DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

AUCTION-BASED CHANNEL ALLOCATION APPROACH IN WIRELESS NETWORKS

by
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November, 2010
İZMİR

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A Thesis Submitted to the Graduate School of Natural and Applied Sciences of Dokuz Eylül University In Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Electrical-Electronic Engineering, Electrical-Electronics Program

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Ph.D. THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **“AUCTION-BASED CHANNEL ALLOCATION APPROACH IN WIRELESS NETWORKS”** completed by **HAKAN MURAT KARACA** under supervision of **YRD. DOÇ. DR ZAFER DİCLE** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy.

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AUCTION-BASED CHANNEL ALLOCATION APPROACH IN WIRELESS NETWORKS

ABSTRACT

We investigate various applications of graph theory and classify them based on 7 layers of OSI. Most of its applications are related to coloring and channel assignment problem (MAC). Other applications can be summarized as routing, topology control, interference reduction, sensing function allocation, trellis and state diagrams. As a cross layer application (MAC and physical layers) of graph theory, we consider the problem of throughput maximization during spectrum allocation under Signal to Interference plus Noise Ratio (SINR) constraint in cognitive radio networks. We propose a novel auction-based channel allocation algorithm, in which graph coloring and bidding theory play an important role and which tries to maximize both total and primary users' utilities while satisfying SINR constraint on primary receivers, without controlling secondary user powers. For comparison, we discuss a greedy algorithm as well, however, which does not consider interference issue. In order to compare results of proposed and greedy algorithms, we propose net throughput by taking into account outage probability of primary receiver. Simulation results show that exposing higher SINR (outage) threshold not only decreases total system and primary users' throughput, but also worsens channel distribution performance. On the other hand, adding auction mechanism significantly increases total gain throughput and primary user' s utility. Especially till SINR threshold values of 20 dBs, auction provides outstanding performance and proposed algorithm has total throughput results close to those of the greedy one even though no interference constraint is applied in the greedy algorithm. Another noticeable point of simulation results is crossover of net throughputs of proposed and greedy algorithms at a SINR threshold level after which results of ABSA-UNIC and NASA-UNIC are much better. This clearly shows superiority of proposed mechanism. Simulations were carried out using matlab 7.7.0 (R2008b) and codes is given in the attached file algorithms.m.

Keywords : Graph theory, OSI layers, Applications, Channel Assignment, Cognitive Radio, Throughput Maximization, Interference Constraint, Auction.

KABLOSUZ AĞLARDA AÇIK ARTTIRMAYA DAYALI KANAL PAYLAŞIMI YAKLAŞIMI

ÖZ

Çizge teorinin çeşitli uygulamalarını inceliyoruz ve bunları OSI modelinin 7 katmanına göre sınıflandırıyoruz. Uygulamaların çoğu renklendirme ve bir MAC katmanı uygulaması olan kanal ataması problemi ile ilgilidir. Diğer uygulamalar yönlendirme, topoloji kontrolü, girişim azaltma, algılama işlev ataması, trellis ve durum diyagramlarıdır. Bir çapraz katman (MAC ve fiziksel katman) uygulaması olarak, kavramsal radyo ağlarda, işaret girişim gürültü oranı (IGGO) kısıtı altında spektrum paylaşımı sırasında toplam çıktıyı maksimize etme problemini ele alıyoruz. Renklendirme, fiyat teklifi ve açık arttırma teorisi üzerine kurulu, hem birincil hem de ikincil kullanıcılar için toplam faydayı maksimize etmeye çalışan, aynı zamanda da birincil alıcıların kesintiye uğramaması için IGGO koşulunun geçerli kalmasını sağlayan yeni bir algoritma önermekteyiz. Önerilen algoritma ile sonuçları karşılaştırabilmek için girişim etkisini hesaba katmayan bir greedy algoritmayı da ele almaktayız. Bunun için, birincil alıcının kesintiye uğrama olasılığını da hesaba katan bir net çıktı önermekteyiz. Simülasyon sonuçları daha yüksek IGGO kısıtı uygulamanın hem toplam hem de birincil kullanıcıların toplam kazançlarını azaltırken, kanal dağılım başarısını da düşürdüğünü göstermektedir. Diğer taraftan açık arttırma yönteminin algoritmaya ilavesi, toplam ve ayrı ayrı kullanıcıların kazançlarını ciddi biçimde arttırmıştır. Özellikle 20 dB IGGO eşik değerlerine kadar, son derece büyük fayda sağladığı görülmüş, sonuçların girişim kısıtı uygulanmayan greedy yönteme yakın performans gösterdiği görülmüştür. Simülasyon sonuçlarında görülen diğer önemli bir nokta da önerilen ve greedy yöntemlerin net çıktılarının bir IGGO eşik değerinde kesişmeleri ve bu değerden sonra önerilen algoritmanın daha iyi sonuçlar vermesidir. Üstelik kesişmenin gerçekleştiği noktanın üstündeki SINR değerleri pratik SINR değerleri ile son derece uyuşmaktadır. Bu da önerilen mekanizmanın açık bir şekilde üstünlüğünü göstermektedir. Simülasyonlar matlab 7.0.7 (R2008b)' de gerçekleştirilmiştir ve ekli dosya algorithms.m' de verilmiştir.

Anahtar sözcükler : Çizge teorisi, OSI katmanları, Uygulamalar, Kanal Atama, Kavramsal Radyo, Çıktı En Büyükleme, Girişim Kısıtlaması, Açık Arttırma.

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CHAPTER ONE
INTRODUCTION

The paper written by Leonhard Euler on the Seven Bridges of Knigsberg and published in 1736 is regarded as the first paper in the history of graph theory. This paper, as well as the one written by Vandermonde on the knight problem, carried on with the analysis situs initiated by Knobloch, Leibniz, & Euler (1991). More than one century after Euler's paper on the bridges of Knigsberg and while Listing introduced topology, Cayley was led by the study of particular analytical forms arising from differential calculus to study a particular class of graphs, the trees. This study had many implications in theoretical chemistry. The involved techniques mainly concerned the enumeration of graphs having particular properties. Enumerative graph theory then rose from the results of Cayley and the fundamental results published by Plya between 1935 and 1937 and the generalization of these by De Bruijn in 1959. Cayley linked his results on trees with the contemporary studies of chemical composition. The fusion of the ideas coming from mathematics with those coming from chemistry is at the origin of a part of the standard terminology of graph theory. In particular, the term graph was introduced by Sylvester in a paper published in 1878 in Nature.

One of the most famous and productive problems of graph theory is the four color problem: "Is it true that any map drawn in the plane may have its regions colored with four colors, in such a way that any two regions having a common border have different colors?" This problem was first posed by Francis Guthrie in 1852 and its first written record is in a letter of De Morgan addressed to Hamilton the same year.

The autonomous development of topology from 1860 and 1930 fertilized graph theory back through the works of Jordan, Kuratowski and Whitney. Another important factor of common development of graph theory and topology came from the use of the techniques of modern algebra. The first example of such a use comes from the work of the physicist Gustav Kirchhoff, who published in 1845 his Kirchhoff's circuit laws for calculating the voltage and current in electric circuits.

The introduction of probabilistic methods in graph theory, especially in the study of Erdös, & Rényi (1959) of the asymptotic probability of graph connectivity, gave rise to yet another branch, known as random graph theory, which has been a fruitful source of graph-theoretic results.

In mathematics and computer science, graph theory is the study of graphs which are mathematical structures used to model pairwise relations between objects from a certain collection. A graph in this context refers to a collection of vertices or nodes and a collection of edges that connect pairs of vertices. A graph may be undirected, meaning that there is no distinction between the two vertices associated with each edge, or its edges may be directed from one vertex to another which is defined by Knobloch, Leibniz, & Euler (1991).

Graphs are represented graphically by drawing a dot for every vertex, and drawing an arc between two vertices if they are connected by an edge. If the graph is directed, the direction is indicated by drawing an arrow defined by Knobloch, Leibniz, & Euler (1991).

A graph G consists of two types of elements, namely vertices and edges. Every edge has two endpoints in the set of vertices, and is said to connect or join the two endpoints. An edge can thus be defined as a set of two vertices (or an ordered pair, in the case of a directed graph). Alternative models of graph exist; e.g., a graph may be thought of as a Boolean binary function over the set of vertices or as a square (0, 1) matrix. A vertex (basic element) is simply drawn as a node or a dot. The vertex set of G is usually denoted by V(G), or V when there is no danger of confusion. The order of a graph is the number of its vertices, i.e. |V(G)|. An edge (a set of two elements) is drawn as a line connecting two vertices, called endvertices, or endpoints. An edge with endvertices x and y is denoted by xy (without any symbol in between). The edge set of G is usually denoted by E(G), or E when there is no danger of confusion. The size of a graph is the number of its edges, i.e. |E(G)| defined by Diesel (2000).

A graph is a pair G graph = (V, E) of sets satisfying E ⊆ $[V]^2$; thus, the elements of E are 2-element subsets of V. The elements of V are the vertex vertices (or nodes, or points) of the graph G, the elements of E are its edge edges (or lines). The usual way to picture a graph is by drawing a dot for each vertex and joining two of these dots by a line if the corresponding two vertices form an edge. Just how these dots and lines are drawn is considered irrelevant: all that matters is the information which pairs of vertices form an edge and which do not.

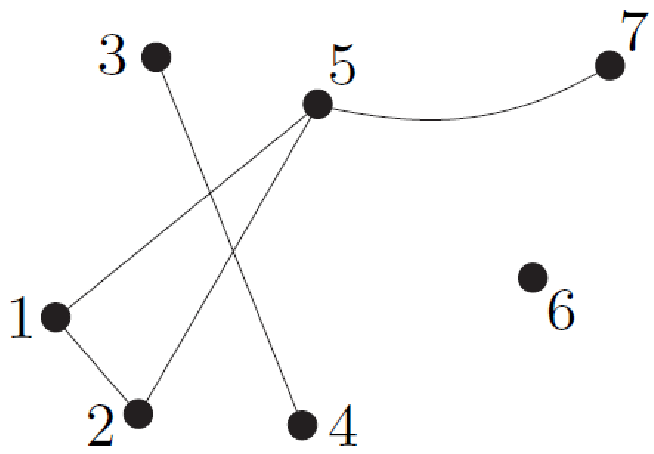

Figure 1.1 The graph on V = {1, . . . , 7} with edge set
E = {{1, 2}, {1, 5}, {2, 5}, {3, 4}, {5, 7}}

A graph with vertex set V is said to be a graph on V. The vertex set of a graph G is referred to as V(G), its edge set as E(G). The number of vertices of a graph G is its order, written as |G|**;** its number of edges is denoted by ||G||. Graphs are finite or infinite according to their order.

A loop is an edge whose endvertices are the same vertex. A link has two distinct endvertices. An edge is multiple if there is another edge with the same endvertices; otherwise it is simple. The multiplicity of an edge is the number of multiple edges sharing the same endvertices; the multiplicity of a graph, the maximum multiplicity of its edges. A graph is a simple graph if it has no multiple edges or loops, a multigraph if it has multiple edges, but no loops, and a multigraph or pseudograph if it contains both multiple edges and loops. When stated without any qualification, a graph is almost always assumed to be simpleone has to judge from the context.

Graph labeling usually refers to the assignment of unique labels (usually natural numbers) to the edges and vertices of a graph. Graphs with labeled edges or vertices are known as labeled, those without as unlabeled. More specifically, graphs with labeled vertices only are vertex-labeled, those with labeled edges only are edge-labeled defined by Knobloch, Leibniz, & Euler (1991).

A subgraph of a graph G is a graph whose vertex set is a subset of that of G, and whose adjacency relation is a subset of that of G restricted to this subset. In the other direction, a supergraph of a graph G is a graph of which G is a subgraph. It is said a graph G contains another graph H if some subgraph of G is H or is isomorphic to H. A subgraph H is a spanning subgraph, or factor, of a graph G if it has the same vertex set as G. It is said H spans G.

A walk is an alternating sequence of vertices and edges, beginning and ending with a vertex, where each vertex is incident to both the edge that precedes it and the edge that follows it in the sequence, and where the vertices that precede and follow an edge are the end vertices of that edge. A walk is closed if its first and last vertices are the same, and open if they are different.

The length l of a walk is the number of edges that it uses. For an open walk, $l = n - 1$, where n is the number of vertices visited (a vertex is counted each time it is visited). For a closed walk, $l = n$ (the start/end vertex is listed twice, but is not counted twice). A trail is a walk in which all the edges are distinct. A closed trail has been called a tour or circuit, but these are not universal, and the latter is often reserved for a regular subgraph of degree two. Traditionally, a path referred to what is now usually known as an open walk. Nowadays, when stated without any qualification, a path is usually understood to be simple, meaning that no vertices (and thus no edges) are repeated. A cycle that has odd length is an odd cycle; otherwise it is an even cycle. A graph is acyclic if it contains no cycles; unicyclic if it contains exactly one cycle; and pancyclic if it contains cycles of every possible length.

A tree is a connected acyclic simple graph. A vertex of degree 1 is called a leaf, or pendant vertex. An edge incident to a leaf is a leaf edge, or pendant edge.

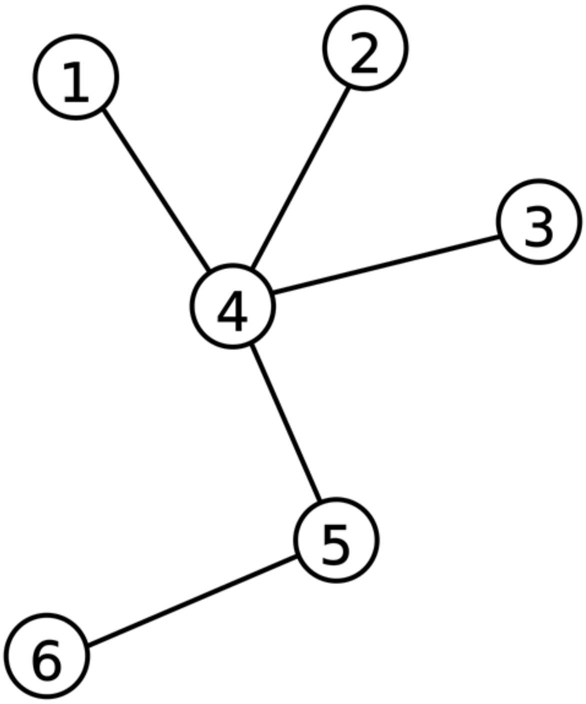

Figure 1.2 A labeled tree with 6 vertices and 5 edges

A subtree of the tree T is a connected subgraph of T. A forest is an acyclic simple graph. A subforest of the forest F is a subgraph of F. A spanning tree is a spanning subgraph that is a tree. Every graph has a spanning forest. But only a connected graph has a spanning tree.

The complete graph of order n is a simple graph with n vertices in which every vertex is adjacent to every other.

A clique in a graph is a set of pairwise adjacent vertices. Since any subgraph induced by a clique is a complete subgraph, the two terms and their notations are usually used interchangeably. A k-clique is a clique of order k. In figure 1.1, vertices 1, 2 and 5 form a 3-clique, or a triangle. A maximal clique is a clique that is not a subset of any other clique.

The clique number (G) of a graph G is the order of a largest clique in G.

In graph theory, degree, especially that of a vertex, is usually a measure of immediate adjacency.

An edge connects two vertices; these two vertices are said to be incident to that edge, or, equivalently, that edge incident to those two vertices. All degree-related concepts have to do with adjacency or incidence.

The degree, or valency, $d_G(v)$ of a vertex v in a graph G is the number of edges incident to v, with loops being counted twice. A vertex of degree 0 is an isolated vertex. A vertex of degree 1 is a leaf. If E is finite, then the total sum of vertex degrees is equal to twice the number of edges.

The total degree of a graph is equal to two times the number of edges, loops included. This means that for a graph with 3 vertices with each vertex having a degree of two (i.e. a triangle) the total degree would be six (e.g. 3 x 2 = 6). The general formula for this is total degree = 2n where n = number of edges.

Two vertices u and v are called adjacent if an edge exists between them. This is denoted by u v or u v. In figure 1, vertices 1 and 2 are adjacent, but vertices 2 and 4 are not. The set of neighbors of v, that is, vertices adjacent to v not including v itself, forms an induced subgraph called the (open) neighborhood of v and denoted N(v). When v is also included, it is called a closed neighborhood and denoted by N[v]. When stated without any qualification, a neighborhood is assumed to be open. In figure 1.1, vertex 1 has two neighbors: vertices 2 and 5. For a simple graph, the number of neighbors that a vertex has coincides with its degree.

A dominating set of a graph is a vertex subset whose closed neighborhood includes all vertices of the graph. A vertex v dominates another vertex u if there is an edge from v to u. A vertex subset V dominates another vertex subset U if every vertex in U is adjacent to some vertex in V. The minimum size of a dominating set is the domination number $\gamma(G)$.

In computers, a finite, directed or undirected graph (with n vertices, say) is often represented by its adjacency matrix: an n-by-n matrix whose entry in row i and column j gives the number of edges from the i-th to the j-th vertex.

In graph theory, the word independent usually carries the connotation of pairwise disjoint or mutually nonadjacent. In this sense, independence is a form of immediate nonadjacency. An isolated vertex is a vertex not incident to any edges. An independent set, or coclique, or stable set or staset, is a set of vertices of which no pair is adjacent. Since the graph induced by any independent set is an empty graph, the two terms are usually used interchangeably.

The independence number $\propto(G)$ of a graph G is the size of a largest independent set of G.

A graph can be decomposed into independent sets in the sense that the entire vertex set of the graph can be partitioned into pairwise disjoint independent subsets. Such independent subsets are called partite sets, or simply parts.

Connectivity extends the concept of adjacency and is essentially a form (and measure) of concatenated adjacency.

If it is possible to establish a path from any vertex to any other vertex of a graph, the graph is said to be connected; otherwise, the graph is disconnected. A graph is totally disconnected if there is no path connecting any pair of vertices. This is just another name to describe an empty graph or independent set.

A cut vertex, or articulation point, is a vertex whose removal disconnects the remaining subgraph. A cut set, or vertex cut or separating set, is a set of vertices whose removal disconnects the remaining subgraph. A bridge is an analogous edge.

If it is always possible to establish a path from any vertex to every other even after removing any k - 1 vertices, then the graph is said to be k-vertex-connected or k-connected. Note that a graph is k-connected if and only if it contains internally disjoint paths between any two vertices (Diesel, 2000). The example graph above is connected (and therefore 1-connected), but not 2-connected. The vertex connectivity or connectivity (G) of a graph G is the minimum number of vertices that need to be removed to disconnect G.

A graph is k-edge-connected if any subgraph formed by removing any k - 1 edges is still connected. The edge connectivity of a graph G is the minimum number of edges needed to disconnect G. One well-known result is that $K(G) \leq K'(G) \leq \delta (G)$.

A component is a maximally connected subgraph. A block is either a maximally 2-connected subgraph, a bridge (together with its vertices), or an isolated vertex. A biconnected component is a 2-connected component.

The distance $d_G(u, v)$ between two (not necessary distinct) vertices u and v in a graph G is the length of a shortest path between them. The subscript G is usually dropped when there is no danger of confusion. When u and v are identical, their distance is 0. When u and v are unreachable from each other, their distance is defined to be infinity.

A weighted graph associates a label (weight) with every edge in the graph. Weights are usually real numbers. They may be restricted to rational numbers or integers. The weight of a path or the weight of a tree in a weighted graph is the sum of the weights of the selected edges. Sometimes a non-edge is labeled by a special weight representing infinity. Sometimes the word cost is used instead of weight. When stated without any qualification, a graph is always assumed to be unweighted. In some writing on graph theory the term network is a synonym for a weighted graph. A network may be directed or undirected, it may contain special vertices (nodes), such as source or sink. The classical network problems include:

- minimum cost spanning tree,
- shortest paths,

- maximal flow (and the max-flow min-cut theorem)

A directed arc, or directed edge, is an ordered pair of endvertices that can be represented graphically as an arrow drawn between the endvertices. In such an ordered pair the first vertex is called the initial vertex or tail; the second one is called the terminal vertex or head (because it appears at the arrow head). An undirected edge disregards any sense of direction and treats both endvertices interchangeably. A loop in a digraph, however, keeps a sense of direction and treats both head and tail identically. A set of arcs are multiple, or parallel, if they share the same head and the same tail. A pair of arcs are anti-parallel if one's head/tail is the other's tail/head. A digraph, or directed graph, or oriented graph, is analogous to an undirected graph except that it contains only arcs. A mixed graph may contain both directed and undirected edges; it generalizes both directed and undirected graphs. When stated without any qualification, a graph is almost always assumed to be undirected.

A digraph is called simple if it has no loops and at most one arc between any pair of vertices. When stated without any qualification, a digraph is usually assumed to be simple.

A directed path, or just a path when the context is clear, is an oriented simple path such that all arcs go the same direction, meaning all internal vertices have in- and out-degrees 1. A vertex v is reachable from another vertex u if there is a directed path that starts from u and ends at v. Note that in general the condition that u is reachable from v does not imply that v is also reachable from u.

A directed cycle, or just a cycle when the context is clear, is an oriented simple cycle such that all arcs go the same direction, meaning all vertices have in- and out-degrees 1. A digraph is acyclic if it does not contain any directed cycle. A finite, acyclic digraph with no isolated vertices necessarily contains at least one source and at least one sink.

The partial order structure of directed acyclic graphs (or DAGs) gives them their own terminology.

If there is a directed edge from u to v, then it is said to be a parent of v and v is a child of u. If there is a directed path from u to v, it is said u is an ancestor of v and v is a descendent of u.

Vertices in graphs can be given colours to identify or label them. Although they may actually be rendered in diagrams in different colours, working mathematicians generally pencil in numbers or letters (usually numbers) to represent the colours.

Given a graph G (V, E) a k-colouring of G is a map $f : V \rightarrow \{1, . . . , k\}$ with the property that $(u, v) \in E \Rightarrow f(u) \neq f(v)$, in other words, every vertex is assigned a colour with the condition that adjacent vertices cannot be assigned the same colour.

The chromatic number $\gamma(G)$ is the smallest k for which G has a k-colouring. Given a graph and a colouring, the colour classes of the graph are the sets of vertices given the same colour.

A graph invariant is a property of a graph G, usually a number or a polynomial, that depends only on the isomorphism class of G.

A graph structure can be extended by assigning a weight to each edge of the graph. Graphs with weights, or weighted graphs, are used to represent structures in which pairwise connections have some numerical values. For example if a graph represents a road network, the weights could represent the length of each road. A digraph with weighted edges in the context of graph theory is called a network which is defined by Knobloch, Leibniz, & Euler (1991).

Networks have many uses in the practical side of graph theory (for example, to model and analyze traffic networks). Within network analysis, the definition of the term “network” varies, and may often refer to a simple graph. Applications of graph theory in the form of network analysis split broadly into three categories. Firstly, analysis to determine structural properties of a network, such as the distribution of vertex degrees and the diameter of the graph. A vast number of graph measures exist, and the production of useful ones for various domains remains an active area of research. Secondly, analysis to find a measurable quantity within the network, for example, for a transportation network, the level of vehicular flow within any portion of it. Thirdly, analysis of dynamical properties of networks which were carried out by Sachs, Stiebits, & Wilson (1988).

In the literature, various applications of graph theory for wireless networks exist. Arbitrary graphs have the advantage of being able to represent all possible network configurations. Certain restricted graphs could give an accurate representation to certain radio network or network scenario; and illuminate some aspects of the problem structure, which might help in solving the problem, and finding the optimal solution, such as finding a chromatic number. Most of its applications are to solve the problem of channel assignment. Especially automatic channel assignment in multi-channel multi-radio wireless mesh networks is a key technique to minimize signal interference and increase network capacity. Tree and planar graphs are most famous restricted arbitrary graphs used in modeling radio networks. Tree is the simplest graphical representation and problems such as message routing and propagation can be well addressed using tree models. However, Sachs, Stiebitz, & Wilson (1988) stated the tree structure is not flexible enough to represent many possible network configurations.

The demand for wireless spectrum has been growing rapidly with the dramatic development of the mobile telecommunication industry in the last decades. Recently, regulatory bodies like the Federal Communications Commissions (FCC) in the United States are recognizing that traditional fixed spectrum allocation can be very inefficient, considering that bandwidth demands may vary highly along the time or space dimension. In order to fully utilize the scarce spectrum resources, with the development of cognitive radio technologies, dynamic spectrum allocation especially distributed spectrum allocation becomes a promising approach to increase the efficiency of spectrum usage. This new wireless networking paradigm, dynamic spectrum access, is also referred to as Next Generation (xG) wireless networks.

xG networks, however, impose several research challenges due to the broad range of available spectrum as well as diverse Quality-of-Service (QoS) requirements of applications. These heterogeneities must be captured and handled dynamically as mobile terminals roam between wireless architectures and along the available spectrum pool. The key enabling technology of xG networks is the cognitive radio. Cognitive radio techniques provide the capability to use or share the spectrum in an opportunistic manner. Dynamic spectrum access techniques allow the cognitive radio to operate in the best available channel.

Main functions of for cognitive radios in xG networks are spectrum sensing, spectrum management, spectrum mobility and spectrum sharing. The ultimate objective of the cognitive radio is to obtain the best available spectrum through cognitive capability and reconfigurability. Since most of the spectrum is already assigned, the most important challenge is to share the licensed spectrum without interfering with the transmission of other licensed users.

The coloring model based on graph theory is an important model to research on channel allocation for cognitive radios, which abstracts the network topology including cognitive users and primary users into a graph and gets the channel lists for each cognitive user according to the result of spectrum sensing. Therefore, the channel allocation problem is formulated as a graph-coloring problem. The cognitive network is generally modelled as a undirected graph G = (V, E), where the vertices represent the secondary users, and edges represent interferences so that no channels can be assigned simultaneously to any adjacent nodes. The graph is referred as the interference graph.

The objective of the channel allocation is to maximize the spectrum utilization, including both primary users' and secondary users'. Nowadays, more and more researchers have already started to study dynamic spectrum allocation via bidding/asking and auction mechanisms. Dynamic channel allocation performance of auctions with collusion and cooperation was analyzed and it was shown that through user cooperation a much better performance is obtained.

Besides spectrum allocation, another important deployment issue of cognitive radio networks is interference management. Many studies concern only dynamic spectrum allocation without considering interference constraints on primary users because of secondery user activities. On the other hand, some studies investigate interference management issue on cognitive networks, whereas they do not consider total gain maximization during channel allocation. There is no work involving both coloring, auction and bidding theory and interference management on primary users without controlling power levels. Existing allocation schemes generally consider either power and channel allocation without considering total gain or only total gain of primary and secondery users without taking interference constraint into consideration.

The thesis is organized as follows. We begin in chapter 2 by giving our literature review results where we investigate graph theory applications and classify them based on 7 layers of OSI. Next, chapter 3 is problem definition part. Here, we give background information about cognitive radio networks, give related work and define the problems of channel allocation for secondary users and interference management. In chapter 4, we give our contribution work which is throughput maximization of auction-based channel allocation for cognitive radio networks under interference constraint. Also, we give our algorithm and simulation results. Finally, conclusions and summary of our contributions are stated in chapter 5.

CHAPTER TWO
OVERVIEW OF GRAPH THEORY APPLICATIONS AND CLASSIFICATION BASED ON OSI LAYERS

In this chapter of the thesis, we investigate applications of graph theory in the literature and classify them based on 7 layers of Open Systems Interconnection model (OSI). Based on this, we look for open areas in the field of cross-layer applications and in the next chapter, we focus on channel assignment and interference management issues in cognitive radio networks which are basically MAC and physical layer applications.

2.1 Transport And Mac Layer Applications

There are various applications of graph theory of various applications of graph theory in different OSI layers. Most of its applications are based on MAC layer.

In the paper "A Client-driven Approach for Channel Management in Wireless LANs", Mishra, Brik, Banerjee, Srinivasan, & Arbaugh (2006) focus on the specific problem of channel assignment to improve application throughput on a per-user basis and for the network as a whole.

Approaches such as Least Congested Channel Search (LCCS) (Mishra et al, 2006) are AP-centric in nature, that is, they capture interference at the APs but do not involve client participation. That s why this type of interference is called as Hidden Interference Problem. In this work, they show that AP-centric approaches lack the ability to detect various similar interference scenarios which can cause serious inefficiencies in the channel utilization. Such observations provide the motivation to innovate clientcentric models and techniques for channel assignment in the context of WLANs. The end goal of this work is to improve application performance. While client-based channel assignment solves a part of the problem, load balancing of clients among APs is also needed for a complete solution. Through application level metrics they show that such a joint solution has significant advantages compared to addressing the two problems independently. They refer to this problem of channel assignment with load balancing as channel management. So, they propose a novel client-centric model of capturing the interference constraints in a WLAN. Based on this model, they develop a centralized technique for addressing the problem of channel management. Such centralized approaches are applicable to managed networks in organizational settings such as airports, hotels, business offices, and centrally managed hotspots. They capture the hidden interference scenarios by constructing a set theoretic model called conflict set coloring.

They use the term conflict to denote scenarios where any two stations (APs or clients) belonging to different BSS interfere with each other by the virtue of sharing the same channel. The goal of channel management based on conflict set coloring is to assign channels/colors in such a way that each client is assigned to APs (chosen from the range set) which suffer from minimum conflict (or are conflict free if possible). They propose a centralized algorithm called CFAssign-RaC (stands for conflict set color assignment using Randomized Compaction) which addresses the joint problem of channel management. A client is said to be conflict-free if its association with an AP on the assigned channel eliminates conflicts at both the AP and the client. If there does not exist such an AP, the client then associates to the AP such that the AP-client link has minimum conflict where conflict on a particular channel can be measured as the number of APs that share the channel.

The goal of channel management over this conflict set system is to assign channels to APs in such a way that it minimizes the conflict for each client. This solution also yields an association mapping of clients to APs, where a client associates to the AP that has the minimum conflict. Here, they model the channel assignment problem in WLANs as a vertex coloring problem cognizant of client interference. A graph is used to represent conflicts or interference between nodes. Such interference can be deduced by obtaining information from the clients. The vertices of this graph correspond to APs, and edges correspond to impact of interference between pairs of APs. The objective is to assign a fixed number of colors (channels) to the vertices (APs) of this graph that minimizes interference.

They define a penalty function to evaluate the degree of interference and in order to achieve low interference at clients, they must choose an assignment of colors to vertices (channels to APs) such that the aggregate value of the penalty functions on all edges is minimized. The goal of a channel assignment scheme is to improve user perceived throughput and network utilization. Apart from suffering interference from other APs and clients associated to other APs, a client shares the medium with clients associated to its own AP. The CFAssign-RaC algorithm makes clients associate to APs that are conflict free i.e., free from inter-AP interference. However, if many clients are already associated to an AP, such clients would experience throughput reduction due to considerable intra-AP load. Thus, the channel assignment solution should associate clients to APs that minimize a combination of both intra-AP load and inter-AP interference. The CFAssign-RaC algorithm (modified to be cognizant of client load) jointly solves both the channel assignment and the load balancing problems as follows: CFAssign-RaC directly outputs the channel assignment for each AP. By using the load-aware objective function to address conflict set coloring, the CFAssign-RaC algorithm implicitly decides the association between the clients and APs (each client is associated to the AP from its range set which has the minimum conflict). This association is a solution to the load balancing problem as well. In the simulations, they study the effect of their algorithms on various metrics such as application level throughput for both UDP and TCP flows and the MAC level collisions. Various metrics were measured to study the effect of their channel management algorithm on different layers of the network stack. First, they measured the application level nthroughput for both FTP/TCP and CBR/UDP flows. Second, they measured the per-packet delay encountered by the CBR/UDP flows at the application layer.

This delay includes the queues at transmitting stations, and the MAC level delay (because of collisions and backoffs). This delay includes the queues at transmitting stations, and the MAC level delay (because of collisions and backoffs). This metric is useful in studying the effect on voice applications where a deadline oriented delivery of packets is more important than reliability. To summarize, they propose a client-based model called conflict set coloring that captures interference at the clients to efficiently utilize spectrum in a wireless LAN. They evaluate a centralized algorithm called CFAssign-RaC based on conflict set coloring which jointly performs channel assignment and load balancing, otherwise called channel management. Through extensive simulations and measurements from deployed testbeds they showed the practical usefulness of such an approach to centrally managed networks. They believe that such client-centric approaches are the key to improved application performance in WLANs and can find wider applicability to newer wireless technologies.

In the paper called “A Self-Managed Distributed Channel Selection Algorithm for WLANs”, Leith, & Clifford (2006) propose a new fully distributed algorithm suited to dynamic channel selection by WLANs. In this scheme each AP employs a simple learning rule to adaptively select the channel to transmit on.

The algorithm does not require direct communication between APs, hence it is referred to as self-managed. The sole information required by the algorithm is feedback to each AP on the presence of interference on a given channel; such feedback is already commonly provided by WLAN protocols such as 802.11. They show that the algorithm is guaranteed to converge to an optimal solution that minimises interference between WLANs provided this is feasible. Moreover, they demonstrate the convergence is, on average, remarkably fast under a wide range of network conditions and topologies.

Let c denote the number of available channels and let each AP maintain a c element state vector p. Let p_i denote the ith element of p with p_i = 1. The following distributed algorithm for updating p is considered.

Algorithm: Distributed Channel Selection

1) Initialise p = [1/c, 1/c, . . . , 1/c]T

2) Toss a weighted coin to select a channel, with p_i the probability of selecting channel i. Measure the quality of the channel: any interference measure can be used

that yields a success when interference/channel noise is within acceptable levels and failure otherwise.

3) On success on channel i, update p as

$p_i = 1$ (1)

$p_j = 0 \ \forall j \neq i$ (2), i.e. on success staying with that channel.

4) On failure on channel i, update p as

$p_i = (1 - b)\, p_i$ (3)

$p_j = (1 - b)\, p_j + b/(c - 1) \ \forall j \neq i$ (4), i.e. on a failed transmission multiplicatively decrease the probability of using that channel, redistributing the probability evenly across the other channels. b is a design parameter, $0 < b < 1$.

5) Return to 2.

Here, choice of Learning Parameter b is important. This parameter determines how quickly an AP discounts previous successes on a channel (or failures on other channels) on experiencing transmission failures on that channel. When b = 0, no action is taken on failures. That is, when b = 0, an AP simply settles forever on the first channel on which it experiences a successful transmission. It is easy to see that this greedy strategy will not, in general, converge to a proper channel allocation. They therefore require b > 0. For b > 0 they have that the algorithm reduces the probability of choosing a channel, uniformly increasing the probability of choosing the remaining channels.

The key issue in assessing the potential gain is the impact of interference on the MAC layer performance. They consider in particular two contrasting examples: (i) a naive centralised MAC scheduler that schedules a transmission in every available slot and (ii) an 802.11 CSMA/CA MAC scheduler. For naive centralised MAC scheduler case, when no channel allocation algorithm is used, network normalised throughput is almost zero since all nodes use the same channel for transmission. However, when algorithm is used, throughput increases. When an 802.11 CSMA/CA MAC scheduler is employed, this time, throughput is low but not zero without algorithm. However, throughput increases again when allocation algorithm is used, but not as much as that with naive centralised MAC scheduler.

In summary, in this paper they consider the problem of a wireless LAN selecting a channel to minimise interference with other WLANs. They introduce a new fully distributed channel selection algorithm that does not require direct communication between APs; that is, the algorithm is self-managing. The sole information required by the algorithm is feedback to each AP on the presence of interference on a chosen channel; such feedback is already commonly provided by WLAN protocols such as 802.11. They establish that convergence of the distributed algorithm is guaranteed provided that the channel allocation problem is feasible. Extensive simulation results are presented that demonstrate rapid convergence under a wide range of network conditions and topologies. While the scope of the present paper is confined to infrastructure networks with static topology, the utility of the proposed algorithm in situations where the network topology is time-varying is briefly discussed.

In the paper called “New graph model for channel assignment in ad-hoc wireless networks”, Cheng, C. Huang, X. Huang, & Wu (2005) consider the zero-interference-minimum-span version of channel assignment problem.

The purpose of channel assignment algorithms is to assign channels to transmitting hosts such that cochannel interference is avoided and the total number of channels used is minimized (Comellas, & Ozón, 1995). There are some other versions of channel assignment problems, for instance, to minimise the total interference for a given set of channels (Murphey, Pardalos, & Resende, 1999).

There are two types of interferences: primary interference and secondary interference. The primary interference is caused by direct collision, due to simultaneous transmissions from hosts that can hear each other. The secondary interference is also called hidden terminal interference, which is caused by hosts outside the hearing range of each other transmitting to the same receiver. In this paper, they present a channel assignment algorithm to eliminate both the primary and secondary interference.

Double disk (DD) graphs (Cheng at al, 2005) are more realistic than the single disk graphs, in which each host is represented as two concentric disks, with the inner disk representing the range of the transmitter (or supply area as it is called in cellular networks) and the outer disk representing the interference area. The region between the outer circle and the inner circle represents the area where the signal is not strong enough to be received successfully, but strong enough to interfere with others. Two hosts are interfering if one hosts interference area intersects with another hosts supply area. However, DD graphs have the problem that they do not distinguish if there exist other hosts in the overlapped area.

In this paper, they propose a new graph model, i.e., two cochannel hosts are considered to be interfering with each other if and only if the receiver of one transmitter is in the interference area of another transmitter. They call it the interference double disk graph model. To avoid confusion with the intersect disk (ID) graph and the double disk (DD) graph, they use FDD to denote it. Similar to a DD graph model, this model considers two concentric disks each representing the transmission range and interference range separately, but it more accurately models wireless networks. In an FDD graph, an edge exists between two vertices if the inner disk of one transmitter overlaps with the outer disk of the other transmitter and there exists another node within the overlapped area. A traditional way to represent the performance ratio of colouring algorithms is to compare the chromatic number X(G) with the clique number W(G), because W(G) is the lower bound for X(G). It has been shown that for any UD graph, the chromatic number is bounded by a clique number times a constant. In this paper, they try to find out if there is an upper bound for w(G)/o(G) in FDD graphs. They prove that this upper bound exists and X(G) ≤ 14 * (W(G) - 1) for FDD graphs.

Given a set of nodes V on the Euclidean plane and each node v is associated with two concentric disks with radii r_v and R_v respectively, where $R_v = c \times r_v$ and constant $c \geq 1$, build an FDD graph on V and assign each node a colour such that no node has the same colour as its adjacent nodes in the FDD graph. To construct the FDD graph, V is used as the vertex set, and the edge set is constructed in such a way that there exists an edge between two nodes x and y if and only if $x \neq y$, and there exists a node $w \in V$ that satisfies $|xw| \leq r_x$, $|yw| \leq R_y$ or $|yw| \leq r_y$, $|xw| \leq R_x$.

They use D(v) and d(v) to denote the area covered by the outer disk and inner disk of node v, respectively. Since w could be x or y, the above statement is equivalent to: x and y are connected by an edge in G if and only if at least one of the following is true:

(i) D(y) covers x

(ii) D(x) covers y

(iii) There exists a node $z \in V \setminus \{x,y\}$ that lies in the overlapped area of d(x) and D(y)

(iv) There exists a node $z \in V \setminus \{x,y\}$ that lies in the overlapped area of D(x) and d(y)

This graph model includes both direct interference edges and indirect interference edges, therefore an appropriate vertex colouring of an FDD graph can eliminate both direct collisions and hidden terminal collisions. The distributed implementation of the channel assignment algorithm would require that each node has knowledge of its two-hop neighbourhood, which is obtained within the first three rounds described below. Each node has three states: initial, colouring and coloured. Heuristic 2 Round 1: A node in initial statewould start by broadcasting its own ID, and learn its onehop neighbours from the information it has received. Round 2: Once a node receives the IDs from all its neighbours, it broadcasts its one-hop neighbours. Based on the information it has received from all its neighbours, each node learns itstwo-hop neighbours, and then computes a local FDD graph that spans over its two-hop neighbours. It then enters the colouring state. Round 3: A node with a stable FDD graph would broadcast its degree (i.e., the number of neighbours in its FDD graph), and relay this information for its one-hop neighbours. Round 4: To decide a channel number, each node would first build a list from its local FDD graph using the smallest last order. To get a list of smallest-last order, start with an empty list, pick a node with the smallest node degree, put it at the head of the list, and remove it from the local FDD graph; repeat until all nodes are in the list. A tie is broken in favour of a smaller node ID. The relative order of two nodes that appear at the list is consistent between each other. The node that finds itself at the head of the list would pick the smallest channel number not used by its FDD neighbours (i.e. nodes that share an edge with it on the FDD graph) and announce its channel immediately, and then go to the coloured state. Other nodes once they hear this announcement will remove it from the list, update the FDD graph, and relay it for one hop. Round 4 is repeated until every node is assigned a channel number. A node in the coloured state would periodically announce its channel number and ID, and relay this information for one hop.

As a conclusion, in this paper, they consider the collision free channel assignment problem in ad-hoc wireless networks. They model the wireless networks by a new class of graphs (interference double disk graphs (FDD)). The problem of minimising the number of channels needed to eliminate interference is a graph colouring problem in FDD graphs. They prove its NP-completeness and provide an upper bound for its chromatic number. They design a centralised channel assignment approximation algorithm and its distributed implementation that can eliminate both direct collisions and hidden terminal collisions. The FDD graph model requires more channels than the containment disk (CD) graph model, and less channels than the intersection disk (ID) and double disk (DD) graph models. FDD graphs model the wireless networks more accurately than CD, ID and DD graphs. The performance ratio of this algorithm on FDD graphs is 14 when the radii of outer disks and inner disks have a constant ratio. For a more general case where $R_i = c_i \times r_i$ and c_i is not constant, the performance ratio of the sequential colouring algorithm is still unknown. The theoretical bound provided in this paper can be used as a worst-case estimation on the total number of channels needed in wireless ad-hoc networks. More efficient channel assignment algorithms to reduce the number of channels needed can be considered. Especially when the traffic pattern is given, or the activity factor of nodes is given, channels can be reused between neighbouring nodes when their activity periods have no overlap, or their intended receivers are not interfered by the others.

Another paper related to channel assignment problem is "An Interference-Aware Channel Assignment Scheme for Wireless Mesh Networks" (Sen, Murthy, Ganguly, & Bhatnagar, 2007). They investigate the following question in this paper: Given the

(i) locations of the wireless mesh routers,

(ii) transmission and interference ranges of the transmitters,

(iii) the number of channels available on each link and

(iv) the number radio interfaces available at each router, what is the largest number of links that can be activated simultaneously subject to interference and radio constraints so that the resulting network is connected? Their goal is to activate all such links and they present an interference-aware channel assignment algorithm that realizes this goal. Their channel assignment scheme is traffic unaware in the sense that the channels are assigned without taking into account traffic pattern or the paths to be taken for establishing connections between source-destination node pairs. In this paper, they show that the Link Interference Graph constructed with the widely used interference model gives rise to a special class of graphs known as Overlapping Double-Disk (ODD) graphs. They prove that the Maximum Independent Set (MIS) computation problem is NP-complete, even for this special class of graphs.

The contributions of this paper are summarized below.

- Novel characterization of the Link Interference Graphs as Overlapping Double-Disk graphs.
- Development of a Polynomial Time Approximation Scheme (PTAS) for computation of MIS of an ODD graph.
- Development of a channel assignment algorithm with an objective of activating the largest number of links subject to interference and radio constraints.
- Comprehensive performance evaluation of the heuristic solution in comparison with the optimal solution obtained by solving an integer linear program.

They view the routers of a WMN as some points ($p_1, \ldots, p_n$) (specified by their x, y coordinates) on a two dimensional plane. Associated with each point p_j, $1 \leq j \leq n$ is a transmission range, R_T and an interference range, R_I, $R_I \subseteq R_T$ (they assume that all routers have identical transmission and interference ranges). At the second level of abstraction, they construct a graph G = (V, E), in which each node represents a point on the plane (router) and there is an edge from node v_i to node v_j if the Euclidean distance between the corresponding points p_i to p_j is less than or equal to the transmission range R_T. This assumption implies that each router has a circular coverage area with the center of the circle at the location of the router. The circular coverage area associated with point p_i (and node v_i) will be referred to as the disk associated with the point p_i (and node v_i).

The graph G = (V, E) will be referred to as the Potential Communication Graph (PCG). A link between any two nodes in this graph indicates that this pair of nodes can communicate with each other if their transmitters and receivers are assigned the same channel. It may be noted that even though these nodes are within the communication range of each other, they may not be able to communicate with each other unless the same channel is assigned to both of them. In the absence of the load information between source-destination pairs in the network, a good channel assignment strategy would be to do channel assignment in such a way that the resulting communication graph can support as many simultaneous active links. The problem considered in this paper is as follows: Given L, the location of the wireless routers R_T, the transmission range R_I, the interference range N, the number of available channels and K, the number of available radios at each of the routers, the problem is to assign channels such that the number of links that can be activated simultaneously is maximized subject to radio, interference constraints and the resulting graph is connected.

Simultaneous transmission on a common channel on two distinct edges e_1 and e_2 of PCG connecting nodes (u_1, v_1) and (u_2, v_2) respectively are said to interfere with each other if minimum $d(u_1, u_2)$, $d(u_1, v_2)$, $d(u_2, v_1)$, $d(v_1, v_2) \leq R_I$, where $d(u_i, u_j)$ indicates the Euclidean distance between the nodes u_i and u_j and R_I indicates the interference range. The Link Interference Graph LIG is constructed as follows: Corresponding to every link in PCG, there is a node in LIG and two nodes in LIG have an edge between them only if the corresponding links interfere with each other. Given the locations (p_1, . . . , p_n) of the routers on a two dimensional plane, they draw a line connecting points p_a and p_b to indicate the link la,b between the routers, if the distance between p_a and p_b less than or equal to R_T. Similarly, they draw a line connecting points p_c and p_d to indicate the link $l_{c,d}$ between the routers if the distance between p_c and p_d less than or equal to R_T. In order to determine if the links $l_{a,b}$ and $l_{c,d}$ interfere with each other, they do the following: They draw a circle with centers at the points p_a, p_b, p_c, p_d with radius $R_I/2$. Since $d(p_a, p_b) \leq R_T$ and $R_I \subseteq R_T$, the circles with centers at p_a and p_b will overlap. The same thing will happen for the circles with centers at p_c and p_d. They refer to this figure as Overlapping Double-Disks (ODD). The mid-point of the line joining the centers of the two disks will be referred to as the center of the double disks. The links $l_{a,b}$ and $l_{c,d}$ will interfere with each other if and only if the corresponding ODDs intersect.

The heuristic for the channel assignment takes as input the location of the routers, transmission radius, interference radius, number of channels, number of radios on each node and outputs the channels assigned to the radios of each router. The heuristic starts by reserving one radio on each node for ensuring connectivity later. The channel assignment heuristic invokes two functions namely, MIS channel assignment and ensure connectivity. At the end of the MIS channel assignment algorithm, the topology resulting from the channel assignment may have several connected components. To ensure connectivity, algorithm uses the single radio that was reserved earlier to connect all the components. The algorithm maintains a set S consisting of a single connected component. At each iteration, all paths that connect S with some component not in S are examined. The interference degree of a path is the largest number of edges interfered by an edge in the path. The path and the channel to be assigned on all nodes of this path that lead to the least interference is computed. Channel assignments are done on this path and the component C_i connected by this path is included into S. This procedure is repeated until all components are merged into S.

As a result, in this paper, they have provided a heuristic for the channel assignment problem in Wireless Mesh Networks. In the process, they characterize the LIG as ODD graphs and provide a PTAS to compute MIS for ODD graphs. Their results demonstrate the effectiveness of the heuristic.

In the paper “Flow-based Channel Assignment in Channel Constrained Wireless Mesh Networks”, Weihuang, Bin, Wang, & Agrawal (2008) first compute the minimum number of channels for a feasible conflict free channel assignment, and then perform assignment adjusting by taking the number of available orthogonal channels into account. Given a WMN and the traffic profile (i.e., traffic demand of each MR), the traffic flows among the MRs are modelled as a Linear Programming (LP) problem, targeting to find the fair flow of each MR so that each MR has the same proportional traffic that can be successfully forwarded to the IGW. Based on the fair flows, a weighted flow-based conflict graph is generated for further usage of channel assignment. They calculate the minimum number of channels by which a flow-based graph can be colored in a way that adjacent edges employ different channels. They observe the minimum number of channels for a conflict free channel assignment in different topologies, which provides the reference for the network design. They also estimate the approximation ratio of the number of channels obtained by the heuristic algorithm and the optimal solution.

If the available channels are not enough for a conflict free assignment, their algorithm performs a priority-based channel assignment to enable high load radios have a dedicated channel for use without conflict with its neighbors. The algorithm performs a procedure of channel mergence to allow a set of light traffic radios to share a common channel even they are neighbors. The selection of radios for channel mergence is based on the traffic load on these radios, which is computed in the flow-based conflict graph. This means the radio having less traffic is more likely to be selected for sharing channels. In addition, the channel can be reassigned to radios if the traffic profile in the network is significantly changed.

Flow-based channel assignment includes three stages: fair flow formulation, conflict-free channel assignment, and channel mergence. Adaptive channel re-assignment mechanism is also provided to meet significant traffic changes.

Fair Flow Formulation: At the beginning, the IGW discovers the network topology and collects the traffic demand information of each MR in the network. To reach the fairness for the MR traffic, they define $\lambda (0 < \lambda \leq 1)$ as the traffic proportion parameter, meaning that each MR can successfully transmit λ proportion of its aggregate traffic. Let s_{uv}^{kt} be a binary variable for edge uv, where t denotes a time slot used for data transmission over a specific channel k. $s_{uv}^{kt} = 1$ indicates link uv is active for the packet transmission in time slot t by employing channel k. Otherwise, $s_{uv}^{kt} = 0$. To determine the flows on the edges, it can be formulated by a LP problem subjecting to the following constraints.

- MR-Radio Constraint: Since the transmission or reception has to employ a radio, the total number of active links of a MR at a given time slot cannot exceed the total number of radios of the MR.
- MR-Interference Constraint: Given the interference distance $R_I = q \times R_T$, the interference region for a pair of transmitting MRs u and v is the union of two circles centered at u and v with radius R_I. The maximum number of simultaneous flows in the interference region within a time slot is c(q), where c(q) denotes the maximum number of simultaneous transmission in the interference region and is determined by the value of q.

By solving LP problem under constraints they obtain the fair flows, with the objective to maximize proportion parameter. According to the calculated traffic flows above, a flow graph $G_f(V, E_f)$ can be further generated from G(V, E) by removing the edges without traffic. When the network traffic is predominantly directed between the MR and the IGW for Internet access, a large number of links in G(V, E) will be removed in generating $G_f(V, E_f)$.

Conflict-free channel assignment: Once the flow-based conflict graph is generated, the channel assignment on flow graph G_f has been transformed to the node coloring problem on conflict graph G_c. The target of conflict free channel assignment on G_f is to assign different channels to all links within the interference region. It is transformed to assign colors (i.e., channels) to the nodes on G_c in a way such that any two adjacent nodes in G_c are assigned different colors.

The optimal node coloring problem is a NP-hard problem. Greedy node coloring algorithm can be implemented to find a near optimal channel assignment. Considering each node in sequence $V_1, \ldots, V_n$, it assigns each node with the first available channel (e.g. the channel that is least used in nodes and not used by any assigned neighbors).

Channel Mergence: If the number of available channels Nch is less than Xgreedy(G_C), the channel assignment may not be accomplished due to the shortage of channels. In this case, it is unavoidable that two adjacent nodes are colored by same color, indicating two neighboring flow links share a common channel. When two neighboring flow links share a common channel, it is necessary for the MAC protocol to avoid the collision in the case of using the shared channel. In this case, it is still needed to reduce the interference in the channel assignment.

For this purpose, they introduce a channel merging algorithm to assign channel in a low interference way. For instance, the network has three orthogonal channels and $f_A > f_B > f_C$. By using the greedy channel assignment algorithm, for example, they assign channels 1, 2, and 3 to nodes A, B, and C, respectively. If there are only two available channels, they have to adjust the assignment of certain node(s). They call such adjustment channel mergence. In this case, they need to consider two questions: (i) which node should be selected for channel mergence, and (ii) which channel should be used for the selected node. In their approach, they choose the node having the minimum weight (i.e., the minimum flow) in the flow-based conflict graph. The reason behind this is that the node having the minimum weight causes less interference on the channel which it is united to.

In order to avoid high interference, it is needed to evaluate the interference on each node and choose the channel having less interference introduced by the newly added node. For example, let us consider channel 1 for node C (i.e. f_C). Denote the interference at node A on channel 1 as $Int_1(A)$, which is interfered by the neighboring nodes of A in the conflict graph. The set of neighboring nodes is denoted by N(A). Then, they have $Int_1(A) = Int_1(C) = f_A + f_A$. It is noted that they approximate the degree of interference by using the traffic flow. The reason behind it is that more traffic to transmit, more interference will be resulted. On the other hand, the introduced interference is $f_B + f_C$ if they assign f_C with channel 2. Due to $f_B + f_C < f_A + f_C$, they finally assign f_C with channel 2. Therefore, they color nodes B and C with the same color, meaning flows B and C share the same channel 2.

In summary, a WMN implements multi-radio and multichannel communication in a multi-hop fashion. In this paper, they address the number of channels for a feasible conflict free channel assignment and observe it in different topologies. The results indicate the reasonable number of channels for a designed WMN. The channels can be assigned to the radios if the number of available channels is enough. Channel mergence procedure is performed if the number of available channels is less than that of required by considering the fair flows in the network.

In the paper “Unit Disk Graph and Physical Interference Model:Putting Pieces Together”, Lebhar & Lotker (2009) propose a novel approach that facilitates the use of sophisticated theoretical algorithmic tools in real networks with proved guarantees of performances and success. They show that it is possible to design an emulation scheme of the UDG topology under the SINR model without controlling the power levels of the nodes.

As a tractable mathematical object, the unit disk graph (UDG) is a popular model that has enabled the development of efficient algorithms for crucial networking problems. In a ρ-UnitDiskGraph, two nodes are connected if and only if their distance is at most ρ, for some $\rho > 0$. However, such a connectivity requirement is basically not compatible with the reality of wireless networks due to the environment of the nodes as well as the constraints of radio transmission. For this purpose, the signal interference plus noise ratio model (SINR) is the more commonly used model (Gupta, & Kumar, 2000). The SINR model focuses on radio interferences created over the network depending on the distance to transmitters. Nevertheless, due to its complexity, this latter model has been the subject of very few theoretical investigations and lacks of good algorithmic features.

In this paper, they demonstrate how careful scheduling of the nodes enables the two models to be combined to give the benefits of both the algorithmic features of the UDG and the physical validity of the SINR. Precisely, they show that it is possible to emulate a $1/\sqrt{(n \ln n)}$-UDG that satisfies the constraints of the SINR over any set of n wireless nodes distributed uniformly in a unit square, with only a O(ln3 n) time and power stretch factor. The main strength of their contribution lies in the fact that the scheduling is set in a fully distributed way and considers non-uniform power ranges, and it can therefore fit the sensor network setting.

They demonstrate that it is possible to emulate a UDG model in which each link satisfies SINR conditions over a set of wireless nodes. The main idea is to force each node to transmit at a particular time slot so as to guarantee that, for some $\rho > 0$, any node belonging to the ρ neighborhood of any transmitter is able to hear the transmitters message under SINR restrictions. This process can be viewed as an emulation of a UDG in the sense that the resulting network behavior is the same as a UDG. Once this scheduling is set in each node, it is possible to apply any algorithm originally designed for a ρ-UDG with the guarantee of successful transmissions over every link under the SINR model. The scheduling of the nodes inherently produces a time stretch factor, one of the difficulties of the emulation process is therefore how to ensure a low time stretch. They demonstrate that it is indeed possible to emulate a UDG with only a polylogarithmic time stretch in the uniform distribution setting. The second crucial obstacle that has to be overcome is how to label the nodes for the scheduling in a fully distributed way to fit the setting of sensor networks, in which nodes have no knowledge of their neighborhood. The labels are necessary to enable the emulation to be repeated in the future. The strength of their contribution lies specifically in the randomized labeling procedure that they use for this purpose.

The ρ-UDG of a set of points S is the graph of a set of vertices S and where there is an edge between two nodes u and v if and only if $d(u, v) \leq \rho$ (d is Euclidean distance) (Barrire, Fraigniaud, & Narayanan, 2001).

A UDG may not always be connected if the threshold is not large enough, depending on the distribution of the nodes in the plane.

They propose a scheduling scheme that emulates a UDG collision-free under the SINR model. The purpose of the scheduling is to force all the nodes transmitting simultaneously to be far apart from one another so as to minimize interferences within the UDG disks.

In the SINR model it is possible to set a security perimeter Δ for simultaneous emissions, which guarantees interference-free transmissions within any balls of some radius R.

The scheduling scheme SCHED is a probabilistic and fully distributed algorithm aiming at forcing nodes emitting simultaneously to be at distance at least from one another, for some Δ. For their scheduling scheme to distribute the appropriate time slots to each node, they first run an original probabilistic labeling scheme at the heart of the emulation operation.

If only one set of cells is allowed to emit simultaneously, with only one node emitting in each of these cells, the scheme guarantees that all simultaneous transmissions are at least Δ-apart and therefore collision-free as long as Δ has been chosen appropriately given the SINR constraints.

The purpose of the probabilistic labeling scheme is to assign a label to each node, such that it is unique within its cell. Since the number of nodes in a cell of side can be large, it is necessary to sub-divide these cells into smaller ones of side R so as to limit the label sizes.

The delicate part of their scheduling scheme is then to assign unique labels to the nodes inside the smaller cells, while there is no central computation available and communications can fail due to interferences. To achieve this goal, their scheme uses randomization. In a first phase, the nodes pick their label uniformly at random. This phase is then repeated a sufficient number of times to ensure that each node has picked a label unique in its small cell, in at least one of the rounds, with high probability. This enables each uniquely labeled node to transmit its position without interference to its neighbors in the cell during the time slot of the round. Description of the algorithm LABEL: In Step 1, the nodes identify to which cell they belong and produce a first part of their label accordingly: $\mu(s)$ for a node s. Steps 2 and 3 guarantee that only distant smaller cells will emit simultaneously by attributing the next part of the labels: k(s) for node s. Steps 4 and 5 consist in repeating random number picking. According to the value of the random number $\lambda(s)$ a node s has picked, they determine the exact time slot when it is allowed to emit. It then sends $\lambda(s)$ along with its geographic position (step 7). If a node receives (i.e. without interference) a position that belongs to the same small cell, it records it (step 8). Step 9 finalizes the labeling process: each node computes its rank in its small cell according to the other positions it has recorded during the execution.

Description of the scheduling scheme.: Once the nodes are labeled, the scheduling scheme simply consists of enforcing that, in each time slot, there is only one of the four sets of cells that is active, and in each cell of the set, at most one node that transmits.

To summarise, this paper proposes a novel approach that facilitates the use of sophisticated theoretical algorithmic tools in real networks with proved guarantees of performances and success. They show that it is possible to design an emulation scheme of the UDG topology under the SINR model without controlling the power levels of the nodes. By developing an original preprocessing phase of randomized labeling, they provide a tool enabling the transition from a one-shot local broadcast to the possibility of unlimited executions on an emulated graph.

In another paper “A Novel Spectrum Allocation Mechanism Based on Graph Coloring and Bidding Theory”, Liu, Xu, & Tan (2009) propose a novel distributed collusion algorithm to allocate channels in the spectrum pool, through which they can obtain the utilities of both the primary users and the cognitive users (who are not owner of the spectrum and bidding for some channels from the primary users.). The efficient allocation is determined by an interference graph, and the utility can be gain from the bids of the secondary users (Akyildiz, Lee, Vuran, & Mohanty, 2006).

Problem Formulation: They abstract the cognitive network as a undirected graph G = (V, E), where the vertices represent the secondary users, and edges represent interferences so that no channels can be assigned simultaneously to any adjacent nodes. They also refer to the graph G as the interference graph. In the follow, they use channel and color interchangeably (Liu, Xu, & Tan, 2009).

The objective of the channel allocation is to maximize the spectrum utilization, including both primary users and secondary users.

The optimal coloring problem is known to be NP-hard. They first discuss two heuristic based approaches that produce good coloring solutions, and then they propose a distributed collusion mechanism to solve the problem mentioned in equation.

- Distributed random algorithm: The main feature of random algorithm is that it needs less of iterations and calculations. In this mechanism, they assume that each node generates a random number uniformly from [0,1]. Within one round, if the node has the highest random number among all the users, it then wins the color, at the same time, the nodes (users) set including this one that cause no interference with each other gains this color, too. After one round, the nodes obtaining colors go to its request list. If one node needs no more colors, then remove this node from the interference graph and the edges connected with it. After that all nodes update their random numbers according to the following mechanism: if it wins one color, it divides its number by 2; otherwise, it keeps invariant (Liu, Xu, & Tan, 2009).

- Distributed greedy algorithm: As discussed in the previous section, the random algorithm may result in low utilities, so, then they discuss a distributed greedy algorithm with the objective of maximization of the utilization. The distributed greedy algorithm handles colors and nodes one by one (Liu, Xu, & Tan, 2009).

- Distributed collusion algorithm: Through the above mechanisms, the system utility is finally obtained. But they have not taken the aspects of the user bids and the revenue of primary users into account. In this part, they propose a novel distributed collusion mechanism with maximal independent set (MIS), through which they can not only gain the assigned channels of each node, but also get the utilities of both primary and cognitive users.

In summary, they focus on the study of the secondary users who purchase some channels for their own communication services. They propose a novel distributed collusion mechanism to allocate channels in the spectrum pool with graph coloring and bidding theory. Distinguishing to the existing mechanisms, such as distributed random and greedy algorithms, simulation results show that the proposed mechanism has a similar performance to the greedy (optimal) one, and through which they can also obtain the utility of primary users (Liu, Xu, & Tan, 2009).

In the paper “Effective Sensing Function Allocation Using a Distributed Graph Coloring and a Slot Allocation Algorithm in Wireless Sensor Networks”, Kawano & Miyazaki (2009) propose an algorithm for sensing function allocation.

The proposed algorithm has the following features:

- Function distribution balancing: The proposed method can carry out sensing function allocation in order to balance the distribution of each sensing function in a target monitoring field. Simple packet and low overhead: The packet structure used in the proposed method is very simple and contains only the source node ID and the color value allocated to the node. Thus, the proposed method can be applied to many sensor network systems even if their wireless bandwidth is narrow.
- Robustness: The proposed method dynamically allocates a sensing function for the current sensor nodes and their networks. Thus, it is robust against the failure or disappearance of the nodes.
- Rich scalability: Because each node needs to communicate only with its neighbors in order to establish the network in the roposed method, this network can be scaled.

In this paper, they consider a WSN model. The WSN consists of a BS, which is the main node, and many sensor nodes. This WSN is organized autonomously as follows: First, many sensor nodes are scattered in the environment. Next, each sensor node negotiates with the neighboring sensor nodes and decides its own sensing task. Then, on the basis of its own sensing task, the node starts transmitting the sensed data to the BS by using a multihop wireless route. The sensing task and the network structure are continuously and automatically maintained by periodical negotiations among the sensor nodes. An observer can obtain the sensed data from the WSN through the BS in order to observe the target monitoring area. Each sensor node has wireless communication and data processing functions as well as some sensing functions. Although the functioning of each sensor node is relatively simple, the observer can obtain environmental information from many networked sensor nodes.

The function allocation problem is defined as the problem of maximizing the number of sensor nodes that satisfy the following constraints C_1 and C_2.

- Constraint C_1: The sensing function of sensor node n_i should be different from that of any sensor nodes in $N_g(n_i)$. Here, $N_g(n_i)$ is a set of the one-hop distanced neighboring sensor nodes of sensor node n_i, $0 \leq i \leq N$, and N is total number of sensor nodes.

- Constraint C_2: All required sensing functions should be allocated to sensor node n_i and the sensor nodes in $N_g(n_i)$.

The algorithm consists of two processing periods, the sensing period and the function allocation period. Each node repeats these two processing periods alternately. In the function allocation period, each sensor node decides its own sensing time and selects its own sensing functions. In contrast, in the sensing period, each sensor node gets the sensed data by using allocated sensing functions. Here, they assume that all sensor nodes are synchronized, and their state transitions are carried out simultaneously.

Sensing Period: In the sensing period, the actual sensing is simply periodically repeated certain times specified by numOfSampling.

Function Allocation Period: The function allocation period consists of two task phases. One is the function decision phase in which each sensor node decides its own sensing task for the next sensing period. The other is the notify/update phase. In this phase, only the color value decided in the function decision phase and sensor node ID are exchanged among the neighboring nodes, and used in the next function decision phase. In their algorithm, the sensing function is dynamically allocated in each function decision phase only using the information of the colors of the neighboring nodes. Thus, each sensor node does not need to keep any status or information of the neighboring nodes.

In order to realize the sensing function allocation, they introduce a distributed graph coloring algorithm and a slot allocation algorithm. The function decision phase has three steps: coloring, slot division, and function decision.

Step 1: Coloring a graph coloring algorithm is used as the function allocation algorithm. This algorithm is based on DP algorithm that was previously proposed by the authors themselves. In coloring algorithm, color refers to the integer number that starts from one. Using a given number of colors, the algorithm colors each node to meet constraint C_1. That is the color allocated to a node should be different from the colors of its neighboring nodes. The policy of the color decision for each sensor node is relatively simple. Each node just changes the color periodically in order to meet the constraint. In general, in distributed graph coloring algorithms, if a sensor node detects a color conflict with the neighboring nodes, it tends to change its color to avoid the conflict. Here, a color conflict refers to the state that the color allocated to a node is the same as the color of one of the neighboring nodes.

However, if the color changing timing is constant and the same among all sensor nodes, the simultaneous color changing in the neighboring nodes may generate new conflicts. In order to avoid this adverse side effect, they have introduced a new probability function that calculates the color changing timing with respect to the number of neighboring nodes. Hence, in many cases, the DP coloring algorithm avoids a simultaneous color change. In addition, even if some conflicts do occur, they can be resolved by the proposed algorithm sooner or later. In the proposed algorithm, each sensor node finds a minimum color that is not used in the one-hop reachable nodes. This coloring policy is obviously different from that of other graph coloring algorithms including DP algorithm. They often perform the graph coloring under the given number of colors. The goal of the proposed coloring algorithm is to realize color allocation for all involved nodes with the lowest possible number of colors in order to meet constraint C_1.

Step 2: A slot division is realized by using the allocated color information for each sensor node. In order to preserve the quality of the environment sensing, the variations in the sensing functions need to be mentioned even if the number of deployed sensor nodes is not sufficient. To do that, they adopt a method as method 3. It is originally developed to realize a dynamic time-slot scheduling for TDMA, which uses a distance-2 colored graph. Here, distance-2 graph coloring means that each sensor node has a color that is different from the colors of the two-hop reachable nodes. Each node carries out the task, and performs the slot allocation in which some sensing functions mapped in the next step should be activated.

Step 3: By using the slot division slcolor and the number of sensing functions obtained in step 2, it is carried out actual sensing function allocation and sensing time allocation in the sensing period. The task overview is as follows: First, with the consideration of the total number of the sensing functions equipped in each sensor node, the initial slot allocation process to get slsensor is executed. The color allocated to each slot in slsensor indicates the actual sensing function. Next, by binding slsensor to slcolor, the final function allocation information is generated. For the binding step, there are three variations considered according to the relation between the number of sensing functions numOfSensors and slot division slcolor realized in step 2, that is,

(1) slcolor is smaller than numOfSensors (numOfSensors > slcolor -> length). In this case, the number of sensing functions in the sensor node is larger than the length of slot list slcolor, i.e., the number of elements in slcolor. Thus, some nodes would be allocated more than one sensing function.

(2) slcolor is as the same as numOfSensors (numOfSensors = slcolor -> length). The slot of color allocated to node n directly indicates the sensing function allocated to the node.

(3) slcolor is bigger than numOfSensors (numOfSensors < slcolor -> length). In this case, because the number of sensing functions is smaller than the length of slot list slcolor, i.e., the number of elements in slcolor, an allocated sensing function could be duplicated with that of some neighboring nodes. In order to avoid this problem, they introduce a time division scheduling method for the nodes allocating the same sensing function.

Consequently, they propose a sensing function allocation method that is based on a distributed graph coloring and a slot allocation algorithm. The method performs a dynamic sensing function allocation in order to balance the distribution of sensing functions in the target monitoring area. In addition, the distributed graph coloring algorithm used in the proposed method is so general that it can commonly be applied to other function allocation and combinatorial optimization problems in sensor network systems and other application fields.

Another paper “A New Channel Assignment Mechanism for Rural Wireless Mesh Networks” (Dutta, Jaiswal, Panigrahi, & Rastogi, 2008) proposes a channel allocation scheme for the edges in a point-to-point mesh, that allows all edges at a node to

1) operate independently of each other, with no synchronization required

2) do full-duplex data transfer at all times, i.e. a node can be simultaneously transmitting and receiving on all its links. They also propose an algorithm that achieves a channel allocation with the above properties on all nodes of the given graph. Formally, it is given a graph G with bi-directional edges for every link. It is required to allocate a channel to each edge (or equivalently color the edge) such that for each node in the graph, the set of colors assigned to its incoming edges is disjoint from the set of colors assigned to its outgoing.

They propose a naive way to directed edge color a bidirectional graph G as follows:

- Vertex color the corresponding undirected graph G, using k colors.
- Color every out-going edge of a node in G with the color of that node in G. As no two adjacent nodes have the same color in G, it is easy to see that for any node in G, no out-going edge has the same color as an incoming edge. Thus, they have obtained a directed edge coloring of G using k colors.

An important contribution of their work is to present a simple directed edge coloring algorithm. Given a vertex-coloring of an undirected graph using k colors, they give a directed edge coloring of the corresponding bidirectional graph. No overlap is allowed between the set of channels being assigned to the incoming links at any node with the set of channels being assigned to the outgoing links. It is desired to minimize the number of channels used for communication in the network under this constraint. They frame this channel allocation problem in terms of edge coloring, and call it the minimum directed edge coloring problem (or DEC, in short).

To summarize, in this paper they describe a simple channel allocation scheme that allows point-to-point links in a rural mesh network to operate in full-duplex mode at all times and completely independent of each other.

In the paper "Timely Sensor Data Collection Using Distributed Graph Coloring", Paradis & Han (2008) present a protocol for sensor applications that require periodic collection of raw data reports from the entire network in a timely manner. They formulate the problem as an NP-hard graph coloring problem. They, then, present TIGRA - a distributed heuristic for graph coloring that takes into account application semantics and special characteristics of sensor networks. TIGRA ensures that no interference occurs and spatial channel reuse is maximized by assigning a specific time slot for each node to transmit.

They consider wireless sensor networks with a single sink and multiple homogeneous data sources for applications that require raw data periodically. In this scenario, each sensor node periodically produces a new value and this value may need to traverse multiple hops to reach the sink. The reading from a node can be combined with the readings from other nodes on its way to the sink. Given a set of sensor values that are generated periodically, the objective is to schedule all the transmissions for each period to be completed in the shortest possible amount of time. Ideally, all the noninterfering transmissions can be scheduled at the same time slot to minimize overall delay. Tree-based collection has been typically used in these applications. If the same routing tree topology is maintained, at each period every sensor node sends the same number of readings upstream to the sink, whether generated at the node or relayed for one of its child nodes.

In many WSN applications, a sensor reading can often be represented with a small number of bytes, so more than one reading can fit into a standard transmission packet. They exploit this property to reduce the number of packets transmitted. Instead of individually sending each sensor reading, the readings are batched or combined at intermediate nodes and forwarded upstream along the tree. They refer to this as batch processing.

The problem is to determine the smallest length conflict-free assignment of time slots during which the reading generated at each node may be combined with readings from other nodes and transmitted to the sink over the routing tree.

Existing distributed graph coloring algorithms cannot be directly applied due to the following reasons. Although the data collection graph (or tree) is generated initially, the interference set needs to be dynamically determined in a decentralized manner given that there is no location information of each node. This implies that the graph to be colored is not fully established before coloring begins and many links related to schedule conflicts need to be gradually discovered during coloring. In addition, wireless links are asymmetric, leading to directed graph. Therefore, they propose a new distributed graph coloring heuristic.

One of the fundamental questions is the number of colors used in the coloring. The length of each round is determined by the number of colors used to color the vertices belonging to that round. The number of colors is determined by the amount of interference between the nodes in the round. Each node only transmits (as a child) during one of the rounds but can potentially be receiving transmissions (as a parent) from its children in any other round, therefore, each node has to actively participate in its own coloring as well as coloring for all of its children. There is no concern about interference between transmissions that are scheduled in different rounds since different rounds are scheduled sequentially; therefore, each node can maintain separate palette of available colors for each round that it participates in either as a sender or as a receiver. The colors are represented by integers corresponding to a time slot assignment within that round. The number of colors in a palette is not predetermined, but new colors are only added when necessary. If all the transmissions were interfering with each other, each node would need a separate slot to transmit and the number of colors across all the palettes would be equal to the number of nodes n. In order to minimize the number of colors, the nodes always try to get the lowest available integer from their palette. As colors become unavailable when nodes overhear other nodes in the same round using them, those colors get deleted and the lowest available remaining color becomes the next candidate. As a result of this color palette mechanism, the coloring with a minimal number of colors will be produced. A top-down coloring approach is more efficient with a tree based collection structure. In TIGRA, a parent node assigns different colors to each of its children; as a result, only conflicts between nonrelated pairs of nodes (i.e., nodes with different parents) have to be resolved.

In summary, TIGRA can eliminate packet collisions and avoid network congestion, two major factors for latency. Other causes for latency include node failures due to battery depletion or environmental influence and link failures due to external objects and conditions. Since recovery from these faults typically involve retransmission, it thereby increases packet delivery latency as well.

Mishra, Banerjeeb, & Arbaugha (2005) define scalable distributed algorithms for the channel assignment problem that tries to optimize user performance in wireless LAN environments with multiple APs in their paper “Weighted Coloring based Channel Assignment for WLANs”.

A channel assignment problem is typically modeled as a graph coloring problem. There is a vertex on the graph corresponding to each AP, an edge on this graph represents potential interference, and the colors represent the number of non-overlapping channels. A goal of the channel assignment problem is to cover all APs (vertices) with the minimum number of channels (colors) such that no two adjacent APs (vertices) use the same channel (color). This is the minimum graph coloring problem. They define a weighted variant of the graph-coloring problem, in which it is permissible to allocate overlapping channels to neighboring APs. The goal in this variant is to minimize the impact of such overlapping channel assignments between neighboring APs on user performance.

The channel assignment problem for WLANs can be modeled as a graph coloring problem in which the APs are the vertices of a graph. A conflict between two APs (due to physical proximity and potential interference) is represented by an edge in the graph. The goal of this graph coloring problem is to assign a set of distinct colors (one corresponding to each available channel). To enable an efficient channel assignment under such circumstances, the above graph theoretic formulation is extended to a weighted graph coloring problem with a certain objective function. In this weighted variant, each vertex corresponds to a distinct AP as before. However, each edge on this graph now has a weight associated with it. The weight of an edge indicates the importance of using different colors (channels) for the corresponding vertices (APs) that are connected by that edge. Here, they assume that the weight of an edge indicates the number of clients associated with the two corresponding APs that are affected if these APs are assigned the same channel. As a result, they formulate channel assignment in WLANs as a weighted vertex coloring problem. They propose two efficient, scalable and fault tolerant distributed algorithms that achieve significantly better performance than the stateof-the-art Least Congested Channel Search (LCCS). Through simulations, they show that the two techniques achieve up to 45.5% and 56% reduction in interference for sparse and dense topologies respectively with 3 non-overlapping channels.

2.2 Network Layer Applications

In the paper “A New Distributed Algorithm for Virtual Backbone in Wireless Sensor Networks with Different Transmission Ranges”, Raei, Fathi, Akhlaghi, & Ahmadipoor (2009) propose a new distributed algorithm and show that the achieved CDS is within a constant factor of the optimal CDS.

Wireless Sensor Networks (WSNs) have attracted a great deal of research attention due to their wide range of potential applications. In WSN, there is no fixed or pre-defined infrastructure. The nodes in a WSN generally communicate with each other, either through a single hop or multiple hops. Although there is no physical backbone infrastructure, a virtual backbone can be formed by constructing a Connected Dominating Set (CDS).

A Dominating Set (DS) of a graph is a subset of nodes such that each node in the graph is either in the subset or adjacent to at least one node in that subset. A CDS is a DS, which induces a connected sub graph. A CDS is a good candidate of a virtual backbone for wireless networks, because any node in the network is less than 1-hop away from a CDS node.

With the help of the CDS, routing is easier and can adapt quickly to network topology changes. Since, only the CDS nodes are responsible for relaying messages for the network, the non-CDS nodes can thus turn off their communication module to save energy when they have no data to be transmitted out.

To reduce the traffic during communication and prolong network lifetime and simplify the connectivity management, it is desirable to construct a Minimum CDS (MCDS). The MCDS problem has been studied intensively in unit disk graph (UDG), in which each node has the same transmission range (Clark, Colbourn, & Johnson, 1990).

To build a MCDS, they compute a Maximal Independent Set (MIS) of the network graph. An independent set (IS) of an undirected graph G(V,E) is a subset of V that no two nodes in the subset have an edge. In other words, if I is a IS and $u \in I$, $v \in I$ then $uv \notin E$. An MIS of a graph is an IS that cannot include any more nodes within V. Thus an MIS is a DS of a graph. However, in practice, the transmission ranges of all nodes are not necessary equal. In this case, a WSN can be modeled using a directed graph G(V,E). The nodes in V are located in a Euclidean plane and each node $v_i \in V$ has a transmission range $r_{min} \leq r_i \leq r_{max}$. A directed edge (v_i, v_j) is a member of E if and only if $d(v_i, v_j) \leq r$ where $d(v_i, v_j)$ denotes the Euclidean distance between v_i and v_j. Such graphs are called disk graphs. An edge (v_i, v_j) is bidirectional if both (v_i, v_j) and (v_j, v_i) are in E, i.e., $d(v_i, v_j) \leq \min(r_i, r_j)$. In this paper, they only study the MCDS problem in disk graphs where all the edges in the network are bidirectional, called Disk Graphs with Bidirectional links (DGB). The main contributions of this paper are as follows:

- The algorithm is fully distributed, which can be easily implemented in WSN.
- The algorithm has constant approximation ratio in DGB, which reduces the overhead of maintaining the backbone and the cost in communication.
- The algorithm for MCDS problem in DGB has time and message complexity of O(n).

They assume that all nodes in WSN are distributed in a two dimensional plane and nodes have different transmission ranges. The network topology is modeled as a Disk Graph with Bidirectional links, DGB in short. They use G(V, E) to represent such networks, where V is the set of sensor nodes and E is the set of edges. Their algorithm consists of two phases. In the first phase, they compute a MIS of the network graph. The second phase of the algorithm is to choose the minimal number of the nodes (called connectors) to make the DS connected, i.e., CDS. Each node v_i has a unique id (ID_i), a state (S_i), a transmission range (R_i). In each phase, they select nodes with the largest transmission range (R_i) among its neighbors, to reduce size of the CDS. In each node v_i, timer (T_i) set by the following formula: $T_i = 1/R_i \times T_{max}$ where T_{max} is maximum time for each timer, then each node with largest R_i terminates the timer faster than its neighbors. The algorithm consists of two phases MIS construction and CDS construction.

As a result, in this paper, the minimum Connected Dominating Set (MCDS) problem in Disk Graphs with only Bidirectional links (DGB) has been studied. The disk graphs can be used to model wireless ad-hoc and sensor networks where nodes have different transmission ranges. They have proposed a new distributed algorithm and shown that the achieved CDS is within a constant factor of the optimal CDS. The main approach in their algorithms is to construct a maximal independent set (MIS) and then connect them. Through the theoretical analysis, they have shown that their algorithm has constant approximation ratio and time and message complexity of O(n).

Another paper is “A Geo-Routing Algorithm in Planar Graph for Ad-hoc Wireless Networks” where Bin Muhammad (2007) presents a fully distributed algorithm to compute a planar subgraph for geo-routing in ad-hoc wireless networks. They consider the idealized unit disk graph model in which nodes are assumed to be connected if, and only if, nodes are within their transmission range. The main contribution of this work is a fully distributed algorithm to extract the connected, planar graph for routing in the wireless networks. This problem shall be formulated in the geometric graph as follows. Let N be a set of nodes deployed in a certain region R. The problem is to build a planar graph G = (N, E) on N such that each node is connected to its closest neighbors. Formally, the edge (u, v) is a member of E if and only if $\delta(u, v) \leq 1$, where $\delta(u,v)$ is the distance between node u and its closest neighbor v. In addition, they present a distributed algorithm for routing on the unit disk graph, which is fundamentally based upon the famous Face Routing algorithm (Kranakis, E., Sing, H. & Urrutia, J., 1999). The main idea of the Face Routing algorithm is that an information packet walks along faces of planar graphs and proceeds along the line connecting the source and destination nodes. The algorithm can be summarized as follows: Start at source s and let F be the face that is intersected by line segment joining source s and destination t, st. Explore the boundary of face F by traversing the edges of F and remember the intersection point p on line st with the edge of F which is nearest to destination t. After traversing all edges, go back to p. If reached the destination while traversing the boundary of F, it is done. Otherwise, it divides the line st into two line segments where line pt is the part of line st not yet traversed. Update face F to be the face which is incident to p and which is intersected by the line segment pt in the region of p and start all over again.

The overall strategy of the algorithm can be divided in two distinct phases. The phase I extracts the connected and planar graph from the given graph while phase II does the actual routing on the graph produced by phase I. In the phase I, they propose the distributed algorithm, which is based on the work by Theoleyre, Schiller, & Duda (2009), on unit disk model. The basic idea of the algorithm is as follows. Each node in the given graph, G, broadcasts its identity and position (coordinates) and gathers identities and positions of their neighbor nodes. Using this information, each node computes the local Delaunay triangulation, LDG, such that edges of the triangles are not larger than one unit. This part of the algorithm is based on the distributed algorithm proposed. Now each node sends the message to its neighboring node to remove the edges which are not Gabriel edges. When node u receives a message REMOVE(edge(u, v)), it accepts if there is no point (some node) lies in the disk of diameter uv, otherwise, rejects it by sending the message REJECT(edge(u, v)). If u and v both send and receive the message REMOVE, then the edge (u, v) will be removed. In other words, if node u has sent the remove message to node v and also received the remove message from node v, then the edge (u, v) is removed from the local Delaunay graph, LDG. Since DT is planar and GG is connected, therefore, the graph produced by the algorithm is planar and connected. In the phase II, the routing algorithm is based on the Face algorithm. The basic idea is as follows. Let f be the face of G with a starting point s on its boundary that intersects line segment (s, t), where t is the destination. Using right-hand rule, traverse the face f in the counterclockwise direction. If the edge (u, v) of the face f intersects with (s, t) at s' and dist(s', t) < dist(s, t), then this intersection s becomes the new starting point s. In the similar fashion traverse faces until s becomes a destination.

Here is the summary of the algorithm:

1. Given planar G, start with source node S.
2. Traverse the face f in counter clockwise direction.
3. If any edge of the f intersects line st (say s'), where t is the distance node, such that d(s, t) < d(s', t), then s' becomes the new starting node.
4. Move to the adjacent face and goto 2.

Consequently, they present a technique to extract the connected, planar subgraph geometric routing algorithms. They consider the idealized unit disk graph model in which nodes are assumed to be connected if, and only if, nodes are within their transmission range. The main contribution of this paper is a fully distributed algorithm to extract the connected, planar graph for routing in the wireless networks.

They have also presented the geometric routing algorithm that is based upon the famous Face Routing algorithm. The algorithm is fully distributed and nodes know only the position of other nodes and can communicate with neighboring nodes in their transmission range.

Another paper is “Efficient Greedy Geographical Non-Planar Routing with Reactive Deflectio” (Theoleyre, Schiller, & Duda, 2009) where they present a novel geographical routing scheme for spontaneous wireless mesh networks and they propose a flexible greedy routing scheme that can be adapted to any variant of geographical routing and works for any connectivity graph, not necessarily unit disk graphs. The main drawback of greedy geographical routing is packet loss at blocked nodes near voids or obstacles. A node must drop a packet when the improvement associated with any of its neighbors is negative. In face routing the left-hand rule tries to go around a void, but it requires the connectivity graph of nodes to be planar. Relative Neighborhood Graphs can yield planar graphs for unit disk graphs (UDG), but in real wireless environments, the conditions for obtaining planar graphs are not satisfied due to asymmetric links and not circular radio coverage. There is no efficient and localized planarization algorithm proposed for a general connectivity graph. A possible solution to this problem is the following method: a border node initiates local flooding to find the next hop closer to the destination. However, it results in long delays and significant overhead.

Here, they use a reactive method: a node becomes blocked with respect to a given destination when it cannot forward a packet to any neighbor closer to the destination. Hence, the part of the network not concerned by forwarding this packet does not generate any control traffic so that this approach is more scalable.

In their approach, a node chooses a neighbor closer to the destination and not blocked for this direction. If a node fails to forward a packet to a given destination, it will consider itself as blocked for this direction. It will advertise backwards a list of blocked directions so that its neighbors will not choose it as a next hop for these directions. If several non blocked neighbors exist, the forwarder chooses the neighbor closest to the destination, i.e. with the best improvement. For advertising blocked directions, they propose to use the notion of blocked sectors: a node N advertises that it is blocked for any destination that falls in sector S (N, $\mathrm{angle}_{\mathrm{min}}$, $\mathrm{angle}_{\mathrm{max}}$, $\mathrm{dist}_{\mathrm{min}}$). This proposed algorithm reduces in the long term the route length as well. However, they need several useless packet transmissions and backtracking before the network converges, and blocked sectors are correctly constructed. They propose a mechanism to accelerate the convergence of this propagation process by extrapolating the location of a blocked area. To detect the border of a void, node N first searches for the blocked k-neighbor closest to the direction of the destination D, i.e. minimizing angle ((N,D), (N,BN)) for all blocked nodes BN. Then, N constructs the Maximum Connected Set of blocked nodes that contains BN: it adds BN to this set, and recursively adds all its blocked neighbors. Finally, N computes the forbidden sector that spans the maximum connected set it extrapolates the blocked area.

Consequently, they propose a scheme for greedy geographical routing with reactive defect detection. The idea is to reactively detect blocked nodes and propagate the defect information by computing a set of blocked sectors. To reduce the route length and accelerate void detection in dense mesh networks, they propose a method to extrapolate void location.

In the paper "Ad-Hoc Networks Beyond Unit Disk Graphs", Kuhn, Wattenhofer, & Zollinger (2003) study a model for ad-hoc networks close enough to reality as to represent existing networks, being at the same time concise enough to promote strong theoretical results. Unit disk graph model does not account for the presence of obstacles, such as walls, buildings, mountains or also weather conditions which might obstruct signal propagation. On the other hand, unit disk graphs are simple enough to promote strong theoretical results (Kuhn, Wattenhofer, & Zollinger, 2003).

In a Quasi unit disk graph, two nodes are connected by an edge if their distance is less than or equal to d, d being a parameter between 0 and 1. Furthermore, if the distance between two nodes is greater than 1, there is no edge between them. In the range between d and 1 the existence of an edge is not specified.

They establish a constructive lower bound for Quasi unit disk graphs showing that basically any algorithm without routing tables requires sending of $\Omega((\frac{c}{d})^2)$ messages to route from a source s to a destination t, where c is the length of the shortest path between s and t. They show that, with the aid of a topology control graph structure, a restricted fooding algorithm is guaranteed not to perform worse and that this technique is consequently asymptotically message-optimal.

If the network nodes are known with information about their own and their neighbors' positions and assume that the message source knows the position of the destination the basic assumptions of geometric routing, a more subtle approach than flooding of the network is possible. They present a combination of greedy routing and restricted flooding. The task of a volatile memory routing algorithm is to transmit a message from a source s to a destination t on a graph, where each node of the graph holds a memory in which O(log n) bits1 may be stored as long as the message is en route. The task of a geometric routing algorithm is to transmit a message from a source s to a destination t on a graph while observing the following rules:

- Every node is informed about its own and all of its neighbors' positions.
- The source of a message knows the position of the message destination.
- A message may contain control information about at most O(1) nodes.
- A node is only allowed to temporarily store a message before retransmission; no other memory is available.

A geometric volatile memory routing algorithm is a volatile memory routing algorithm additionally observing the first three rules of the definition of geometric routing algorithms. They give a lower bound on the message complexity of any volatile memory routing algorithm. Then they describe how to obtain a subgraph of a given Quasi unit disk graph which forms the basis for their algorithms matching the lower bound. Although asymptotically message-optimal, a flooding-based algorithm is prohibitively expensive in most networks for practical purposes. Previous work shows that this problem can often be tackled by combining a correct routing algorithm (that is guaranteed to find the destination) with a greedy routing scheme. They therefore describe a geometric volatile memory routing algorithm that tries to leverage the advantages of a greedy routing approach with respect to both conceptual simplicity and message-efficiency: In order to route a message, a node simply forwards it to its neighbor closest to the destination. Greedy routing can however run into a local minimum with respect to the distance to the destination, that is a node without any neighbors closer to t. In their case such a local minimum is circumvented by employment of restricted flooding, in particular by the aid of the geometric Echo algorithm.

Their algorithm GEcho combines both greedy routing and flooding in two modes: Generally the message is forwarded in greedy mode as long as possible. Whenever running into a local minimum, the algorithm switches to echo mode. In order to keep the cost of flooding-based echo low, the algorithm tries to fall back to greedy mode as early as possible. The fallback criterion is chosen such that the combined routing algorithm is asymptotically optimal with respect to message complexity. In particular, the Echo algorithm does not terminate only when finding t, but already when finding a node v which is significantly closer to t than the local minimum. Consequently, after showing a lower bound on message complexity, they show a flooding algorithm based on Quasi UDG matches this lower bound. Moreover, they propose an alternative construction of a planar graph which can be used to perform geometric routing.

In the paper “A Simple Improved Distributed Algorithm for Minimum CDS in Unit Disk Graphs”, Funke, Kesselman, Meyer, & Segal (2005) propose an improved distributed 6:91-approximation algorithm for computing a connected dominating set in unit disk graphs. Although a wireless ad-hoc network has no physical backbone infrastructure, a virtual backbone can be formed by nodes in a connected dominating set (CDS) of G. A CDS of G is a subset $S \subseteq V$ such that each node in V is adjacent to some node in S and the communication graph induced by S is connected. They denote by OPT a minimum CDS in G. The problem is to find a minimum CDS in unit disk graphs. They present a very simple 6:91-approximation algorithm for computing a minimum CDS in unit disk graphs. The main contribution of this paper is an improved analysis of the relationship between the size of a maximal independent set and a minimum CDS in a unit disk graph, which yields better bounds for many previous algorithms.

A maximal independent set is also a dominating set, which only needs to be connected to obtain a CDS. Here, they construct a connected set S and an independent set $I \subseteq S$. In a nutshell, they color a node (without connection to D2-coloring which is used for an assignment of time slots to the nodes such that no interference occurs) with the following colors: black, the node is a part of I; blue, it is not in S but adjacent to a node in I; grey, it is in S but not in I, red, it is neither black, grey, nor blue, but a neighbor to a grey or blue node; and white, it is neither black nor grey nor blue, nor a neighbor to a grey or blue node. Initially, one node is colored red (this node can be chosen by running a leader election algorithm) and all other nodes are colored white. Each red node u (except the first one) keeps its parent grey node. The execution of the algorithm is divided into rounds. Each round consists of three phases and in each phase they use a conflict-free time slots assignment so that each node is able to transmit once. Basically, in a round each red node with minimum ID among its red neighbors joins I and its blue parent joins S. Then the colors of the relevant nodes are updated accordingly. The algorithm terminates when there remain no white or red nodes.

In the paper “A Polynomial Time Solution to Minimum Forwarding Set Problem in Wireless Networks under Unit Disk Coverage Model”, Baysan, Sarac, Chandrasekaran, & Bereg (2009) investigate Minimum Forwarding Set Problem in Wireless Networks.

Energy-efficient broadcast problem has received a significant attention from the research community and a large number of studies have been published in the area.

One promising approach that was proposed for energy-efficient broadcast is the neighbor designation approach where the goal is to prevent unnecessary transmission of broadcast packets for energy efficiency (Qayyum, Viennot, & Laouiti, 2002). Each node collects 2-hop neighborhood information and then identifies a subset of its 1-hop neighbors as forwarding nodes for relaying a broadcast message toward its 2-hop neighbors. The efficiency of neighbor designation approach depends on finding a minimum size forwarding node set among the 1-hop neighbors. This problem is referred to as minimum forwarding set problem (MFSP) (Qayyum, Viennot, & Laouiti, 2002).

The MFSP becomes a geometrical problem when unit disks is used to model the coverage area of wireless transmissions. Unit disk graphs (UDGs) are neither perfect nor planar graphs. Thus, efficient algorithms proposed for planar and perfect graphs cannot be applied to UDGs. MFSP under the unit disk coverage assumption resembles to the well-known Minimum Dominating Set (MDS) problem. MDS problem for UDGs has been studied extensively. The problem is shown to be NP complete for UDGs (Clark, Colbourn, & Johnson, 1990).

Another related problem to MFSP is the well-known Disk Cover (DC) problem that tries to find a minimal size set of disks (from a given set of disks) to cover a given set of points on a plane (Hochbaum, & Maass, 1985). MFSP is a special instance of the DC problem where disks are selected from a given set of 1-hop nodes. In this work, they assume a unit disk coverage model for wireless transmissions. In addition, as most local knowledge based broadcast approaches, their approach requires the availability of 2-hop neighborhood information. The required information includes 1) the identities of the 1-hop and 2-hop neighbors and 2) a radial ordering of the 2-hop neighbors with respect to the broadcasting node. The availability of the position information for the nodes is sufficient to compute the radial ordering of the 2-hop neighbors. One simple way of acquiring the position information is to use a GPS unit at each node. They present the first polynomial time algorithm to solve the MFSP under unit disk coverage model for wireless transmission. First, they introduce two properties named as Two-Set Property and

Noninterleaving Property. They, then, present an algorithm that uses a dynamic programming approach to build an optimal solution and prove its correctness.

In summary, they have studied the MFSP in the context of WANETs. Leveraging the practical characteristics of the application environment, they have proposed a polynomial time algorithm to build an optimal solution to the MFSP under the unit disk coverage model for wireless transmission. This can be used as a basis on the design and development of new algorithms for several wireless network applications including energy-efficient multicast and broadcast protocols, energy-efficient topology control protocols, and energy-efficient virtual backbone construction protocols for WANETs and sensor networks.

2.3 Physical Layer Applications

In the paper "A Hybrid Interference Model-based Topology Control Algorithm", Liu, Zhang, Liu, & Dai (2008) address the interference problem in wireless ad-hoc networks with the objective of minimizing the interference. Topology control protocol plays an important role in ad-hoc networks. It is used for saving energy and increasing network capacity in the network perspective. Nodes make local choices (such as setting the transmit power level and sleeping a node) with the goal of achieving a network property, such as connectivity, symmetry, sparsity, low interference. In wireless ad-hoc networks, communication between nodes takes place over radio channels. As long as all nodes use the same frequency band for communication, any node-to-node transmission will add to the level of interference experienced by other users. There are two techniques to compute interference effect:

(a) purely geometric measurement on topology;

(b) mechanisms from communication theory, such as radio fading, path loss, encoding, and modulation. But these interference measures are simple and does not account for multihop communications. Reducing interference in the network leads to fewer collisions and packet retransmissions, which directly extends the capacity of the network and indirectly reduces the power consumption. Therefore, reducing the interference in the reduced graph is an important goal for topology control algorithms.

They integrate SINR-based model into graph-based model. They consider a wireless ad-hoc network with all nodes V, distributed in a two dimensional plane. Each node has its own transmission power which can be adjusted between 0 and Pmax, where Pmax denotes the common maximum transmission power corresponding to the transmission range dmax.

Unit disk graph, G = (V, E), is often employed to model the original topology of an ad-hoc network (Clark, Colbourn, & Johnson, 1990). The communication graph G = (V, E) defines the network topology. V is the set of nodes. E is denoted the set of wireless links that the nodes in V can use to communicate with each other. Topology control is done by selecting a subset of the available links in the network graph G to form the reduced graph G_{t_c}= (V, E_{t_c}). The resulting topology G_{t_c} should have the following properties: symmetry, connectivity, sparseness. The resulting topology G_{t_c} should be symmetric, that is, node u is a neighbor of node v if and only if node v is a neighbor of node u. Two nodes u and v are connected if there is a path from u to v, potentially through multiple hops. If two nodes are connected in G, then they should still be connected in G_{t_c}. Each node in a sparse network has a small number of neighbors.

A successful message transmission is divided into two phases: DATA frame and ACK frame. So the interference of a bidirectional link can be denoted by Linter (u, v) = Linter<u, v>+Linter<v, u>. This definition only calculates one hops interference. Currently, they take into account the length of paths in the graph. The path interference of a path p = $\{e_1, e_2, \ldots, e_m\}$ is defined as the sum of the interference of all links in the path. The graph interference is the maximal path interference among all interference optimal paths. A resulting topology can be required to reduce the graph interference of networks. They describe a topology control algorithm HIMTC (Hybrid Interference Model-based Topology Control). The algorithm consists of three phases: create RNG of networks, set the transmitting power and reduce interference. In the simulations, they use the following metrics: the graph interference and the network capacity.

Instead of using a long, energy-inefficient and high interference edge, communication can take place along a multi-hop path composed of short edges that connects the two endpoints of the long edge. The maximum power communication graph can be properly pruned in order to maintain only energy inefficient and capacity efficient edges in their algorithm. As a result of reducing the link interference, links have a higher success rate to access the wireless channel. Thereby, HIMTC also reduces the path relay and throughput. Consequently, in this paper they address the interference problem in wireless ad-hoc network with the objective of minimizing the interference. First, they present the excellent feature and the weak point on existing interference models. The graph-model based topology control captures interference inadequately under the physical model. Then, they propose a hybrid interference model that can be easily used in practical network protocols to measure the amount of interference in a wireless network. Finally, based on their interference model, they study topology control which was considered as an inherently graph-theoretic notion in previous literature in a general way. In this paper they explicitly analyze topology control with special emphasis on the physical definition of interference, or more specifically the SINR.

Wang & Zeng (2007) propose a trellis and a Wiberg-like graph (Wiberg, 2006) for a Bose-Chaudhuri-Hochquenghem (BCH) code in frequency domain in the paper "Graph representations of BCH codes in frequency domain" and thus the concept of codes defined on graphs is extended from time domain to frequency domain. There is not a representing approach based on graphs for BCH codes in frequency domain. Here, they propose a novel trellis diagram and the corresponding Wiberg-like graph for BCH codes in frequency domain. A trellis for a block code C of length in frequency domain is defined as an edge labeled directed graph, which has the following properties:

(1) all states can be reached from the starting state;

(2) the terminating state can be reached from all states;

(3) the number of edges traversed in passing from the starting state to the terminating state along any path is n; and

(4) the set of n tuples obtained by reading off the edge labels encountered in traversing all paths from the starting state to the terminating state is the codeword C in frequency domain.

They introduce a Wiberg-like graph for a BCH code in frequency domain. On the whole, the Wiberg-like graph bears a strong resemblance to its counterpart in time domain and their only difference is that the visible sites and hidden sites are variables of a Galois field. However, the structure of frequency-domain trellis is greatly different from its counterpart in time domain. A particular character of the trellis diagram in frequency domain is that error-correcting capacity of the code can be observed from its trellis architecture directly.

As a result, a novel trellis diagram and a Wiberg-like graph to represent a BCH code in frequency domain are proposed and it confirms us that a BCH code not only can be defined on graphs in time domain, but also can be defined on graphs in frequency domain. Trellis structure of BCH codes in frequency domain is investigated as well.

In the paper “Wheel Codes: Turbo-like Codes on Graphs of Small Order”, Radebaugh, Koetter, & Powell (2003) investigate a specific class of codes on graphs called wheel codes, where the underlying graph of a wheel code is constructed by the wheel construction. They present some results from the beginning of an indepth study of the wheel codes. In particular, they discuss in detail the wheel construction used to generate the underlying graphs for these wheel codes. Especially, they focus on cubic Hamiltonian graphs generated by the wheel construction. After that they discuss the codes that they generate using these graphs, and in doing so show why the wheel construction is an especially useful construction. Factor graphs provide an excellent framework on which to run iterative decoding algorithms and have therefore been used extensively in the area of Turbo codes and the factor graph representation of a wheel code is given directly by the underlying cubic Hamiltonian graph (Blahut, 2003). A cubic Hamiltonian graph obtained from the wheel construction readily provides a simple labeling of the information bits, so they give a procedure to label the bits. Then, they discuss the implementation of an encoder and decoder for the wheel codes. Since wheel codes behave like Turbo codes for a certain window of time, they decode a wheel code using a sliding window version of a forward-backward algorithm such as the MAP algorithm.

In summary, they discuss a class of codes on graphs called wheel codes, which they construct using the wheel construction. With this construction, they produce graphs with girths of 6 to 14, allowing a sliding window of length at most 13. They give some examples of generalized spoke vectors, several of which are based on Exoos cubic cage constructions, that allow for a wide range of possible wheel code lengths. Implementations of the wheel codes and a corresponding sliding window decoder are discussed. Finally, they note that as more capacity-approaching codes are presented, one naturally becomes interested in code implementations. The flexibility and simple implementation that the wheel codes offer make them attractive, in spite of their distance from the Shannon channel capacity.

Reggiani, Tartara, & Maggio (2001) propose a state partitioning for reducing the complexity of SISO (Soft Input Soft Output) detectors in their paper “A Reduced-State Soft Input Soft Output Algorithm Based on State Partitioning”. This procedure consists of partitioning the set of original states in order to generate a new set of super-states. This new version of the SISO algorithm, called Scaled SISO (SC-SISO), does not require determination of path survivors or feedback operations. This approach finds its justification in the theory of symbolic dynamics. Namely, the dynamical evolution of the code or channel can be seen through a reduced degree of complexity, a sort of coarse description of the original Markov process. The simplest application of this technique consists of merging the states which have the most recent n bits in common, with n smaller than the total memory of the original trellis. Consequently, the scaled trellis will contain 2^n states. They observe that in this case the partition preserves the one-to-one correspondence between the new transitions and the input labels. The key concept behind a scaled SISO is quite simple: merging two or more states produces a new super-state and a standard SISO algorithm can follow the system evolution in terms of this new description. After partitioning the original states of the trellis, the complete graph G reduces to a less complex graph GR. They observe that, although a reduced graph GR is still a topological Markov chain, the outputs associated with each transition depend also on past input values. This follows from observing the original Markov process with reduced memory and is a clear sign of the SC-SISO suboptimality. In the modified SISO, scaled graph performance is related to the reduced Euclidean distances involved in the new trellis. A problem observed in some codes and channels concerns catastrophic behaviors of scaled trellises: merging two states could produce a couple of branches with one output label in common between 0 and 1 input, reducing the minimum Euclidean distance even to zero. This means that the scaling procedure turns out to be effective until the reduced trellis exhibits catastrophic behavior.

Consequently, they present a reduced version of a Soft Input Soft Output algorithm based on state merging. The simulation results for the examples considered confirm that the approach is effective in reducing the computations without dramatically affecting the performance. The main advantages are a good flexibility due to the state partitioning technique and the absence of path survivors and modifications to the standard SISO algorithm. On the other hand, this implementation requires a pre-computation of the probabilities of the sets of outputs that are originated by state merging. This operation is an average of the probabilities of the single outputs, possibly weighted in the selfiteration process.

Cancellieri, Ferro, & Mazzone (1996) introduce a novel kind of state diagram in their paper “State Diagram for Cyclic Block Codes”, based on the so called de Bruijn graphs, for the description of cyclic block codes. A state diagram for any cyclic block code is a compact method for listing in proper order all its q^k codewords, being q the dimension of the symbol alphabet (Wolf, 1978).

From a state diagram which can be designed with general rules and whose complexity is low, it is then trivial to obtain a trellis diagram, like that necessary to perform soft-decision decoding and it is also possible to define a new algebraic decoding procedure using some properties strictly linked to this state diagram.

Given the number r of control symbols, they show that the state diagram topology, which appears as a de Bruijn graph (a regular strongly-connected graph, with q^r nodes representing as many states, and q^{r+1} arcs representing transitions among them), is, unique, even in node labelling, for the original cyclic code and for all the possible shortened and lengthened codes obtained from it. They also introduce a new algebraic decoding procedure. The decoding operation is based on the property that all codewords generate closed paths in the state diagram, which start from state 0 and come back to state 0 after n steps, being n the code length. In this way, if the path produced by the received sequence ends in a state X different from 0, an error has occured and the arrival state X is univocally linked to the occurred error.

Consequently, they propose the use of a properly defined state diagram for the description of cyclic block codes, and of all the shortened and lengthened codes obtained from them. They also introduce a new algebraic decoding procedure for correcting random errors. The features of this new approach can be employed for the search of new codes and decoding procedures. In particular, some expedients are dedicated expressly for facing error bursts and patterns of phased errors.

Another paper is “Graph-Theoretic Construction of Low-Density Parity-Check Codes” where Djurdjevic, Lin, & Abdel-Ghaffar (2003) present a novel graph theoretic method for constructing low-density parity check (LDPC) codes from connected graphs without the requirement of large girth. The method is based on finding a set of paths in a connected graph, which satisfies the constraint that any two paths in the set are either disjoint or cross each other at one and only one vertex.

Let G = (V, E) be an undirected, connected graph with vertex set V of size q and edge set E. For $1 \leq L < q$, let P be a set of n paths of length L, which satisfies the constraint that any two paths in P are either disjoint (i.e., have no vertex in common), or singularly crossing (ie, have exactly one vertex in common). This constraint is called the disjoint-crossing (DC) constraint. Let matrix H over GF(2) display the incidence relationship between the paths in P and the vertices in G. It follows from the constraints on the paths in P that no two columns (or two rows) in H can have more than one 1-component in common. This ensures that the Tanner graph of H does not contain cycles of length 4. If L is chosen to be much smaller than the number of vertices in G, then H is a sparse matrix. Given a graph G, its paths of length L can be represented by a trellis T_L of L sections with L + 1 levels of nodes. Each level consists of q nodes which are the vertices of G. For $0 \leq k < L$, a node v_i at the kth level and a node v_j at the (k + 1)th level are connected by a branch if and only if (v_i, v_j) is an edge in E. To find a set P of paths of length L in G that satisfies the DC constraint, an extend-select-eliminate (ESE) algorithm is devised to parse the path trellis TL of G. The algorithm consists of L steps. At the end of each step i, $1 \leq i \leq L$, a set of paths of length i that satisfies the DC constraint is obtained. For a large connected graph G, it may be prohibitively complex to process its path trellis by the ESE algorithm. To overcome this problem, they take a divide-and-conquer approach. To find two paths in the set that are either disjoint or cross each other at one and only one vertex, two trellis-based algorithms for finding these paths are devised. Good LDPC codes of practical lengths are constructed and they perform well with iterative decoding.

Esmaeili & Khandani (2000) discuss the maximum likelihood decoding of linear block codes by Wagner rule decoding in the paper called “Acyclic Tanner graphs and maximum-likelihood decoding of linear block codes”.

The well known graphical models presented for linear codes are trellis diagram, Tanner graph (TG), and Tanner-Wiberg-Loeliger graph (Blahut, 2003). A TG of a linear code C is a bipartite graph obtained from a set of parity check equations representing C. The two sets of vertices are called variable nodes and check nodes. Projection of linear block codes on maximal acyclic Tanner graphs provides the basis for the application of the Wagner rule to develop an efficient soft decision decoding algorithm. Using this projection, a given linear block code is represented by a combination of a trellis and a Tanner graph, where the efficiency of the decoding algorithm lies in the ability to exploit the structure of the underlying trellis diagram. It has been shown that the best maximum likelihood techniques known so far for the decoding of many important codes such as Hamming codes, Reed-Muller codes, hexacode, the extended Golay codes, and the QR code are in fact based on this kind of projection (Richard, 2003). The application of this approach on an arbitrary linear block code depends on the identification of relatively uniform acyclic subcodes of the code.

The paper "State Diagram Connectivity and its Effects on the Decoding of Shift-Register-Based Codes" (Collins, 1995) determines lower limits on the amount of information which must flow in the Viterbi decoding of codes based on shift-register encoders, e.g., trellis codes or convolutional codes. The challenge of building a graph-partition based (fixed state) decoder is to divide a de Bruijn graph into regions (sets of nodes having no members in common) such that the number of lines crossing region boundaries is as small as possible. If the partition size is fixed (e.g., by available VLSI technology), then, no matter how large the total graph is, only a constant number of partitions can have nonunique paths from their input to their outputs.

This paper does not only obtain the scaling laws which prescribe the size of the decoding engines that are now being built but also establishes the extent of the potential for improving the combination of shift-register-based codes and Viterbi decoding. For the construction of efficient decoding hardware, this paper provides the practical guidance that the graph partition approach is a good solution, at least for the current generation of convolutional and trellis codes. The reason follows from the efficiency of the inner mechanism of the processing modules. At all scales of division the communications efficiency of the graph partition design is always within a factor of three of the absolute limit. This paper has dealt only with communication between modules, but keeping the modules small is important as well. The hardware savings which the graph partition approach makes possible are sufficient to offset its slight communications disadvantage. Certain choices of timevarying code could, however, allow these same methods to be applied to a state variable decoder. If a general-purpose parallel processor whose elements are much faster than their connecting switch is to be programmed to become a decoder, then the state variable approach is also likely to be superior.

Obviously, a time-invariant (state fixed) mapping, from the large network to the smaller number of physical processors, is superior to a time-varying assignment, because it greatly eases the difficulty of context switching, as each physical processor is time-shared among the nodes of the network which it is simulating. The technique of time-varying graph rearrangement used in this paper, although not as useful for the parallel-processing problem, is more easily transportable to other types of large interconnection networks than are the graph-partitioning techniques. This paper answers the partitioning question for parallel computers based on the de Bruijn graph and provided tools which may be useful for deriving similar bounds for other interconnection topologies.

In summary, in this chapter we make a review of applications of graph theory in the literature and we classify them based on the 7 layers of OSI. We see applications mostly belong to MAC and transport layer, network layer and physical layer. Examples of transport layer applications are congestion prevention, flow control; network layer applications are data routing, topology control; applications on MAC layer generally include channel allocation and coloring problems and physical layer applications are mostly related to interference reduction, trellis, state diagrams and graph representation of codes.

CHAPTER THREE
SPECTRUM ALLOCATION AND INTERFERENCE MANAGEMENT ISSUES IN COGNITIVE RADIO NETWORKS

3.1 Cognitive Radio Networks

In recent years, the development of intelligent, adaptive wireless devices called cognitive radios, together with the introduction of secondary spectrum licensing, has led to a new paradigm in communications: cognitive networks. Cognitive networks are wireless networks that consist of several types of users: often a primary user (the primary licenseholder of a spectrum band) and secondary users (cognitive radios). These cognitive users employ their cognitive abilities to communicate without harming the primary users.

3.1.1 Motivation and Definition of a Cognitive Radio Network

Cognitive networks are initiated by the apparent lack of spectrum under the current spectrum management policies. The right to use the wireless spectrum in the United States is controlled by the Federal Communications Commission (FCC). Most of the frequency bands useful to wireless communication have already been licensed by the FCC. However, the FCC has designated a few unlicensed bands, most notably the industrial scientific and medical (ISM) bands, over which the immensely popular WiFi devices transmit. These bands are filling up fast, and, despite their popularity, the vast majority of the wireless spectrum is in fact licensed. Currently, the primary license holders obtain from the FCC the exclusive right to transmit over their spectral bands. Since most of the bands have been licensed, and the unlicensed bands are also rapidly filling up, it would appear that a spectral crisis is approaching. This, however, is far from the case. Recent measurements have shown that for as much as 90% of the time, large portions of the licensed bands remain unused. As licensed bands are difficult to reclaim and release, the FCC is considering dynamic and secondary spectrum licensing as an alternative to reduce the amount of unused spectrum. Bands licensed to primary users could, under certain negotiable conditions, be shared with nonprimary users without having the primary license release its own license. Whether the primary users would be willing to share their spectrum would depend on a number of factors, including the impact on their own communication.

The limited available spectrum and the inefficiency in the spectrum usage necessitate a new communication paradigm to exploit the existing wireless spectrum opportunistically. Dynamic spectrum access is proposed to solve these current spectrum inefficiency problems. DARPA’ s approach on Dynamic Spectrum Access network, the so-called NeXt Generation (xG) program aims to implement the policy based intelligent radios known as cognitive radios (Lassila, & Penttinen, 2008).

NeXt Generation (xG) communication networks, also known as Dynamic Spectrum Access Networks (DSANs) as well as cognitive radio networks, will provide high bandwidth to mobile users via heterogeneous wireless architectures and dynamic spectrum access techniques. The inefficient usage of the existing spectrum can be improved through opportunistic access to the licensed bands without interfering with the existing users. xG networks, however, impose several research challenges due to the broad range of available spectrum as well as diverse Quality-of-Service (QoS) requirements of applications. These heterogeneities must be captured and handled dynamically as mobile terminals roam between wireless architectures and along the available spectrum pool. The key enabling technology of xG networks is the cognitive radio. Cognitive radio techniques provide the capability to use or share the spectrum in an opportunistic manner. Dynamic spectrum access techniques allow the cognitive radio to operate in the best available channel.

Once a cognitive radio supports the capability to select the best available channel, the next challenge is to make the network protocols adaptive to the available spectrum. Hence, new functionalities are required in an xG network to support this adaptivity. In summary, the main functions for cognitive radios in xG networks can be summarized as follows (Lassila, & Penttinen, 2008):

- Spectrum sensing: Detecting unused spectrum and sharing the spectrum without harmful interference with other users. It is an important requirement of the Cognitive Radio network to sense spectrum holes. Detecting primary users is the most efficient way to detect spectrum holes.
- Spectrum management: Capturing the best available spectrum to meet user communication requirements. Cognitive radios should decide on the best spectrum band to meet the Quality of service requirements over all available spectrum bands, therefore spectrum management functions are required for Cognitive radios. These management functions are spectrum analysis and spectrum decision.
- Spectrum mobility: It is defined as the process when a cognitive radio user exchanges its frequency of operation. Cognitive radio networks target to use the spectrum in a dynamic manner by allowing the radio terminals to operate in the best available frequency band, maintaining seamless communication requirements during the transition to better spectrum.
- Spectrum sharing: Providing the fair spectrum scheduling method among coexisting xG users. One of the major challenges in open spectrum usage is the spectrum sharing. It can be regarded to be similar to generic media access control MAC problems in existing systems.

The application of cognitive networks, however, is not limited to just fixing the current spectrum licensing. Other applications abound in shared spectra, such as the ISM band (where different devices need to coexist without inhibiting each other), sensor networks (where the sensors may need to operate in a spectrum with higher power devices), and current services such as the cellular network (where the operator may want to offer different levels of services to different types of users).

Cognitive radio technology is the key technology that enables an xG network to use spectrum in a dynamic manner. The term, cognitive radio, can formally be defined as follows: A ‘‘Cognitive Radio’’ is a radio that can change its transmitter parameters based on interaction with the environment in which it operates (Lassila, & Penttinen, 2008)

From this definition, two main characteristics of the cognitive radio can be defined:

- Cognitive capability: Cognitive capability refers to the ability of the radio technology to capture or sense the information from its radio environment. This capability cannot simply be realized by monitoring the power in some frequency band of interest, but more sophisticated techniques are required in order to capture the temporal and spatial variations in the radio environment and avoid interference to other users. Through this capability, the portions of the spectrum that are unused at a specific time or location can be identified. Consequently, the best spectrum and appropriate operating parameters can be selected.
- Reconfigurability: The cognitive capability provides spectrum awareness whereas reconfigurability enables the radio to be dynamically programmed according to the radio environment. More specifically, the cognitive radio can be programmed to transmit and receive on a variety of frequencies and to use different transmission access technologies supported by its hardware design.

The ultimate objective of the cognitive radio is to obtain the best available spectrum through cognitive capability and reconfigurability as described before. Since most of the spectrum is already assigned, the most important challenge is to share the licensed spectrum without interfering with the transmission of other licensed users as illustrated in figure 3.1.

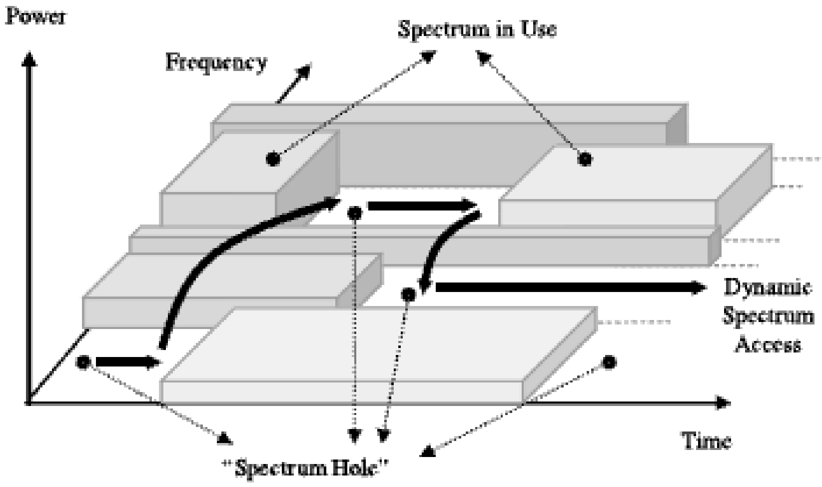

Figure 3.1 Dynamic spectrum usage

The cognitive radio enables the usage of temporally unused spectrum, which is referred to as spectrum hole or white space. If this band is further used by a licensed user, the cognitive radio moves to another spectrum hole or stays in the same band, altering its transmission power level or modulation scheme to avoid interference as shown in figure 3.1.

Reconfigurability is the capability of adjusting operating parameters for the transmission on the fly without any modifications on the hardware components. This capability enables the cognitive radio to adapt easily to the dynamic radio environment. There are several reconfigurable parameters that can be incorporated into the cognitive radio as explained below:

- Operating frequency: A cognitive radio is capable of changing the operating frequency. Based on the information about the radio environment, the most suitable operating frequency can be determined and the communication can be dynamically performed on this appropriate operating frequency.
- Modulation: A cognitive radio should reconfigure the modulation scheme adaptive to the user requirements and channel conditions. For example, in the case of delay sensitive applications, the data rate is more important than the error rate. Thus, the modulation scheme that enables the higher spectral

efficiency should be selected. Conversely, the loss-sensitive applications focus on the error rate, which necessitate modulation schemes with low bit error rate.

- Transmission power: Transmission power can be reconfigured within the power constraints. Power control enables dynamic transmission power configuration within the permissible power limit. If higher power operation is not necessary, the cognitive radio reduces the transmitter power to a lower level to allow more users to share the spectrum and to decrease the interference.
- Communication technology: A cognitive radio can also be used to provide interoperability among different communication systems.

The transmission parameters of a cognitive radio can be reconfigured not only at the beginning of a transmission but also during the transmission. According to the spectrum characteristics, these parameters can be reconfigured such that the cognitive radio is switched to a different spectrum band, the transmitter and receiver parameters are reconfigured and the appropriate communication protocol parameters and modulation schemes are used.

3.1.1.1 The XG Network Architecture

Existing wireless network architectures employ heterogeneity in terms of both spectrum policies and communication technologies. Moreover, some portion of the wireless spectrum is already licensed to different purposes while some bands remain unlicensed. For the development of communication protocols, a clear description of the xG network architecture is essential. In this section, the xG network architecture is presented such that all possible scenarios are considered.

The components of the xG network architecture can be classified in two groups as the primary network and the xG network. The elements of the primary and the xG network are defined as follows:

- Primary network: An existing network infrastructure is generally referred to as the primary network, which has an exclusive right to a certain spectrum band. Examples include the common cellular and TV broadcast networks. The components of the primary network are as follows:
    - Primary user: Primary user (or licensed user) has a license to operate in a certain spectrum band. This access can only be controlled by the primary base-station and should not be affected by the operations of any other unlicensed users. Primary users do not need any modification or additional functions for coexistence with xG base-stations and xG users.
    - Primary base-station: Primary base-station (or licensed base-station) is a fixed infrastructure network component which has a spectrum license such as base-station transceiver system (BTS) in a cellular system. In principle, the primary base-station does not have any xG capability for sharing spectrum with xG users. However, the primary base-station may be requested to have both legacy and xG protocols for the primary network access of xG users, which is explained below.
- xG network: xG network (or cognitive radio network, Dynamic Spectrum Access network, secondary network, unlicensed network) does not have license to operate in a desired band. Hence, the spectrum access is allowed only in an opportunistic manner. xG networks can be deployed both as an infrastructure network and an ad-hoc network. The components of an xG network are as follows:
    - xG user: xG user (or unlicensed user – xG user: xG user (or unlicensed user, cognitive radio user, secondary user) has no spectrum license. Hence, additional functionalities are required to share the licensed spectrum band.
    - xG base-station: xG base-station (or unlicensed base-station, secondary base-station) is a fixed infrastructure component with xG capabilities. xG base-station provides single hop connection to xG users without spectrum access license. Through this connection, an xG user can access other networks.

- Spectrum broker: Spectrum broker (or scheduling server) is a central network entity that plays a role in sharing the spectrum resources among different xG networks. Spectrum broker can be connected to each network and can serve as a spectrum information manager to enable coexistence of multiple xG Networks.

The reference xG network architecture is shown in figure 3.2, which consists of different types of networks: a primary network, an infrastructure based xG network, and an ad-hoc xG network. xG Networks are operated under the mixed spectrum environment that consists of both licensed and unlicensed bands. Also, xG users can either communicate with each other in a multihop manner or access the base-station. Thus, in xG networks, there are three different access types as explained next:

- xG network access: xG users can access their own xG base-station both on licensed and unlicensed spectrum bands.
- xG ad-hoc access: xG users can communicate with other xG users through ad-hoc connection on both licensed and unlicensed spectrum bands.
- Primary network access: The xG users can also access the primary base-station through the licensed band.

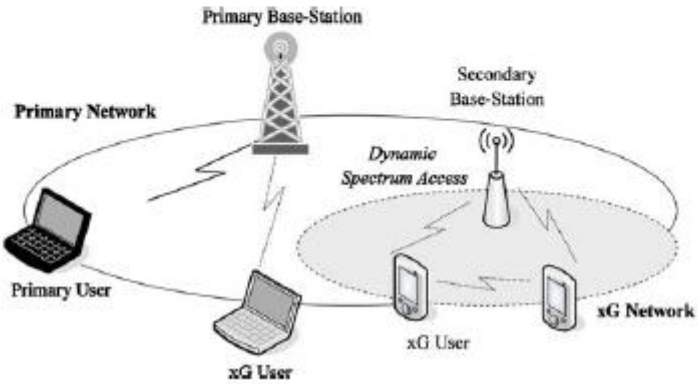

Figure 3.2 Cognitive radio network example

3.2 Cognitive Networks: Models and Design Issues

3.2.1 Interference Management

Cognitive radios have recently been studied intensively as they provide strategies to use the transmission spectrum more efficiently by enabling the cognitive secondary users (SUs) to use the transmission bands allocated to the licensed primary users (PUs) while causing no or only limited (or tolerable) interference to them. Interference is controlled by having the cognitive SUs be aware of the environment (e.g., through channel sensing) and adapt their transmission strategies accordingly.

The central challenge for the cognitive SUs is to control their interference levels. In general, interference management needs to be performed under uncertainty as channel sensing done by the SUs may result in false alarms and miss-detections. In such an interference limited scenario, cognitive SUs should also satisfy their own quality of service (QoS) requirements by transmitting at high rates and limiting the delay experienced by the data in the buffers. This, too, has to be achieved under channel uncertainty since wireless channel conditions, which vary over time randomly due to mobility and changing environment, can only be estimated imperfectly through training techniques. Note also that providing QoS guarantees is especially more challenging for SUs as they have to take into account both the changing channel conditions and varying primary user activity. These considerations are critical for the successful deployment of cognitive radio systems in practice.

Managing the interference is one of the most significant parts in cognitive radio networks since secondary user can reuse the spectrum of the primary user only under the condition that the primary services are not harmfully interrupted. There is a question how much is the harmful interference that ultimately depends on the application. There are two approaches to avoid harmful interference below:

- Overlay Approach (Interference-Free Approach) In this approach, the secondary users access the portion of the spectrum that is not used by primary users. As a result, there is virtually no interference to the primary users.

- Underlay Approach (Interference-Tolerant Approach) In this approach, the secondary users access the network by spreading their signals over a wide frequency band. The underlay approach imposes severe constraints on the transmission power of secondary users. Operating below the noise floor of primary users, the secondary users are allowed to interfere with primary users up to a certain tolerable level.

The former approach is not very practical if we take into account cognitive radio technique's inherent need to increase the spectrum utilization. So the latter allowing the second user to use the spectrum band while the primary user is operating on the spectrum is more appropriate.

Interference analysis has been studied by a number of authors (Gastpar (2007), Marthur, Haleem, Chandramouli, & Subbalakshmi (2007), Ghasemi & Sousa (2008), Digham (2008), Chen, Iellamo, Coupechoux, & Godlewski (2010), Weng, Peng, & Wang (2010), Vu, Ghassemzadeh, & Tarokh (2008)). The interference depends on the locations of the cognitive users, which are random, and on the random channel fading. Hence this interference is random.

As an example case, we assume that all nodes in figure 3.3 experience Rayleigh fading which are independent from node to node. The received signal y can be expressed as

$$y = \sqrt{\gamma_i}\, h_i\, x + \omega_i \qquad (3.2.1)$$

where x denotes the transmitted signal. h_i denotes the channel fading coefficient, γ_i is the average channel gain and ω_i is additive white Gaussian noise.

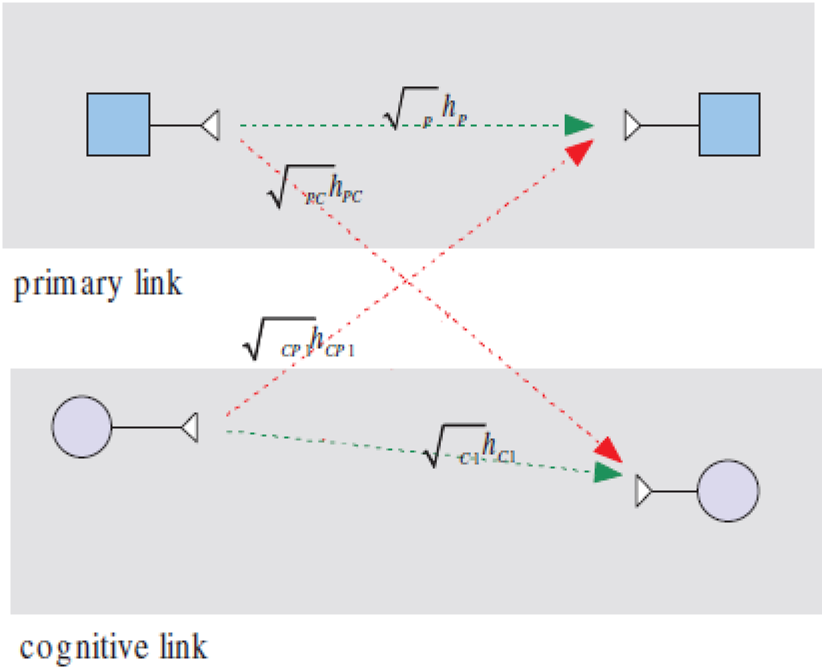

Figure 3.3 A simple cognitive scenario with one primary link and one cognitive link using one transmission antenna

Here stands “P” for the primary link, “C1” for the channel between the antenna of the cognitive transmitter and the cognitive receiver, “CP1” for the channel between the antenna of the cognitive transmitter and the primary receiver, and “PC” for the channel between the primary transmitter and the cognitive receiver.

Because of channel fading, the cognitive user may make mistake in detecting the primary user, which will cause interference to the primary user’s transmission (Huang, Zhang, Cheng, Yu, & Qiu, 2007). We use false alarm probability P_{false} and missed detection probability P_{miss} to show the performance of the detector. Let P_P be the transmission power of the primary transmitter, and P_C denote the transmission power allocated to the antenna of the cognitive transmitter respectively. For simplicity, we assume these parameters are fixed given the detection method used by the cognitive user. Moreover, we assume that the secondery user can learn the statistics of the channels (fading parameters) and the system parameters (P_{miss}, P_{false}, β_P, β_C) by scanning the environment and listening to the primary link’ s signal during a long period. Here, β_P and β_C show instantaneous received signal to interference and noise ratio (SINR) thresholds at primary and cognitive receivers respectively, over which the transmitted packet will be successfully received. Using these parameters, it can select the optimum transmission power (P_C) to maximize its own stable throughput under the following constraints:

1) the queue of the primary user remains stable when the cognitive link is active,
2) the total power allocated to the two transmission antennas should not exceed the maximum transmission power.

3.2.1.1 Ideal System

In an ideal system, the cognitive user can detect the activity of the primary user without any error. It transmits its own packets only when the slot is idle. Therefore, the cognitive link and the primary link transmit in different time slots, and will not cause any interference to each other. Under this situation, the maximum stable throughput of the link equals the average departure rate μi, and the packet queue is stable if the average arrival rate λ_i is less than μ_i.

We assume that there is a given threshold β_i at each receiver, and the transmitted packet will be successfully received if the instantaneous received signal to interference and noise ratio (SINR) is above this threshold. This threshold depends on the link's transmission mode. Therefore the outage (unsuccessful packet reception) probability on the primary link reads

$$P_{out,i} = \mathrm{Prob}[SINR_i < \beta_i] \tag{3.2.2}$$

where the subscript i can be either "P" or "C".

The power decreases exponentially with increasing distance from transmitter, so at the point of receiver it is defined as

$$P_i \,/\, d_{(T,P)}{}^{\alpha} \tag{3.2.3}$$

where $d_{(T,P)}$ represents distance between transmitter and receiver and α is path loss coefficient which is generally between 2 and 4. So, $SINR_i$ becomes,

$$SINR_i = \frac{P_i \,/\, {d_{(T,P)}}^{\alpha}}{\omega_i} \tag{3.2.4}$$

As the primary link and the cognitive link are active in different time slot, both of them can communicate using the maximum transmission power $P_{P,max}$ and $P_{C,max}$. This results in that no power constraint is imposed on the cognitive transmitter.

3.2.1.2 Real System

In real system, there may be errors in the detection process as mentioned above. So the cognitive user may transmit in slots already occupied by the primary user. The packets transmitted in these slots by both the primary user and the cognitive user will be successfully received with lower probability, which in turns reduces the actual throughput.

If the cognitive user successfully detects that the channel is occupied, the primary link can communicate without interference from the cognitive link. If the detection is unsuccessful, the cognitive link transmits in the same slot as the primary link with possibility P_{miss}. In this case, the received signals of the primary receiver contains not only signals from the primary transmitter and the noise, but also interference from the cognitive transmitter. Therefore, the instantaneous received SINR of the primary receiver is:

$$SINR_P = \frac{P_P \,/\, {d_{(T,P)}}^{\alpha}}{\omega_P + P_C \,/\, {d_{(C,P)}}^{\alpha}} \tag{3.2.5}$$

where $d_{(T,P)}$ represents distance between primary transmitter and primary receiver and $d_{(C,P)}$ represents distance between cognitive transmitter and primary receiver. So, if this $SINR_P$ value is under the outage SINR threshold β_P, packets will not be successfully delivered to the primary receiver.

$$P_{out,P} = \text{Prob}[SINR_P < \beta_P] \quad (3.2.6)$$

This means, a power constraint is imposed on the cognitive user so that the SINR of the incumbent primary receiver does not drop below its minimal SINR requirement denoted by β_P that is determined by the QoS requirement of the primary user on channel.

Generally, the CR network consists of an access point (AP) controlling different CRs (users), as illustrated in figure 3.4.

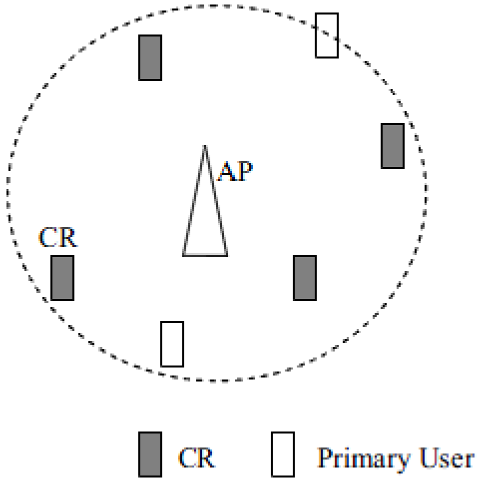

Figure 3.4 Access point with its cognitive users

Here, figure 3.4 shows a CR configuration wherein an AP controls the transmission of SUs lying within its range of coverage and also collects reports about the activities of primary users (PUs) that SUs may interfere with. The following knowledge is available at the AP:

- the set of vacant channels that are not currently utilized by PUs and are free for SUs to use (this set is the outcome of the scanning and sensing process which is not addressed here) and
- the power gains of this channel set corresponding to each of the contending users.

Channels are assumed to be independent and identically distributed (IID).

3.2.2 Spectrum Allocation among Cognitive Users

After detection of vacant channels, another important problem is spectrum allocation between secondery users (cognitive users). As hosts in a wireless network, if secondery users in a cognitive network use a single shared channel, transmission from one secondery user will interfere with other users within its propagation range. Collisions can be avoided by partitioning the given radio spectrum into a set of disjointed channels and assigning channels to transmitters appropriately. This is called the channel assignment problem or the frequency assignment problem. Here, we assume interchannel interference is small, so only cochannel interference is considered. Since radio transmission has a limited propagation range, two hosts can use the same channel provided that the two hosts are spaced sufficiently apart. There is a one-to-one correspondence between the channel assignment problem and the vertex colouring problem in graph theory. Formally, the channel assignment problem can be modelled as an appropriate colouring problem on an undirected graph representing the network topology, where vertices correspond to hosts and edges correspond to pairs of hosts that cannot use the same channel. The purpose of channel assignment algorithms is to assign channels to transmitting hosts such that cochannel interference is avoided. Besides this, either the total number of channels used or total interference for a given set of channels is minimised.

Channel allocation not only depends on spectrum availability, but it is also determined based on internal (and possibly external) policies. Hence, the design of a spectrum allocation policy to improve the performance of a node is an important research topic. Here, we study on auction-based allocation of channels in cognitive radio networks for throughput maximization of both primary and secondery users when interference constraints are also satisfied. These include two adjacent cognitive users should not use the same channel and also interference constraint on the primary receivers so that transmitted packets are successfully received.

The demand for wireless spectrum has been growing rapidly with the dramatic development of the mobile telecommunication industry in the last decades. Recently, regulatory bodies like the Federal Communications Commissions (FCC) in the United States are recognizing that traditional fixed spectrum allocation can be very inefficient, considering that bandwidth demands may vary highly along the time or space dimension (Ileri, Samardzija, & Mandayam, 2005). In order to fully utilize the scarce spectrum resources, with the development of cognitive radio technologies, dynamic spectrum allocation especially distributed spectrum allocation becomes a promising approach to increase the efficiency of spectrum usage. This new wireless networking paradigm, dynamic spectrum access, is also referred to as Next Generation wireless networks (Akyildiz, Lee, Vuran, & Mohanty, 2006).

Traditionally, network-wide spectrum assignment is carried out by a central server, namely, spectrum broker (Buddhikot, 2005). Recently, distributed spectrum allocation approaches have been studied to enable efficient spectrum sharing only based on local observations (Etkin, Parekh, & Tse, 2005). Peng, Zheng, & Zhao (2006) reduced the allocation problem to a variant of the graph coloring problem, and provided a general approximation methodology through vertex labeling. Moreover, researchers have already started to study distributed spectrum allocation via graph coloring and auction theories (Ileri, Samardzija, & Mandayam, 2005, Peng, Zheng, & Zhao, 2006, Xin, Xie, & Shen, 2005). Ileri, Samardzija, & Mandayam (2005) proposed a demand responsive pricing framework to maximize the profits of legacy spectrum operators while considering the users' response model to the operators' pricing strategy.

Nowadays, more and more researchers have already started to study dynamic spectrum allocation via bidding/asking and auction mechanisms. Xuezhi, Yutao, & Guisen (2009) focus on the study of the secondary users who purchase some channels for their own communication services. They proposed a novel distributed collusion mechanism to al-locate channels in the spectrum pool with graph coloring and bidding theory. Distinguishing to the previous mechanisms, through the proposed algorithm they also obtain the utility of primary users. Dynamic channel allocation performance of auctions with collusion and cooperation was analyzed and it was shown that through user cooperation a much better performance is obtained. Huang, Berry, & Honig (2006) proposed an auction-based mechanism to efficiently share spectrum among the users in interference-limited systems.

The above studies all concern only dynamic spectrum allocation without considering interference constraints on primary users because of secondery user activities. Following studies investigate interference management issue on cognitive networks, however, they do not consider total gain maximization during channel allocation. Digham (2008) proposed a near optimal scheme for jointly allocating channels and power levels among different users contending for a channel access in a CR network. A conservative design was considered wherein a constraint on the SINR at the primary users is imposed, by controlling power levels of cognitive users. Ghasemi & Sousa (2008) developed a statistical model of interference aggregation in spectrum-sensing cognitive radio networks by taking into account the random variations in the number, location and transmitted power of the cognitive radios. They also studied the effect of cooperative sensing on the distribution of the aggregate interference under i.i.d. fading channels and highlighted the tradeoff between local signal processing (i.e., individual sensitivity) and cooperation. In the scheme investigated by Vu, Ghassemzadeh, & Tarokh (2008), they studied a network consisting of a primary user and multiple cognitive users. They investigated the case the primary user sends a beacon prior to each transmission to silence the cognitive users and claim the spectrum and they formulate this interference power as a function of the beacon threshold, the number of cognitive users, the primary and cognitive transmit powers, the distance between the primary transmitter and receiver, and the receiver protected radius. On the other hand, sending a beacon signal and silencing secondery users can be thought of being contrary to the idea of cognitive networks that primary users do nothing and are not affected by secondery users.

A collaborative scheme for secondary nodes was developed to compute the approximate maximum interference-free transmit power (MIFTP) in the presence of a primary transmitter with unknown location and transmit power (Mark & Nasif, 2009). Ma & Tsang (2010) considered a multihop multi-channel CR network and they present a cross-layer optimization framework by jointly designing the spectrum sharing and routing with the SINR constraints. Distinguished from the previous studies, they adopt a more realistic SINR model to capture the conflict relationships among the links, rather than using the Protocol Model, with the objective to maximize the minimum end-to-end flow throughput. However, they did not consider again utilities of primary and secondery users during spectrum sharing.

Although the existing distributed spectrum allocation schemes have achieved some success on enhancing the spectrum efficiency through coloring algorithm design and market mechanisms, there has been very few people adopt both graph coloring and bidding theory at the same time to solve the spectrum allocation problem in cognitive radio networks and, to the best of our knowledge, there is no work involving both coloring, auction and bidding theory and interference management on primary users without controlling power levels. Existing allocation schemes generally consider either power and channel allocation without considering total gain or only total gain of primary and secondery users without taking SINR threshold into consideration.

In this thesis work, we focus on the study of both of the gain of primary users who own licensed spectrum and secondary users who purchase some channels for their own communication services, in terms of both total gain and interference management. Here, secondary users refer to cognitive users who are not owner of the spectrum and bidding for some channels from the primary users. Through greedy algorithm with interference graph, we can easily gain the spectrum allocation list and the total utility of the cognitive users, but the primary users' utility is difficult to obtain if more than one spectrum holder exist. Furthermore, if detection of the vacant channels by secondery users cannot be carried out correctly, packets may not be delivered to the primary receiver because of total interference effect on it. Our work is unique in the sense of both auction-based channel allocation, total gain maximization and interference management to satisy related SINR threshold (quality of service (QoS) requirement of primary receiver), without controlling secondery user power levels. Our novel algorithm allocates channels in the spectrum pool, through which it tries to maximize utilities of both the primary users and cognitive users and also manage interference so that packets from primary transmitters are successfully delivered to related primary receivers.

To summarize, in chapter 3 firstly we explain the need for cognitive radio networks and give background information. Next, we investigate model and issues in design of cognitive radio networks which include channel allocation and interference management problems. Then, we continue with related work and open areas in literature and we conclude the chapter by giving some preliminary information about our work.

CHAPTER FOUR
AUTION-BASED THROUGHPUT MAXIMIZATION IN COGNITIVE RADIO NETWORKS UNDER INTERFERENCE CONSTRAINT

4.1 System Model and Utility Functions

A cognitive radio network can consist of many primary (spectrum holders) and secondery users simultaneously. In this thesis, we consider a wireless system with a few primary users and multiple secondary users, these users operate simultaneously. Our model considers the case that primary users are fixed and their positions are known a priori and they consist of one primary transmitter and one primary receiver. Secondery users are randomly distributed according to the model parameters and fixed, and get the list of current available channels according to the working states, position distributions, and power covering ranges of primary users, so it is much more suitable for the actual demand of cognitive radio systems.

We have the following assumptions,

- Primary system and secondary cell apply the same multicarrier modulation scheme (OFDM or FBMC)
- Error occurrs during the detection process of vacant channels, so the cognitive users transmit in slots already occupied by the primary user.

In our system model, we assume all users are selfish and rational, that is, their objectives are to maximize their own utility, and not to cause damage to other users. Generally speaking, in order to have the rewards of achieving certain communication goals, the secondary users want to utilize more spectrum resources.

In figure 4.1, an example primary network together with secondery network consisting of an access point and its cognitive users around is given.

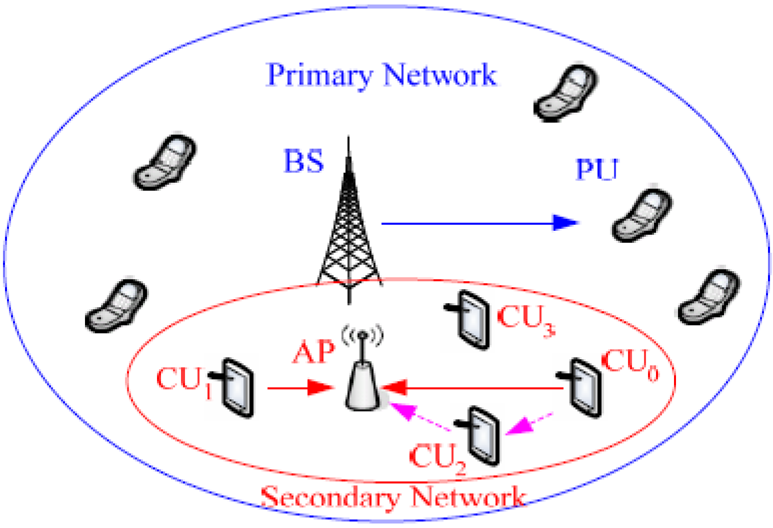

Figure 4.1 System model with coexistence of primary and cognitive radio network

We consider the collection of the available spectrums from all primary users as a spectrum pool, which totally consists of N non-overlapping channels where N < K. Assume there are J primary users and K secondary users, indicated by the set $\{p_1, p_2, \ldots, p_J\}$ and $\{s_1, s_2, \ldots, s_K\}$ respectively. So,

$$N = \sum_{i=1}^{J} n_i \tag{4.1.1}$$

The channels belonging to primary user p_i can be represented by a vector $A_i=\{a_i^j\}_{j\in\{1,2,\ldots,n_i\}}$, where a_i^j represents the channel index in the spectrum pool and n_i is the total number of channels belonging to user p_i. Moreover, denote the payoff of primary user p_i when leases its jth channel as b_i^j, with i = {1, 2, . . . , J}, j = {1, 2, ..., n_i}.

A mathematic description of the model has been given by graph theory, and the channel allocation model has been abstracted as a coloring problem. Then, it can be expressed as an undirected graph G = (V, E), where the vertices represent the secondary users. V = {s_i, i = 1, 2, . . . , K} represents the set of all secondery users which want to obtain channels. |V| =K represents the total number, and s_i represents each one of secondery users. E is the edge set. So, e_{ij} shows an edge between vertices (secondery users) s_i and s_j which means distance between them is shorter than a defined value and those nodes interfere. So, they cannot use the same channel at the same time. We also refer to the graph G as the interference graph. We use channel and color interchangeably. An example interference graph with few primary users and multiple secondery users is given in figure 4.2, where nodes interfering with each other are shown connected by blue lines.

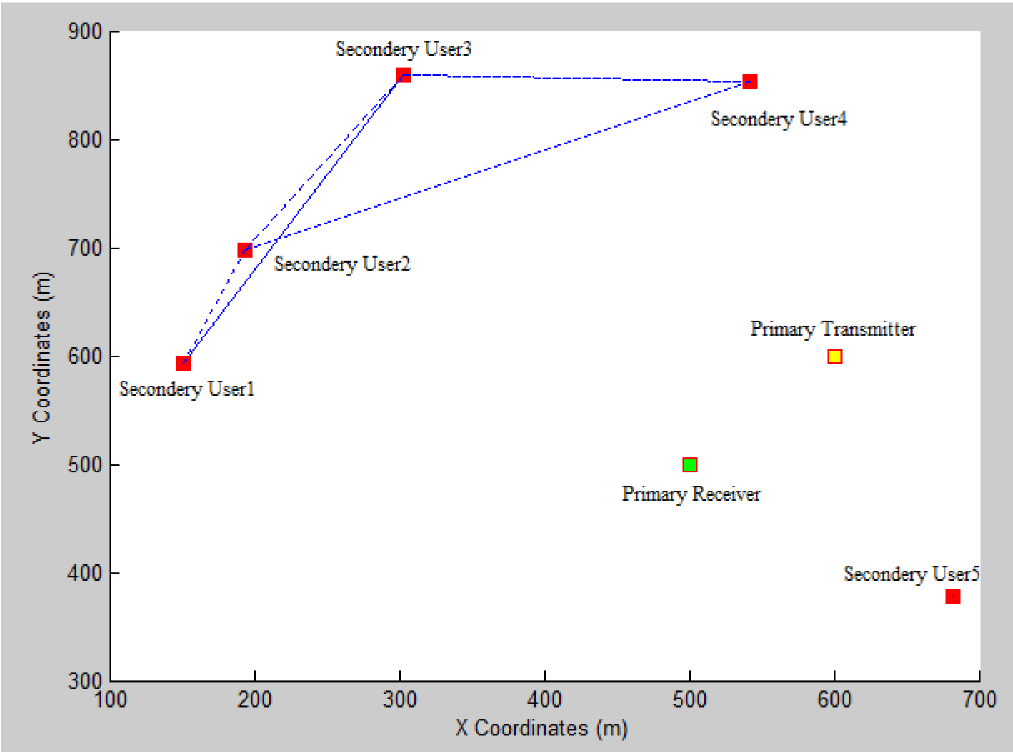

Figure 4.2 Sample interference graph with few primary users and multiple secondery users

We define the revenue vector of user s_i as $R_i=\{r_i^j\}_{j\in\{1,2,\ldots,N\}}$, where the jth element is the revenue if this user successfully leases the jth channel in the spectrum pool. Similarly, denote the bid vector of user s_i as $B_i=\{b_i^j\}_{j\in\{1,2,\ldots,N\}}$, where b_i^j means this user's bid of the jth channel in the spectrum pool.

We assume user s_i has the same utility for all the channels in the spectrum pool, $s_i \in S$, and it needs at most n_i^s channels, so, the utility function of this user can be modelled as follows.

$$U_{s_i}(B_A, \alpha_i^A) = \sum_{j=1}^{N}(r_i^j - b_i^j) * \alpha_i^j\ , \quad \sum_{j=1}^{N}\alpha_i^j \leq n_i^s \tag{4.1.2}$$

where $B_A = \{b_i^j\}_{j\in\{1,2,\ldots,N\}}$, $\alpha_i^A = \{\alpha_i^j\}_{j\in\{1,2,\ldots,N\}}$. Here, $\alpha_i^j \in \{0,1\}$ and shows if secondary user s_i successfully leases the jth channel in the spectrum pool or not. When the secondary users' performance achieve the best, the primary user p_j' s payoff can be written as

$$U_{P_j} = \sum_{k=1}^{n_j}(b_i^k)^*\ ,\ j\in\{1, 2, \ldots, J\} \tag{4.1.3}$$

where $(b_i^k)^*$ means the optimal bidding of secondary user s_i.

The objective of the channel allocation is to maximize the spectrum utilization, including both primary users and secondary users while satisfying necessary interference condition on the primary receivers. This problem can be formally represented as the following non-linear programming problem.

$$\text{maximize } \{ \sum_{i=1}^{K} U_{s_i}(B_A, \alpha_i^A) + \sum_{j=1}^{J} U_{P_j}(B) \}$$

$$\text{Subject to } \{ \sum_{i=1}^{K} \frac{P_{C_i}}{(d_{C_i,P_R})^{\alpha}} \leq P_{U_{th}} \} \tag{4.1.4}$$

where B is the set of bids for all the secondary users and $P_{U_{th}}$ is tolerable interference threshold at the primary receiver P_R.

$$\text{SINR} = \frac{\frac{P_{P_T}}{(d_{P_T,P_R})^{\alpha}}}{\text{No} + \sum_{i=1}^{K}\frac{P_{C_i}}{(d_{C_i,P_R})^{\alpha}}} \geq \beta_{P_R} \tag{4.1.5}$$

where β_{P_R} is tolerable signal to interference plus noise ratio (SINR) threshold at P_R (SINR value when total interference is $P_{U_{th}}$) so that corresponding primary receiver can receive primary transmitter packets successfully and No is additive white Gaussian noise for the receiver, with zero mean. This is equivalent to guaranteeing a minimum rate for primary receiver. Because there are more than one secondery transmitter, total interference is sum of all interferences at the point of receiver. In order for the SINR value to be greater than β_{P_R}, total interference should satisfy,

$$\sum_{i=1}^{K} \frac{P_{C_i}}{\left(d_{C_i,P_R}\right)^{\alpha}} \leq P_{U_{th}} \tag{4.1.6}$$

By monitoring the activity of the primary user, the cognitive link (base stations in our model) can communicate when it senses an idle slot. However, as mentioned in chapter (3.2.1), due to impairments on the wireless fading channel, the detection process may incur in errors in real system, and interference thereby will be generated from the cognitive link to the primary link. After assumption of false detection, our channel allocation algorithm assigns channels to cognitive users so that no adjacent nodes will be assigned the same channel and also total amount of interference on the primary receiver stays under the outage threshold, as stated in equation (4.1.6), so that the QoS requirement of the primary user on channel is satisfied.

Our proposed algorithm does channel allocation with the goal of maximizing total gain throughput and also satisfying interference constraint on primary receivers. Moreover, we modify the algorithm so that it does allocation without and with auction. Auction is carried out such that from two nodes competing for a channel the one which has higher revenue has higher priority and gets the channel.

Interference constraint is supplied by dynamically checking interference level before assigning a channel to all users in a maximum independent set. If it is not satisfied, first, interference is decreased by discarding assigning channel to the user which has lowest revenue and interference ratio, that means node satisfying condition

$$\text{Min } \left\{ U_{s_i} / \frac{P_{C_i}}{\left(d_{C_i,P_R}\right)^{\alpha}},\ i=1,\ldots,K \right\} \tag{4.1.7}$$

If this does not suffice, it goes on by discarding node(s) with lowest revenue from nodes who already obtained maximum number of channels so far. Then, if still insufficient, it excludes node(s) with lowest revenue from nodes who already obtained one channel less then maximum number of channels obtained so far and goes on like this, till interference condition is satisfied.

We assume node locations are static. We also assume the set of available channels at each secondary user is static. This corresponds to the applications with a slow varying spectrum environment (e.g., TV broadcast bands). We assume that there exists a centralized server in the CR network. Each secondary user reports its location and the set of available channels to the spectrum server. The spectrum management and flow routing, therefore, is simple and coordinated. During the channel allocation process, secondery users need to interact with control center (centralized server), and the center has to feed back the allocation results to users.

Here, we assume that links using different channels do not interfere with each other. Interference only occurs among the links sharing the same channel.

Our model consists of one primary receiver located in the center of a square area with side 1km and primary transmitter 141,4m away from its primary receiver, so J=2. Their positions are fixed and known, (500m,500m) and (600m,600m) respectively. There are K secondery users (we vary K in the range [10, 30] for different test cases) random uniformly distributed in a 1000m × 1000m area (figure 4.3), similar to the work of Cuiran & Chengshu (2008). Each secondary user represents an access point (base station), so each of them may need more than one channel (cognitive users around it) which are all random uniformly distributed around primary users in the square area and they are assumed to be fixed. The unused spectrum from primary users form a spectrum pool with 5 orthogonal frequency channels (N=5), for our test cases. The revenues of cognitive users and primary users are scores which are assumed to be i.i.d. random variables uniformly distributed in the ranges [10, 30] and [5, 7] respectively. Each secondery users' channel need varies randomly in the range [1, 3].

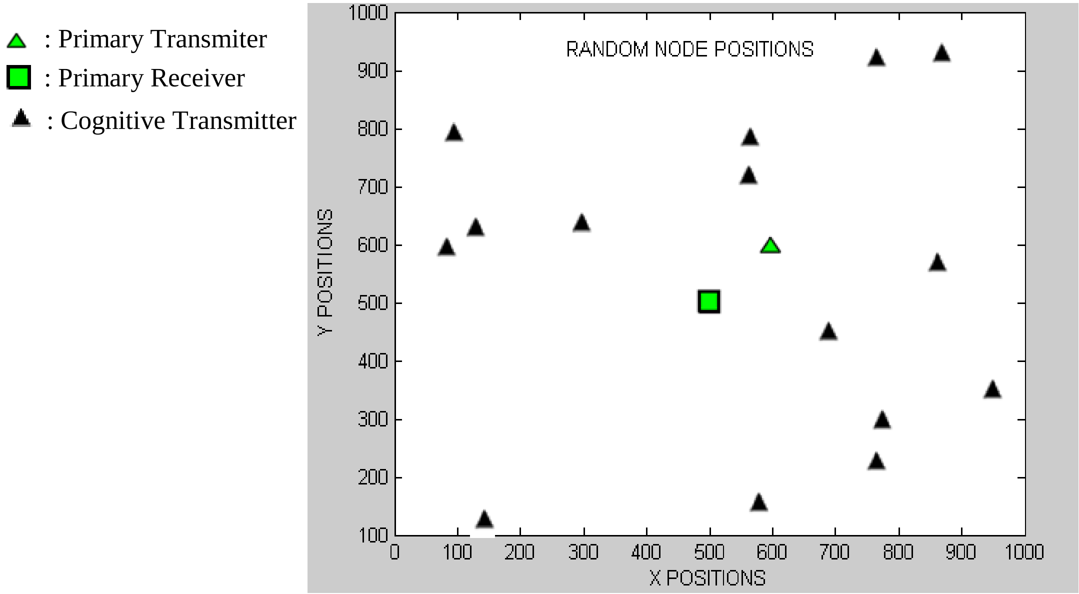

Figure 4.3 Our simulation model

In our model, we assume primary transmitter power is 1W (30 dBm). Power level at receiver d(m) away from its transmitter decreases exponentially with distance, as we already mentioned. So, the power at the primary receiver is,

$$\frac{P_{Transmitter}}{(d_{P_T,P_R})^{\alpha}} = \frac{1000}{\sqrt{((600-500)^2+(600-500)^2)}^{4}} = 2,5 \times 10^{-6} \text{ mW (-56 dBm)}$$

Total interference on the primary receiver depends on the secondery user positions and their power levels. We assume $P_{C_i} = P_{Cognitive}$ for all secondery users.

$$\sum \text{Interference} = \sum_{i=1}^{K} \frac{P_{C_i}}{(d_{C_i,P_R})^{\alpha}} = P_{Cognitive} \times \sum_{i=1}^{K} \frac{1}{(d_{C_i,P_R})^{\alpha}} \qquad (4.1.8)$$

When secondery transmitter coincides with the primary receiver, d_{C_i,P_R} gets its minimum value which is 0 and in this case interference goes to infinity.

Since secondery users are random uniformly distributed, the total interference power at the point of the primary receiver is actually random as well. We can calculate mean value of the total interference power. For this purpose, we can use the model given in figure 4.4.

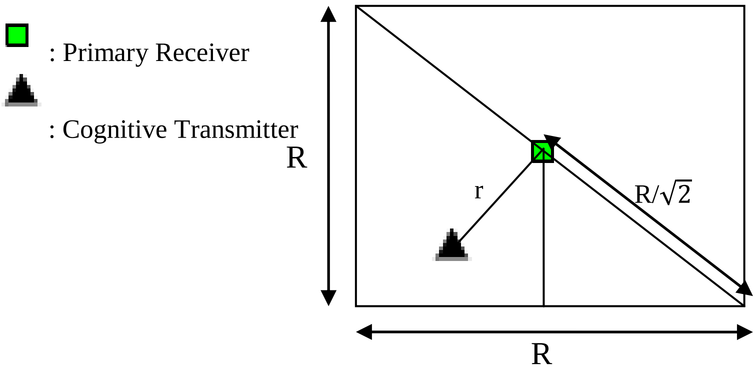

Figure 4.4 Model for calculation of interference mean value

For uniformly distributed cognitive users, r has cumulative distribution function (CDF)

$$F(r) = r^2 / R^2 \tag{4.1.9}$$

So, probability density function of r is

$$f(r) = 2\,r / R^2\,, \qquad 0 \le r \le R/\sqrt{2} \tag{4.1.10}$$

Therefore, the mean value of the total interference on the primary receiver is

$$E[I] = \sum_{i=0}^{K} \int_0^{R/\sqrt{2}} \frac{P_{C_i}}{r^{\alpha}}\, f(r)dr \tag{4.1.11}$$

where P_{C_i} = P because we assume all cognitive users transmit with the same power. Therefore,

$$E[I] = 2KP \int_0^{R/\sqrt{2}} r^{1-\alpha} dr \tag{4.1.12}$$

If we assume K is 15, P is 100 mW and α is 4, then

$$E[I] = 2KP \times 1/2 \times ((\frac{1}{0})^2 - (\frac{1}{R/\sqrt{2}})^2) = \infty \tag{4.1.13}$$

As it is seen, mean value of the total interference is infinity since cognitive users can coincide with the primary receiver.

Therefore, to limit the interference from each cognitive transmitter, it is sometimes assumed that the cognitive transmitters must be at least a distance $\in$ from the primary receiver, for some $\in > 0$. This practical constraint basically disallows the interfering transmitter to be at the same point as the interfered receiver. This region is called as primary-exclusive region (PER). In our simulations, we do not make such an assumption. However, we can recalculate mean value of the total interference according to this assumption. If we assume that, cognitive users cannot be in a square area with one side of $\in$ m around the primary receiver, the lower boundary of the integral in (4.1.11) becomes $\in/2$ instead of 0.

$$\mathrm{E[I]} = \sum_{i=0}^{K} \int_{\in/2}^{R/\sqrt{2}} \frac{P_{C_i}}{r^{\alpha}} \, f(r) \, dr \qquad (4.1.14)$$

For $\alpha = 4$,

$$\mathrm{E[I]} = 2KP \times 1/2 \times \left(\left(\frac{1}{\in/2}\right)^2 - \left(\frac{1}{R/\sqrt{2}}\right)^2 \right) \qquad (4.1.15)$$

In our model R is 1000m, P is 100 mW (in one of simulation scenarios) and if we assume $\in$ to be 25 m,

$$\mathrm{E[I]} = 15 \times 100 \times \left(\left(\frac{1}{25/2}\right)^2 - \left(\frac{1}{1000/\sqrt{2}}\right)^2 \right) = 9{,}597 \cong 9{,}82 \text{ dBm}$$

d_{C_i,P_R} is maximum when secondery transmitters lie exactly at the corners of the area, meaning resulting interference is minimum (since all transmitters coincide, this is practically impossible case, but gives lower bound of total interference). In this case, distance between secondery transmitters and primary receiver is

$$\sqrt{R/2^2 + R/2^2} = \frac{R}{\sqrt{2}}$$

$$0 < d_{C_i,P_R} \leq \frac{R}{\sqrt{2}} \tag{4.1.16}$$

Therefore, total interference value when all secondery transmitters lie at $\frac{R}{\sqrt{2}}$ distance is

$$\text{Interference}_{\text{min}} = P_{\text{Cognitive}} \times L \times \frac{1}{(\frac{R}{\sqrt{2}})^{\alpha}} \tag{4.1.17}$$

where L shows at most how many nodes use simultaneously the same channel.

As an example case, let us consider, P_{Primary} = 30 dBm, $P_{\text{Cognitive}}$ = 20 dBm, K = 15, R = 1000m, No = -100 dBm and $\alpha = 4$ (values we used in one of our experiments). According to simulation results of our model with 15 secondery users, mostly 9 nodes transmit simultaneously using the same channel (L = 9), so interference becomes

$$\text{Interference}_{\text{Total}} = 100 \times 9 \times \frac{1}{(\frac{1000}{\sqrt{2}})^4} = 3{,}6 \times 10^{-9}\text{mW} \cong -86{,}44 \text{ dBm.}$$

and the corresponding SINR value gets,

$$\text{SINR} = \frac{\frac{P_T}{(d_{P_T,P_R})^{\alpha}}}{\text{No} + \sum_{i=1}^{K} \frac{P_{C_i}}{(d_{C_i,P_R})^{\alpha}}} = \frac{2{,}5\times10^{-6}}{10^{-10} + 3{,}6\times10^{-9}} \cong 675{,}67 \cong 28{,}3 \text{ dB}$$

(This value corresponds to a QAM-64 Coding rate=3/4 Wimax system SINR need.)

As secondery users lie nearer to the primary receiver, interference increases and finally when they reach it, it goes to infinity (upper bound of total interference when no primary-exclusive region around primary receiver is used) which means certain outage of primary receiver (as we already calculated in equation (4.1.13), mean value of total inteference is infinity in case without PER).

4.2 Proposed Spectrum Allocation Mechanism for Cognitive Radio Networks Under Interference Constraint

Flow diagram of proposed algorithm for auction based spectrum allocation in cognitive radio networks under interference constraint is given in figure 4.5 (a). We call it as ABSA-UNIC, consisting of initials of the name of the algorithm. The version without auction is given in figure 4.5 (b) which we call as NASA-UNIC (no auction).

Both ABSA-UNIC and NASA-UNIC first create an interference graph corresponding to the cognitive network and copy of it. Then, they calculate maximum independent set of the copy graph using linear programming. After that, they find least used channel and try to assign it to the members of MIS. When doing this, it checks whether total interference exceeds interference threshold (corresponding SINR threshold of primary receiver). When threshold is exceeded, it tries to satisfy threshold by decreasing interference level by several ways. On ABSA-UNIC, first, interference is decreased by discarding assigning channel to the user which has lowest revenue and interference ratio (given in equation (4.1.7)). If this does not suffice, it goes on by discarding node(s) with lowest revenue from nodes who already obtained maximum number of channels so far. Then, if still insufficient, it excludes node(s) with lowest revenue from nodes who already obtained one channel less then maximum number of channels obtained so far and goes on like this, till interference condition is satisfied. NASA-UNIC decreases interference by discarding channel to the nodes creating highest interference till threshold is satisfied. Nodes which did not get any channel so far have more priority than nodes obtained some channels.

After assigning channel to the members, total revenue and primary users' revenues are updated accordingly. Nodes obtained all channels they need are removed from the copy and main interference graph. If number of nodes in the copy graph is greater than 1, channel assignment procedure is repeated until number of nodes decreases to 1. After that, new copy graph is generated, maximum independent

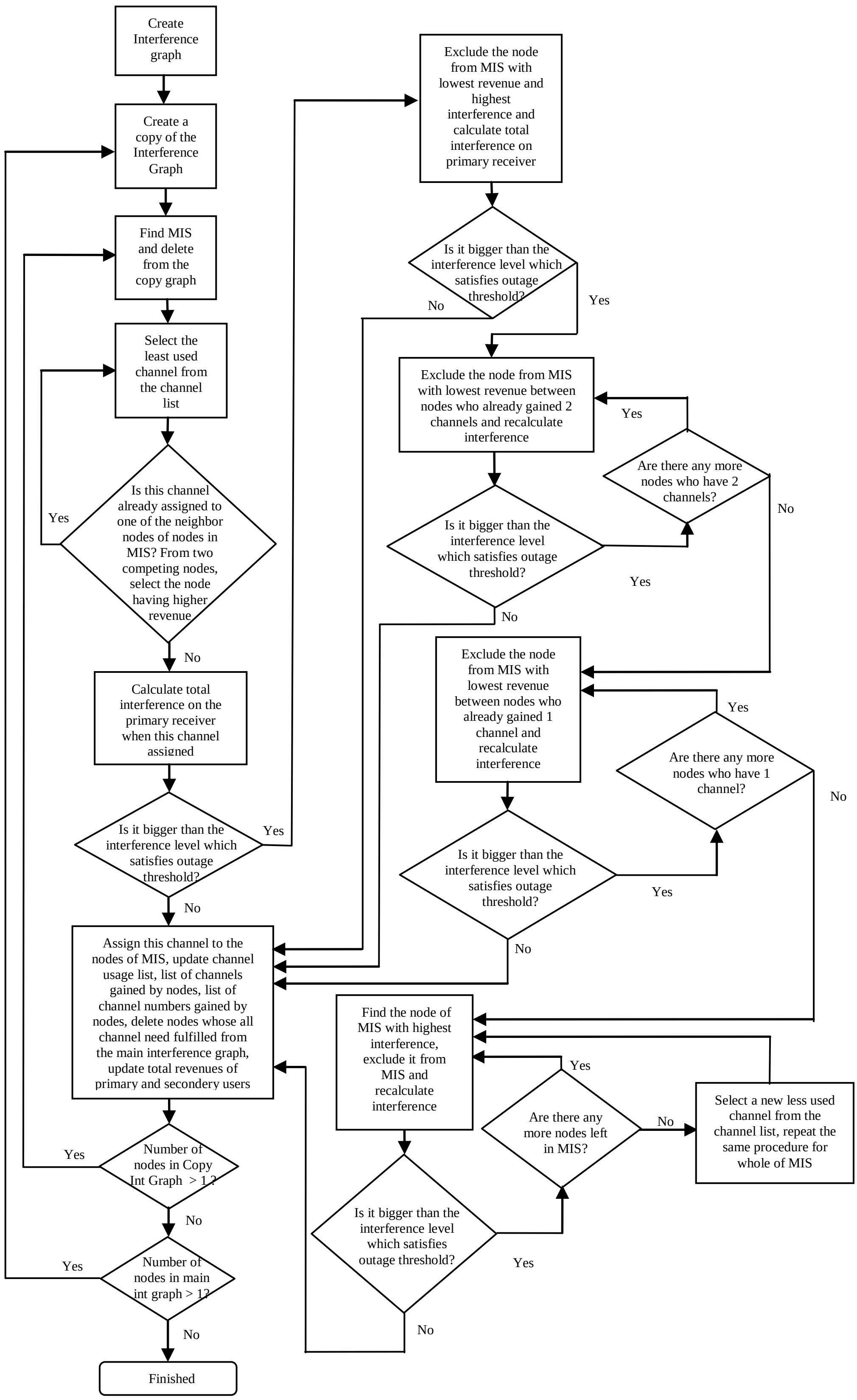

Figure 4.5 (a) Flow diagram of proposed auction-based spectrum allocation algorithm in cognitive radio networks under interference constraint (ABSA-UNIC)

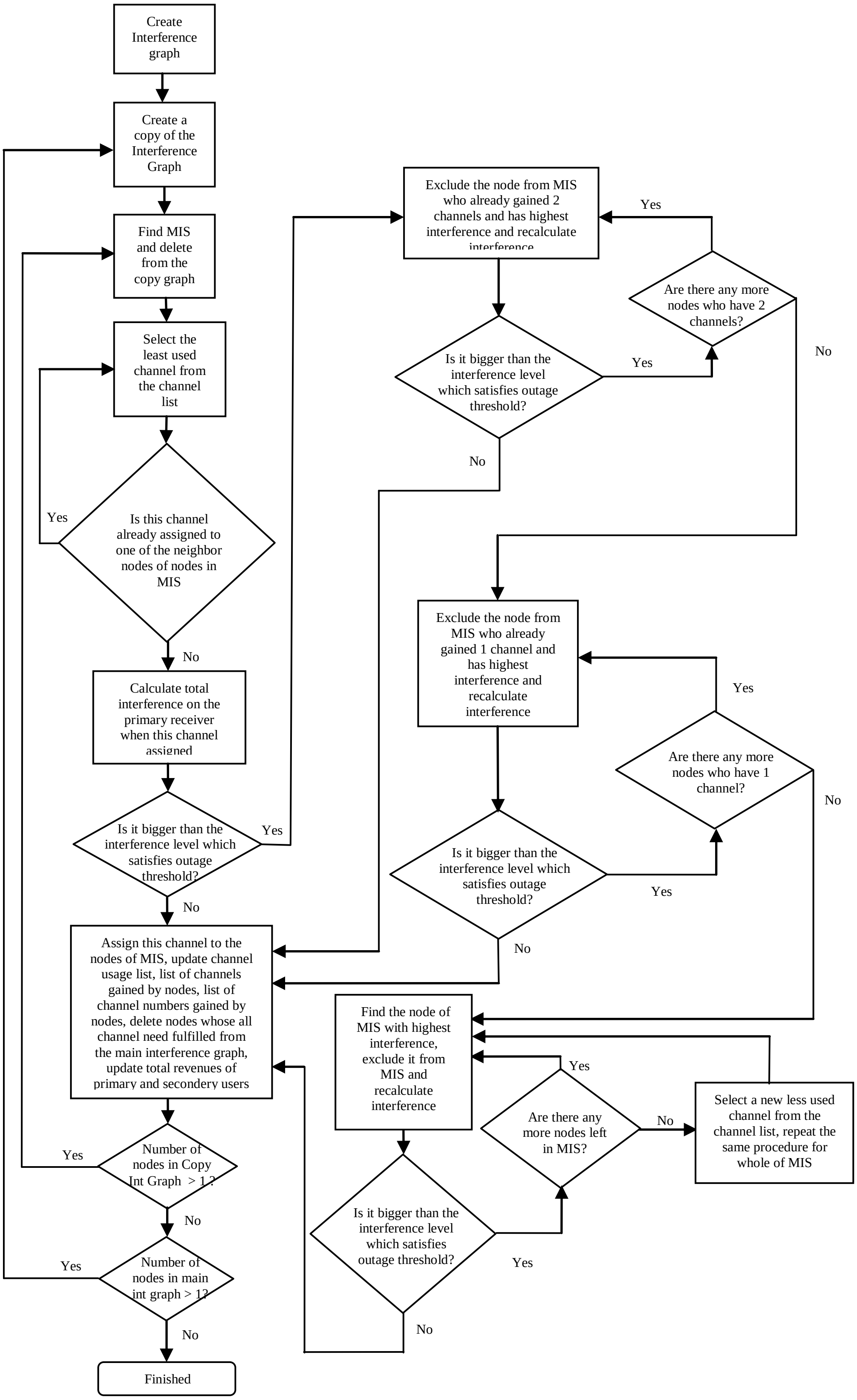

Figure 4.5 (b) Flow diagram of proposed spectrum allocation algorithm in cognitive radio networks under interference constraint (NASA-UNIC)

set is found and assignment procedure is repeated until number of nodes in the main graph decreases to 1. As a result, total sytem throughput gain and total gain of primary users is obtained and channel distribution metric is calculated.

To compare results of our algorithm, we use a greedy algorithm which produces good coloring results. However, it does not take into account interference management (SINR threshold). On greedy allocation algorithm, first, interference graph is created and nodes are sorted based on their degrees. Least used channel is assigned to the lowest degree node. From two nodes with the same degree, the node with higher revenue has higher priority to get channel. Nodes not creating interference with this node (not being neighbor of that node) are assigned the same channel as well. Moreover, each time when channel to be assigned, neighbor nodes are checked whether already having same channel or not. If yes, that channel is not assigned. Nodes obtaining all channels they need are deleted from the interference graph. After that, nodes are sorted and channels are assigned the same way until all channels are assigned or no more channel can be assigned. At every channel assignment, primary and secondery payoffs are added and finally total system throughput and throughput of primary users are calculated separately. Figure 4.6 shows flow diagram of greedy allocation algorithm.

Since interference constraint is not considered in greedy algorithm, depending on SINR threshold value, primary receiver goes under outage in a portion of all iteration runs. Therefore, in order to compare results of proposed algorithm with results of greedy algorithm, we should take into account outage probability during the calculation of greedy throughputs. For this purpose, we propose the following net throughput definition.

$$\mathrm{NetThroughput}\ (\mathrm{SINR_{Thr}}) = \mathrm{TotalThroughput} \times (1 - \mathrm{Prob_{Out}}(\mathrm{SINR_{Thr}})) \qquad (4.1.18)$$

Here, $\mathrm{Prob_{Out}}$ represents ratio of number of outages to number of iterations of greedy algoritm. Since proposed algorithms (ABSA-UNIC and NASA-UNIC) consider interference constraint, we should compare results of proposed algorithms with results of greedy algorithm taking this into consideration.

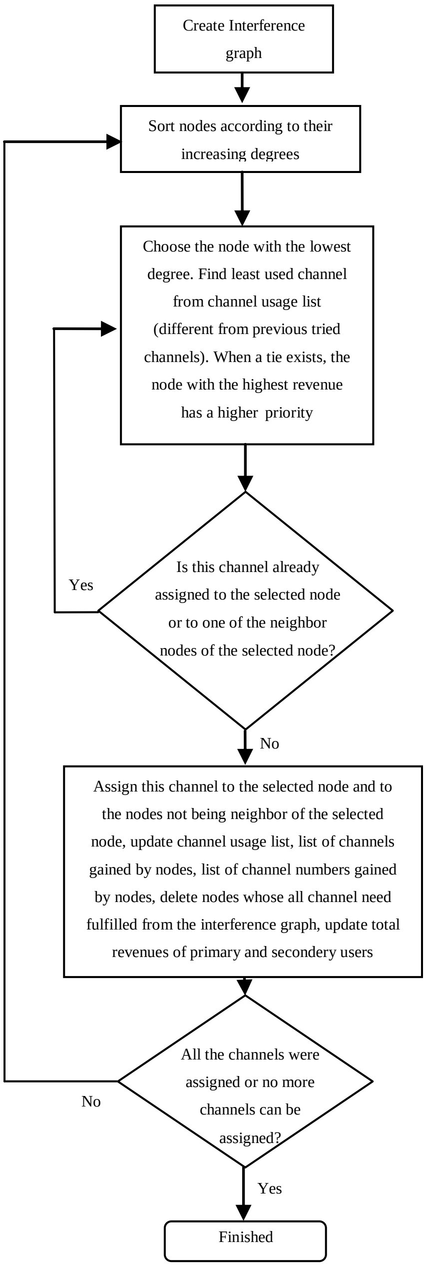

Figure 4.6 Greedy spectrum allocation algorithm

In order to show channel distribution fairness of each experiment, we propose the following metric:

$$\text{ChDistFairness} = \sum\{(\text{Ch}_{\text{Need}} - \text{Ch}_{\text{Obt}}) / (\text{Ch}_{\text{Need}} \times \text{K})\} \tag{4.1.19}$$

where K= (Number of Cognitive Users) and ChDistFairness, Ch_{Need} and Ch_{Obt} show channel distribution fairness, series of number of channels needed and series of number of channels obtained.

Similar to definition given in equation (4.1.18), we have to take into account interference effect on calculation of channel distribution fairness of greedy algorithm as well. So, we propose the following equation for channel distribution fairness as well.

$$\mathrm{ChDistFairness}_{Gre} = \sum\{(Ch_{Need} - Ch_{Obt}) / (Ch_{Need} \times K)\} \times (1 - Prob_{Out}(SINR_{Thr})) + 1/K \times Prob_{Out}(SINR_{Thr}) \qquad (4.1.20)$$

Here, $\mathrm{ChDistFairness}_{Gre}$ shows channel distribution fairness of greedy algorithm, Clearly, since $Prob_{Out} = 0$ for proposed algorithms NASA-UNIC and ABSA-UNIC, the first term remains. If all nodes get all channels they need and $Prob_{Out} = 0$, metric becomes 0. If none of the nodes gets channel and $Prob_{Out} = 0$, metric becomes 1/K which is upper limit of the metric for our proposed algorithms. For $Prob_{Out} > 0$, the second term increases directly proportionally to it which is true for greedy algorithm.

4.3 Simulation Parameters and Simulation Results

Table 4.1 System model parameters

Node Index	Position	
Cognitive Users: 1-K	Random in [0-1000,0-1000]	
Primary Receiver	(500,500)	
Primary Transmitter	(600,600)	
Average AWGN Power	-100	dBm
path Loss coefficient	4	

We examined total throughput by changing SINR threshold and also coverage area of cognitive transmitters, which means changing transmitting power. Simulations were carried out using matlab 7.7.0 (R2008b).

Table 4.2 Simulation parameters

Primary User Transmit Power	Secondery User Transmit Power	SINR Threshold β (Interference Susceptibility)
30 dBm	20 dBm	-30 dB
	23 dBm	-3.01 dB
	27 dBm	0 dB
		6,99 dB
		10 dB
		13,01 dB
		14,77 dB
		16,99 dB
		18.75 dB
		20 dB
		23,01 dB
		26,99 dB
		30 dB

4.3.1 Test Cases

For every test case, we use an interference graph like in figure 4.7 to abstract the cognitive network where the vertices represent the secondary users, and edges represent interferences so that no channels can be assigned simultaneously to any adjacent nodes. Moreover, throughput gain calculated in test cases shows total score calculated at the end of iterations, that means, throughput has unit of score.

In all cases, primary and secondery users lie in an open area of R (m) × R (m), as shown in figure 4.3. *(For comparison of our simulation parameters to values in practive, in 802.22, to achieve a coverage area of 33 km, the transmission power level considered is around 4 watts. Here, we consider lower transmit power levels resulting in smaller coverage areas).*

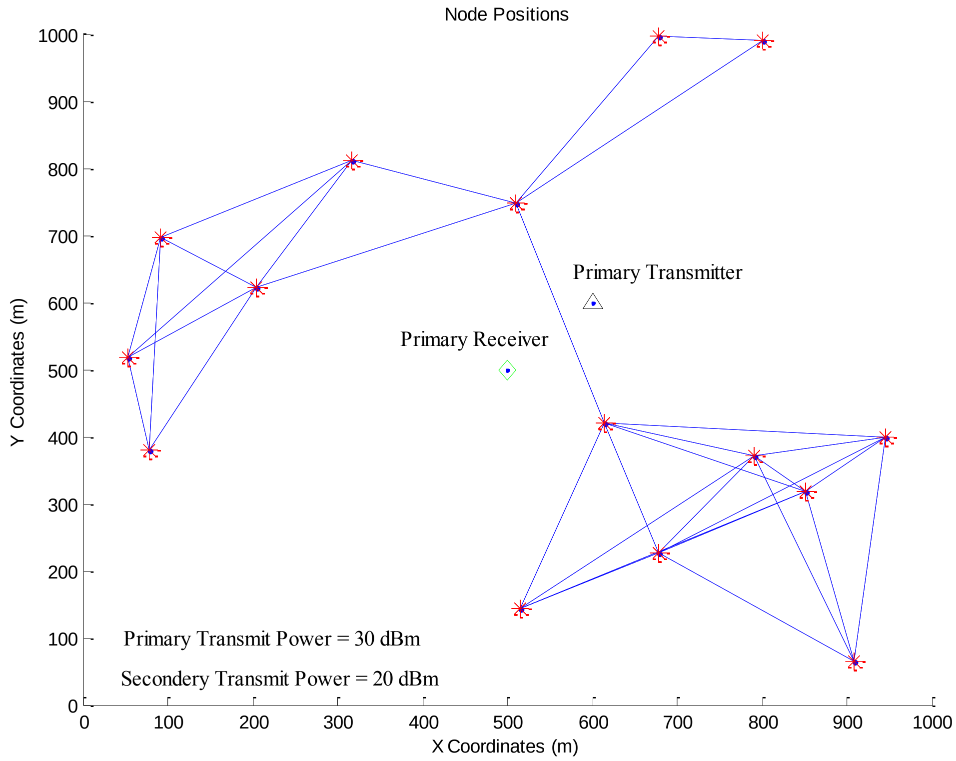

Figure 4.7 Interference graph with 15 secondery users and 2 primary users. Triangle and diamond represent primary transmitter and receiver respectively and stars represent secondery users. Edges represent interferences between secondery users.

1. Effect of SINR threshold on total gain of primary + secondery users, no auction (algorithm NASA-UNIC): Primary transmitter is a base station with transmit power of 30dBm (1000mW) and each secondery user is a base station station with transmit power of 20dBm (100mW). Secondery user has coverage of around 200m, which means that any two SUs interfere with each other when distance between them is less than 400m (shown in test case no 1 of table 4.3). We investigate total gain throughputs for every value of SINR threshold (outage threshold) from -3.01dB to 30db. We run 1000 simulations for each value of SINR threshold and get mean of total gain throughputs of both primary and secondery users. Furthermore, we also check throughput limit of greedy algorithm which indicates results without considering interference effect and results obtained applying equation (4.1.18) in order to see interference effect on greedy results. All results are given in figure 4.8. We give utility of primary user only (figure 4.9) and channel distribution versus SINR threshold value (figure 4.10) as well.

Table 4.3 Test Cases

Test Case No	Transmitter Type	Power	Unit	Coverage Area	Number of Primary Users (J)	Number of Secondery Users (K)	Auction Based Allocation
1	Primary Transmitter	30	dBm	373m (0.373R)	2	15	NO
	Cognitive Transmitter	20	dBm	200m (0.2R)			
2	Primary Transmitter	30	dBm	373m (0.373R)	2	15	NO
	Cognitive Transmitter	23	dBm	250m (0.25R)			
3	Primary Transmitter	30	dBm	373m (0.373R)	2	10-30	NO
	Cognitive Transmitter	20	dBm	200m (0.2R)			
4	Primary Transmitter	30	dBm	373m (0.373R)	2	15	YES
	Cognitive Transmitter	20	dBm	200m (0.2R)			
5	Primary Transmitter	30	dBm	373m (0.373R)	2	15	YES
	Cognitive Transmitter	23	dBm	250m (0.25R)			
6	Primary Transmitter	30	dBm	373m (0.373R)	2	10-30	YES
	Cognitive Transmitter	20	dBm	200m (0.2R)			
7	Primary Transmitter	30	dBm	373m (0.373R)	2	15	YES
	Cognitive Transmitter	20	dBm	200m (0.2R)			
	Cognitive Transmitter	23	dBm	250m (0.25R)			
	Cognitive Transmitter	27	dBm	300m (0.3R)			

2. Effect of secondery transmitter power increase (coverage area) on total gain of primary + secondery users versus SINR threshold, no auction (algorithm NASA-UNIC): Primary transmitter is a base station which has transmit power of 30dBm (1000mW) and each secondery user is a base station station with transmit power of 23dBm (200mW). Secondery user has coverage of around 250m, which means that any two SUs interfere with each other when distance between them is less than 500m (shown in test case no 2 of table 4.3). Like test case 1, we investigate total throughputs for every value of SINR threshold (outage threshold) from -3.01dB to 30dB. Again, we run 1000 simulations for each value of SINR threshold and get mean total gain throughputs of both primary and secondery users. We give these

results with greedy results applied interference effect in figure 4.11. As in test case 1, we give gain of primary user only (figure 4.12) and channel distribution success versus SINR threshold value (figure 4.13). Furthermore, figure 4.14 gives comparison of total gain throughputs of NASA-UNIC for secondery users' transmit powers of 100 mW and 200 mW (figure 4.14).

3. Effect of number of secondery users on total gain of primary + secondery users, no auction (algorithm NASA-UNIC): Primary transmitter is a base station with transmit power of 30dBm (1000mW) while each secondery user is a base station station with transmit power of 20dBm (100mW). We increase number of secondery users (K) from 10 to 30 and investigate total gain throughput variation. Results are shown in figure 4.15.

4. Effect of SINR threshold on total gain of primary + secondery users (algorithm ABSA-UNIC): This case is the same as the first one, except allocation is carried out based on auction theory. Total and primary users' gain versus SINR threshold are given in figures 4.16 and figure 4.17 and channel distribution versus SINR threshold is given in figure 4.18.

5. Effect of secondery transmitter power increase (coverage area) on total gain of primary + secondery users versus SINR threshold (algorithm ABSA-UNIC): This case is the same as the second one, however, here again, allocation is implemented based on auction. Results are given in figure 4.19, figure 4.20 and figure 4.21. Comparison of total gain throughput of our algorithm for secondery users' transmit powers of 100 mW and 200 mW is given in figure 4.22. Moreover, to show significant advantage of applying auction, we present comparison of total gain throughputs with and without auction for SINR threshold value of 100 in figure 4.23.

6. Effect of number of secondery users on total gain of primary + secondery users (algorithm ABSA-UNIC): This is the same as the third case, except, auction-based allocation is used. Results are shown in figure 4.24.

7. Effect of increasing coverage area on total gain throughput (algorithm ABSA-UNIC): In this case, we investigate how total gain throughput is affected by setting secondery user transmit power to 20dBm, 23dBm and 27dBm which means changing coverage area in [200, 250, 300]m, respectively. Figure 4.25 depicts corresponding results.

4.3.2 Simulation Results

In the first experiment, we investigate effect of increasing SINR threshold on total gain throughput and primary user gain, without applying auction (NASA-UNIC). As seen in figures 4.8 and 4.9, total gain throughput always decreases with increasing SINR threshold as expected. As SINR threshold increases, primary receivers will be more susceptible to interference and this results in there will be less number of secondery users accessing the same channel at the same time. Therefore, channel distribution performance gets worse and total throughput decreases. We give throughput limit (without interference) of greedy algorithm and greedy net throughput results calculated by using equation (4.8.11). It is noticed throughput value of NASA-UNIC is much lower than throughput value of greedy algorithm when SINR threshold value is 0. This is because user bidding higher revenue has higher priority for acquiring a channel in greedy algorithm as well (auction). Another noticeable point seen is results of NASA-UNIC and greedy algorithm crossover at SINR threshold value of approximately 2-3 dB. After this point (which fairly match to practical SINR threshold values given in figure 4.32), net throughputs of NASA-UNIC are higher than those of greedy mechanism because outages occurring during a portion of iterations result in a negative effect on net throughout. Obviously, this effect becomes more dominant as SINR threshold increases. For instance, greedy net throughput decreases to 0 at around 20 dB of SINR threshold level, however, NASA-UNIC has total gain throughputs of about 450 and 500 respectively at that level. Consequently, this clearly shows superiority of proposed algorithm. Moreover, figure 4.10 depicts channel distribution fairness from which we can notice channel distribution performance worsens with increasing SINR threshold. Besides this, we again see there is a crossover of results in accordance with previous results.

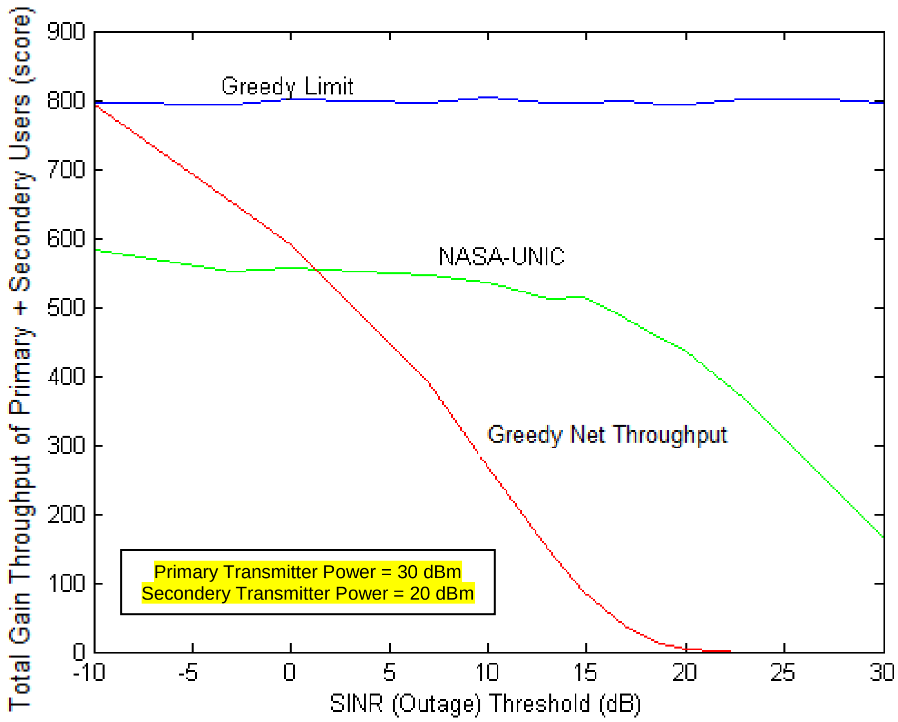

Figure 4.8 Total gain throughput versus increasing SINR threshold (with secondery transmit power = 100 mW)

In the second case, we compare again NASA-UNIC with greedy algorithm. However, we increase secondery users' transmit power from 100 to 200 mW which means increasing coverage area from 200 to around 250 meter. Since neighbor nodes more interfere with each other, there will be fewer users who can access to the same channel, consequently channel distribution performance decreases and so total and primary user throughputs decrease as well, as depicted in figures 4.11 and 4.12. Similar to the first test case, net throughputs of NASA-UNIC and greedy mechanism have a crossover point after which results of NASA-UNIC are clearly much better. Figure 4.13 shows channel distribution fairness which is expected to be worse than the first case. Besides this, figure 4.14 gives comparison of total gain throughputs of the first and second experiments from which it is seen total utilities drop with increasing coverage area, as expected.

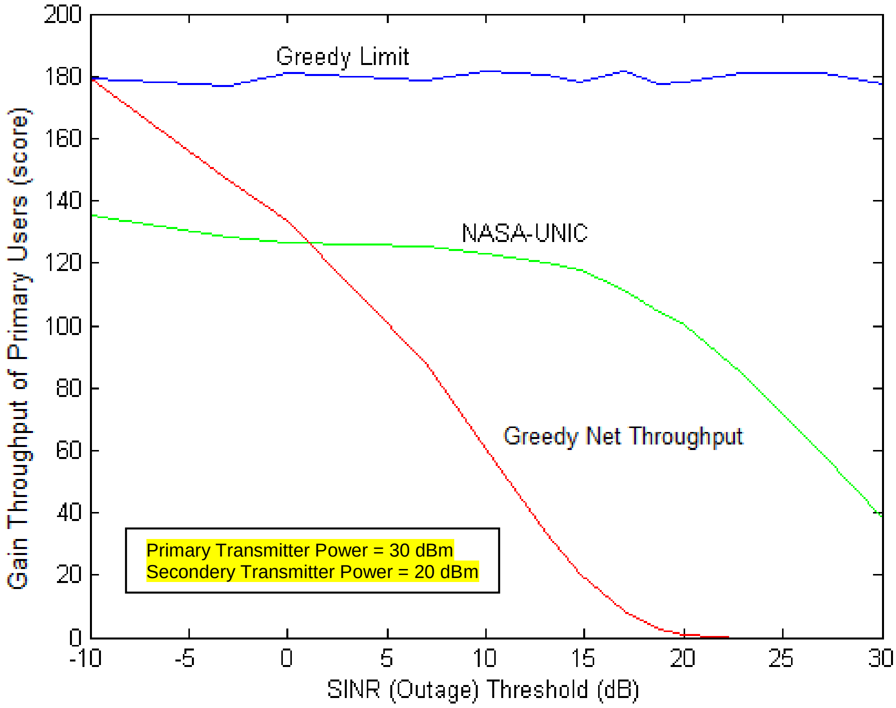

Figure 4.9 Gain throughput of primary user versus increasing SINR threshold

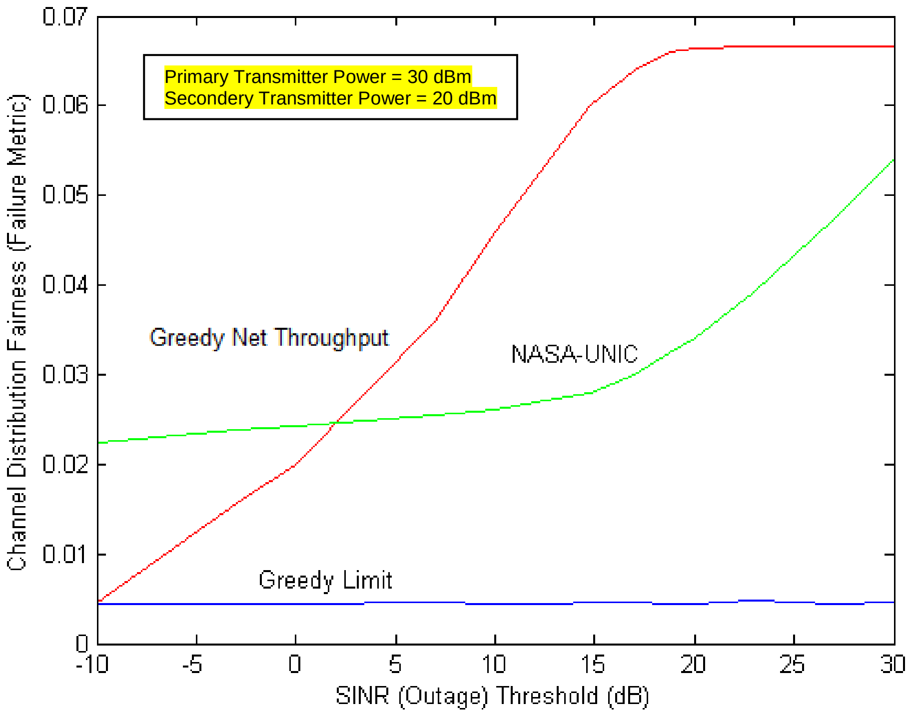

Figure 4.10 Channel distribution fairness versus increasing SINR threshold

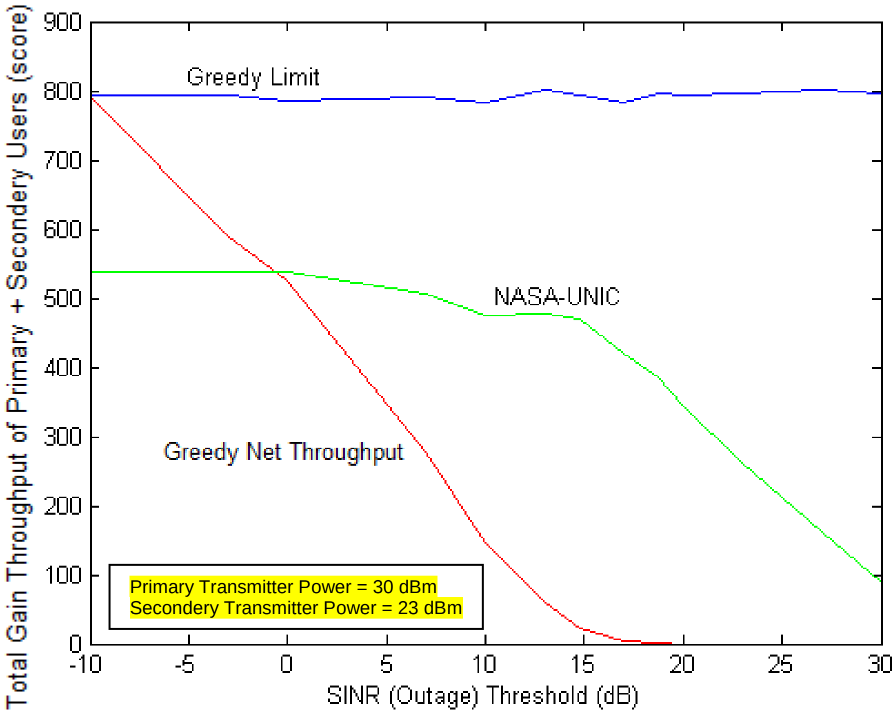

Figure 4.11 Total gain throughput versus increasing SINR threshold (with secondery transmit power = 200 mW)

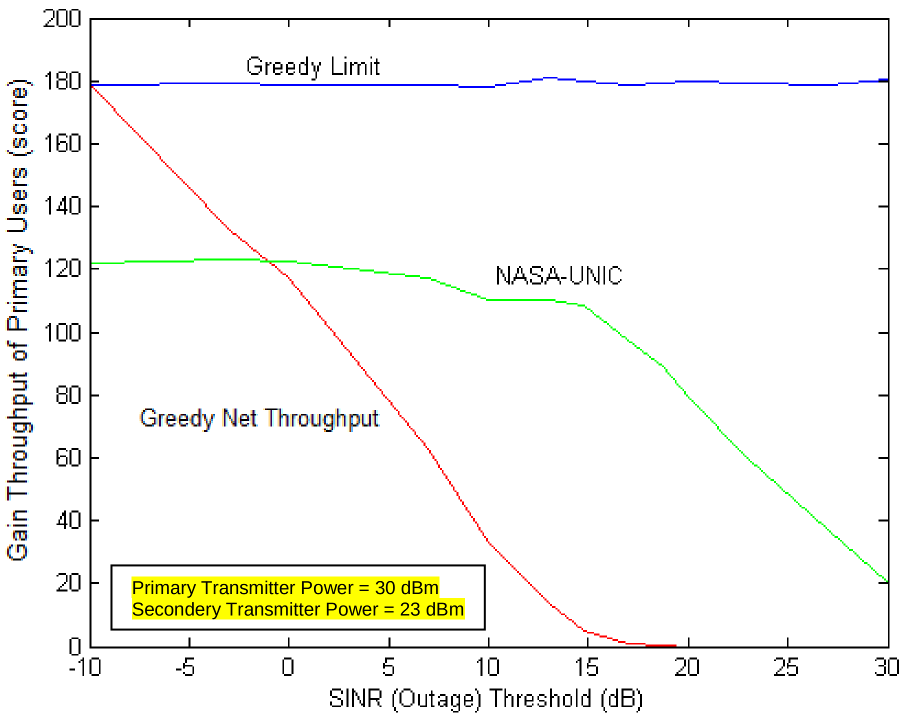

Figure 4.12 Gain throughput of primary user versus increasing SINR threshold

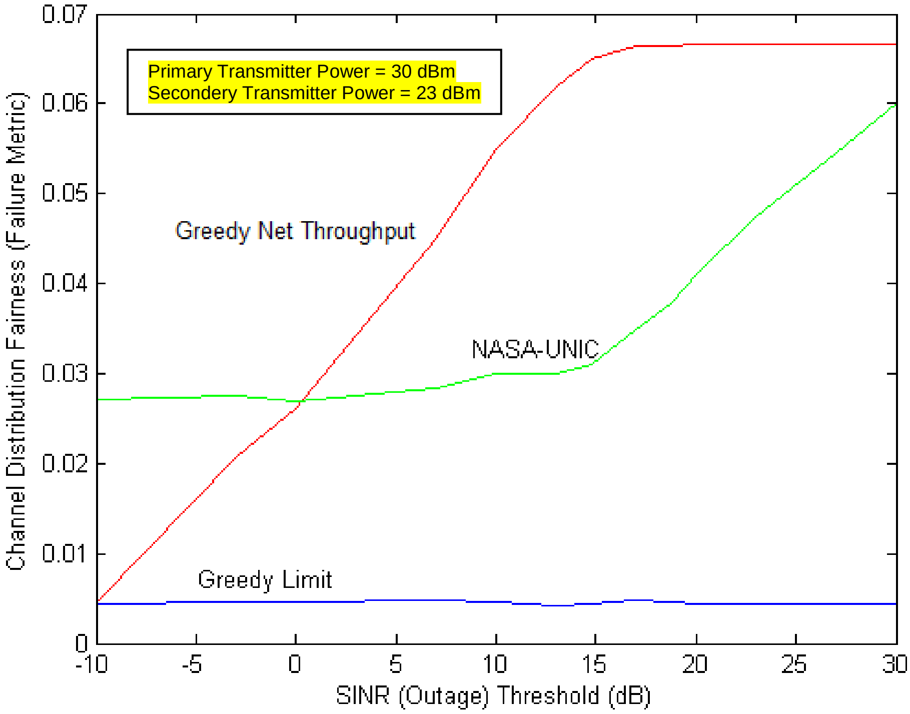

Figure 4.13 Channel distribution fairness versus increasing SINR threshold

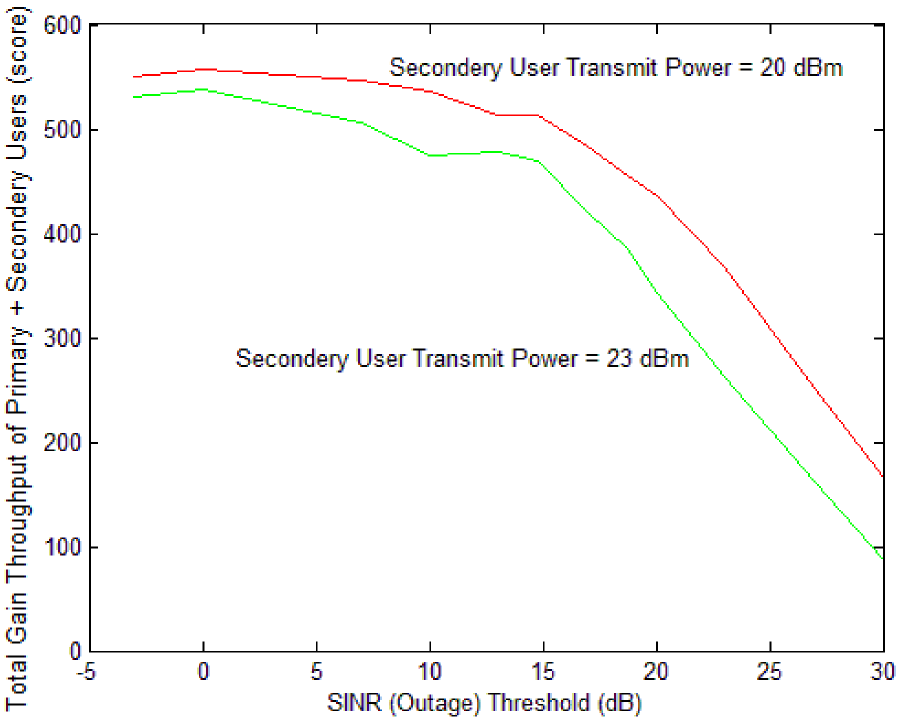

Figure 4.14 Comparison of total gain throughput of proposed algorithm with with increasing secondery user transmit power (coverage area)

In the third experiment, we change number of secondery users and observe how gain throughputs vary. Figure 4.15 shows total utilities of proposed (NASA-UNIC) and greedy algorithms versus number of cognitive users, from which we can notice throughput increases almost directly proportional to the number of secondery users since number of users obtaining channel increases. We also see that for SINR theshold value of 0, NASA-UNIC has lower performance than the greedy one. However, for higher values of SINR threshold, it is obvious that NASA-UNIC will exceed and then have higher level net throughput curves.

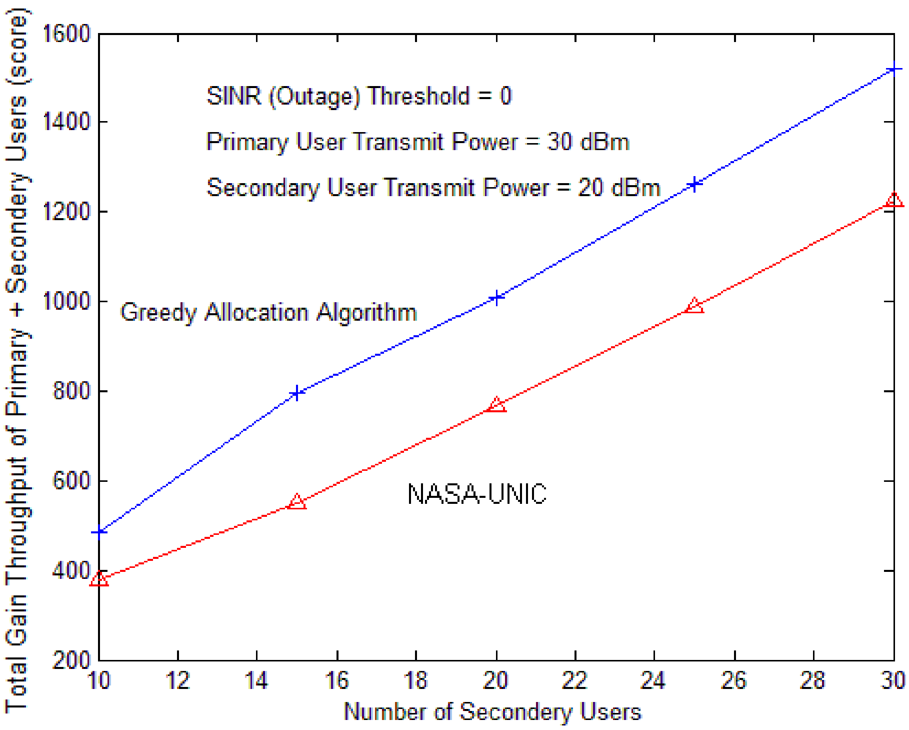

Figure 4.15 Total gain throughput versus number of secondery users

The fourth scenario is the same as the first one, except for allocation is carried out on auction (ABSA-UNIC). Figures 4.16 and 4.17 show total and primary user utilities versus SINR threshold where significant advantage of auction can be seen. Total and primary user utilities almost approximate to greedy values for SINR threshold value of 0. As expected, utilities decrease with increasing SINR threshold, whereas they are still much higher than those without auction. Moreover, again throughputs of proposed algorithm and greedy intersect and this time, because of auction, intersection occurs at very low SINR threshold values which means ABSA-

UNIC shows great performance even at low SINR threshold values. Figure 4.18 shows channel distribution versus SINR threshold with auction based allocation, where again results are much better than the case without auction (NASA-UNIC).

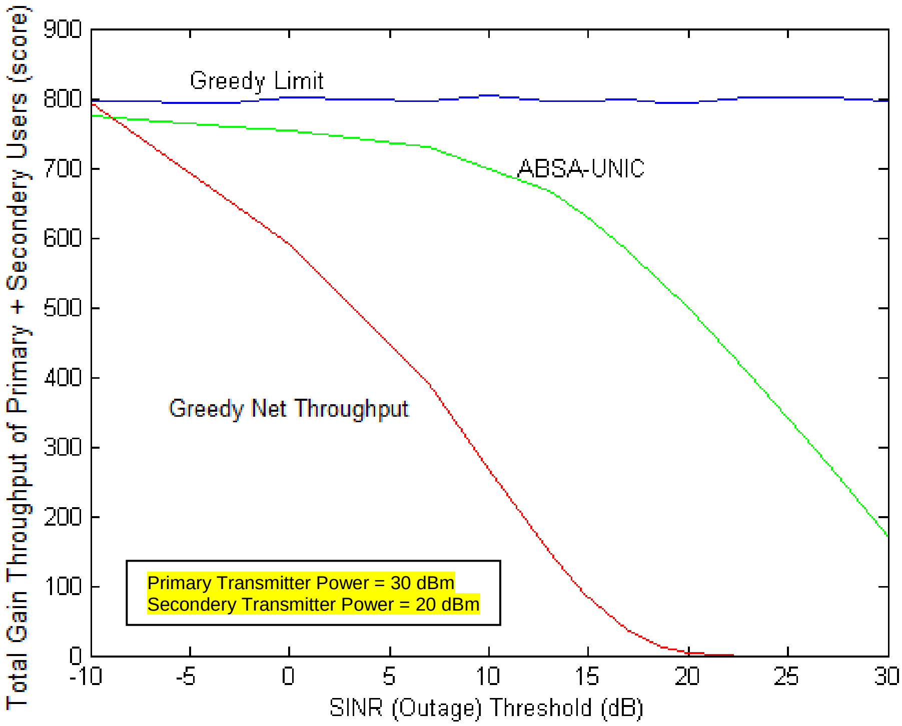

Figure 4.16 Total gain throughput versus increasing SINR threshold, case with auction

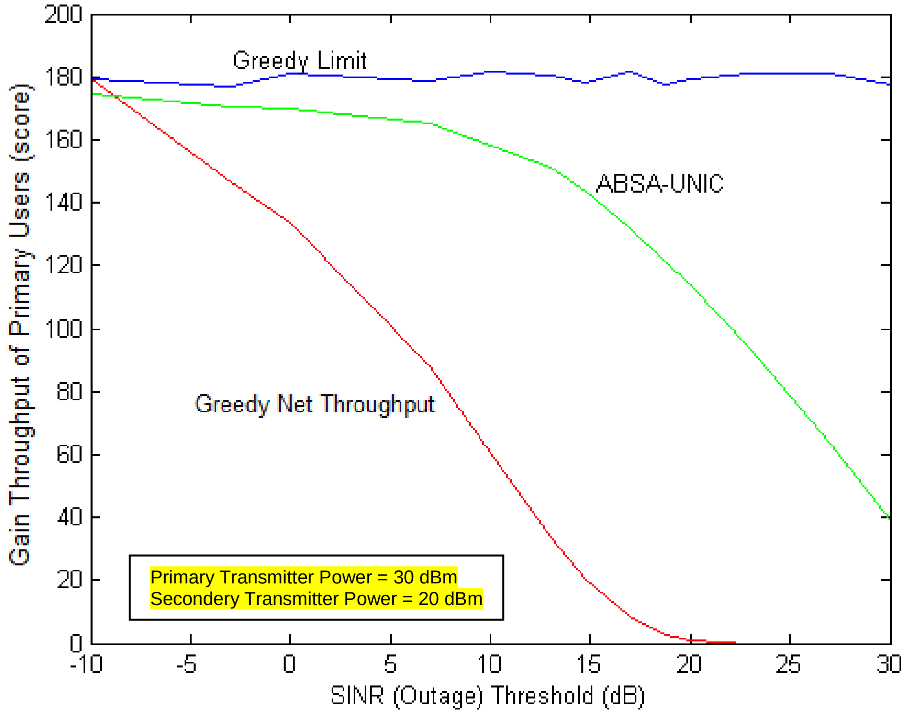

Figure 4.17 Gain throughput of primary user versus increasing SINR threshold, case with auction

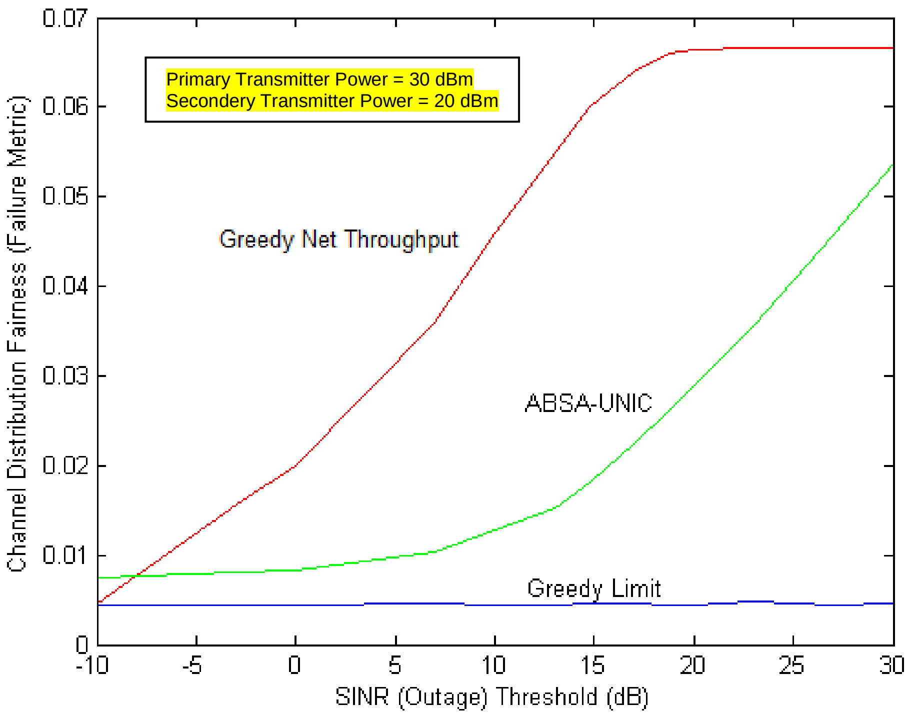

Figure 4.18 Channel distribution fairness versus increasing SINR threshold, case with auction

Fifth experiment is again the auction version of the second experiment (ABSA-UNIC). Figures 4.19, 4.20 and 4.21 depict corresponding results, from which significant advantage of auction can be seen again. Similar to previous results, net throughputs intersect at low SINR threshold values. Figure 4.22 compares total utilities of the third and fourth experiments. Different from the case without auction, outstanding point of this case is utilities are almost the same till values of SINR threshold 10 (10dB), after which the difference increases slowly. In figure 4.23, we notice total utility with auction is clearly higher, especially under values of 25 dB of SINR threshold.

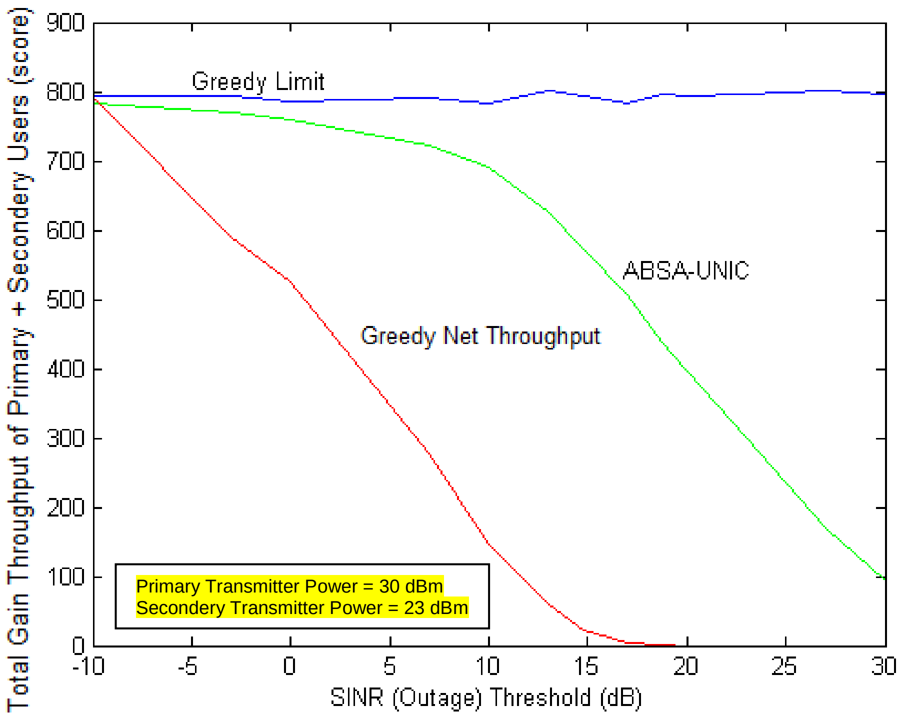

Figure 4.19 Total gain throughput versus increasing SINR threshold, case with auction (with secondery transmit power = 200 mW)

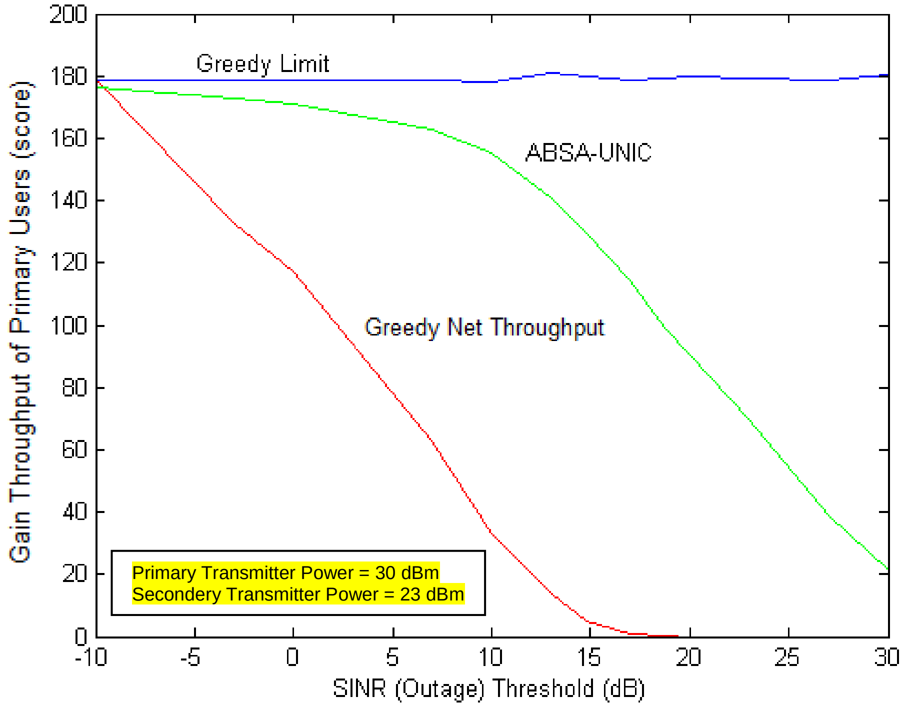

Figure 4.20 Gain throughput of primary user versus increasing SINR threshold, case with auction (with secondery transmit power = 200 mW)

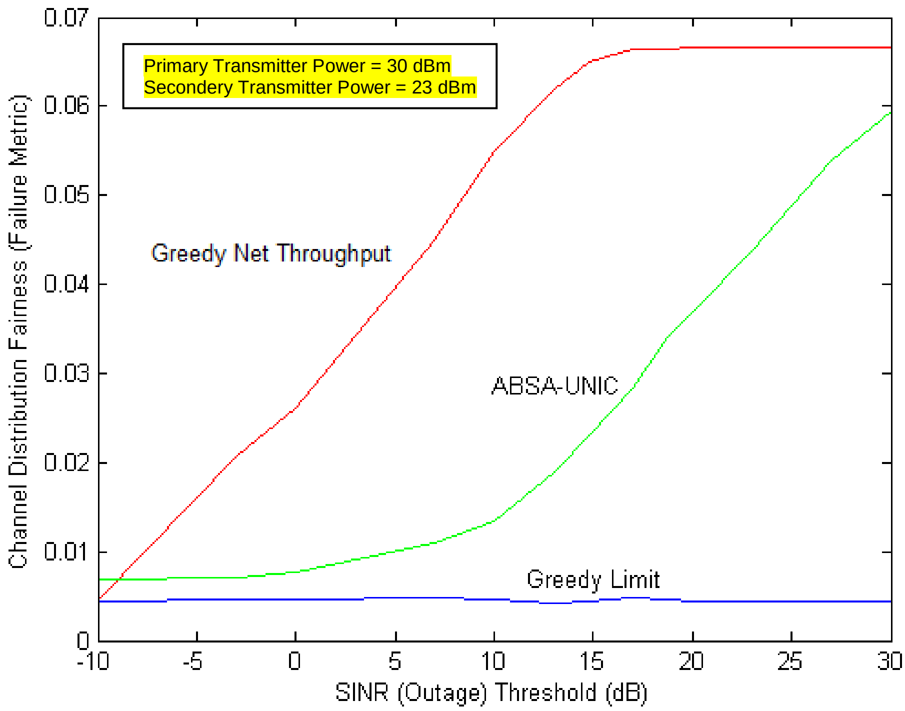

Figure 4.21 Channel distribution versus increasing SINR threshold, case with auction allocation (with secondery transmit power = 200 mW)

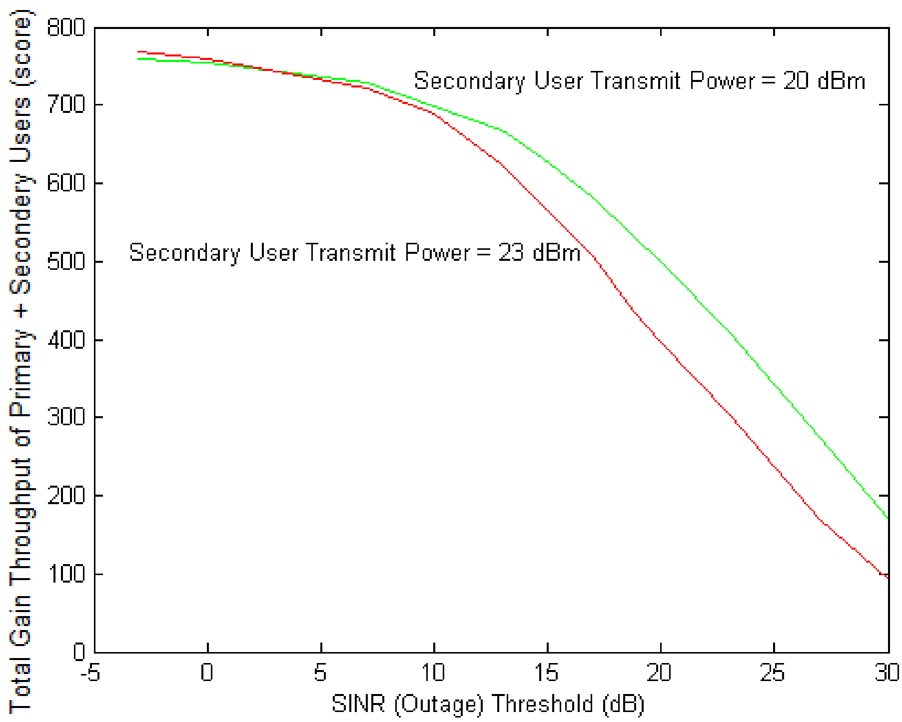

Figure 4.22 Comparison of total gain throughput of proposed allocation algorithm with increasing secondery user transmit power (coverage area), case with auction

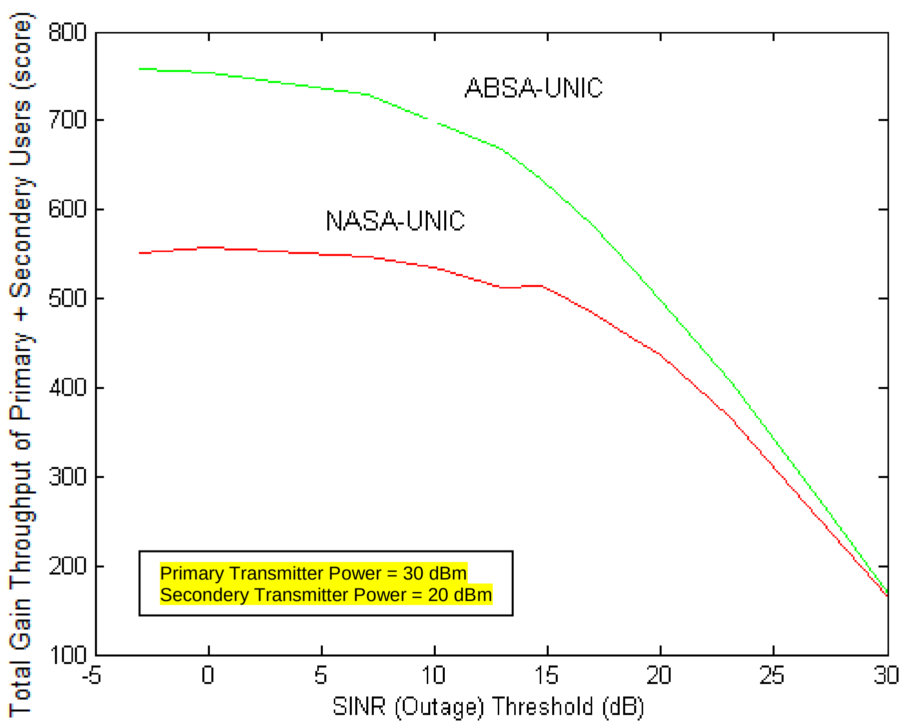

Figure 4.23 Comparison of total gain throughputs versus SINR threshold with and without auction

In the sixth test case, similar to the third experiment, total throughput with auction (ABSA-UNIC) versus number of secondery users are investigated which is given in figure 4.24. Still, total throughput increases with number of secondery users, furthermore, results are seen to be too close to the results of greedy algorithm. We also notice that for SINR threshold value of 0, ABSA-UNIC has very close performance to the greedy one and for higher values of SINR threshold, it is obvious that ABSA-UNIC will exceed and then have higher level net throughput curves.

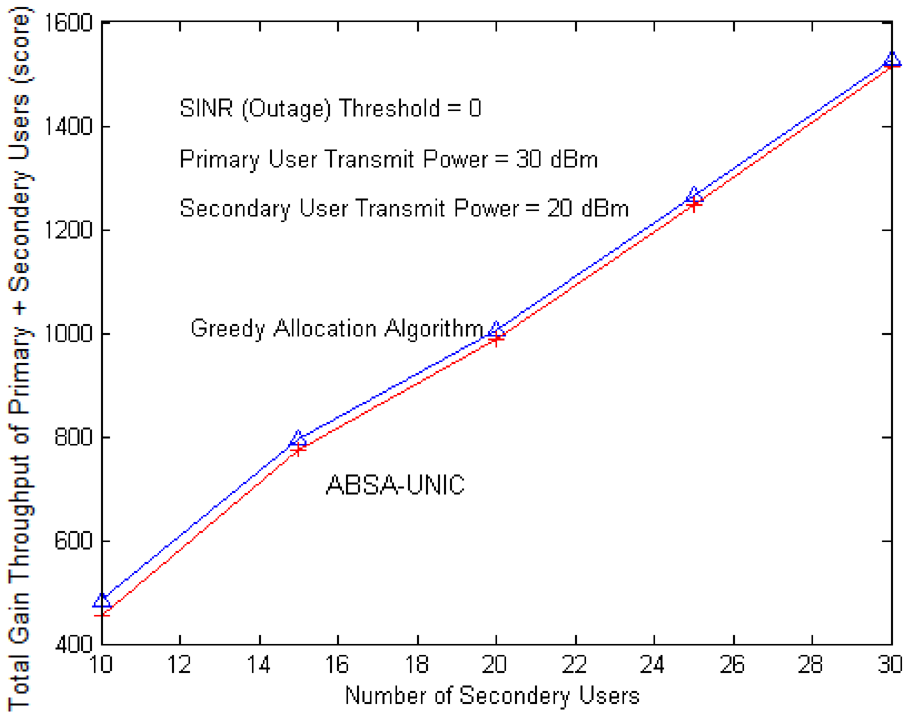

Figure 4.24 Total gain throughput versus number of secondery users, with auction

In the last experiment, we investigate effect of changing secondery user transmit power (therefore coverage area) on the total utility, in case with auction (ABSA-UNIC). Figure 4.25 shows results from which we see setting transmit power to 20dBm, 23dBm and 27dBm decreases throughput especially at higher values of SINR threshold since number of interfering users (therefore conflict between them) increase and SINR threshold constraint becomes more dominant at higher values.

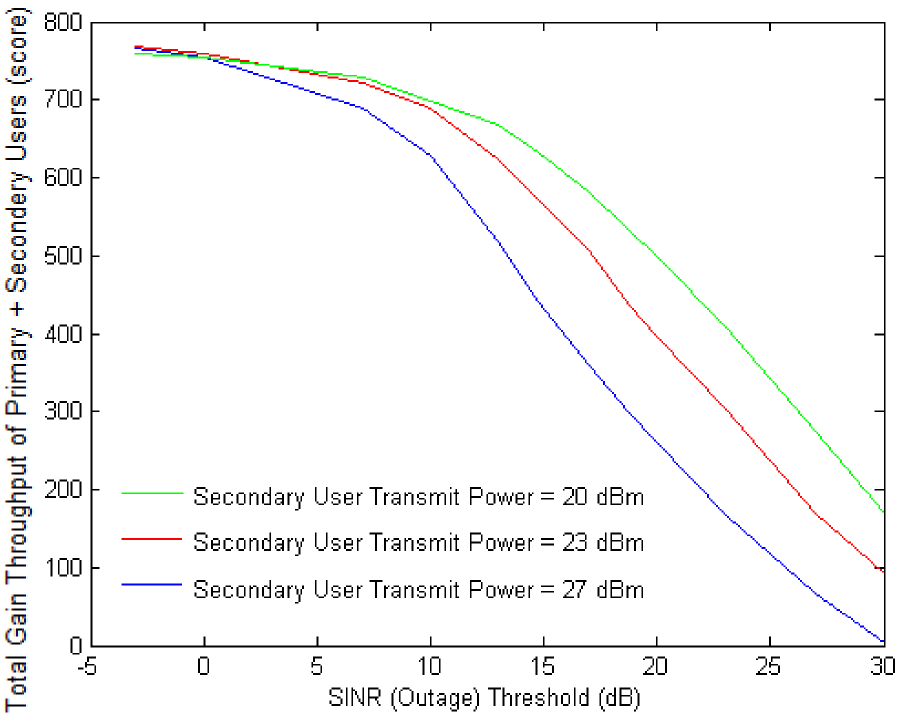

Figure 4.25 Total gain throughput versus SINR threshold with increasing coverage area (ABSA-UNIC)

In order to do comparison of channel distribution fairness versus secondery user transmit power, we fix primary user transmit power to 30 dBm while changing secondery user transmit power in [20, 23, 27]dBm, meaning coverage radius change in [200,250,300]m, respectively. We fix number of secondery users to 15 as well. Figure 4.26 shows results from which we see, with increasing of the coverage radius, channel distribution results worsens, because there will be fewer users who can access to the same channel. Also we can see that the proposed mechanism has similar performance to the greedy one for SINR threshold value of 0. On the other hand, we can imagine that channel distribution fairness of greedy algorithm will be worse for higher SINR threshold values, that means, the blue bars will exceed the red bars after a SINR threshold level and then the higher the SINR threshold value, the higher the blue bars than the red bars.

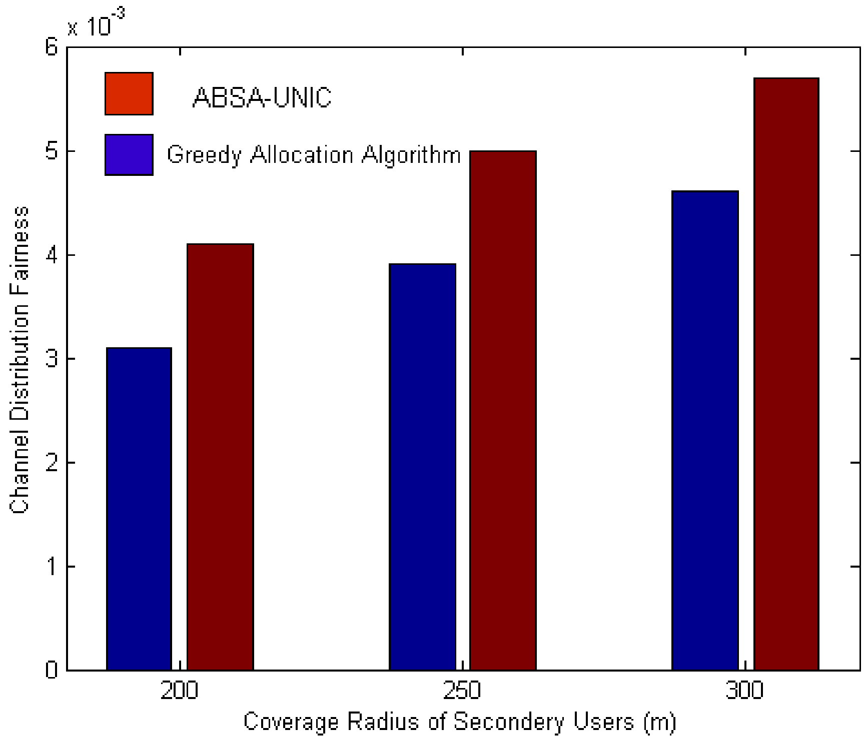

Figure 4.26 Channel distribution fairness versus coverage area, with primary user transmit power fixed to 30 dBm, secondery user transmit power change in [20, 23, 27] dBm and SINR threshold = 0

In order to obtain heuristic maximum throughputs (scores) of ABSA-UNIC algorithm for every SINR threshold value, first we listed all possible independent sets of an interference graph with 15 secondery users, not only the maximum one, for primary user transmit power of 30 dBm and secondery user transmit power of 20 dBm. Next, we ran ABSA-UNIC starting with each of them one by one and noted final total throughput gains. At the end, we obtained the maximum value of all throughputs as heuristic maximum. At the same time, we ran our algorithm starting with the maximum independent set as well and finally compared the results which is given in figure 4.27. We notice heuristic maximum throughput scores are higher than all other throughput levels because it shows the best scores of all iterations over all SINR threshold values. Furthermore, as expected, it falls under the greedy limit level after around 20 dB, since greedy limit does not include SINR effect as we already explained.

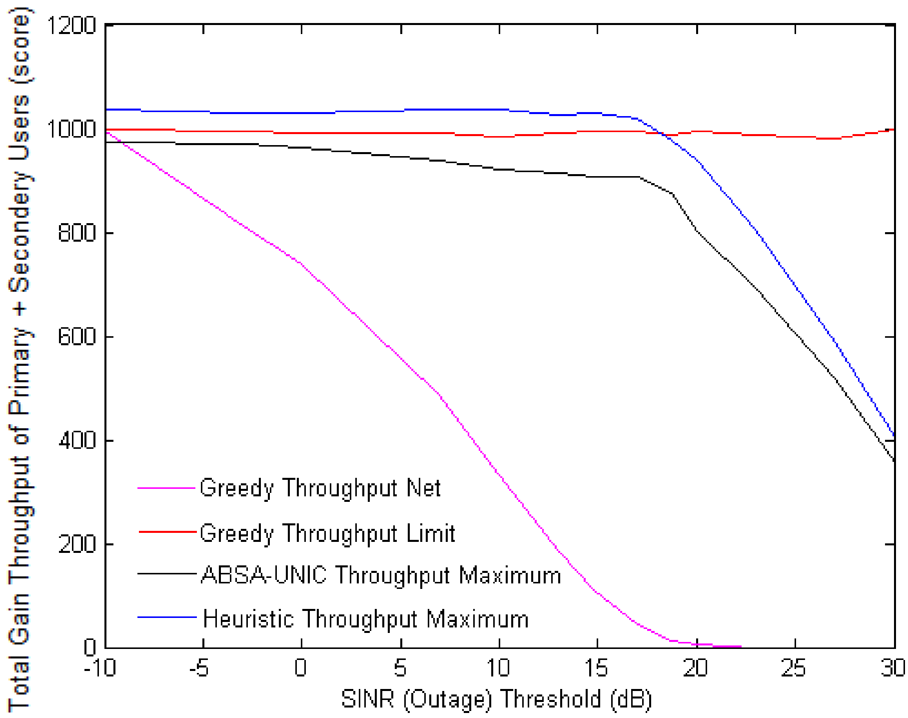

Figure 4.27 Heuristic and proposed algorithm maximum throughputs versus SINR threshold

To compare maximum throughput values (scores) of Brute Force, greedy algorithm and our algorithm ABSA-UNIC, we compare maximum values reached by running greedy and ABSA-UNIC for 1000 times, for SINR threshold values of 0 and 100.

During these simulations, we fixed primary and secondery user biddings to following scores in order for comparison to give more meaningful results:

$P_{\text{secondery}}$= [22, 24, 20, 23, 11, 20, 15, 18, 21, 20, 12, 11, 11, 16, 24]
P_{primary} = [5, 5, 6, 7, 5, 6, 5, 5, 5, 6, 7, 7, 5, 6, 7]

Table 4.4 presents results for SINR threshold value of 0. As seen on the table, maximum total throughput of Brute Force is 784, it is 768 with greedy and 784 with ABSA-UNIC.

Figures 4.28 and 4.29 show distributions of all throughput values of ABSA-UNIC and greedy algorithm respectively, at the end of 1000 iterations.

Table 4.4 Maximum throughput values (scores) for SINR threshold = 0

SINR threshold = 0		Brute Force	Greedy Algorithm	ABSA-UNIC	Greedy (Primary User)	ABSA-UNIC (Primary User)
Over 1000 Iterations	Maximum Gain Throughput Value (score)	784	768	784	183	188
Secondery User Payoff fixed to 30 Primary User Payoff fixed to 7	Maximum Gain Throughput Percentage		2,5%	0,7%	0,5%	0,7%
	Outage Percentage	0%		0%		
	Channel Distribution Metric (minimum)	0		0		
	Total Gain Throughput > 750	55,5%		24,5%		

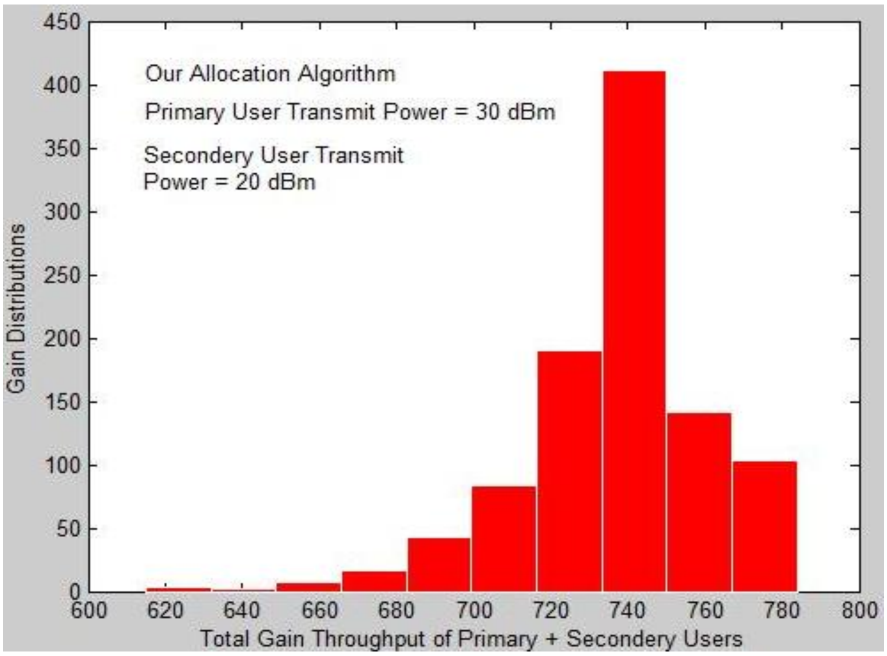

Figure 4.28 Distributions of total gain throughputs for SINR threshold = 0 (ABSA-UNIC)

In table 4.5, results are given for SINR threshold value of 100. Again, it is seen maximum total gain throughputs of Brute Force and greedy are 784 and 768, respectively. It is 768 with ABSA-UNIC, as well. Moreover, maximum primary user gains of greedy algorithm and ABSA-UNIC are 183 and 182 respectively. Because of interference outage threshold, on 99,7% of 1000 iterations of greedy algorithm, primary receiver got outage whereas never outage occurred with ABSA-UNIC which shows our algorithm’ s advantage.

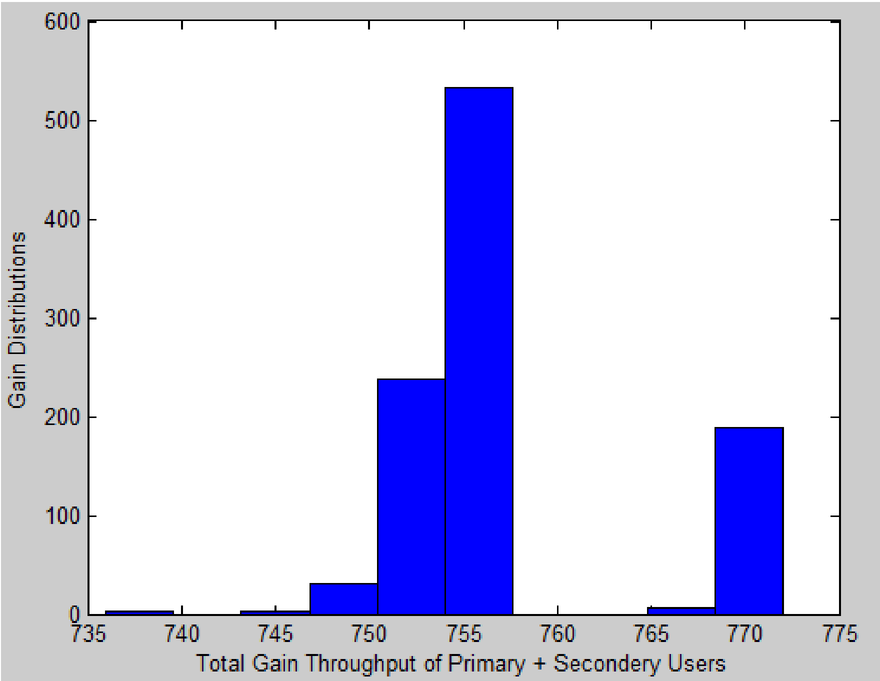

Figure 4.29 Distributions of total gain throughputs for SINR threshold = 0 (greedy algorithm)

Table 4.5 Maximum throughput values (scores) for SINR threshold = 100 (20dB)

SINR threshold = 100		Brute Force	Greedy Algorithm	ABSA-UNIC	Greedy (Primary User)	ABSA-UNIC (Primary User)
Over 1000 Iterations	Maximum Gain Throughput Value (score)	784	768	768	183	182
Secondery User Payoff fixed to 30 Primary User Payoff fixed to 7	Maximum Gain Throughput Percentage		0,24%	0,01%	0,07%	0,01%
	Outage Percentage	99 ,7%		0%		
	Channel Distribution Metric (minimum)	0,0009		0,0018		
	Total Gain Throughput > 750	55,9%		0,2%		

Figures 4.30 and 4.31 show distributions of all throughput values of ABSA-UNIC and greedy algorithm respectively, at the end of 1000 iterations.

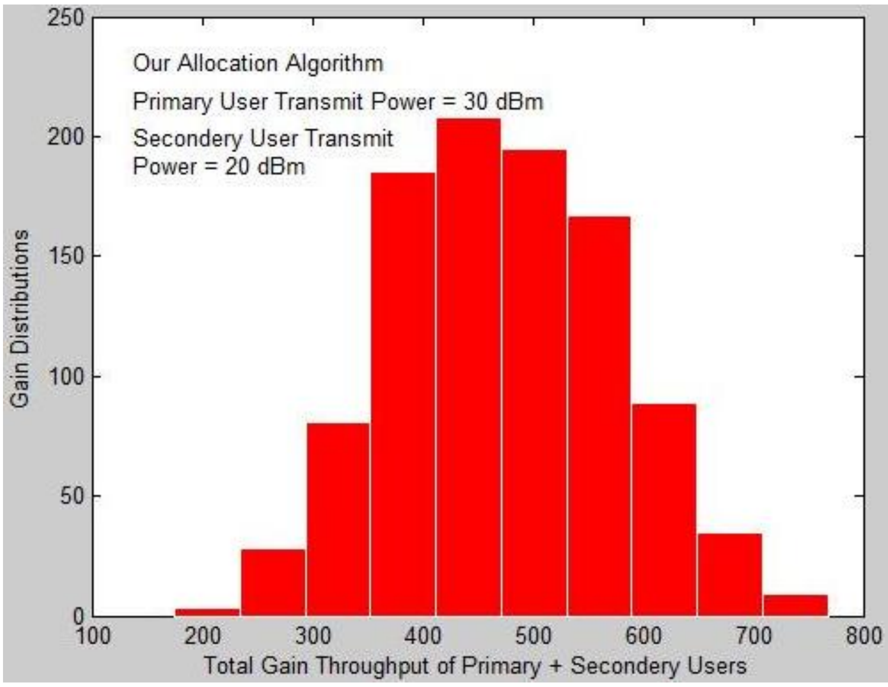

Figure 4.30 Distributions of total gain throughputs for SINR threshold = 100 (ABSA-UNIC)

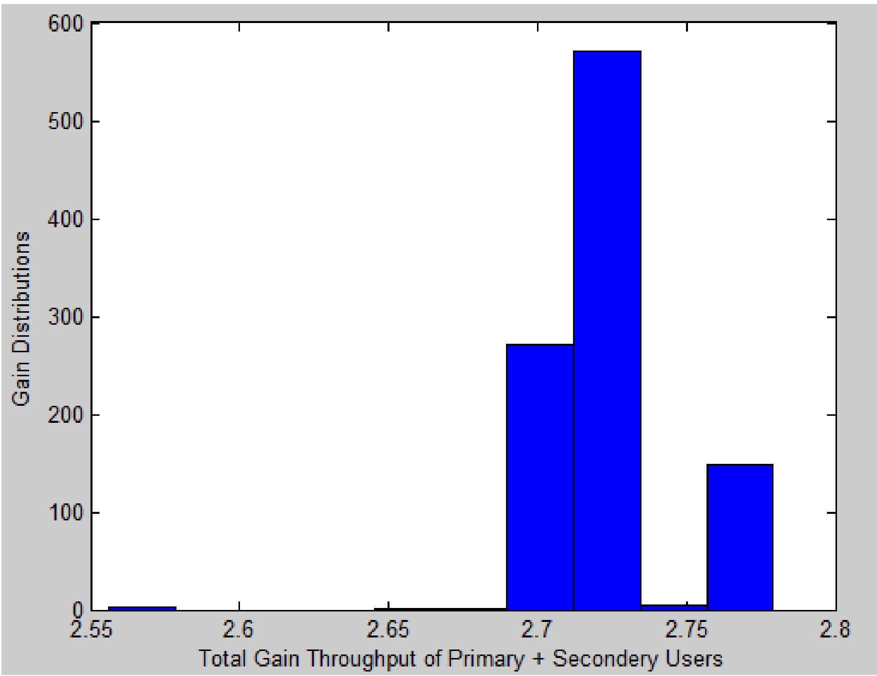

Figure 4. 31 Distributions of total gain throughputs for SINR threshold = 100 (greedy algorithm with interference effect)

In order to compare SINR thresholds we used in our experiments with practical values, we can check values in figure 4.32. It compares the performance of various wireless access technologies and puts them in contrast to the shannon physical limitation.

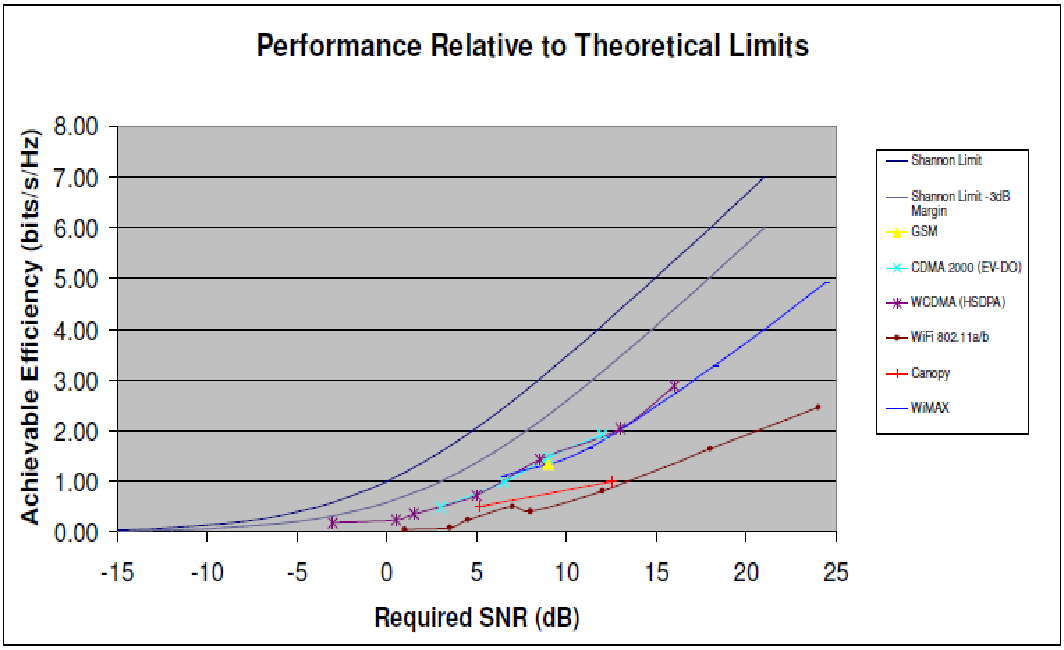

Figure 4.32 Shannon Limits of various wireless access technologies

Following table shows some SINR limits for WIMAX system:

Table 4.6

System	Coding Rate	SINR Value
WIMAX	BPSK (Coding rate=1/2)	6.4 dB
	QPSK (Coding rate=1/2)	9.4 dB
	QPSK (Coding rate=3/4)	11.2 dB
	QAM-16 (Coding rate=1/2)	16.4 dB
	QAM-16 (Coding rate=3/4)	18.2 dB
	QAM-64 (Coding rate=1/2)	22.7 dB
	QAM-64 (Coding rate=3/4)	24.4 dB

In this chapter, first we give our model and utility functions. Then, we represent the problem as a non-linear programming problem. After that, we explain our proposed algorithm (version with auction called as ABSA-UNIC and version with no auction called as NASA-UNIC) and a greedy algorithm for channel allocation which we use for comparison. Then, we give our simulation model, parameters and test cases. Finally, we present simulation results graphically and comment about them.

CHAPTER FIVE
CONCLUSIONS

In this thesis work, we study graph theory and its various applications in the field of computer networks. In mathematics and computer science, graph theory is the study of graphs which are mathematical structures used to model pairwise relations between objects from a certain collection. A graph in this context refers to a collection of vertices or nodes and a collection of edges that connect pairs of vertices. Graphs are represented graphically by drawing a dot for every vertex, and drawing an arc between two vertices if they are connected by an edge.

Networks have many uses in the practical side of graph theory (for example, to model and analyze traffic networks). Within network analysis, the definition of the term network varies, and may often refer to a simple graph. Applications of graph theory in the form of network analysis split broadly into three categories. Firstly, analysis to determine structural properties of a network, such as the distribution of vertex degrees and the diameter of the graph. A vast number of graph measures exist, and the production of useful ones for various domains remains an active area of research. Secondly, analysis to find a measurable quantity within the network, for example, for a transportation network, the level of vehicular flow within any portion of it. Thirdly, analysis of dynamical properties of networks.

In the literature, various applications of graph theory for wireless networks exist. Arbitrary graphs have the advantage of being able to represent all possible network configurations. Certain restricted graphs could give an accurate representation for certain radio network or network scenario. They may illuminate some aspects of the problem structure, which might help in solving the problem, and finding the optimal solution, such as finding a chromatic number. On the other hand, most of the applications of graph theory are to solve the problem of channel assignment. Especially automatic channel assignment in multi-channel multi-radio wireless mesh networks is a key technique to minimize signal interference and increase network capacity. Tree and planar graphs are most famous restricted arbitrary graphs used in modeling radio networks. Tree is the simplest graphical representation and problems such as message routing and propagation can be well addressed using tree models.

After studying graph theory in the introduction part of the thesis, we investigate its various applications in the field of networks and classify them based on 7 layers of OSI. Most of the applications are seen regarding MAC, transport, network and physical Layers.

Applications on MAC layer are mostly related to channel assignment, coloring and scheduling problems. Furthermore, new graph models are introduced for interference minimization and better spectrum utilization. Especially, unit disk and double disk graph models are widely used and researchers propose many algorithms based on these models. On the other hand, examples of transport layer applications are congestion prevention and flow control algorithms.

On network layer, most of applications are related to routing problems. Routing algorithms generally aim to transmit messages from a source to a destination on a graph with minimum number of hops and in a faster way. Also, there are applications related to topology control and flooding issues.

Physical layer applications are mostly regarding trellis and state diagrams. Those diagrams are generally used for graph representation of codes in time and frequency domain. Applications for interference management based on SINR models exist as well.

After having a detailed look at graph theory applications, we see spectrum allocation is an important open area in wireless networks because of scarcity of available frequency spectrum. The demand for wireless spectrum has been growing rapidly with the dramatic development of the mobile telecommunication industry in the last decades. In order to fully utilize the scarce spectrum resources, with the development of cognitive radio technologies, dynamic spectrum allocation becomes a promising approach to increase the efficiency of spectrum usage. Cognitive radio techniques provide the capability to use or share the spectrum in an opportunistic manner. Dynamic spectrum access techniques allow the cognitive radio to operate in the best available channel.

The ultimate objective of the cognitive radio is to obtain the best available spectrum through cognitive capability and reconfigurability. Since most of the spectrum is already assigned, the most important challenge is to share the licensed spectrum without interfering with the transmission of other licensed users.

The coloring model based on graph theory is an important model to research on channel allocation for cognitive radios, which models the network topology including cognitive users and primary users as an interference graph so that the channel allocation problem is formulated as a graph-coloring problem.

Another important issue in cognitive radio networks is that spectrum owners should not be affected by usage of spectrum holes by secondery users. In literature there are many studies concerning only dynamic spectrum allocation without considering interference constraints, whereas some of them only consider interference and power management issues without taking into account throughput maximization during channel allocation.

As a result of these observations, we see there is no work on cross-layer applications of graph theory and cognitive radio is a good field for this purpose. We study on a MAC-physical cross layer application of graph theory which includes both spectrum allocation and SINR management in cognitive radio networks. We consider sum of revenues of both primary and secondery users as throughput of channel allocation mechanism and we investigate primary users' total revenue separately as well. We not only aim to maximize throughput but also satisfying SINR limit on primary receivers so that their operation is not affected. We also investigate effect of auction theory on total throughput. For this purpose, we propose a novel algorithm to allocate channels in the spectrum pool with graph coloring and bidding theory, which compares total interference on primary receiver with outage interference threshold and takes actions accordingly. Distinguishing from related works, both auction-based throughput maximization and interference management are handled together, without controlling secondery user power levels. We call the auction version of our algorithm as ABSA-UNIC and the no-auction version as NASA-UNIC. For comparison, we also use a greedy algorithm for channel allocation which does not take interference effect into consideration. Therefore, primary receiver goes under outage in a portion of iterations depending on SINR threshold value whereas never outage occurs with proposed algorithm. Because of this, in order to compare results, we define net throughput by taking into account outage probability together with throughput obtained at the end of simulations.

In our simulations, we consider a model with one primary transmitter and its corresponding receiver with variable number of secondery users. Moreover, we use transmission power levels and SINR threshold values in accordance with realistic systems. We investigate how total throughput is affected by modifying several parameters. These include number of secondery users, transmission power levels (which means changing coverage area) and also SINR threshold level. We also simulate channel allocation without and with auction. On the other hand, we check channel distribution success of proposed algorithm by defining a distribution fairness metric as well.

At the end of simulations, we investigate total gain throughput versus SINR threshold level, coverage area and number of secondery users. Simulation results show that exposing higher SINR (outage) threshold always decreases total gain and primary users' utilities. The higher the SINR threshold, the more susceptible to interference the primary receivers will be and this results in there will be less number of secondery users accessing the same channel at the same time. Consequently, channel distribution performance gets worse and total throughput decreases. On the other hand, adding auction significantly increases total gain throughput and primary user' s utility. Especially till SINR threshold values of 20 dBs, auction provides outstanding performance and ABSA-UNIC has total throughput results close to those of the greedy one even though no interference constraint is applied to the greedy algorithm. Consequently, since users bidding higher have higher priority to get channel, auction-based allocation mechanism significanlty increases total gain throughput. Another noticeable point seen in simulation results is crossover of net throughputs of proposed and greedy algorithms at a SINR threshold value. After this point, results of NASA-UNIC and ABSA-UNIC are better than those of greedy mechanism because outages occurring during a portion of iterations result in a negative effect on net throughout. Obviously, this effect gets more dominant as SINR threshold increases. In summary, this result clearly shows advantage of proposed algorithm.

5.1 Summary of Contributions

In this thesis, our study mainly concerns applications of graph theory in wireless networks. Our original work is a MAC-physical cross-layer application implementation which is related to spectrum allocation problem in cognitive radio networks.

In this work, we study on allocation of vacant channels which are not used by primary users (spectrum holders) to secondery users who pay money for those channels. Cognitive radio is modelled as an undirected graph for this purpose where vertices represent secondery users and edges represent interferences so that no channels can be assigned simultaneously to any adjacent nodes. The objective of allocation is to maximize spectrum utilization for both primary and secondery users. That means maximizing total revenue of both primary and secondery users. It is important to allocate channel to a user who bid more than the other users for the same channel. So, auction-based allocation improves total throughput. Considering this, for throughput maximization, we propose an allocation algorithm based on auction. On the other hand, it is important channels occupied by cognitive users are not used by primary users at the same time. Otherwise, primary receivers are affected because of co-channel interference resulting from secondery users. If interference level goes above a threshold value, primary users go under outage. Based on this, we handle interference management in our allocation algorithm as well and improve it so that primary receivers do not go under outage when maximizing total throughout during auction-based allocation. Consequently, the algorithm we propose in this thesis is unique which is based on graph coloring, bidding and auction theory at the same time and it not only tries to maximize total throughput which is sum of revenues of all users but also satisfies SINR condition on primary receivers. Moreover, proposed algorithm gives much better net throughputs than greedy algorithm because of outages occurring with greedy mechanism.

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