values (Badiru & Sieger, 1998).

The third type approach in project evaluation is to spread a fixed sum of money among possible investment policies. The investment problem here is to determine how much money should be allocated to each possible investment policy so that the objective is minimized or maximized. In the literature, there are some studies based on this approach related with investment evaluation under uncertainty. Some of the methods used in these studies are dynamic programming (Ammar & Khalifa, 2005), fuzzy logic and genetic algorithms (Huang, 2007).

The fourth and the last approach in project evaluation and selection process is trying to evaluate the investment projects by analyzing their project risk. For example, according to Van Groenedaal (1998), the variability in the NPV of an investment project is an indication of the project's risk. There are many methods used in investment project evaluation to identify and assess the level of perceived project risk. Mostly used methods are payback period method (Lefley, 1996), sensitivity analysis (Jovanovic, 1999; Borgonovo & Peccati, 2004), probability analysis and simulation (Lefley, 1997).

As a result of literature review, it is specified that; there are four main types of approaches recently considered and studied which have similar targets in project evaluation and selection under uncertainty or risk. The summary of these approaches is shown in Table 2.1.

Table 2.1 The summary of the approaches recently considered in project evaluation and selection under uncertainty or risk

Approaches	**The methods used in these approaches**
Multi-criteria or multi-objective investment evaluation and selection approach under uncertainty	Analytic hierarchy process, goal programming, multi-attribute utility models, group decision making, fuzzy multi-criteria/multi-objective programming
Simulation and post-simulation analysis based investment evaluation and selection approach	Computer simulation, simulation metamodeling
Capital allocation approach (determining how much money should be allocated to each possible investment policy so that the objective is minimized or maximized)	Dynamic programming, fuzzy logic, genetic algorithms
Risk assessment approach (trying to evaluate the investment projects by analyzing their project risk)	Sensitivity analysis, probability analysis, computer simulation

Table 2.2 Mostly used investment project evaluation and selection techniques

	Techniques	Advantages	Disadvantages
Economic Analysis Methods	- Internal Rate of Return (IRR) - Net Present Value (NPV) - Net Present Value Ratio (NPVR) - Return on Investment (ROI) - Benefit-cost Analysis - Simple Payback Period - Discounted Payback Period - Decision Tree Analysis	- Ease in data collection - Intuitive appeal	- Do not take into account qualitative (strategic and non-economic) benefits - Consider a single objective of cash flows, and ignore other benefits such as quality and flexibility
Strategic Approaches	- Technical Importance - Business Objectives - Competitive Advantage - Research and Development	- Require less technical data - Use the general objectives of the firm	- Necessity to use these techniques with economic or analytic ones since they consider only long-term intangible benefits

Analytic Methods	- Scoring models
	  - Analytic Hierarchy Process
	  - Outranking Methods
	- Mathematical Programming
	  - Integer Programming
	  - Goal Programming
	  - Data Envelopment Analysis
	- Stochastic Methods
	  - Game Theoretical Models
	  - Multi-attribute Utility Models
	  - Fuzzy Linguistic Methods
	  - Expert Systems
	  - Simulation
	- Fuzzy Set Theory
		- Uncertainty of the future and the multi-objectivity can be incorporated
		- Subjective criteria can be introduced in the modeling phase
			- Require more data
			- Usually more complex than the economic analysis

As a result of the literature survey, the investment project evaluation methods can be classified into three main categories such as; economic analysis methods, strategic approaches and analytic methods. The classification of the investment project evaluation and selection techniques according to these three categories, and also advantages and disadvantages of these techniques are demonstrated in Table 2.2. The classification table has been constructed by extending the table given in Karsak & Tolga (2001).

In the following section, gaps in the existing project evaluation and selection literature will be discussed and the need for the proposed research will be explained.

2.7 Gaps in the Existing Literature and the Need for the Proposed Research

The previous chapter included the summary of important studies on evaluation and selection of investment projects for the past 30 years. As mentioned before, this dissertation proposes a novel methodology and this proposed methodology consists of three stages. Each stage deals with the problems that should be solved during the process of evaluation of investment projects, and models or methods are proposed for solving to these problems.

In order to properly comprehend the models developed and methods used during these three stages of proposed methodology, the following chapters include detailed information regarding modeling techniques and methods. In other words, the chapters preceding the sixth chapter, which discusses the proposed methodology in detail, deal with modeling and decision making techniques used during the stages of proposed methodology.

For example, the project evaluation and selection problem considered in the first stage of the proposed methodology is a type of MCDM problem and it is better addressed through the classification/sorting problematic. Therefore, the third chapter discusses the problem of MCDM and multi-criteria classification in detail and explains the method used during classification of projects in the first stage of proposed methodology and why this method is employed. Same phenomenon is valid for the fourth and fifth chapters.

Besides the literature review given in the previous section, the studies in which the modeling techniques and methods covered by the proposed methodology have been considered separately and some of the studies which seem to be important have been summarized in relevant chapters. That is, in this dissertation, apart from the literature review section, all chapters include review of literature regarding the issues discussed. In this way, we both conducted the review of literature about project evaluation and selection, and examined and reported the literature thought to be related to the models developed and methods used at each stage of the proposed methodology.

This comprehensive literature review suggests that there is a need for an integrated methodology regarding the evaluation and selection of investment projects. Up to present, in their studies, researchers have preferred to seek for solution by dealing with the problems encountered during the process of evaluation of investment projects individually rather than collectively. However, investment project evaluation process is an integrated study. Thus, considering the problems that should be solved together will increase the accuracy of results obtained during the evaluation of projects.

For instance, the first analysis to be conducted during the project evaluation is to determine the ones among potential project proposals that will satisfy the needs of an investor or entrepreneur. In this way, the investor will not have to conduct a detailed feasibility study for all investment ideas. As mentioned, the reason for this is that conducting a detailed feasibility study is a time-consuming and costly process. Furthermore, an attempt to conduct feasibility studies for many project proposals at the same time may result in missing a lot of profitable investment opportunities.

Therefore, first of all, it is necessary to classify the project proposals during the process of investment evaluation. Of course, it will not be realistic to consider only one criterion during this classification. Then, the problem of project classification is a MCDM problem. On the other hand, the values of evaluation criteria determined during the classification of investments could be specified without needing much detailed analyses.

In literature, there are many studies on classification of investment project proposals in accordance with certain criteria. These studies are summarized in both Section 2.6 and Chapter Three. However, the review of literature indicates that researchers have classified the investment proposals through the use of a classification technique which requires a training sample. In other words, they have determined the parameter values of the classification method by using a reference set and made the classification on the basis of these parameters values. However, the use of a training sample during the evaluation or classification of investments will often be an inaccurate way. Similarly, it is also inaccurate to obtain data from the feasibility report of a similar project when one conducts a feasibility study for any investment project. The reason for this is that each investment has its own distinctive characteristics.

Therefore, when one makes an attempt to identify the investment proposals appropriate for the needs of entrepreneurs or investors, it is necessary to classify them through a multi-criteria classification method which does not require a training sample. However, it could not be encountered with any study in literature in which investment proposals are classified via such a method. Also, it could not be found any study which is based on the principle of determining only the promising ones among investment proposals and conducting feasibility studies only for them. In the studies included in literature, investment proposals have been classified through methods requiring a training sample. In these studies, the values of the some of the selected criteria could be obtained only through detailed feasibility studies. Therefore, classification has often been carried out on the basis of limited number of project proposals. However, when the number of project proposals increase, the efficiency of classification methods proposed by researchers will get lost.

When investment proposals are classified in accordance with certain criteria, it is necessary to determine the values obtained by each investment proposal according to each evaluation criterion. At this stage, it is also essential to consider that the values of the selected criteria involve risks when the superiority of one investment proposal over another is identified. Then, the uncertainty and risk should be considered during the classification of investment proposals on the basis of certain criteria.

The literature includes studies based on analyses the results of feasibility studies of investments. In these studies, generally, the NPVs of project proposals have been analyzed. When determining the NPVs of project proposals, these studies have been used the traditional NPV formulation mentioned in Section 2.4.2.1 and NPVs of project proposals are obtained in accordance with this formulation. In these studies, it is often assumed that the effect of inflation is same both on project inflows and outflows, so the effect of inflation on project inflows and outflows is not taken into account. But it is obvious that inflation effect will be different for cost and revenue components, and it should be considered in project evaluation process. The use of traditional NPV formulation in studies on project evaluation and the fact that inflationist effects are generally not taken into account are regarded as a drawback.

As stated before, because of the uncertainty and risk of the future, the values of the project parameters can not be estimated with complete certainty. Therefore, uncertainty and risk phenomena should be considered while evaluating projects. Several methods have been presented in the literature to handle the analysis of the investment projects under uncertainty or risk. One of these methods is simulation. In literature, there are many studies in which simulation method is used during the evaluation of investment projects under uncertainty and risk, and some of these studies have been mentioned in Section 2.6 and Chapter Four. As known, in simulation based project evaluation approaches, the risky project parameters are defined as probability distributions. The expected profitability of the project is calculated via simulation approach.

However, one of the most important drawbacks of simulation-based project evaluation studies is the use of traditional NPV formulation for determining the profitability of projects. In these studies, only the net cash flows or gross cash inflows and cash outflows are simulated in order to provide a sufficient number of NPVs and to develop the NPV distribution. In the event that the existence of inflation is not taken into consideration, simulating only the net cash flows or gross cash inflows and cash outflows may seem a right choice. However, when inflation is prevalent and it is thought that each individual inflow and outflow components which composes cash flows of projects such as sales volume, sale price, revenues, material cost, and labor cost will display varied increases from one year to another, individual inflow and outflow components should be defined as probability distributions. In such a situation, it is necessary to develop a new NPV formulation that eliminates the weakness of using the traditional formulation of NPV while evaluating the projects.

As a result of the literature survey, it could not be encountered any studies which considers the individual inflow and outflow components separately during the process of project evaluation, defines the risky components as probability distributions, and determines the expected profitability of projects by considering that the values of different components will display different variation. This has been regarded as a gap in the literature.

As mentioned in Section 1.1, during any time period, most enterprises, especially public enterprises, have to make a ranking and selection among a number of investment project alternatives. Some of these alternatives may be promising and can allow the enterprises and entrepreneurs to realize their objectives. However, the budgets of the enterprises are generally not enough to implement all of these investment proposals which have high expected utility level at the same time. In these cases, the enterprises prefer to implement the investment project proposals at the number allowed by the size of their budgets. On the other hand, the lack of budget is not the only reason for the fact that some of the investment project proposals with high expected utility level are selected and performed and the others are not. The other reasons of this complexity may be some technical limitations such as earliest and latest start dates and precedence relations.

However, in today's high competitive environments, enterprises, especially public enterprises, have to act well-planned. The first step of acting well-planned is to determine a planning horizon and to predict how much budget to allocate for carrying out investment projects each period over that planning horizon. In this way, there will not be the cases in which some of the project proposals, evaluated at the beginning of each period and predicted to have high expected utility level, cannot carried out due to the lack of budget allocated for investments for that period. If some of the project proposals cannot be put into practice owing to lack of budget, these project proposals will have the chance of being carried out at other periods in the planning horizon. The reason for this is that the budget estimate regarding each period in the planning horizon is certain. In this new case, the main objective of the enterprises is to maximize the expected utility of all investment projects which are carried out over the planning horizon.

Also, the review of literature indicates that in the studies related with the evaluation and selection of investment projects, project proposals are evaluated on the basis of certain methods or models over any time period and some of them are selected and put into practice. However, it is often inaccurate to select the projects with highest profitability over any time period and to eliminate other projects which will be able to prove profitable due to lack of budget, other limitations or different reasons.

Instead, especially public enterprises should prefer to determine a planning horizon and the budgets to be allocated for investments for all years within this horizon rather than allocating investment budgets for one single year. In this way, it will be possible to make a decision at which period the project proposals, which cannot be carried out at the beginning of planning horizon owing to lack of budget or any other limitation but are thought to be profitable, will be put into practice within the planning horizon. On the other hand, some projects whose economic evaluation in the current time period is unfavorable may, as a result of expected population and income growth, fare much better at a later date.

The literature survey shows that there is only one study in which projects are evaluated in accordance with the approach described above and scheduled over a planning horizon. This study which was written by Medaglia et al. has been published in 2008. There are some similarities between the problem considered in the third stage of the proposed methodology within the scope of this dissertation and the problem discussed in that study found in literature. However, the models developed in the third stage of proposed methodology have important distinctive characteristics when compared to the models developed in the study found in literature.

First of all, in the study found in literature, NPVs of projects are determined through the traditional method. Another difference is that the study found in literature assumes that the values of some risky constraints to be deterministic. For example, the budget to be allocated for any time period within the planning horizon may be flexible. Assuming this value to be deterministic may result in rejection of a project with high profitability owing to lack of budget. Therefore, it will be necessary to develop new models in the event that values of some constraints are flexed. Within the scope of this dissertation, these kinds of models have also been developed. Another difference is that the developed models include some additional constraints.

When one schedules the projects within a planning horizon, it should be made an attempt to select the projects that have the highest profitability and lower risk as much as possible. However, in the study found in literature, such kind of a phenomenon has not been analyzed. This is a drawback. Another significant drawback is that the study found in literature does not take into account the fact that monetary value of some project parameters may change due to inflationary effects over the course of time, as is the case for calculating the profitability through traditional NPV formulation without taking the inflation effect into consideration. For example, it is a drawback to assume that initial investment cost of a project will remain the same regardless of the year of the planning horizon in which it is put into practice.

It has already been stated that it will be inaccurate to consider only the objective of maximum profitability while determining the projects that will be realized in the planning horizon. In this case, it will be impossible to select the set of projects with a little lower profitability but much less risk when compared to the set of projects with the maximum profitability. For that reason, instead of committing to an objective of selecting the set of projects with maximum profitability, there should be a model for selecting the set of projects with a degree of profitability higher than an acceptable level and with the least risk. It could not be found any such study in the literature and this has been regarded as a deficiency and gap.

2.8 Summary of Chapter

In this chapter, an overview of investment project evaluation and selection has been presented. At the beginning of the chapter, basic concepts and definitions related with investment project evaluation and selection process have been explained, and investment project cycle has been discussed. Then, investment project evaluation and selection methods under certainty have been considered and also investment project evaluation and selection process under risky and uncertain environments have been presented, separately. At the end of the chapter, the survey of the related literature is provided, and gaps in the existing literature and the need for the proposed research have been explained.

CHAPTER THREE
MULTI-CRITERIA DECISION MAKING: MULTI-CRITERIA CLASSIFICATION

Because the investment project evaluation and selection is a multi-criteria decision making (MCDM) problem in nature, the main objective of this chapter is to present the methodological approaches for MCDM, to provide an overview of MCDM methods previously used in investment project evaluation and selection problems, and to explain the new multi-criteria sorting (MCS) procedure, named as PROMSORT, which has not been applied to the investment project evaluation and selection problems. In this dissertation, this new MCS procedure has been adapted to these types of problems. Because, it does not require a training sample, and by using this procedure, the inherent risk and uncertainty associated with the values of evaluation criteria can be handled.

3.1 Introduction

Decision making problems, according to their nature, the policy of the decision maker, and the overall objective of the decision, may require the choice of an alternative solution, the ranking of the alternatives from the best to the worst ones of the assignment of the considered alternatives into predefined homogeneous classes. This last type of decision problem is referred to as *classification* or *sorting* (Doumpos & Zopounidis, 2002).

While both classification and sorting refer to the assignment of a set of alternatives into predefined groups, they differ with respect to the way that the groups are defined. In that sense, classification refers to the case where the groups are defined in a nominal way. On the contrary, sorting (a term which is widely used by MCDM researches) refers to the case where the groups are defined in an ordinal way starting from those including the most preferred alternatives to those including the least preferred alternatives (Zopounidis & Doumpos, 2002). For simplicity reasons, henceforth generally the term "classification" will be used in this dissertation. The distinction will be made between sorting and classification when required.

Classification problems are often encountered in a variety of fields including finance, marketing, environmental and energy management, human resources management, medicine, etc. The major practical interest of the classification problem has motivated researchers in developing an arsenal of methods for studying such problems, in order to develop mathematical models achieving the higher possible accuracy and predicting ability (Doumpos & Zopounidis, 2002). For several decades, many statistical and econometric classification methods, which constitute the traditional approach to develop classification models, have dominated this field.

The most well known statistical and econometric methods include; discriminant analysis, logit analysis, and probit analysis. Discriminant analysis has been the first multivariate statistical classification method. Logit and probit analysis are actually special forms of regression analysis in cases where the dependent variable is discrete. However, they are several shortcomings due to the restrictive statistical assumptions. A significant drawback of all these methods is the exclusion of qualitative criteria such as quality of management, market position etc. (Zopounidis & Doumpos, 1999).

In order to overcome these shortcomings, alternative classification models have been developed by researchers. The recent research in developing classification models is based on operations research and artificial intelligence techniques. Methodologies such as neural networks, machine learning, rough sets, fuzzy sets and MCDM are considered by researchers both at the theoretical and practical levels. The research made at the theoretical level focuses on different aspects of the model development and validation process. At the practical level, researchers focus on the use of classification methodologies to analyze real-world problems and provide decision support, or on the investigation of the performance of different methodologies using real-world data (Zopounidis & Doumpos, 2002).

In this dissertation, a novel methodology for risky investment projects evaluation is proposed. The first stage of the proposed methodology includes opportunity and pre-feasibility studies. The aim of this stage is to identify the investment opportunities, and to carry out preliminary election of project ideas and to give prominence to ideas which have the highest chance of attaining the goals planned by entrepreneurs and investors. Therefore, the project evaluation and selection problem considered in the first stage of the proposed methodology is better addressed through the classification/sorting problematic.

While determining the promising project ideas among other ones according to the goals of entrepreneurs, in most of time, there are multiple evaluation criteria that should be taken into account. In other words, the preliminary project selection is a MCDM problem in nature. It is obvious that when more than one evaluation criterion exists in the problem, decision making becomes more complex.

For these reasons, while all methodologies such as neural networks, machine learning, rough sets, fuzzy sets that are used in developing classification models have advantages and disadvantages, their discussion is out of the scope of this dissertation. Therefore, this chapter focuses on the MCDM methods for developing classification models.

Compared to alternative approaches, MCDM research does not focus solely on developing “automatic” procedures for analyzing an existing data set in order to construct a classification model. MCDM researchers also emphasize on the development of efficient preference modeling methodologies that will enable the decision analyst to incorporate the decision maker’s preferences in the developed classification model (Zopounidis & Doumpos, 2002).

Because the preliminary project selection is a MCDM problem in nature, the first objective of this chapter is to provide an overview of the methodological approaches for MCDM that are used in investment project selection problems. Then, since the project evaluation and selection problem considered in the first stage of the proposed methodology is better addressed through the classification/sorting problematic, the second objective of this chapter is to provide a brief discussion of multi-criteria classification (MCC) problem, to review the existing MCDM methods for classification problems. At the end of this chapter, the new MCS procedure, named as PROMSORT, which has not been applied to the investment project evaluation and selection problems is explained in details. Because, in this dissertation, this new MCS procedure has been adapted to these types of problems.

Although several methodologies have been developed to deal with classification/sorting problems, most of them assume that adequate number of reference alternatives have already been determined and use these reference alternatives as training samples to infer some of the model parameter. However, in the preliminary project selection problem presented in this dissertation, most of time, a set of training sample may not be possible. For these reasons, in the scope of this dissertation, this method will be used in order to assign project alternatives to predefined ordered categories in the first stage of the proposed methodology.

This MCS procedure does not require a training sample, and by using this method, the inherent risk and uncertainty associated with the values of evaluation criteria can be handled. On the other hand, it is more important point that this MCS procedure has not been applied to the investment project evaluation and selection problems, until this dissertation. This thesis and our proposed novel methodology is the first one that uses this MCS procedure, PROMSORT, in order to assign project alternatives to predefined ordered categories.

3.2 Decision Making Problematics

Decision science is a very broad and rapidly evolving research field at theoretical and practical levels. Today, the range of decision making problems has been extended. The nature of these problems is widely diversified in terms of their complexity, the type of solutions that should be investigated, as well as the methodological approaches that can be used to address them. The classification of decision making problems is a difficult task depending upon the scope of the classification. However, Doumpos & Zopounidis (2002) classify decision making problems into the two following categories;

(1) Discrete problems involving the examination of a discrete set of alternatives. Each alternative is described along some attributes. Within the decision making context these attributes have the form of evaluation criteria.

(2) Continuous problems involving cases where the number of possible alternatives is infinite. In such cases one can only outline the region where the alternatives lie (feasible region), so that each point in this region corresponds to a specific alternative. Resource allocation is a representative example of this form of problems.

The investment project evaluation and selection problems are as a type of discrete decision making problems. These problems can be shown as in Table 3.1.

Table 3.1 Discrete decision making problems

		Criteria			
		g_1	g_2	…	g_n
Alternatives	x_1	g_{11}	g_{12}	…	g_{1n}
	x_2	g_{21}	g_{22}	…	g_{2n}
	x_3	g_{31}	g_{32}	…	g_{3n}
	.	.	.	…	.
	.	.	.		.
	.	.	.		.
	x_m	g_{m1}	g_{m2}	…	g_{mn}

Roy (1996) states that when considering a discrete decision making problem, there are four different kinds of analyses (decision making problematics) that can be performed in order to provide significant support to decision makers. These are as follows;

(1) to identify the best alternative or select a limited set of the best alternatives,

(2) to construct a rank-ordering of the alternatives from the best to the worst ones,

(3) to classify/sort the alternatives into predefined homogenous groups,

(4) to identify the major distinguishing features of the alternatives and perform their description based on these features.

Doumpos & Zopounidis (2002) represent these decision making problematics graphically as in Figure 3.1.

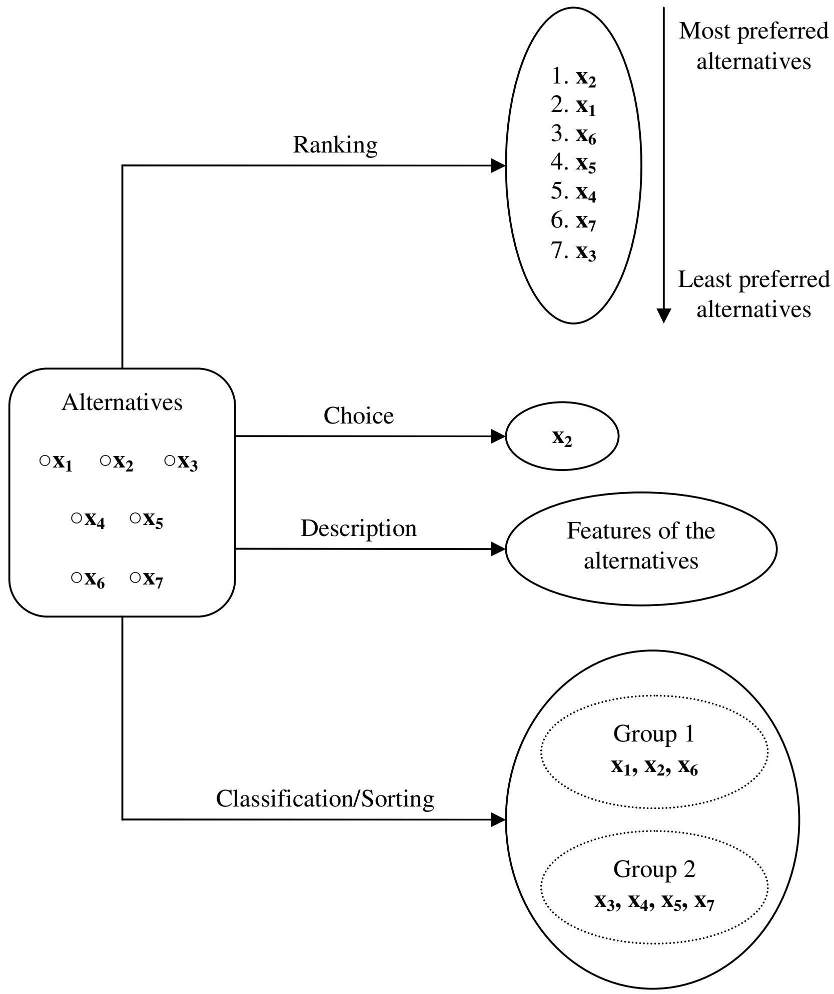


Figure 3.1 Decision making problematics (Doumpos & Zopounidis, 2002; p.3)

Doumpos & Zopounidis (2002) states that the first three forms of decision making problems (choice, ranking, classification) lead to a specific result regarding the evaluation of the alternatives. Both choice and ranking are based on relative judgments, involving pair-wise comparisons between the alternatives. Consequently, the evaluation result depends on the considered set of alternatives. On the other hand, the classification problem is based on absolute judgments. Since the groups are usually specified independently of the alternatives under consideration, the classification of the alternatives requires their comparison to some reference profiles that distinguish the groups.

In the literature, clustering and classification terms are used almost synonymously. However, they have different meaning in the MCDM literature. Therefore, it should be emphasized the difference between them. Clustering is an analytical technique for developing meaningful subgroups (clusters) of alternatives. Specifically, the objective of clustering is to classify alternatives into a small number of mutually exclusive groups based on the similarities among the alternatives. In clustering, unlike classification, the groups are not predefined. Instead, the technique is used to identify the groups (clusters). However, in classification the groups are defined a priori (Hair et al., 1998).

Figure 3.2 outlines the difference between classification and clustering terms in a graphical way.

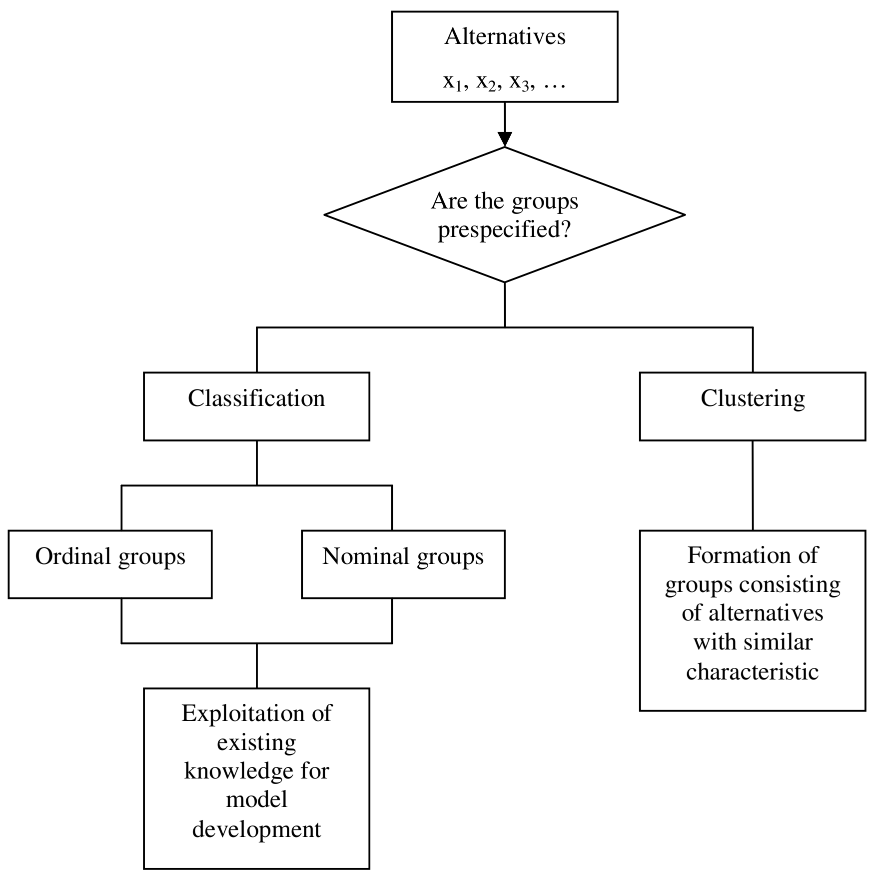


Figure 3.2 The classification versus clustering problem (Doumpos & Zopounidis, 2002; p.5)

If the categories are defined in a nominal way, which means that the categories are not ordered from the best to the worst, the problem is called as nominal classification problem. On the contrary, the categories are defined in an ordinal way in ordinal classification or sorting problems. Within the MCDM context, MCC problems are generally studies as sorting problematic (Araz & Ozkarahan, 2007). As mentioned, in the scope of this dissertation, the project evaluation and selection problem considered in the first stage of the proposed methodology is a type of sorting problems.

Both nominal and ordinal classification problems have numerous practical applications. Doumpos & Zopounidis (2002) state some characteristic examples as follows;

(1) *Medicine:* medical diagnosis to assign the patients into groups (diseases) according to the observed symptoms (Belacel, 2000),

(2) *Pattern recognition:* Human characteristics or physical object are recognized and classified into properly defined groups (Ripley, 1996; Nieddu & Patrizi, 2000),

(3) *Human resources management:* Personnel are assigned into appropriate occupation groups according to their skills (Gochet et al., 1997)

(4) *Production management:* monitoring the operation of complex production systems for fault diagnosis purposes (Catelani & Fort, 2000; Shen et al., 2000);

(5) *Marketing:* marketing policies for penetration to new market are categorized and selected, customers are categorized based on their characteristic, etc. (Siskos et al., 1998);

(6) *Environmental management and energy policy:* analysis and in time diagnosis of environmental impacts, examination of the effectiveness of energy policy measures (Diakoulaki et al., 1999);

(7) *Financial management and economics;* bankruptcy prediction, credit risk assessment, country risk assessment (Zopounidis & Doumpos, 1998; Zopounidis & Doumpos, 1999; Araz et al., 2006)

A number of methods for addressing such classification problems have been developed from a variety of research disciplines, including statistics/econometrics, artificial intelligent, and operations research. Zopounidis & Doumpos (2002) review the research conducted on the framework of the MCDM. The review covers different forms of MCDM classification models, different aspects of the model development process, as well as real-world applications of MCDM classification techniques and their software implementations.

3.3 General Outline of Classification Methods

Lots of classification methods have been proposed in the literature in order to develop classification models. Most of them operate on the basis of a regression philosophy, trying to exploit the knowledge that is provided through the a priori definition of the groups. Doumpos & Zopounidis (2002) present a general outline of the procedure used to develop a classification model as in Figure 3.3. This procedure is common to most of the existing classification methods.

In traditional regression, the objective is to identify the functional relationship between a dependent variable Y and a vector of independent variables X given a sample of existing observations. Most of the existing classification methods address the classification problem in a similar approach. The only actual difference between the statistical regression and the classification problem is that the dependent variable is not a real valued variable, but a discrete one.

Henceforth, the dependent variable that determines the classification of the alternatives is denoted by C, while its discrete levels (groups) will be denoted by C_1, C_2, …, C_q, where q is the number of groups. Similarly, g is used to denote the vector of independent variables, i.e., g=(g_1, g_2, …, g_n). The independent variables are referred to as criteria or attributes. Both terms are quite similar. However, an attribute defines a nominal description of the alternatives, whereas a criterion defines an ordinal description (Doumpos & Zopounidis, 2002).

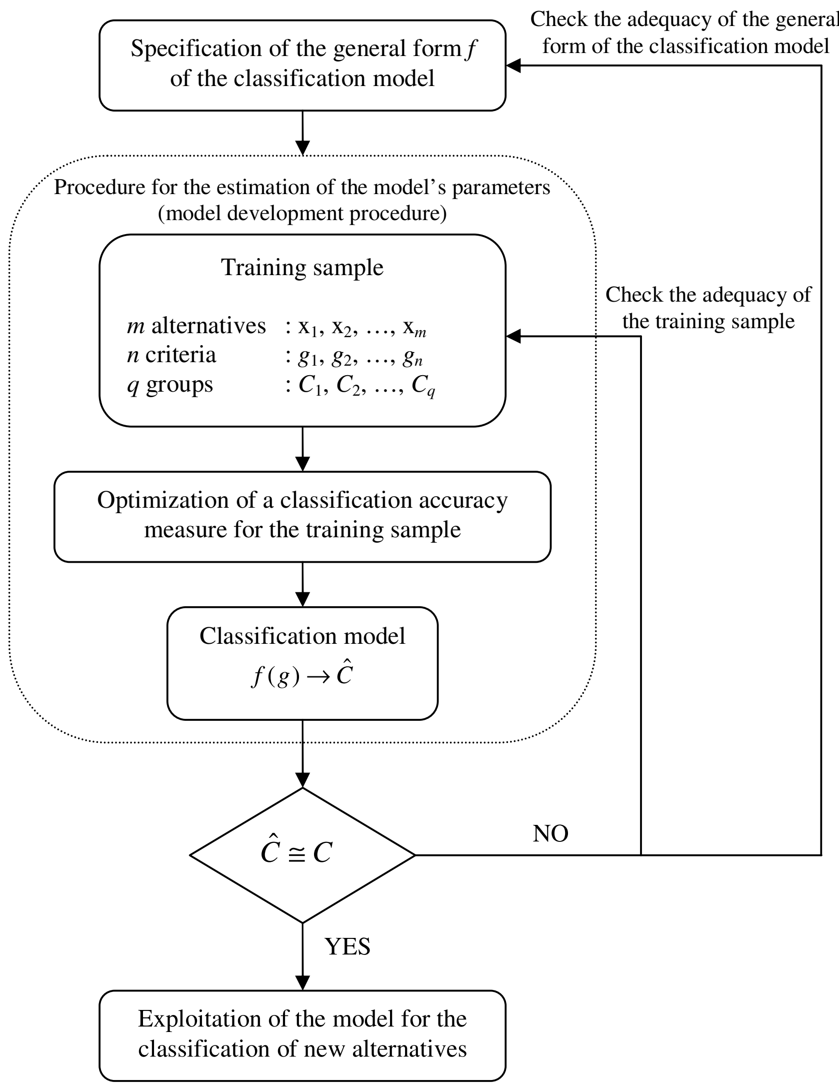


Figure 3.3 General outline of the model development process for classification problems (Doumpos & Zopounidis, 2002; p.7)

As seen in Figure 3.3, the sample of observations is needed in order to construct the classification model. In the MCDM literature, it is called as the training sample. The observations in the training sample are referred to as alternatives. According to the figure, there are *m* alternatives in the training sample. Each alternative is considered as a vector consisting of the performance of the alternative on each criterion.

On the basis of these notations, addressing the classification problem involves the development of a model of the form $f(g) \rightarrow \hat{C}$, which can be used to determine the classification of the alternatives given their characteristics described using the criteria vector g. the development of such a model is performed so that a predefined measure of the differences between the a priori classification C and the estimated classification $\hat{C}$ is minimized. If the developed model performs satisfactorily in the training sample, it can be used to decide upon the classification of any new alternative that becomes under consideration. This is the major point of interest in implementing the above process: to be able to organize the knowledge embodied in the training sample so that it can be used for real-time decision making purposes (Doumpos & Zopounidis, 2002).

As a summary, several methodologies have been proposed for classification models development, and most of them require adequate number of reference alternatives named as training sample. In other words, in order to use the developed classification model for real-time decision making purposes, at first, it performs satisfactorily in the training sample. However, in the preliminary project selection problem presented in this dissertation, most of time, a set of training sample may not be possible. Therefore, in the scope of this dissertation, in order to assign project alternatives to predefined ordered classes in the first stage of the proposed methodology, the MCS procedure, PROMSORT, that does not require a training sample has been used. As mentioned before, besides it does not require a training sample, also, it takes into consideration the inherent risk and uncertainty associated with the values of evaluation criteria; and this thesis and our proposed novel methodology is the first one that uses this MCS procedure, in order to assign project alternatives to predefined ordered classes.

3.4 Methodological Approaches for Multi-Criteria Decision Making

Among the different methodologies approaches proposed for addressing classification problems, MCDM is an advanced field of operations research providing several advantages from the research and practical points of view. MCDM provides lots of methodologies for addressing decision making problems. Many of these approaches are well-suited to the nature of the classification problem. The major characteristic shared by all MCDM classification approaches is their focus on the modeling and addressing of sorting problems (Doumpos & Zopounidis, 2002).

In the literature, methodological approaches for MCDM are categorized by different ways. For example, according to the features of the developed models, Roy (1996) identifies three major methodological streams;

(1) Single synthesis criterion approaches, where incomparability is excluded,
(2) Outranking synthesis approaches, where incomparability is accepted,
(3) Interactive local judgment approaches with trial and error iterations.

Pardalos et al. (1995) did not consider only the features of the developed models; they also considered the features of the model development process. They categorized the methodological approaches for MCDM as follows;

(1) Multi-objective Mathematical Programming,
(2) Multi-attribute Utility Theory,
(3) Outranking Relation Theory,
(4) Preference Disaggregation Analysis.

As known, multi-attribute utility theory, outranking relation theory, and preference disaggregation analysis are traditionally used in discrete problem. However, multi-objective mathematical programming is mostly used in continuous problems.

The following sections briefly outline the main concepts and features of these MCDM approaches.

3.4.1 Multi-objective Mathematical Programming

Multi-objective mathematical programming (MOMP) is the theoretical and methodological framework for dealing with mathematical programs with multiple objective functions. If the mathematical Programming (MP) problems contains only one objective function, the output obtained after solving these problems is the optimal solution and all the relevant information about the values of the decision variables.

However, in MOMP, there are more than one objective functions and there is no single optimal solution that simultaneously optimizes all the objective functions. In these cases the decision makers are looking for the "most preferred" solution. In MOMP the concept of optimality is replaced with that of efficiency or Pareto optimality. The efficient (or Pareto optimal, non-dominated) solutions are the solutions that cannot be improved in one objective function without deteriorating their performance in at least one of the rest (Mavrotas, 2008).

The general formulation of a MOMP problem is as follows;

$$\begin{aligned} &Max/Min \quad \{f_1(x), f_2(x), ..., f_n(x)\} \\ &Subject\ to \qquad x \in B \end{aligned} \tag{3.1}$$

where x is the vector of the decision variables; f_1, f_2, …, f_n are the objective functions to be optimized. They may be linear or non-linear, and B is the set of feasible solutions.

Mavrotas (2008) defines the efficient solution mathematically without loss of generality assumes that all the objective functions are for maximization as follows; A feasible solution x of a MOMP problem is efficient if there is no other feasible solution x' such as $f_i(x') \geq f_i(x)$ for every $i = 1,2,...,n$ with at least one strict inequality. Every efficient solution corresponds to a non-dominated or non-improvable vector in the criterion space. If the condition $f_i(x') \geq f_i(x)$ is replaced with $f_i(x') > f_i(x)$, the weakly efficient solutions are obtained. Weakly efficient solutions are not usually pursued in MOMP because they may be dominated by other efficient solutions.

As mentioned, in MOMP framework, the objective functions are of conflicting nature and it is not possible to find a solution that optimizes simultaneously all the objective functions. Therefore, the rational decision maker is looking for the most preferred solution among the efficient solutions of the MOMP. In the absence of any other information, none of these solutions can be said to be better than the other. Usually a decision maker is needed to provide additional preference information and to identify the "most preferred" solution.

Hwang & Masud (1979) categorize the methods for solving MOMP problems into three categories according to the phase in which the decision maker involves in the decision making process expressing his/her preferences. These categories are as follows;

(1) The priori methods,
(2) The interactive methods,
(3) The generation or posteriori methods.

In a priori methods the decision maker expresses his/her preferences before the solution process (e.g. setting goals or weights for the objective functions). The criticism about the a priori methods is that it is very difficult for the decision maker to know beforehand and to be able to accurately quantify (either by means of goals or weights) his/her preferences.

In the interactive methods phases of dialogue with the decision maker are interchanged with phases of calculation and the process usually converges after a few iterations to the most preferred solution. The decision maker progressively drives the search with his answers towards the most preferred solution. The drawback is that he never sees the whole picture (the set of efficient solutions) or an approximation of it.

Hence, the most preferred solution is “most preferred” in relation to what he/she has seen and compare so far.

In a posteriori methods (or generation methods) the efficient solutions of the problem (all of them or a sufficient representation) are generated and then the decision maker involves, in order to select among them, the most preferred one (Mavrotas, 2008).

In Equation 3.1, the objective functions to be optimized and also the constraints may be linear or non-linear. If the objective functions and constraints are formulated linearly, them MOMP model becomes a multi-objective linear programming (MOLP) model. In other words, the problem to optimize multiple conflicting linear objective functions simultaneously under the given linear constraints is called the MOLP problem. In the literature, several methodologies have been proposed for addressing MOMP, especially MOLP, problems. Because, most of the MOMP models in the literature are formulated as a MOLP model.

Goal Programming (GP) approach founded by Charnes & Cooper (1961) is perhaps the oldest methodology in the field of MOMP. The concept of goal is different from that of objective. An objective simply defines a search direction (e.g. profit maximization). On the other hand, a goal defines a target against which the attained solutions are compared (Keeney & Raiffa, 1993).

The procedure to formulate a GP model starts with specifying a target value for each objective in order to transform all objectives into goals. In GP, unwanted deviations from this set of target values are then minimized in the resultant objective function, termed the achievement function in GP. In other words, an objective function of an MOMP formulation is transformed into a constraint within the context of a GP formulation. The right hand side of these constraints includes the target values of the goals, which can be defined either as some satisfactory values of the goals or as their optimal values (Doumpos & Zopounidis, 2002).

Despite its advantages, GP has a few drawbacks. A major one is that decision makers must specify the goals and their priorities a priori. Another drawback of GP is the lack of a systematic approach to set priorities and trade-offs among objectives. Although, it has a few drawbacks, GP is the one of the most powerful and well known MOMP solution methodology, and several variants of GP have been proposed to address MOMP problems. A survey of GP methods and applications can be seen in Romero (1986), Tamiz et al. (1998), and Aouni & Kettani (2001).

3.4.2 Multi-attribute Utility Theory

Multi-attribute utility theory (MAUT) extends the traditional utility theory to the multidimensional case. Utility theory is a branch of decision analysis that involves the building of mathematical models to describe the behavior of a decision maker when faced with making a choice among alternatives in the presence of risk. The basic assumption of utility theory is that people make decisions with the objective of maximizing their expected utility (Badiru & Pulat, 1995).

The objective of MAUT is to model and represent the decision maker's preferential system into a utility function *U(g),* where *g* is the vector of the evaluation criteria $g = (g_1, g_2, ..., g_n)$. Generally, the utility function is a non-linear function defined on the criteria space, such that (Doumpos & Zopounidis, 2002);

$$\begin{aligned} U(g_x) > U(g_{x'}) \Leftrightarrow x \succ x' \quad & \text{(alternative } x \text{ is preferred to } x'\text{)} \\ U(g_x) = U(g_{x'}) \Leftrightarrow x = x' \quad & \text{(alternative } x \text{ is indifferent to } x'\text{)} \end{aligned} \tag{3.2}$$

The key point in using utility theory and also MAUT is the proper choice of utility models. There are two simple but widely used utility models in the literature: the *additive utility model* and the *multiplicative utility model.*

The additive model is the most commonly used form of utility function. The additive utility of a combination of outcomes for *n* evaluation criteria is expressed as follows (Doumpos & Zopounidis, 2002);

$$U(g) = w_1 u_1(g_1) + w_2 u_2(g_2) + \ldots + w_n u_n(g_n) \quad (3.3)$$

In this model, $u_1, u_2, \ldots, u_n$ are the marginal utility functions corresponding the evaluation criteria. Each marginal utility function $u_i(g_i)$ defines the utility of the alternatives for each individual criterion g_i. $w_1, w_2, \ldots, w_n$ are constants representing the trade-off that the decision maker is willing to take on a criterion in order to gain one unit on criterion g_i. These constants are often considered to represent the weights of the criteria and they are defined such that they sum-up to one;

$$\sum_{i=1}^{n} w_i = 1 \quad (3.4)$$

Doumpos & Zopounidis (2002) states that the form of the utility function is quite similar to simple weighted aggregation models. Actually, such models are a special form of an additive utility function, where all marginal utilities are defined as linear functions on the criteria's values.

For interested reader, a detailed description about MAUT and its applications is presented in the book of Keeney & Raiffa (1993), Badiru & Pulat (1995) and Shtub et al. (2005).

3.4.3 Outranking Relation Theory

One of the most widely used criteria aggregation model in the MCDM context is the outranking relation theory (ORT). The outranking relation is defined as a binary relation that enables the assessment of the outranking degree of an alternative *a* over an alternative *b*. The outranking relation allows to conclude that *a* outranks *b* if there are enough arguments to confirm that *a* is at least as good as *b*, while there is no essential reason to refute this statement.

The ORT techniques compare all couples of alternatives. Instead of building complex utility functions, they determine which alternatives are being preferred to the others by systematically comparing them on each criterion.

All ORT techniques operate in two major stages. The first stage involves the development of an outranking relation, whereas the second stage involves the exploitation of the outranking relation in order to perform the evaluation of the alternatives for choice, ranking, classification/sorting purposes (Doumpos & Zopounidis, 2002).

Doumpos & Zopounidis (2002) states that there are two main differences between MAUT framework and ORT;

(1) Although the evaluations obtained through the development of utility function in MAUT are transitive, the outranking relation is not transitive.

(2) In the MAUT framework only the preference and indifference relations are considered. In addition to these two relations, ORT introduces the incomparability relation.

The widely used ORT techniques are the ELECTRE (**EL**imination **E**t **C**hoix **T**raduisant la **RE**alité – Elimination and Choice Expressing the Reality) family of methods and PROMETHEE (**P**reference **R**anking **O**rganization **METH**od for **E**nrichment **E**valuations) family of methods. There are several variants of these two families of methods that are suitable for addressing choice, ranking, and classification/sorting problems. The methods and their variants will be described in the following sub sections.

3.4.3.1 ELECTRE Family of Methods

The family of ELECTRE methods initially introduced by Roy (1968) is the most extensively used ORT techniques. Several versions of ELECTRE methods exist in the literature. Roy (1968) presented ELECTRE I which is based on a relational representation of the decision maker's preferences. ELECTRE I aims to choose the set of the best alternatives. The methods ELECTRE II, III and IV were developed and presented respectively by Roy & Bertier (1973), Roy (1978), Roy & Hugonnard

(1982). These methods aim to rank the alternatives from the best to the worst. ELECTRE III and ELECTRE IV methods take into account indifference and preference thresholds. All the above methods are using the idea of using weights to determine the relative importance of criteria, except from ELECTRE IV which makes the assumption that there is no importance relation on the criteria, as no criterion is unimportant in relation to any other. At last, Yu (1992) presented ELECTRE TRI, which is based on ELECTRE III and specially devoted to the classification/sorting problems.

As mentioned before, in this dissertation, a novel methodology for risky investment projects evaluation is proposed, and the first stage of the proposed methodology includes opportunity and pre-feasibility studies. The aim of this stage is to identify the investment opportunities, and to carry out preliminary election of project ideas and to give prominence to ideas which have the highest chance of attaining the goals planned by entrepreneurs and investors. Therefore, the project evaluation and selection problem considered in the first stage of the proposed methodology is better addressed through the classification/sorting problematic. For that reason, ELECTRE TRI which is specially devoted to the classification/sorting problems will be explained more detailed in the later sections. In this section, ELECTRE III, which is the base of ELECTRE TRI, will be briefly explained. For more information about ELECTRE methods and applications, we refer the interested reader Figueira et al. (2004), Rogers & Bruen (2000), and Karagiannidis & Moussiopoulos (1997).

There are two important concepts in ELECTRE methods. These are thresholds and outranking. Assume that F represents a set of criteria, g_j, j=1,2,…,r and A represents a set of potential alternatives. Such alternatives are not necessarily exclusive, i.e., they can be put into operation jointly. $g_j(a)$ is called the jth performance of alternative a. If we consider only the point of view reflected by the jth criterion, traditional preference modeling assumes the following three relations hold for two alternatives $(a,b) \in A$;

aPb	(a is preferred to b)	$g_j(a) > g_j(b)$	
aIb	(a is indifferent to b)	$g_j(a) = g_j(b)$	(3.5)
aJb	(a cannot be compared to b)		

For example, let us assume that there are two cups of tea; one has 12 mg of sugar and the other has 13 mg of sugar. Traditional preference modeling says that because the amount of sugar is not equal, then one will be preferred over the other.

In contrast to the traditional approach, ELECTRE introduces the concept of an *indifference threshold* associated with the jth criterion, q_j, and the preference relationships with respect to the jth criterion are redefined as follows;

aPb	(a is preferred to b)	$g_j(a) > g_j(b) + q_j$	
aIb	(a is indifferent to b)	$\lvert g_j(a) - g_j(b) \rvert \le q_j$	(3.6)
aJb	(a cannot be compared to b) remains		

The indifference threshold for criterion j, q_j, is the largest difference of performances significant for indifference and it is specified by the decision maker. In other words, the indifference threshold can be defined either with respect to the uncertainty of the criteria values or as a threshold at which the differences become perceptible to decision maker (Rogers & Bruen, 1998). Maystre et al. (1994) defined the indifference threshold as the minimum margin of uncertainty.

While the introduction of this threshold goes some way toward incorporating how a decision maker actually does feel about realistic comparisons, a problem remains. There is a point at which a decision maker changes from indifference to strict preference. Conceptually, there is a good reason to introduce a buffer zone between indifference and strict preference, an intermediary zone where a decision maker hesitates between preference and indifference. This zone of hesitation is referred to as a *weak preference*; it is also a binary relation like P and I above, and is modeled by introducing a *preference thresholds*, p_j. The preference threshold represents the largest difference of performances not significant for a strict preference. Maystre et al. (1994) defined the preference threshold as the maximum margin of uncertainty with respect to different criteria. Therefore, the preference threshold implies that there is no doubt that a certain alternative is better than the other.

Thus, we have a double threshold model, with an additional binary relation *Q* that measures weak preference. That is (Buchanan & Sheppard, 1998);

$$\begin{array}{lll} aPb & (a \text{ is strongly preferred to } b) & g_j(a) - g_j(b) > p_j \\ aQb & (a \text{ is weakly preferred to } b) & q_j < g_j(a) - g_j(b) \leq p_j \\ aIb & (a \text{ is indifferent to } b) & \left| g_j(a) - g_j(b) \right| \leq q_j \end{array} \tag{3.7}$$

The choice of thresholds intimately affects whether a particular binary relationship holds. The choice of appropriate thresholds is not easy.

Using thresholds, the ELECTRE method seeks to build an outranking relation *S*. *aSb* means that according to the global model of DM preferences, there are good reasons to consider that "*a* is at least as good as *b*" or "*a* is not worse than *b*." It should be noted that these binary relationships are applied to each of the *r* criteria; that is; aS_jb means that “"*a* is at least as good as *b* with respect to the *j*th criterion". In order to develop this outranking relationship, two further definitions are required: that of concordance and discordance (Buchanan & Sheppard, 1998).

By definition, the *j*th criterion is in concordance with the assertion *aSb* if and only if aS_jb. In other words, the *j*th criterion is in concordance with the assertion *aSb* if and only if $g_j(a) \geq g_j(b) - q_j$. Thus, even if $g_j(a)$ is less than $g_j(b)$ by an amount up to q_j, it does not contravene the assertion aS_jb and therefore is in concordance.

The *j*th criterion is in discordance with the assertion *aSb* if and only if bP_ja. In other words, the *j*th criterion is in discordance with the assertion *aSb* if and only if $g_j(b) \geq g_j(a) + p_j$. That is, if *b* is strictly preferred to *a* for criterion *j*, then it is clearly not in concordance with the assertion that *aSb*.

These two concepts of concordance and discordance can be thought of as "harmony" and "disharmony". For each criterion *j* we are looking to see whether, for every pair of alternatives (*a,b*), there is harmony or disharmony with the assertion *aSb*; that is, *a* is at least as good as *b* (Buchanan & Sheppard, 1998).

With concordance and discordance concepts, it is possible to obtain a measure of the strength of the assertion *aSb*. This measure is called as concordance index, $C(a,b)$. By definition, the concordance index $C(a,b)$ characterizes the strength of the positive arguments able to validate the assertion *aSb*. Therefore, for a given pair of alternatives $(a,b) \in A$, the concordance index $C(a,b)$, regarding the whole set of criteria, can be defined as follows;

$$C(a,b) = \frac{\sum_{j=1}^{r} k_j c_j(a,b)}{\sum_{j=1}^{r} k_j} \tag{3.8}$$

where k_j is the importance coefficient or weight of criterion *j*, and the concordance degree $c_j(a,b)$ states the degree of the claim that alternative *a* is at least as good as alternative *b* in terms of criterion *j*. $c_j(a,b)$ is also called as the local concordance index, and it can be calculated as follows;

$$c_j(a,b) = \begin{cases} 0 & \text{if} \quad g_j(b) - g_j(a) \geq p_j \\ 1 & \text{if} \quad g_j(b) - g_j(a) \leq q_j \\ \dfrac{p_j - [g_j(b) - g_j(a)]}{p_j - q_j} & \text{otherwise} \end{cases} \tag{3.9}$$

At this point, in order to produce a final ranking of alternatives from the pairwise outranking information, the discordance index should be calculated. In order to calculate discordance, a further threshold called the *veto threshold* is defined. The veto threshold, v_j, allows for the possibility of *aSb* to be refused totally if, for any one criterion *j*, $g_j(b) > g_j(a) + v_j$. The discordance index for each criterion *j*, $d_j(a,b)$ is calculated a follows;

$$d_j(a,b) = \begin{cases} 0 & if \quad g_j(b) - g_j(a) \le p_j \\ 1 & if \quad g_j(b) - g_j(a) > v_j \\ \dfrac{g_j(b) - g_j(a) - p_j}{v_j - p_j} & otherwise \end{cases} \qquad (3.10)$$

The discordance indices of different criteria are not aggregated using the weights, since one discordant criterion is sufficient to discard outranking.

After calculating a concordance and discordance measure for each pair of alternatives $(a,b) \in A$, the last step for producing a final ranking is to combine these two measures to produce a measure of the degree of outranking. This measure assesses the strength of the assertion that "*a* is at least as good as *b*". The degree of outranking for each pair $(a,b) \in A$ is defined by $S(a,b)$ and can be calculated as follows;

$$S(a,b) = \begin{cases} C(a,b) & if \quad d_j(a,b) \le C(a,b) \quad \forall j \in F \\ C(a,b) * \prod_{j \in J(a,b)} \dfrac{1 - d_j(a,b)}{1 - C(a,b)} & otherwise \end{cases} \qquad (3.11)$$

where $J(a,b)$ is the set of criteria such that $d_j(a,b) > C(a,b)$ (Miettinen & Salminen, 1999).

According to Equation 3.11, if the strength of the concordance exceeds that of the discordance, then the value of concordance should not be modified. On the other hand, if the discordance is 1.0 for any $(a,b) \in A$ and any criterion *j*, then we have no confidence that *aSb*; therefore, $S(a,b) = 0.0$.

The ranking algorithm of ELECTRE III uses the degree of outranking, and the ranking of the alternatives is normally carried out by a distillation procedure, where the alternatives are ranked based on their qualification from the best to the worst (descending distillation) and from the worst to the best (ascending distillation). The final partial order of the alternatives is built based on these two complete orders. Another way of producing a ranking based on the matrix of the outranking degrees is to use a *minimum-procedure*. The alternatives are ranked based on the minimum outranking degree $S(a,b)$ of each alternative *a*. The alternative having the highest minimum is ranked the first, and so on (Pirlot, 1995; Miettinen & Salminen, 1999).

3.4.3.2 PROMETHEE Family of Methods

The PROMETHEE method is one of the most recent MCDM methods was developed by Brans (1982) and future extended by Brans & Vincke (1985). It is an outranking method for a finite set of alternative actions that are to be ranked and selected among criteria are often conflicting. It is also a quite simple ranking method in conception and application compared with the other methods for multi-criteria analysis (Brans et al. 1986).

The PROMETHEE family of outranking methods including the PROMETHEE I for partial ranking of the alternatives and the PROMETHEE II for complete ranking of the alternatives were developed by J.P. Brans in 1982. A few years later, several versions of the PROMETHEE methods such as the PROMETHEE III for ranking based on interval, the PROMETHEE IV for complete or partial ranking of the alternatives when the set of viable solutions is continuous, the PROMETHEE V for problems with segmentation constraints (Brans & Mareschal, 1992), the PROMETHEE VI for the human brain representation (Brans & Mareschal, 1995), the PROMETHEE GDSS for group decision making based on the PROMETHEE II method, (Macharis et al. 1998) and the visual interactive module GAIA (Geometrical Analysis for Interactive Aid) for graphical representation (Mareschal & Brans, 1988, Brans & Mareschal, 1994) were developed to help more complicated decision-making situations (Brans & Mareschal, 2005).

Recently, Figueira et al. (2004) proposed two extended approaches on PROMETHEE, called as the PROMETHEE TRI for dealing with sorting problems and the PROMETHEE CLUSTER for nominal classification. Since the project evaluation and selection problem considered in the first stage of the proposed methodology is better addressed through the classification/sorting problematic. PROMETHEE TRI which is devoted to the sorting problems will be explained more detailed in the later sections.

Let A be a set of alternatives and $g_j(a)$ represent the value of criterion g_j $(j = 1,2,...,r)$ of alternative $a \in A$. As the first step in PROMETHEE, for each pair of actions, a preference function $F_j(a,b)$ that represents preference level of a to b on criterion j can be defined as follows (Araz, 2007);

$$\begin{array}{lll} F_j(a,b) = 0 & \textit{iff} & g_j(a) - g_j(b) \le q_j \\ F_j(a,b) = 1 & \textit{iff} & g_j(a) - g_j(b) \ge p_j \\ 0 < F_j(a,b) < 1 & \textit{iff} & q_j < g_j(a) - g_j(b) < p_j \end{array} \qquad (3.12)$$

where q_j and p_j are indifference and preference thresholds for jth criterion. For each criterion, the preference function, $F_j(a,b)$, translates the difference between the evaluations obtained by two alternatives into a preference degree ranging from 0 to 1. In order to facilitate the selection of a specific preference function, Brans & Vincke (1985) proposed six basic types, namely: (1) Usual criterion, (2) U-shape criterion, (3) V-shape criterion, (4) Level criterion, (5) V-shape with indifference criterion and (6) Gaussian criterion. These six types are particularly easy to define. For each criterion the value of an indifference threshold q, the value of a strict preference threshold p, or the value of s an intermediate value between p and q has to be fixed. s is only used with the Gaussian criterion (Brans & Mareschal, 1992). In each case these parameters have a clear significance for the decision maker.

If a is better than b according to jth criterion, $F_j(a,b) > 0$, otherwise $F_j(a,b) = 0$. After specifying a preference function, using the weights w_j assigned to each criterion (where $\sum w_j = 1$), the outranking degree $\Pi(a,b)$ for each pair of alternatives (a,b) is computed as follows;

$$\Pi(a,b) = \sum_{j=1}^{r} w_j F_j(a,b) \tag{3.13}$$

If the number of alternatives is more than two, overall ranking is done by aggregating the measures of pairwise comparisons. For each alternative $a \in A$, the following two outranking dominance flows can be obtained with respect to all the other alternatives $x \in A$ (Araz et al., 2007);

$$\phi^{+}(a) = \frac{1}{n-1} \sum_{x \in A} \Pi(a,x) \quad \textit{leaving flow} \tag{3.14}$$

$$\phi^{-}(a) = \frac{1}{n-1} \sum_{x \in A} \Pi(x,a) \quad \textit{entering flow} \tag{3.15}$$

The leaving flow is the sum of the values of the arcs leaving node *a* and therefore provide a measure of the outranking character of *a*. The larger $\phi^{+}(a)$, the more alternative *a* dominates the other alternatives of *A*. The entering flow measures the outranked character. The smaller $\phi^{-}(a)$, the better alternative *a*.

In PROMETHEE I, alternative *a* is preferred to alternative *b*, *aPb*, if the leaving flow of alternative *a* is greater than the leaving flow of alternative *b* and entering flow of alternative *a* is smaller than the entering flow of alternative *b*. This situation can be shown as follows;

$$aPb \quad if: \quad \phi^{+}(a) \geq \phi^{+}(b) \quad and \quad \phi^{-}(a) \leq \phi^{-}(b) \tag{3.16}$$

According to PROMETHEE I, alternative *a* is indifferent to alternative *b*, *aIb,* if two alternatives *a* and *b* have the same leaving and entering flows. This situation can be shown as follows;

$$aIb \quad if: \quad \phi^{+}(a) = \phi^{+}(b) \quad and \quad \phi^{-}(a) = \phi^{-}(b) \tag{3.17}$$

In the case where the leaving flows indicate alternative *a* is better than alternative *b*, while the entering flows indicate the reverse the two alternatives are considered incomparable. In other words, according to PROMETHEE I, two alternatives are considered incomparable, *aRb*, if alternative *a* is better than alternative *b* in terms of leaving flows, while the entering flows indicate the reverse. Therefore, alternative *a* and *b* are incomparable if;

$$\begin{aligned} aRb \quad if: \quad & \phi^{+}(a) > \phi^{+}(b) \quad and \quad \phi^{-}(a) > \phi^{-}(b) \quad or \\ & \phi^{+}(a) < \phi^{+}(b) \quad and \quad \phi^{-}(a) < \phi^{-}(b) \end{aligned} \tag{3.18}$$

In PROMETHEE I, the partial preorder is obtained from the two rankings given by ϕ^{+} and ϕ^{-}. With this, certain alternatives may remain incomparable. In PROMETHEE II, the complete ranking can be obtained by using the net flows. The net flow of alternative *a* is determined as follows;

$$\phi(a) = \phi^{+}(a) - \phi^{-}(a) \tag{3.19}$$

Therefore, in PROMETHEE II, alternative *a* outranks alternative *b* if its net flow is greater than that of *b*; the consideration of ϕ leads to complete ranking. In this case, all the alternatives are comparable and there is no incomparability. This situation can be shown as follows;

$$\begin{aligned} aPb \quad if \quad & \phi(a) > \phi(b) \\ aIb \quad if \quad & \phi(a) = \phi(b) \end{aligned} \tag{3.20}$$

For each alternative *a* it can also be determined the net flow for each criterion separately. Therefore, the single criterion net flow for criterion g_j is calculated as follows (Mareschal & Brans, 1988);

$$\phi_j(a) = \frac{1}{n-1} \sum_{x \in A} \left(F_j(a,x) - F_j(x,a) \right) \tag{3.21}$$

$\phi_j(a)$ measures the strength of alternative *a* over all the other alternatives on criterion *j*. The larger the single criterion net flows $\phi_j(a)$, the better alternative *a* on criterion g_j (Figueira et al., 2004).

3.4.4 Preference Disaggregation Analysis

The implementation of several MCDM methods requires the decision maker to explicitly define a considerable amount of specific preferential information, such as the relative importance of the criteria, preference, indifference thresholds, etc. Obtaining such information from the decision maker is not an easy task. For example, both MAUT and ORT support the decision maker in aggregating different evaluation criteria on the basis of a pre-specified modeling form (utility function or outranking relation). This is a forward process performed on the basis of the direct interrogation of the decision maker. Here, the decision maker specifies all the model parameters.

Preference disaggregation analysis (PDA) has been extensively used over the past two decades to resolve this difficulty. Instead of asking the decision maker to provide details on his/her preferential system; PDA employs a regression-like process to infer the required information through the analysis of the decision maker's judgments on some reference alternatives. In other words, PDA does not require the decision maker to provide specific information on how the decisions are taken; it rather asks the decision maker to express his/her actual decisions. The decision maker provides a global evaluation of these alternatives, usually expressed either by ranking them from the most preferred to the last preferred ones, or by assigning them to preference classes. Such a global evaluation is implicitly based on the preferential system of the decision maker. Thus, the identification of the criteria aggregation model that best fits the decision maker's global judgments on the reference alternatives should be equivalent to the direct specification of detailed preferential information by the decision maker (Doumpos & Zopounidis, 2007, 61-62).

In their paper, Doumpos & Zopounidis (2007) states that this PDA process is formulated as an optimization problem. Given the general form of a criteria aggregation model $f(A)$ defined by a set of parameters A, the objective is to identify the optimal values for the parameters in A that minimize the observed deviations between the model's outputs ad the decision maker's global evaluation of the reference alternatives. In an outranking relation model, *A* may involve the criteria weights, preference, indifference and veto thresholds, etc. (Roy, 1991).

As a consequence, it can be said that the focus in PDA is the development of a general methodological framework, which can be used to analyze the actual decisions taken by the decision maker so that an appropriate model can be constructed representing the decision maker system of preferences, as consistently as possible (Doumpos & Zopounidis, 2002).

In the literature, several PDA techniques have been proposed for different kinds of problem formulations. However, in this dissertation, the new MCS procedure that requires the direct interrogation of the decision maker will be used in order to assign project alternatives to predefined ordered categories in the first stage of the proposed methodology. For that reason, PDA paradigm is beyond the scope of the research proposed in this thesis. A more detailed explanation about the PDA paradigm and an extensive review of existing PDA techniques can be found in Doumpos & Zopounidis (2002), and Lagrèze & Siskos (2001).

Since the project evaluation and selection problem considered in the first stage of the proposed methodology is better addressed through the classification/sorting problematic, in Section 3.5.2, the most known MCC methods that uses PDA paradigm in the model development will be briefly explained.

3.5 Multi-Criteria Decision Making Methods for Classification Problems

After defining the context of MCDM and methodological approaches for MCDM, this section focuses on the review of the most characteristic MCDM methods proposed for addressing classification problems. A complete survey of all methods that have been proposed for MCC can be found in the work of Doumpos & Zopounidis (2002). They classified the MCC methods into two categories;

(1) Methods Based on the Direct Interrogation of the Decision Maker,
(2) Preference Disaggregation Classification Methods.

In the following sections, most known MCC methods for each category will be briefly explained.

3.5.1 Methods Based on the Direct Interrogation of the Decision Maker

The methods classified into this category require the decision maker to define specific information on the parameters of the developed model. The required information includes technical and non-technical parameters such as the weights of the evaluation criteria, preference, indifference and veto thresholds, etc. The direct specification of these parameters by the decision maker ensures that the developed sorting model fits his/her judgment policy.

The most known MCC methods that require the direct interrogation of the decision maker are the analytic hierarchy process, the ELECTRE TRI and the PROMETHEE TRI. Besides these methods, various MCC methods that require the direct interrogation of the decision maker have also been proposed in the literature such as N-TOMIC and PROAFTN. The following sections discuss the model development aspects of these methods.

3.5.1.1 The Analytic Hierarchy Process (AHP) Method

The analytic hierarchy process (AHP) developed by Thomas L. Saaty (1980) is designed to solve complex multi-criteria decision problems. AHP requires the decision maker to provide judgments about the relative importance of each criterion and then specify a preference for each decision alternative using each criterion. The output of AHP is a prioritized ranking of the decision alternatives based on the overall preferences expressed by the decision maker (Anderson et al., 2005).

The first step of AHP is to develop a graphical representation of the problem in terms of the overall objective, the criteria and sub-criteria to be used, and the decision alternatives. Such a graph depicts the hierarchy for the problem. Figure 3.4 presents a simple example of a decision hierarchy for decision alternatives.

In an AHP hierarchy, the top level reflects the overall objective of the decision problem. The evaluation criteria on which the final objective is dependent are listed at intermediate levels in the hierarchy. The lowest level in the hierarchy contains the competing alternatives through which the final objective might be achieved (Badiru & Pulat, 1995). A simple AHP model seen in Figure 3.4 has three levels (goal, criteria and alternatives). Four evaluation criteria are represented as *C1, C2, C3* and *C4*, three alternatives are represented as *A1, A2* and *A3*. Though the simple model with three levels shown in Figure 3.4 is the most common AHP model, more complex models containing more than three levels are also used in the literature. For example, criteria can be divided further into sub-criteria and these sub-criteria can be divided into sub-sub-criteria (Ramanathan, 2006).

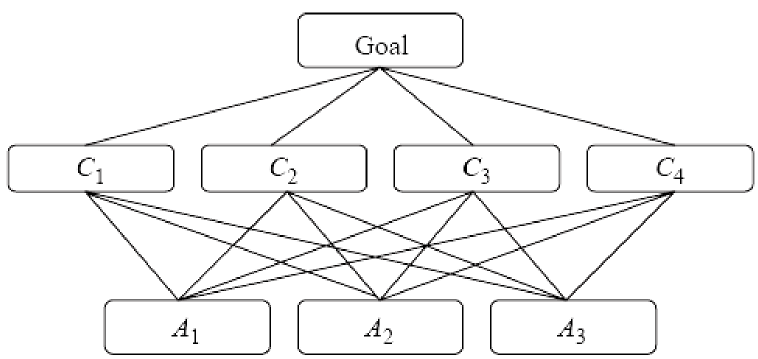


Figure 3.4 AHP for decision alternatives

Within the context of a classification problem the elements of the final level of the hierarchy represent the choices (groups) available to the decision maker regarding the classification of the alternatives (Doumpos & Zopounidis, 2002).

After the hierarchy has been constructed, the decision maker performs pairwise comparisons at each level to determine the relative importance of each element at that level with respect to each element at the next higher level in the hierarchy. The objective of all these comparisons is to assess the relative significance of all elements of the hierarchy in making the final decision according to the initial objective. In pairwise comparisons, AHP uses a scale with values from 1 to 9. Table 3.2 shows how the decision maker's verbal descriptions of the relative importance between the two criteria are converted into a numerical rating.

Table 3.2 The AHP weight scale for comparing elements *a* and *b* in an AHP hierarchy

Scale	**Definition**
1	Both *a* and *b* are of equal importance (indifferent)
3	Moderate preference for *a* over *b*
5	Strong preference for *a* over *b*
7	Very Strong preference for *a* over *b*
9	Extreme preference for *a* over *b*
2, 4, 6, 8	Intermediate values

The results of the comparisons made by the decision maker are used to form a nxn matrix W for each level of the hierarchy, where n denotes the number of elements in that level. The matrix below shows the general layout for pairwise comparisons.

$$W = \begin{bmatrix} w_{11} & w_{12} & \cdots\cdots & w_{1n} \\ w_{21} & w_{22} & \cdots\cdots & w_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ w_{n1} & w_{n2} & \cdots\cdots & w_{nn} \end{bmatrix} \qquad (3.22)$$

where, W is the matrix of pairwise comparisons, w_{ij} is the relative preference of the decision maker for i to j. As known, w_{ij} should be equal to $1/w_{ji}$ and w_{ii} should be equal to 1.

An important consideration in this process is the consistency of the pairwise judgments provided by the decision maker. For example, if criterion A is preferred to criterion B by a scale of 2 and criterion B is preferred to criterion C by a scale of 3, perfect consistency of criterion A compared to criterion C would have a scale of 6. In other words, if $w_{ik} * w_{kj} = w_{ij}$ for all i, j, k, then matrix W is said to be perfectly consistent.

In practical situations, perfect consistency is difficult to achieve. Thus, a tolerance level for consistency was developed by Saaty. The tolerance level, referred to as consistency ratio, is acceptable if it is less then 0.10 (10%). If a consistency ratio is unacceptable, the decision maker should review and revise the pairwise comparisons. For further details on the procedure for computing the consistency ratio, readers should refer to Saaty (1980).

In AHP, after constructing the pairwise comparison matrix, the priority of each criterion in terms of its contribution to the overall goal should be calculated. This aspect of AHP is referred to as synthesization. For doing this, Anderson et al. (2005) present three-step procedure that provides a good approximation of the synthesization results. It should be emphasized that the priorities of each criterion are valid for only the particular goal specified in the AHP model for the problem. If another goal is specified, the criteria would need to be reevaluated with respect to that new goal (Badiru & Pulat, 1995).

After the priorities of the evaluation criteria are obtained, the next step is to evaluate the alternatives on the basis of the criteria. In this step, a relative evaluation rating is obtained for each decision alternative with respect to each criterion. The procedure for the pairwise comparison of the alternatives is similar to the procedure for comparing the criteria. Therefore, if there are n evaluation criteria in the problem, we would have n separate matrices of pairwise comparisons of the alternatives, one matrix for each criterion.

The last stage of the AHP method involves determining the overall evaluation of the decision alternatives. In order to compute the overall score of a decision alternative, the following equation is used;

$$S_j = \sum_i w_i k_{ij} \tag{3.23}$$

where S_j represents the overall score for decision alternative j; w_i represents the weight for criterion i, and k_{ij} represents the rating (local weight) for decision alternative j with respect to criterion i. therefore, $w_i k_{ij}$ represents the global evaluation (weight) of decision alternative j with respect to criterion i.

For a classification problem the global evaluation for the elements in the last level of the hierarchy are used to decide upon the classification of a decision alternative. Since, the elements of the last level correspond to the prespecified groups; an alternative is assigned to the group for which the evaluation of the corresponding element is higher (Doumpos & Zopounidis, 2002).

3.5.1.2 The ELECTRE TRI Method

The family of ELECTRE methods initially introduced by Roy (1968) is founded on the ORT concepts. These methods are the most extensively used ORT techniques. Several versions of ELECTRE methods exist in the literature. The ELECTRE TRI method (Yu, 1992; Mousseau et al., 2000) is a member of this family of methods, developed for addressing classification problems. It is based on the framework of the ELECTRE III method (Roy, 1991).

ELECTRE TRI assigns a discrete set of alternatives $A=\{a_1, a_2, ..., a_m\}$ into q groups $C_1, C_2, ..., C_q$. Each alternative a_j is considered as a vector $g_j = (g_{j1}, g_{j2}, ..., g_{jn})$ consisting of the performance of alternative a_j on the set of evaluation criteria g. The groups are defined in an ordinal way. In other words, group C_1 contains the most preferred alternatives and C_q contains the least preferred alternatives. In ELECTRE TRI, a fictitious alternative r_k is introduced as the boundary among each pair of consecutive groups C_k and C_{k+1}. Any fictitious alternative is called as *reference profile* or simply *profile.* The profile r_k and r_{k+1} are the lower bound and the upper bound of the group C_k, respectively. Each profile r_k is a vector consisting of partial profiles defined for each criterion $r_k = (r_{1k}, r_{2k}, ..., r_{nk})$. Since the groups are defined in an ordinal way, each partial profile must satisfy the condition $r_{ik} > r_{i,k+1}$ for all $k = 1,2,...,q-1$ and $i = 1,2,...,n$. Figure 3.5 represents the reference profiles in ELECTRE TRI (Doumpos & Zopounidis, 2002).

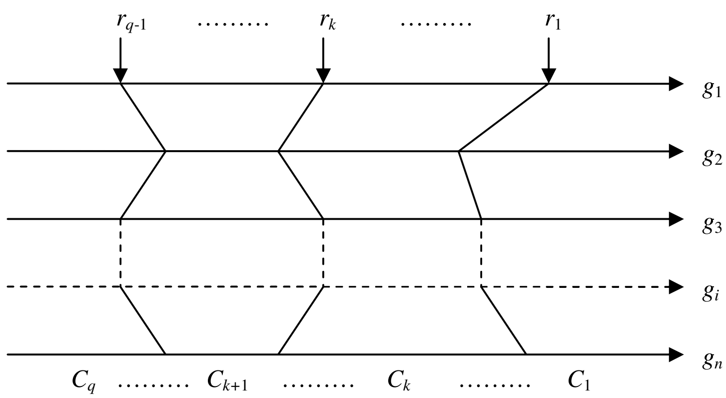


Figure 3.5 The reference profiles in ELECTRE TRI (Doumpos & Zopounidis, 2002, P.60)

ELECTRE TRI assigns the alternatives into the pre-specified groups following two consecutive steps (Mousseau et al., 2000);

(1) Construction of an outranking relation *S* used to decide on whether an alternative outranks a profile or not.
(2) Exploitation of the relation *S* to assign each alternative to a specific group.

The development of the outranking relation *S* is based on the comparison of the alternatives with the reference profiles. These comparisons are performed for all pairs (a_j, r_k), $j = 1,2,...,m$ and $k = 1,2,...,q-1$. Generally, the comparison of an alternative a_j with a profile r_k is accomplished in two stages, involving the concordance and discordance test respectively. In order to make these tests, the concordance index and discordance index should be determined. In Section 3.4.3.1, the detailed information about how to determine the concordance and discordance indices has been given. As seen in this section, in order to determine these indices, ELECTRE TRI requires some parameters such as weights, preference, indifference and veto thresholds. Because, it is based on ELECTRE III.

After calculating a concordance and discordance measure, the next step is to combine the two indices so that an overall estimation of the strength of the outranking degree of an alternative a_j over the profile r_k can be estimated considering all the evaluation criteria. This measure assesses the strength of the assertion that "alternative a_j is at least as good as profile r_k according to all criteria". The detailed information about how to determine this measure, $S(a_j, r_k)$, has been also given in Section 3.4.3.1.

The degree of outranking provides the means to decide whether an alternative a_j outranks profile r_k $(a_j S r_k)$ or not. However, as the assignment of alternatives to groups does not result directly from the relation S, an exploitation phase is necessary. Therefore, the outranking relation is considered to hold if $S(a_j, r_k) > \lambda$. The cut-off point λ is defined by the decision maker. It takes the values between 0.5 and 1 (Mousseau et al., 2000; Doumpos & Zopounidis, 2002).

After determining cut-off point, three possible outcomes of the comparison of an alternative a_j with a profile r_k can be established as follows (Doumpos & Zopounidis, 2002);

(1) Indifference (I): $(a_j I\, r_k) \Leftrightarrow (a_j S r_k) \wedge (r_k S a_j)$

(2) Preference (P): $(a_j P\, r_k) \Leftrightarrow (a_j S r_k) \wedge (not\ r_k S a_j)$

(3) Incomparability (R): $(a_j R\, r_k) \Leftrightarrow (not\ a_j S r_k) \wedge (not\ r_k S a_j)$

The above three relations provide the basis for developing the classification rule. In ELECTRE TRI, two assignment procedures are available: the optimistic procedure and the pessimistic procedure. Both procedures begin by comparing an alternative a_j to the lowest (worst) profile $r_{q\text{-}1}$. If $(a_j P\, r_{q-1})$, then the procedure continues with the comparison of a_j to the next profile $r_{q\text{-}2}$. The same procedure continues until one of the two following situations appears (Doumpos & Zopounidis, 2002);

(1) $(a_j P\, r_k) \wedge (r_{k-1} P a_j) \vee (a_j I r_{k-1})$

(2) $(a_j P\, r_k) \wedge (a_j R\, r_{k-1}) \wedge (a_j R\, r_{k-2}) \wedge \ldots \wedge (a_j R\, r_{k-l}) \wedge (r_{k-l-1} P a_j)$

In the first case, both the optimistic and the pessimistic procedures will assign the alternative a_j into group C_k. In the second case, however, the pessimistic procedure will assign the alternative into group C_k, whereas the optimistic procedure will assign the alternative into group C_{k-l}.

In ELECTRE TRI, the assignment procedures depend on the value of cut-off point. As mentioned, it ranges between 0.5 and 1. When the cut-off point decreases, the pessimistic and optimistic characters of these rules are weakened (Mousseau et al., 2000).

3.5.1.3 The PROMETHEE TRI Method

PROMETHEE TRI was proposed by Figueira et al. (2004), and it is a member of the family of PROMETHEE methods. It has been designed for sorting problems. As mentioned in Section 3.1, when there is no preference relation among the groups we refer to the nominal classification problem. If the groups are ordered from the worst to the best one, or vice-versa, the problem is called an ordinal sorting problem.

When assigning an alternative *a* to a certain group, PROMETHEE TRI makes use of the concept of reference alternatives instead of profile limits as in ELECTRE TRI. A *reference* or *central alternative,* r_h, is a typical element which can be used to characterize a group C_h. A reference alternative is not necessarily an actual alternative; it can be, and in general it is, a fictitious one. It should be noticed that a group can be characterized by one or several reference alternatives. But for the sake of simplicity, Figueira et al. (2004) suggest that a group can be characterized by unique alternative. In many practical situations decision makers prefer to give a typical element of a certain group instead of defining lower and upper profile limits of groups. PROMETHEE TRI is particularly adequate to deal with such a kind of situations.

In comparison with ELECTRE TRI, PROMETHEE TRI performs the classification into two phases. During the first phase, the single criterion net flows are computed for each alternative and reference alternative. The detailed information about how to compute the single criterion net flows has been given in Section 3.4.3.2. The quality of a given alternative *a* can be appreciated by the decision maker through the definition of the profile of *a*. This profile is drawn from the single criterion net flow, $\phi_j(a)$, that measures the strength of alternative *a* over all the other alternatives on criterion *j*. The larger the single criterion net flows $\phi_j(a)$, the better alternative *a* on criterion g_j (Figueira et al., 2004).

The profile of alternative *a* and reference alternative r_h are shown in Figure 3.6 and Figure 3.7, respectively.

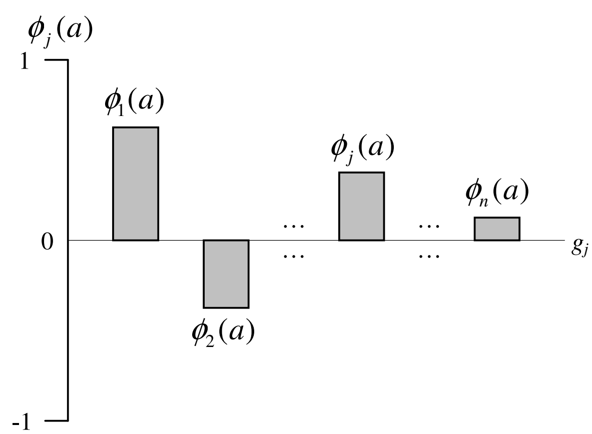


Figure 3.6 The profile of the alternative *a*

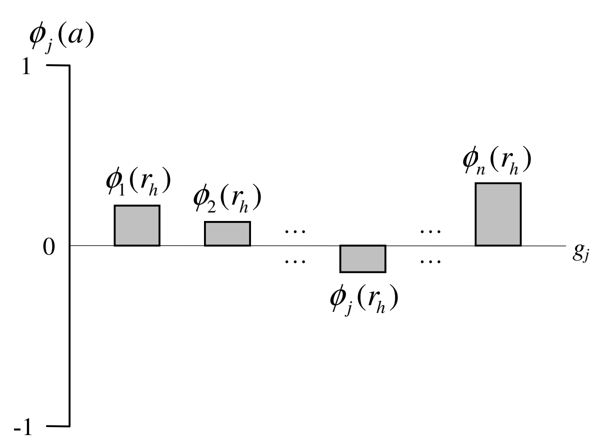


Figure 3.7 The profile of the reference alternative r_h

In the second phase of the PROMETHEE TRI, in order to assign alternatives into the groups, the profiles of the alternatives and reference alternatives should be compared, and the deviation between the alternatives and reference alternatives should be computed. The deviation between alternative *a* and reference alternative r_h is defined as the summation of the deviations for all the criteria. Therefore, this deviation can be computed by using the following equation (Figueira et al., 2004);

$$e(a, r_h) = \sum_{j \in J} \left| \phi_j(a) - \phi_j(r_h) \right| w_j \tag{3.24}$$

where *J* denotes the set of the criteria indices. The absolute deviation between alternative *a* and reference alternative r_h is shown in Figure 3.8.

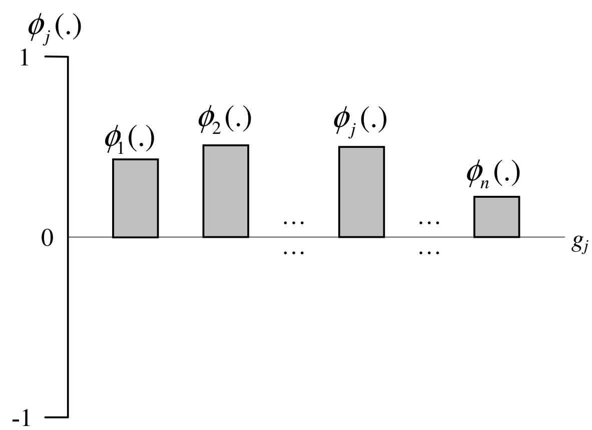


Figure 3.8 The absolute deviation between alternative *a* and reference alternative r_h

Therefore, the second phase of the PROMETHEE TRI consists of assigning alternatives to the group with the smallest deviation. In other words, an alternative *a* is assigned to group *l*, if the deviation is minimum. If there are *q* groups, this situation can be shown as follows;

$$a_i \in C_l \quad if \quad e(a, r_l) = \min_{h=1,\dots,q} \{e(a, r_h)\} \tag{3.25}$$

Compared to the existing approaches, a major distinctive feature of the PROMETHEE TRI is the use of central reference alternatives to characterize the groups, instead of limit profiles. However, PROMETHEE TRI may assign an alternative *a* to a worse group than alternative *b*'s, although the alternative a is preferred to b according to PROMETHEE results. Since PROMETHEE TRI does not use the outranking relation between two alternatives obtained by PROMETHEE, it uses only single criterion net flows as inputs. It is obvious that the use of single criterion net flow may not always give the ordered categories. Furthermore, it is important to note that it works with central reference alternatives such a nominal classification method (Araz & Özkarahan, 2007).

3.5.1.4 Other Outranking Classification Methods

Besides The ELECTRE TRI and The PROMETHEE TRI, the use of outranking relation in the development of classification methods has also been considered by other researchers and various MCC methods that require the direct interrogation of the decision maker have also been proposed in the literature. The N-TOMIC method presented by Massaglia & Ostanello (1991) assigns the alternatives into nine pre-specified groups. These nine groups actually define a trichotomic classification of the alternatives, i.e. good alternatives (high performance), uncertain alternatives and bad alternatives (low performance). Two reference profiles are used in order to assign the alternatives into the groups. These two profiles define the concepts of a good and bad alternative (Doumpos & Zopounidis, 2002).

The N-TOMIC method use the assignment procedure that includes three stages, and it uses the limit profiles, concordance and discordance concepts discussed previously for the ELECTRE TRI method to assign the alternatives into three main sets. These are Good, Uncertain and Bad. Each set contains three sub groups. The N-TOMIC method requires the decision maker to explicitly define a considerable amount of specific preferential information, such as the relative importance of the criteria, preference, indifference, and veto thresholds. For detailed information, the reader can refer to Massaglia & Ostanello (1991) and Doumpos & Zopounidis (2002).

The other kind of outranking classification method considered in this section is the PROAFTN method (Belacel, 2000). The N-TOMIC method and the other methods are suitable for addressing sorting problems where the groups are defined in an ordinal way. The major distinguishing feature of the PROAFTN method is its applicability in classification problems with nominal groups. Each group is characterized by a reference profile as in the PROMETHEE TRI method. The PROAFTN method, then, develop a fuzzy indifference relation measuring the strength of the affirmation "alternative a_j is indifferent to profile r_k". The development of the fuzzy indifference relation is based on the concordance and discordance concepts discussed for the ELECTRE TRI. The assignment of an alternative is performed by comparing it to all reference alternatives in terms of fuzzy indifference relation and assigning into the most similar group. Comprehensive description about the method is provided in the works of Belacel (2000).

3.5.2 Preference Disaggregation Classification Methods

The implementation of MCC methods requires the decision maker to explicitly define a considerable amount of specific preferential information, such as the relative importance of the criteria, preference, indifference thresholds, etc. On the other hand, PDA does not require the decision maker to provide specific information on how the decisions are taken. They provide the framework for developing a classification models through the analysis of the global judgment of the decision maker using mathematical programming techniques. Doumpos & Zopounidis (2002) states that PDA's objective is to analyze the training sample in order to specify the parameters of the model as consistently as possible with the judgment policy of the decision maker. Therefore, preference disaggregation classification methods assume that the reference set is known a priori.

As mentioned in Section 3.4.4, PDA paradigm is beyond the scope of the research proposed in this thesis. Because, in this dissertation, the new MCS procedure that requires the direct interrogation of the decision maker will be used in the first stage of the proposed methodology. A more detailed explanation about the PDA paradigm and an extensive review of existing PDA techniques can be found in Doumpos & Zopounidis (2002), and Lagrèze & Siskos (2001).

This section presents the most known MCC methods that use PDA paradigm in the model development. The considered methods include the UTADIS method (**UT**ilités **A**dditives **DIS**criminantes), the MHDIS method (**M**ulti-group **H**ierarchical **DIS**crimination) and the PAIRCLASS method (PAIRwise CLASSification). The UTADIS and the MHDIS methods combine a utility function-based framework with the preference disaggregation paradigm. On the other hand, the PAIRCLASS method uses an outranking relation for classification purposes.

3.5.2.1 The UTADIS Method

The UTADIS method is a variant of the well-known UTA method (**UT**ilités **A**dditives) for addressing classification problems. The UTA method employs the PDA paradigm to develop an additive utility function. The UTADIS method, on the other hand, extends the ordinal regression framework of the UTA method in cases where the objective is not to rank the alternatives from the best to the worst ones, but to sort them into predefined homogenous groups (Doumpos & Zopounidis, 2004a).

The objective of the UTADIS method is to develop a criteria aggregation model used to assign the alternatives into q predefined ordered groups. In this method, the alternatives of group C_1 receive the highest scores and the alternatives of group C_q receive the worst scores. The criteria aggregation model is expressed as an additive utility function (Doumpos & Zopounidis, 2002);

$$U(g) = \sum_{i=1}^{n} w_i u_i(g_i) \qquad (3.26)$$

In this model, g (g_1, g_2, …, g_n) is the vector of the evaluation criteria; $u_i(g_i)$ is the marginal utility function of criterion g_i; w_i is a scaling constant indicating the significance of criterion g_i. These constants are often considered to represent the weights of the criteria and they are defined such that they sum-up to one.

The global utility serves as an index used to decide upon the sorting of the alternatives into the predefined groups. The global utility of an alternative a_j represents a measure of the overall performance of the alternative considering its performance on all criteria. The global utilities range in the interval [0, 1] and they constitute the criterion used to decide upon the classification of the alternatives. The classification is performed through the comparison of the global utilities of the alternatives to some utility thresholds that define the lower bound of each class. In the general case where q groups are considered, the classification of the alternatives is performed through the following classification rules (Zopounidis & Doumpos, 2002);

$$\left.\begin{array}{ll} U(a_j) \geq u_1 & \Rightarrow \; a_j \in C_1 \\ u_2 \leq U(a_j) < u_1 & \Rightarrow \; a_j \in C_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\ U(a_j) < u_{q-1} & \Rightarrow \; a_j \in C_q \end{array}\right\} \tag{3.27}$$

where u_1, u_2, …, u_{q-1} denote the utility thresholds separating the group. Each utility threshold u_k separates two consecutive groups C_k and C_{k+1}.

Therefore, the development of the sorting model through the UTADIS method requires the determination of the marginal utility functions to obtain the specific form of the global utility function, as well as the selection of the utility thresholds (Doumpos & Zopounidis, 2004a).

Given the classification of the alternatives in the reference set, an additive utility model and a set of utility thresholds developed in the UTADIS method should minimize the classification error rate. The error rate refers to the differences between the estimated classification defined through the developed model and the pre-specified classification for the alternatives of the reference set. The development of the additive utility model that minimizes these errors is performed though the solution of the linear program that is explained in Doumpos & Zopounidis (2004a).

The detailed information about the UTADIS method can be found in Doumpos & Zopounidis (2002).

3.5.2.2 The MHDIS Method

The main difference between the MHDIS method and the UTADIS method is that the MHDIS method uses a sequential/hierarchical process in order to classify the alternative to groups using available information. A second major difference between the two methods involves the mathematical programming framework used to develop the classification models. The development of the additive utility model in UTADIS is performed though the solution of the linear program. In MHDIS, the model development process is performed using two linear programs and a mixed integer one that gradually calibrate the developed model so that it accommodates two objectives: The first is the minimization of the total number of misclassifications, and the second is the maximization of the clarity of the classification. The model development process is explained in details in Doumpos & Zopounidis (2002).

The common feature shared by both MHDIS and UTADIS involves the form of the criteria aggregation model that is used to model the decision maker's preferences in classification problems. Therefore, both methods employ a utility based framework.

In MHDIS, if there are q groups, the hierarchical discrimination process consists of q-1 stages. Therefore, in each stage k, it is wanted to discriminate the alternatives of group C_k from the alternatives of the other groups. In other words, in stage k, the hierarchical discrimination process should decide whether the alternative belongs into group C_k, or it belongs at most in the group C_{k+1}. It belongs into one of the groups C_{k+1} to C_q. Within this framework, the procedure starts from group C_1 (most preferred alternatives). The aim of this stage is to specify the alternatives belonging into group C_1. Then, the alternatives found to belong into this group are excluded from further consideration, and the second stage begins. The same procedure continues until all alternatives are classified into the predefined groups (Doumpos & Zopounidis, 2002).

Since at each stage k of the hierarchical discrimination process there are choices available for the classification of an alternative, two additive utility functions should be developed for each stage. The first utility function $U_k(g)$ denotes the utility of classifying any alternative into group C_k on the basis of the alternative's performance on the set of criteria g. It provides a measure of the similarity of the alternatives to the characteristics of group C_k. The second utility function $U_{\sim k}(g)$ describes the remaining alternatives at stage k and it measures the utility of the second choice (i.e., classification at most into group C_{k+1}). Therefore, in the MHDIS method, instead of developing a single additive utility function, $2(q\text{-}1)$ additive utility functions are developed.

The sorting rule used to decide upon the assignment of the alternatives has the following form (Doumpos & Zopounidis, 2002);

$$\left.\begin{array}{llll} \textit{if} & U_k(a_j) > U_{\sim k}(a_j) & \textit{then} & a_j \in C_k \\ \textit{if} & U_k(a_j) < U_{\sim k}(a_j) & \textit{then} & a_j \in \{C_{k+1}, C_{k+2}, \ldots, C_q\} \end{array}\right\} \quad (3.28)$$

The case $U_k(a_j) = U_{\sim k}(a_j)$ is considered to be a misclassification. As mentioned, both UTADIS and MHDIS employ a utility based modeling framework. However, the marginal utility functions in MHDIS do not indicate the performance of an alternative with regard to an evaluation criterion; they rather serve as a measure of the conditional similarity of an alternative a_j to the characteristics of group C_k (on the basis of a specific criterion) when the choice among C_k and all the worse groups $C_{k+1},\ldots, C_q$ is considered. The detailed information about the estimation of utility functions, and model extrapolation can be found in Doumpos & Zopounidis (2002).

3.5.2.3 The PAIRCLASS Method

This method is proposed by Doumpos & Zopounidis (2004b). The PAIRCLASS method involves pairwise comparisons based on the MCDM paradigm. The basis of this method is a preference relation that is used to perform pairwise comparisons among the alternatives. It employs concepts from the outranking relations framework in order to compare the alternatives to be classified.

The PAIRCLASS method is based on the use of a reference set X consisting of m of alternatives, evaluated over n criteria. The training sample incorporates the necessary preferential information on the judgment policy of the decision-maker. The reference alternatives are classified by the decision maker in q ordered classes $C_1 \succ C_2 \succ ... \succ C_q$. Here, let us consider the simple two-class case, $C_1 \succ C_2$.

The alternatives of the reference set and their classification should be taken into account in order to classify the alternatives. Therefore, the alternatives of the reference set X provide the basis to which any alternative $a_k \notin X$ is compared in order to decide its classification. In particular, the classification of any a_k is decided on the basis of the pairwise comparisons (a_k, a_i), for all $a_i \in X$. The results of these comparisons lead to the estimation of the outranking and the outranked character of a_k as opposed to the reference alternatives. In the PAIRCLASS method is used for classification purposes (Doumpos & Zopounidis, 2004b).

Here, let us consider the simple two-class case, $C_1 \succ C_2$, and assumes that any alternative a_k will be assigned to one of these groups, and the reference alternatives that belong to class C_1 $(a_i \in X \cap C_1)$ and the reference alternatives that belong to class C_2 $(a_l \in X \cap C_2)$ exist. Therefore, according to the PAIRCLASS method, the classification of the alternative a_k is based on its comparison to the reference alternatives that belong to class C_1 and class C_2, separately.

In order to determine the intensity of preference of the decision maker for the set of reference alternatives a_i that belong to class C_1 over a_k, the preference index, P_{ik}, should be computed. It is specified as the weighted average of the preference of a_i over a_k on each criterion g_j, and can be shown as follows (Doumpos & Zopounidis, 2004b);

$$P_{ik} = \sum_{j=1}^{n} w_j \, F_j(a_i, a_k) \tag{3.29}$$

where $w_j \in [0,1]$ is the weight of criterion g_j, and $F_j(a_i, a_k)$ represents the preference function that indicates the strength of the preference of the decision maker for a_i over a_k defined on the basis of the performances of the two alternatives on the criterion g_j. As discussed in Section 3.4.3.2, in the PROMETHEE methods, a preference function $F_j(a_i, a_k)$ represents the preference level of a_i to a_k on criterion g_j. For each criterion, the preference function, $F_j(a_i, a_k)$, translates the difference between the evaluations obtained by two alternatives into a preference degree ranging from 0 to 1. This difference $\left(d_j^{ik}\right)$ can be shown as follows;

$$d_j^{ik} = g_j(a_i) - g_j(a_k) \tag{3.30}$$

Therefore, in the PAIRCLASS method, the each preference function $F_j(a_i, a_k)$ can be defined as follows (Doumpos & Zopounidis, 2004b);

$$F_j(a_i, a_k) = \begin{cases} 0 & \text{if} \quad d_j^{ik} < 0 \\ h_j(d_j^{ik}) & \text{if} \quad d_j^{ik} \geq 0 \end{cases} \tag{3.31}$$

Generally, the preference functions $F_j(a_i, a_k)$ may have different forms depending on the form of the functions h_j. For example, in the PROMETHEE methods, In order to facilitate the selection of a specific preference function, Brans & Vincke (1985) proposed six basic types. Nevertheless, the PAIRCLASS method does not use these basic types of preference functions. Doumpos & Zopounidis (2004b) proposed a linear programming approach to obtain the preference functions and the weights from a set of reference alternatives.

Then, the preference index, P_{kl}, that indicates the intensity of preference of the decision maker for the alternative a_k over the set of reference alternatives a_l that belong to class C_2 is computed as follows;

$$P_{kl} = \sum_{j=1}^{n} w_j \, F_j(a_k, a_l) \tag{3.32}$$

The classification rule used is based on the difference between the leaving and entering flow for the alternative a_k. This difference defines a net flow f_k for a_k (Doumpos & Zopounidis, 2004b);

$$f_k = \frac{1}{m_2} \sum_{a_l \in C_2} P_{kl} - \frac{1}{m_1} \sum_{a_i \in C_1} P_{ik} \tag{3.33}$$

where m_1 and m_2 denote the number of reference alternatives belonging to classes C_1 and C_2, respectively. The net flow can take the values between -1 and 1. For example, if a net flow $f_k = -1$, it indicates that the alternative a_k does not outrank any reference alternative from class C_2, while being strictly outranked by all reference alternatives from class C_1. Similarly, if a net flow $f_k = 1$, it indicates that a_k strictly outranks all reference alternatives from class C_2, while not being outranked by any reference alternative from class C_1. Finally, if a net flow $f_k \approx 0$, it indicates an ‘‘average’’ alternative, and the classification is not clear enough.

As a consequence, according to the PAIRCLASS method, the classification rule is as follows (Doumpos & Zopounidis, 2004b);

$$\left.\begin{array}{lll} f_k > b & \Rightarrow & a_k \in C_1 \\ f_k < b & \Rightarrow & a_k \in C_2 \end{array}\right\} \tag{3.34}$$

Here, b is a cut-off point. It can be specified by the decision maker or it can be estimated by the data that the reference alternatives provide.

Further details about the PAIRCLASS method can be found in (Doumpos & Zopounidis, 2004b).

3.6 PROMSORT

PROMSORT, which is a MCS method that assigns alternatives to predefined ordered categories, is proposed by Araz & Ozkarahan (2005) for financial classification problems. They firstly applied this method to the business failure risk problem, which is one of the major problems in the field of finance. PROMSORT was also used to solve the country risk assessment problem by Araz et al. (2006) and the strategic supplier selection problem by Araz et al. (2007). PROMSORT is based on the methodological framework of PROMETHEE methodology.

As mentioned before, in this dissertation, a novel methodology for risky investment projects evaluation is proposed. The first stage of the proposed methodology includes opportunity and pre-feasibility studies. The aim of this stage is to identify the investment opportunities, and to carry out preliminary election of project ideas and to give prominence to ideas which have the highest chance of attaining the goals planned by entrepreneurs and investors. Therefore, the project evaluation and selection problem considered in the first stage of the proposed methodology is better addressed through the classification/sorting problematic.

There are many statistical and econometric classification methods, which constitute the traditional approach to develop classification models. However, they are several shortcomings due to the restrictive statistical assumptions. In order to overcome these shortcomings, several methodologies such as ELECTRE TRI, PROMETHEE TRI, UTADIS and MHDIS have been developed to deal with classification/sorting problems. On the other hand, most of them assume that adequate number of reference alternatives have already been determined and use these reference alternatives as training samples to infer some of the model parameter.

However, in the preliminary project selection problem presented in this dissertation, most of time, a set of training sample may not be possible. For these reasons, in the scope of this dissertation, PROMSORT will be used in order to assign project alternatives to predefined ordered classes in the first stage of the proposed methodology. This MCS procedure does not require a training sample, and by using this method, the inherent risk and uncertainty associated with the values of evaluation criteria can be handled. On the other hand, it is important to point out that this MCS procedure has not been applied to the investment project evaluation and selection problems, until this dissertation. This thesis and our proposed novel methodology is the first one that uses this MCS procedure, PROMSORT, in order to assign project alternatives to predefined ordered categories.

Let F denote the set of indices of the criteria $g_1, g_2, ..., g_j$ $[F = (1,2,..., j)]$ and B the set of indices of the profiles defining (k+1) categories $[B = (1,2,...,k)]$, b_h being the upper limit of category C_h and the lower limit of category C_{h+1}, $h = 1,2,...,k$. Assume that $C_2 > C_1$ means that Category 2 outranks Category 1, the set of profiles $[B = (b_1, b_2,...,b_k)]$ must have the following property (Araz & Ozkarahan, 2005; Araz et al., 2006);

$$[b_k \, P \, b_{k-1}], [b_{k-1} P \, b_{k-2}], ..., [b_2 \, P \, b_1] \qquad (3.35)$$

PROMSORT assigns alternatives to categories following the three consecutive steps:

(1) Construction of an outranking relation using PROMETHEE I,
(2) Exploitation of the outranking relation in order to assign alternatives to specific categories except the incomparability and indifference situations,
(3) Final assignment of the alternatives based on pairwise comparison.

These consecutive steps are explained in the following sub sections.

3.6.1 Construction of an Outranking Relation Using PROMETHEE I

In PROMSORT, categories are defined by lower and upper limits like ELECTRE TRI The comparison of an action a with a profile limit b_h is defined in the following way;

$$
\begin{aligned}
&(aPb_h) \text{ iff } &&\begin{bmatrix} \phi^+(a) > \phi^+(b_h) \text{ and } \phi^-(a) < \phi^-(b_h), \text{ or} \\ \phi^+(a) = \phi^+(b_h) \text{ and } \phi^-(a) < \phi^-(b_h), \text{ or} \\ \phi^+(a) > \phi^+(b_h) \text{ and } \phi^-(a) = \phi^-(b_h) \end{bmatrix} \\
&(aIb_h) \text{ iff } &&\left[\phi^+(a) = \phi^+(b_h) \text{ and } \phi^-(a) = \phi^-(b_h)\right] \qquad (3.36) \\
&(aRb_h) \text{ iff } &&\begin{bmatrix} \phi^+(a) > \phi^+(b_h) \text{ and } \phi^-(a) > \phi^-(b_h), \text{ or} \\ \phi^+(a) < \phi^+(b_h) \text{ and } \phi^-(a) < \phi^-(b_h) \end{bmatrix}
\end{aligned}
$$

3.6.2 Assignment of the Alternatives

The assignment of alternatives to categories results directly from the outranking relation (Assume that $C_2 > C_1$ means that Category 2 outranks Category 1).

(1) Compare alternative *a* successively to b_i, for $i = k, k-1, \ldots, 1$

(2) b_h being the first profile such that aPb_h

(3) b_t being the first profile such that aRb_t or aIb_t

(4) If *h*>*t*, assign *a* to category C_{h+1}

(5) Otherwise do not assign *a* to any category (it is not certain that alternative *a* should be assigned to category *t* or $t+1$)

After the second phase, it is possible that some alternatives could not have been assigned to a category, since outranking relation indicates that these alternatives are indifferent or incomparable to a profile limit and could not be assigned to a category directly. On the other hand, some alternatives could be assigned to the categories. In the third stage, these alternatives are used as the reference actions of the categories to be able to assign the alternatives which have not yet been assigned.

3.6.3 Final Assignment

In second phase, some alternatives are assigned in $h+1$ ordered categories $C_{h+1} > C_h > \ldots > C_1$. Now, these alternatives are the reference alternatives for ordered categories. Suppose that a reference set X_h consisting of *m* alternatives for category *h*, i.e., $X = \{x_1, x_2, ..., x_m\}$ and an alternative *a* which has not yet been assigned to a category. Similar as Doumpos & Zopounidis (2004b), in this phase, at first, a distance should be determined. It is calculated by using the following equation;

$$\left.\begin{aligned} d_k &= \frac{1}{n_t} d^+{}_k - \frac{1}{n_{t+1}} d^-{}_k \\ d^+{}_k &= \sum_{x \in X_t} [\phi(a) - \phi(x)] \\ d^-{}_k &= \sum_{x \in X_{t+1}} [\phi(x) - \phi(a)] \end{aligned}\right\} \tag{3.37}$$

where $d^+{}_k$ represents the outranking character of *a* over all alternatives assigned to category C_t, $d^-{}_k$ represents the outranked character of *a* by all alternatives belong to category C_{t+1}, n_t and n_{t+1} are the numbers of reference alternatives of category C_t and C_{t+1}, respectively, and $\phi(a)$ is the net flow of alternative *a*.

In this phase, then, a cut-off point *s* should be assigned. *s* can be specified by the decision maker and reflects the decision maker's point of view: pessimistic or optimistic. If the distance is greater than the cut-off point, assign alternative *a* to the category C_{t+1}, otherwise assign to the C_t. This rule can be shown as follows (Araz & Ozkarahan, 2005; Araz et al., 2006);

$$\left.\begin{aligned} &if \quad d_k \geq s \quad a \in C_{t+1} \\ &if \quad d_k < s \quad a \in C_t \end{aligned}\right\} \tag{3.38}$$

3.6.4 Illustrative Case study: Country Risk Assessment

In this section, the performance of the PROMSORT procedure is explored through an application to country risk evaluation. The main aim of this numerical example to be illustrated in this section is to evaluate economical and financial performances of the countries members and candidates of EU in terms of eight evaluation criteria given in Table 3.3.

Table 3.3 Parameters for PROMETHEE and profile limits for PROMSORT

Code	Evaluation Criteria	Obj.	Weight	*q*	*p*	b_2	b_1
g_1	Inflation, GDP deflator (annual %)	Min	15	-	1.0	2.5	5.0
g_2	GNI Per Capita ($)	Max	30	-	10000	20000	8000
g_3	GDP average annual % growth (2000-2004)	Max	5	-	1.0	4	1.5
g_4	Export/Import Ratio	Max	15	-	10.0	110.0	90.0
g_5	Cost % of per capita income	Min	5	-	3.0	8.0	15.0
g_6	Market Capitalization (%GDP)	Max	5	-	10.0	75.0	20.0
g_7	High-technology exports (% of manufactured exports)	Max	15	-	5.0	20.0	10.0
g_8	Expenditures for R&D (% of GDP)	Max	10	-	0.5	2.0	1.0

The application involves 26 countries that were classified in three predefined classes:

i: Class 1: High risk [the worst category];
ii: Class 2: Medium risk;
iii: Class 3: Low risk [the best category].

All of the countries included in the analysis are given in Table 3.4.

Table 3.4 Countries included in the analysis

EU Members						Candidates
Germany	Denmark	Netherlands	Sweden	Hungary	Slovenia	Turkey
Austria	Estonia	U.Kingdom	Italy	Poland	Greece	Croatia
Belgium	Finland	Ireland	Latvia	Portugal		Bulgaria
Czech R.	France	Spain	Lithuania	Slovak R.		Romania

The data (year 2004) are gathered from the World Bank's World Developments Indicators 2006 (World Bank, 2006). Some countries are excluded from the analysis because of the lack of information (e.g. Luxemburg). The parameters of PROMETHEE methodology and limit profiles of classes can be shown in Table 3.2. Following the methodology described above, PROMSORT assignments were given in Table 3.5.

Table 3.5 PROMSORT classifications

Class	PROMSORT Classification
C3	Germany, Austria, Belgium, Denmark, Finland, France, Netherlands, United Kingdom, Ireland, Sweden
C2	Czech Rep., Estonia, Spain, Italy, Hungary, Portugal, Slovenia, Slovak Rep., Greece, Croatia, Lithuania,
C1	Poland, Latvia, Turkey, Bulgaria, Romania,

It can be easily seen from the results that most of the countries categorized as high-income by World Bank are assigned to the low risk class. However, some of high-income countries are evaluated as medium risk class. This is probably because only the EU members and candidates are considered in the evaluation. Furthermore, World Bank classifies all countries into classes by considering the GNI per capita criterion. It should also be noted that all candidate countries, except Croatia, are assigned to the high risk class. In order to compare the risk classes obtained from PROMSORT, single criterion net flows of PROMETHEE can be helpful. Average single criterion net flows for each group were determined. Figure 3.9 illustrates the comparison of the groups by means of average single criterion net flows.

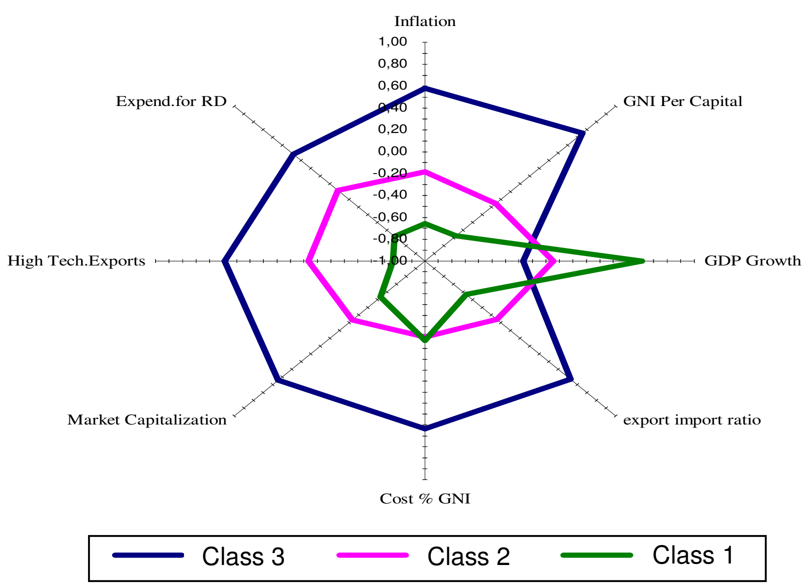


Figure 3.9 The comparison of the classes by means of average single criterion net flows

As seen in the Figure 3.9, the average performance of each risk class in terms of each criterion is substantially different from the others. In summary, candidate countries should highly improve their economical performances, except GDP growths, before being a member of EU. In the same manner, each country can be compared with profile limits b_1 and b_2.

Figure 3.10 shows the comparison of Turkey, which is assigned to Class 1, with profile limits in terms of single criterion net flows. It should be highlighted that Turkey has relatively good performance on Market Capitalization, GDP growth and Export/Import ratio. However, Turkey is quite weak on the GNI per capita and Expenditures for R&D criteria. On the other hand, the main shortcomings of Turkey are “Inflation Rate”, “High Tech. Exports” and “cost of % GNI”.

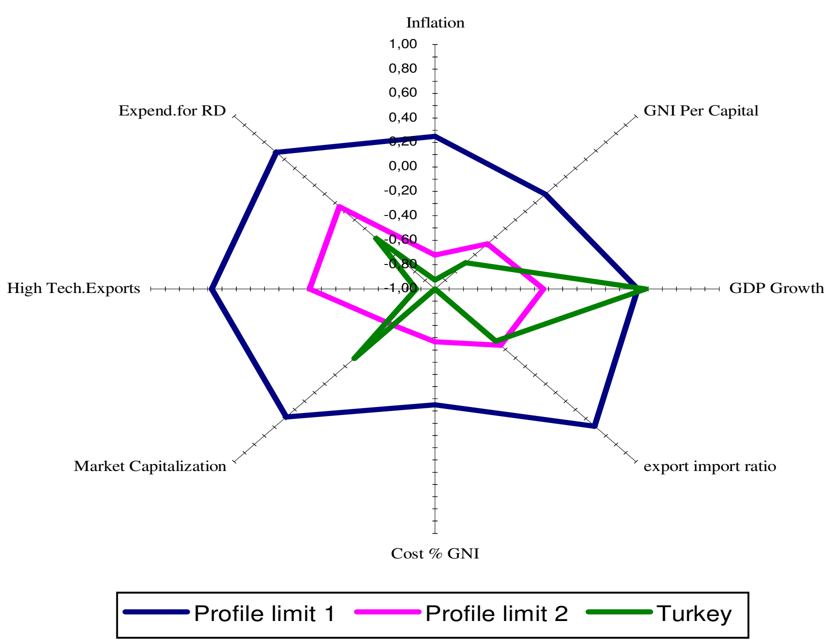


Figure 3.10 The comparison of Turkey and profile limits by means of single criterion net flows

In this paper, one of the major problems in the field of international economics is studied. Country risk assessment problem of European Union members and candidates is considered by using a MCS procedure, PROMSORT. In the evaluation phase, a limited number of criteria, which are frequently used in the literature, are selected.

3.7 Summary of Chapter

The main objective of this chapter is to present the methodological approaches for MCDM, to provide an overview of MCDM methods previously used in investment project evaluation and selection problems, and to explain the new MCS procedure, named as PROMSORT, which has not been applied to the investment project evaluation and selection problems. In this chapter, taxonomy of the multi-criteria decision making problems has been described and some methods used for solving these problems have been reviewed. This chapter also provides a comprehensive overview of multi-criteria classification problem and reviews some methods to solve these problems. At the end of this chapter, one of the multi-criteria sorting procedures, called PROMSORT, which will be used to assign the project alternatives to predefined ordered categories in the first stage of the proposed methodology, has been presented in details.

CHAPTER FOUR

SIMULATION IN RISKY INVESTMENT PROJECT EVALUATION AND SELECTION PROCESS

The main objective of this chapter is to give basic information about the simulation in risky investment project evaluation and selection process. After describing Monte Carlo simulation of a risky investment project, the computer simulation modeling of a risky investment project which will be used in the proposed methodology will be explained in details. This section covers computer simulation model building, simulation software packages types, and output analysis of simulation. At the end of this chapter, advantages of using simulation in risky investment projects evaluation and the key points that should be taken into account in investment project evaluation via the simulation method will be presented.

4.1 Introduction

Simulation is one of the most widely used quantitative approaches to decision making. In general, it is a method for learning about a real system by experimenting with a model that represents the system. The simulation model contains the mathematical expressions and logical relationships that describe how to compute the value of the outputs given the values of the inputs. Any simulation model has two inputs; controllable inputs and probabilistic inputs. Figure 4.1 shows a conceptual diagram of a simulation model (Anderson et al., 2005).

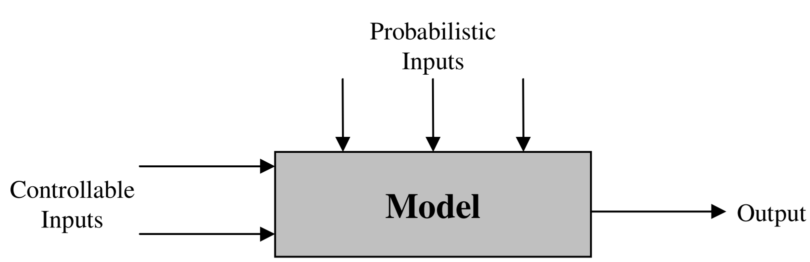


Figure 4.1 Diagram of a Simulation Model

In conducting a simulation experiment, an analyst selects the value, or values, for the controllable inputs. Then values for the probabilistic inputs are randomly generated. The simulation model uses the values of the controllable inputs and the values of the probabilistic inputs to compute the value, or values, of the output. By conducting a series of experiments using a variety of values for the controllable inputs, the analyst learns how values of the controllable inputs affect or change the output of the simulation model. After reviewing the simulation results, the analyst is often able to make decision recommendations for the controllable inputs that will provide the desired output for the real system (Anderson et al., 2005).

Computers have made it both feasible and relatively inexpensive to apply simulation methods to economic decisions. Computer simulation seems to be one of the most effective tolls for risky investment project appraisal. Simulation is based on repeated calculation of project effectiveness for randomly selected input parameters, and the probability distribution of the effectiveness measure thus calculated. Consequently, the probability of occurrence of unfavorable values of the effectiveness indicator and also measures of its variability can be determined. As an example, when simulation is used in the evaluation of risky investment projects, it requires that estimates be made of the probability distributions of risky project parameters. These probability distributions are then put into the simulation model, and the model is replicated several times until a sufficient number of the effectiveness measure such as the NPV of a project are available to define its distribution.

The results of these replications are then used to determine the probability distribution of the project's NPV and to compute the expected value and a standard deviation of returns. This information provides the decision maker with an estimate of a project's expected returns as well as its risk. Given this information, it is possible to compute the probability of achieving a NPV that is greater or less than any particular value (McGuigan et al., 2002, pp.56-58).

Recently, the usage of simulation in investment project evaluation under uncertain and risky environments has been increasing. Because, simulation based project evaluation approaches enables to make more reliable investment decision since they permits including future uncertainty and risk in analyze process. In this dissertation, an integrated multi-criteria decision making methodology for risky investment projects evaluation is proposed. As mentioned, the proposed methodology consists of three main stages. In the second stage of the proposed methodology, two different computer simulation models based on a new NPV formulation explained in Section 6.2.2 are developed by using ARENA simulation software. The first model is developed in order to calculate the expected NPV of each project proposal and their standard deviations, and the second model is developed in order to calculate the expected cash flows for each project in each period. The outputs of these models are used in the third stage of the proposed methodology.

Hence, the main objective of this chapter is to give basic information about the simulation in risky investment project evaluation and selection process. In the following sections, firstly, Monte Carlo simulation of a risky investment project will be described. Then, the computer simulation modeling of a risky investment project which is used in the proposed methodology will be explained in details. Lastly, advantages of using simulation in risky investment projects evaluation process will be told.

4.2 Monte Carlo Simulation of a Risky Investment Project

In recent years, an increasing trend has been seen in project evaluation studies under uncertainty and risk. Due to possibility of a deviation in some expected project parameter values during its life cycle, it is necessary to consider uncertainty and risk phenomena while evaluating projects.

In such situations, values of the risky project parameter can be defined as probability distributions. Then project profitability is calculated by using random values of project parameters which has been generated from their own probability distributions. Hence every random value generated for a risky project parameter causes to calculate a different profitability value. By this way, the effects of the changes in the values of risky parameters on the project profitability can be determined. This fact is also the main goal of sensitivity analysis. As known, sensitivity analysis is used for observing the change in the objective function, while there is a change only in a one factor but the other factors are constant, with the aim of determining the sensitivity of the function to one factor. This is the weakness of sensitivity analysis; it is not possible to determine explicitly the cause of change in objective function when the values of risky parameters change simultaneously. Here, by the simulation approach and developed simulation models, it is possible to determine the effects of the simultaneous changes in the value of risky parameters on the project feasibility.

Due to the advantages that are introduced above, recently the usage of simulation in project evaluation under uncertainty and risk has been increasing. The expected profitability of the project is calculated via simulation approach. It is well known that, project profitability is generally determined by checking NPV. In literature, much of the studies that use simulation approach to calculate the expected NPV of the project are used the traditional formulation of NPV expressed in Equation (2.7).

In some studies net cash flows, in some discount rate, and in others, both of them are defined as probability distributions, but almost all of these studies used Monte Carlo simulation. As depicted in Section 2.5.3, while evaluating projects under risk and uncertainty, parameters affecting project profitability are not taken as a constant value. They are defined as *random variables* which have a variation range and a probability distribution within this range. Once a distribution for a random variable that affects project profitability is defined; it is needed to determine ways to generate samples from this distribution. Monte Carlo simulation is a specific type of simulation method in which a random sample of outcomes is generated for a specified probability distribution.

Park (2002) states that the sampling process is the key part of the analysis. It must be done such that the sequence of values sampled will be distributed in the same way as the original distribution. To accomplish this objective, it is needed a source of independent, identically distributed uniform random numbers between 0 and 1. It can be used a table of random numbers but most digital computers have programs available to generate "equally likely (uniform)" random decimals between 0 and 1. It is used $U(0,1)$ to denote such a statistically reliable uniform random number generator, and uniform random numbers generated by this routine are represented by $U_1, U_2, \ldots, U_n$.

For any given random numbers, the question is, how are they used to sample a distribution in a simulation analysis? The first task is to convert the distribution into its corresponding cumulative frequency distribution. Then, the random number generated is set equal to its numerically equivalent percentile and is used as the entry point on the $F(x)$ axis of the cumulative frequency graph. The sampled value of the random variable is x value corresponding to this cumulative percentile entry point.

This method of generating random values works because choosing a random decimal between 0 and 1 is equivalent to choosing a random percentile of the distribution. Then, the random value is used to convert the random percentile to a particular value. The method is general and can be used for any cumulative probability distribution, either continuous or discrete (Park, 2002, pp.858-859).

As a consequence, a Monte Carlo simulation analysis consists of a series of repetitive computations of NPV. To perform the sequence of repeated simulation trials, it is generated a sample observation for each random variable in the model and these values are substituted into the NPV equation. The trials are continued until a sufficient number of NPVs are available to define the NPV distribution.

Then, the probability distribution of the NPV can be used to calculate the expected NPV, the standard deviation of the distribution of NPV and the coefficient of variation.

In the literature, there are lots of studies in which the Monte Carlo simulation approach is used in order to evaluate the risk investment projects and determine the project risk. Most of these studies are used the traditional formulation of NPV expressed in Equation (2.7). Also, in these studies, it is often assumed that the effect of inflation is same both on project inflows and outflows, so the effect of inflation on project inflows and outflows is not taken into account. But it is obvious that inflation effect will be different for cost and revenue components, and it should be considered in project evaluation process.

The other important point is; in most of these studies, only the net cash flows or gross cash inflows and cash outflows are simulated in order to provide a sufficient number of NPVs and to develop the NPV distribution.

However, determining only the net cash flows or gross cash inflows and cash outflows by a probability distributions can make the calculation process easy, but it is not realistic. Instead of it, risky individual inflow and outflow components, such as sales volume, sale price, revenues, material cost, and labor cost should be defined as probability distributions. In such situation, the formulation of NPV will be changed. In this dissertation, a new NPV formulation has been developed that eliminates the weakness of using the traditional formulation while evaluating the projects. The developed NPV formulation will be explained in details in Section 6.2.2.

In this new situation, the number of parameters, which are defined as probability distributions and used for calculation of expected NPV, will increase. Therefore, using Monte Carlo simulation approach for modeling the new developed NPV formulation will cause some complexities. Hence, it is a necessity to develop a computer simulation model for new NPV formulation by using computer simulation software. By the help of this model, all parameters affecting the NPV of the project can be defined as discrete and continuous probability distributions if required.

In this dissertation, a computer simulation models has been developed by using simulation software. The developed computer simulation models will be explained in details in Section 6.2.2. Therefore, in the next section, the general information about computer simulation modeling of a risky investment project will be given.

4.3 Computer Simulation Modeling of a Risky Investment Project

As mentioned in the previous sections, simulation is a statistics based behavioral approach that applies predetermined probability distributions and random numbers to estimate risky outcomes. The first step in simulation is the construction of a mathematical model of the managerial decision making situation that we seek to simulate. Secondly, a sample observation for each random variable in the model is generated and these values are substituted into the mathematical model. Then, the sequence of repeated simulation trials is performed. The trials are continued until a sufficient number of outputs are available. Monte Carlo simulation approach enables to generate a sample observation for each random variable in the model. However, if the numbers of the random variables in the mathematical model increases, providing a sufficient number of outputs would be more difficult by using Monte Carlo simulation. In this situation, it is needed to estimate or specify the probability distribution of each random variable in the model.

As it is well known that before the economics of a risky investment project can be evaluated, it is necessary to reasonably estimate the various inflow and outflow components that describe the project. Investment projects may range from something as simple as the purchase of a new machine to the design and construction of very expensive process or resource recovery complex. In evaluating investment projects, we are concerned the project cash flows that result directly from the investment (Park, 2002).

The first step of the simulation method during the process of evaluating risky investment projects is again to develop a mathematical model regarding the criterion of project evaluation. As mentioned, an important and mostly used criterion to evaluate an investment project is its NPV. The mathematical models that represents how to calculate the NPV of a project is explained in details in Section 2.4.2.1. In the event that these mathematical models are used for calculating the NPV of a project while evaluating risky investment projects through simulation method, the decision maker needs to determine the probability distribution of limited number of parameters. For instance, in the event that one uses the mathematical model contained in Equation 2.7 in simulation trials, the project proposal can be evaluated under risk by defining only periodical net cash flows and discount rate as probability distributions.

However, in feasibility studies, it is necessary firstly to determine the components of net cash flows separately in order to be able to determine the net cash flows. Therefore, it is not a rational behavior to define only net cash flows as probability distributions while evaluating risky investment projects via simulation method. Therefore, instead of defining only the net cash flows as probability distributions, it is necessary to define the probability distribution of each risky individual inflow and outflow components, such as revenues, material cost, labor cost, overhead cost, depreciation, taxes, and so on.

On the other hand, when the values of individual inflow and outflow components are defined as a constant-money unit expression, inflation should be taken into consideration with the matter of fact that it has important effect on the project cash flows. As known, constant money units represent constant purchasing power independent of the passage of time. In order to calculate actual-money unit expression for these components, anticipated changes in amounts caused by inflationary or deflationary effects should be estimated for each component. Because, especially in the inflation periods, price of the inflow and outflow components of the project will increase.

Naturally, the price of inflow or outflow components of the investment may increase with different rate from the general inflation rate. Also the increase rates for inflow or outflow components can show variety to each other. So, some additional arrangements should be performed on the traditional NPV formula by considering inflation, and its effect on the project parameters.

In this dissertation, the new NPV formulation has been developed by considering all factors described above. In this formulation, there are lots of project parameters that can be defined as probability distributions. When these parameters are defined as probability distributions, it is rather difficult to determine the expected NPVs of projects by using Monte Carlo simulation. Today, besides Monte Carlo simulation, computer simulation models can be developed by using some simulation programs with the aim of calculating the NPVs of projects. Some parameters of these projects are defined as probability distributions. The developed computer simulation models provide great convenience for finding out expected profitability values of projects especially in the event that many project parameters are defined as probability distributions. In their studies, Armaneri et al. (2005) and Armaneri & Yalçınkaya (2006) have developed simulation models by using the simulation program ARENA (Kelton et al., 1997) for calculation of the NPV of an investment project.

By the help of these developed simulation models, all parameters affecting the NPV of the project can be defined as probability distributions if required. Moreover, with these models, it is also possible to define risky project parameters with specific distribution types such as uniform distribution, normal distribution and exponential distribution. For example, with the aim of presenting how their proposed approach is applied in the process of evaluating any investment project, Armaneri et al. (2005) firstly analyzed a hypothetic investment project and defined the periodical net cash flows of that project, salvage value and initial investment amount as uniform distributions. In this way, at each run, the simulation model will generate random values for these three parameters suitable for their own distributions and the NPV of the project will be recorded as model output for each replication.

In order to determine the random value for a project parameter in Monte Carlo simulation, the decision maker has to generate random numbers with the help of a computer or a calculator. However, computer simulation models determine the different random values for project parameters at each run itself and calculate the NPV of the project without entailing such a necessity.

In the scope of this dissertation, in order to calculate the expected NPV of a project proposal and its standard deviation, a new computer simulation model has been developed by using ARENA simulation software for a new NPV formulation. As mentioned, the developed NPV formulation and its computer simulation model will be explained in details in Section 6.2.2. In this dissertation, in order to calculate the expected cash flows for each project in each period, another computer simulation model has been developed. The outputs of these models are used in linear programming models which are developed in the third stage of the proposed methodology.

In the following sections, computer simulation model building, simulation software packages types and simulation output analysis will be briefly explained.

4.3.1 Computer Simulation Model Building

As simulation models get progressively more complex, it becomes virtually impossible to perform them manually, thus making the computer a necessity. It is well known that the use of computer simulation is not restricted to simulating a physical phenomenon. In recent years, techniques for testing the results of some investment decision before they are actually executed have been developed. As a result, many phases of business investment decisions have been simulated with considerable success. In computer simulation model building for risky investment proposals, the general approach is to assign a subjective a subjective or objective probability distribution to each unknown factor and to combine these into a probability distribution for the profitability as a whole. The essential idea is that, if we can simulate the actual state of nature for unknown investment variables on a computer, we may be able to obtain the resulting NPV distribution.

The following logical steps are often suggested for a computer program that simulates risky investment proposals (Park, 2002);

(1) Identify all the variables that affect the measure of investment worth (e.g., NPV),

(2) Identify the relationships among all the variables. The relationships of interest here are expressed by the equations or the series of numerical computations by which we compute the NPV of an investment project. These equations make up the model we are trying to analyze,

(3) Classify the variables into two groups: the parameters whose values are known with certainty and the random variables for which exact values cannot be specified at the time of decision making,

(4) Define distributions for all the random variables,

(5) Select (or generate) one observation from each random variable by using random sampling,

(6) Calculate the NPV based on these sampled values,

(7) Repeat this random sampling many times to obtain the NPV distribution.

In computer simulation modeling, the number of variables that can be considered is practically unlimited, and the distributions used to define the possible values for each random variable can be of any type and any shape. The distributions can be based on statistical data if they are available, or, more commonly, on subjective judgment.

An important aspect of any simulation study involves confirming that the simulation model accurately describes the real system. In accurate simulation models cannot be expected to provide worthwhile information. Thus, before using simulation results to draw conclusions about a real system, one must take steps to verify and validate the simulation model.

Verification is the process of determining that the computer procedure that performs the simulation calculations is logically correct. Verification is largely a debugging task to make sure that no errors are in the computer procedure that implements the simulation. In some cases, an analyst may compare computer results for a limited number of events with independent hand calculations. In other cases, tests may be performed to verify that the probabilistic inputs are being generated correctly and that the output from the simulation model seems reasonable. The verification step is not complete until the user develops a high degree of confidence that the computer procedure is error free (Anderson et al., 2005).

Validation is the process of ensuring that the simulation model provides an accurate representation of a real system. Validation requires an agreement among analysts and managers that the logic and the assumptions used in the design of the simulation model accurately reflect how the real system operates. The first phase of the validation process is done prior to, or in conjunction with, the development of the computer procedure for the simulation process. Validation continues after the computer program has been developed with the analyst reviewing the simulation output to see whether the simulation results closely approximate the performance of the real system. If possible, the output of the simulation model is compared to the output of an existing real system to make sure that the simulation output closely approximates the performance of the real system. If this form of validation is not possible, an analyst can experiment with the simulation model and have one or more individuals experienced with the operation of the real system review the simulation output to determine whether it is a reasonable approximation of what would be obtained with the real system under similar conditions (Anderson et al., 2005).

Verification and validation are not tasks to be taken lightly. They are key steps in any simulation study and are necessary to ensure that decisions and conclusions based on the simulation results are appropriate for the real system.

As such in simulation of real systems, in simulation of mathematical models, developed computer simulation models should be verified and validated by considering the principles described above.

4.3.2 Simulation Software Packages Types

Because simulation is one of the most widely used quantitative analysis techniques, various software tools have been developed to help analysts implement a simulation model on a computer. The use of spreadsheets for simulation has grown rapidly in recent years, and third-party software vendors have developed spreadsheet add-inns that make building simulation models on a spreadsheet much easier. These add-in packages provide an easy facility for generating random values from a variety of probability distributions and provide a rich array of statistics describing the simulation output. Although spreadsheets can be a valuable tool for some simulation studies, they are generally limited to smaller, less complex systems.

With the growth of simulation applications, both users of simulation and software developers began to realize that computer simulations have many common features: model development, generating values from probability distributions, maintaining a record of what happens during the simulation, and recording and summarizing the simulation output. A variety of special-purpose simulation packages are available, including GPSS®, SIMSCRIPT®, SLAM®, and ARENA®. These packages have built-in simulation clocks, simplified methods for generating probabilistic inputs, and procedures for collecting and summarizing the simulation output. Special-purpose simulation packages enable quantitative analysts to simplify the process of developing and implementing the simulation model.

Simulation models can also be developed using general-purpose computer programming languages such as BASIC, FORTRAN, PASCAL, C, and C++. The disadvantage of using these languages is that special simulation procedures are not built in. One command in a special-purpose simulation package often performs the computations and record-keeping tasks that would require several BASIC, FORTRAN, PASCAL, C, and C++ statements to duplicate. The advantage of using a general-purpose programming language is that they offer greater flexibility in terms of being able to model more complex systems.

To decide which software to use, an analyst will have to consider the relative merits of a spreadsheet, a special-purpose simulation package, and a general-purpose computer programming language. The goal is to select the method that is easy to use while still providing an adequate representation of the system being studied (Anderson et al., 2005, pp.618-619).

4.3.3 Simulation Output Analysis

In many simulation studies a great amount of time and money is spent on model development and programming, but little effort is made to analyze the simulation output data appropriately. As a matter of fact, a very common mode of operation is to make a single simulation run of somewhat arbitrary length and then to treat the resulting simulation estimates as the "true" model characteristics. Since random samples from probability distributions are typically used to drive a simulation model through time, these estimates are just particular realizations of random variables that may have large variances. As a result, these estimates could, in a particular simulation run, differ greatly from the corresponding true characteristics for the model. The net effect is, of course, that there could be a significant probability of making erroneous inferences about the system under study (Law, 2007).

If all of the inputs to the simulation were deterministic, the outputs would be deterministic. However, many simulations include some sort of randomness, which can arise in a variety of ways. Because of the randomness in the components driving a simulation, the output from the simulation is also random, so statistical techniques must be used to analyze the results (Nakayama, 2002).

The objective of output analysis is to estimate the value(s) of one or more unknown parameters by applying appropriate statistical techniques to the data collected from the simulation. The options available for designing and analyzing simulation experiments depend on whether the simulation of interest is terminating or non-terminating, which depends on whether there is an obvious way for determining the simulation run length.

A terminating simulation is one for which there is a "natural" event E that specifies the length of each run (replication). Since different runs use independent random numbers and the same initialization rule, this implies that comparable random variables are independent or identically distributed (IID). A non-terminating simulation is one for which there is no natural event E to specify the length of a run.

This often occurs when we are designing a new system or modifying an existing system, and we are interested in the behavior of the system in the long run when it is operating normally (Law, 2007).

The simulation of a risky investment projects is a kind of terminating simulation. Because, in this kind of simulation, the simulation is run until the stopping criterion is met. As known, the stopping criterion depends upon the purpose of the analysis. In simulating of a risky investment projects, the main stopping criterion is the number of replications. Therefore, in this section, the output analysis of terminating simulations will be considered. The detail information about the output analysis of non-terminating simulations can be found in Seila (1991), Nakayama (2002), and Law (2007).

The initial conditions for a terminating simulation generally affect the desired measures of performance. Since the value of the parameter depends upon the initial conditions, data must be generated by independently replicating the simulation run using the same initial conditions to start each replication. In other words, in terminating simulations, each replication uses the same initial conditions, and the statistical counters for the simulation are reset at the beginning of each replication. If the run is replicated n times, with each replication producing a single observation, X_i, the data will consist of n observations $X_1, X_2, ... X_n$. If the runs are made using independent random number seeds, the observations will be IID, and the techniques that are normally applied to IID data can be applied here.

In this dissertation, the actual state of nature for risky project parameters has been simulated on a computer in order to obtain the resulting NPV distribution. Therefore, simulation analysis consists of a series of repetitive computations of NPV. The simulation trials have been continued until a sufficient number of NPVs are available to define the NPV distribution. After a sufficient number of repetitive simulation trials have been run, the simulation analysis essentially completed. The only remaining tasks are to tabulate the computed NPVs to determine the expected value, standard deviation and to make various graphic displays useful to management. In the scope of this dissertation, after completing simulation trials, the expected NPV for each project and their standard deviations have been calculated by using *Output Analyzer Module* of ARENA simulation software, and also, an approximate 95% confidence interval for the expected NPV for each project has been determined. Lastly, the coefficient of variation for each project has been calculated.

As mentioned before, in this dissertation, another computer simulation model has been developed for calculating the expected cash flows for each project in each period. Thus, the replications of this simulation model have been continued until a sufficient number of cash flows are available to define the cash flow distribution for each project in each period. After completing simulation trials, the expected cash flows for each project in each period have been calculated.

All outputs of the simulation models constructed in this dissertation are used in linear programming models which are developed in the third stage of the proposed methodology.

4.4 Advantages of Using Simulation in Risky Investment Projects Evaluation

Simulation has become an increasingly important management science method in recent years. Various surveys have shown simulation to be one of the methods most widely applied to real-world problems. As the same manner, recently, the usage of simulation in investment project evaluation under uncertain and risky environments has been increasing. Because, simulation based project evaluation approaches enables to make more reliable investment decision since they permits including future uncertainty and risk in analyze process.

As mentioned in Section 2.4, certainty concept in project evaluation means that the values of all project parameters are assumed to be known with complete certainty; the project analysis is concerned with measuring the economic worth of projects and selecting the best investment projects. However, it is nearly impossible to know the values of these parameters with complete certainty before the project is realized. Due to possibility of a deviation in some expected values of project parameters during its life cycle, it is necessary to consider uncertainty and risk phenomena while evaluating projects.

In this situation, the values of project parameters can not be estimated with certainty. Any wrong value that is estimated by the decision maker will directly affect the return and the profitability of the project. In addition, sometimes the wrong alternative can be accepted and implemented because of this wrong estimation. Therefore, it is necessary to define and locate the investment decision-making problem in its real conditions. Simulation approach provides to define the risky project parameters as probability distributions instead of single deterministic values. In this way, the effect of uncertainty and risk on project evaluation is decreased to some extent, although not totally.

As mentioned before, if values of the risky project parameters are defined as probability distributions, then project profitability is calculated by using random values of these project parameters which has been generated from their own probability distributions. Hence every random value generated for a risky parameter causes to calculate a different profitability value. By this way, the effects of the changes in parameters on the project profitability can be determined. This fact is also the main goal of sensitivity analysis. However, the sensitivity analysis does not provide information about how the objective function is affected when two or more of the variables defined as probability distributiona change at the same time. Thanks to the simulation method, it is possible to determine the effects of individual or concurrent changes of the values of the project parameters on feasibility of the project.

As a conclusion, today, the simulation method has become a method commonly used for evaluating investment projects under risky conditions. However, despite the significant advantages it enjoys, there are two important points which require attention during the evaluation of projects through the simulation method.

As known that the one of the key step in developing a simulation model is to generate a random number. There are numerous subroutines available on practically every computer system that generates random numbers. These random numbers are generated by mathematical processes as opposed to a physical process, such as spinning a roulette wheel. For this reason, they are referred to as *pseudo random numbers.* They are not true random numbers. True random numbers can be produced only by a physical process, such as spinning a roulette wheel over and over. However, a physical process, such as spinning a roulette wheel, cannot be conveniently employed in a computerized simulation model. Thus, there is a need for a numerical method that artificially creates random numbers (Taylor, 1999).

When random numbers are generated through any methods, one can enter a cycle following a certain amount of replication since these numbers are not true random numbers. In other words, after a while, formerly generated numbers may start to be generated once again. If this happens, the NPVs to be calculated will be the same since the same random numbers will be generated at each cycle all the time. In order to prevent this situation, the first way is to make replication at a number which will not lead to a cycle. The second way is to use the functions of random number generation programs which allow generating different numbers at each replication (Eski & Armaneri, 2006).

When risky investment projects are simulated, the random variables affecting the NPV of a project are generally assumed to be independent to each other. However, it must be recognized that some of the random variables affecting the NPV of a project may be related to one another. If they are, it is needed to sample from distributions of the random variables in a manner that accounts for any dependency. This issue can be critical, as the results obtained from a simulation analysis can be misleading if the analysis does not account for the dependent relationships. The sampling techniques for these dependent random variables are beyond the scope of this section, but can be found in many simulation textbooks. In this dissertation, while developing the simulation models, the dependencies between some project parameters have been taken into account.

4.5 Summary of Chapter

This chapter has been devoted to describe the usage of simulation in risky investment project evaluation and selection process. In this chapter, firstly, Monte Carlo simulation of a risky investment project has been described. Then, the computer simulation modeling of a risky investment project which will be used in the proposed methodology has been explained in details. This section covers computer simulation model building, simulation software packages types, and output analysis of simulation. At the end of this chapter, advantages of using simulation in risky investment projects evaluation and the key points that should be taken into account in investment project evaluation via the simulation method have been presented.

CHAPTER FIVE
FUZZY MATHEMATICAL PROGRAMMING: AN OVERVIEW

The main objective of this chapter is to review the basic concepts of fuzzy set theory (FST) proposed by Zadeh (1965), and to present an overview of fuzzy mathematical programming, which will be used in the proposed methodology in this research. In this chapter, a brief overview of fuzzy sets is first presented. Then, decision making in fuzzy environments is examined. In section 5.3, Fuzzy linear programming (FLP) and fuzzy multi-criteria analysis are reviewed.

5.1 Introduction

In project evaluation and selection process, certainty indicates that it is assumed the parameters of the project to be definitely known, and that there are no doubts about their values or their occurrence. However, project parameters such as costs, revenues, project completion time, interest rates, inflation rates, budget availability normally change during a life cycle of the project. Therefore, an existence of a deviation or gap between forecasted values and actual values is inevitable. Because of the uncertainty and risk of the future, the parameters of alternative projects can not be estimated with complete certainty. Any wrong value that is estimated by the decision maker will directly affect the return and the profitability of the project. For instance, if the cash flows of any project alternative are estimated higher than it is, real profitability of the project will be under than expected. In addition, sometimes the wrong alternative can be accepted and implemented because of this wrong estimation. In this situation, the entrepreneur will face waste use of resources for the project, which are already limited.

It is important how to express the distributions of project parameters. For the purpose of modeling, it is assumed that each parameter will be entered into the model by three ways; a single deterministic value, with probability distribution, and with fuzzy numbers.

If a single deterministic value for each parameter is used, the reliability of the analyses depends upon the accuracy of these deterministic values. A fundamental limitation of this assumption is that the various investment project parameters cannot be practically assumed a higher degree of certainty. The value of each parameter is affected by a myriad of risks and uncertainties which are often difficult to quantify. Because of that reason, it is necessary to express the project parameters as probability distributions or fuzzy numbers in order to analysis of risk and uncertainty.

In probability theory and statistics, a probability distribution describes the range of possible values that a random variable can attain and the probability that the value of the random variable is within any subset of that range. As known, a random variable is a parameter of variable that can have more than one possible value. The value of a random variable at any one time is unknown until the event occurs, but the probability that the random variable will have a specific value is known in advance. As mentioned in Section 2.5.3, random variables are classified as either discrete or continuous.

In risky environments, the value of a risky project parameter at any one time is unknown until the event occurs. However, the probability that the project parameter will have a specific value is known. From this point of view, the risky project parameters can be defined as random variables and expressed as probability distributions. On the other hand, in uncertain environments, there is no way to assign any probabilities to future random events. While probability theory can be a powerful tool in the appropriate circumstances, some times the type of uncertainty encountered in investment projects does not fit the axiomatic basis of probability theory. Simply because, uncertainty in the projects is usually caused by the inherent fuzziness of the parameter estimate rather than randomness (Choobineh & Behrens, 1992).

One way to alleviate this shortcoming is to use the FST where the user needs only to determine a possible range, and perhaps even a most likely value for each investment parameter, without the input of each factor’s relative frequency.

In the middle of the previous century, operations research began to be applied to real-world decision making problems and thus became one of the most important fields in science and engineering. Unfortunately, real-world problems are often not deterministic. Thus, precise mathematical models are not enough to tackle all practical problems. To deal with imprecision/uncertainty, concepts and techniques of probability theory have been reconsidered and criticized when modeling practical problems. Around the same time, FST was developed by Zadeh (1965). Since then, it has been applied to the fields of operations research, management science, artificial intelligence/expert systems, control theory, statistics and many other fields (Lai & Hwang, 1994).

In operations research, FST has been applied to techniques of linear and nonlinear programming, integer programming, dynamic programming, reliability, quality control, queuing theory, multiple criteria decision making, group decision making, decision support systems, expert systems and so on. It helps to improve oversimplified (crisp) models and provides more robust and flexible models for real-world complex systems, especially those involving human aspects. Thereby, decision makers must not only consider the existing alternatives under given constraints, but also develop new alternatives by considering all possible situations (Lai & Hwang, 1994).

Since past three decades, the FST is widely used in investment project analysis and applied to an appraisal of investment project risk. In literature, a number of works is appeared which relate to the application of fuzzy sets in cash flow analysis (Buckley, 1987; Kuchta, 2000; Mohamed & McCowan, 2001). Calzi (1990) presented principles of widened methods for financial mathematics with fuzzy numbers. Buckley (1992) applies fuzzy numbers for calculating the NPV. In his calculations, a discount rate and a discounting period are expressed in the form of fuzzy numbers. Choobineh & Behrens (1992) present possibility distributions used in issues of economic analyses. Chiu & Park (1994) apply fuzzy numbers to an analysis of cash flows generated by an investment project. They present methods for selecting the best project among a set of projects that are mutually exclusive in a situation when cash flows are described with the fuzzy numbers used. Esogbue & Hearnes (1998) present fuzzy numbers and possibility distributions used in fixed asset replacement calculations. Interestingly, they use a theorem of fuzzy sets for defining the economic life cycle of fixed assets. Kuchta (2000) presents an application of fuzzy sets for making decisions when qualifying investment projects for approval or rejection. Kahraman et al. (2002) analyze methods of calculating various measures of effectiveness when parameters of calculation of effectiveness are presented in the form of fuzzy numbers. Rebiasz (2007) suggests a method for quantification of project risk. This study discusses methods for integrating probability distribution and possibility distribution into a description of the uncertainty of project parameters in calculations of effectiveness and investment project risk.

As emphasized in the first four chapters, this dissertation proposes an integrated multi-criteria decision making methodology for risky investment projects evaluation. The proposed methodology consists of three main stages. In the third stage of the proposed methodology, multi-objective mathematical models are developed. These models are constructed by two ways; *multi-objective linear programming model* and *fuzzy multi-objective linear programming models*. Hence, the main objective of this chapter is to review the basic concepts of FST, and to present an overview of fuzzy mathematical programming, which will be used in the proposed methodology in this research.

Mathematical models are very useful tools for making decisions. They are designed to find the best available solution to a problem subject to a certain set of constraints. For example, a linear programming model can be used to determine the optimal product mix in a production environment. However, when modeling any problem, estimating exact values of the coefficients, the right hand size values of constraints, the target values of goals are difficult tasks. Even if all information can be provided by a decision maker, the uncertainty still exists in the problem. Therefore, in order to reflect this uncertainty, it is needed to construct a model with inexact parameters, constraints and goals (Wang & Wang, 1997).

Two major different kinds of uncertainties, ambiguity and vagueness exist in the real life. While ambiguity is associated with one to many relations, that is, situations in which the choice between two or more alternatives is left unspecified, vagueness is associated with the difficulty of making sharp or precise distinctions in the world; that is, some domain of interest is vague if it cannot be delimited by sharp boundaries (Inuiguchi & Ramik, 2000). Some uncertain descriptions show ambiguities of the true values. For example, *about 5 minutes* shows that one value around 5 is true but not known exactly. On the other hand, some uncertain descriptions show vagueness, e.g., *substantially larger than* $ 1000 does not define a sharp boundary of a set of satisfactory values but shows that values around 1000 and larger than 1000 are to some extent and completely satisfactory, respectively.

Inuiguchi & Ramik (2000) also classified the fuzzy mathematical programming into three categories in view of the kinds of uncertainties treated in the method;

(1) Fuzzy mathematical programming with vagueness,
(2) Fuzzy mathematical programming with ambiguity,
(3) Fuzzy mathematical programming with vagueness and ambiguity.

The fuzzy mathematical programming in the first category treats decision making problem under fuzzy goals and constraints. The fuzzy goals and constraints represent the flexibility of the target values of objective functions and elasticity of constraints, respectively. The second category in fuzzy mathematical programming treats ambiguous coefficients of objective functions and constraints but does not treat fuzzy goals and constraints. The last type of fuzzy mathematical programming treats ambiguous coefficients as well as vague DM's preference.

There are a lot of fuzzy mathematical programming types. It would take a lot of space and time to introduce all those types of fuzzy mathematical programming. Thus, after giving the basic concepts of FST, in the following sections, fuzzy mathematical programming types which are used in the proposed methodology will be described.

5.2 Fuzzy Sets

In 1965, L.A. Zadeh published his famous paper "Fuzzy sets" providing a new mathematical tool which enables us to describe and handle vague or ambiguous notions such as "a set of all real numbers which are much greater than 1", or "the set of tall men". Since then, FST has been rapidly developed by Zadeh himself and numerous researchers, and an increasing number of successful real applications of this theory in a wide variety of unexpected fields have been appearing. The main idea of FST is quite intuitive and natural: instead of determining the exact boundaries as in an ordinary set, a fuzzy set allows no sharply defined boundaries because of a generalization of a characteristic function to a membership function (Sakawa, 1993).

In real life, some information can only be approximately determined. For instance, "The processing time is about 13 min" shows that one value around 13 is true but not known exactly. This situation can be defined by an ordinary set in which the set of numbers L from 12 to 14 is crisp, and can be written as, $L = \{r \in \Re | 12 \leq r \leq 14\}$ and also the characteristic function of this set is as follows (Bezdek, 1993);

$$C_L(r) = \begin{cases} 1 & 12 \leq r \leq 14 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad C_L : \Re \rightarrow \{0,1\} \tag{5.1}$$

The values of C_L are equal to 1, when r is in L; otherwise C_L is equal to zero. So ordinary sets correspond to two-valued logic; is or isn't, black or white, 1 or 0. Unlike two-valued conventional logic, fuzzy logic is multi-valued. It deals with degrees of membership and degrees of truth. Fuzzy logic uses the continuum of logical values between 0 and 1. Instead of just black and white, it employs the spectrum of colours, accepting that things can be partly true and partly false at the same time (Bezdek, 1993).

In other words, in ordinary set, an element of the universe either belongs to or does not belong to the set. That is, the membership of an element is crisp-it is either yes (in the set) or no (not in the set). A fuzzy set is a generalization of an ordinary set in that it allows the degree of membership for each element to range over the unit interval [0, 1]. Thus, the membership function of a fuzzy set maps each element of the universe of discourse to its range space which, in most cases, is set to the unit interval. One of the biggest differences between crisp and fuzzy sets is that the former always have unique membership functions, whereas every fuzzy set has an infinite number of membership functions that may represent it (Lin & Lee, 1996).

As its name suggests, fuzzy logic is the logic underlying modes of reasoning which are approximate rather than exact. The importance of fuzzy logic derives from the fact that most modes of human reasoning and especially common sense reasoning are approximate in nature.

The essential characteristics of fuzzy logic are defined as follows (Zadeh, 1965):

(1) In fuzzy logic, exact reasoning is viewed as a limiting case of approximate reasoning.
(2) In fuzzy logic everything is a matter of degree.
(3) Any logical system can be fuzzified.
(4) In fuzzy logic, knowledge is interpreted as a collection of elastic or, equivalently, fuzzy constraint on a collection of variables.
(5) Inference is viewed as a process of propagation of elastic constraints.

For understanding the difference between ordinal (crisp) sets and fuzzy sets, suppose that ages are denoted by a numerical-valued variable which ranges over the interval X=[0, α). Then the set of ages less than or equal to 20 is obviously an ordinary (crisp) set. However, the set of "young ages" has no sharply defined boundaries and can be interpreted as a fuzzy set A of X.

The difference between conventional set theory and FST can be easily seen from the temperature of a room example (Aziz & Parthiban, 1996). Conventional set theory can be somewhat limiting if we wish to describe s humanistic problem mathematically. Figure 5.1 illustrates how conventional sets characterize the temperature of a room.

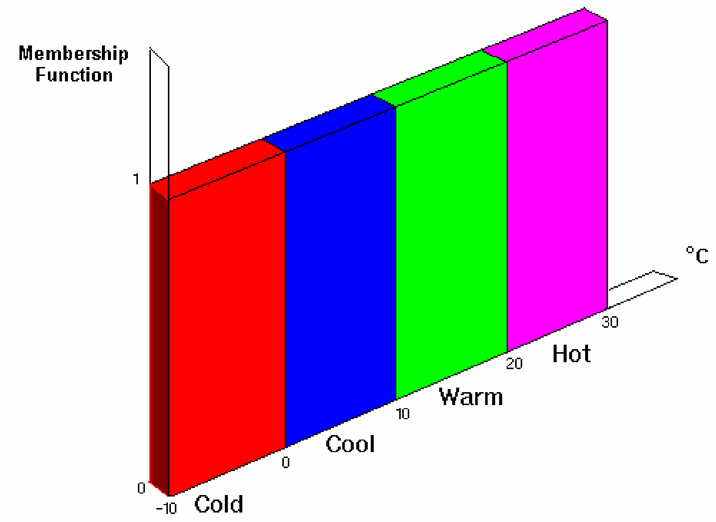


Figure 5.1 Conventional sets to characterize the temperature of a room

As seen from Figure 5.1, the conventional set theory is not sufficient to define a transition from warm to hot by the increment of one degree of centigrade of heat. In the real world a smooth drift from warm to hot would occur. This natural phenomenon can be described more accurately by FST (Aziz & Parthiban, 1996). Figure 5.2 shows how fuzzy sets quantifying the same information can describe this natural drift.

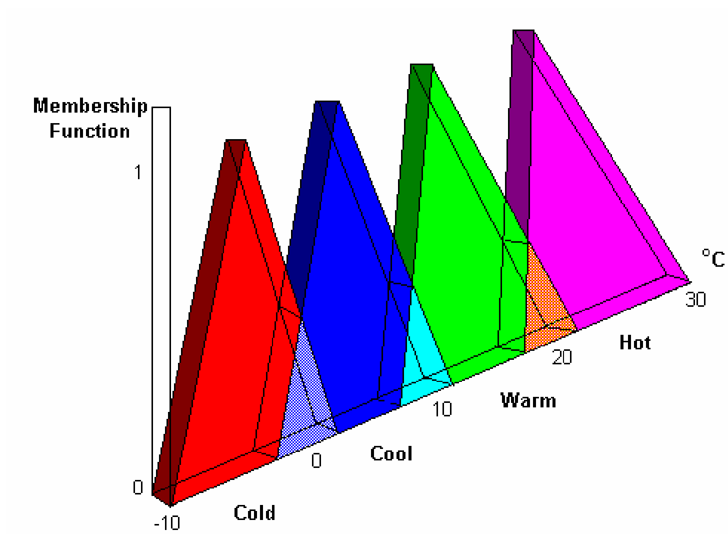


Figure 5.2 Fuzzy sets to characterize the temperature of a room

In general, a fuzzy set initiated by Zadeh (1965) is defined as follows (Sakawa, 1993, p.7)

Let X denote a universal set. Then a fuzzy subset $\tilde{A}$ of X is defined by its membership function;

$$\mu_{\tilde{A}} : X \rightarrow [0,1] \tag{5.2}$$

which assigns to each element $x \in X$ a real number $\mu_{\tilde{A}}(x)$ in the interval [0,1], where the value of $\mu_{\tilde{A}}(x)$ at x represents the grade of membership of x in $\tilde{A}$. Thus, the nearer the value of $\mu_{\tilde{A}}(x)$ is unity, the higher the grade of membership of x in $\tilde{A}$.

A fuzzy subset $\tilde{A}$ can be characterized as a set of ordered pairs of element x and grade $\mu_{\tilde{A}}(x)$ and is often written;

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\} \tag{5.3}$$

When the membership function $\mu_{\tilde{A}}(x)$ contains only the two points 0 and 1, then $\mu_{\tilde{A}}(x)$ is identical to the characteristic function $C_A : X \rightarrow \{0,1\}$, and hence, $\tilde{A}$ is no longer a fuzzy subset, but an ordinary set *A*. As is well known, an ordinary set *A* is expressed as;

$$A = \{x \in X | C_A(x) = 1\} \tag{5.4}$$

Through its characteristic function

$$C_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases} \tag{5.5}$$

A fuzzy subset is always defined as a subset of a universal set *X*. for the sake of convenience; a fuzzy subset is usually called a fuzzy set by omitting the term “sub”. A fuzzy set is often denoted by $\tilde{A}, \tilde{B}, \tilde{C}, \ldots,$ but it is sometimes written as *A, B, C,*…for simplicity in the notation.

When X is a finite set whose elements are x_1, x_2,…, x_3, a fuzzy set A on X is expressed as follows (Sakawa, 1993);

$$A = \{(x_1, \mu_A(x_1)), (x_2, \mu_A(x_2)), ..., (x_n, \mu_A(x_n))\} \quad (5.6)$$

According to the notation proposed by Zadeh (1965), this fuzzy set A on X is written as;

$$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + ... + \mu_A(x_n)/x_n \quad (5.7)$$

or more simply

$$A = \sum_{i=1}^{n} \mu_A(x_i)/x_i \quad (5.8)$$

When X is infinite, a fuzzy set A is frequently written as

$$A = \int_X \mu_A(x)/x \quad (5.9)$$

The operations "+" and "∑" do not refer to the ordinary addition but the set-theoretic "or". The integral "∫" can be views as a natural extension of "∑".

5.2.1 Basic Set-Theoretic Operations for Fuzzy Sets

The membership function is obviously the crucial component of a fuzzy set. In is therefore not surprising that operations with fuzzy sets are defined via their membership functions. Several set-theoretic operations involving fuzzy sets originally proposed by Zadeh (1965) are as follows (Sakawa, 1993; Lootsma, 1997; Zimmermann, 2001);

(1) Equality

The fuzzy sets *A* and *B* on *X* are equal, denoted by *A*=*B*, if and only if their membership functions are equal everywhere on *X*;

$$A = B \Leftrightarrow \mu_A(x) = \mu_B(x) \quad \textit{for all} \quad x \in X \tag{5.10}$$

(2) Containment

The fuzzy set *A* is contained in *B* (or a subset of *B*), denoted by $A \subseteq B$, if and only if their membership function is less or equal to that of *B* everywhere on *X*;

$$A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x) \quad \textit{for all} \quad x \in X \tag{5.11}$$

(3) Complementation

The complement of a fuzzy set *A* on *X*, denoted by $\overline{A}$, is defined by

$$\mu_{\overline{A}}(x) = 1 - \mu_A(x) \quad \textit{for all} \quad x \in X \tag{5.12}$$

The complement operation in FST is shown in Figure 5.3.

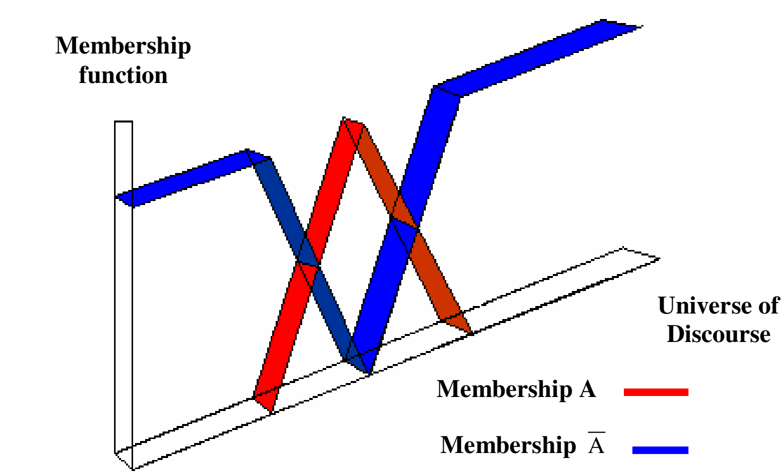


Figure 5.3 The Complement operation in FST

(4) Intersection

The intersection of two fuzzy sets *A* and *B* on *X*, denoted by $A \cap B$, is defined by;

$$\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\} \quad \textit{for all} \quad x \in X \qquad (5.13)$$

The intersection $A \cap B$ is the largest fuzzy set which is contained in both *A* and *B*. The intersection operation in FST is shown in Figure 5.4.

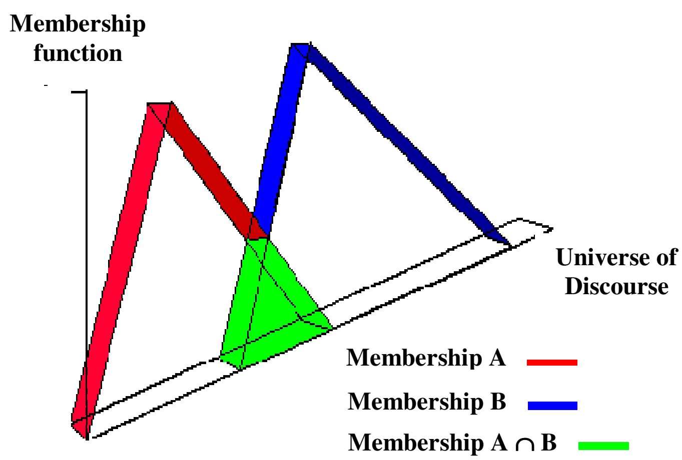


Figure 5.4 The Intersection operation in FST

(5) Union

The union of two fuzzy sets *A* and *B* on *X*, denoted by $A \cup B$, is defined by;

$$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\} \quad \textit{for all} \quad x \in X \qquad (5.14)$$

The union $A \cup B$ is the smallest fuzzy set containing both *A* and *B*. The union operation in FST is shown in Figure 5.5.

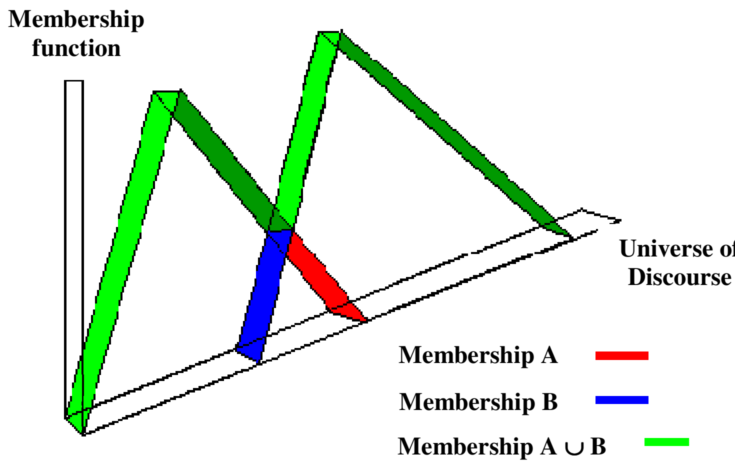


Figure 5.5 The Union operation in FST

5.2.2 α – Level Set of a Fuzzy Set

The concept of α - level sets serves as an important transfer between ordinary sets and fuzzy sets. It also plays an important role in the construction of a fuzzy set by a series of ordinary sets.

The α - level set of a fuzzy set A is defined as an ordinary set A_α for which the degree of its membership function exceeds the level α (Sakawa, 1993):

$$A_\alpha = \{x \mid \mu_A(x) \geq \alpha\}, \quad \alpha \in [0, 1]. \tag{5.15}$$

Observe that the α - level set A_α can be defined by the characteristic function

$$c_{A_\alpha} = \begin{cases} 1 & if \;\; \mu_A(x) \geq \alpha \\ 0 & if \;\; \mu_A(x) < \alpha \end{cases} \tag{5.16}$$

since it is an ordinary set. Actually, an α - level set is an ordinary set whose elements belong to the corresponding fuzzy set to a certain degree α.

It is clear that the following evident property holds for the α - level set;

$$\alpha_1 \le \alpha_2 \Leftrightarrow A_{\alpha_1} \supseteq A_{\alpha_2} \tag{5.17}$$

This relationship is illustrated in Figure 5.6.

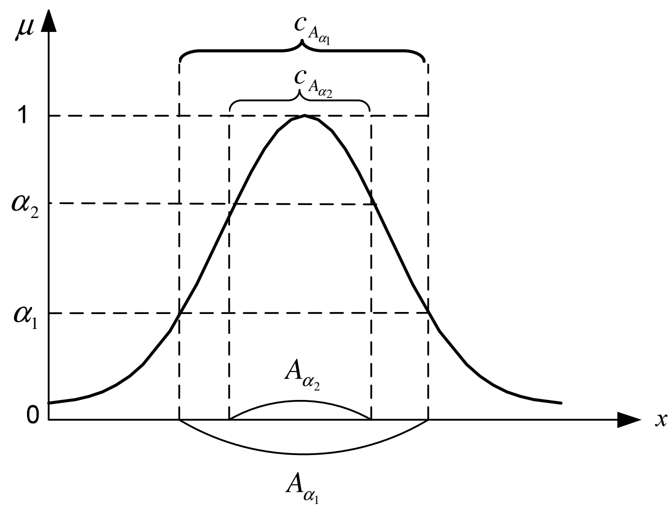


Figure 5.6 Examples of α - level sets (Sakawa, 1993)

5.2.3 Fuzzy Numbers

Before introducing the definition of fuzzy numbers, an extension of ordinary convex sets to fuzzy sets should be considered. Naturally, a convex fuzzy set is defined through its membership function in a real *n*-dimensional Euclidean space R^n.

A fuzzy set A in X=R^n is said to be a convex fuzzy set if and only if its α - level sets are convex. In other words, a fuzzy set A is convex if and only if (Sakawa, 1993)

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \ge \min(\mu_A(x_1), \mu_A(x_2)) \tag{5.18}$$

for all $x_1, x_2 \in X$ *and* $\lambda \in [0,1]$.

Among fuzzy sets, numbers such as "approximately *m*" or "about *n*" can be defined as fuzzy sets of the real line R^1. Such fuzzy numbers are formally defined as follows (Dubois & Prade, 1980; Zimmermann, 2001);

A fuzzy number is a convex normalized fuzzy set A of the real line R^1 whose membership function is piecewise continuous.

A membership function of a fuzzy number has the functional value $\mu_A(x) = 1$ at precisely one element.

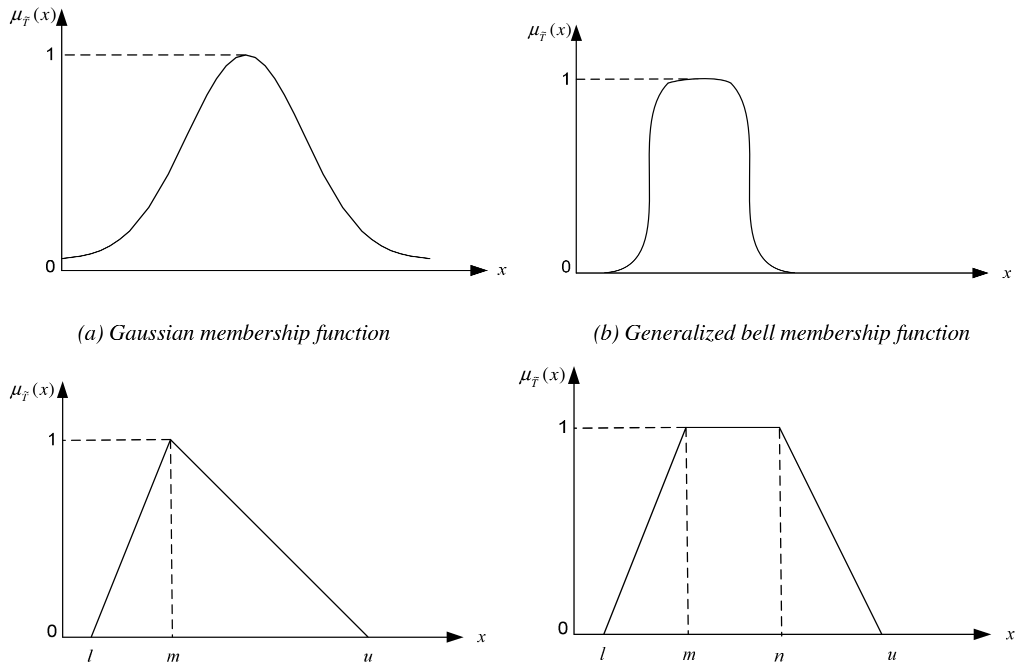
In general, a fuzzy number has a membership function which increases monotonically from 0 to 1 on the left-hand side; thereafter, there is a single top or a plateau at the level 1; and finally, the membership function decreases monotonically to 0 on the right-hand side (Lootsma, 1997).

A fuzzy number is a quantity whose value is imprecise, rather than exact as is the case with "ordinary" (single-valued) numbers. In many respects, fuzzy numbers depict the physical world more realistically than ordinary numbers.

Various types of membership functions can be used to represent the fuzzy numbers. Functional forms of the basic membership functions namely Gaussian, Generalized Bell, Triangular and Trapezoidal membership functions are shown in Figure 5.7.

5.3 Decision Making in a Fuzzy Environment

The term *decision* can have very many different meanings. In classical (normative, statistical) decision theory, a decision can be characterized by a set of decision alternatives (the decision space); a set of states of nature (the state space); a relation assigning to each pair of a decision and state a result; and finally, the utility function that orders the results according to their desirability. When deciding under certainty, the decision maker knows which state to expect and chooses the decision alternative with the highest utility, given the prevailing state of nature.



(a) Gaussian membership function
(b) Generalized bell membership function
(c) Triangular membership function
(d) Trapezoidal membership function

Figure 5.7 Types of membership functions

In 1970, Bellman & Zadeh considered the classical model of a decision and suggested a model for decision making in a fuzzy environment that has served as a point of departure for most of the authors in "fuzzy" decision theory. They consider a situation of decision making under uncertainty, in which the goal as well as the constraint(s) are fuzzy, and argue as follows: The goal is characterized by its membership function, and so are the constraints. Since it is wanted to satisfy the goal as well as the constraints, a decision in a fuzzy environment is defined by analogy to nonfuzzy environments as the selection of activities that simultaneously satisfy goal(s) *and* constraints. The "decision" in a fuzzy environment can therefore be viewed as the intersection of fuzzy constraints and fuzzy goal(s) (Zimmermann, 2001, pp.329-331).

Sakawa (1993) introduce the conceptual framework for decision-making in a fuzzy environment as follows;

Let X be a given set of possible alternatives which contains the solution of a decision-making problem under consideration.

A fuzzy goal G is a fuzzy set on X characterized by its membership function

$$\mu_G : X \to [0,1] \tag{5.19}$$

A fuzzy constraint C is a fuzzy set on X characterized by its membership function

$$\mu_C : X \to [0,1] \tag{5.20}$$

Realizing that both the fuzzy goal and the fuzzy constraint are desired to be satisfied simultaneously, Bellman & Zadeh (1970) defined the fuzzy decision D resulting from the fuzzy goal G and fuzzy constraint C as the intersection of G and C. To be more explicit, the fuzzy decision of Bellman & Zadeh is the fuzzy set D on X defined as

$$D = G \cap C \tag{5.21}$$

and is characterized by its membership function

$$\mu_D(x) = \min(\mu_G(x), \mu_C(x)) \tag{5.22}$$

If the decision maker wants to have a crisp decision proposal, it seems appropriate to suggest the dividend with the highest degree of membership in the fuzzy decision. This is called as maximizing decision. It is then defined as;

$$\max_{x \in X} \quad \mu_D(x) = \max_{x \in X} \quad \min(\mu_G(x), \mu_C(x)) \tag{5.23}$$

More generally, suppose that we have k fuzzy goals $G_1, \ldots, G_k$ and m fuzzy constraints $C_1, \ldots, C_k$, then the fuzzy decision D is the intersection of the given goals $G_1, \ldots, G_k$ and the given constraints $C_1, \ldots, C_k$. That is

$$D = G_1 \cap G_2 \cap \ldots \cap G_k \cap C_1 \cap C_2 \cap \ldots \cap C_m \tag{5.24}$$

and the corresponding maximizing decision is defined as

$$\max_{x \in X} \quad \mu_D(x) = \max_{x \in X} \quad \min\left(\mu_{G_1}(x), \ldots, \mu_{G_k}(x), \mu_{C_1}(x), \ldots, \mu_{C_m}(x)\right) \tag{5.25}$$

It is significant to realize here that in the fuzzy decision, the fuzzy goals and the fuzzy constraints enter into the expression for D in exactly the same way. In other words, in the definition of the fuzzy decision, there is no longer a difference between the fuzzy goals and the fuzzy constraints (Sakawa, 1993).

However, depending on the situations, other aggregation patterns for the fuzzy goal G and the fuzzy constraint C may be worth considering. When fuzzy goals and fuzzy constraints have unequal importance, Bellman & Zadeh (1970) also suggested the convex fuzzy decision defined by

$$\mu_D^{co}(x) = \sum_{i=1}^{k} \alpha_i \mu_{G_i}(x) + \sum_{j=1}^{m} \beta_j \mu_{C_j}(x), \tag{5.26}$$

$$\sum_{i=1}^{k} \alpha_i + \sum_{j=1}^{m} \beta_j = 1, \quad \alpha_i, \beta_j \geq 0 \tag{5.27}$$

where the weighting coefficients reflect the relative importance among the fuzzy goals and constraints.

As an example of an alternative definition of a fuzzy decision, the product fuzzy decision defined by

$$\mu_D^{pr}(x) = \left(\prod_{i=1}^{k} \mu_{G_i}(x)\right) . \left(\prod_{j=1}^{m} \mu_{C_j}(x)\right) \tag{5.28}$$

has been proposed (Sakawa, 1993).

5.3.1 Linear Programming and Fuzzy Linear Programming

Linear programming is an algebraic method used solves sets of linear equations. The formal methodology was developed around 1947. Linear programming models shall be considered as a special kind of decision model: The decision space is defined by the constraints; the goal is defined by the objective function; and the type of decision is decision making under certainty.

The classical linear programming problem is written in the following form:

Minimize the linear objective function

$$z = c_1x_1 + c_2x_2 + \ldots + c_nx_n \tag{5.29}$$

subject to the m linear inequality constraints

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &\le b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &\le b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots& \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &\le b_m \end{aligned}\right\} \tag{5.30}$$

and nonnegativity conditions for all variables

$$x_j \ge 0, \qquad j = 1,\ 2, \ldots, n \tag{5.31}$$

where the a_{ij}, b_i and c_j are given constants.

By introducing an n-dimensional row vector $c = (c_1, \ldots, c_n)$, an n-dimensional column vector $x = (x_1, \ldots, x_n)^T$, an m-dimensional column vector $b = (b_1, \ldots, b_m)^T$, and an $m \times n$ matrix $A = \lfloor a_{ij} \rfloor$, this problem can be expressed in a more compact vector-matrix form.

$$\left.\begin{array}{l}\text{minimize } Z = cx \\ \text{subject to } Ax \le b \\ x \ge 0.\end{array}\right\} \qquad (5.32)$$

In contrast to conventional linear programming problem, Zimmermann (1976) proposed to soften the rigid requirements of the DM to strictly minimize the objective function and to strictly satisfy the constraints. Namely, by considering the imprecision or fuzziness of the DM's judgment, he softened the usual linear programming problem into the following fuzzy version (Sakawa, 1993):

$$\left.\begin{array}{l} cx \prec z_0 \\ Ax \prec b \\ x \ge 0 \end{array}\right\} \qquad (5.33)$$

where the symbol "$\prec$" denotes a relaxed or fuzzy version of the ordinary inequality "$\le$". To be more explicit, these fuzzy inequalities representing the DM's fuzzy goal and fuzzy constraints mean that "the objective function *cx* should be essentially smaller than or equal to an aspiration level (target value of the objective function) z_0 of the DM" and "the constraints *Ax* should be essentially smaller than or equal to *b*", respectively. On the other hand, the symbol "$\succ$" denotes a fuzzy version of the ordinary inequality "$\ge$", and can be read as "essentially greater than or equal to".

In the same spirit as the fuzzy decision of Bellman & Zadeh (1970), considering the fuzzy goal and fuzzy constraints as equally important, Zimmermann (1976) expressed the problem as follows:

$$\left.\begin{array}{l} Bx \prec b' \\ x \ge 0 \end{array}\right\} \qquad (5.34)$$

where

$$B = \begin{bmatrix} c \\ A \end{bmatrix}, \qquad b' = \begin{bmatrix} z_0 \\ b \end{bmatrix}. \qquad (5.35)$$

For treating the ith fuzzy inequality $(Bx)_i \prec b_i'$, i = 0,…, m, of the DM's fuzzy inequalities $Bx \prec b'$, he proposed the following linear membership function:

$$\mu_i((Bx)_i) = \begin{cases} 1 & ; \ (Bx)_i \leq b_i' \\ \dfrac{(b_i' + d_i) - (Bx)_i}{d_i} & ; \ b_i' < (Bx)_i \leq b_i' + d_i \\ 0 & ; \ (Bx)_i > b_i' + d_i \end{cases} \tag{5.36}$$

where each d_i is a subjectively chosen constant expressing the limit of the admissible violation of the ith inequality. It is assumed that the ith membership function should be 1 if the ith constraint is well satisfied, 0 if the ith constraint is violated beyond its limit d_i, and linear from 0 to 1. Such a linear membership function is illustrated in Figure 5.8.

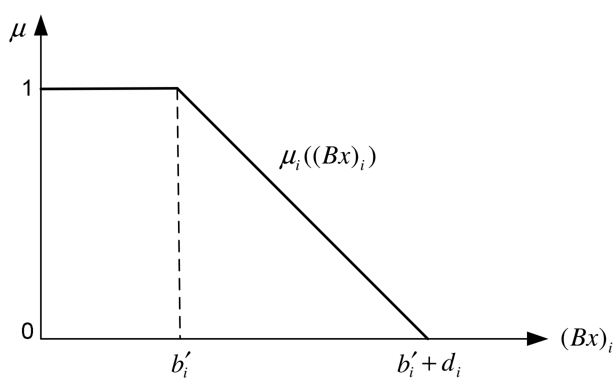


Figure 5.8 "$\prec$" type linear membership function

Following the fuzzy decision of Bellman & Zadeh (1970) together with the linear membership functions, the problem of finding the maximum decision is to choose x^* such that

$$\mu_D(x^*) = \max_{x \geq 0} \min_{i=0,\dots,m} \{\mu_i((Bx)_i)\}. \tag{5.37}$$

In other words, the problem is to find the $x^* \geq 0$ which maximizes the minimum membership function values (Sakawa, 1993).

By introducing the auxiliary variable λ, which is the overall satisfactory level of compromise, the following equivalent model can be obtained:

$$\left.\begin{array}{ll} \text{maximize} & \lambda \\ \text{subject to} & \mu_i((Bx)_i) \geq \lambda, \quad i = 1,\ldots,m \\ & \lambda \geq 0, \quad x \geq 0. \end{array}\right\} \tag{5.38}$$

The fuzzy decision of Bellman & Zadeh (1970) is sometimes called the "*min operator*". As can be seen, this operator aims to maximize the minimum membership grade in the related fuzzy model.

Sommer & Pollastschek (1978) proposed the adoption of the add operator for aggregating the DM's fuzzy goal and fuzzy constraints instead of the minimum operator. Their add operator can be viewed as a special case of the convex fuzzy decision by setting all α_i and β_j equal to 1.

Using the linear membership functions for representing the DM's fuzzy goal and fuzzy constraints and adopting the add operator instead of the minimum operator, the fuzzy version of the original linear programming problem becomes

$$\left.\begin{array}{ll} \text{maximize} & \sum_{i=0}^{m} \mu_i((Bx)_i) \\ \text{subject to} & x \geq 0 \end{array}\right\} \tag{5.39}$$

5.3.2 Fuzzy Multi-criteria Analysis

In the recent past, it has become more and more obvious that comparing the desirability of different means of action, judging the suitability of products, or determining "optimal" solutions in decision problems cannot be done in many cases by using a single criterion or a single objective function. A major concern is that almost all decision problems have multiple, usually conflicting, criteria. Research on how to solve such problems has been enormous. Two major areas have evolved, both of which concentrate on decision making with several criteria;

(1) Multi-objective Decision Making (MODM),
(2) Multi-attribute Decision Making (MADM).

The main difference between these two directions is that the former concentrates on continuous decision spaces, primarily on mathematical programming with several objective functions, and the latter focuses on problems with discrete decision spaces. From a practical viewpoint, MADM is associated with problems whose number of alternatives has been predetermined. The decision maker is to select/prioritize/rank a finite number of courses of action. On the other hand, MODM is not associated with problems in which alternatives have been predetermined. The DM's primary concern is to design a most promising alternative with respect to limited resources (Lai & Hwang, 1994; Zimmermann, 2001).

Generally, real-world decision making problems occur in a somehow uncertain environment. The performance of alternatives, constraints of the problem and goals of decision makers may not be known precisely. FST has contributed to MODM as well as to MADM. Besides MODM problems, FST also successfully applied to a variety of MADM problems. In the following sections, fuzzy MODM and fuzzy MADM will be explained briefly.

5.3.2.1 Fuzzy Multi-objective Decision Making

In the last three decades, MODM techniques have been applied to solve many practical problems such as academic planning, econometrics and development planning, financial planning, capital budgeting, healthcare planning, manpower planning, production planning, transportation planning, traffic management, public administration, water resource management and so on. Symbolically, a general linear MODM problem with K objectives may be stated as (Lai & Hwang, 1994; Selim, 2006);

$$\left.\begin{array}{ll} \max/\min & Z = Cx \\ \text{subject to} & Ax \leq b \\ & x \geq 0 \end{array}\right\} \tag{5.40}$$

where $Z = (z_1, z_2,..., z_k)^T$ is the vector of objectives, C is a $K \times N$ matrix of constants, x is an $N \times 1$ vector of the decision variables, A is an $M \times N$ matrix of constants, and b is an $M \times 1$ vector of constants.

In practice, input data of *C, b, A* are usually fuzzy/imprecise because of incomplete or non-obtainable information. For instance, available labor hours and available material (*b*) may be "*around 1000*" hours and "*about 1500*" units respectively. Similarly, unit profits (*c*) of products may be expected to be "*about $ 25*" per unit and estimates of technological coefficients (*A*) may be "*around 5*" units per labor hour. Imprecise *C*, *b*, and *A*, are described by linguistic terms, but not by the chance concept. Thus, conventional probability theory may not be a correct way to model this imprecise nature. FST, on the other hand, provides better tools to represent a MODM problem with fuzzy input data.

To formulate fuzzy numbers, membership functions or possibility distributions can be used depending on specific problems. The grade of a membership function indicates a subjective degree of satisfaction within given tolerances. On the other hand, the grade of possibility indicates the subjective or objective degree of occurrence of an event (Lai & Hwang, 1994).

A membership function (μ), assigns to each object of a domain its grade of membership in fuzzy set *A*. The nearer the value of membership function to unity, the higher the grade of membership of element or object in a fuzzy set *A*. As mentioned in Section 5.2.3, various types of membership functions can be used to represent the fuzzy set and functional forms of the basic membership functions are shown in Figure 5.7.

It has been shown that use of linear membership functions can provide similar solution quality to that using more complicated nonlinear membership functions (see e.g. Delgado et al., 1993; Sakawa, 1993; Liu & Sahinidis, 1997). Thus, in this dissertation, linear membership functions are adopted. As mentioned, in the third stage of the proposed methodology, fuzzy multi-objective linear programming models have been developed. In these models, the fuzzy goals are characterized by linear membership function which can be seen in Figure 5.8 and Equation (5.36).

The fuzzy constraints are characterized by triangular membership functions which can be seen in Figure 5.7. A triangular fuzzy number can be expressed as $\tilde{T} = (l, m, u)$. When l >0, then $\tilde{T}$ is a positive triangular fuzzy number. The membership function of positive triangular fuzzy number $\tilde{T}$ is defined as:

$$\mu_{\tilde{T}}(x) = \begin{cases} \dfrac{x-l}{m-l} & ; \ l < x \le m \\ \dfrac{u-x}{u-m} & ; \ m < x \le u \\ 0 & ; \ \text{otherwise} \end{cases} \tag{5.41}$$

where $l > 0$.

5.3.2.1.1 Fuzzy Multi-objective Linear Programming. Zimmermann (1978) extended his fuzzy linear programming approach to the general linear MODM problem with *K* objective functions, given in Equation (5.40). Fuzzy multi-objective linear programming model with *maximization* objective is as follows;

$$\left.\begin{aligned} &Cx \succ Z \\ &Ax \prec b \\ &x \ge 0. \end{aligned}\right\} \tag{5.42}$$

Zimmermann (1978) suggests using a linear membership function for each of the objective functions $z_k(x) = c_k(x),\ k = 1,...,K$, of this problem. Then the corresponding linear membership function $\mu_k(z_k(x))$ is defined as

$$\mu_k(z_k(x)) = \begin{cases} 1 & ; \quad z_k(x) > z_k^{\max} \\ \dfrac{z_k(x) - z_k^{\min}}{z_k^{\max} - z_k^{\min}} & ; \quad z_k^{\min} < z_k(x) \le z_k^{\max} \\ 0 & ; \quad z_k(x) \le z_k^{\min}. \end{cases} \tag{5.43}$$

Linear membership function $\mu_i(Ax)_i$ for the *i*th constraint is defined as

$$\mu_i((Ax)_i) = \begin{cases} 1 & ; \quad (Ax)_i \le b_i \\ \dfrac{(b_i + d_i) - (Ax)_i}{d_i} & ; \quad b_i < (Ax)_i \le b_i + d_i \\ 0 & ; \quad (Ax)_i > b_i + d_i. \end{cases} \tag{5.44}$$

In Equations (5.43) and (5.44), $\mu_k(z_k(x))$ and $\mu_i(Ax)_i$ denotes the degree of the membership of the goals and the constraints, respectively. As mentioned before, the membership degree expresses the satisfaction of the DM with the solution.

According to Zimmermann (1978), fuzzy multi-objective linear programming model with *minimization* objective is as follows;

$$\left.\begin{array}{l} Cx \prec Z \\ Ax \prec b \\ x \ge 0. \end{array}\right\} \tag{5.45}$$

Then the corresponding linear membership function $\mu_k(z_k(x))$ for each of the objective functions $z_k(x) = c_k(x),\ k = 1,...,K$, is defined as

$$\mu_k(z_k(x)) = \begin{cases} 1 & ; \quad z_k(x) \le z_k^{\min} \\ \dfrac{z_k^{\max} - z_k(x)}{z_k^{\max} - z_k^{\min}} & ; \quad z_k^{\min} < z_k(x) \le z_k^{\max} \\ 0 & ; \quad z_k(x) > z_k^{\max}. \end{cases} \tag{5.46}$$

Figure 5.9 illustrates both types of membership functions for each of the objective functions.

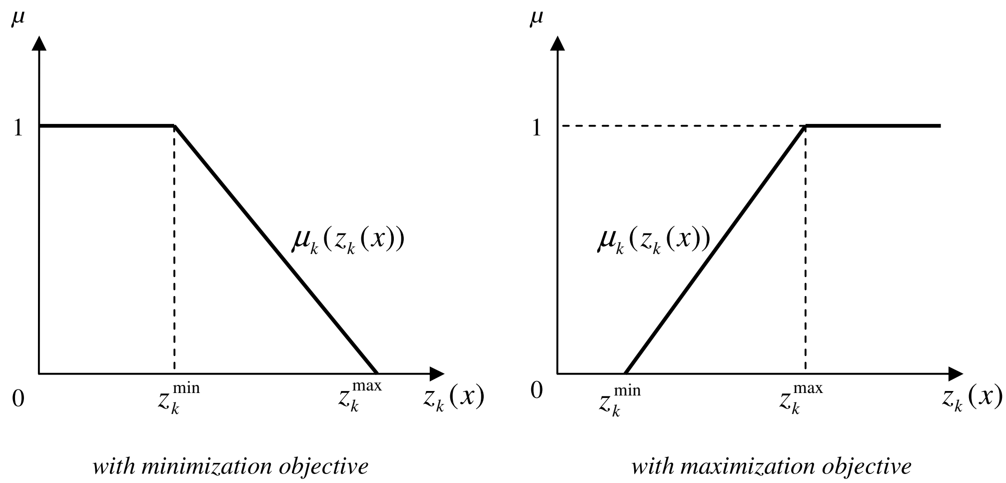


Figure 5.9 The linear membership functions for each of the objective functions

In order to build a fuzzy multi-objective programming model, the DM may establish aspiration levels e.g. $z_k^{\min}$, $z_k^{\max}$ in advance that he or she wants to achieve for the values of the objective functions to be minimized and maximized, respectively, as well as each of the constraints modeled as a fuzzy set by a specific membership function. Hence, the conventional distinction between objectives and constraints no longer applies in fuzzy multi objective linear programming models (Chang & Wang, 1997).

Using the linear membership functions and following the fuzzy decision of Bellman & Zadeh (1970), the original multi-objective linear programming problem can be interpreted as follows (Sakawa, 1993).

$$\text{maximize} \quad \min_{\substack{k=1,\dots,K \\ i=1,\dots,M}} \{\mu_k(z_k(x)),\ \mu_i(Ax)_i\}. \tag{5.47}$$

By introducing the auxiliary variable λ, this problem can be reduced to the following conventional linear programming problem;

$$\left.\begin{array}{lll} \text{maximize} & \lambda & \\ \text{subject to} & \mu_k(z_k(x)) \geq \lambda, & k = 1,\ldots, K \\ & \mu_i(Ax)_i \geq \lambda, & i = 1,\ldots, M \\ & \lambda \geq 0, \quad x \geq 0. & \end{array}\right\} \tag{5.48}$$

As mentioned in Section 5.2.3, various types of membership functions can be used to support the fuzzy analytical framework although the fuzzy description is hypothetical and membership values are subjective (Chang & Wang, 1997).

Chang & Wang (1997) states that the use of FST to improve the multi-objective linear programming model may present at least four contributions:

(1) Fuzzy uncertainties embedded in the model parameters can be directly reflected into the optimization processes.

(2) The variation or vagueness of the DM's aspiration level in the fuzzy multi objective linear programming model can further be incorporated and thereby generate a more confident solution set for policy decision making.

(3) Regardless of the orientations of the DM's aspiration level (i.e., maximization or minimization of specific targets), each objective or goal may have its own independent membership function and different aspiration levels.

(4) The solution procedure of fuzzy multi objective linear programming is dramatically simplified when compared with conventional multi-objective programming. The fuzzy multi objective linear programming configuration does not have to search for the satisfactory solution in a set of noninferior solutions by distance based criteria, as required by the conventional solution procedure of the deterministic multi-objective programming model.

5.3.2.1.2 Fuzzy Multi-objective Modeling Approaches. In the following sections, six fuzzy multi-objective modeling approaches will be presented.

- *Fuzzy Goal Programming*

Goal programming (GP) was originally proposed by Charnes & Cooper (1961). It has been further developed by Lee (1972), Ignizio (1976) and Romero (1991) among others. This method deals with multiobjective linear programming problems that assumed the decision maker could specify goals or aspiration levels for the objective functions. It is one of the most powerful MODM approaches in practical decision making. The key idea behind GP is to minimize the deviations from goals or aspiration levels set by the decision maker.

A standard GP formulation requires that the target values of the goals and the parameters of the constraints are precisely known a priori. However, one of the major drawbacks for a decision maker in using GP is to determine precisely the goal value of each objective function (Arıkan & Güngör, 2001).

GP focuses to minimize the distance between Z_k and an aspiration level or target value of the objective function $\overline{Z}_k$, In GP, the distance between Z_k and $\overline{Z}_k$ is expressed by the deviational variables. In FGP, membership function values of the each objective replace by the deviational variables (Mohamed, 1997).

FST in GP was first considered by Narasimhan (1980). Narasimhan & Rubin (1984), Hannan (1981), Ignizio (1982b) and Tiwari et al. (1986, 1987) extended the FST to the field of GP. Ramik (2000), Rao et al. (1988), Wang & Fu (1997), Mohamed (1997)**,** Ohta & Yamaguchi (1996), Abd El-Wahed & Abo-sinna (2001) and Mohammed (2000) have investigated various aspects of decision problems using FGP theoretically.

The main difference between fuzzy GP and fuzzy linear programming is that the fuzzy linear programming uses the definite intervals determined from solutions of the linear programming models and so the solution does not change from decision maker to decision maker, whereas in fuzzy GP, aspiration levels are specified by decision maker and reflect relative flexibility (Arıkan & Güngör, 2001).

A typical fuzzy GP problem formulation can be stated as follows:

$$\left.\begin{array}{ll} Find \quad x_i\,, & i=1,\ldots,n \\ Z_m(x_i) \prec \overline{Z}_m & m=1,\ldots,M \\ Z_k(x_i) \succ \overline{Z}_k & k=M+1,\ldots,K \\ g_j(x_i) \le b_j & j=1,\ldots,J \\ x_i \ge 0 & i=1,\ldots,n \end{array}\right\} \quad (5.49)$$

where,

$Z_m(x_i)$ = the *m*th goal constraint,

$Z_k(x_i)$ = the *k*th goal constraint,

$\overline{Z}_m(x_i)$ = the target value of the *m*th goal,

$\overline{Z}_k(x_i)$ = the target value of the *k*th goal,

$g_j(x_i)$ = the *j*th inequality constraint,

b_j = the available resource of inequality constraint *j*.

In formulation (5.49), the symbols "$\prec$ and $\succ$" denote the fuzzified versions of "$\le$ and $\ge$" and can be read as "approximately less / greater than or equal to". These two types of linguistic terms have different meanings. Under "approximately less than or equal to" situation, the goal *m* is allowed to be spread to the right-hand-side of $\overline{Z}_m$ ($\overline{Z}_m = l_m$ where l_m denote the lower bound for the *m*th objective) with a certain range of r_m ($\overline{Z}_m + r_m = u_m$, where u_m denote the upper bound for the *m*th objective). Similarly, with "approximately greater than or equal to", p_k is the allowed left side of $\overline{Z}_k$ ($\overline{Z}_k$ - $p_k = l_k$, and $\overline{Z}_k = u_k$) (Wang & Fu, 1997).

As can be seen, GP and fuzzy GP have some similarities. Both of them need an aspiration level for each objective, which is determined by decision maker. In addition to the aspiration levels of the goals, fuzzy GP needs max-min limits (u_k, l_k) for each goal (Mohamed, 1997).

While the decision makers decide the max-min limits, the linear programming results are starting points and the intervals are covered by these results. Generally, the decision makers find estimates of the upper (u) and lower (l) values for each goal using payoff table as seen in Table 5.1. Therefore, the feasibility of each fuzzy goal is guaranteed (Selim, 2006).

Table 5.1 The payoff table

	$Z_1(X)$	$Z_2(X)$	$\ldots$	$Z_M(X)$
$X^{(1)}$	Z_{11}	Z_{12}	$\ldots$	Z_{1M}
$X^{(2)}$	Z_{21}	Z_{22}	$\ldots$	Z_{2M}
$\vdots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
$X^{(M)}$	Z_{M1}	Z_{M2}	$\ldots$	Z_{MM}

Here, $Z_m(X)$ denotes the *m*th objective function, and $X^{(m)}$ is the optimal solution of the *m*th single objective problem. Solving the problem with $X^{(m)}$ (m = 1,…, M) for each objective, a payoff matrix with entries $Z_{pm} = Z_m(X^{(p)})$, m, p=1,…, M can be formulated as presented in Table 5.1. Here, $u_m = \max(Z_{1m}, Z_{2m}, ..., Z_{Mm})$ and $l_m = \min(Z_{1m}, Z_{2m}, ..., Z_{Mm})$, m= 1,…, M.

After constructing fuzzified aspiration levels with respect to the linguistic terms of “approximately less than or equal to”, and “approximately greater than or equal to”, the membership functions can be developed for each goal as follows:

For “approximately less than or equal to”;

$$\mu_{z_m}(x) = \begin{cases} 1 & ; \quad Z_m(x) \le l_m \\ \dfrac{u_m - Z_m(x)}{u_m - l_m} & ; \quad l_m < Z_m(x) \le u_m \\ 0 & ; \quad Z_m(x) > u_m . \end{cases} \tag{5.50}$$

For “approximately greater than or equal to”;

$$\mu_{z_k}(x) = \begin{cases} 1 & ; \; Z_k(x) > u_k \\ \dfrac{Z_k(x) - l_k}{u_k - l_k} & ; \; l_k < Z_k(x) \le u_k \\ 0 & ; \; Z_k(x) \le l_k. \end{cases} \tag{5.51}$$

Using Belman & Zadeh (1970)’s fuzzy decision theorem, the fuzzy solution set is obtained by the intersection of all membership functions representing the fuzzy goals. The membership function $\mu_F(x)$ which characterizes the fuzzy solution can be defined as follows (Sakawa, 1993);

$$\mu_F(x) = \mu_{Z_1}(x) \cap \mu_{Z_2}(x) .... \cap \mu_{Z_k}(x) = \min[\mu_{Z_1}(x), \mu_{Z_2}(x),...., \mu_{Z_k}(x)] \tag{5.52}$$

Then the optimum decision is one that maximizes the minimum membership function values (Sakawa, 1993);

$$\max_{x \in F} \; \mu_F(x) = \max_{x \in F} \; \min[\mu_{Z_1}(x), \mu_{Z_2}(x),..., \mu_{Z_k}(x)] \tag{5.53}$$

By introducing the auxiliary variable λ, which is the overall satisfactory level of compromise, formulation (5.52) can be transformed to the following conventional linear programming problem;

$$\left.\begin{array}{lll} \text{maximize} & \lambda & \\ \text{subject to} & \lambda \le \mu_{Z_k}(x) & k = 1,...,K \\ & g_j(x_i) \le b_j & i = 1,..., n, \quad j = 1,..., J \\ & x_i \ge 0 & i = 1,..., n \\ & \lambda \in [0,1]. & \end{array}\right\} \tag{5.54}$$

- *Tiwari et al.'s Weighted Additive Approach*

Consideration of different relative importance and priority of the goals in the fuzzy GP problem is important because some goals are more important than others (Chen & Tsai, 2001). The preemptive structure in fuzzy environment and the different relative importance of the goals have been investigated by some researches. In order to reflect the relative importance of the goals, the weighted average of membership function values was used by Hannan in 1981. Tiwari et al. (1987) proposed a weighted additive model that incorporates each goal's weight into the objective function in an additive fashion. The weighted additive model proposed by Tiwari et al. (1987) is as follows;

$$\left.\begin{array}{ll} \text{maximize} & \sum_i w_i \mu_{Z_i}(x) \\ \text{subject to} & \mu_{Z_i}(x) \in [0,1], \quad \forall i \\ & x \geq 0. \end{array}\right\} \qquad (5.55)$$

and other system constraints.

where w_i denotes the weight of the ith goal. The weighted additive model maximizes the weighted sum of the achievement levels of the fuzzy goals. To determine the weights of the goals, there are some good approaches in the literature such as analytic hierarchy process (Saaty, 1980), and weighted least square method (Chu et al., 1979). Also, there are some fuzzy approaches for finding crisp weights in fuzzy environment. For more information about these approaches, the reader can refer to see Lai & Hwang (1994).

- *Werners' "Fuzzy and" Operator*

As mentioned before, the fuzzy decision of Bellman & Zadeh (1970) is sometimes called the "*min operator*". As can be seen, this operator aims to maximize the minimum membership grade in the related fuzzy model. It is not a compensatory operator. That is, goals with a high degree of membership are not traded off against goals with a low degree of membership. Therefore, some computationally efficient compensatory operators can be used in setting the objective function in fuzzy programming to investigate better results (Selim, 2006).

One criterion used to evaluate the performance of compensatory operators in fuzzy optimization is *monotonicity*. Among the compensatory operators which are well suited in solving multi-objective programming problems, Werners' (1988) "*fuzzy and*" operator has an advantage of being a strongly monotonically increasing function. That is, it is positively related with the compensation rate. Furthermore, Werners' "*fuzzy and*" operator is easy to handle, and has generated reasonable consistent results in applications (Canz, 1996).

The "*fuzzy and*" operator is formulated as follows (Werners, 1988);

$$\mu_{\mathrm{D}}(x) = \mathrm{Max}\left\{\gamma \min_{k=1}^{K}(\mu_k(x)) + (1-\gamma)(1/K)\sum_{k=1}^{K}\mu_k(x)\right\} \qquad (5.56)$$

where K is the total number of objectives, $\mu_k(x)$ is the membership function of goal k, and γ is the coefficient of compensation defined within the interval [0,1]. By adopting "min operator" into Equation (5.56), the following linear programming problem can be formed (Selim, 2006);

$$\left.\begin{array}{lll} \text{maximize} & \gamma\lambda + (1-\gamma)(1/K)\sum_{k=1}^{K}\lambda_k & \\ \text{subject to} & \mu_k(x) \ge \lambda + \lambda_k\ , & \forall k \in K,\ \forall x \in X \\ & \lambda,\ \lambda_k,\ \gamma \in [0,1]. & \end{array}\right\} \qquad (5.57)$$

and other system constraints.

- *Li's Two-Phase Approach*

Li (1990) proposed a two-phase approach. The solution process is divided into two phases. In the first phase, Zimmermann's *"min operator"* is used to search for an optimal value of λ and to find a possible solution. If the possible solution is unique (usually, we do not know) in phase one, it will be an optimal solution. Otherwise, in phase two, a new program will be formulated to maximize the arithmetic mean value of all memberships restricted by original constraints and $\lambda_k \geq \lambda', \forall k$. Obviously, phase two yields an efficient solution. Its formulation is as follows;

$$\left.\begin{array}{ll} \text{maximize} & \sum_{k=1}^{K} \lambda_k / K \\ \text{subject to} & \lambda' \leq \lambda_k \leq \mu_k(x) \quad k = 1, \ldots, K, \ , x \in X \\ & \lambda', \lambda_k \in [0, 1] \end{array}\right\} \tag{5.58}$$

and other system constraints.

In Li's Two-Phase Approach, λ' is the solution of the problem with "min operator". Li stated that by using the two phase approach we can always obtain an efficient solution. The above equation is essentially equivalent to;

$$\left.\begin{array}{ll} \text{maximize} & \left[\sum_{k=1}^{K} \mu_k(x)\right] / K \\ \text{subject to} & \lambda' \leq \mu_k(x) \quad k = 1, \ldots, K, \ , x \in X \\ & \lambda' \in [0, 1] \end{array}\right\} \tag{5.59}$$

and other system constraints.

- *Lai & Hwang's Approach*

Lai & Hwang (1993) unified both objectives of Equations (5.59) and (5.54) as follows;

$$\left.\begin{aligned} &\text{maximize} && \lambda + \delta\left[\sum_{k=1}^{K} \mu_k(x)\right] / K \\ &\text{subject to} && \lambda \le \mu_k(x) \quad k = 1,\ldots,K, \quad ,x \in X \\ & && \lambda \in [0,1] \end{aligned}\right\} \qquad (5.60)$$

and other system constraints.

where δ is a sufficiently small positive number. However, weights between objectives are not equal. For that reason, Lai & Hwang (1993) proposed the following general form;

$$\left.\begin{aligned} &\text{maximize} && \lambda + \delta\left[\sum_{k=1}^{K} w_k \mu_k(x)\right] \\ &\text{subject to} && \lambda \le \mu_k(x) \quad k = 1,\ldots,K, \quad ,x \in X \\ & && \lambda \in [0,1] \end{aligned}\right\} \qquad (5.61)$$

and other system constraints.

where w_k is the relative weight of the *k*th objective and $\sum_k w_k = 1$.

- *Lin's Weighted Max-Min Approach*

When the decision maker provides relative weights for fuzzy goals with corresponding membership functions, the ratio of the achieved levels should be as close to the ratio of the objective weights as possible to reflect their relative importance (Lin, 2004). Weighted additive approach gives objectives of heavy weight higher achieved levels than others. However, the ratio of the achieved levels is not necessarily the same as that of the objective weights. Thus, Lin (2004) proposed a *weighted max–min model* as follows;

$$\left.\begin{array}{ll} \text{maximize} & \beta \\ \text{subject to} & w_k \beta \leq \mu_k(x), \quad k = 1, 2, \ldots, K, \\ & \mu_k \in [0,1], \end{array}\right\} \tag{5.62}$$

and other system constraints.

Lin's weighted max–min approach aims to find an optimal solution within the feasible area such that the ratio of the achieved levels is as close to the ratio of the weights as possible (Selim, 2006).

5.3.2.2 Fuzzy Multi-attribute Decision Making

In MADM, each alternative is described by using multiple attributes. For a given set o alternatives, MADM models try to choose the best alternative among them, rank the alternatives from the best to the worst or classify them into classes. The general MADM model can be defined as follows (Zimmermann, 2001);

Let $X = \{x_i | i = 1,2,...,n\}$ be a (finite) set of decision alternatives and $G = \{g_j | j = 1,2,...,m\}$ a (finite) set of goals according to which the desirability of an action is judged. Determine the optimal alternative x^0 with the highest degree of desirability with respect to all relevant goals g_j.

Most approaches in MADM consist of two stages;

(1) The aggregation of the judgments with respect to all goals and per decision alternative, and

(2) The rank ordering of the decision alternatives according to the aggregated judgments.

In crisp MADM models, it is usually assumed that the final judgments of the alternatives are expressed as real numbers. In this case, the second stage does not pose any particular problems and suggested algorithms concentrate on the first stage. Fuzzy models are sometimes justified by the argument that the goals, g_j, or their attainment by the alternatives, x_i, respectively, cannot be defined or judged crisply but only as fuzzy sets. In this case, the final judgments are also represented by fuzzy sets, which have to be ordered to determine the optimal alternative. Then, the second stage is, of course, by far not trivial (Zimmermann, 2001).

Many fuzzy methods and models have been suggested to solve the MADM problem. They differ by their assumptions concerning the input data and by the measures used for aggregation and ranking. Also, they concentrate either on the first step (aggregation of ratings), or the second step (ranking), or both. Obviously all of them have advantages and disadvantages. However, they will not be discussed here.

5.4 Summary of Chapter

In this chapter, an overview of fuzzy mathematical programming has been presented. Basic concepts and definitions of fuzzy set theory, fuzzy linear programming, fuzzy multi-objective and fuzzy multi-attribute decision making have been explained in this concern. Fuzzy multi-objective linear programming and fuzzy multi-objective modeling approaches which will be employed in the computational experiments performed in this dissertation have been also explained in this chapter.

CHAPTER SIX
AN INTEGRATED MULTI-CRITERIA DECISION MAKING METHODOLOGY FOR RISKY INVESTMENT PROJECTS EVALUATION

This chapter proposes an integrated multi-criteria decision making methodology for risky investment projects evaluation that consists of three main stages called as opportunity and pre-feasibility studies, feasibility study, and investment project evaluation and decision. In this chapter, computational experiments will be presented in order to explore the application of the proposed methodology. In our experiments, several different well known multi-objective modeling approaches are employed in the third stage of the proposed methodology.

6.1 Introduction

As discussed in the previous chapters, economic resources at disposal are not enough for satisfying all needs and realizing all targets of enterprises or countries. Naturally, limited resources constitute an obstacle to finance all investment alternatives possible and perform all these investment alternatives at the same time. For this reason, both enterprises and countries are supposed to carry out appropriate investments so as to use their resources in a proper and rational manner. Investments are the basic factors which lead notable changes in economy of any country from a macro perspective and enable enterprises to attain their goals and maintain their existence from a micro perspective. In order to provide a success in economic growth, analyzing investment proposals in accordance with scientific fundamentals and taking investment decisions on the basis of results obtained from these analyses is very important.

In this chapter, we propose a novel methodology for risky investment projects evaluation. This proposed methodology consists of three stages. Each stage deals with the problems that should be solved during the process of evaluation of investment projects. Comprehensive literature review suggests that there is a need for an integrated methodology regarding the evaluation and selection of investment projects. Up to present, in their studies, researchers have preferred to seek for solution by dealing with the problems encountered during the process of evaluation of investment projects individually rather than collectively. However, investment project evaluation is an integrated process.

Different from the approaches proposed in the literature, in the methodology proposed in this dissertation, at first, investment projects are classified by using a MCS method which does not require a training sample, and takes into account the inherent risk and uncertainty associated with the values of evaluation criteria. Preliminary election of project ideas and to give prominence to ideas which are promising among other ones is inherently a multi-criteria decision making problem. Therefore, the proposed methodology is based on the multi-criteria evaluation of the investment project alternatives. Second, in order to determine the profitability of the projects, a new NPV formulation that eliminates the weakness of using the traditional formulation of NPV is used.

As mentioned before, it is necessary to consider uncertainty and risk phenomena while evaluating projects. For that reason, in the proposed methodology, in order to calculate the expected profitability of the project and determine the risk level of the projects, the computer simulation model is developed for a developed NPV formulation. Also, the second simulation model is developed in order to calculate the expected cash flows for each project in each period. The last contribution of this dissertation is to construct multi-objective mathematical models such as multi-objective linear programming model and fuzzy multi-objective linear programming models in order to solve the optimal project selection and scheduling problem that was explained in details in the Section 1.1.

This chapter organized as follows; the next section is devoted to explain the proposed integrated methodology for risky investment projects evaluation. Third section demonstrates how the proposed methodology can be applied to the project evaluation and selection problem by means of a hypothetical example. Finally, the last section presents the summary of the chapter.

6.2 Proposed Methodology for Risky Investment Projects Evaluation

In this section, proposed integrated multi-criteria decision making methodology for risky investment projects evaluation that consists of three main stages called as opportunity and pre-feasibility studies, feasibility study, and investment project evaluation and decision is explained in details.

As mentioned in Section 2.3, the researchers divide cycle of any investment project into several phases. The development of an industrial investment project from the stage of the initial idea until the plant is in operation can be shown in the form of a cycle comprising three distinct phases; the pre-investment, the investment, and the operational phases. Each of these three phases is divisible into stages, some of which constitute important consultancy, engineering and industrial activities. These phases and their covered stages can be shown in Figure 2.1, and detailed information about these phases can be found in Section 2.3.

By the fact that the methodology proposed in this dissertation is based on the evaluation of risky investment projects, the investment and the operational phases of the investment project cycle is out of the scope of this dissertation. The pre-investment phase which means the time period between the birth of investment idea and decision-making to invest consists of several stages. The literature review indicates that these stages are generally called as follows; identification of investment opportunities (opportunity studies), pre-feasibility studies, feasibility studies, and investment project evaluation and decision. Section 2.3.1 provides detailed information about these stages. As a consequence, the stages of the proposed methodology have been defined as follows:

- Stage 1: Opportunity and Pre-Feasibility Studies,
- Stage 2: Feasibility Study,
- Stage 3: Investment Project Evaluation and Decision.

The flow diagram of the proposed methodology is shown in Figure 6.1. In the following sections, the stages of the proposed methodology will be explained.

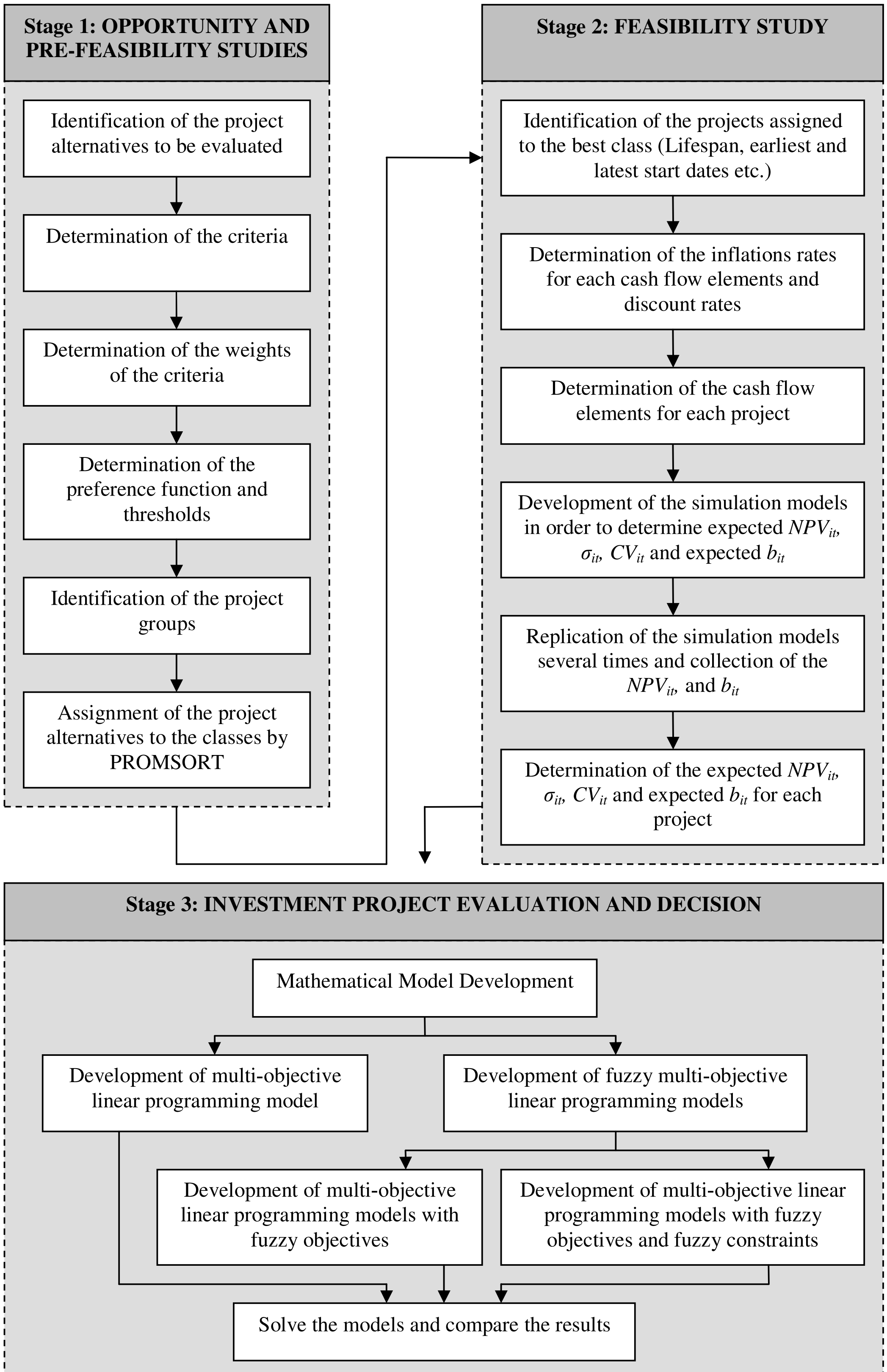


Figure 6.1 Flow diagram of the proposed methodology

6.2.1 Stage 1: Opportunity and Pre-Feasibility Studies

The first stage of the proposed methodology includes opportunity and pre-feasibility studies. The aim of this stage is to identify the investment opportunities, and to carry out preliminary election of project ideas and to give prominence to ideas which have the highest chance of attaining the goals planned by entrepreneurs and investors. Therefore, the project evaluation and selection problem considered in the first stage of the proposed methodology is better addressed through the classification/sorting problematic. Sorting problematic involves the assignment of a set of alternatives in homogenous classes defined in a preference order.

As mentioned in the previous chapters, while determining the promising project ideas among other ones according to the goals of entrepreneurs, in most of time, there are multiple evaluation criteria that should be taken into account. In other words, the preliminary project selection is a MCDM problem in nature. It is obvious that when more than one evaluation criterion exists in the problem, decision making becomes more complex.

Several methodologies have been developed to deal with classification/sorting problems. However, most of them assume that adequate number of reference alternatives have already been determined and use these reference alternatives as training samples to infer some of the model parameter. On the other hand, in the preliminary project selection problem presented in this dissertation, most of time, a set of training sample may not be possible. For these reasons, in the scope of this dissertation, the new MCS procedure, named as PROMSORT, will be used in order to assign project alternatives to predefined ordered classes in the first stage of the proposed methodology.

As mentioned Section 3.6, this MCS procedure does not require a training sample, and by using this method, the inherent risk and uncertainty associated with the values of evaluation criteria can be handled. On the other hand, it is important to point out that this MCS procedure has not been applied to the investment project evaluation and selection problems, until this dissertation. This thesis and our proposed novel methodology is the first one that uses this MCS procedure, PROMSORT, in order to assign project alternatives to predefined ordered classes.

The PROMSORT method and its differences from the other MCS methods have been explained in details in Section 3.6. As seen in this section, PROMSORT, which is a MCS method that assigns alternatives to predefined ordered categories, is proposed by Araz & Ozkarahan (2005) for financial classification problems.

Let F denote the set of indices of the criteria $g_1, g_2, ..., g_j$ $[F = (1,2,..., j)]$ and B the set of indices of the profiles defining (k+1) categories $[B = (1,2,...,k)]$, b_h being the upper limit of category C_h and the lower limit of category C_{h+1}, $h = 1,2,...,k$. Assume that $C_2 > C_1$ means that Category 2 outranks Category 1, the set of profiles $[B = (b_1, b_2,...,b_k)]$ must have the following property (Araz & Ozkarahan, 2005; Araz et al., 2006);

$$[b_k\, P\, b_{k-1}], [b_{k-1} P\, b_{k-2}], ..., [b_2\, P\, b_1] \quad (6.1)$$

PROMSORT assigns alternatives to categories following the three consecutive steps:

(1) Construction of an outranking relation using PROMETHEE I,
(2) Exploitation of the outranking relation in order to assign alternatives to specific categories except the incomparability and indifference situations,
(3) Final assignment of the alternatives based on pairwise comparison.

These consecutive steps have been explained in Section 3.6. In PROMSORT, categories are defined by lower and upper limits like ELECTRE TRI and both profile limits and reference alternatives are used to assign an alternative to a category. In order to determine the reference alternatives, firstly all alternatives are compared with the profile limits using the PROMETHEE outranking relation. In the second step, the assignment of alternatives to categories results directly from the outranking relation. After the second phase, it is possible that some alternatives could not have been assigned to a category, since outranking relation indicates that these alternatives are indifferent or incomparable to a profile limit and could not be assigned to a category directly. On the other hand, some alternatives could be assigned to the categories. In the third stage, these alternatives are used as the reference actions of the categories to be able to assign the alternatives which have not yet been assigned. In Section 3.6.4, the performance of the PROMSORT procedure is explored through an application to country risk evaluation as an illustrative case study.

6.2.2 Stage 2: Feasibility Study

After assigning of the project alternatives to the classes by PROMSORT, the second stage of the proposed methodology called as feasibility study begins. A feasibility study should provide all data necessary for an investment decision. The commercial, technical, financial, economic and environmental prerequisites for an investment project should therefore be defined and critically examined on the basis of alternative solutions already reviewed in the pre-feasibility study.

As known, mostly used criterion to evaluate an investment project is its NPV. Before the economics of a risky engineering project can be evaluated, in order to determine the NPV of a project, it is necessary to reasonably estimate the various cost and revenue components that describe the project. Engineering projects may range from something as simple as the purchase of a new machine to the design and construction of very expensive process or resource recovery complex. In evaluating investment projects, we are concerned the project cash flows that result directly from the investment.

Once the cash flow elements are determined (both inflows and outflows), they may be grouped into three areas (Park, 2002);

(1) Cash flow elements associated with operations,
(2) Cash flow elements associated with investment activities (capital expenditures),
(3) Cash flow elements associated with project financing (such as borrowing)

The main purpose of grouping cash flows this way is to provide information about the operating, investing and financing activities of a project.

A generic version of a cash flow statement is shown in Figure 6.2 where we first determine the net income from operations and then adjust the net income by adding any non-cash expenses, mainly depreciation (or amortization).

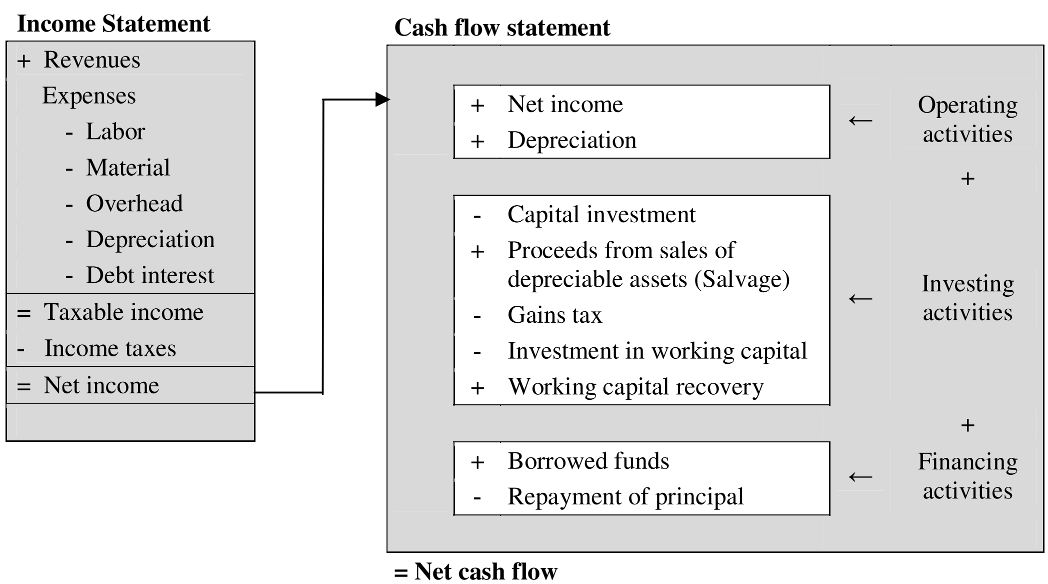


Figure 6.2 A popular format used for presenting a cash flow statement (Park, 2002)

As known, cash flow classified as financing activities include (1) the amount of borrowing and (2) the repayment of principal. Recall that interest payments are tax deductible expenses so that they are classified as operating, not financing, activities.

In the scope of this dissertation, it is tried to select and schedule the risky investment projects. It is wanted to allocate the available budget to each possible risky project in order to achieve the objectives. Therefore, for these types of studies, there are not any financial activities. Because, it is assumed that projects are not financed with borrowed funds. When projects require only operating and investing activities, income statement and cash flow statement of the projects should be generated as shown in the following table.

Table 6.1 Income statement and cash flow statement

Year	**0**	**1**	**2**	**3**	**……**	**N**
Income Statement						
Revenues						
Expenses						
Labor						
Material						
Overhead						
Depreciation						
Taxable Income						
Income Taxes (%)						
Net Income						
Cash Flow Statement						
Operating activities						
Net Income						
Depreciation						
Investment activities						
Investment						
Salvage						
Gains Tax						
Net cash flow						

In the inflation periods, price of the items that compose project incomes and expenses, will increase and also the relative prices of all items will change. Therefore, it is inevitable to make wrong decisions about investment if inflation is not taken into consideration with the matter of fact that it has important effect on the project net cash flows. To introduce the effect of inflation into our economic analysis, we need to define several inflation-related terms (Park, 2002);

- *Actual money units (A_t):* Actual money units are estimated of future cash flows for year *t* that take into account anticipated changes in amount caused by inflationary or deflationary effects.

- *Constant money units (A_t'):* Constant money units represent constant purchasing power independent of the passage of time. In situations where inflationary effects were assumed when cash flows were estimated, these estimates can be converted to constant money units (base year money units) by adjustment using accepted general inflation rate. Here, it is assumed that the base year is 0.

Since constant money units represent money unit amounts expressed in terms of purchasing power of the base year, it can be found the equivalent money units in year *t* using general inflation rate ($\bar{f}$);

$$A_t = A_t'(1+\bar{f})^t \tag{6.2}$$

where A_t' is constant money unit expression for the cash flow occurring at the end of the year *t*, and A_t is actual money unit expression for the cash flow at the end of year *t*. If the estimated general inflation rates are different for each year, A_t should be calculated as follows;

$$A_t = A_t'(1+\bar{f}_0)(1+\bar{f}_1)(1+\bar{f}_2)...(1+\bar{f}_t) \tag{6.3}$$

$$A_t = A_t'\prod_{z=0}^{t}(1+\bar{f}_z) \tag{6.4}$$

where $\bar{f}_0, \bar{f}_1, \bar{f}_2, ..., \bar{f}_t$ represent the estimated general inflation rates for year 0, 1, 2, …*t*, respectively.

Naturally, the unit prices of the individual inflow and outflow components, such as revenues, material cost, labor cost, and overhead cost may increase with different rate from the general inflation rate. Also the increase rates for the price of inflow or outflow items can be different one another. For that reason, some additional arrangements should be performed on the traditional NPV formulation by considering inflation, and its effect on the individual inflow and outflow components.

In order to construct the new NPV formulation that takes into account the different inflation effects on the cash flow elements, at first, inflow and outflow components should be defined separately. Let us assume that the decision maker defines the individual inflow and outflow components and their symbols as in the following;

REV_t : Constant money unit expression for the *revenues* occurring at the end of the year *t*,
LAB_t : Constant money unit expression for the *labor expenses* occurring at the end of the year *t*,
MAT_t : Constant money unit expression for the *material expenses* occurring at the end of the year *t*,
OVE_t : Constant money unit expression for the *overhead expenses* occurring at the end of the year *t*,
DEP_t : Actual money unit expression for the *depreciation expenses* at the end of year *t*.
IIC_t : Actual money unit expression for the *investment cost* at the end of year *t*.
SAL : Constant money unit expression for the *salvage value* occurring at the end of the year *t*,
GTA : Actual money unit expression for the *gains tax amounts* at the end of year *t*.
TAX : Constant *Income Tax Rate* for every year

As mentioned, the new NPV formulation is based on three principles. Firstly, the inflation effect on project inflows and outflows may be different one another. Secondly, the increase rates for the price of inflow or outflow items can be different each other, and the last, the inflation rates for specific cash flow element may be different for each year. Therefore, it should be define different inflation rates for each individual inflow and outflow components, such as revenues, labor cost, material cost, overhead cost, and salvage value. Also, different inflation rates should be defined for each year in the planning horizon. As in inflation rates, the discount rate may be different for each year. So, discount rate should be determined for each year. As a consequence, let us assume that the decision maker defines the inflation rates, discount rate and their symbols as in the following;

e_t : Inflation rate for *revenues* for year *t*.
γ_t : Inflation rate for *labor expenses* for year *t*.
ω_t : Inflation rate for *material expenses* for year *t*.
τ_t : Inflation rate for *overhead expenses* for year *t*.
Ω_t : Inflation rate for *salvage value* for year *t*.
i_t : Discount rate for year *t*.

Here, it is important how to express the distributions of the project parameters. For the purpose of modeling, it is assumed that each parameter will be entered into the model as follows; (1) a single deterministic value, (2) with probability distribution, and (3) with possibility distribution.

If a *single deterministic value* for each parameter is used, the reliability of this stage's output depends upon the accuracy of these deterministic values. A fundamental limitation of this assumption is that the various project parameters cannot be practically assumed a higher degree of certainty. The value of each parameter is affected by a myriad of risks and uncertainties which are often difficult to quantify. Because of that reason, it is necessary to express the risky project parameters as *probability* or *possibility distributions* in order to analyze risk and uncertainty.

As mentioned above, REV_t, LAB_t, MAT_t, OVE_t and SAL represent constant money unit expression occurring at the end of the year t. We calculate actual money unit expressions at the end of year t ($AMUE_t$) for these cash flow elements as follows;

$$AMUE_t \text{ for the revenues} = REV_t \prod_{z=0}^{t} (1+e_z) \tag{6.5}$$

$$AMUE_t \text{ for the labor expenses} = LAB_t \prod_{z=0}^{t} (1+\gamma_z) \tag{6.6}$$

$$AMUE_t \text{ for the material expenses} = MAT_t \prod_{z=0}^{t} (1+\omega_z) \tag{6.7}$$

$$AMUE_t \text{ for the overhead expenses} = OVE_t \prod_{z=0}^{t} (1+\tau_z) \tag{6.8}$$

$$AMUE_t \text{ for the salvage value} = SAL \prod_{z=0}^{t} (1+\Omega_z) \tag{6.9}$$

After estimating cash flow elements and other project parameters such as inflation rates and discount rate, net cash flow from operation determines as follows;

Taxable Income in Period t (TI_t):

$$TI_t = REV_t \prod_{z=0}^{t}(1+e_z) - \left[LAB_t \prod_{z=0}^{t}(1+\gamma_z) + MAT_t \prod_{z=0}^{t}(1+\omega_z) + OVE_t \prod_{z=0}^{t}(1+\tau_z) + DEP_t \right] \tag{6.10}$$

Income Taxes in Period t (IT_t):

$$IT_t = \left\{ REV_t \prod_{z=0}^{t}(1+e_z) - \left[LAB_t \prod_{z=0}^{t}(1+\gamma_z) + MAT_t \prod_{z=0}^{t}(1+\omega_z) + OVE_t \prod_{z=0}^{t}(1+\tau_z) + DEP_t \right] \right\} TAX \tag{6.11}$$

Net Income in Period t (NI_t):

$$NI_t = REV_t \prod_{z=0}^{t}(1+e_z) - \left[LAB_t \prod_{z=0}^{t}(1+\gamma_z) + MAT_t \prod_{z=0}^{t}(1+\omega_z) + OVE_t \prod_{z=0}^{t}(1+\tau_z) + DEP_t \right]$$

$$- \left\{ REV_t \prod_{z=0}^{t}(1+e_z) - \left[LAB_t \prod_{z=0}^{t}(1+\gamma_z) + MAT_t \prod_{z=0}^{t}(1+\omega_z) + OVE_t \prod_{z=0}^{t}(1+\tau_z) + DEP_t \right] \right\} TAX$$

$$NI_t = \left\{ REV_t \prod_{z=0}^{t}(1+e_z) - \left[LAB_t \prod_{z=0}^{t}(1+\gamma_z) + MAT_t \prod_{z=0}^{t}(1+\omega_z) + OVE_t \prod_{z=0}^{t}(1+\tau_z) + DEP_t \right] \right\} [1-TAX] \tag{6.12}$$

Net Cash Flow from Operation in Period t (b_{it}):

$$b_{it} = \left\{ REV_t \prod_{z=0}^{t}(1+e_z) - \left[LAB_t \prod_{z=0}^{t}(1+\gamma_z) + MAT_t \prod_{z=0}^{t}(1+\omega_z) + OVE_t \prod_{z=0}^{t}(1+\tau_z) + DEP_t \right] \right\} [1-TAX] + DEP_t$$

$$b_{it} = \left\{ REV_t \prod_{z=0}^{t}(1+e_z) - \left[LAB_t \prod_{z=0}^{t}(1+\gamma_z) + MAT_t \prod_{z=0}^{t}(1+\omega_z) + OVE_t \prod_{z=0}^{t}(1+\tau_z) \right] \right\} [1-TAX] + DEP_t * TAX \tag{6.13}$$

In this situation, the main objective is to determine in which projects to invest and when to invest. For that reason, it should be determined $NPV_{it(i)}$ that represents the net present value of the project *i* given that it starts in the year *t(i)* of the planning horizon. As a consequence, the final structure of the $NPV_{it(i)}$ formulation can be obtained as given in the equation below;

$$NPV_{it(i)} = \sum_{t=t(i)+u_i}^{t(i)+v_i-1} \frac{\left\{REV_t \prod_{z=0}^{t}(1+e_z) - \left[LAB_t \prod_{z=0}^{t}(1+\gamma_z) + \prod_{z=0}^{t} MAT_t(1+\omega_z) + \prod_{z=0}^{t} OVE_t(1+\tau_z)\right]\right\}[1-TAX] + DEP_t * TAX}{\prod_{z=0}^{t}(1+i_z)}$$
$$- \sum_{t=t(i)}^{t(i)+u_i-1} \frac{IIC_t}{\prod_{z=0}^{t}(1+i_z)} + \frac{SAL \prod_{z=0}^{t(i)+v_i-1}(1+\Omega_z) + GTA}{\prod_{z=0}^{t(i)+v_i-1}(1+i_z)} \quad (6.14)$$

In this equation, u_i represents the length of the project's investment period and v_i represents lifespan of the project. As mentioned, the aim of this dissertation and this proposed methodology is to evaluate risky investment projects. For that reason, it is not logical to determine the project parameters such as; cash flows, discount rates, and completion time in deterministic values, because they generally has stochastic nature. Therefore, in order to analyze risk and uncertainty, the risky project parameters should be expressed as *probability* or *possibility distributions*. The concept of risk most widely used in the evaluation of risky projects is the variability/volatility of the NPV. The variability is often measured in terms of variance.

As easily known that if some of project parameters are expressed as probability or possibility distribution, it should be tried to determine expected $NPV_{it(i)}$, the standard deviation of each $NPV_{it(i)}$ $[\sigma_{it(i)}]$. In this stage of the proposed methodology, also, coefficient of variation $[CV_{it(i)}]$ and expected cash flows of the project *i* given that it proceeds in the year *t* of the planning horizon $[b_{it}]$ should be determined. Because these values will be used in the third stage of the methodology.

When the distributions of risky project parameters are known, expected $NPV_{it(i)}$, $\sigma_{it(i)}$, $CV_{it(i)}$ and expected b_{it} can easily be obtained by means of the simulation technique and simulation model developed for the evaluation of the project.

Simulation approach provides the project parameters to be defined as ***probability distributions*** with their intervals instead of single values. The focus of the simulation technique for the project evaluation is to calculate the variation of profitability of the project by using random numbers generated for project parameters according to their probability distributions. For each run of the simulation a different $NPV_{it(i)}$ is obtained and for the sufficient number of runs, the distribution of $NPV_{it(i)}$ can be constructed. Recent computer technology and software for statistics and simulation have ability to generate random numbers and process them rapidly. Therefore the number of replications is not a time consuming and costly activity for decision process and the higher the number of replications, the more significant parameter values for all combinations to be analyzed (Eski & Armaneri, 2006).

The output of simulation provides an excellent basis for decision making, because it enables the decision maker to view a continuum of risk-return tradeoffs rather than a single point estimate (Keat & Young, 2000).

As mentioned in Section 4.2, Monte Carlo simulation approach enables to generate a sample observation for each risky project parameters in the model. However, if the numbers of the risky project parameters in the evaluation process increases, providing a sufficient number of NPVs would be more difficult by using Monte Carlo simulation. In order to determine the random value for risky project parameter in Monte Carlo simulation, the decision maker has to generate random numbers with the help of a computer or a calculator. However, computer simulation models determine the different random values for project parameters at each run itself and calculate the NPV of the project without entailing such a necessity.

Therefore, in the scope of this dissertation, in order to calculate expected $NPV_{it(i)}$, the standard deviation of each $NPV_{it(i)}$ $[\sigma_{it(i)}]$, and coefficient of variation $[CV_{it(i)}]$, a new computer simulation model has been developed by using ARENA 10.0 simulation software for a new NPV formulation. In other words, the $NPV_{it(i)}$ expression given in Equation 6.14 is executed through the ARENA 10.0 simulation program (Kelton et al., 1997).

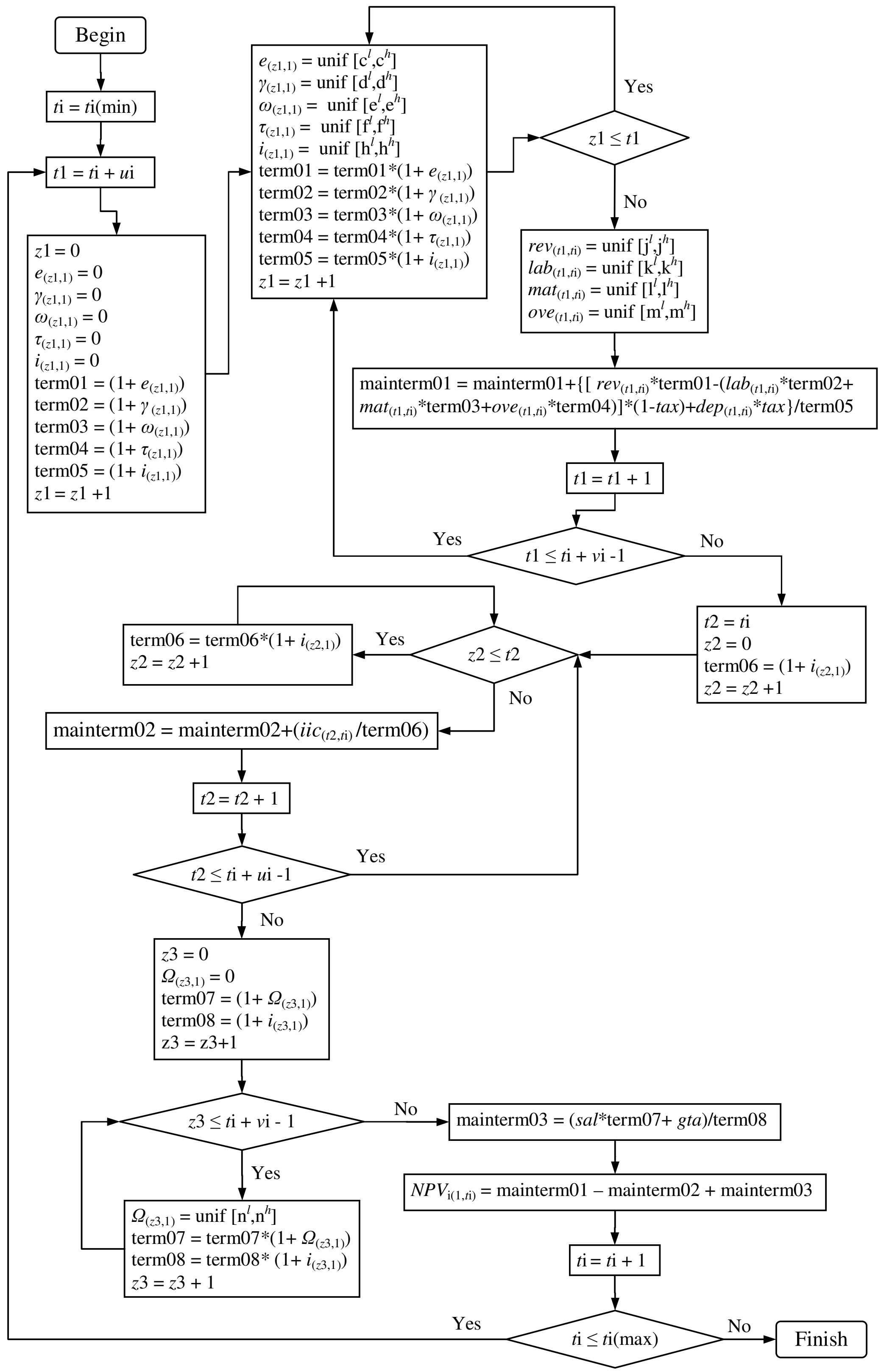


Figure 6.3 The Flowchart of the $NPV_{it(i)}$ calculation event

The synchronization of the random numbers is performed by using one of variance reduction techniques; common random numbers (CRN), and furthermore the verification of the models are tested through the constant numbers entered for each variable. The flowchart of the $NPV_{it(i)}$ calculation event is given in Figure 6.3. Siman report of a computer simulation model for a $NPV_{it(i)}$ formulation can be shown in Appendix A.

In this stage, in order to determine expected cash flows of the project *i* given that it proceeds in the year *t* of the planning horizon $[b_{it}]$, another simulation model has been constructed. The *"net cash flow from operation"* expression is executed through the ARENA 3.0. The flowchart of the b_{it} calculation event is given in Figure 6.4. Siman report of a computer simulation model for a b_{it} formulation can be shown in Appendix B.

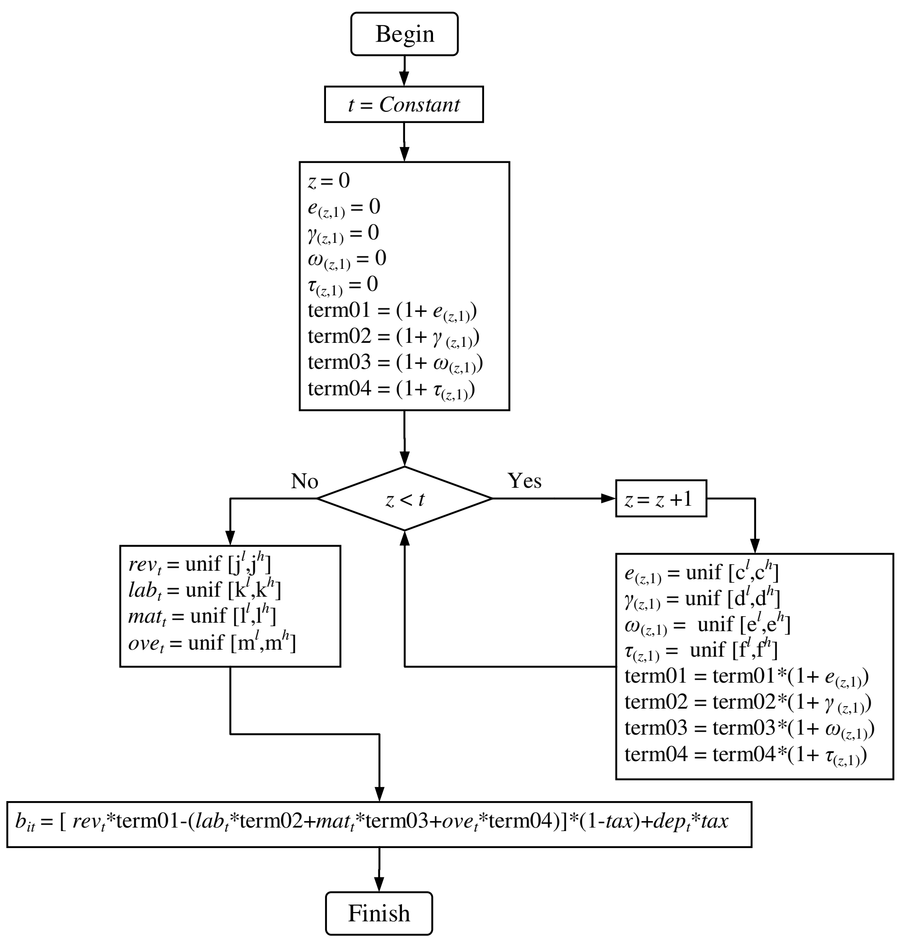


Figure 6.4 The flowchart of the b_{it} calculation event

By the help of these constructed simulation models, all risky project parameters affecting the $NPV_{it(i)}$ of the project can be defined as deterministic values or probability distributions if required. Moreover, with these models, it is also possible to define risky project parameters as specific distribution types such as uniform distribution, normal distribution and exponential distribution. However, if the project parameters are defined as possibility distributions, at first, each of these distributions should be converted into the nearest probability distribution for the further detailed analysis. A practical method is proposed for data transformation from fuzzy number to uniform distribution (Dubois & Prade, 2005; Armaneri et al., 2008). According to this method, to construct sets, uniform random numbers are generated as the membership values and corresponding boundaries are calculated. By generating the significant number of *α-cut* sets, then using averages, single uniform distribution is developed for this fuzzy set.

In order to run the developed simulation models, there are several data related with the investment project alternatives. The necessary simulation inputs and obtained outputs after running the model will be shown in the computational experiments section.

6.2.3 Stage 3: Investment Project Evaluation and Decision

At the previous stage of the proposed methodology, firstly, the expected profitability of each project proposal is determined. At this stage, not only one single profitability value is determined for each project proposal. The reason for this is that, as specified above, while the proposed methodology tries to determine which projects will be carried out within the planning horizon, it also seeks an answer for when the project proposal will be realized. Therefore, here, each project proposal can be carried out at any time between its earliest and latest starting dates. So, for each project proposal, it should be determined the expected profitability values for each year in which project proposal can be realized.

It is a known fact that parameter values of project proposals involve risk. Therefore, while determining the expected profitability values for each year in which project proposal can be realized, it should also be determined the deviation of these values from their expected values. These values are found out through simulation models developed at the previous stage.

The values of coefficient of variation are found in order to determine the extent to which expected profitability values may deviate per unit.

As a result of using the proposed methodology, the decision will be made about which projects will be carried out over the planning horizon. One of the basic assumption on which the methodology is based is that the budget planned to be allocated for each year in the planning horizon is certain. When one makes a decision about which projects will be realized each year over the planning horizon, naturally, the budget allocated for that year will be taken into consideration. However, there is an important point that should be noted here. There may be some projects started, completed and carried out within the planning horizon. Certain amount of income will be generated by the projects previously carried out. Therefore, incomes generated by the investment projects previously carried out should be added to the amount of investment budget concerning the years following the completion of the project. The reason for this is that when a decision is made about which projects will be realized in any year within the planning horizon, it should be considered the incomes generated by the projects started, completed and carried out in previous years. Thus, in the second stage of the proposed methodology, finally, it is determined that how much income a project proposal would have generated if it had been carried out in any year.

All of these values are the inputs of the mathematical models which will be explained in the following sections. In the mathematical models developed in the third stage of the proposed methodology, there are two objectives. First objective is trying to maximize sum of NPVs of the chosen projects, and the second objective is trying to minimize sum of coefficient of variations of the chosen projects. These mathematical models have developed for optimization of linear objective functions subject to linear equality and linear inequality constraints. For these reasons, they are called as multi-objective linear programming models.

As a consequence, in this stage, mathematical models with three different structures and three different purposes were developed and they will be explained in detail in the following sections.

6.2.3.1 Multi-objective Linear Programming Model

At the third stage of the proposed methodology, firstly a multi-objective linear programming model has been developed without taking the potential uncertainties about the objectives and some constraints into consideration. This conventional (crisp) mathematical model has been constructed in order to solve the optimal project selection and scheduling problem that was explained in details in the Section 1.1. It combines the project selection and scheduling decisions, while considering risk and profitability as optimization criteria. The developed model will help to take a decision about when and which project proposals will be carried out over the planning horizon.

It is assumed that two objectives are developed by the decision maker. First objective is maximizing sum of NPVs of the chosen projects, and the second objective is minimizing sum of coefficient of variations of the chosen projects. The second objective provides minimizing the variability of the NPVs of the chosen projects. Therefore, the model maximizes the sum of NPVs of the chosen projects while minimizing their deviation. This crisp multi-objective linear programming model is coded using LINGO 8.0 (LINDO, 2003) format and LINGO code for this model is presented in Appendix C. The details of the mathematical model are given in the following;

<u>*Notations;*</u>

- P represents the set of investment projects to be considered,
- T represents the planning horizon,
- NPV_{it} represents the net present value of the project i that starts in the year t of the planning horizon.

- CV_{it} represents the coefficient of variation of the project i that starts in the year t of the planning horizon.
- u_i represents the length of the project's investment period,
- v_i represents the lifespan of the project,
- t_i^- represents the earliest starting date for project i, meaning the earliest period in which the project can be started,
- t_i^+ represents the latest starting date for project i, meaning the tardiest period in which the project can be started, $[t_i^- \leq t_i^+]$
- K_L represents the minimum number of projects to be carried out,
- K_U represents the maximum number of projects to be carried out,
- g_{ij} represents the gap allowed between precedence relations, [if project i precedes project j, then $(i,j) \in A$, g_{ij} indicates the number of periods of separation or overlap between them.]
- r_t^0 represents the amount of available budget for investment for year t (t= 1, ..., T);
- c_{ik} represents the investment cost (negative cash flow) for project i in period k (k = 1, …, u_i),
- b_{it} represents the expected financial income (positive cash flow) generated by project i in period t.
- Y_{it} represents the binary decision variable that takes the value of 1 if project i $(i \in P)$ starts on year t $\left(t = t_i^-,...,\min\left[t_i^+, T - u_i + 1\right]\right)$; it takes the value of 0, otherwise.
- X_{ikt} represents the binary decision variable that takes the value of 1 if the period k (k= 1,…, v_i) is assigned to year t in the planning horizon $t = t_i^-,...,\min\left(t_i^+ + v_i - 1, T\right)$; otherwise it takes the value of 0.
- R_t represents the amount of investment resources not used at the end of year t, and carried over as budget for the next year t+1.
- d_z represents the set of mutually exclusive projects in situation z
- MAX_z represents the maximum acceptable number of mutually exclusive projects in situation z

Objective Functions;

$$\max \quad \sum_{i\in P} \sum_{t=t_i^-}^{\min(t_i^+, T-u_i+1)} NPV_{it} Y_{it} \tag{6.15}$$

$$\min \quad \sum_{i\in P} \sum_{t=t_i^-}^{\min(t_i^+, T-u_i+1)} CV_{it} Y_{it} \tag{6.16}$$

Model Constraints [System Constraints]

$$\sum_{t=t_i^-}^{\min\left(t_i^+, T-u_i+1\right)} Y_{it} \leq 1, \quad \left(i \in P\right) \tag{6.17}$$

$$K_L \leq \sum_{i\in P} \sum_{t=t_i^-}^{\min\left(t_i^+, T-u_i+1\right)} Y_{it} \leq K_U \tag{6.18}$$

$$Y_{it} \leq X_{ik(t+k-1)}, \qquad (i \in P), \quad k = 1,\ldots,v_i, \qquad t = t_i^-,\ldots,\min\left(t_i^+, T-u_i+1\right) \tag{6.19}$$

$$\sum_{t=t_i^-}^{\min\left(t_i^+ + v_i - 1, T\right)} X_{ikt} \leq 1, \qquad i \in P, \quad k = 1,\ldots,v_i \tag{6.20}$$

$$R_t = R_{t-1} + r_t^0 - \sum_{i\in P}\sum_{k=1}^{u_i} c_{ik}(1+e)^{t-k} X_{ikt} + \sum_{i\in P}\sum_{k=u_i+1}^{v_i} b_{it} X_{ikt}, \qquad t = 2,3\ldots,T \tag{6.21}$$

$$R_1 = r_1^0 - \sum_{i\in P}\sum_{k=1}^{u_i} c_{ik} X_{ik1} + \sum_{i\in P}\sum_{k=u_i+1}^{v_i} b_{i1} X_{ik1}, \qquad t = 1 \tag{6.22}$$

$$Y_{jt} \leq \sum_{t'=t_i^-}^{t-u_i-g_{ij}} Y_{it'}, \qquad (i,j) \in A, \quad t - u_i - g_{ij} \geq 0; \quad t = t_j^-,\ldots,t_j^+ \tag{6.23}$$

$$Y_{jt} = 0, \qquad (i,j) \in A, \quad t - u_i - g_{ij} < 0; \quad t = t_j^-,\ldots,t_j^+ \tag{6.24}$$

$$\sum_{i\in d_z} \sum_{t=t_i^-}^{\min(t_i^+, T-u_i+1)} Y_{it} \leq MAX_z, \quad d_z \in B, \quad z \in (1,2,\ldots,m), \quad d_z \subset P \tag{6.25}$$

$$X_{ikt} \in \{0,1\} \qquad i \in P, \quad k = 1,\ldots,v_i, \quad t = t_i^-,\ldots,t_i^+ + v_i - 1 \tag{6.26}$$

$$Y_{it} \in \{0,1\} \qquad i \in P, \quad t = t_i^-,\ldots,\min(t_i^+, T-u_i+1) \tag{6.27}$$

$$R_t \geq 0 \qquad t = 1,2,\ldots,T \tag{6.28}$$

As seen in (6.15), the first objective of the model seeks to maximize the sum of NPVs of the chosen projects, and the second objective seeks to minimize sum of coefficient of variations of the chosen projects. The second objective provides minimizing the variability of the NPVs of the chosen projects.

The set of constraints in (6.17) assures that every project may or may not be selected, and allows the model to select at most one start date for each investment project. The set of constraints in (6.18) allows the number of selected projects to fall between a lower and upper bounds K_L and K_U, respectively. The set of constraints in (6.19) provides the activitation of the corresponding periods of investment and income generation, once a project starts on a given period. The set of constraints in (6.20) assures that every period of investment project is assigned to a given year in the planning horizon at most once. Constraints (6.21) and (6.22) show the set of budget constraints. This set of constraints includes the amount of available budget for investment for a given year, the resources coming from the previous year, and the income generated by the investment projects previously carried out. Without loss of generality, it is assumed that the resources coming from the previous year for the first year is equal to 0. The set of constraints in (6.23) and (6.24) shows the modeling of precedence relations. The set of constraints in (6.25) represents how the mutually exclusive projects are modeled. The set of constraints in (6.26) and (6.27) are the binary restrictions on the decision variables. Lastly, the set of constraints in (6.28) establishes non-negativity conditions for the available resources at the end of year t.

As seen above, in this crisp multi-objective linear programming model there are two objectives. First objective is trying to maximize sum of NPVs of the chosen projects, and the second objective is trying to minimize sum of coefficient of variations of the chosen projects. However, it is not possible to optimize both objectives, simultaneously. In other words, there is no guarantee that the project set obtained as a result of the model run will consist of projects with both the maximum profitability and lowest risk. Perhaps, any project set will provide the highest profitability while involving a considerably high risk at the same time. Therefore, the developed mathematical model should be solved in two phases. In the first phase, one of the objectives is optimized subject to all constraints. In the second phase, the second objective is optimized including an additional restriction that avoids the deterioration of the first objective, guaranteeing Pareto optimality (Steuer, 1986). Let NPV_{opt} and CV_{opt} be the optimal values for each objective obtained in the first phase. (6.29) and (6.30) represent the constraints used in the second phase, depending on the objective selected in the first phase. It should be noticed that one of them will be used in the second phase.

$$\sum_{i \in P} \sum_{t=t_i^-}^{\min(t_i^+, T-u_i+1)} NPV_{it} Y_{it} \geq NPV_{opt} \tag{6.29}$$

$$\sum_{i \in P} \sum_{t=t_i^-}^{\min(t_i^+, T-u_i+1)} CV_{it} Y_{it} \leq CV_{opt} \tag{6.30}$$

Within the scope of this dissertation, it has been assumed that the primary objective of investors will be to select the projects with maximum total expected profitability. In the computational experiments section, the mathematical model specified above has been solved and the objective in (6.15) has been optimized at the first phase. Therefore, at the first phase, maximum total profitability value (NPV_{opt}) has been determined. At the second phase, NPV_{opt} found at the first phase has been written in (6.29), thus it has been added a new constraint to the model and the objective in (6.16) has been optimized. In this way, one can select the project set with the lowest risk among the project sets that have maximum total profitability. If there is only one single project set that has maximum total profitability, the project set to be obtained at the end of both phases during the solution process will be same.

6.2.3.2 Fuzzy Multi-objective Linear Programming Models

As mentioned in the previous section, developed crisp multi-objective linear programming model can be solved in two phases. In the scope of this dissertation, it has been assumed that the primary objective of the decision maker will be to determine the project set which will maximize the total profitability. In this case, at the end of running of the model, one is able to select the project set with the lowest risk among the project sets that have maximum total profitability value found out during the first phase. Yet, two important problems arise here;

(1) There may be another project set with a little less total profitability than the maximum total profitability value specified at the first phase of the solution process but with very low risk. However, at the end of the solution of crisp mathematical model, only the project set with the lowest risk can be selected among the project sets with maximum total profitability. On the other hand, it may be possible to select a far less risky project set if the decision maker agrees to make a profit a little below the maximum total profitability value. However, crisp mathematical model does not provide this opportunity.

(2) In crisp mathematical model, all constraints have been assumed to be deterministic. However, some constraints may be flexible. One of the most significant constraints which may be flexible in the model is the budget constraint. The assumption that the amount of budget allocated for each year is deterministic may lead to an inability to carry out the projects which may be very profitable owing to little inadequacy of budget. If the decision maker determines upper and lower bounds of the amount of budget planned to allocate for each year, this problem is overcome.

The following sections explain the models developed for solving these two problems. Firstly, the models towards solving the first problem above will be explained. Then, it will be described the models towards solving both problems, simultaneously.

6.2.3.2.1 Multi-objective Linear Programming Model with Fuzzy Objectives. By using multi-objective linear programming model with fuzzy objectives, uncertainty of the decision makers' aspiration levels for the goals is treated, and consequently, the preferred compromise solution can be determined. With this way, the first type of problem described in the previous section is solved.

In order to develop fuzzy multi-objective linear programming models, at first, the conventional (crisp) linear programming model should be developed. Therefore efficient extreme solutions of the problem can be obtained. For doing this, once the multi-objective linear programming model is developed, it is solved with each of the objective functions by themselves. In other words, at first, Equation (6.15) is set as the objective and the model is solved. Then, Equation (6.16) is set as the objective and the model is solved, and the payoff table (see Table 6.2) is exhibited. Each row of Table 6.2 refers to an efficient extreme solution which can be selected as a compromise solution by the decision maker.

Table 6.2 The payoff table

	$Z_1(X)$	$Z_2(X)$	…	$Z_M(X)$
$X^{(1)}$	Z_{11}	Z_{12}	…	Z_{1M}
$X^{(2)}$	Z_{21}	Z_{22}	…	Z_{2M}
⋮	…	…	…	…
$X^{(M)}$	Z_{M1}	Z_{M2}	…	Z_{MM}

As discussed in Section 5.3.2.1.2, $Z_m(X)$ denotes the *m*th objective function, and $X^{(m)}$ is the optimal solution of the *m*th single objective problem. Solving the problem with $X^{(m)}$ (*m* = 1,…, *M*) for each objective, a payoff matrix with entries $Z_{pm} = Z_m(X^{(p)})$, *m*, *p*=1,…, M can be formulated as presented in Table 6.2. Decision makers generally obtain efficient extreme solutions for constructing the membership functions of the objectives. By considering the efficient extreme solutions, the lower and upper bounds of the objectives can be determined. According to the payoff table, the upper bound and the lower bound of the *m*th objective function are determined as $u_m = \max(Z_{1m}, Z_{2m}, ..., Z_{Mm})$ and $l_m = \min(Z_{1m}, Z_{2m}, ..., Z_{Mm})$, *m*= 1,…, *M*, respectively. Therefore, in the developed mathematical model, there are two objectives, and the lower and upper bounds of these objectives are shown as in Table 6.3. Here, the first objective in (6.15) is named as *NPV,* and the second objective in (6.16) is named as *CV.*

Table 6.3 Lower and upper bounds of the objectives

Objectives	Lower Bound	Upper Bound
NPV	NPV_{lb}	NPV_{ub}
CV	CV_{lb}	CV_{ub}

Membership functions of fuzzy objectives can be defined now using corresponding upper and lower bounds as follows;

$$\mu_{NPV} = \begin{cases} 0 & if \quad NPV \le NPV_{lb} \\ \dfrac{NPV - NPV_{lb}}{NPV_{ub} - NPV_{lb}} & if \quad NPV_{lb} < NPV \le NPV_{ub} \\ 1 & if \quad NPV > NPV_{ub} \end{cases} \tag{6.31}$$

$$\mu_{CV} = \begin{cases} 1 & if \quad CV \le CV_{lb} \\ \dfrac{CV_{ub} - CV}{CV_{ub} - CV_{lb}} & if \quad CV_{lb} < CV \le CV_{ub} \\ 0 & if \quad CV > CV_{ub} \end{cases} \tag{6.32}$$

These linear membership functions for each objective function are illustrated in Figure 6.5.

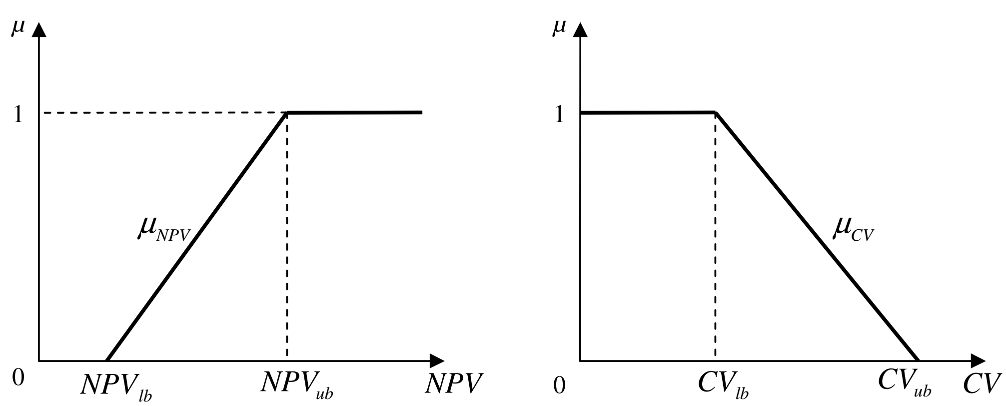


Figure 6.5 The linear membership functions for each objective function

In this section of the dissertation, in order to construct multi-objective linear programming model with fuzzy objectives, five fuzzy multi-objective modeling approaches have been used. These approaches are Tiwari et al.'s weighted additive approach (Tiwari et al. 1987), Werners' "fuzzy and" operator approach (Werners, 1988), Li's two-phase approach (Li, 1990), Lai & Hwang's approach (Lai & Hwang, 1993), Lin's weighted max-min approach (Lin, 2004).

- *Tiwari et al.'s Weighted Additive Approach*

Tiwari et al.'s Weighted Additive Approach is presented in Section 5.3.2.1.2. Using this approach, the problem can be formulated as follows;

$$\left.\begin{aligned} &\textit{maximize} \quad w_{NPV}\mu_{NPV} + w_{CV}\mu_{CV} \\ &\textit{subject to} \\ &\qquad \mu_{NPV}, \mu_{CV} \in [0,1], w_{NPV}, w_{CV} \geq 0 \\ &\text{and others system constraints (6.17 to 6.28)} \end{aligned}\right\} \qquad (6.33)$$

- *Werners' "Fuzzy and" Operator Approach*

Werners' "Fuzzy and" Operator Approach is presented in Section 5.3.2.1.2. Using this approach, the problem can be formulated as follows;

$$\left.\begin{aligned} &\textit{maximize} \quad \gamma\lambda + (1-\gamma)\left[(\lambda_1 + \lambda_2)/2\right] \\ &\textit{subject to} \\ &\qquad \lambda + \lambda_1 \leq \mu_{NPV} \\ &\qquad \lambda + \lambda_2 \leq \mu_{CV} \\ &\qquad \mu_{NPV}, \mu_{CV}, \lambda, \lambda_1, \lambda_2, \gamma \in [0,1] \\ &\text{and others system constraints (6.17 to 6.28)} \end{aligned}\right\} \qquad (6.34)$$

- *Li's Two-Phase Approach*

As explained in Section 5.3.2.1.2, Li (1990) proposed a two-phase approach in which the first phase is to use *"min operator"* to search for an optimal value of λ. Using this approach, the problem can be formulated as follows;

$$\left.\begin{aligned}&\textit{maximize} \quad (\mu_{NPV} + \mu_{CV})/2\\&\textit{subject to}\\&\qquad \lambda \le \mu_{NPV}\\&\qquad \lambda \le \mu_{CV}\\&\qquad \mu_{NPV}, \mu_{CV}, \lambda \in [0,1]\\&\text{and others system constraints (6.17 to 6.28)}\end{aligned}\right\} \qquad (6.35)$$

where λ is the solution of the problem with *"min operator"*.

- *Lai & Hwang's Approach*

Lai & Hwang's Approach is explained in Section 5.3.2.1.2. Using this approach, the problem can be formulated as follows;

$$\left.\begin{aligned}&\textit{maximize} \quad \lambda + 0.001(0.50\mu_{NPV} + 0.50\mu_{CV})\\&\textit{subject to}\\&\qquad \lambda \le \mu_{NPV}\\&\qquad \lambda \le \mu_{CV}\\&\qquad \mu_{NPV}, \mu_{CV}, \lambda \in [0,1]\\&\text{and others system constraints (6.17 to 6.28)}\end{aligned}\right\} \qquad (6.36)$$

where δ=0.001 is a sufficiently small positive number, and it is assumed that weights of objectives are equal.

- *Lin's Weighted Max-min Approach*

As stated in Section 5.3.2.1.2, Lin's weighted max–min approach aims to find an optimal solution within the feasible area such that the ratio of the achieved levels is as close to the ratio of the weights as possible. Using this approach, the problem can be formulated as follows;

$$\left.\begin{array}{l} \textit{maximize} \quad \beta \\ \textit{subject to} \\ \qquad w_{NPV}\beta \leq \mu_{NPV} \\ \qquad w_{CV}\beta \leq \mu_{CV} \\ \qquad \mu_{NPV}, \mu_{CV} \in [0,1], w_{NPV}, w_{CV} \geq 0 \\ \\ \text{and others system constraints (6.17 to 6.28)} \end{array}\right\} \qquad (6.37)$$

6.2.3.2.2 Multi-objective Linear Programming Model with Fuzzy Objectives and Fuzzy Constraints. By using multi-objective linear programming model with fuzzy objectives and fuzzy constraints, besides the uncertainty of the decision makers' aspiration levels for the goals, also uncertainty for some of the constraints is treated. During the development of mathematical models in this section, it is assumed that the estimated amounts of budget for each year are flexible. In this way, thanks to the developed models, it is likely that the project sets with little less profitability than maximum total profitability but with very low risks will be selected. Furthermore, the situation in which the projects that may be very profitable are not selected owing to little inadequacy of budget is avoided.

In the mathematical models developed in this section, besides the membership functions of the fuzzy objectives, the membership function of the fuzzy constraint should be determined. In the previous section, it has been explained how to determine the membership functions of the fuzzy objectives. So, in this section, it will not be discussed again. As a consequence, membership functions of fuzzy objectives can be defined as follows;

$$\mu_{NPV} = \begin{cases} 0 & if \quad NPV \le NPV_{lb} \\ \dfrac{NPV - NPV_{lb}}{NPV_{ub} - NPV_{lb}} & if \quad NPV_{lb} < NPV \le NPV_{ub} \\ 1 & if \quad NPV > NPV_{ub} \end{cases} \tag{6.38}$$

$$\mu_{CV} = \begin{cases} 1 & if \quad CV \le CV_{lb} \\ \dfrac{CV_{ub} - CV}{CV_{ub} - CV_{lb}} & if \quad CV_{lb} < CV \le CV_{ub} \\ 0 & if \quad CV > CV_{ub} \end{cases} \tag{6.39}$$

It should be noticed that fuzzy objectives are assumed to have linear membership functions.

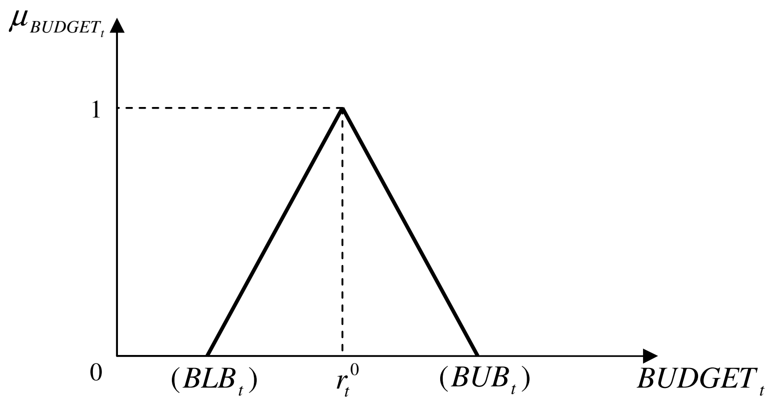


Figure 6.6 Membership function of the budget for each year

As mentioned, in this section, the uncertainty of the budget for each year is treated in addition to the uncertainty of the decision makers' aspiration levels for the goals. That is, the budget for each year is altered within a range. Therefore, it is stated that the budget for each year as fuzzy parameters using triangular membership function as illustrated in Figure 6.6. As known, the membership functions of fuzzy constraints can be defined using corresponding upper and lower bounds. In the computational experiments, the lower and upper bounds of the budget for each year are assumed 90% and 110% of their corresponding expected budget (r_t^0) values, respectively.

In Figure 6.6, BLB_t represents the lower bound of the budget for year *t*, and BUB_t represents the upper bound of the budget for year *t.* Therefore, membership function of the budget for each year can be defined using corresponding upper aand lower bounds as follows;

$$\mu_{BUDGET_t} = \begin{cases} \dfrac{BUDGET_t - BLB_t}{r_t^0 - BLB_t} & if \quad BLB_t < BUDGET_t \leq r_t^0 \\ \dfrac{BUB_t - BUDGET_t}{BUB_t - r_t^0} & if \quad r_t^0 < BUDGET_t \leq BUB_t \\ 0 & if \quad otherwise \end{cases} \tag{6.40}$$

In order to construct multi-objective linear programming model with fuzzy objectives and fuzzy constraints, same fuzzy multi-objective modeling approaches have been used.

- <u>*Tiwari et al.'s Weighted Additive Approach*</u>

Using this approach, the problem can be formulated as follows;

$$\left.\begin{array}{l} maximize \quad w_{NPV}\mu_{NPV} + w_{CV}\mu_{CV} + \sum_{t=1}^{T}\mu_{BUDGET_t} \\ subject\ to \\ \qquad \mu_{NPV}, \mu_{CV} \in [0,1], w_{NPV}, w_{CV} \geq 0 \\ \\ Fuzzy\ Budget\ Constraints \\ \\ \mu_{BUDGET_t} \leq (BUDGET_t - BLB_t)/(r_t^0 - BLB_t), \quad \forall t \\ \mu_{BUDGET_t} \leq (BUB_t - BUDGET_t)/(BUB_t - r_t^0), \quad \forall t \\ \mu_{BUDGET_t} \leq 1, \quad \forall t \\ \lambda \leq \mu_{BUDGET_t}, \quad \forall t \\ \\ \text{and others system constraints (6.17 to 6.28)} \end{array}\right\} \tag{6.41}$$

- *Werners' "Fuzzy and" Operator Approach*

Using this approach, the problem can be formulated as follows;

$$\left.\begin{array}{ll}
\textit{maximize} & \gamma\lambda + (1-\gamma)\left[(\lambda_1+\lambda_2)/2\right] \\
\textit{subject to} & \\
& \lambda + \lambda_1 \le \mu_{NPV} \\
& \lambda + \lambda_2 \le \mu_{CV} \\
& \mu_{NPV}, \mu_{CV}, \lambda, \lambda_1, \lambda_2, \gamma \in [0,1] \\
\textit{Fuzzy Budget Constraints} & \\
\mu_{BUDGET_t} \le (BUDGET_t - BLB_t)/(r_t^0 - BLB_t), \quad \forall t & \\
\mu_{BUDGET_t} \le (BUB_t - BUDGET_t)/(BUB_t - r_t^0), \quad \forall t & \\
\mu_{BUDGET_t} \le 1, \quad \forall t & \\
\lambda \le \mu_{BUDGET_t}, \quad \forall t & \\
\text{and others system constraints (6.17 to 6.28)} &
\end{array}\right\} \qquad (6.42)$$

- *Li's Two-Phase Approach*

Using this approach, the problem can be formulated as follows;

$$\left.\begin{array}{ll}
\textit{maximize} & (\mu_{NPV} + \mu_{CV})/2 \\
\textit{subject to} & \\
& \lambda \le \mu_{NPV} \\
& \lambda \le \mu_{CV} \\
& \mu_{NPV}, \mu_{CV}, \lambda \in [0,1] \\
\textit{Fuzzy Budget Constraints} & \\
\mu_{BUDGET_t} \le (BUDGET_t - BLB_t)/(r_t^0 - BLB_t), \quad \forall t & \\
\mu_{BUDGET_t} \le (BUB_t - BUDGET_t)/(BUB_t - r_t^0), \quad \forall t & \\
\mu_{BUDGET_t} \le 1, \quad \forall t & \\
\lambda \le \mu_{BUDGET_t}, \quad \forall t & \\
\text{and others system constraints (6.17 to 6.28)} &
\end{array}\right\} \qquad (6.43)$$

where λ is the solution of the problem with *"min operator"*.

- *Lai & Hwang's Approach*

Using this approach, the problem can be formulated as follows;

$$
\left.
\begin{array}{l}
\textit{maximize} \quad \lambda + 0.001(0.50\mu_{NPV} + 0.50\mu_{CV}) \\
\textit{subject to} \\
\qquad \lambda \le \mu_{NPV} \\
\qquad \lambda \le \mu_{CV} \\
\qquad \mu_{NPV}, \mu_{CV}, \lambda \in [0,1] \\
\textit{Fuzzy Budget Constraints} \\
\mu_{BUDGET_t} \le (BUDGET_t - BLB_t)/(r_t^0 - BLB_t), \quad \forall t \\
\mu_{BUDGET_t} \le (BUB_t - BUDGET_t)/(BUB_t - r_t^0), \quad \forall t \\
\mu_{BUDGET_t} \le 1, \quad \forall t \\
\lambda \le \mu_{BUDGET_t}, \quad \forall t \\
\text{and others system constraints (6.17 to 6.28)}
\end{array}
\right\} \qquad (6.44)
$$

where δ=0.001 is is a sufficiently small positive number, and it is assumed that weights between objectives are equal.

- *Lin's Weighted Max-min Approach*

Using this approach, the problem can be formulated as follows;

$$
\left.
\begin{array}{l}
\textit{maximize} \quad \beta \\
\textit{subject to} \\
\qquad w_{NPV}\beta \le \mu_{NPV} \\
\qquad w_{CV}\beta \le \mu_{CV} \\
\qquad \mu_{NPV}, \mu_{CV} \in [0,1], w_{NPV}, w_{CV} \ge 0 \\
\textit{Fuzzy Budget Constraints} \\
\mu_{BUDGET_t} \le (BUDGET_t - BLB_t)/(r_t^0 - BLB_t), \quad \forall t \\
\mu_{BUDGET_t} \le (BUB_t - BUDGET_t)/(BUB_t - r_t^0), \quad \forall t \\
\mu_{BUDGET_t} \le 1, \quad \forall t \\
\lambda \le \mu_{BUDGET_t}, \quad \forall t \\
\text{and others system constraints (6.17 to 6.28)}
\end{array}
\right\} \qquad (6.45)
$$

6.3 Computational Experiments

In this section, it is tried to show the applicability of the proposed novel methodology in the process of risky investment projects evaluation. For doing this, a hypothetical investment project evaluation and selection problem covering all stages of the proposed methodology was developed.

In this hypothetic problem, it was assumed that 40 investment project alternatives were determined by the investor. Again, the investor plans to carry out the selected projects among over a 13 year planning horizon. Therefore, the primary objective of the investor is to make a decision about which projects will be carried out at which time over the planning horizon. While doing so, the investor wants to select the investment projects that have maximum total profitability and the lowest risk.

The following sections include an individual application of each stages of the novel methodology proposed within the scope of this dissertation and an interpretation of the obtained results.

- ✓ *Stage 1: Opportunity and Pre-Feasibility Studies*

As mentioned in Section 6.2.1, the first stage of the proposed methodology includes opportunity and pre-feasibility studies. The aim of this stage is to identify the investment opportunities, and to carry out preliminary election of project ideas and to give prominence to ideas which have the highest chance of attaining the goals planned by entrepreneurs and investors. In this stage, PROMSORT is used in order to assign project alternatives to predefined ordered classes in the first stage of the proposed methodology.

Therefore, at first, the promising project proposals are determined which have the highest chance of attaining the goals planned by entrepreneurs and investors among 40 investment proposals included in the hypothetic problem. For doing this, the project alternatives are classified with respect to some evaluation criteria. Therefore, it is assumed that these investment project proposals are classified in three predefined classes;

a) Class 1 (C1): High risk [the worst category],
b) Class 2 (C2): Medium risk,
c) Class 3 (C3): Low risk [the best category].

Table 6.4 indicates the evaluation criteria specified by the decision maker for assessing investment projects in the hypothetic problem. As can be seen, 5 evaluation criteria were determined. For instance, according to the first evaluation criterion (g_1), the projects with high values are regarded as better projects while according to the third evaluation criterion (g_3) the projects with lower values are defined as better projects. Therefore, two criteria have to be minimized (g_3, g_5), and three criteria have to be maximized (g_1, g_2, g_4). Besides the evaluation criteria, the parameters of PROMETHEE methodology and limit profiles of classes can be shown in Table 6.4.

Table 6.4 Parameters for PROMETHEE and profile limits for PROMSORT

Code	**Evaluation Criteria**	**Obj.**	**Weight**	***q***	***p***	b_1	b_2
g_1	Availability of production factors	Max	20	-	1	2	4
g_2	Export possibility	Max	25	-	1	2	4
g_3	Competition level	Min	15	-	1	4	2
g_4	Possibilities for diversification	Max	15	-	1	2	4
g_5	Completion time required	Min	25	-	1	5	3

Table 6.5 shows the data for 40 investment project proposals and two profile limits for PROMSORT.

PROMSORT assigns alternatives to classes following the three consecutive steps. The first step is the construction of an outranking relation using PROMETHEE I. For that reason, all data has been entered to the Decision Lab 2000 software and PROMETHEE I partial ranking results have been obtained. As known, in PROMETHEE I, ranking of alternatives is performed using the leaving and entering flows. Therefore, after running the Decision Lab 2000 software, the values of leaving and entering follows of each project and the profile limits are determined as in Table 6.6. In this table, the net flows are also presented.

Table 6.5 Project alternatives included in the analysis

		Criteria				
		g_1	g_2	g_3	g_4	g_5
Project Alternatives	***P1***	3	2	3	2	4
	P2	3	3	3	3	4
	P3	2	3	4	2	3
	P4	2	2	3	3	4
	P5	5	4	2	3	2
	P6	3	2	4	1	4
	P7	3	2	3	3	4
	P8	2	1	3	2	4
	P9	4	4	2	5	3
	P10	5	5	2	4	1
	P11	3	3	3	3	4
	P12	3	3	3	3	4
	P13	3	2	4	4	3
	P14	2	3	4	3	5
	P15	2	2	3	2	4
	P16	5	5	1	4	3
	P17	3	3	4	4	3
	P18	3	2	3	2	4
	P19	2	3	5	1	5
	P20	2	1	4	2	6
	P21	4	5	2	4	1
	P22	4	5	1	4	1
	P23	4	5	1	3	2
	P24	4	4	2	4	1
	P25	3	2	4	2	4
	P26	2	3	4	2	3
	P27	2	2	4	3	4
	P28	3	2	3	2	5
	P29	3	2	4	1	4
	P30	2	2	4	2	6
	P31	2	4	4	1	4
	P32	3	3	4	2	5
	P33	3	2	3	2	3
	P34	4	5	2	5	1
	P35	3	3	3	3	4
	P36	5	4	1	4	2
	P37	3	3	4	3	3
	P38	3	3	4	2	4
	P39	3	3	5	1	5
	P40	3	3	3	4	3

The second step of the PROMSORT is the exploitation of the outranking relation in order to assign alternatives to specific classes except the incomparability and indifference situations. The assignment of alternatives to classes results directly from the outranking relation. At the end of this stage, project alternatives have been classified as in Table 6.7. As seen from this table, only one project alternative (P19) could not have been assigned to a class.

Table 6.6 Leaving flows, entering flows and net flows of the projects and profile limits

		ϕ^+	ϕ^-	ϕ			ϕ^+	ϕ^-	ϕ
Project Alternatives and Profile Limits	***P1***	0.20	0.45	-0.24	***Project Alternatives and Profile Limits***	***P22***	0.85	0.03	0.82
	P2	0.34	0.32	0.02		***P23***	0.79	0.09	0.69
	P3	0.27	0.44	-0.17		***P24***	0.78	0.08	0.70
	P4	0.20	0.50	-0.30		***P25***	0.15	0.49	-0.35
	P5	0.76	0.13	0.63		***P26***	0.27	0.44	-0.17
	P6	0.13	0.55	-0.42		***P27***	0.14	0.55	-0.41
	P7	0.25	0.41	-0.15		***P28***	0.17	0.54	-0.38
	P8	0.13	0.62	-0.49		***P29***	0.13	0.55	-0.42
	P9	0.74	0.12	0.62		***P30***	0.04	0.72	-0.68
	P10	0.85	0.02	0.83		***P31***	0.24	0.52	-0.28
	P11	0.34	0.32	0.02		***P32***	0.19	0.51	-0.31
	P12	0.34	0.32	0.02		***P33***	0.30	0.39	-0.08
	P13	0.33	0.36	-0.02		***P34***	0.86	0.03	0.82
	P14	0.19	0.56	-0.37		***P35***	0.34	0.32	0.02
	P15	0.15	0.54	-0.39		***P36***	0.82	0.07	0.75
	P16	0.80	0.06	0.74		***P37***	0.38	0.31	0.07
	P17	0.42	0.27	0.15		***P38***	0.23	0.41	-0.18
	P18	0.20	0.45	-0.24		***P39***	0.17	0.62	-0.45
	P19	0.11	0.71	-0.60		***P40***	0.48	0.22	0.25
	P20	0.03	0.81	-0.78		b_1	0.05	0.68	-0.63
	P21	0.82	0.04	0.78		b_2	0.70	0.13	0.58

Table 6.7 The obtained results at the end of the second step of the PROMSORT

Project alternatives	**Assignments**	**Project alternatives**	**Assignments**
P1	Assigned to C2	*P21*	Assigned to C3
P2	Assigned to C2	*P22*	Assigned to C3
P3	Assigned to C2	*P23*	Assigned to C3
P4	Assigned to C2	*P24*	Assigned to C3
P5	Assigned to C3	*P25*	Assigned to C2
P6	Assigned to C2	*P26*	Assigned to C2
P7	Assigned to C2	*P27*	Assigned to C2
P8	Assigned to C2	*P28*	Assigned to C2
P9	Assigned to C3	*P29*	Assigned to C2
P10	Assigned to C3	*P30*	Assigned to C1
P11	Assigned to C2	*P31*	Assigned to C2
P12	Assigned to C2	*P32*	Assigned to C2
P13	Assigned to C2	*P33*	Assigned to C2
P14	Assigned to C2	*P34*	Assigned to C3
P15	Assigned to C2	*P35*	Assigned to C2
P16	Assigned to C3	*P36*	Assigned to C3
P17	Assigned to C2	*P37*	Assigned to C2
P18	Assigned to C2	*P38*	Assigned to C2
P19	*INCOMPARABLE*	*P39*	Assigned to C2
P20	Assigned to C1	*P40*	Assigned to C2

After the second step of the PROMSORT, one project alternative (P19) could not have been assigned to a class, since outranking relation indicates that this alternative is incomparable to a profile limit and could not be assigned to a class directly. On the other hand, the other alternatives could be assigned to the classes. In the third step, these alternatives are used as the reference actions of the classes to be able to assign the alternatives which have not yet been assigned. Therefore, in the third step, at first, a distance should be determined. As mentioned in Section 3.6.3, it is calculated by using the following equation;

$$\left.\begin{aligned} d_k &= \frac{1}{2}d^+{}_k - \frac{1}{27}d^-{}_k \\ d^+{}_k &= \sum_{x\in X_1}\left[\phi(a)-\phi(x)\right] \\ d^-{}_k &= \sum_{x\in X_2}\left[\phi(x)-\phi(a)\right] \end{aligned}\right\} \qquad (6.46)$$

In this equation, a reference set X_1 consists of 2 alternatives for C1, and a reference set X_2 consists of 27 alternatives for C2. *a* represents a project alternative which has not yet been assigned to a class.

After the necessary calculations were made, the value of d_k^+ is determined as 0.13, and the value of d_k^- is determined as 0.40. Therefore, the value of d_k is found as -0.27.

Table 6.8 PROMSORT Classifications

Class	**PROMSORT Classification**		
	(s = -1)	(s = 0)	(s = 1)
C3	*P5, P9, P10, P16, P21, P22, P23, P24, P34, P,36*	*P5, P9, P10, P16, P21, P22, P23, P24, P34, P,36*	*P5, P9, P10, P16, P21, P22, P23, P24, P34, P,36*
C2	*P1, P2, P3, P4, P6, P7, P8, P11, P12, P13, P14, P15, P17, P18, P19, P25, P26, P27, P28, P29, P31, P32, P33, P35, P37, P38, P39, P40*	*P1, P2, P3, P4, P6, P7, P8, P11, P12, P13, P14, P15, P17, P18, P25, P26, P27, P28, P29, P31, P32, P33, P35, P37, P38, P39, P40*	*P1, P2, P3, P4, P6, P7, P8, P11, P12, P13, P14, P15, P17, P18, P25, P26, P27, P28, P29, P31, P32, P33, P35, P37, P38, P39, P40*
C1	*P20, P30*	*P19, P20, P30*	*P19, P20, P30*

As discussed previously, in order to identify the differences among the investment project classes and alternative investment projects shortcomings of investment projects compared with limit profiles or alternative investment projects with regard to each criterion, single criterion net flows of PROMETHEE can be used.

The single criterion net flows for each alternative and limit profiles are shown in Table 6.9 and 6.10, respectively.

Table 6.9 Single criterion net flows for investment project alternatives

		Criteria				
		g_1	g_2	g_3	g_4	g_5
Project Alternatives	**P1**	0.0244	-0.5854	0.1707	-0.439	-0.2439
	P2	0.0244	0.0976	0.1707	0.1707	-0.2439
	P3	-0.7317	0.0976	-0.5366	-0.439	0.3902
	P4	-0.7317	-0.5854	0.1707	0.1707	-0.2439
	P5	0.9268	0.5854	0.6585	0.1707	0.7073
	P6	0.0244	-0.5854	-0.5366	-0.9024	-0.2439
	P7	0.0244	-0.5854	0.1707	0.1707	-0.2439
	P8	-0.7317	-0.9756	0.1707	-0.439	-0.2439
	P9	0.6585	0.5854	0.6585	0.9756	0.3902
	P10	0.9268	0.878	0.6585	0.6829	0.9024
	P11	0.0244	0.0976	0.1707	0.1707	-0.2439
	P12	0.0244	0.0976	0.1707	0.1707	-0.2439
	P13	0.0244	-0.5854	-0.5366	0.6829	0.3902
	P14	-0.7317	0.0976	-0.5366	0.1707	-0.7805
	P15	-0.7317	-0.5854	0.1707	-0.439	-0.2439
	P16	0.9268	0.878	0.9268	0.6829	0.3902
	P17	0.0244	0.0976	-0.5366	0.6829	0.3902
	P18	0.0244	-0.5854	0.1707	-0.439	-0.2439
	P19	-0.7317	0.0976	-0.9756	-0.9024	-0.7805
	P20	-0.7317	-0.9756	-0.5366	-0.439	-0.9756
	P21	0.6585	0.878	0.6585	0.6829	0.9024
	P22	0.6585	0.878	0.9268	0.6829	0.9024
	P23	0.6585	0.878	0.9268	0.1707	0.7073
	P24	0.6585	0.5854	0.6585	0.6829	0.9024
	P25	0.0244	-0.5854	-0.5366	-0.439	-0.2439
	P26	-0.7317	0.0976	-0.5366	-0.439	0.3902
	P27	-0.7317	-0.5854	-0.5366	0.1707	-0.2439
	P28	0.0244	-0.5854	0.1707	-0.439	-0.7805
	P29	0.0244	-0.5854	-0.5366	-0.9024	-0.2439
	P30	-0.7317	-0.5854	-0.5366	-0.439	-0.9756
	P31	-0.7317	0.5854	-0.5366	-0.9024	-0.2439
	P32	0.0244	0.0976	-0.5366	-0.439	-0.7805
	P33	0.0244	-0.5854	0.1707	-0.439	0.3902
	P34	0.6585	0.878	0.6585	0.9756	0.9024
	P35	0.0244	0.0976	0.1707	0.1707	-0.2439
	P36	0.9268	0.5854	0.9268	0.6829	0.7073
	P37	0.0244	0.0976	-0.5366	0.1707	0.3902
	P38	0.0244	0.0976	-0.5366	-0.439	-0.2439
	P39	0.0244	0.0976	-0.9756	-0.9024	-0.7805
	P40	0.0244	0.0976	0.1707	0.6829	0.3902

Table 6.10 Single criterion net flows for investment profile limits

	Criteria				
	g_1	g_2	g_3	g_4	g_5
b1	-0.7317	-0.5854	-0.5366	-0.439	-0.7805
b2	0.6585	0.5854	0.6585	0.6829	0.3902

In order to compare the invesment project classes obtained as a result of PROMSORT, the average single criterion net flows for each class are determined. The average single criterion net flows are presented in Table 6.11.

Table 6.11 The average single criterion net flows for each class

	Criteria				
	g_1	g_2	g_3	g_4	g_5
Class 3 (C3)	0.7658	0.7610	0.7658	0.6390	0.7414
Class 2 (C2)	-0.1996	-0.2276	-0.2123	-0.1798	-0.1590
Class 1 (C1)	-0.7317	-0.4878	-0.6829	-0.5935	-0.9106

Figure 6.7 illustrates the comparison of the classes by means of the average single criterion net flows.

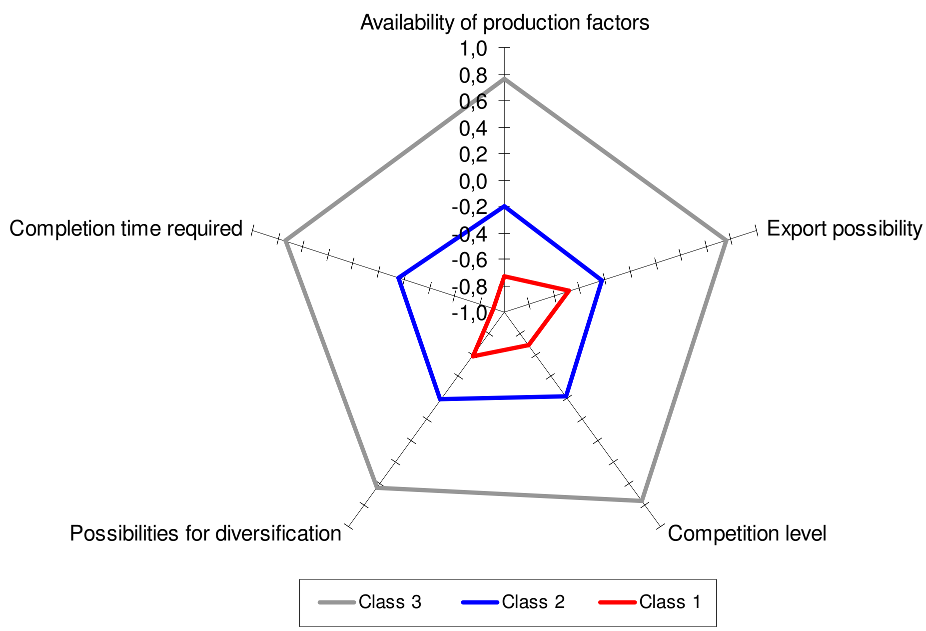


Figure 6.7 The comparison of the invesment project classes by means of average single criterion net flows

According to the results given in Figure 6.7, one can conclude that the investment project alternatives assigned to the third class, which represents the best category, are superior on all evaluation criteria compared to other investment project alternatives. On the other hand, the investment project alternatives assigned to the first class, which represents the worst category, are weak on all evaluation criteria compared to other investment project alternatives.

In the same manner, each investment alternative can be compared with profile limits in terms of single criterion net flows. For example, Figure 6.8, 6.9, 6.10 and 6.11 represent the comparison of investment project 5, 15, 25, and 35 to limit profiles, respectively.

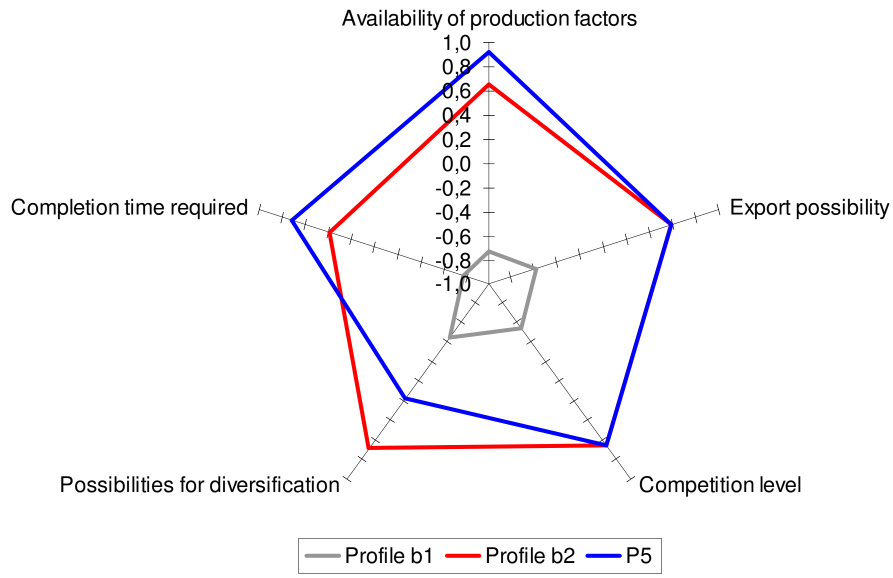


Figure 6.8 The comparison of investment project 5 with profiles by means of single criterion net flows

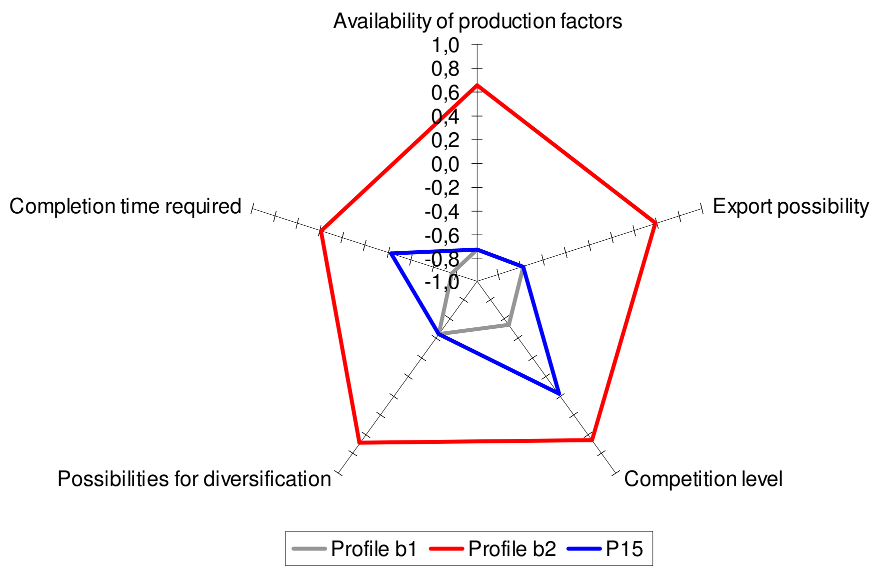


Figure 6.9 The comparison of investment project 15 with profiles by means of single criterion net flows

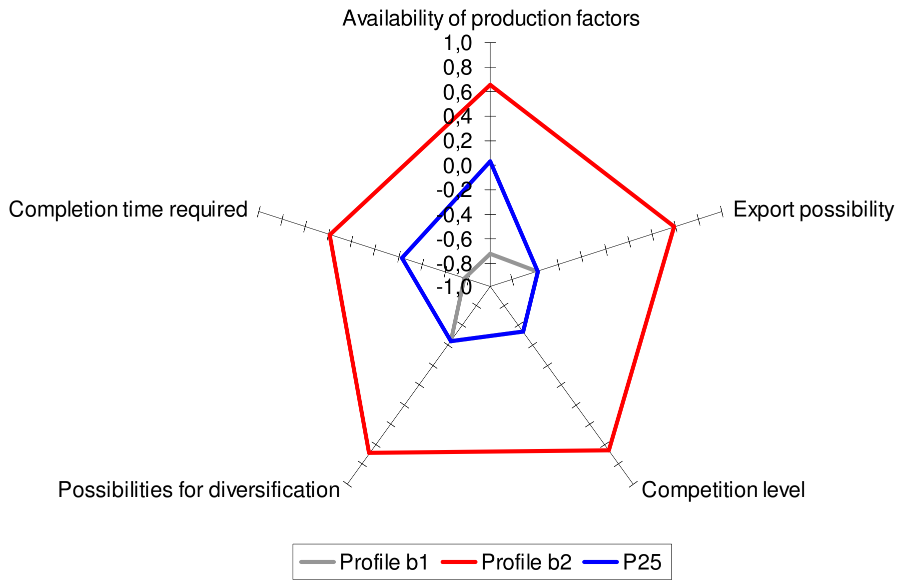


Figure 6.10 The comparison of investment project 25 with profiles by means of single criterion net flows

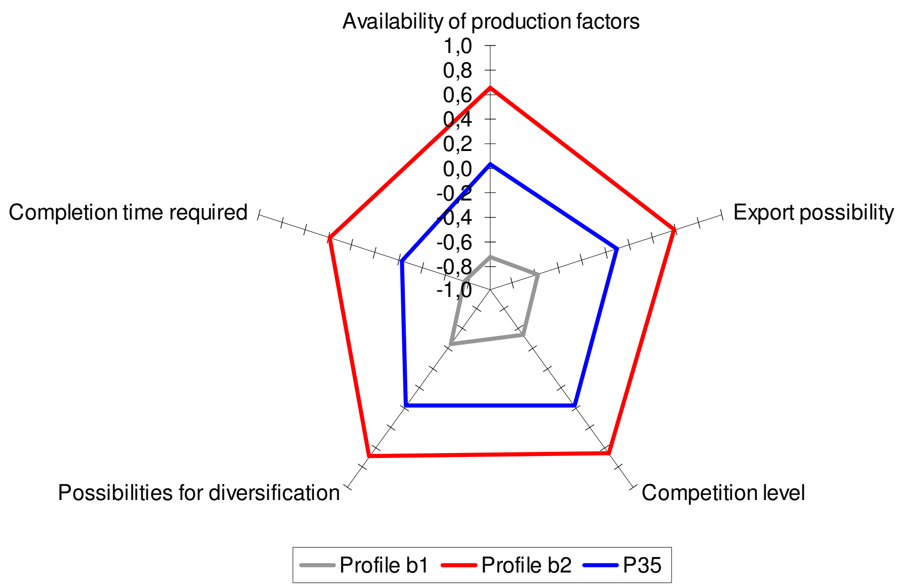


Figure 6.11 The comparison of investment project 35 with profiles by means of single criterion net flows

As seen from Figure 6.8, the investment project 5 is superior on “availability of production factors”, “completion time required”, “export possibility” and competition level”. However, it is weak a little in the “possibilities for diversification”.

When investment project 15 is considered, it can be seen from Figure 6.9 that the investment project's performance is too low on "availability of production factors", "export possibility" and "possibilities for diversification". Also, it is weak in the "completion time required" and "competition level".

As seen from Figure 6.10 that the perforamance of the investment project 15 quite weak in the "export possibility", "competition level" and "possibilities for diversification". Also, it is weak in the "availability of production factors" and "completion time required".

At last, according to the Figure 6.11, one can conclude that the performance of the investment project 35 is medium all evaluation criteria compared to the limit profiles.

- ✓ *Stage 2: Feasibility Study*

At the end of the first stage of the proposed methodology, through 40 investment project alternatives, ten of them have been found promising among other ones, and it has been decided to conduct the feasibility studies for these project alternatives. Therefore, the second stage of the proposed methodology includes conducting feasibility studies for the selected project alternatives. As seen in Table 6.12, these selected alternatives have been numbered, again.

As known, in order to determine in which projects to invest and when to invest in the planning horizon, it should be determined $NPV_{it(i)}$ that represents the net present value of the project i given that it starts in the year $t(i)$ of the planning horizon. Therefore, in the scope of the proposed methodology, at the end of the second stage, all $NPV_{it(i)}$s for each project should be determined. In this stage, also, the standard deviations of each $NPV_{it(i)}$ $[\sigma_{it(i)}]$, coefficient of variation $[CV_{it(i)}]$ and expected cash flows of the project i given that it proceeds in the year t of the planning horizon $[b_{it}]$ should be determined. Because these values will be used in the third stage of the methodology.

Now, let us assume that the investment period lengths (u_i) and lifespan (v_i) of the selected project alternatives are determined as in Table 6.12, and it can be seen in the second column, the highest investment period length of alternatives is determined as 3 years, and in third column the highest investment lifespan of alternatives is determined as 13 years.

Table 6.12 The investment period length and lifespan of the projects (years)

Project	$\boldsymbol{u_i}$	$\boldsymbol{v_i}$
Pro01	1	8
Pro02	1	9
Pro03	1	5
Pro04	2	12
Pro05	3	13
Pro06	1	10
Pro07	2	10
Pro08	3	9
Pro09	2	12
Pro10	1	6

As mentioned in Section 6.2.3, while the proposed methodology tries to determine which projects will be carried out within the planning horizon, it also seeks an answer for when the project proposal will be realized. Each project proposal can be carried out at any time between its earliest and latest starting dates. Therefore, the earliest and latest start dates of the projects should be determined. Let us assume that these values are determined as in Table 6.13.

Table 6.13 The earliest and latest start dates of the projects

Project	t_i^-	t_i^+
Pro01	2009	2021
Pro02	2009	2021
Pro03	2009	2021
Pro04	2009	2015
Pro05	2010	2010
Pro06	2012	2019
Pro07	2011	2016
Pro08	2009	2010
Pro09	2009	2012
Pro10	2010	2014

As known, before the economics of a risky investment project can be evaluated, it is necessary to reasonably estimate the various inflow and outflow components that describe the project. Let us assume that the inflow and outflow components which appear every working period of the project alternatives are shown in Table 6.14. Note that the values of all components are estimated in constant-money units. As mentioned, constant money units represent constant purchasing power independent of the passage of time. For instance, project alternative one (Pro01) is expected to create a revenue for every working periods, that is uniformly distributed between 100 TL and 120 TL in today's monetary value. Also the project is expected to create a labor expense that is uniformly distributed between 30 TL and 40 TL, a material expense which is uniformly distributed between 20 TL and 30 TL, and an overhead expense that is uniformly distributed between 10 TL and 20 TL for every working period in today's monetary value. All these values are in constant-money units.

Table 6.14 The inflow and outflow components of the projects

Project	REV_t	LAB_t	MAT_t	OVE_t
Pro01	Unif [100, 120]	Unif [30, 40]	Unif [20, 30]	Unif [10, 20]
Pro02	Unif [90, 100]	Unif [25, 35]	Unif [20, 30]	Unif [10, 20]
Pro03	Unif [140, 160]	Unif [50, 60]	Unif [30, 40]	Unif [15, 25]
Pro04	Unif [180, 200]	Unif [40, 50]	Unif [40, 50]	Unif [20, 30]
Pro05	Unif [120, 130]	Unif [20, 30]	Unif [25, 30]	Unif [10, 15]
Pro06	Unif [80, 90]	Unif [20, 30]	Unif [15, 25]	Unif [10, 15]
Pro07	Unif [130, 150]	Unif [30, 50]	Unif [20, 40]	Unif [10, 20]
Pro08	Unif [160, 180]	Unif [40, 60]	Unif [40, 50]	Unif [20, 40]
Pro09	Unif [220, 240]	Unif [70, 80]	Unif [50, 60]	Unif [30, 40]
Pro10	Unif [100, 110]	Unif [20, 30]	Unif [20, 30]	Unif [10, 20]

It should be noticed that, the developed simulation model gives us a chance to define the distributions of inflow and outflow components in all kind of distributions such as uniform distribution, normal distribution and exponential distribution. Here, in our case, only the uniform distribution is used. This distribution was randomly selected.

Also let us assume that the investment costs materialized in investment period are exhibited in Table 6.15. For instance, project alternative five (Pro05) has 3 years investment period length and 13 years lifespan, and investment costs are 50, 50 and 40 TL (Turkish Liras) in respectively.

Table 6.15 Investment costs of the projects

	Periods of Investment		
Project	***First Year***	***Second Year***	***Third Year***
Pro01	80	0	0
Pro02	110	0	0
Pro03	91	0	0
Pro04	60	60	0
Pro05	50	50	40
Pro06	63	0	0
Pro07	80	40	0
Pro08	40	40	40
Pro09	80	80	0
Pro10	70	0	0

It should be noted two important points here. The investment cost values shown in Table 6.15 indicates the cost values to be faced when project alternatives get started in the first year of the planning horizon. However, project alternatives may as well get started both at the beginning of the planning horizon and in the following periods. In the event that project alternatives get started in the following periods of the planning horizon, the cost values in this table should be updated by taking the effect of inflation in consideration. For that reason, investment cost values of each project alternative were entered into the simulation model in the matrix form. In this matrix, it was determined what kind of investment cost is caused on the basis of the year in which any project alternative gets started.

Table 6.16 presents the matrix developed with the aim of entering investment cost values of the first project alternative (Pro01) into the simulation model. The first column of the table indicates the investment cost to be faced in the event that project alternatives get started in the first year of the planning horizon. The second column indicates the investment cost to be faced in the event that these project alternatives get started in the second year of the planning horizon. As can be seen, the investment cost to be faced in the event that project alternatives get started in the second year is 10% higher when compared to the one to be faced in the event that they get started in the first year. The reason for this is that the following principle was adopted here: The investment cost to be faced in the event that a project alternative gets started in other years of the planning horizon rather than the first year was assumed to be 10% higher than the investment cost determined for the previous year.

Tablo 6.16 Initial investment cost matrix for the first project alternative (Pro01)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	…	20
0	80.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	⋯	0.0
1	0.0	88.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	⋯	0.0
2	0.0	0.0	96.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	⋯	0.0
3	0.0	0.0	0.0	106.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	⋯	0.0
4	0.0	0.0	0.0	0.0	117.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	⋯	0.0
5	0.0	0.0	0.0	0.0	0.0	128.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	⋯	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	141.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	⋯	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	155.9	0.0	0.0	0.0	0.0	0.0	0.0	⋯	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	171.5	0.0	0.0	0.0	0.0	0.0	⋯	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	188.6	0.0	0.0	0.0	0.0	⋯	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	207.5	0.0	0.0	0.0	⋯	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	228.2	0.0	0.0	⋯	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	251.1	0.0	⋯	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	⋯	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	⋯	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	⋯	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	⋯	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	⋯	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	⋯	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	⋯	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	⋯	0.0

Tablo 6.17 Initial investment cost matrix for the fourth project alternative (Pro04)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	…	20
0	60.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
1	60.0	66.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
2	0.0	66.0	72.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
3	0.0	0.0	72.6	79.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
4	0.0	0.0	0.0	79.9	87.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
5	0.0	0.0	0.0	0.0	87.8	96.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
6	0.0	0.0	0.0	0.0	0.0	96.6	106.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	106.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0

Tablo 6.18 Depreciation amounts matrix for the first project alternative (Pro01)

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	…	**20**
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
1	11.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
2	11.4	12.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
3	11.4	12.6	13.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
4	11.4	12.6	13.8	15.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
5	11.4	12.6	13.8	15.2	16.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
6	11.4	12.6	13.8	15.2	16.7	18.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
7	11.4	12.6	13.8	15.2	16.7	18.4	20.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
8	0.0	12.6	13.8	15.2	16.7	18.4	20.2	22.3	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
9	0.0	0.0	13.8	15.2	16.7	18.4	20.2	22.3	24.5	0.0	0.0	0.0	0.0	0.0	…	0.0
10	0.0	0.0	0.0	15.2	16.7	18.4	20.2	22.3	24.5	26.9	0.0	0.0	0.0	0.0	…	0.0
11	0.0	0.0	0.0	0.0	16.7	18.4	20.2	22.3	24.5	26.9	29.6	0.0	0.0	0.0	…	0.0
12	0.0	0.0	0.0	0.0	0.0	18.4	20.2	22.3	24.5	26.9	29.6	32.6	0.0	0.0	…	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	20.2	22.3	24.5	26.9	29.6	32.6	35.9	0.0	…	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	22.3	24.5	26.9	29.6	32.6	35.9	0.0	…	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	24.5	26.9	29.6	32.6	35.9	0.0	…	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	26.9	29.6	32.6	35.9	0.0	…	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	29.6	32.6	35.9	0.0	…	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	32.6	35.9	0.0	…	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	35.9	0.0	…	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0

Tablo 6.19 Depreciation amounts matrix for the fourth project alternative (Pro04)

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	…	**20**
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
2	12.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
3	12.0	13.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
4	12.0	13.2	14.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
5	12.0	13.2	14.5	16.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
6	12.0	13.2	14.5	16.0	17.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
7	12.0	13.2	14.5	16.0	17.6	19.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
8	12.0	13.2	14.5	16.0	17.6	19.3	21.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
9	12.0	13.2	14.5	16.0	17.6	19.3	21.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
10	12.0	13.2	14.5	16.0	17.6	19.3	21.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
11	12.0	13.2	14.5	16.0	17.6	19.3	21.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
12	0.0	13.2	14.5	16.0	17.6	19.3	21.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
13	0.0	0.0	14.5	16.0	17.6	19.3	21.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
14	0.0	0.0	0.0	16.0	17.6	19.3	21.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
15	0.0	0.0	0.0	0.0	17.6	19.3	21.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
16	0.0	0.0	0.0	0.0	0.0	19.3	21.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	21.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	…	0.0

As another example, Table 6.17 indicates the investment cost matrix of the fourth project alternative (Pro04) which has a two-year investment period.

Also annual depreciation amounts, the salvage value at the end of lifespan, gains tax amounts, and income tax rate for every year should be determined in order to calculate the $NPV_{it(i)}$ of the project *i*. To calculate the annual depreciation amounts, straight line depreciation method is used in this case, and it is assumed that the whole value of the investment cost is depreciated in the lifespan of the project. So, the gains tax amounts will be zero. It is also suitable to use different depreciation methods while calculating the annual depreciation amounts. Since the investment cost values of project alternatives depend on the year in which they get started, depreciation amounts should be determined in accordance with the starting year of the projects, too. Therefore, depreciation amounts of each project alternative were determined again in the matrix form. For example, Table 6.18 includes the matrix developed for depreciation amounts of the first project alternative (Pro01).

As another example, Table 6.19 indicates the matrix for depreciation amounts of the fourth project alternative (Pro04). Here, it should be noticed that the straight line depreciation method was used and the effect of inflation was ignored during the determination of depreciation amounts included in both tables. However, if required, depreciation amounts can be determined by taking the effect of inflation into account. In this case, only the values included in the matrix will change.

It is needed to determine the inflation rate for every component for every year in order to calculate the values of inflow and outflow components which include inflationary effects. Inflation rates can be different for every component, and also a component may have different rates for different years. The developed NPV formulation depicted in Equation 6.14 and computer simulation model permit to implement different inflation rate effects. By this way, inflow and outflow components can be determined in actual money units. After determining all components in projects' working periods as an actual money unit, we need to determine a discount rate in order to discount the expected future values to today's values. As in inflation rates, discount rates also can be determined in different values for each year. These decisions are in authorization of decision maker. As a consequence, it is assumed that the inflation and discount rates determined for each cost and revenue component is depicted in Table 6.20.

Table 6.20 Multiple Inflation Rates and Discount rate for each project

Item	Symbol	Value	Description
Inflation rate for *revenues*	e_t	Unif [0.06, 0.08]	Determine for each year (assume e_0=0)
Inflation rate for *labor expenses*	γ_t	Unif [0.04, 0.06]	Determine for each year (assume γ_0=0)
Inflation rate for *material expenses*	ω_t	Unif [0.04, 0.05]	Determine for each year (assume ω_0=0)
Inflation rate for *overhead expenses*	τ_t	Unif [0.03, 0.05]	Determine for each year (assume τ_0=0)
Inflation rate for *salvage value*	Ω_t	Unif [0.07, 0.09]	Determine for each year (assume Ω_0=0)
Discount rate	i_t	Unif [0.12, 0.14]	Determine for each year (assume i_0=0)

For instance, inflation rate for revenues is uniformly distributed in 6% and 8%, and it is determined for each year. The developed simulation model will create inflation rates from this stochastic distribution for each year. However, decision maker can take a constant inflation rate for revenues. In order to calculate the profitability of the project, the base year inflation and discount rates are assumed to be zero in the simulation model. Additionally, in this case, income tax rate is assumed to be 25% for each year.

As a consequence, for each project alternative, in order to determine expected $NPV_{it(i)}$, the standard deviation of each $NPV_{it(i)}$ $[\sigma_{it(i)}]$, and coefficient of variation $[CV_{it(i)}]$, all of these values have been entered to the developed computer simulation model for the $NPV_{it(i)}$ expression given in Equation 6.14, and the model has been run 100 times for each year in the planning horizon. The obtained outputs after running this model are shown in Table 6.21, 6.22, and 6.23, respectively.

In this stage, in order to determine expected cash flows of the project *i* given that it proceeds in the year *t* of the planning horizon $[b_{it}]$, the second simulation model has been run. The obtained outputs after running this model are shown in Table 6.21. Siman report of both computer simulation models can be shown in Appendix A and B. It should be noticed that, in all calculations of this stage, the years of the planning horizon were numbered 0 to 12, respectively.

Table 6.21 The expected NPV_{it} of the project alternatives

year→ *i*↓	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Pro01	108	213	318	421	520	617	713	803	891	977	1061	1142	1219
Pro02	55	111	169	225	280	336	391	444	496	548	599	647	693
Pro03	47	93	141	190	240	288	336	384	431	476	520	563	604
Pro04	380	750	1107	1449	1782	2102	2411						
Pro05		231											
Pro06				112	221	327	429	529	625	717	806		
Pro07			178	350	516	676	829	976					
Pro08	196	396											
Pro09	458	909	1240	1561									
Pro10		138	205	271	334	396							

Table 6.22 Standard deviation of NPV_{it} [σ_{it}] of the project alternatives

year→ *i*↓	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Pro01	13	20	25	28	29	31	33	34	35	36	38	38	40
Pro02	11	16	20	23	24	25	26	27	28	29	31	32	33
Pro03	10	15	18	23	25	26	26	28	28	29	30	30	31
Pro04	20	28	35	38	42	42	48						
Pro05		12											
Pro06				10	15	17	18	20	20	22	23		
Pro07			17	25	29	31	35	36					
Pro08	6	22											
Pro09	22	30	38	42									
Pro10		9	12	15	18	19							

Table 6.23 The CV_{it} of the project alternatives

year→ *i*↓	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Pro01	0.120	0.092	0.080	0.066	0.055	0.050	0.046	0.042	0.040	0.037	0.036	0.034	0.033
Pro02	0.200	0.142	0.121	0.100	0.085	0.075	0.066	0.062	0.057	0.053	0.052	0.049	0.047
Pro03	0.213	0.160	0.130	0.121	0.104	0.089	0.078	0.072	0.066	0.061	0.058	0.054	0.051
Pro04	0.053	0.037	0.031	0.026	0.023	0.020	0.020						
Pro05		0.054											
Pro06				0.090	0.066	0.052	0.043	0.037	0.032	0.031	0.029		
Pro07			0.098	0.071	0.056	0.046	0.042	0.037					
Pro08	0.031	0.057											
Pro09	0.048	0.033	0.031	0.027									
Pro10		0.062	0.060	0.056	0.053	0.047							

Table 6.24 The expected b_{it} of the project alternatives

year→ *i*↓	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Pro01	29	33	36	40	44	49	53	59	64	71	78	85	93
Pro02	23	26	29	32	35	39	43	47	52	57	63	69	76
Pro03	36	40	45	50	55	61	67	74	81	90	98	107	119
Pro04	60	66	73	79	87	96	105	114	125	137	150	163	179
Pro05	48	53	58	62	68	74	80	87	95	103	112	121	132
Pro06	23	25	28	31	34	38	41	45	50	55	60	66	72
Pro07	45	50	55	60	66	72	78	85	93	102	111	121	132
Pro08	39	44	50	55	61	68	75	82	91	101	111	121	134
Pro09	53	60	67	74	83	92	101	112	124	138	151	165	183
Pro10	34	38	42	45	50	54	59	64	70	77	84	91	100

✓ *Stage 3: Investment Project Evaluation and Decision*

In this stage, the decision will be made about which projects will be carried out over the planning horizon. Therefore, firstly a multi-objective linear programming model has been developed without taking the potential uncertainties about the objectives and some constraints into consideration. One of the basic assumptions here is that the budget planned to be allocated for each year in the planning horizon is certain. Table 6.25 shows the allocated budget for each year in the planning horizon.

Table 6.25 Allocated investment budget for each year of the planning horizon

Year	**Allocated Budget (TL)**
2009	150
2010	150
2011	200
2012	200
2013	200
2014	300
2015	300
2016	350
2017	350
2018	200
2019	150
2020	250
2021	250

In the hypothetic problem, it is assumed that project 1 precedes project 10, and the number periods of separation between these projects is equal to 1. There are no precedence relations between the other projects taken into consideration. Therefore, Table 6.26 shows the precedence relations between projects.

Table 6.26 The precedence relations between projects [g_{ij}]

$j\rightarrow$ $i\downarrow$	**Pro01**	**Pro02**	**Pro03**	**Pro04**	**Pro05**	**Pro06**	**Pro07**	**Pro08**	**Pro09**	**Pro10**
Pro01										1
Pro02										
Pro03										
Pro04										
Pro05										
Pro06										
Pro07										
Pro08										
Pro09										
Pro10										

It is also assumed in the hypothetic problem that there are two situations that represent the projects which are mutually exclusive. For example, in situation 1, project 6 and project 7 are mutually exclusive, and the maximum acceptable number of these mutually exclusive projects is equal to 1. Therefore, all mutually exclusive projects can be shown in Table 6.27.

Table 6.27 Mutually exclusive projects

d_z↓	Pro01	Pro02	Pro03	Pro04	Pro05	Pro06	Pro07	Pro08	Pro09	Pro10	MAX_Z
d_1						1	1				1
d_2								1	1		1

Now, let us assume that the primary objective of investors is to select the projects with maximum total expected profitability. In this situation, the conventional mathematical model given in (6.15) to (6.28) has been solved and the objective in (6.15) has been optimized at the first phase. Therefore, at the first phase, maximum total profitability value has been determined as 3537 TL. At the second phase, this value has been written in (6.29), thus it has been added a new constraint to the model and the objective in (6.16) has been optimized. In this way, one can select the project set with the lowest risk among the project sets that have maximum total profitability. However, in the hypothetic problem, there is only one project set that has maximum total profitability (3537 TL), so, the project set obtained at the end of both phases during the solution process is same. As a consequence, the optimal scheduling of the investment project alternatives in the planning horizon are determined as follows;

Table 6.28 Optimal scheduling of the investment project alternatives in the planning horizon

year→ *i*↓	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Pro01				■									
Pro02					■								
Pro03									■				
Pro04		■											
Pro05													
Pro06													
Pro07				■									
Pro08													
Pro09		■											
Pro10						■							

Now, let us assume that the uncertainty of the decision makers' aspiration levels for the goals is wanted to be treated. In this situation, at first, the conventional (crisp) linear programming model should be developed. The crisp formulation of the problem has been developed using the equations 6.15 - 6.28. Then, this model was solved with each of the objective functions by themselves. In other words, at first, Equation (6.15) was set as the objective and the model was solved. Then, Equation (6.16) was set as the objective and the model was solved. As a consequence, the obtained efficient extreme solutions of the problem are presented in Table 6.29.

Table 6.29 The payoff table

	NPV	*CV*
NPV	3537	0.405
CV	2194	0.247

Considering the efficient extreme solutions given in Table 6.29, the lower and upper bound of the objectives can be determined. Therefore, the lower and upper bounds of the objectives are shown as in Table 6.30.

Table 6.30 Lower and upper bounds of the objectives

Objectives	**Lower Bound**	**Upper Bound**
NPV	2194	3537
CV	0.247	0.405

Membership functions of fuzzy objectives can be defined now using corresponding upper and lower bounds as follows;

$$\mu_{NPV} = \begin{cases} 0 & if \quad NPV \leq 2194 \\ \dfrac{NPV - 2194}{3537 - 2194} & if \quad 2194 < NPV \leq 3537 \\ 1 & if \quad NPV > 3537 \end{cases} \tag{6.47}$$

$$\mu_{CV} = \begin{cases} 1 & if \quad CV \leq 0.247 \\ \dfrac{0.405 - CV}{0.405 - 0.247} & if \quad 0.247 < CV \leq 0.405 \\ 0 & if \quad CV > 0.405 \end{cases} \tag{6.48}$$

As mentioned, in order to construct multi-objective linear programming model with fuzzy objectives, five fuzzy multi-objective modeling approaches have been used. The solution results of the models with these five modeling approaches are summarized in the following tables.

Table 6.31 Solution results of the model with Tiwari et al.'s weighted additive approach

w_i	NPV	CV	μ_{NPV}	μ_{CV}
[0.5; 0.5]	3057	0.257	0.642	0.936
[0.8; 0.2]	3537	0.405	1.000	0.000

Table 6.32 Solution results of the model with Werners' "fuzzy and" operator approach

γ	λ	NPV	CV	μ_{NPV}	μ_{CV}
0.5	0.000	3057	0.257	0.642	0.936
0.8	0.642	3057	0.257	0.642	0.936

Table 6.33 Solution results of the model with Li's two-phase approach

λ	NPV	CV	μ_{NPV}	μ_{CV}
0.0	3057	0.257	0.642	0.936

Table 6.34 Solution results of the model with Lai & Hwang's approach

δ	w_k	λ	NPV	CV	μ_{NPV}	μ_{CV}
0.001	[0.5; 0.5]	0.642	3057	0.257	0.642	0.936
0.001	[0.8; 0.2]	0.642	3057	0.257	0.642	0.936

Table 6.35 Solution results of the model with Lin's weighted max-min approach

w_k	β	NPV	CV	μ_{NPV}	μ_{CV}
[0.5; 0.5]	1.285	3057	0.257	0.642	0.642
[0.8; 0.2]	1.064	3337	0.342	0.851	0.213

As seen in the solution results tables above, while the developed fuzzy multi-objective linear programming models are solved by using five fuzzy multi-objective modeling approach, randomly selected parameter values and weights of objectives have been used. For example, at first, the weights of the objectives were assumed to be equal. Secondly, the weights of the first and second objectives were assumed to be

0.8 and 0.2, respectively. According to the solution results, the obtained values by using the first four fuzzy multi-objective approaches indicates that if the decision maker accepts approximately 13% decrease of total NPV, it can be selected another project set that has approximately 36% lower risk. Solution results of the model with Lin's weighted max-min approach also indicates that if the decision maker accepts approximately 5% decrease of total profitability, it can be selected another project set that has approximately 16% lower risk.

Now, let us assume that, in the hypothetic investment project evaluation problem, the estimated amount of budget allocated for each year is flexible. In this situation, besides the uncertainty of the decision makers' aspiration levels for the goals, also uncertainty for this constraint is treated.

In the mathematical models developed in this situation, besides the membership functions of the fuzzy objectives, the membership function of the fuzzy constraint should be determined. As known, the membership functions of fuzzy objectives can be defined using corresponding upper and lower bounds. In order to determine the lower and upper bounds of the objectives, the payoff table should be constructed again. The obtained payoff table of the problem is presented in Table 6.36.

Table 6.36 The payoff table

	NPV	*CV*
NPV	3537	0.405
CV	2194	0.247
NPV	3537	0.405
CV	2194	0.247

The upper part of the payoff table given in Table 6.36 is constructed by solving the problem considering the individual objective functions subject to fuzzy constraint set while the crisp constraint set is considered in the lower part. As seen from this table, the efficient extreme solutions of the upper part of the payoff table are the same with the lower part.

Considering the efficient extreme solutions given in Table 6.36, the lower and upper bounds of the objectives were determined as in Table 6.37.

Table 6.37 Lower and upper bounds of the objectives

Objectives	**Lower Bound**	**Upper Bound**
NPV	2194	3537
CV	0.247	0.405

As a consequence, membership functions of fuzzy objectives are the same with (6.47) and (6.48).

In order to treat the uncertainty of the budget for each year in addition to the uncertainty of the decision makers' aspiration levels for the goals, the budget for each year is altered within a range. Therefore, it is stated that the budget for each year as fuzzy parameters using triangular membership function as illustrated in Figure 6.12. Here, the lower and upper bounds of the budget for each year are assumed 90% and 110% of their corresponding expected budget (r_t^0) values, respectively.

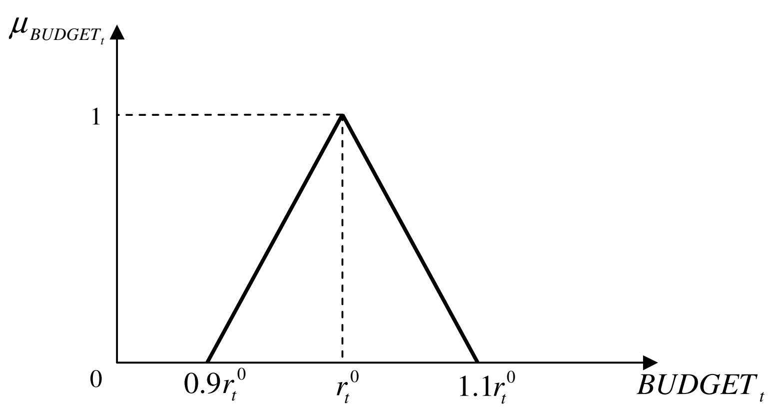


Figure 6.12 Membership function of the budget for each year

Therefore, membership function of the budget for each year can be defined using corresponding upper and lower bounds as follows;

$$\mu_{BUDGET_t} = \begin{cases} \dfrac{BUDGET_t - 0.9r_t^0}{r_t^0 - 0.9r_t^0} & if \quad 0.9r_t^0 < BUDGET_t \le r_t^0 \\ \dfrac{1.1r_t^0 - BUDGET_t}{1.1r_t^0 - r_t^0} & if \quad r_t^0 < BUDGET_t \le 1.1r_t^0 \\ 0 & if \quad otherwise \end{cases} \tag{6.49}$$

In order to construct multi-objective linear programming model with fuzzy objectives and fuzzy constraints, same fuzzy multi-objective modeling approaches have been used. The solution results of the models with these five modeling approaches are summarized in the following tables.

Table 6.38 Solution results of the model with Tiwari et al.'s weighted additive approach

w_i	*NPV*	*CV*	μ_{NPV}	μ_{CV}
[0.5; 0.5]	3057	0.257	0.642	0.936
[0.8; 0.2]	3537	0.405	1.000	0.000

Table 6.39 Solution results of the model with Werners' "fuzzy and" operator approach

γ	λ	*NPV*	*CV*	μ_{NPV}	μ_{CV}
0.5	0.000	3057	0.257	0.642	0.936
0.8	0.642	3057	0.257	0.642	0.936

Table 6.40 Solution results of the model with Li's two-phase approach

λ	*NPV*	*CV*	μ_{NPV}	μ_{CV}
0.0	3057	0.257	0.642	0.936

Table 6.41 Solution results of the model with Lai & Hwang's approach

δ	w_k	λ	*NPV*	*CV*	μ_{NPV}	μ_{CV}
0.001	[0.5; 0.5]	0.642	3057	0.257	0.642	0.936
0.001	[0.8; 0.2]	0.642	3057	0.257	0.642	0.936

Table 6.42 Solution results of the model with Lin's weighted max-min approach

w_k	β	*NPV*	*CV*	μ_{NPV}	μ_{CV}
[0.5; 0.5]	1.285	3057	0.257	0.642	0.642
[0.8; 0.2]	1.064	3337	0.342	0.851	0.213

As seen, the obtained results in Table 6.31 to 6.35 are the same with the results in Table 6.38 to 6.42. In other words, treating the uncertainty of the budget for each year in addition to the uncertainty of the decision makers' aspiration levels for the goals has not been changed the results of the multi-objective linear programming models with fuzzy objectives. This situation indicates that the second problem structure described in Section 6.2.3.2 has not been occurred in the hypothetic problem. Therefore, in this hypothetic problem, there is no profitable project which can not be carried out owing to little inadequacy of budget.

6.4 Summary of Chapter

This chapter proposed an integrated multi-criteria decision making methodology for risky investment projects evaluation. This proposed methodology consists of three stages. Each stage deals with the problems that should be solved during the process of evaluation of investment projects. Different from the approaches proposed in the literature, in the methodology proposed in this dissertation, at first, investment projects is classified by using a MCS method which does not require a training sample, and takes into account the inherent risk and uncertainty associated with the values of evaluation criteria. Second, in order to determine the profitability of the projects, a new NPV formulation that eliminates the weakness of using the traditional formulation of NPV is used.

In the proposed methodology, in order to calculate the expected profitability of the project and determine the risk level of the projects, the computer simulation model was developed for a developed NPV formulation. Also, the second simulation model was developed in order to calculate the expected cash flows for each project in each period. The last contribution of this dissertation is to construct multi-objective mathematical models such as multi-objective linear programming model and fuzzy multi-objective linear programming models in order to solve the optimal project selection and scheduling problem that was explained in details in the Section 1.1.

CHAPTER SEVEN
CONCLUSION

7.1 Summary and Concluding Remarks

Investments play an important role for enterprises in order to fulfill their objectives and attain their goals. Ability of enterprises to maintain their existence in the long run and adapt themselves to changes in economic, technical and social environment mainly depends on the investments they plan and make. Changes in economic, technologic, social and cultural environment lead enterprises to plan their investments. Selecting the most suitable one among different investment areas is of great importance for enterprises. Besides large funds required by investments, investment decisions affect an enterprise's policy across a long time period. The reason for this is that enterprises are bound by selection of manufacturing area, selection of technology, selection of location for an enterprise to be founded or extended, and determination of production capacity throughout an investment's life cycle. So, planning and evaluation of investments is vital for enterprises.

Since the ability of enterprises to adapt themselves to fast-changing economic and technological environment and their achievements in the long run depend on investment projects they try to make, preparation and evaluation of investment projects are highly important for enterprises and countries. Projects which are prepared on the basis of incomplete or inaccurate data without detailed consideration and analysis lead to wrong investment decisions. Therefore, investment projects should be prepared and evaluated in accordance with scientific principles so as to take consistent and reliable decisions about the evaluation and selection of investment projects.

As mentioned in the previous chapters, economic resources at disposal are not enough for satisfying all needs and realizing all targets of enterprises or countries. Naturally, limited resources constitute an obstacle to finance all investment alternatives possible and perform all these investment alternatives at the same time.

For this reason, both enterprises and countries are supposed to carry out appropriate investments so as to use their resources in a proper and rational manner. Otherwise, already limited resources would be wasted. Therefore, it is necessary to make a choice between competing investment proposals, to list them in accordance with certain evaluation criteria and give up some of the investment proposals at least for a while should the problem of lack of resources arise.

In every investment, all or some of the available capital is used for purchasing several assets. An enterprise, for instance, may use some of its available capital for purchasing fixed assets such as machinery, equipment and vehicles. This enterprise, therefore, allocates some of its capital to such assets. These assets have been purchased with the aim of using them for long periods and providing benefits or advantages to the enterprise. In other words, it is often impossible to convert these purchased assets into cash in the short term.

When investment decisions are taken, one analyzes whether the expected advantages or benefits of investments will be achieved in return for total investment costs required for these investments. For example, before an enterprise makes an investment decision about founding a new production plant, it has to analyze whether the income to be obtained from the products to be manufactured in the new plant will exceed the investment cost to be suffered in order to found this new plant.

As known, investment decisions are taken by making a prediction before they are put into practice about the costs to be suffered over the investment's life cycle and incomes it will bring about. In other words, before making investment decisions, a prediction is made on costs and incomes regarding yet unrealized periods, and decisions are taken with a comparison of these predictions of costs and incomes with each other. However, there is no certainty that these predictions on future periods will be actually realized values. A decision maker will always encounter with uncertainties during the prediction and planning of the future. While making predictions on monetary values of the future, the decision maker observes the course and tendency of past happenings and phenomena existing up to that time and assumes that this tendency will also continue in the future. However, as already mentioned, there is no certainty and guarantee that future happenings in economic and social life will be a repetition of and similar to past happenings. The reason for this is that future is full of uncertainties, and many factors beyond the decision maker's control cause deviations in estimated monetary values.

We should note here that, the phenomenon "uncertainty" in investment decisions presents itself not only in predictions of monetary values regarding the elements of cost or income but also in amount-related predictions. For example, during the process of taking an investment decision, one tries to predict also the amount of demand by the market for the products to be manufactured as a result of the investment likely to be realized. However, it should be remembered that these predictions of demand are liable to deviations owing to uncertain and risky nature of the future.

In project management, it is common to refer to very high levels of uncertainty as sources of risk. Risk is present in most investment projects. As mentioned in Section 2.5, the term *risk* is used to describe an investment project whose cash flow is not known in advance with absolute certainty, but for which an array of alternative outcomes and their probabilities are known. For example, when the product of a project deviates from the predicted value of its market price in the future, this will directly affect predicted project incomes and therefore project cash flows. Then, this situation affects the accuracy and consistence of investment decisions.

The prices of elements which provide incomes or lead to costs in projects will change in the course of time owing to the inflation. Inflation is one of the most significant reasons why the prices of the products of a project deviate from the predicted values in the future. Inflation leads to a change not only in product prices but also in prices of all income and expense items.

Therefore, inflation proves an element which causes many project parameters such as project incomes and expenses to deviate from the predicted values in the future. In the light of this information, it is possible to call the inflation as a risk element which causes the project parameters to deviate from the predicted values and also leads to changes in estimated project cash flows. Furthermore, inflation rates in the future period can be estimated only through prediction.

In inflationary periods, there is a change in the prices of items which compose the project incomes or expenses. In case of inflation, the prices of income and expense items will absolutely change and so will the relative prices of all elements. These variations in project income and expense items will naturally affect project net cash flows. As known, for an evaluation of investment projects, at first, it is necessary to find out the estimated net cash flows of project alternatives. Then, projects are evaluated on the basis of certain evaluation methods through the use of cash flows determined and a decision is taken regarding the investment. Since inflation has a great impact on the net cash flows of projects, it is inevitable that wrong investment decisions will be made if inflation is not taken into consideration during the project evaluation.

The development of an industrial investment project, no matter of what kind or for what duration, from the stage of the initial idea until the plant, is in operation can be shown in the form of a cycle comprising three distinct phases. These phases are called as the pre-investment phase, the investment phase, and the operational phase. The detail analyses of these phases were discussed in the previous sections.

By the fact that the methodology proposed in this dissertation is based on the evaluation of risky investment projects, the investment and the operational phases of the investment project cycle is out of the scope of this dissertation. The pre-investment phase which means the time period between the birth of investment idea and decision making to invest consists of several stages. The literature review indicates that these stages are generally called as follows; identification of investment opportunities (opportunity studies), pre-feasibility studies, feasibility studies, and investment project evaluation and decision.

In the scope of this dissertation, an integrated multi-criteria making methodology for risky investment projects evaluation that was developed by considering all the facts described above has been proposed. The proposed methodology consists of three main stages. Each stage deals with the problems that should be solved during the process of evaluation of investment projects. Comprehensive literature review suggests that there is a need for an integrated methodology regarding the evaluation and selection of investment projects. Up to present, in their studies, researchers have preferred to seek for solution by dealing with the problems encountered during the process of evaluation of investment projects individually rather than collectively. However, investment project evaluation is an integrated process.

The first stage of the methodology includes opportunity and pre-feasibility studies. The aim of this stage is to identify the investment opportunities, to carry out preliminary election of project ideas and to give prominence to ideas which have the highest chance of attaining the goals planned by entrepreneurs and investors. The preliminary project selection is a MCDM problem in nature. Therefore, in the first stage, different from the approaches proposed in the literature, the investment projects are classified by using a MCS method which does not require a training sample, and takes into account the inherent risk and uncertainty associated with the values of evaluation criteria. This MCS method named as PROMSORT that assigns alternatives to predefined ordered categories was proposed for financial classification problems. It was also used to solve the strategic supplier selection problem. In the scope of this dissertation, this new MCS procedure has been adapted to the investment project evaluation and selection problems.

After assigning of the project alternatives to the classes by PROMSORT, the second stage of the proposed methodology called as feasibility study begins. In the second stage of the proposed methodology, a new NPV formulation that eliminates the weakness of using the traditional formulation of NPV while evaluating the projects has been developed. The literature includes studies based on analyses the results of feasibility studies of investments. In these studies, it is often assumed that the effect of inflation is same both on project inflows and outflows, so the effect of inflation on project inflows and outflows is not taken into account. But it is obvious that inflation effect will be different for cost and revenue components, and it should be considered in project evaluation process. For that reason, the developed NPV formulation takes into account the different inflation effects on the cash flow elements. Therefore, the new NPV formulation is based on three principles. Firstly, the inflation effect on project inflows and outflows may be different one another. Secondly, the increase rates for the price of inflow or outflow items can be different each other, and the last, the inflation rates for specific cash flow element may be different for each year.

As mentioned, in uncertain and risky environments, the values of the project parameters can not be estimated with complete certainty, and it is necessary to consider uncertainty and risk phenomena while evaluating projects. The risky project parameters are defined as probability distributions by using simulation models. Also, the expected profitability of the project is calculated via simulation. For these reasons, in the second stage, a computer simulation model for new NPV formulation has been developed by using computer simulation software. By the help of this model, all parameters affecting the NPV of the project can be defined as discrete and continuous probability distributions if required. Also, the second simulation model has been developed in order to calculate the expected cash flows for each project in each period.

During any time period, most enterprises, especially public enterprises, have to make a ranking and selection among a number of investment project alternatives. Some of these alternatives may be promising and can allow the enterprises and entrepreneurs to realize their objectives. However, the budgets of the enterprises are generally not enough to implement all of these investment proposals which have high expected utility level at the same time. In these cases, the enterprises prefer to implement the investment project proposals at the number allowed by the size of their budgets. On the other hand, the lack of budget is not the only reason for the fact that some of the investment project proposals with high expected utility level are selected and performed and the others are not. The other reasons of this complexity may be some technical limitations such as earliest and latest start dates and precedence relations.

However, in today's high competitive environments, enterprises, especially public enterprises, have to act well-planned. The first step of acting well-planned is to determine a planning horizon and to predict how much budget to allocate for carrying out investment projects each period over that planning horizon. In this way, there will not be the cases in which some of the project proposals, evaluated at the beginning of each period and predicted to have high expected utility level, cannot carried out due to the lack of budget allocated for investments for that period. If some of the project proposals cannot be put into practice owing to lack of budget, these project proposals will have the chance of being carried out at other periods in the planning horizon. The reason for this is that the budget estimate regarding each period in the planning horizon is certain. In this new case, the main objective of the enterprises is to maximize the expected utility of all investment projects which are carried out over the planning horizon.

In the scope of this dissertation, this type of problem is called as optimal project selection and scheduling problem and it was explained in detail in Section 1.1. This problem is complex. Because, investment project should be considered according to multiple objectives, project cash flows are uncertain, the estimated budget for each time period can be flexible, and there are several limitations. The last original contribution of this dissertation is to construct multi-objective mathematical models such as multi-objective linear programming model and fuzzy multi-objective linear programming models in order to solve this optimal project selection and scheduling problem.

In the mathematical models developed in the third stage of the proposed methodology, there are two objectives. First objective is trying to maximize sum of NPVs of the chosen projects, and the second objective is trying to minimize sum of coefficient of variations of the chosen projects. They combine project selection and scheduling decisions, while considering risk and profitability as optimization criteria.

At the third stage of the proposed methodology, firstly a multi-objective linear programming model has been developed without taking the potential uncertainties about the objectives and some constraints into consideration. The developed model will help to take a decision about when and which project proposals will be carried out over the planning horizon. The model tries to maximize the sum of NPVs of the chosen projects while minimizing their deviation.

However, it is not possible to optimize both objectives, simultaneously. In other words, there is no guarantee that the project set obtained as a result of the model run will consist of projects with both the maximum profitability and lowest risk. Perhaps, any project set will provide the highest profitability while involving a considerably high risk at the same time. Therefore, the crisp multi-objective linear programming model can be solved in two phases. In the first phase, one of the objectives is optimized subject to all constraints. In the second phase, the second objective is optimized including an additional restriction that avoids the deterioration of the first objective, guaranteeing Pareto optimality. Within the scope of this dissertation, it has been assumed that the primary objective of investors will be to select the projects with maximum total expected profitability. In this case, at the end of running of the model, one is able to select the project set with the lowest risk among the project sets that have maximum total profitability value found out during the first phase. However, two important problems, described in detail in Section 6.2.3.2, arise.

In order to solve the first type of problem described in Section 6.2.3.2, multi-objective linear programming models with fuzzy objectives have been developed. By using these models, uncertainty of the decision makers' aspiration levels for the goals is treated, and consequently, the preferred compromise solution can be determined.

In order to treat the uncertainty for some of the constraints besides the uncertainty of the decision makers' aspiration levels for the goals, also multi-objective linear programming models with fuzzy objectives and fuzzy constraints have been developed. During the development of these mathematical models, it is assumed that the estimated amounts of budgets for each year are flexible. In this way, thanks to the developed models, it is likely that the project sets with little less profitability than maximum total profitability but with very low risks will be selected. Here, besides the membership functions of the fuzzy objectives, the membership function of the fuzzy constraint should be determined.

At the third stage of the proposed methodology, in order to construct fuzzy multi-objective linear programming models, five fuzzy multi-objective modeling approaches have been used. These approaches are Tiwari et al.'s weighted additive approach, Werners' "fuzzy and" operator approach, Li's two-phase approach, Lai & Hwang's approach, and Lin's weighted max-min approach.

Finally, in order to show the applicability of the proposed novel methodology in the process of risky investment projects evaluation, a hypothetical investment project evaluation and selection problem covering all stages of the proposed methodology was developed. In this hypothetic problem, it was assumed that 40 investment project alternatives were determined by the investor. Again, the investor plans to carry out the selected projects among over a 13 year planning horizon. Therefore, the primary objective of the investor is to make a decision about which projects will be carried out at which time over the planning horizon. While doing so, the investor wants to select the investment projects that have maximum total profitability and the lowest risk. Section 6.3 includes an individual application of each stages of the novel methodology proposed within the scope of this dissertation and an interpretation of the obtained results. Results of the computational experiments show that the proposed methodology can effectively be used in evaluation of risky investment projects.

7.2 Contributions of the Thesis

As mentioned in Section 1.3, the following list summarizes the original contributions to be achieved with this dissertation to the investment project evaluation and selection literature.

(1) The major contribution of this dissertation is to propose an integrated multi-criteria decision making methodology for risky investment projects evaluation which includes all the facts described in the previous sections. This integrated methodology has been explained in details in Chapter Six.

(2) The second original contribution of this dissertation is to classify the investment projects by using a MCS method which does not require a training sample, and takes into account the inherent risk and uncertainty associated with the values of evaluation criteria. This MCS method named as PROMSORT that assigns alternatives to predefined ordered categories was proposed by Araz & Ozkarahan (2005) for financial classification problems. It was also used to solve the strategic supplier selection problem by Araz et al. (2007). This new MCS procedure has been adapted to the investment project evaluation and selection problems. In the scope of this dissertation, this method has been used in order to assign project alternatives to predefined ordered categories in the first stage of the proposed methodology.

(3) The third original contribution of this dissertation is to develop a new NPV formulation that eliminates the weakness of using the traditional formulation of NPV, which has been expressed in Section 2.4.2.1, while evaluating the projects. The developed NPV formulation has been explained in details in Section 6.2.2.

(4) The fourth original contribution of this dissertation is to develop a computer simulation model for new NPV formulation by using computer simulation software. As mentioned, in uncertain and risky environments, the values of the project parameters can not be estimated with complete certainty, and it is necessary to consider uncertainty and risk phenomena while evaluating projects. The risky project parameters are defined as probability distributions by using simulation models. Also, the expected profitability of the project is calculated via simulation. In the developed

NPV formulation, the numbers of the random variables have been increased. So, providing a sufficient number of NPVs to define the NPV distribution would be more difficult by using Monte Carlo simulation. In this dissertation, the second simulation model is developed in order to calculate the expected cash flows for each project in each period. The developed computer simulation models have been explained in details in Section 6.2.2.

(5) The fifth original contribution of this dissertation is to construct multi-objective mathematical models such as multi-objective linear programming model and fuzzy multi-objective linear programming models in order to solve the optimal project selection and scheduling problem that was explained in details in the Section 1.1. The constructed multi-objective mathematical models have been explained in details in Section 6.2.3.

7.3 Suggestions for Future Research

In the scope of this dissertation, we propose a novel methodology for risky investment projects evaluation. This proposed methodology consists of three stages. Each stage deals with the problems should be solved during the process of evaluation of investment projects.

In the first stage of the proposed methodology, the new MCS procedure, named as PROMSORT, has been used in order to assign project alternatives to predefined ordered classes. However, the major drawback of PROMSORT, like other MCS method, is that the decision maker must satisfy the considerable amount of information. The decision maker should assign values to profiles, weights and thresholds. In the implementation of this method, the parameters have been determined subjectively. Therefore, one of the further research studies should be develop an indirect estimation procedure for the parameters specified by the decision maker using a set of traning samples.

In the application of the PROMSORT method, it is assumed that the performances of an alternative on a set of criteria are known exactly. However, in the project evaluation and selection process, some criteria may be impractical to evaluate, information may be difficult to obtain, complex to analyze or there may not be sufficient time to perform these issues. When the performances of alternatives can be only approximately determined, FST comes in handy to model these uncertainities and imprecision. Therefore for further research, the fuzzy PROMSORT developed by Araz (2007) can be used in order to classify the project alternatives into predefined classes in the first stage of the proposed methodology.

In the second stage of the proposed methodology, while the feasibility studies for the selected projects are conducted, it is focused on to determine the project profitability which is the mostly used project evaluation criterion. However, besides this criterion, there are some other project evaluation criteria. For further research, in this stage, different computer simulation models can be developed in order to determine the values of these criteria.

In the computer experiments, while the profitability and coefficient of variations of the selected projects are tried to be found in the second stage, the situation in which the values of the project parameters are defined as stochastic distributions, have been analyzed. However, the solution process in the situation in which the values of the project parameters are defined as fuzzy numbers has been briefly described. For further research, when the values of the some project parameters defined as fuzzy numbers, the investment problem that covers how to use the simulation method in this situation can be considered.

In the third stage of the proposed methodology, in order to find which projects should be carried out and when they should be realized in the planning horizon, the multi-objective mathematical models have been developed. In these developed models, two objectives have been considered. However, in the project evaluation and selection process, besides these objectives, different objectives can be handled. For further research, the new mathematical models can be developed that considers these objectives.

While the developed multi-objective mathematical models with fuzzy objectives and fuzzy constraints are solved in the third stage, the most common fuzzy multi-objective modeling approaches have been used. For further research, the new fuzzy multi-objective modeling approach can be developed and these multi-objective models can be solved by using this approach.

In the scope of this dissertation, in order to demonstrate the applicability of the proposed methodology, the hypothetic investment project evaluation and selection problem has been developed, and in the computational experiments, this problem has been solved by using the proposed methodology. Therefore, one of the future research studies can be the implementation of the proposed methodology, whose applicability was proved in this dissertation, to the real life investment project evaluation and selection problems.

The systematic methodology for investment project evaluation and selection presented in this dissertation can be extended to the analysis of other management decision problems.

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APPENDIX A

SIMAN REPORT OF A COMPUTER SIMULATION MODEL FOR A $NPV_{it(i)}$ FORMULATION

```
0$      CREATE,     1:,1:NEXT(24$);
24$     ASSIGN:     t_i=t_i_min:
                    mt_i=t_i+1;
1$      ASSIGN:     t1=t_i+u_i:
                    mt1=t1+1;
2$      ASSIGN:     z1=0:
                    mz1=z1+1:
                    e(mz1,1)=0:
                    y(mz1,1)=0:
                    w(mz1,1)=0:
                    t(mz1,1)=0:
                    i(mz1,1)=0:
                    term01=(1+e(mz1,1)):
                    term02=(1+y(mz1,1)):
                    term03=(1+w(mz1,1)):
                    term04=(1+t(mz1,1)):
                    term05=(1+i(mz1,1)):
                    z1=z1+1:
                    mz1=z1+1;
3$      ASSIGN:     e(mz1,1)=e_exp:
                    y(mz1,1)=y_exp:
                    w(mz1,1)=w_exp:
                    t(mz1,1)=t_exp:
                    i(mz1,1)=i_exp:
                    term01=term01*(1+e(mz1,1)):
                    term02=term02*(1+y(mz1,1)):
                    term03=term03*(1+w(mz1,1)):
                    term04=term04*(1+t(mz1,1)):
                    term05=term05*(1+i(mz1,1)):
                    z1=z1+1:
                    mz1=z1+1;
4$      BRANCH,     1:
                    If,z1<=t1,3$,Yes:
                    Else,5$,Yes;
5$      ASSIGN:     REV(mt1,mt_i)=REV_exp:
                    LAB(mt1,mt_i)=LAB_exp:
                    MAT(mt1,mt_i)=MAT_exp:
                    OVE(mt1,mt_i)=OVE_exp;
6$      ASSIGN:     additional=
                    ((REV(mt1,mt_i)*term01-
                    (LAB(mt1,mt_i)*term02+MAT(mt1,mt_i)*term03+OVE(mt1,mt_i)*term0
                    4))*(1-TAX)+DEP(mt1,mt_i)*TAX)/term05:
                    mainterm01=mainterm01+additional;
7$      ASSIGN:     t1=t1+1:
                    mt1=t1+1;
8$      BRANCH,     1:
                    If,t1<=(t_i+v_i-1),3$,Yes:
                    Else,9$,Yes;
9$      ASSIGN:     t2=t_i:
```

```
                        mt2=t2+1;
10$     ASSIGN:         z2=0:
                        mz2=z2+1:
                        term06=(1+i(mz2,1)):
                        z2=z2+1:
                        mz2=z2+1;
11$     BRANCH,         1:
                        If,z2<=t2,12$,Yes:
                        Else,13$,Yes;
12$     ASSIGN:         term06=term06*(1+i(mz2,1)):
                        z2=z2+1:
                        mz2=z2+1:NEXT(11$);
13$     ASSIGN:         mainterm02=mainterm02+(IIC(mt2,mt_i)/term06);
14$     ASSIGN:         t2=t2+1:
                        mt2=t2+1;
15$     BRANCH,         1:
                        If,t2<=(t_i+u_i-1),11$,Yes:
                        Else,16$,Yes;
16$     ASSIGN:         z3=0:
                        mz3=z3+1:
                        o(mz3,1)=0:
                        term07=(1+o(mz3,1)):
                        term08=(1+i(mz3,1)):
                        z3=z3+1:
                        mz3=z3+1;
17$     BRANCH,         1:
                        If,z3<=(t_i+v_i-1),18$,Yes:
                        Else,19$,Yes;
18$     ASSIGN:         o(mz3,1)=o_exp:
                        term07=term07*(1+o(mz3,1)):
                        term08=term08*(1+i(mz3,1)):
                        z3=z3+1:
                        mz3=z3+1:NEXT(17$);
19$     ASSIGN:         mainterm03=(SAL*term07+GTA)/term08;
20$     ASSIGN:         NPVi(1,mt_i)=mainterm01-mainterm02+mainterm03;
22$     ASSIGN:         t_i=t_i+1:
                        mt_i=t_i+1;
23$     BRANCH,         1:
                        If,t_i<=t_i_max,1$,Yes:
                        Else,21$,Yes;
21$     DISPOSE:        No;



VARIABLES:   1,t_i,CLEAR(System),CATEGORY("None-None"):
             2,mt_i,CLEAR(System),CATEGORY("None-None"):
             3,t_i_min,CLEAR(System),CATEGORY("None-None"),0:
             4,t_i_max,CLEAR(System),CATEGORY("None-None"),12:
             5,u_i,CLEAR(System),CATEGORY("None-None"),1:
             6,v_i,CLEAR(System),CATEGORY("None-None"),8:
             7,t1,CLEAR(System),CATEGORY("None-None"):
             8,mt1,CLEAR(System),CATEGORY("None-None"):
             9,t2,CLEAR(System),CATEGORY("None-None"):
             10,mt2,CLEAR(System),CATEGORY("None-None"):
             11,z1,CLEAR(System),CATEGORY("None-None"):
             12,mz1,CLEAR(System),CATEGORY("None-None"):
             13,z2,CLEAR(System),CATEGORY("None-None"):
```

```
14,mz2,CLEAR(System),CATEGORY("None-None"):
15,z3,CLEAR(System),CATEGORY("None-None"):
16,mz3,CLEAR(System),CATEGORY("None-None"):
17,TAX,CLEAR(System),CATEGORY("None-None"),0.25:
18,SAL,CLEAR(System),CATEGORY("None-None"),0:
19,GTA,CLEAR(System),CATEGORY("None-None"),0:
20,mainterm01,CLEAR(System),CATEGORY("None-None"):
21,mainterm02,CLEAR(System),CATEGORY("None-None"):
22,mainterm03,CLEAR(System),CATEGORY("None-None"):
23,term01,CLEAR(System),CATEGORY("None-None"):
24,term02,CLEAR(System),CATEGORY("None-None"):
25,term03,CLEAR(System),CATEGORY("None-None"):
26,term04,CLEAR(System),CATEGORY("None-None"):
27,term05,CLEAR(System),CATEGORY("None-None"):
28,term06,CLEAR(System),CATEGORY("None-None"):
29,term07,CLEAR(System),CATEGORY("None-None"):
30,term08,CLEAR(System),CATEGORY("None-None"):
31,additional,CLEAR(System),CATEGORY("None-None"):
40,e(21,1),CLEAR(System),CATEGORY("None-None"):
70,y(21,1),CLEAR(System),CATEGORY("None-None"):
100,w(21,1),CLEAR(System),CATEGORY("None-None"):
130,t(21,1),CLEAR(System),CATEGORY("None-None"):
160,i(21,1),CLEAR(System),CATEGORY("None-None"):
190,o(21,1),CLEAR(System),CATEGORY("None-None"):
220,REV(21,13),CLEAR(System),CATEGORY("None-None"):
520,LAB(21,13),CLEAR(System),CATEGORY("None-None"):
820,MAT(21,13),CLEAR(System),CATEGORY("None-None"):
1120,OVE(21,13),CLEAR(System),CATEGORY("None-None"):

1500,DEP(21,21),CLEAR(System),CATEGORY("NoneNone"),0,11.42857143,11.4
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0,0,0,0,0,0,18.40582857,18.40582857,18.40582857,18.40582857,18.40582857,18.4
0582857,18.40582857,0,0,0,0,0,0,0,0,0,0,0,0,0,0,20.24641143,20.24641143,20.24
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83,24.49815783,24.49815783,24.49815783,24.49815783,0,0,0,0,0,0,0,0,0,0,0,0,0,
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0,0,0,0,0,0,0,0,0,0,0,0,0,0,0:


2000,IIC(21,21),CLEAR(System),CATEGORY("NoneNone"),80,0,0,0,0,0,0,0,0,0,0
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```

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              0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,128.8408,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1
              41.72488,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,155.897368,0,0,0,0,0,0,0,0,0,0,0,0,
              0,0,0,0,0,0,0,0,0,171.4871048,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,188.6358153,0
              ,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,207.4993968,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
              ,0,0,0,228.2493365,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,251.0742701,0,0,0,0,0,0,
              0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
              0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
              0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
              0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0:

              2500,NPVi(1,21),CLEAR(System),CATEGORY("None-None");

SEEDS:        11,12000,Common:
              12,13000,Common:
              13,14000,Common:
              14,15000,Common:
              15,16000,Common:
              16,17000,Common:
              17,18000,Common:
              18,19000,Common:
              19,20000,Common:
              20,21000,Common;

OUTPUTS:      1,NPVi(1,1),"NPVi_0.dat",NetPresentValue_i_0:
              2,NPVi(1,2),"NPVi_1.dat",NetPresentValue_i_1:
              3,NPVi(1,3),"NPVi_2.dat",NetPresentValue_i_2:
              4,NPVi(1,4),"NPVi_3.dat",NetPresentValue_i_3:
              5,NPVi(1,5),"NPVi_4.dat",NetPresentValue_i_4:
              6,NPVi(1,6),"NPVi_5.dat",NetPresentValue_i_5:
              7,NPVi(1,7),"NPVi_6.dat",NetPresentValue_i_6:
              8,NPVi(1,8),"NPVi_7.dat",NetPresentValue_i_7:
              9,NPVi(1,9),"NPVi_8.dat",NetPresentValue_i_8:
              10,NPVi(1,10),"NPVi_9.dat",NetPresentValue_i_9:
              11,NPVi(1,11),"NPVi_10.dat",NetPresentValue_i_10:
              12,NPVi(1,12),"NPVi_11.dat",NetPresentValue_i_11:
              13,NPVi(1,13),"NPVi_12.dat",NetPresentValue_i_12:
              14,NPVi(1,14),"NPVi_13.dat",NetPresentValue_i_13:
              15,NPVi(1,15),"NPVi_14.dat",NetPresentValue_i_14:
              16,NPVi(1,16),"NPVi_15.dat",NetPresentValue_i_15:
              17,NPVi(1,17),"NPVi_16.dat",NetPresentValue_i_16:
              18,NPVi(1,18),"NPVi_17.dat",NetPresentValue_i_17:
              19,NPVi(1,19),"NPVi_18.dat",NetPresentValue_i_18:
              20,NPVi(1,20),"NPVi_19.dat",NetPresentValue_i_19:
              21,NPVi(1,21),"NPVi_20.dat",NetPresentValue_i_20;

REPLICATE,    100,0.0,,Yes,Yes,0.0,,,24.0,Hours,No,No,,,Yes;

EXPRESSIONS: 1,REV_exp,UNIF(100,120,11):
              2,LAB_exp,UNIF(30,40,12):
              3,MAT_exp,UNIF(20,30,13):
              4,OVE_exp,UNIF(10,20,14):
              5,e_exp,UNIF(0.06,0.08,15):
              6,y_exp,UNIF(0.04,0.06,16):
              7,w_exp,UNIF(0.04,0.05,17):
              8,t_exp,UNIF(0.03,0.05,18):
              9,i_exp,UNIF(0.12,0.14,19):
              10,o_exp,UNIF(0.07,0.09,20);
```

APPENDIX B

SIMAN REPORT OF A COMPUTER SIMULATION MODEL FOR A b_{it} FORMULATION

```
0$      CREATE,     1:,1;
1$      ASSIGN:     z=0:
                    maz=z+1:
                    e(maz,1)=0:
                    y(maz,1)=0:
                    w(maz,1)=0:
                    t(maz,1)=0:
                    term01=(1+e(maz,1)):
                    term02=(1+y(maz,1)):
                    term03=(1+w(maz,1)):
                    term04=(1+t(maz,1));
3$      BRANCH,     1:
                    If,z<t1,7$,Yes:
                    Else,4$,Yes;
7$      ASSIGN:     z=z+1:
                    maz=z+1;
2$      ASSIGN:     e(maz,1)=e_exp:
                    y(maz,1)=y_exp:
                    w(maz,1)=w_exp:
                    t(maz,1)=t_exp:
                    term01=term01*(1+e(maz,1)):
                    term02=term02*(1+y(maz,1)):
                    term03=term03*(1+w(maz,1)):
                    term04=term04*(1+t(maz,1)):NEXT(3$);
4$      ASSIGN:     REVt=REV_exp:
                    LABt=LAB_exp:
                    MATt=MAT_exp:
                    OVEt=OVE_exp;
5$      ASSIGN:     bit=(REVt*term01-(LABt*term02+MATt*term03+OVEt*term04))*(1-
        TAX)+DEPt*TAX;
6$      DISPOSE;



VARIABLES:  1,t1,12:
            2,z:
            3,maz:
            4,TAX,0.25:
            5,term01,:
            6,term02,:
            7,term03,:
            8,term04,:
            10,e(13,1):
            30,y(13,1):
            50,t(13,1),:
            70,w(13,1):
            90,REVt:
            91,LABt:
```

```
                92,MATt:
                93,OVEt:
                94,DEPt,7:
                95,bit;

SEEDS:          11,12000,Common:
                12,13000,Common:
                13,14000,Common:
                14,15000,Common:
                15,16000,Common:
                16,17000,Common:
                17,18000,Common:
                18,19000,Common;

OUTPUTS:        1,bit,"bit.dat",bitValue;

REPLICATE,      5,0.0,,Yes,Yes,0.0;

EXPRESSIONS:    1,REV_exp,UNIF(80,90,11):
                2,LAB_exp,UNIF(20,30,12):
                3,MAT_exp,UNIF(15,25,13):
                4,OVE_exp,UNIF(10,15,14):
                5,e_exp,UNIF(0.06,0.08,15):
                6,y_exp,UNIF(0.04,0.06,16):
                7,w_exp,UNIF(0.04,0.05,17):
                8,t_exp,UNIF(0.03,0.05,18);
```

APPENDIX C

LINGO CODE FOR THE DEVELOPED MATHEMATICAL MODELS

C.1 Lingo Code for the Multi-objective Linear Programming Model

```
SETS:
PROJECTS/1..10/:tie,tia,enk,enk2,u,v;
YEARS/1..13/:R,ro;
MYEARS/1..30/;
INVYEARS/1..3/;
ASS1(PROJECTS,YEARS): Y,b,npv, vit;
ASS2(PROJECTS,YEARS,MYEARS):X;
ASS3(PROJECTS,PROJECTS):g;
ASS4(PROJECTS, INVYEARS):c;
ME/1..10/:degn;
ASS5(ME, PROJECTS):deg;

ENDSETS

DATA:
tie = @OLE("kitap1.xls", tie);
tia = @OLE("kitap1.xls", tia);
enk = @OLE("kitap1.xls", enk);
ro = @OLE("kitap1.xls", rto);
g = @OLE("kitap1.xls", gij);
c = @OLE("kitap1.xls", cik);
b = @OLE("kitap1.xls", bit);
v = @OLE("kitap1.xls", vi);
u = @OLE("kitap1.xls", ui);
npv = @OLE("kitap1.xls", npv);
enk2 = @OLE("kitap1.xls", enk2);
vit = @OLE("kitap1.xls", Vit);
@OLE("kitap1.xls", yit)=Y;
KL=5;
KU=10;
deg = @OLE("kitap1.xls", deg);
degn = @OLE("kitap1.xls", degn);
e=0.1;

ENDDATA

@FOR(ME(M):@SUM(PROJECTS(I)|deg(M,I)#eq#1:@SUM(YEARS(T):Y(I,T)))<=de
gn(M));

!max = @SUM(PROJECTS(I):@SUM(YEARS(T): npv(I,T)*Y(I,T)));

min = @SUM(PROJECTS(I):@SUM(YEARS(T): vit(I,T)*Y(I,T)));

!@SUM(PROJECTS(I):@SUM(YEARS(T): npv(I,T)*Y(I,T)))>=3537;

@SUM(PROJECTS(I):@SUM(YEARS(T): npv(I,T)*Y(I,T)))=maxnpv;
@SUM(PROJECTS(I):@SUM(YEARS(T): vit(I,T)*Y(I,T)))=varkat;
```

```
@FOR(PROJECTS(I):@SUM(YEARS(T)|T#GE#tie(I)#AND#T#LE#enk(I):Y(I,T))<=
1);

@SUM(PROJECTS(I):@SUM(YEARS(T)|T#GE#tie(I)#AND#T#LE#enk(I):Y(I,T)))>
=KL;

@SUM(PROJECTS(I):@SUM(YEARS(T)|T#GE#tie(I)#AND#T#LE#enk(I):Y(I,T)))<
=KU;

@FOR(PROJECTS(I):@FOR(YEARS(K)|K#LE#v(I):@FOR(YEARS(T)|T#GE#tie(I)#A
ND#T#LE#enk(I):Y(I,T) = X(I,K,T+K-1))));

@FOR(ASS3(I,J)|g#NE#500:@FOR(YEARS(T)|T#GE#tie(J)#AND#T#LE#tia(J):@S
UM(YEARS(TT)|TT#GE#tie(I)#AND#TT#LE#(T-u(I)-g(I,J)):
Y(I,TT))>=Y(J,T)));

@FOR(YEARS(T)|T#GE#2:R(T) = R(T-1)+ ro(T) -
@SUM(PROJECTS(I):@SUM(INVYEARS(K)|K#LE#u(I):c(I,K)*(1+e)^(T-
K)*X(I,K,T)))+
@SUM(PROJECTS(I):@SUM(YEARS(K)|K#GE#(u(I)+1)#AND#K#LE#V(I):b(I,T)*X(
I,K,T))));

R(1) = ro(1) -
@SUM(PROJECTS(I):@SUM(INVYEARS(K)|K#LE#u(I):c(I,K)*X(I,K,1)))+
@SUM(PROJECTS(I):@SUM(YEARS(K)|K#GE#(u(I)+1)#AND#K#LE#V(I):b(I,1)*X(
I,K,1)));

@FOR(PROJECTS(I):@FOR(YEARS(K)|K#LE#v(I):@SUM(YEARS(T)|T#GE#tie(I)#A
ND#T#LE#enk2(I):X(I,K,T)) <=1));

@FOR(PROJECTS(I):@FOR(YEARS(K):@FOR(MYEARS(T)|T#LT#tie(I)#OR#T#GT#en
k2(I):X(I,K,T) =0)));
@FOR(PROJECTS(I):@FOR(YEARS(T)|T#LT#tie(I)#OR#T#GT#enk(I):Y(I,T)=0))
;

@FOR(PROJECTS(I):
@SUM(YEARS(K):@SUM(YEARS(T):X(I,K,T)))<=@SUM(YEARS(T):Y(I,T))*1000);

@FOR(ASS1(I,T): @BIN(Y(I,T)));
@FOR(ASS2(I,K,T):@BIN(X(I,K,T)));
```

C.2 Lingo Code for the Multi-objective Linear Programming Models with Fuzzy Objectives

```
SETS:
PROJECTS/1..10/:tie,tia,enk,enk2,u,v;
YEARS/1..13/:R,ro;
MYEARS/1..30/;
INVYEARS/1..3/;
ASS1(PROJECTS,YEARS): Y,b,npv, vit;
ASS2(PROJECTS,YEARS,MYEARS):X;
ASS3(PROJECTS,PROJECTS):g;
ASS4(PROJECTS, INVYEARS):c;
ME/1..10/:degn;
ASS5(ME, PROJECTS):deg;

ENDSETS

DATA:
tie = @OLE("kitap1.xls", tie);
tia = @OLE("kitap1.xls", tia);
enk = @OLE("kitap1.xls", enk);
!ro = @OLE("kitap1.xls", rto);
g = @OLE("kitap1.xls", gij);
c = @OLE("kitap1.xls", cik);
b = @OLE("kitap1.xls", bit);
v = @OLE("kitap1.xls", vi);
u = @OLE("kitap1.xls", ui);
npv = @OLE("kitap1.xls", npv);
enk2 = @OLE("kitap1.xls", enk2);
vit = @OLE("kitap1.xls", Vit);
@OLE("kitap1.xls", yit)=Y;
KL=5;
KU=10;
deg = @OLE("kitap1.xls", deg);
degn = @OLE("kitap1.xls", degn);
e=0.1;

ENDDATA

!*****************************************************************;
!Tiwari et al.'s weighted additive approach;

max= 0.5*MNPV+0.5*MVARKAT;

MNPV<=((maxnpv-npvlb)/(npvub-npvlb));
MVARKAT<=((varub-varkat)/(varub-varlb));

MNPV<=1;
MVARKAT<=1;

!*****************************************************************;
!Werners' “fuzzy and” operator approach;

max= 0.8*LAMDA + 0.2*((LAMDA1+LAMDA2)/2);

MNPV<=((maxnpv-npvlb)/(npvub-npvlb));
```

```
MVARKAT<=((varub-varkat)/(varub-varlb));

LAMDA+LAMDA1<=MNPV;
LAMDA+LAMDA2<=MVARKAT;

MNPV<=1;
MVARKAT<=1;

!*****************************************************************;
!Li's two-phase approach;

max= (MNPV+MVARKAT)/2;

MNPV<=((maxnpv-npvlb)/(npvub-npvlb));
MVARKAT<=((varub-varkat)/(varub-varlb));

LAMDA<=MNPV;
LAMDA<=MVARKAT;

MNPV<=1;
MVARKAT<=1;

!*****************************************************************;
!Lai & Hwang's approach;

max= LAMDA + 0.001*(0.50*MNPV+0.50*MVARKAT);

MNPV<=((maxnpv-npvlb)/(npvub-npvlb));
MVARKAT<=((varub-varkat)/(varub-varlb));

LAMDA<=MNPV;
LAMDA<=MVARKAT;

MNPV<=1;
MVARKAT<=1;

!*****************************************************************;
!Lin's weighted max-min approach;

max= BETA;

MNPV<=((maxnpv-npvlb)/(npvub-npvlb));
MVARKAT<=((varub-varkat)/(varub-varlb));

0.50*BETA<=MNPV;
0.50*BETA<=MVARKAT;

MNPV<=1;
MVARKAT<=1;

!*****************************************************************;

npvlb=2194;
npvub=3537;
varlb=0.247;
varub=0.405;
```

```
@FOR(ME(M):@SUM(PROJECTS(I)|deg(M,I)#eq#1:@SUM(YEARS(T):Y(I,T)))<=de
gn(M));

!max = @SUM(PROJECTS(I):@SUM(YEARS(T): npv(I,T)*Y(I,T)));

!min = @SUM(PROJECTS(I):@SUM(YEARS(T): vit(I,T)*Y(I,T)));

!@SUM(PROJECTS(I):@SUM(YEARS(T): npv(I,T)*Y(I,T)))>=1180;

@SUM(PROJECTS(I):@SUM(YEARS(T): npv(I,T)*Y(I,T)))=maxnpv;
@SUM(PROJECTS(I):@SUM(YEARS(T): vit(I,T)*Y(I,T)))=varkat;

@FOR(PROJECTS(I):@SUM(YEARS(T)|T#GE#tie(I)#AND#T#LE#enk(I):Y(I,T))<=
1);

@SUM(PROJECTS(I):@SUM(YEARS(T)|T#GE#tie(I)#AND#T#LE#enk(I):Y(I,T)))>
=KL;
@SUM(PROJECTS(I):@SUM(YEARS(T)|T#GE#tie(I)#AND#T#LE#enk(I):Y(I,T)))<
=KU;

@FOR(PROJECTS(I):@FOR(YEARS(K)|K#LE#v(I):@FOR(YEARS(T)|T#GE#tie(I)#A
ND#T#LE#enk(I):Y(I,T) = X(I,K,T+K-1))));

@FOR(ASS3(I,J)|g#NE#500:@FOR(YEARS(T)|T#GE#tie(J)#AND#T#LE#tia(J):@S
UM(YEARS(TT)|TT#GE#tie(I)#AND#TT#LE#(T-u(I)-g(I,J)):
Y(I,TT))>=Y(J,T)));

@FOR(YEARS(T)|T#GE#2:R(T) = R(T-1)+ ro(T)-
@SUM(PROJECTS(I):@SUM(INVYEARS(K)|K#LE#u(I):c(I,K)*(1+e)^(T-
K)*X(I,K,T)))+
@SUM(PROJECTS(I):@SUM(YEARS(K)|K#GE#(u(I)+1)#AND#K#LE#V(I):b(I,T)*X(
I,K,T))));

R(1)=ro(1)-
@SUM(PROJECTS(I):@SUM(INVYEARS(K)|K#LE#u(I):c(I,K)*X(I,K,1)))+
@SUM(PROJECTS(I):@SUM(YEARS(K)|K#GE#(u(I)+1)#AND#K#LE#V(I):b(I,1)*X(
I,K,1)));

@FOR(PROJECTS(I):@FOR(YEARS(K)|K#LE#v(I):@SUM(YEARS(T)|T#GE#tie(I)#A
ND#T#LE#enk2(I):X(I,K,T)) <=1));

@FOR(PROJECTS(I):@FOR(YEARS(K):@FOR(MYEARS(T)|T#LT#tie(I)#OR#T#GT#en
k2(I):X(I,K,T) =0)));
@FOR(PROJECTS(I):@FOR(YEARS(T)|T#LT#tie(I)#OR#T#GT#enk(I):Y(I,T)=0))
;

@FOR(PROJECTS(I):
@SUM(YEARS(K):@SUM(YEARS(T):X(I,K,T)))<=@SUM(YEARS(T):Y(I,T))*1000
);

@FOR(ASS1(I,T): @BIN(Y(I,T)));
@FOR(ASS2(I,K,T):@BIN(X(I,K,T)));
```

C.3 Lingo Code for the Multi-objective Linear Programming Models with Fuzzy Objectives and Fuzzy Constraints

```
SETS:
PROJECTS/1..10/:tie,tia,enk,enk2,u,v;
YEARS/1..13/:R,ro,FARKA,FARKU,BALTL,BUSTL,MBUDGET;
MYEARS/1..30/;
INVYEARS/1..3/;
ASS1(PROJECTS,YEARS): Y,b,npv, vit;
ASS2(PROJECTS,YEARS,MYEARS):X;
ASS3(PROJECTS,PROJECTS):g;
ASS4(PROJECTS, INVYEARS):c;
ME/1..10/:degn;
ASS5(ME, PROJECTS):deg;

ENDSETS

DATA:
tie = @OLE("kitap1.xls", tie);
tia = @OLE("kitap1.xls", tia);
enk = @OLE("kitap1.xls", enk);
!ro = @OLE("kitap1.xls", rto);
g = @OLE("kitap1.xls", gij);
c = @OLE("kitap1.xls", cik);
b = @OLE("kitap1.xls", bit);
v = @OLE("kitap1.xls", vi);
u = @OLE("kitap1.xls", ui);
npv = @OLE("kitap1.xls", npv);
enk2 = @OLE("kitap1.xls", enk2);
vit = @OLE("kitap1.xls", Vit);
@OLE("kitap1.xls", yit)=Y;
KL=5;
KU=10;
deg = @OLE("kitap1.xls", deg);
degn = @OLE("kitap1.xls", degn);
e=0.1;

!********************************;
BALTL= @OLE("kitap1.xls", BALTL);
BUSTL= @OLE("kitap1.xls", BUSTL);
FARKA= @OLE("kitap1.xls", FARKA);
FARKU= @OLE("kitap1.xls", FARKU);
!********************************;

ENDDATA

!*****************************************************************;
!Tiwari et al.'s weighted additive approach;

max= MNPV+MVARKAT+@SUM(YEARS(T):MBUDGET(T));

MNPV<=((maxnpv-npvlb)/(npvub-npvlb));
MVARKAT<=((varub-varkat)/(varub-varlb));

MNPV<=1;
MVARKAT<=1;
```

```
!****************************************************************;
!Werners' "fuzzy and" operator approach;

max= 0.5*LAMDA + 0.5*((LAMDA1+LAMDA2)/2);

MNPV<=((maxnpv-npvlb)/(npvub-npvlb));
MVARKAT<=((varub-varkat)/(varub-varlb));

LAMDA+LAMDA1<=MNPV;
LAMDA+LAMDA2<=MVARKAT;

MNPV<=1;
MVARKAT<=1;

!****************************************************************;
!Li's two-phase approach;

max= (MNPV+MVARKAT)/2;

MNPV<=((maxnpv-npvlb)/(npvub-npvlb));
MVARKAT<=((varub-varkat)/(varub-varlb));

LAMDA<=MNPV;
LAMDA<=MVARKAT;

MNPV<=1;
MVARKAT<=1;

!****************************************************************;
!Lai & Hwang's approach;

max= LAMDA + 0.001*(0.50*MNPV+0.50*MVARKAT);

MNPV<=((maxnpv-npvlb)/(npvub-npvlb));
MVARKAT<=((varub-varkat)/(varub-varlb));

LAMDA<=MNPV;
LAMDA<=MVARKAT;

MNPV<=1;
MVARKAT<=1;

!****************************************************************;
!Lin's weighted max-min approach;

max= BETA;

MNPV<=((maxnpv-npvlb)/(npvub-npvlb));
MVARKAT<=((varub-varkat)/(varub-varlb));

0.50*BETA<=MNPV;
0.50*BETA<=MVARKAT;

MNPV<=1;
MVARKAT<=1;

!****************************************************************;
```

```
npvlb=2194;
npvub=3537;
varlb=0.247;
varub=0.405;

@FOR(ME(M):@SUM(PROJECTS(I)|deg(M,I)#eq#1:@SUM(YEARS(T):Y(I,T)))<=de
gn(M));

!max = @SUM(PROJECTS(I):@SUM(YEARS(T): npv(I,T)*Y(I,T)));

!min = @SUM(PROJECTS(I):@SUM(YEARS(T): vit(I,T)*Y(I,T)));

!@SUM(PROJECTS(I):@SUM(YEARS(T): npv(I,T)*Y(I,T)))>=1180;

@SUM(PROJECTS(I):@SUM(YEARS(T): npv(I,T)*Y(I,T)))=maxnpv;
@SUM(PROJECTS(I):@SUM(YEARS(T): vit(I,T)*Y(I,T)))=varkat;

@FOR(PROJECTS(I):@SUM(YEARS(T)|T#GE#tie(I)#AND#T#LE#enk(I):Y(I,T))<=
1);

@SUM(PROJECTS(I):@SUM(YEARS(T)|T#GE#tie(I)#AND#T#LE#enk(I):Y(I,T)))>
=KL;
@SUM(PROJECTS(I):@SUM(YEARS(T)|T#GE#tie(I)#AND#T#LE#enk(I):Y(I,T)))<
=KU;

@FOR(PROJECTS(I):@FOR(YEARS(K)|K#LE#v(I):@FOR(YEARS(T)|T#GE#tie(I)#A
ND#T#LE#enk(I):Y(I,T) = X(I,K,T+K-1))));

@FOR(ASS3(I,J)|g#NE#500:@FOR(YEARS(T)|T#GE#tie(J)#AND#T#LE#tia(J):@S
UM(YEARS(TT)|TT#GE#tie(I)#AND#TT#LE#(T-u(I)-g(I,J)):
Y(I,TT))>=Y(J,T)));

@FOR(YEARS(T)|T#GE#2:R(T) = R(T-1)+ ro(T)-
@SUM(PROJECTS(I):@SUM(INVYEARS(K)|K#LE#u(I):c(I,K)*(1+e)^(T-
K)*X(I,K,T)))+
@SUM(PROJECTS(I):@SUM(YEARS(K)|K#GE#(u(I)+1)#AND#K#LE#V(I):b(I,T)*X(
I,K,T))));

R(1)=ro(1)-
@SUM(PROJECTS(I):@SUM(INVYEARS(K)|K#LE#u(I):c(I,K)*X(I,K,1)))+
@SUM(PROJECTS(I):@SUM(YEARS(K)|K#GE#(u(I)+1)#AND#K#LE#V(I):b(I,1)*X(
I,K,1)));

@FOR(PROJECTS(I):@FOR(YEARS(K)|K#LE#v(I):@SUM(YEARS(T)|T#GE#tie(I)#A
ND#T#LE#enk2(I):X(I,K,T)) <=1));

@FOR(PROJECTS(I):@FOR(YEARS(K):@FOR(MYEARS(T)|T#LT#tie(I)#OR#T#GT#en
k2(I):X(I,K,T) =0)));
@FOR(PROJECTS(I):@FOR(YEARS(T)|T#LT#tie(I)#OR#T#GT#enk(I):Y(I,T)=0))
;

@FOR(PROJECTS(I):
@SUM(YEARS(K):@SUM(YEARS(T):X(I,K,T)))<=@SUM(YEARS(T):Y(I,T))*1000);

@FOR(ASS1(I,T): @BIN(Y(I,T)));
@FOR(ASS2(I,K,T):@BIN(X(I,K,T)));
```

```
!*************************************************************;
!Fuzzy Budget constraints;

@FOR(YEARS(T):MBUDGET(T)<=((ro(T)-BALTL(T))/FARKA(T)));
@FOR(YEARS(T):MBUDGET(T)<=(BUSTL(T)-(ro(T))/FARKU(T)));
@FOR(YEARS(T):MBUDGET(T)<=1);

@FOR(YEARS(T):LAMDA<=MBUDGET(T));

!*************************************************************;
```