DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

ANALYSIS OF STRUCTURAL ACOUSTIC COUPLING OF PLATES

by
Özgür ARPAZ

September, 2006
İZMİR

ANALYSIS OF STRUCTURAL ACOUSTIC COUPLING OF PLATES

A Thesis Submitted to the
Graduate School of Natural and Applied Sciences of Dokuz Eylül University
In Partial Fulfillment of the Requirements for the Degree of Master of Science
in Mechanical Engineering, Machine Theory and Dynamics Program

by
Özgür ARPAZ

September, 2006
İZMİR

M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **“ANALYSIS OF STRUCTURAL ACOUSTIC COUPLING OF PLATES”** completed by **ÖZGÜR ARPAZ** under supervision of **PROF. DR. A. SAİDE SARIGÜL** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

....

Prof. Dr. A. Saide SARIGÜL

Supervisor

....

Prof. Dr. Hira KARAGÜLLE

(Jury Member)

....

Prof. Dr. Arif KURBAK

(Jury Member)

Prof.Dr. Cahit HELVACI
Director
Graduate School of Natural and Applied Sciences

ACKNOWLEDGMENTS

I would like to express my deep gratitude to Prof. Dr. A. Saide SARIGÜL for her supervision, valuable guidance and unlimited patience trough the thesis.

I would also like to thank Res. Assist.Abdullah Seçgin for his significant contributions to my thesis, for his great assistance, and for continuous encouragement from the beginning of my study.

I would also like to thank Assist. Prof. Dr. Zeki Kıral, Lecturer Kemal VAROL, Res. Assist. Murat Akdağ for their assistance and moral support in BATÜL Laboratory.

Finally, I wish to express sincere thanks to my family and to my friends for their confidence, patience and support.

Özgür ARPAZ

ANALYSIS OF STRUCTURAL ACOUSTIC COUPLING OF PLATES

ABSTRACT

In vibration and noise control engineering, vibration of a thin plate backed by a cavity, the sound in this cavity and mutual effects of vibration and sound are of considerable importance in the accurate and realistic design of various machines and vehicles. Due to the mechanical, acoustical or both type of excitation on the thin plate-cavity system, a coupling phenomenon occurs between vibration and sound. This thesis mainly focused on a parametric study based on a vibro-acoustic model including a plate-cavity system in order to examine the coupling effects. All analyses were performed by using I-DEAS Vibro-Acoustic module that uses coupled FEM/BEM approach. The parametric free vibration study includes six cases composed of different cavity depths and plate thicknesses. Forced frequency response analysis covers five different excitations, including harmonic, almost periodic, random structural and acoustical excitations. In this regard, free and forced uncoupled and coupled analyses were evaluated by structural, acoustical and energy point of views.

Keywords: Vibro-acoustics, Fluid-structure interactions, Coupled BEM/FEM, I-DEAS

PLAKALARIN YAPISAL-AKUSTİK BAĞLAŞIKLIK ANALİZİ

ÖZ

Titreşim ve ses kontrol mühendisliğinde; arka tarafında boşluk bulunan ince bir plakanın titreşimi, bu boşluktaki ses ve titreşim ile sesin karşılıklı etkileşimleri çeşitli makine ve araçların doğru ve gerçekçi tasarımında büyük ölçüde önem taşır. Plaka-boşluk sisteminde mekanik, akustik veya her iki tipteki zorlamalar nedeniyle, titreşim ve ses arasında bir bağlaşıklık durumu ortaya çıkar. Bu çalışma esas olarak, plaka ve hacimden oluşmuş basit bir bağlaşık vibro-akustik modelin bağlaşıklık etkilerinin parametrik olarak incelenmesine yoğunlaşmıştır. Bütün analizler, bağlaşık BEM/FEM yaklaşımını kullanan I-DEAS Vibro-Akustik modül kullanılarak gerçekleştirilmiştir. Parametrik serbest titreşim analizi, plakaların kalınlıklarının ve boşluk derinliğinin değiştirilmesinden oluşan altı farklı durumu içerir. Zorlanmış frekans tepki analizi, harmonik, yaklaşık periyodik, gelişigüzel yapısal ve akustik zorlamalar olmak üzere beş farklı zorlamayı içerir. Bu bağlamda serbest ve zorlanmış, bağlaşık olmayan ve bağlaşık analizlerin sonuçları, yapısal, akustik ve enerji bakış açısından değerlendirilmiştir.

Anahtar Kelimeler: Vibro-akustik, Akışkan-yapı etkileşimleri, Bağlaşık BEM/FEM, I-DEAS

CONTENTS

Page

Page

CHAPTER ONE

INTRODUCTION

This thesis deals with the vibro-acoustic analysis of acoustically coupled structural systems. If an enclosure formed by thin walls is subjected to any acoustic or mechanical excitation, a coupling phenomenon occurs due to the mutual interaction of sound and vibration. Acoustic pressure waves within the enclosure created by a structural vibration induce a back mechanical load to the structure. With this additional load on the vibrating structure, the interior acoustic pressure waves change. This loop explains the nature of the structural-acoustic coupling phenomenon. The behaviour of sound in small volumes and vibration of volume walls are of considerable importance to noise and vibration control engineers. Machines or systems including a cavity such as washing machines, refrigerators, small rooms, passenger cabins, air and marine vehicles etc., are a few example that this coupling effect should be taken into consideration. Disregarding the coupling effect yields unrealistic results in the vibration or acoustic analysis of cavity systems.

If for a fluid loaded vibrating structure, the direct influence of the fluid (air, water, oil etc.,) on the vibration and sound distribution of the structure is assumed as negligible, such an analysis is known as “uncoupled analysis”. The solution in uncoupled analysis is performed at two discrete stages; firstly, vibration of the structure is treated and then, sound radiation caused by this structural vibration is considered. However, in this case, there may be some uncertainty due to disregarding the back loop effect of the fluid loading. Therefore, an accurate analysis takes into account of vibration and acoustic radiation simultaneously with their mutual effects. This type of analysis is known as “coupled analysis”. Particularly, for dynamic systems in which a small acoustic cavity is enclosed by a thin-walled structure, this mutual interaction should not be neglected.

Many researchers have investigated vibro-acoustic behaviour from different points of views, by using different numerical techniques, different models and different cavities.

Lyon (1963) has examined the noise attenuation of a box with one flexible wall. The noise reduction has been computed in this study theoretically. The study considers the usual practical situation in which the flexible panel’s natural frequency is lower than the natural frequency of the first cavity mode. The frequency range is broken into three distinct parts according to the panel and acoustic volume’s resonance behaviour, regarding noise reduction calculations.

Pretlove (1965) has studied the effect of a closed cavity on the free vibrations of a flexible panel. He developed a mathematical model and compared this mathematical model’s results with experimental results. Pretlove examined the effect of relation between the plate dimension and the depth of the cavity on coupling. Pretlove used two different cavities, and concluded that the coupling effect should be taken into account for shallow cavities.

Pretlove (1966) has modified his free vibration theory for plate-cavity system which is excited by external random acoustic pressures. He concluded that large effects of coupling only occur for thin panels covering shallow cavities.

Guy and Bhattacharya (1973) have concerned with the effect of a cavity on the back of a panel on sound transmission and the vibration of the panel. The phenomena, such as negative transmission loss, combined panel and cavity resonance have been examined. In this study, two models have been used in the theoretical analysis and experiments performed for comparison.

Mariem and Hamdi (1987) have developed a mixed formulation based on coupling the functionals of structure and fluid to study the fluid-structure interactions. By using this formulation the discretization of the fluid domain required in finite element method has been avoided. Mariem and Hamdi have applied this formulation to the analyses of a baffled plane and a flexible panel backed by a cavity to find the transmission loss factor and the dominant mode in noise transmission.

Frendi and Robinson (1993) have studied the effect of acoustic coupling on plate vibrations using two numerical models for random and harmonic excitations.

They concluded that the coupling is important at high excitation levels for both random and harmonic excitations, and therefore should be taken into account for accurate prediction of the plate response. They also stated that the coupling is important for high frequencies, nonlinear structural response and acoustic radiation.

Kopuz (1995) have studied an integrated FEM/BEM approach for the prediction of interior noise fields of cavities. In this work, Kopuz has used the commercial program SYSNOISE and examined open ended and closed cavities with a coupled analysis. Sound pressure levels produced inside the structure were calculated and the uncoupled, coupled and experimental results were compared. Although, Kopuz has stated that coupled analysis requires much more computer memory and time compared to uncoupled analysis; and concluded that a coupled analysis may not be justified when the cost of making this type of analysis is considered, the coupling results exhibit considerable stiffness effect caused by the cavity.

Ding and Chen (2001) has established a symmetrical finite element model for structural-acoustic coupling analysis of an elastic, thin-walled cavity under the excitation of interior acoustic sources and exterior structural loading. They have deviced on experimental set-up and measured sound pressure amplitudes in the cavity to validate the correctness of the proposed method.

Lee (2002) has studied the effect of structural-acoustic coupling on the nonlinear natural frequency of a rectangular box which consists of one flexible plate. The analysis has been performed by using a finite element multi-mode approach and the effect of the cavity depth on the natural frequencies and convergence studies have been discussed. It has been concluded that, air cavity formed by a rectangular box have a negative stiffness effect on the flexible plate whereas a positive stiffness effect is present when the air pressure acting on plate is out of phase to the plate motion.

Li and Cheng (2006) have developed a coupled vibro-acoustic model to examine the structural and acoustic coupling of a flexible panel backed by a cavity with a tilted wall. The model is based on integro-modal approach. They showed the geometrical distortion effect on the vibro-acoustic behaviour of the coupled system. They have used the averaged sound pressure level inside the enclosure and the averaged quadratic velocity of the vibrating plate for showing the distortion effect on the vibro-acoustic behaviour. They concluded that even a small distortion highly affects the acoustic pressure distribution in the cavity. They found also that a slight distortion of the enclosure may affect noticeably the structural–acoustic coupling, and hence inside acoustic field.

There are many studies and results on coupled and uncoupled vibro-acoustic analyses in literature. These results sometimes conflict with each other for example on the subject of the superiority of the coupled solution on the uncoupled solution and the factors affecting the necessity of coupled solution. In this study, modal and frequency response analysis of structural-acoustic coupling problem is presented by using box models with one flexible wall. Through the analyses, I-DEAS™ software, which uses the theory of integrated Boundary Element Method/Finite Element Method (BEM/FEM), was used. The analysis procedure requires a geometric model to be created by I-DEAS™ Master Modeler and Meshing Task and a vibro-acoustic model to be built by I-DEAS Vibro-Acoustics™ Task. Firstly, some problems for which the results are present in literature were performed for the verification of use of I-DEAS software. Secondly, free vibration analyses, including six cases obtained by different combinations of the thickness of the plate and the depth of the cavity, were done. At last, forced vibration analyses, for five types of structural excitations, harmonic, almost periodic, random, the combination of random force with an acoustic source, random force with a lower amplitude, both for uncoupled and coupled cases were performed. The results are evaluated by structural, acoustical and energy point of views. The cavity effects on the plate vibrations and sound pressures are shown. Differences between coupled and uncoupled analysis are shown by frequency and amplitude changes in the frequency spectra. Since uncoupled analysis neglects the mutual interaction of sound and vibration, it yields unrealistic behaviour and unacceptable results in box like structures even at low frequencies.

CHAPTER TWO

THEORETICAL BACKGROUND

In this chapter, a general theory of wave motion, which is the theoretical base of most vibro-acoustic methods, is presented at the beginning of this section. Next, basic principles of most commonly used prediction techniques in low frequencies such as Finite Element Method (FEM) and Boundary Element Method (BEM) are given. The algorithm of a hybrid approach, which is known as coupled FEM/BEM is presented at the end of this section.

2.1 Wave Motion

2.1.1. Wave Motion in an Unbounded Medium

The equation of motion of a homogeneous, isotropic, linearly elastic body may be described in terms of the displacement vector $\boldsymbol{u}$ and body force $\boldsymbol{f}$ per unit mass of material (Graff, 1973);

$$(\lambda + \mu)\nabla\nabla \cdot \boldsymbol{u} + \mu\nabla^2 \boldsymbol{u} + \rho \boldsymbol{f} = \rho \ddot{\boldsymbol{u}} \tag{2.1}$$

where λ and μ are the Lamè constants, ρ is the mass density. This equation is the vector equivalent of *Navier's equations*. In terms of rectangular scalar notation, this represents the three equations

$$(\lambda + \mu)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z}\right) + \mu\nabla^2 u + \rho f_x = \rho \frac{\partial^2 u}{\partial t^2}, \tag{2.2}$$

$$(\lambda + \mu)\left(\frac{\partial^2 u}{\partial y \partial x} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z}\right) + \mu\nabla^2 v + \rho f_y = \rho \frac{\partial^2 v}{\partial t^2}, \tag{2.3}$$

$$(\lambda + \mu)\left(\frac{\partial^2 u}{\partial z \partial x} + \frac{\partial^2 v}{\partial z \partial y} + \frac{\partial^2 w}{\partial z^2}\right) + \mu\nabla^2 w + \rho f_z = \rho \frac{\partial^2 w}{\partial t^2}. \tag{2.4}$$

Here *u, v, w* are the particle displacements in the *x, y, z* directions. A simpler set of equations can be obtained by introducing the scalar and vector potentials ϕ and $\boldsymbol{H}$ for

u; ψ and ***F*** for ***f***, respectively, using Helmholtz decomposition theorem (Graff, 1973);

$$\boldsymbol{u} = \nabla\phi + \nabla\times\boldsymbol{H} \;;\; \nabla\cdot\boldsymbol{H} = 0 \tag{2.5}$$

$$\boldsymbol{f} = \nabla\psi + \nabla\times\boldsymbol{F} \;;\; \nabla\cdot\boldsymbol{F} = 0\,. \tag{2.6}$$

Substituting Equations (2.5) and (2.6) in Equation (2.1) and by use of the relation $\nabla\cdot\nabla\phi = \nabla^2\phi$, the fact that $\nabla^2(\nabla\phi) = \nabla(\nabla^2\phi)$ and the fact that $\nabla\cdot\nabla\times\boldsymbol{H} = 0$, one may obtain;

$$\nabla\left\{(\lambda + 2\mu)\nabla^2\phi + \rho\psi - \rho\ddot{\phi}\right\} + \nabla\times(\mu\nabla^2\boldsymbol{H} + \rho\boldsymbol{F} - \rho\ddot{\boldsymbol{H}}) = 0 \tag{2.7}$$

From equation (2.7), one can obtain following equations;

$$\nabla^2\phi - \frac{1}{\boldsymbol{c_L}^2}\ddot{\phi} = -\frac{\rho}{(\lambda + 2\mu)}\psi\,, \tag{2.8}$$

$$\nabla^2\boldsymbol{H} - \frac{1}{\boldsymbol{c_T}^2}\ddot{\boldsymbol{H}} = -\frac{\rho}{\mu}\boldsymbol{F}\,. \tag{2.9}$$

In equation (2.8); c_L is the propagation velocity of longitudinal waves and defined as $c_L = \sqrt{\frac{\lambda + 2\mu}{\rho}}$, also called primary (P) waves. In equation (2.9); c_T is the propagation velocity of shear waves and defined as $c_T = \sqrt{\frac{\mu}{\rho}}$, also called secondary (S) waves. Equations (2.8) and (2.9) are known as inhomogeneous wave equations.

2.2 Prediction Techniques for Low-Frequency Analysis

2.2.1. *Finite Element Method (FEM)*

In engineering practice, to find analytical solution of equation of motion may be very hard, therefore, approximate solutions are generally required. The finite element method is the most commonly used method for solving dynamic problems. In this method, the vibrating body is divided into small elements.

When harmonic structural vibration of a dynamic system is described by the finite element equations, the system of equations may be written in the following form,

$$\left[-\omega^2[M]+i\omega[C]+[K]\right]\{d\}=\{f_e\} \tag{2.10}$$

Here $[M]$ is the structural mass matrix, $[C]$ is the structural damping matrix, $[K]$ is the structural stiffness matrix, $\{d\}$ is the nodal structural displacement vector and $\{f_e\}$ is the external excitation force vector applied at structural nodes.

2.2.2 Boundary Element Method (BEM)

Since the sound propagation in fact is a wave motion, this motion is formulated in the form of homogenous three-dimensional wave equation in terms of the acoustic pressure *p*:

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \,. \tag{2.11}$$

Here c is the speed of sound. Acoustic field around the vibrating arbitrarily shaped body with the surface S and the outward normal ***n*** may be calculated by using the Helmholtz integral equation [Sarıgül, 1990],

$$C(x)p(x) = \int_S \left\{ p(y)\frac{\partial G(R,k)}{\partial n(y)} + iz_o k u_n(y) G(R,k) \right\} ds(y) \tag{2.12}$$

where, u_n is the normal surface velocity, y is any point on the surface *S*, *x* is any point in space, $R = |x - y|$, $i=\sqrt{-1}$, *k* is wave number ($k = \omega/c$) and $e^{i\omega t}$ time dependence is assumed. *ds* is differential boundary element, $G(R,k) = e^{(-ikR)}/R$ is the three-dimensional free-space Green's function, $z_0=\rho_0 c$ is the characteristic impedance of the medium, ρ_0 is the density of the fluid. *C(x)* has different values in accordance with the position of the field point *x*. When the boundary element method is implemented to Equation (2.12), a system of equations,

$$[L]\{p\} = [H]\{u_n\} \tag{2.13}$$

is obtained. Here $\{p\}$ and $\{u_n\}$ are $N \times 1$ vectors including the acoustic pressure and normal velocity of N nodes used in the boundary element meshes, respectively. $[L]$ and $[H]$ are $N \times N$ square matrices.

2.2.3 Coupled FEM/BEM

In FEM and BEM, dynamic variables within each element are expressed in terms of simple shape functions. In order to represent the spatial variation of the dynamic response accurately within each element, very large number of elements, and therefore, subsequent computational effort are required. Since at high frequencies structures have low and variable vibration amplitudes, the element size should be decreased as the frequency increases. This condition restricts practical usage of FEM and BEM in high frequencies.

In low frequency vibro-acoustic problems, BEM has some advantages compared to FEM. BEM needs smaller effort to model infinite domain. In the finite element analysis the entire source volume is discretized whereas in the BEM only the surface of the source is discretized. Besides, the FEM is computationally more efficient for solving vibro-acoustic problems, defined in a bounded fluid domain. In fluid-structure interaction problems, especially, in low frequencies, FEM and BEM may be implemented simultaneously to the structure-fluid system by hybridizing both methods using their superiorities. Coupled FEM/BEM technique is successfully used for coupled structural-acoustic problems in vibro-acoustics (Meriam and Hamdi, 1987; Kopuz, 1995; Everstine, 1997; Chen et al, 1998; Vlahopoulos et al, 1999; Coyette, 1999).

Harmonic structural vibrations described by the finite element equations are given in Equation (2.10). If an additional acoustic loading term is added, Equation (2.10) may be written as follows:

$$\left[-\omega^2[M]+i\omega[C]+[K]\right]\{d\}=\{f_e\}-[G_c]\{p\}. \tag{2.14}$$

Here $[G_c]$ is the transformation matrix including surface area induced by the acoustic pressures, and term $[G_c]\{p\}$ represents the additional structural loading created by the acoustic pressure field on the structure.

The boundary element implementation of acoustic radiation in Equation (2.13) may be rewritten in terms of the displacement vector d instead of the particle velocity u_n in order to simulate acoustic-fluid interaction using same global variables. Using the particle velocity u_n and the density of the fluid in the medium ρ_0, the momentum equation is written as,

$$\rho_0 \frac{\partial u_n}{\partial t} = -\nabla p\,. \tag{2.15}$$

Assuming a harmonic change for the acoustic particle velocity $u_n = Ue^{i\omega t}$

$$\frac{\partial u_n}{\partial t} = i\omega u_n \tag{2.16}$$

Substituting Equation (2.15) in Equation (2.16), the following equality is obtained:

$$\frac{\partial p}{\partial n} = -i\rho_o \omega u_n\,. \tag{2.17}$$

Here u_n now is the normal surface velocity which is equal to the particle velocity at the interaction boundary. By using Equation (2.17), Equation (2.13) may be rewritten as

$$[L]\{p\} = \frac{i}{\rho_0 \omega}[H]\left\{\frac{\partial p}{\partial n}\right\}. \tag{2.18}$$

Setting $\left[R\right] = -\frac{i}{\rho_0 \omega}\left[H\right]$ into Equation (2.18), the following equality is obtained:

$$\left[L\right]\{p\} + \left[R\right]\left\{\frac{\partial p}{\partial n}\right\} = 0 \tag{2.19}$$

The relation between normal surface velocity and vibration displacement at the boundary may be written as;

$$d = \int u_n dt = \frac{1}{i\omega} u_n \text{ and } u_n = i\omega d \,. \tag{2.20}$$

Substituting Equation (2.20) in Equation (2.17), one can obtain,

$$\frac{\partial p}{\partial n} = \rho_o \omega^2 d \,. \tag{2.21}$$

Finally, the boundary element implementation of acoustic radiation in Equation (2.13) may be rewritten in order to simulate coupling interaction using same global variables with an additional acoustic excitation such as a monopole source (Q is strength of the monopole source):

$$[L]\{p\} + \rho_0 \omega^2 [R]\{d\} = \{Q\} \tag{2.22}$$

If Equation (2.14) and Equation (2.22) are settled correspondingly on to the global variables, a general matrix form may be obtained as follows;

$$\begin{bmatrix} \left[K\right] + i\omega\left[C\right] - \omega^2\left[M\right] & \left[G_c\right] \\ \rho_0 \omega^2 \left[R\right] & \left[L\right] \end{bmatrix} \begin{Bmatrix} d \\ p \end{Bmatrix} = \begin{Bmatrix} f_e \\ Q \end{Bmatrix} \tag{2.23}$$

Equation (2.23) is known as general coupled FEM/BEM matrix representation. Although, Equation (2.23) can be solved simultaneously for coupled vibration and acoustic response analyses, the homogenous form of this equation can be used in the prediction of coupled acoustic and structural natural frequencies with their coupled modes by solving following equation,

$$\begin{bmatrix} [K]+i\omega_j[C]-\omega_j^{\,2}M & [G_c] \\ \rho_0\omega_j^{\,2}[R] & [L] \end{bmatrix} \begin{Bmatrix} d_j \\ p_j \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{2.24}$$

where ω_j is the coupled natural frequency ($j = 1, 2, \cdots n$, n is the number of frequencies solved), $\{d_j\} = \{d_j^{(1)}, d_j^{(2)}, \cdots d_j^{(N)}\}$ and $\{p_j\} = \{p_j^{(1)}, p_j^{(2)}, \cdots p_j^{(N)}\}$ are a set of coupled structural mode shapes, and coupled acoustic mode shapes respectively.

CHAPTER THREE

BUILDING A VIBRO-ACOUSTIC MODEL BY I-DEAS™

In this thesis, structural-acoustic behavior of acoustic enclosure-plate systems under free, structurally and acoustically forced conditions is examined. The Box-like model used in this work is illustrated in Figure 3.1. The model was formed in different dimensions by using I-DEAS Vibro-Acoustics™ software.

3.1 The Steps of Building a Vibro-Acoustic Model by I-DEAS

In this chapter, vibro-acoustic analysis by I-DEAS™ is introduced by building a Box-like model which simulates the coupling between thin plate and acoustic enclosure. The analysis chart of Box-like model is demonstrated in Figure 3.2. In I-DEAS™, a vibro-acoustic model is built by utilizing different tasks (or modules) which are I-DEAS™ Master Modeler Task, I-DEAS™ Meshing Task, I-DEAS™ Boundary Conditions Task and I-DEAS™ Modal Solution Task. After the building of the model, analysis can be performed by I-DEAS™ Vibro-Acoustics Task whose solution procedure is based on the coupled BEM/FEM theory. The analysis is mainly performed by the following steps:

1. Creating geometric model
2. Meshing the geometry
3. Defining boundary conditions
4. Performing post-processing
    - Computing modal parameters
    - Calculating frequency response

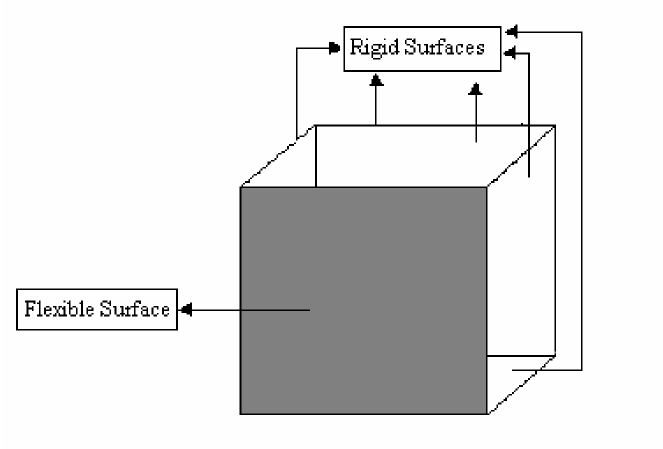


Figure 3.1 Box-like Model.

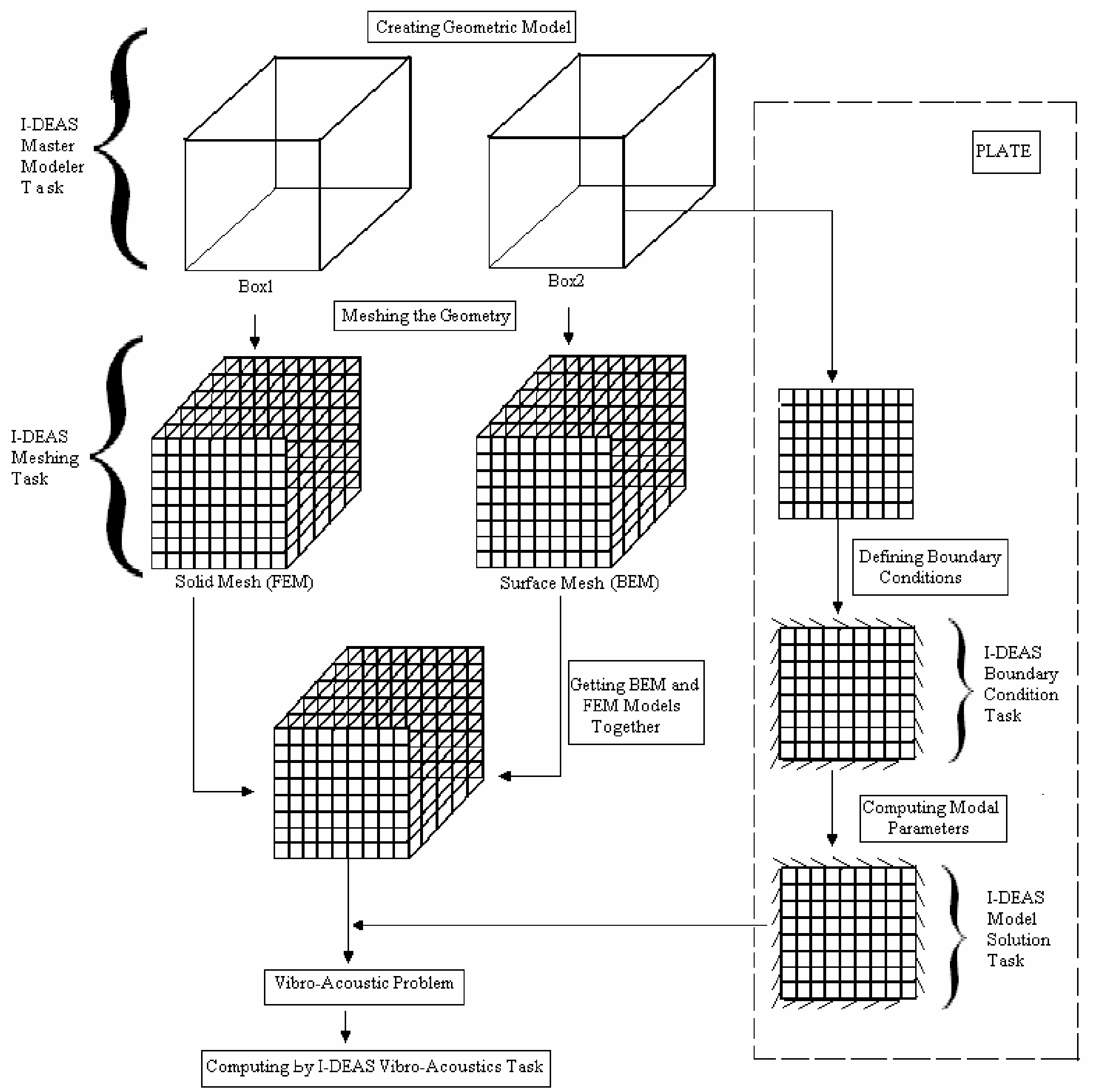


Figure 3.2 Schematic representation of structural-acoustic coupling analysis by using I-DEAS™ for a Box-like model.

3.2 Creating Geometric Model

Building the geometry of model is the first step of defining a vibro-acoustic problem. Since the solution methodology (Coupled BEM/FEM) of I-DEAS requires two different mesh types, two identical box models are created by I-DEAS™ Master Modeler Task and named as “Box1” and “Box2”, respectively.

3.3 Meshing the Geometry

The next step is meshing the geometric models named as “Box1” and “Box2” by using I-DEAS™ Meshing Task. In this task, two kinds of meshes are required for this purpose;

- **Solid (Volume) mesh**: Volume meshes are used to model 3D interior problems using the Finite and Infinite Element Methods. Here, FEM type of mesh was used to create internal domain.
- **Surface mesh:** Surface meshes are used to model 3D problems using the Boundary Element Method in order to create the external domain.

3.3.1 Meshing the “Box1”

The model geometry “Box1” is defined by using solid finite elements. The operation procedure for defining solid mesh is given as follows;

1. Click “Get” icon, and call the geometry named “Box1” in the I-DEAS™ Meshing Task. By selecting “Define Solid Mesh” icon, define the mesh type as volume elements and give a name to this meshed model. (Name: “FEM”)
2. By using the right button of mouse, “*-All_done” is selected for which all elements defining “Box1” are lighted.
3. From the menu which appears on the screen, “Mapped options” is chosen (free mesh would generate tetrahedral elements while the mapped mesh generates solid bricks) then “Define Elements/Side” is chosen in order to define mesh size.

4. Finally, the volume mesh is generated by utilizing “generate solid mesh” icon.

3.3.2 Creating Volume

After meshing the “Box1”, the internal fluid of the cavity must be formed by using these independent volume elements of “Box1”. By selecting “create group” icon in I-DEAS™ Meshing Task, the volume finite elements are grouped and named. (Name: “VFEM”).

3.3.3 Creating Surface Coating of Elements

In order to associate different mesh types such as finite and boundary elements at the same layer, creation of surface coating of elements of “VFEM” is required. A bounding layer is created to serve as the fluid boundary. Surface coating option is chosen from menu (Ctrl+m/ element/ multiple create/ surface coating). This operation makes the solid elements layered through the volume and volume elements are converted to thin shell elements.

3.3.4 Creating a Group for Volume Finite Elements

As indicated in Figure 3.1, front face of the box is flexible whereas other faces of the box are rigid. In this regard, a group composed of thin shells was created and this group was named as “FEM Rigid”. Secondly, another group composed of thin shells on the front face of the box was created and this group was named as “FEM Flexible”. Finally, “FEM Flexible” was removed from “FEM Rigid” by using “remove option” at “Groups” icon.

When creating groups, there is a pop-up menu (accessed by using the right-hand mouse button) that is called filter. This option allows the user to set up a filter to select a subset of the entire display.

3.3.5 Meshing the "Box2"

The model geometry "Box2" is defined by using shell mesh elements. The operation procedure for defining shell mesh is given as:

1. Click "Get" icon, and call the geometry named "Box2" in the I-DEAS™ Meshing Task. Define the mesh type as shell mesh elements and give a name (Name: "BEM").
2. By using the right button of mouse, "*-All_done" is selected.
3. From the menu, first "Mapped options", and then "Define Elements/Side" are chosen sequentially in order to define mesh size.
4. The shell mesh is generated by utilizing "Generate Shell Mesh" icon.

3.3.6 Creating a Group for Boundary Elements

Firstly, a group composed of thin shell elements was created and named as "BEM Rigid". Secondly, another group composed of thin shell elements on the front face of the box was created and named as "BEM Flexible". Finally "BEM Flexible" was removed from "BEM Rigid".

3.3.7 Appending the BEM Model to the FEM Model

In order to use two different mesh types in the same analysis, an appending operation is required. This can be done by getting these two different mesh types together with the following procedure;

1. From the menu, select "Copy Options" and set the append options (Ctrl+m /Manage/ Copy Options).
2. Then "Append" is chosen in the manage control menu (Ctrl+m /Manage/ Append) and finally, "BEM" as "Source FE Model" and "FEM" as "destination FE Model" are signed.

3.4 Creating the Structural FE Model

In order to solve a coupled vibro-acoustic problem, firstly, a structural finite element model has to be built for computing its modal parameters, and specifying the mechanical loading. The procedure for creating structural FE Model is given as follows:

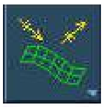
1. From “Box2” part, a new mesh is created and named as “Plate”.
2. “Shell mesh” is defined on the front face of “Box2”.
3. The number of elements per each side was defined from “Define Elements/Side” on the “Mapped options”.
4. Shell mesh is generated by “Generate shell mesh” icon.
5. Thickness may be changed by “Modify” icon (In this study, two values were used: one is 0.5 mm and the other is1mm) .
6. Boundary conditions are implemented to the “Plate” in I-DEAS Boundary Conditions™. “Normal Mode Dynamics” is chosen and four sides of “Plate” are restrained.
7. In I-DEAS Model Solution™ task, after a solution set is created and “Solve” command is performed, modal solution can be obtained.

3.5 Solving Vibro-Acoustic Problems by I-DEAS Vibro-Acoustics™

Modeling vibro-acoustic problems requires creating specific entities, such as acoustic materials, surface contours of fluids, etc. These specific entities are defined by using the dedicated icons of the I-DEAS Vibro-Acoustics™ task. These are: Fluid surface, Fluid volume, Acoustic fluid, Acoustic domain, Acoustic material, Acoustic property, Plane, Structure model, Spectrum, Acoustic loads, Mechanical loads, Load set, Domain points, Solution set. The definitions of these icons and their usages will be introduced in the following sections.

3.5.1 “Fluid Surface” Icon

A “Fluid surface” is a set of shell elements with the same physical acoustic property and wetted by the same fluid on one or both sides. Fluid surfaces can be defined by “Quick Create” button on the menu as shown in Figure 3.3. If groups have been created previously in meshing task, one can define these groups as a fluid surface. In this analysis, fluid surfaces are “BEM FLEXIBLE”, “BEM RIGID”, “FEM FLEXIBLE” and “FEM RIGID” as indicated previous sections.



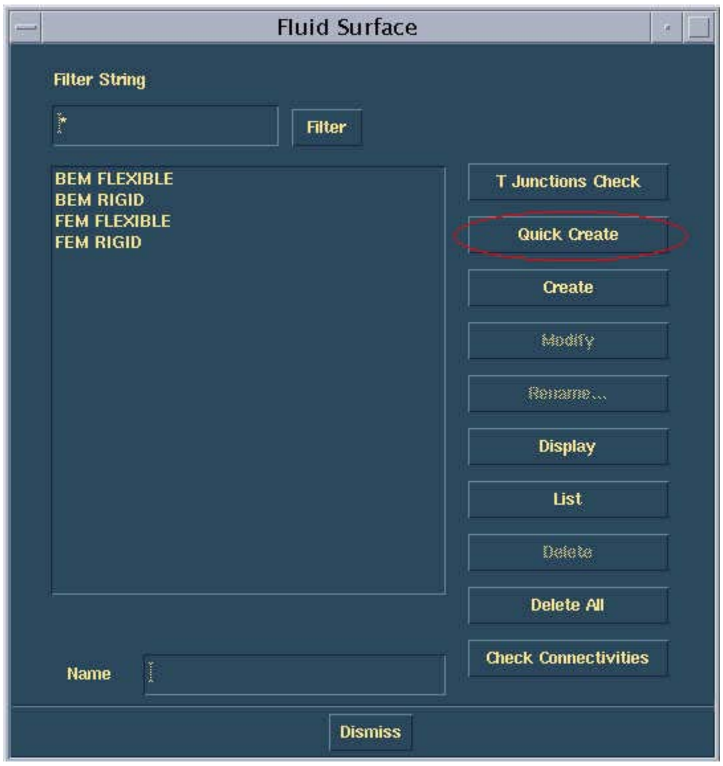


Figure 3.3 Fluid Surface icon and menu.

3.5.2 "Fluid Volume" Icon

A fluid volume is a set of volume finite elements that define a fluid in the Box-like model. Fluid volume can be created by "Quick Create "button on the menu as shown in Figure 3.4. In this analysis, fluid volume is "VFEM" as created in I-DEAS™ Meshing Task before.

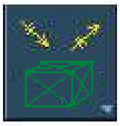
3.5.3. "Fluid Domain" Icon

An acoustic domain is the association of sides of fluid surfaces and fluid volumes (when the finite or infinite element methods are used).

In this vibro-acoustic problem, two fluid domains, which would be created outside and inside of the box, are required. Fluid Domain creating icon and menu is given in Figure 3.5. The fluid domain creation procedure is presented as follows;

- Click "Fluid Domain" icon then select "Create" for creating internal domain and change the domain name as "Internal" and default fluid as "Air".

- For internal domain, the fluid surfaces “FEM Flexible” and “FEM Rigid” are sent to “Selected Surfaces” in the fluid domain menu. Since the positive side of each surface directs to the inside of the box, the control “Selected side” is changed from “Both” to “Plus”. “VFEM” in the “Available Volume” is sent to “Selected Volume”.
- Utilizing “Create” button again, new fluid domain is created and named as “External”. In “External” domain, “BEM Flexible” and “BEM Rigid” are sent to “Selected Surfaces”. Since the negative side of each surface directs to the outside of the box, the control “Selected side” is changed from “Both” to “Minus”. The “Default Fluid” is air in “External” domain.



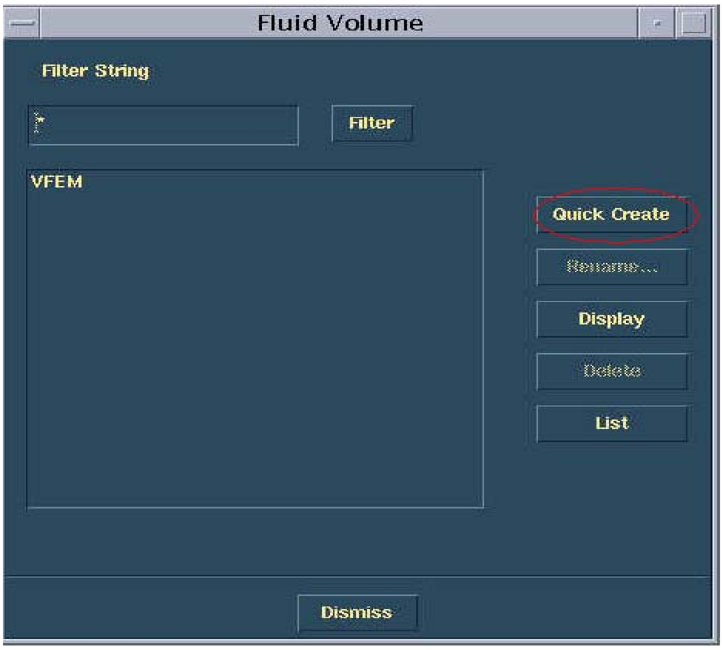


Figure 3.4 Fluid Volume icon and menu.

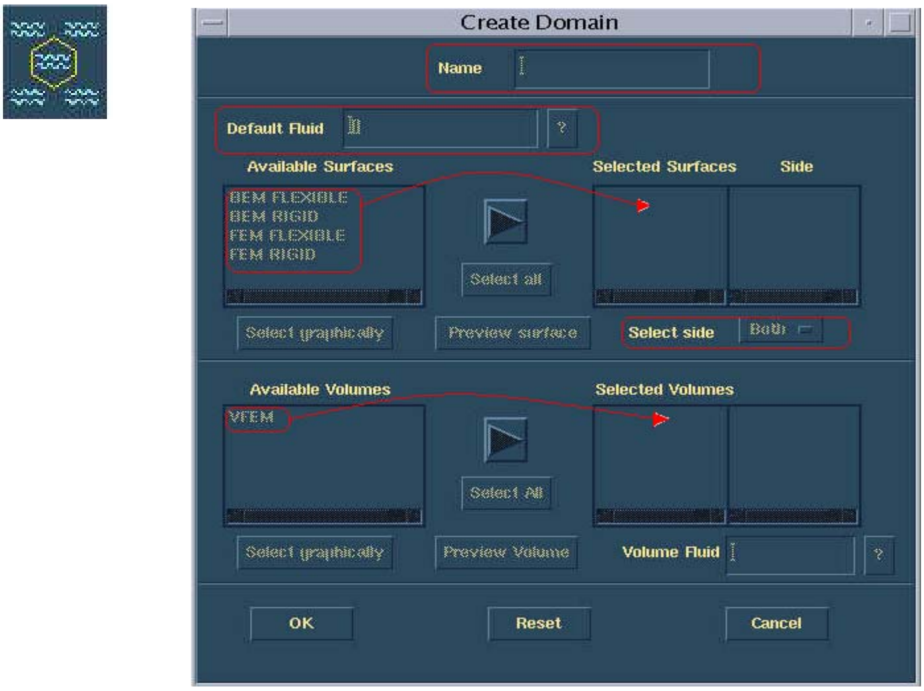


Figure 3.5 Create Domain icon and menu.

3.5.4 “Acoustic Properties” Icon

One of the most important definitions of the fluid model is the acoustic property assigned to the fluid surface. Different properties can be assigned to surfaces, such as rigid, elastic, prescribed acoustic acceleration, prescribed acoustic velocity, prescribed acoustic displacement, prescribed pressure, transmission hole and fluid hole.

The procedure for associating acoustic properties to fluid surfaces can be given as follows:

- Select “Domain Filter” as “External” and then “BEM Flexible” is selected as “Elastic” type as illustrated in Figure 3.6.
- Select “Domain Filter” as “Internal” and then “FEM Flexible” is selected as “Elastic”.

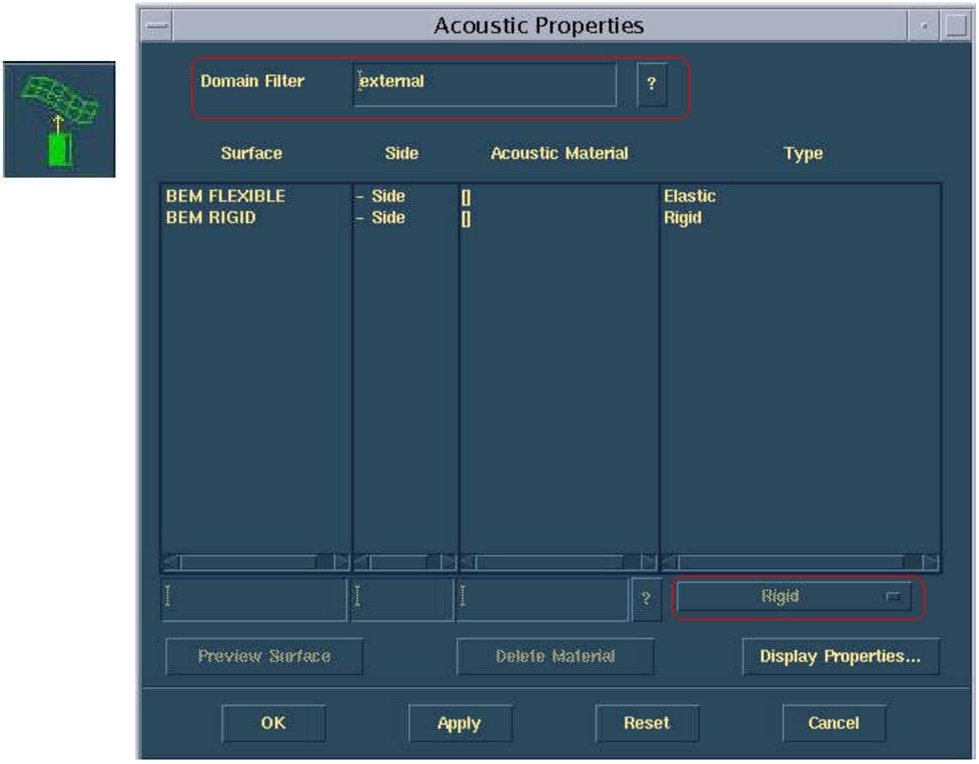


Figure 3.6 Acoustic Properties icon and menu.

3.5.5 " Structure Model" Icon

Structural model "Plate" built in I-DEAS™ Master Modeler must be imported to I-DEAS™ Vibro-Acoustics Task after obtaining modal solutions performed in I-DEAS™ Model Solution Task. The procedure of importing the structure model is given as follows;

- Click "Create" button in the menu in Figure 3.7.
- From menu in Figure 3.8, name is given (Name: "str") then "Get FEM" menu is opened and FE Model is selected (FE Model is the model which is built and named at I-DEAS™ Master Modeler Task). By this menu, nodal forces which are created at I-DEAS™ Boundary Condition Task can be assigned as mechanical loading.

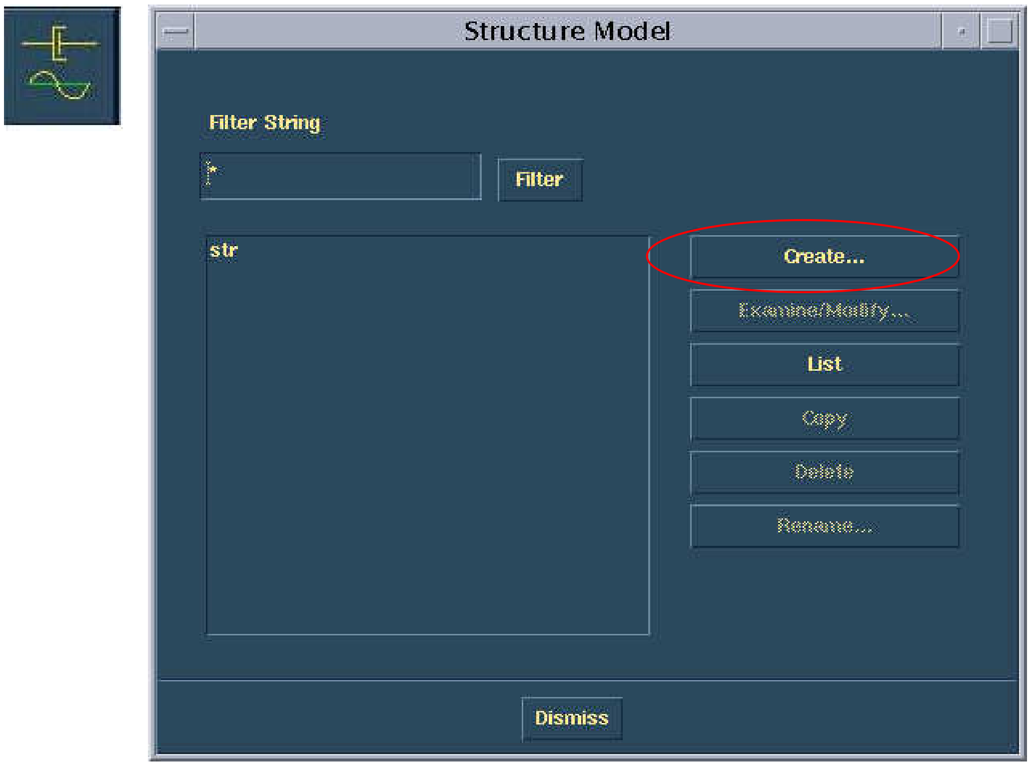


Figure 3.7 Structure Model icon and menu.

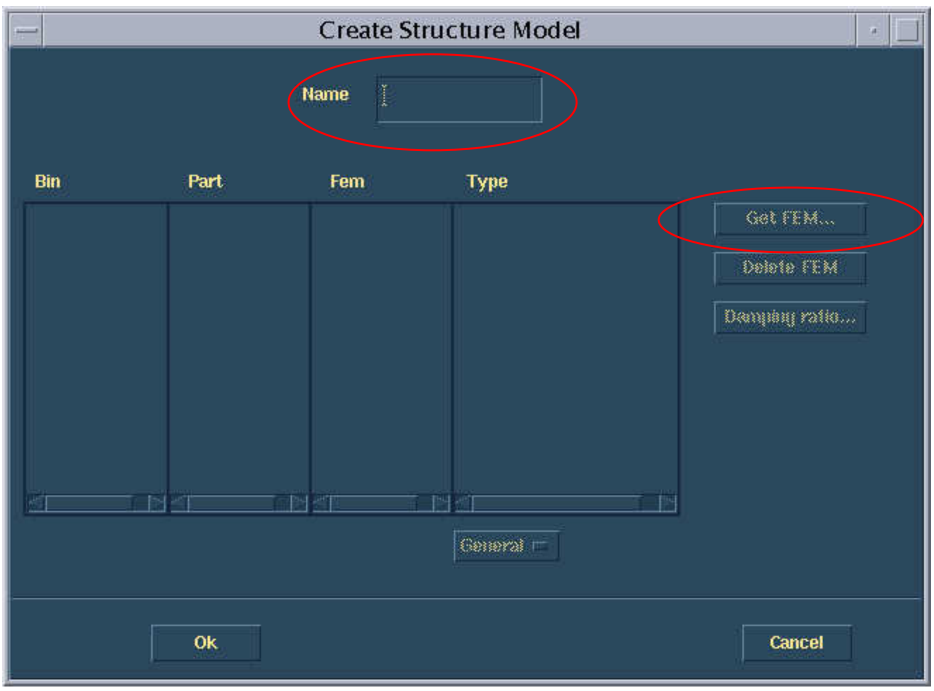


Figure 3.8 Structure Model create menu.

3.5.6 Creating a Spectrum

Vibro-acoustic task uses only frequency dependent functions to solve problems. Spectra describe the variation with frequency of an entity, such as the amplitude of an acoustic source, amplitude of force or admittance of a material. Icons that are shown in Figure 3.9 are used for creating Spectra in I-DEAS Vibro-Acoustics™ Task.

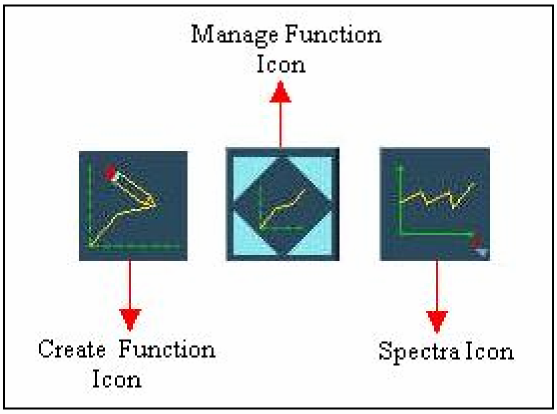


Figure 3.9 Icons for creating Spectra.

The procedure to define a spectrum is as in follows;

- Use “Create functions” icon to define the function. The type of the function must be Spectrum (type 13). Also function should be complex. The abscissa of the function must be frequency and the ordinate depends on the physical quantity.
- Use “Manage function” icon, to transfer the function array that you have just created. Give a record name to the function.
- In the form which comes after clicking “Spectra” icon, select the library in which the spectrum will be stored.
- Click “Add”, give name to the spectrum and choose the record that you have just created.

3.5.7 "Acoustic Loads" Icon

In I-DEAS™ Vibro-Acoustic Task, four types of acoustic sources can be specified as shown in Figure 3.10. In analyses and in the problem for verification of I-DEAS™ Vibro-Acoustic software in the next sections, "Monopole" source and "Surface Load" were used respectively.

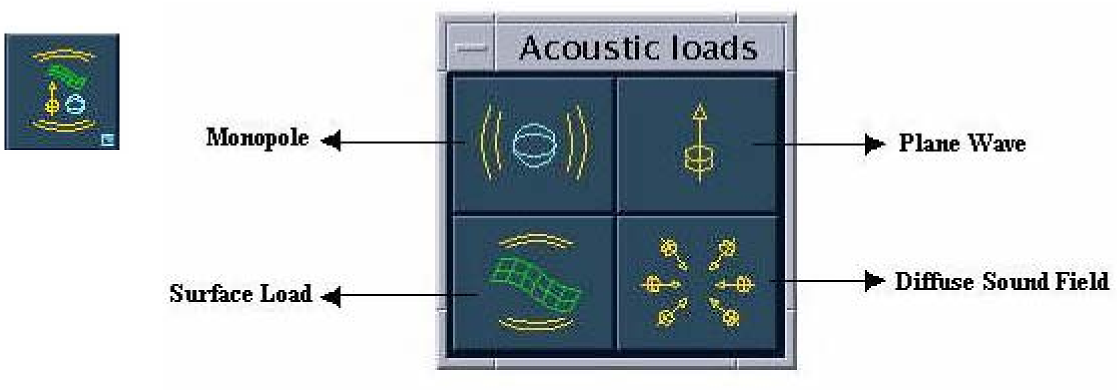


Figure 3.10 Acoustic Loads icon and menu.

Creating a "Monopole" source for our analyses is as in the following;

- "Monopole" menu is opened from the "Monopole" button in Figure 3.10.
- In the monopole menu (Figure 3.11), "Monopole" name is given. A node for monopole source is selected by the button named as "Select a node".
- Domain is selected where the monopole source is located.
- Specify whether the monopole is random or deterministic.
- If the monopole is deterministic, spectrum for monopole is selected which represents the variation with respect to frequency of the amplitude of the monopole (spectrum must be in I-DEAS™ Library or must be created and added to library).

In analyses, the monopole is located as in Figure 3.12. The name of the source was given as "monopole" and the domain was assigned as "internal".

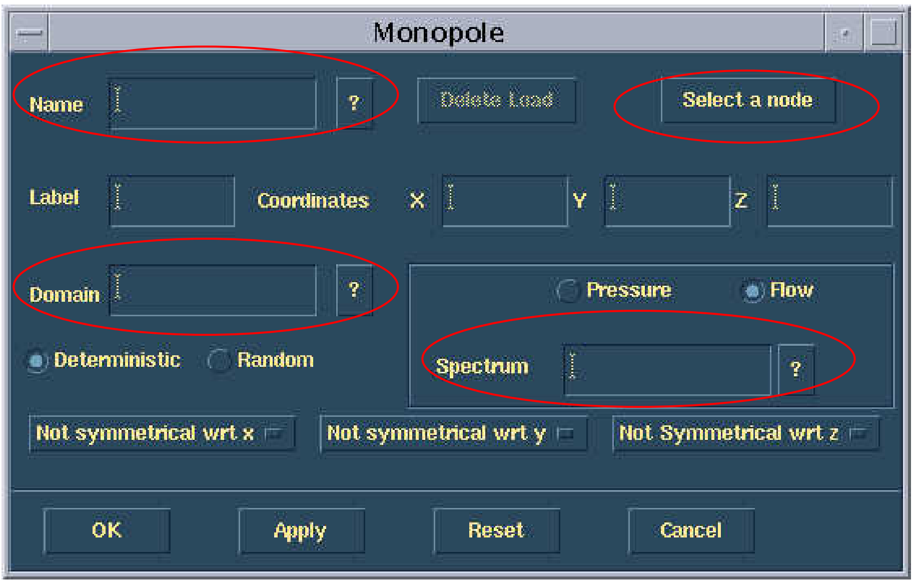


Figure3.11 Monopole Menu.

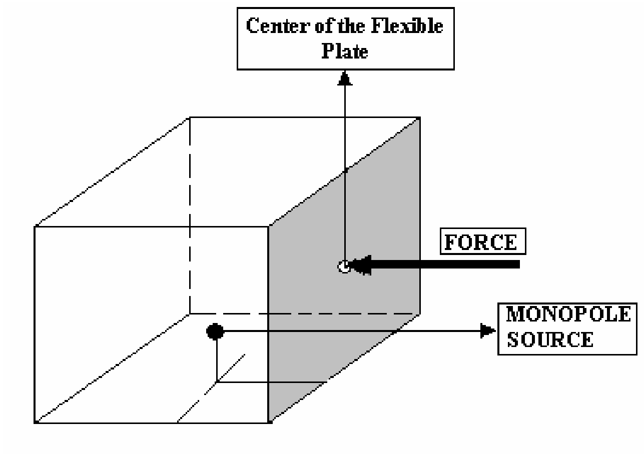


Figure 3.12 Location of monopole source and mechanical force.

Creating "Surface load" for Vibro-Acoustic problems is as follows;

- "Surface load" menu is opened from the "Surface Load" button in Figure 3.10.
- In the surface load menu (Figure 3.13), a name is given.
- Domain and load type are selected. In this part, four types of load can be chosen; acoustic pressure, acoustic acceleration, acoustic velocity, acoustic displacement. For the uncoupled problem in Chapter 4, "acoustic velocity" was selected as "load type".
- A fluid surface is selected for a surface load.

- Spectrum for surface load was selected as "spatially constant".

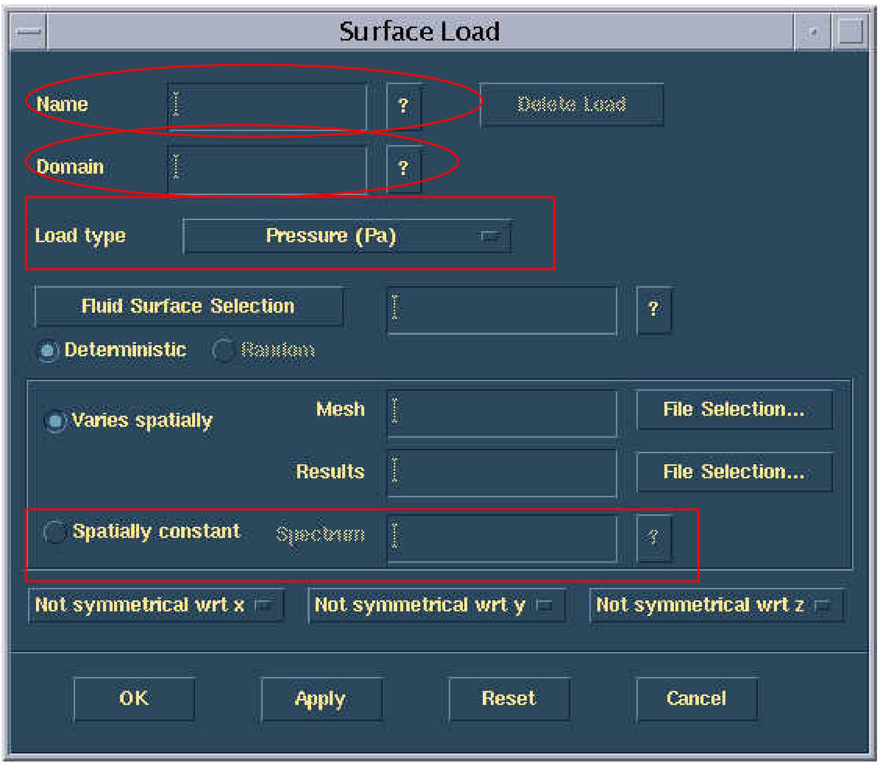


Figure 3.13 Surface Load menu.

3.5.8 Creating a Nodal Force in Vibro-Acoustics Task

Mechanical nodal forces can be created in Modal Response Task, Boundary Conditions Task and Vibro-Acoustics Task. Here the procedure for creating nodal force on I-DEAS Vibro-Acoustics Task will be presented.

The procedure for the nodal forces is as in the following;

- Name of the nodal force is given (Figure 3.14).
- Structural model used in analyses is selected.
- Type of nodal force is selected (two possible types; single nodal force and nodal force set, the latter one let you define more than one nodal force at once).
- In single nodal force mode, enter spectra.
- The node, on which the force is defined, is selected graphically by "Select Node" icon.

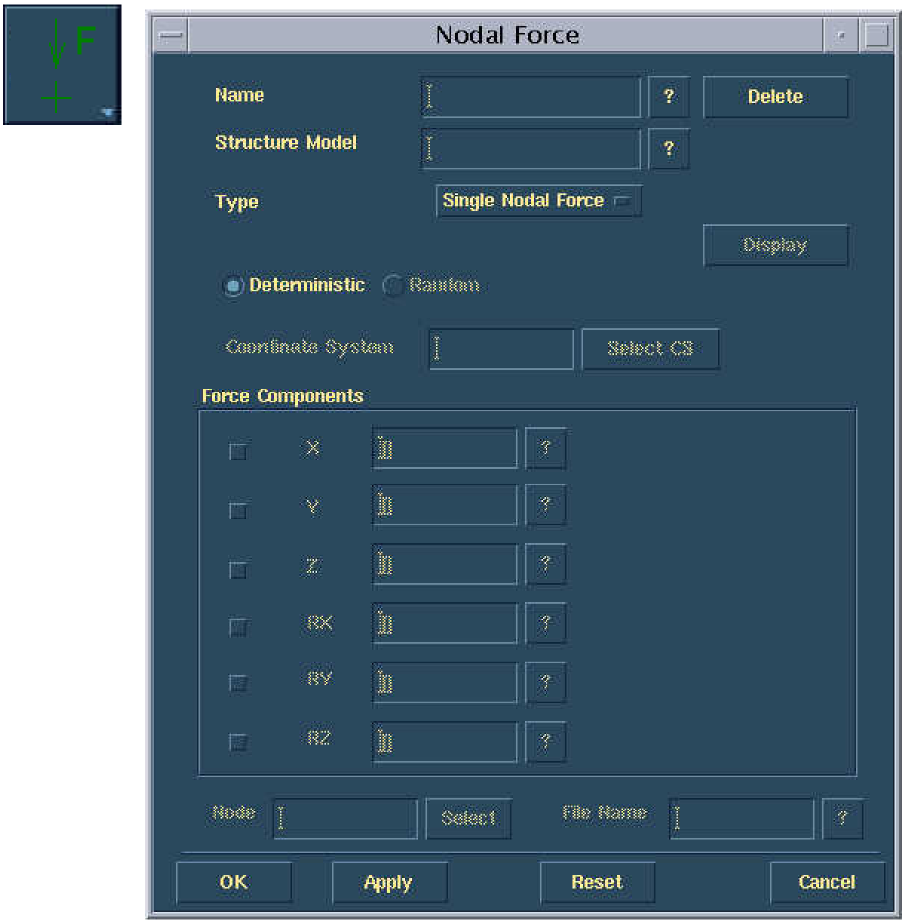


Figure 3.14 Nodal Force icon and menu.

3.5.9. "Load Set" Icon

Various types of Vibro-acoustic excitations can be applied for modeling in I-DEAS Vibro-Acoustics™. These excitations may be defined as mechanical loads (associated with the structure model), acoustical loads (associated to the fluid model), or the combination of these loads.

In this thesis, mechanical load and acoustical load (monopole source) were used separately and together. Therefore more than one load set was created. Procedure for creating a load set is as follows;

- In the menu shown in Figure 3.15 "Create" is chosen.
- In the menu shown in Figure 3.16, name of the "load set" is given.
- Loads formerly created by "Acoustic load" menu or by "Nodal force" menu are selected as "Selected Loads".

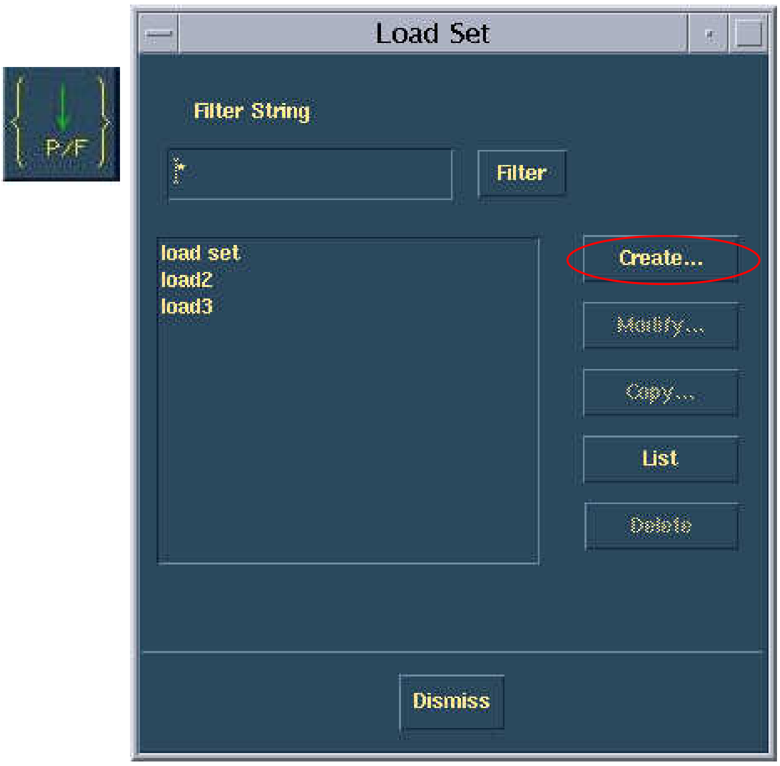


Figure 3.15 Load set menu.

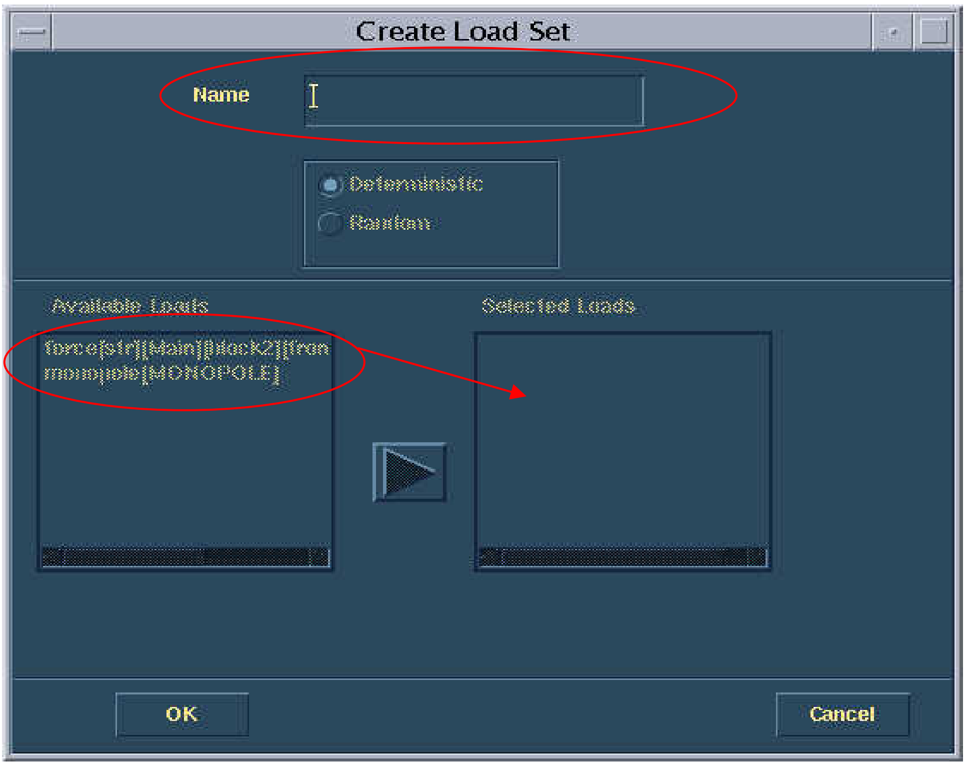


Figure 3.16 Create Load set menu.

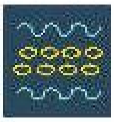
3.5.10 "Domain Points" Icon

Domain points define points in the fluid to evaluate acoustic field quantities. Domain points are defined as nodes of a finite element mesh. Domain points are put together to create domain point groups.

Before creating the domain point groups in I-DEAS Vibro-Acoustics™, a geometry must be created in I-DEAS Master Modeler Task, meshed with "Shell mesh" and appended to the fluid model. Nodes on this geometry are used as domain points.

The procedure for creating domain points group is as follows;

- In the menu in Figure 3.17, "Create" is chosen for using "Create domain points group" menu.
- In the menu in Figure 3.18, name of domain points group is given and domain is selected by "Domain" button.
- Nodes for domain points are selected graphically by "Select Nodes" button.



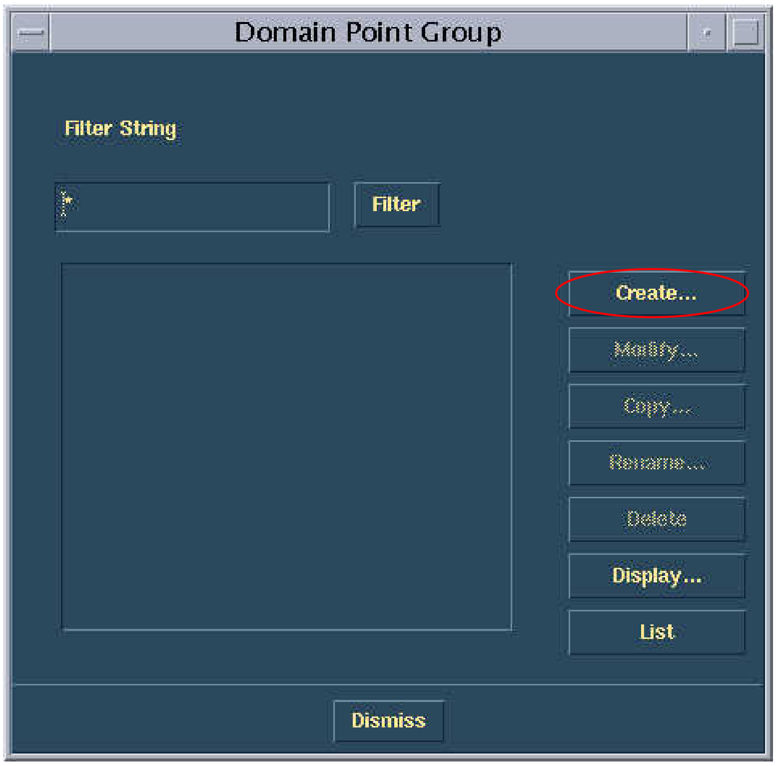


Figure 3.17 Domain Point group icon and menu.

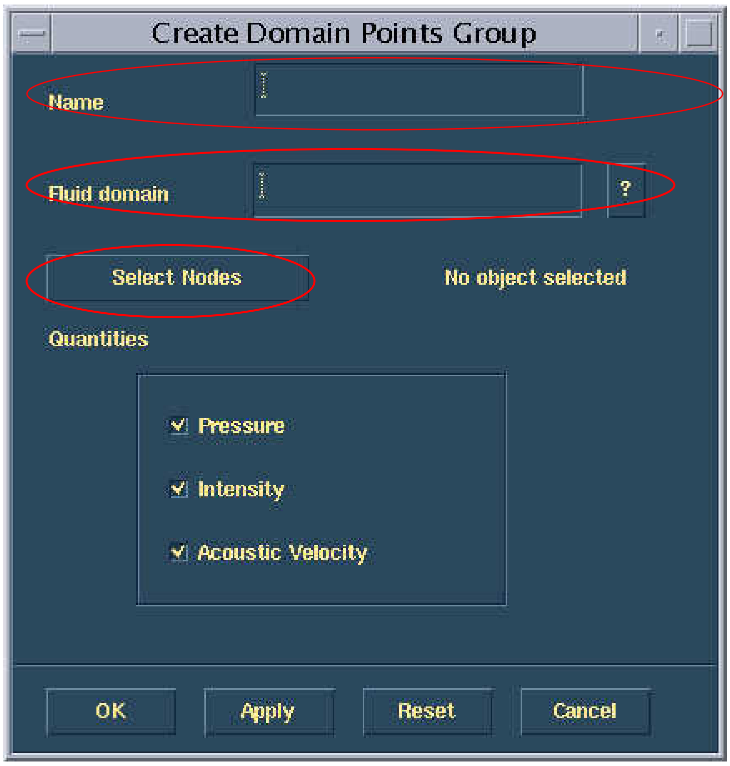


Figure 3.18 Create domain points group menu.

3.5.11 "Solution Set" Icon

A solution set contains all necessary information for the computation. When a solution set is created, the type of computation must be selected. The types of solutions are as follows:

- *Pure acoustic modes computation(Lanczos or SVI Method*): Computation of the acoustic modes of the cavity
- *Elasto-acoustic modes computation*: Computation of the coupled modes between structure and the cavity.
- *Pure acoustic response:* Computation of the response due to only acoustic excitations.
- *Coupled response:* Computation of the response of the fluid/structure coupled systems.
- *Radiation impedance computation:* Computation of the operators of an external domain, to be used with frequency interpolation techniques.

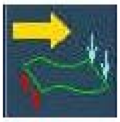
- *Uncoupled elasto-acoustic response*: Computation of acoustical response using an uncoupled approach. This is done by two steps as follows:
    - o *structural acceleration computation due to the mechanical excitations.*
    - o *Acoustic response computation due to the previous structural acceleration.*
- *Pure acoustic response-simplified method*: Computation of approximate acoustical response

Pure acoustic modes computation, elasto-acoustic modes computation and coupled response were used in this thesis. Firstly, pure acoustic mode computation was used for computing acoustic modes then elasto-acoustic mode computation was used to find coupled modes of the system, lastly uncoupled response or coupled response was used for computing the response of the system according to the excitation.

The procedure for creating a solution set is as follows;

- Click the “Create” button on the “Solution Set” menu which is shown in Figure 3.19. “Create Solution Set” menu (shown in Figure 3.20) is opened.
- On the “Create Solution Set” menu, a solution set menu name is given (also solution set name may be given by I-DEAS™ automatically).
- Type of solution is chosen from the menu. Pure acoustic mode computation must be performed before the elasto-acoustic modes computation or coupled response. If the pure acoustic mode computation or elasto-acoustic mode computation are chosen, number of modes can be settled by “Options”.
- Type of solver is chosen (I-DEAS Vibro-Acoustics™ chooses the best type of solver according to the model).

- Domains are selected. To describe the internal and external domains in analysis is an important step. This can be done by changing the domain type.
- Structure model is selected.
- Activating the "Load set" button, load set can be settled by the menu which is shown in Figure 3.21.
- Activating the "Modes" button, cavity modes and structural modes which will be used in analyses can be settled (Figure 3.22)
- Activating the "Response Spectrum" button, the form which is shown in Figure 3.23 appears. From this menu response spectrum of analysis can be settled.



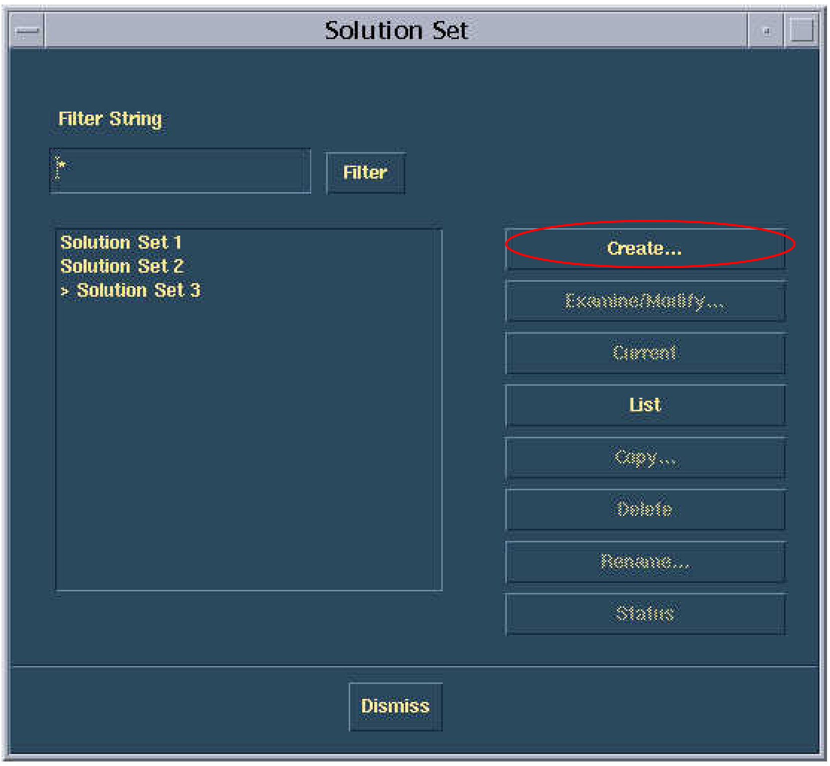


Figure 3.19 Solution set menu.

Figure 3.20 Create solution set menu.

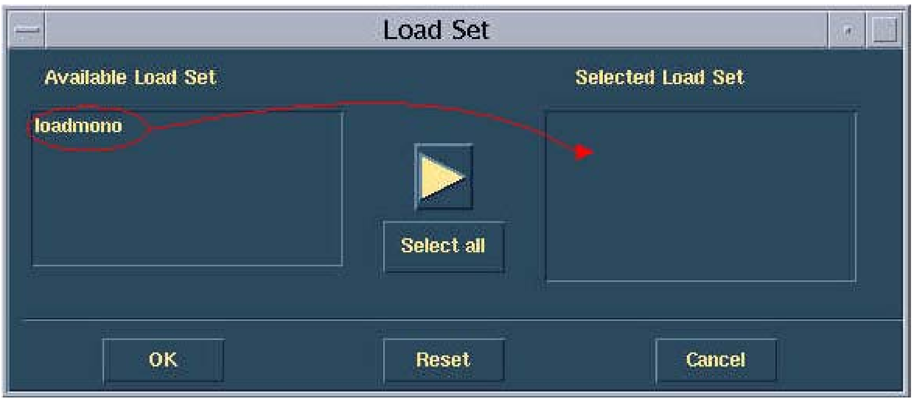


Figure 3.21 Load set Menu.

Figure 3.22 Modes menu.

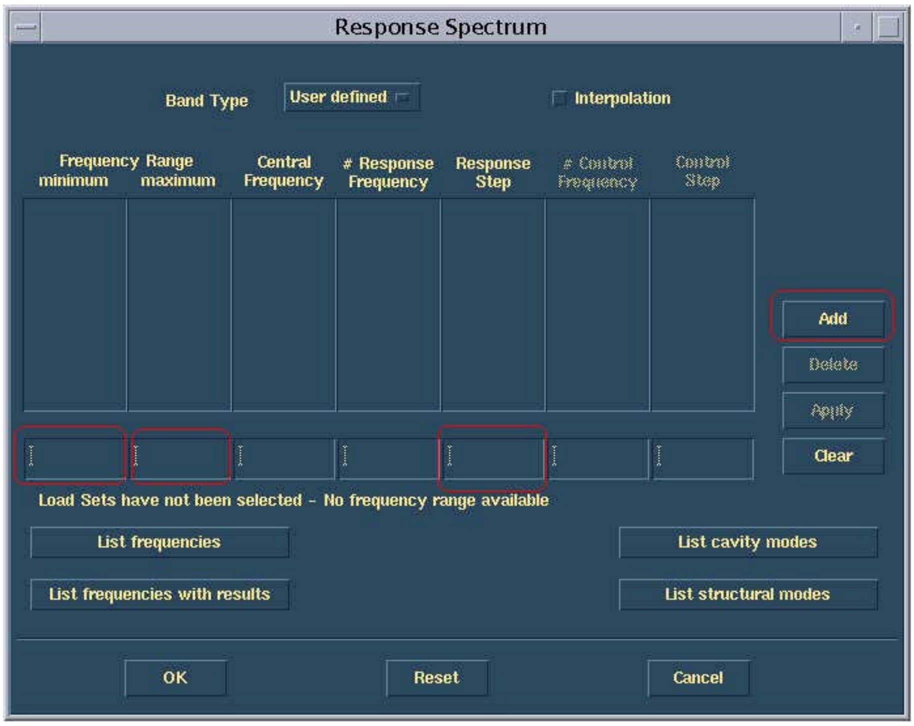


Figure 3.23 Response Spectrum menu.

3.5.12 Rayon Launching

The Rayon solver is the I-DEAS Vibro-Acoustics™ solver. By clicking icon "Rayon launching", one of the solver (based on the type of solution and the type of solver required in the current "solution set") is selected automatically. "Rayon Launching" menu is given in Figure 3.24. In this menu, only thing that has to be done is to select "Interactive" and press "Ok".

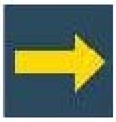


Figure 3.24 Rayon launching icon and menu.

3.5.13 "Acoustic Results Selection" Icon

By using this icon in Figure 3.25, the preferred results can be selected. The procedure for result selection is as follows;

- In acoustic results selection form, select the solution set for which results must be displayed.
- Select the "Domain" for which you wish to post process.
- Select the quantity to post process.
- Select "Frequency" for which you want to display the results
- Select "Fluid surface" where you want results to be displayed.
- Select "Load set" for which the results must be displayed.

- If the results are required in X-Y plot form, an Associated Data File (ADF) name has to be given to the results.

Figure 3.25 Results Selection menu.

In this thesis, firstly, “Pressures on surface” were chosen from “Acoustic Results” as X-Y plots to display pressures. Secondly, “Acceleration on fluid model” was chosen from “Structural Results” as X-Y plots to display accelerations.

CHAPTER FOUR

VERIFICATION OF THE USE OF I-DEAS™ VIBRO-ACOUSTIC SOFTWARE

Before the intended analyses were performed, our use of I-DEAS™ Vibro-Acoustic Software should have been verified. For this reason, two problems which have analytical results in literature were solved by I-DEAS™ Vibro-Acoustic Software and then the solutions were compared with analytical ones. An overview of numerical analysis methods in I-DEAS Vibro-Acoustics™, the comparison problems and the results are presented in this chapter.

4.1 Overview of I-DEAS™ Vibro-Acoustic Software

In this thesis, all analyses were performed by I-DEAS Vibro-Acoustics™ software. I-DEAS Vibro-Acoustics™ is a comprehensive module for solving linear vibro-acoustic problems in the frequency domain. I-DEAS Vibro-Acoustics™ can be used to study the vibro-acoustic behavior of complex, three dimensional structures coupled to one or more fluids, and subjected to mechanical and/or acoustic loads.

I-DEAS Vibro-Acoustics™ includes three principal numerical techniques implemented in frequency domain: Boundary Element Method (BEM), Volume Finite Element Method (FEM), Infinite and Finite Elements Method (IFEM). Also in I-DEAS Vibro-Acoustics™, mixed types of these numerical techniques like coupled Boundary Element Method/Finite Element Method (BEM/FEM) can be used. The numerical techniques used in this work will be explained briefly on the following paragraphs.

A) I-DEAS Vibro-Acoustics™'s Boundary Element Method (BEM) is a general method solving internal and external domain problems. The method is based on an integral equation that couples the Boundary Element for modeling the fluid(s), to the standard Finite Element for modeling the structure. As this method needs only surface meshes to describe the fluid domain, it offers distinct advantages over other techniques used to date:

- It simplifies the work of the engineer by avoiding volume meshing in the fluid, thus reducing the number of unknowns, especially for external problems.
- It avoids geometric singularities and leads to small, symmetric, linear algebraic systems of equations. This makes it possible to take advantage of customized, powerful solvers developed for the classical finite element.
- External domain is naturally taken into account.

B) I-DEAS Vibro-Acoustics™ Volume Finite Element Method (FEM) is a classical finite element method, which needs the solid meshing of the fluid domain. It is the most efficient way to solve an internal problem by using a modal response scheme. Volume or solid finite element method can also very efficiently be used when combined with the boundary element method for problems including both an external and an internal fluid. It allows you to compute;

- Modes and frequencies of acoustic cavities, or coupled modes and frequencies of acoustic cavities coupled with a vibrating structure
- Acoustic response of acoustic cavities or vibro-acoustic response of acoustic cavities coupled to vibrating structure.

C) I-DEAS Vibro-Acoustics™'s Infinite and Finite element method (IFEM) is a very efficient way of solving acoustic and vibro-acoustic radiation problems. The most common type of domain problems is the "free-field" problem in which the source is surrounded by an infinite medium. Infinite elements are used for modeling infinite, unbounded fluid domains. IFEM is based on spheroidal formulation and needs to build a spheroid that surrounds the structure. The region between the structure and the spheroid must be meshed with volume mesh elements.

A mixed method, coupled Boundary Element Method/Finite Element Method (BEM/FEM) is very efficient for a problem that involves closed cavities coupled to external fluids. The finite element method is used to solve the internal cavity problem and boundary element method is used for solving the external problem.

I-DEAS Vibro-Acoustics™ includes a method for coupling the operators computed by FEM for the internal problem with those computed by BEM for the external problem. The advantages using this method are the followings:

- Availability for the modal parameters of the problem like cavity modes and coupled modes for vibro-acoustic problems.
- It can solve internal/external problem with a reduced CPU time.

In this thesis, coupled BEM/FEM method was used. This method requires two independent meshes in order to define two sets of fluid surfaces one for BEM and the other for FEM.

4.2 Dilating Sphere (An Uncoupled Case)

In order to validate our use of I-DEAS™ Vibro-Acoustic software, a problem which has an analytical solution was solved. Acoustic field of a dilating sphere was computed for this reason.

Analytical solution for the surface and field pressures of a dilating sphere is (Graff, 1973):

$$p = \frac{a}{R'} U_0 \frac{iz_0 ka}{1 + ika} e^{-ik(R' - a)} \tag{4.1}$$

Here a is the radius, U_0 is the uniform velocity of the sphere and R' is the radial distance from the center of the sphere to a field location.

In this example, sphere and domain points which are 2m and 6m away from the sphere were created by I-DEAS Master Modeler™ and acoustic pressures were computed by I-DEAS Vibro-Acoustics™ at these domain points. This process is presented below in sequence:

- A sphere was created in I-DEAS Master Modeler™ task.
- This sphere was partitioned to 4 quarter parts. Because in I-DEAS™ software, structures which have single continuous surfaces can not be meshed.

- The quarter spheres were meshed with surface mesh.
- For domain points, two blocks (1m x1m x 1m) which are 2m and 6m away from the origin, were created and their front faces were meshed with surface mesh.
- Sphere and blocks were appended in I-DEAS™ Meshing Task.
- In I-DEAS Vibro-Acoustics™, outer surfaces of quarter spheres were chosen as fluid surfaces and air was chosen as fluid domain.
- These surfaces were defined as "prescribed acoustic velocity" in acoustic properties.
- Acoustic source was "surface load". By using this source, a constant velocity (1m/s) was given to sphere's surfaces. In this problem, one surface load was defined for each surface.
- Centers of the blocks were chosen as domain points.
- At the end of these steps vibro-acoustic analyses were performed.

Theoretical and numerical solutions are given in Figures 4.1 and 4.2. As it is seen, these two different solutions are exactly equal to each other. These results show the validity of the execution of I-DEAS Vibro-Acoustics™ software.

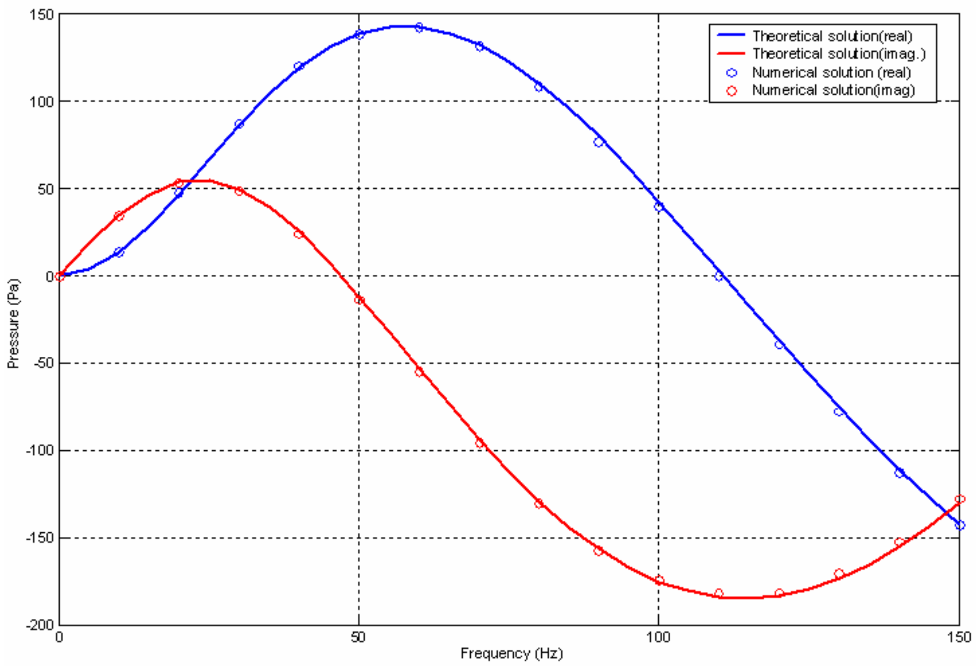


Figure 4.1 Solutions for domain point located 2m away from the sphere.

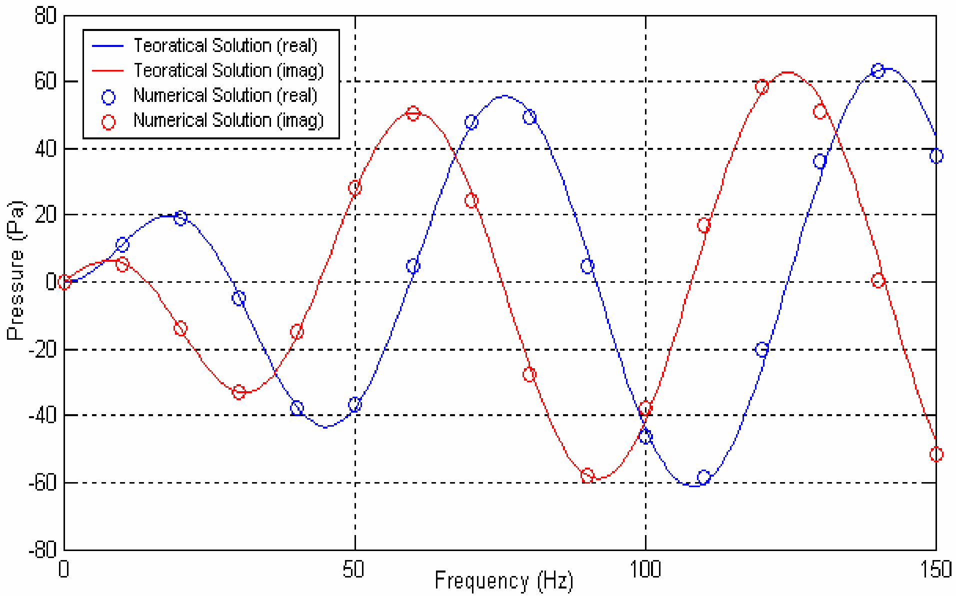


Figure 4.2 Solutions for domain point located 6m away from the sphere.

4.3 Rectangular Box (A Coupled Case)

In this study, another problem which had been solved by Pretlove (1965) was performed by I-DEAS™ Vibro-Acoustic software in order to validate the coupled solution. Pretlove (1965) has developed a mathematical theory for the problem of the effect of a backing cavity on panel vibrations. Pretlove (1965) has used a box model which has a flexible panel on the top of the box as shown in Figure 4.3. The other walls of the box were assumed to be hard (i.e. perfect acoustic reflectors) and the box was filled with air. In this problem, the effect of the air outside on the vibration of the flexible plate was neglected compared with the effect of the air inside. The flexible panel was an aluminum, simply supported plate and its dimensions were 12” (0.3048 m) in length, 6” (0.1524 m) in breadth and 0.064” (1.6256×10^{-4} m) in thickness. Box model dimensions were 12” (0.3048m) in length, 6” (0.1524 m) in breadth and 6” (0.1524 m) in depth. This box was modeled in I-DEAS™ Vibro-Acoustic software and free vibration solutions were compared with Pretlove’s (1965) results.

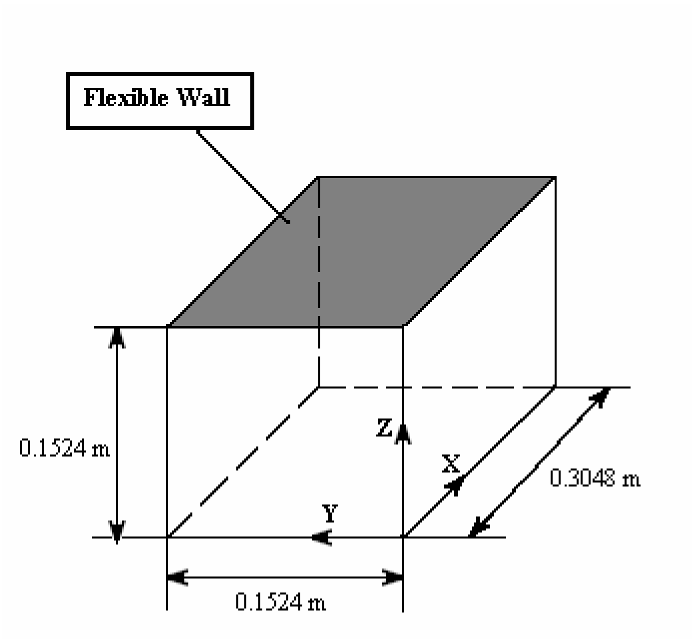


Figure 4.3 Pretlove's (1965) box model with one flexible wall.

First of all, analytical result and FEM results were compared to find an optimal mesh size. Structural and acoustic uncoupled mode frequencies were calculated analytically by using the following equations:

- Structural mode frequencies are (Harris, C.M., & Piersol, A.G. (Eds.), 2002);

$$\omega_{(m,n)} = \sqrt{\frac{D}{\rho h}\pi^2\left[(m/a)^2 + (n/b)^2\right]} \tag{4.2}$$

$$f_{str} = \omega_{(m,n)} / (2\pi) \tag{4.3}$$

where

$$D = \frac{\left(Eh^3\right)}{12\left(1-\mu^2\right)} \tag{4.4}$$

Here, m and n are subscripts showing mode numbers, ρ is density of the plate material, h is thickness, a and b are width and height of the plate, respectively. E is the modulus of elasticity, μ is the Poisson's ratio and D is the plate stiffness. In this problem, values of ρ, h, a, b, E and μ were chosen as the values in (Pretlove, 1965) for the comparison purposes (Here E = 6.6993x10^{10} Pa, μ = 0.33, ρ =2700 kg/m^3, h=1.6256x10^{-3} m).

- Acoustic mode frequencies are (Harris, C.M., & Piersol, A.G. (Eds.), 2002);

$$\omega_{(k,m,n)} = (\nu/2)\sqrt{\frac{k^2}{a^2}+\frac{m^2}{b^2}+\frac{n^2}{c^2}} \tag{4.5}$$

where, ν is the speed of sound in the medium (340 m/s); k, m and n are subscripts of the cavity mode number; a, b and c are length, breadth and depth of the cavity respectively .

Tables 4.1, 4.2 and 4.3 show the results of I-DEAS™ software and comparisons of structural, cavity and coupled mode frequencies respectively with analytical results. It can be seen in Table 4.1 and Table 4.2 that, I-DEAS™ software results approximate to analytical results by decreasing the mesh size. However, due to computational time and computer's memory limitations the mesh size can not be further decreased. In this example, 16×32 meshes for structural modes and $16\times32\times16$ meshes for cavity and coupled modes were chosen as optimal sizes since they have considerable sensitivity. The finite element (mesh) sizes should be nearly the same in all directions in order to find more accurate results. For example, length and breath of the plate are so divided that mesh size is the same in both directions. In this case, the length/breath ratio of the plate was used to find the number of mesh ratio in these directions.

There are four types of modes according to symmetry and antisymmetry in both x and y directions and these four types of modes are uncoupled to each other. One type of structural modes is "Volume Displacing Mode". Pretlove (1965) calculated only these modes since they "are the only modes to be seriously affected the acoustic cavities". Characteristics of these modes are;

1. These modes comprise one quarter of all modes.
2. Symmetric about the panel center lines in both x and y directions.
3. They are composed of odd values of m and n (e.g. 1,1; 3.1; 5,1; 1,3; 1,5; etc.)
4. Only these modes may be excited by plane acoustic waves normally incident upon the flexible wall.

Table 4.1 Structural mode frequencies of the flexible wall of the box model in Figure 4.3 (Hz)

Mode				
m	**n**	**Analytical** (Harris, C.M., & Piersol, A.G. (Eds.), 2002)	**I-DEAS (16x 16 Meshes)**	**I-DEAS (16x 32 Meshes)**
1	1	209.34	207.41	208.51
2	1	334.94	328.77	332.21
3	1	544.28	533.63	539.36
1	2	711.76	708.12	710.84
4	1	837.36	819.69	830.23
2	2	837.36	823.59	830.23
3	2	1046.7	1007.7	1030.2
5	1	1214.2	1200.9	1205.1
4	2	1339.8	1274.6	1311.4
1	3	1549.1	1546.2	1551.5

Table 4.2 Cavity mode frequencies of the cavity of the box model in Figure 4.3 (Hz)

Mode					
k	**m**	**n**	**Analytical** (Harris, C.M., & Piersol, A.G. (Eds.), 2002)	**I-DEAS (16x 16 x 16 Meshes)**	**I-DEAS (16x 32 x 16 Meshes)**
0	0	0	0	0	0
1	0	0	557.71	558.71	558.12
0	1	0	1115.5	1117.3	1117.4
0	0	1	1115.5	1117.3	1117.4
2	0	0	1115.5	1122.7	1117.4
1	1	0	1247.2	1249.2	1248.9
1	0	1	1247.2	1249.2	1248.9
0	1	1	1577.5	1580.1	1580.1
2	1	0	1577.5	1583.9	1580.1
2	0	1	1577.5	1583.9	1580.1

Table 4.3 Coupled Mode Frequencies of the flexible wall of the box model in Figure 4.3 (Hz)

Mode		**Analytical**	**I-DEAS**	**I-DEAS**
m	**n**	**(Pretlove,1965)**	**(16x 16 Meshes)**	**(16x 32 Meshes)**
1	1	216.39	214.72	215.80
			325.76	329.07
3	1	541.60	532.57	538.22
			562.39	561.84
			705.55	708.24
			818.5	828.97
			823.67	830.29
			1006.2	1028.3
			1119.6	1117.5
			1120.2	1120.4
			1123.6	1120.4
5	1	1212.39	1202.4	1206.6
			1247.5	1249.1
			1252.4	1251.7
			1278.1	1313.1
1	3	1543.82	1545.9	1550.9

Structural and coupled mode frequencies computed by Pretlove (1965) and their ratios are presented in Table 4.4. Structural, cavity and coupled mode frequencies for the same box model computed by I-DEAS software are presented in Table 4.5 with coupled/uncoupled frequency ratios. Comparison of ratios in Table 4.4 and Table 4.5 shows that all solutions of the I-DEAS™ Vibro-Acoustic software are very accurate. Besides, the same variation tendency may be observed in both sets of ratios.

Table 4.4 Structural and coupled mode frequencies of the flexible wall of the box model in Figure 4.3 presented by Pretlove (1965) and their ratios

Structural Mode Frequencies (Hz)	Coupled Mode Frequencies (Hz)	Ratio (Coupled/Uncoupled)
209.34	216.39	1.0337
544.30	541.60	0.9950
1214.20	1212.39	0.9985
1549.15	1543.82	0.9966

Table 4.5 Structural, cavity and coupled mode frequencies of the box model in Figure 4.3 computed by I-DEAS and their ratios

Structural Mode Frequencies (Hz)	Cavity Mode Frequencies	Coupled Mode Frequencies	Ratio
(16x 32 Meshes)	(16x 32x 16 Meshes)	(16x 32x 16 Meshes)	Coupled/Uncoupled
	0		
208.51		215.80	1.0350
332.21		329.07	0.9906
539.36		538.22	0.9979
	558.12	561.84	1.0067
710.84		708.24	0.9963
830.23		828.97	0.9985
830.23		830.29	1.0001
1030.2		1028.3	0.9982
	1117.4	1117.5	1.0001
	1117.4	1120.4	1.0027
	1117.4	1120.4	1.0027
1205.1		1206.6	1.0012
	1248.9	1249.1	1.0002
	1248.9	1251.7	1.0022
1311.4		1313.1	1.0013
1551.5		1550.9	0.9996

In a structural-cavity system with a structural fundamental mode frequency less than the first cavity mode frequency, the fundamental frequency increases when coupling effect is taken into account as shown in Tables 4.4 and 4.5 (Pretlove ,1965). It can be seen in Tables 4.4 and 4.5, the fundamental mode of the plate is the most affected mode by the cavity. The increase in the fundamental structural mode may be expressed by the stiffness effect of the cavity. That is, in this mode cavity acts like a stiffness and increases the uncoupled natural frequency of the plate. Table 4.5 shows the small increases in all cavity modes of the coupled system. Therefore it may be said that coupling decreases the flexibility of the cavity.

In this problem the general purpose was not to compute exactly the structural and cavity mode frequencies of Pretlove's box model. The aim was to compute the ratio of coupled mode to uncoupled mode frequencies calculated by I-DEAS and to compare these ratios with Pretlove's ratios as done in (Lee, 2002). Since Pretlove's (1965) and our methods are different, comparison of frequency ratios may be more reliable than comparison of frequencies.

CHAPTER FIVE

FREE VIBRATION OF PLATES BACKED BY A CAVITY

In this thesis, cavity effect on the plate vibration for both free and forced vibrations is discussed. Therefore, three cavities with different dimensions and two plates with different thicknesses were used for numerical calculations.

Pretlove (1965) has used an approach for showing the effect of cavity on the plate vibrations. In this approach, cavity dimensions were chosen according to the vibrating plate dimensions and Pretlove (1965) has used three different cavity depths in his work. The first cavity depth was less than one quarter of the larger dimension of the plate. The second cavity depth was less than one-half of the larger dimension of the plate. The last cavity depth was greater than the larger dimension of the plate.

Pretlove (1965) has shown the results of cavity effects and explained the acoustic modes, according to these cavity depths. When the depth of cavity is less than one quarter and one-half of the larger plate dimension;

- The lowest acoustic mode is the transverse mode and it is important.
- Cavity acts like a stiffness. Due to the stiffness effect, uncoupled fundamental mode frequency of the plate increases in the coupled case.

When the depth of the cavity is greater than the larger dimension of the flexible plate;

- The first two acoustic modes are important. These acoustic modes are called as open-ended acoustic mode and closed-end mode, respectively.
- According to the order of occurrence of these acoustic frequencies and uncoupled fundamental frequency of the plate, two situations may arise;
    1. If the fundamental frequency of the plate is less than the open-ended acoustic mode frequency, the cavity acts as a stiffness.
    2. If the fundamental frequency of the plate falls between these two acoustic mode frequencies then cavity acts as negative stiffness and

natural frequency of the plate is decreased. This phenomenon is called as “virtual mass” effect.

In this thesis, plate dimensions were chosen as 31 cm in length; 31 cm in height and thicknesses were taken as 0.5 mm and 1 mm. The plate was assembled to the cavity by clamped boundary conditions. For showing the effect of the cavity, on the vibration behaviour of the flexible plate, three different cavity depths were chosen. These are;

1. 60 mm. That is, less than one quarter of the larger dimension of the plate (310 mm).
2. 120 mm. That is, less than one-half of the larger dimension of the plate (310 mm).
3. 400 mm. That is, greater than the larger dimension of the plate (310 mm).

Free vibration results computed by Ideas™ vibro-acoustic software, for different cavities and plates are presented in Tables 5.1-5.8. These tables contain structural modes; cavity modes, coupled modes and ratios (coupled mode value/structural mode value or coupled mode value/cavity mode value).

Table 5.1 Modal frequencies of the plate with 310mmx310mmx0.5mm dimensions backed by a cavity of 310mmx310mmx60mm dimensions

Structural Mode Frequencies (Hz)	**Cavity Mode Frequencies (Hz)**	**Coupled Mode Frequencies (Hz)**	**Ratio (Coupled/Uncoupled)**
	0		
46.164		91.987	1.9926
94.160		93.070	0.9884
94.160		93.149	0.9893
138.33		137.77	0.9959
169.01		168.51	0.9970
169.84		179.85	1.0589
210.71		210.06	0.9969
210.71		210.09	0.9971
270.90		269.71	0.9956
270.90		269.85	0.9961
279.72		279.77	1.0002
309.39		309.39	1.0000
310.77		309.98	0.9975
375.94		375.59	0.9991
375.94		375.62	0.9991
398.31		397.87	0.9989
398.68		400.10	1.0036
435.94		434.96	0.9978
435.94		434.98	0.9978
468.56		468.42	0.9997
498.08		498.08	1.0000
499.73		499.76	1.0001
	548.73	569.60	1.0380
	548.73	569.61	1.0381
552.41		540.57	0.9786
552.41		540.58	0.9786
587.75		587.17	0.9990
587.75		587.78	1.0000
587.78		588.56	1.0013
588.64		588.57	0.9999
648.40		648.68	1.0004
648.40		648.68	1.0004

Table 5.2 Modal frequencies of the plate with 310mmx310mmx0.5mm dimensions backed by a cavity of 310mmx310mmx120mm dimensions

Structural Mode Frequencies (Hz)	Cavity Mode Frequencies (Hz)	Coupled Mode Frequencies (Hz)	Ratio (Coupled/Uncoupled)
	0		
46.164		74.403	1.6117
94.160		93.531	0.9933
94.160		93.651	0.9946
138.33		138.05	0.9979
169.01		168.76	0.9985
169.84		174.17	1.0255
210.71		210.35	0.9983
210.71		210.40	0.9985
270.90		270.26	0.9976
270.90		270.37	0.9980
279.72		279.79	1.0002
309.39		309.39	1.0000
310.77		310.37	0.9987
375.94		375.76	0.9995
375.94		375.78	0.9996
398.31		398.09	0.9994
398.68		399.26	1.0015
435.94		435.40	0.9988
435.94		435.44	0.9989
468.56		468.49	0.9998
498.08		498.08	1.0000
499.73		499.67	0.9999
552.41		542.69	0.9824
552.41		542.70	0.9824
	548.83	563.05	1.0259
	548.84	563.16	1.0261
587.75		587.78	1.0000
587.75		587.89	1.0002
587.78		588.03	1.0004
588.64		588.05	0.9990
648.40		648.45	1.0001
648.40		648.47	1.0001

Table 5.3 Modal frequencies of the plate with 310mmx310mmx0.5mm dimensions backed by a cavity of 310mmx310mmx400mm dimensions

Structural Mode Frequencies (Hz)	Cavity Mode Frequencies (Hz)	Coupled Mode Frequencies (Hz)	Ratio (Coupled/Uncoupled)
	0		
46.164		56.227	1.2180
94.160		93.760	0.9957
94.160		93.842	0.9966
138.33		138.12	0.9985
169.01		169.01	1.0000
169.84		170.05	1.0012
210.71		210.55	0.9993
210.71		210.58	0.9994
270.90		270.50	0.9985
270.90		270.57	0.9988
279.72		279.59	0.9995
309.39		309.39	1.0000
310.77		310.47	0.9990
375.94		375.87	0.9998
375.94		375.88	0.9998
398.31		396.21	0.9947
398.68		398.31	0.9991
	425.51	432.12	1.0155
435.94		435.68	0.9994
435.94		435.71	0.9995
468.56		468.56	1.0000
498.08		498.08	1.0000
499.73		499.90	1.0003
552.41		545.56	0.9876
552.41		545.68	0.9878
	548.95	556.77	1.0142
	548.98	557.14	1.0149
587.75		587.78	1.0000
587.75		587.78	1.0000
587.78		587.83	1.0001
588.64		588.64	1.0000
648.40		648.30	0.9998
648.40		648.44	1.0001

Table 5.4 Modal frequencies of the plate with 310mmx310mmx1mm dimensions backed by a cavity of 310mmx310mmx60mm dimensions

Structural Mode Frequencies (Hz)	Cavity Mode Frequencies (Hz)	Coupled Mode Frequencies (Hz)	Ratio (Coupled/Uncoupled)
	0		
92.321		109.96	1.1911
188.29		187.10	0.9937
188.29		187.18	0.9941
276.61		275.98	0.9977
337.94		337.40	0.9984
339.61		341.48	1.0055
421.30		420.22	0.9974
421.30		420.24	0.9975
541.62		531.42	0.9812
541.62		531.55	0.9814
559.25		558.98	0.9995
	548.73	562.95	1.0259
	548.73	562.98	1.0260
618.55		618.55	1.0000
621.30		619.47	0.9971
751.54		751.48	0.9999
751.54		751.52	1.0000
	775.92	779.82	1.0050
796.25		795.43	0.9990
796.98		797.11	1.0002

Table 5.5 Modal frequencies of the plate with 310mmx310mmx1mm dimensions backed by a cavity of 310mmx310mmx120mm dimensions

Structural Mode Frequencies (Hz)	**Cavity Mode Frequencies (Hz)**	**Coupled Mode Frequencies (Hz)**	**Ratio (Coupled/Uncoupled**
	0		
92.321		101.52	1.0996
188.29		187.61	0.9964
188.29		187.73	0.9970
276.61		276.30	0.9989
337.94		337.67	0.9992
339.61		340.45	1.0025
421.30		420.72	0.9986
421.30		420.77	0.9987
541.62		534.82	0.9875
541.62		534.88	0.9876
	548.83	557.62	1.0160
	548.84	557.67	1.0161
559.25		559.18	0.9999
618.55		618.55	1.0000
621.30		620.37	0.9985
751.54		751.51	1.0000
751.54		751.53	1.0000
	776.00	777.96	1.0025
796.25		795.84	0.9995
796.98		796.91	0.9999

Table 5.6 Modal frequencies of the plate with 310mmx310mmx1mm dimensions backed by a cavity of 310mmx310mmx400mm dimensions

Structural Mode Frequencies (Hz)	**Cavity Mode Frequencies (Hz)**	**Coupled Mode Frequencies (Hz)**	**Ratio (Coupled/Uncoupled)**
	0		
92.321		94.863	1.0275
188.29		187.86	0.9977
188.29		187.95	0.9982
276.61		276.38	0.9992
337.94		337.94	1.0000
339.61		338.76	0.9975
421.30		421.04	0.9994
421.30		421.08	0.9995
	425.51	428.04	1.0060
541.62		538.33	0.9939
541.62		538.39	0.9940
	548.95	552.44	1.0064
	548.98	552.44	1.0063
559.25		559.29	1.0001
618.55		618.55	1.0000
621.30		620.71	0.9991
	694.26	695.39	1.0016
	694.32	695.45	1.0016
751.54		751.68	1.0002
751.54		751.70	1.0002

Table 5.7 Comparison of the mode frequencies of plate with 310mmx310mmx0.5mm dimensions backed by different cavity depths

S Structural Modes (Ideas)	C1 Coupled Modes (310mmx 310mmx60 mm)	C2 Coupled Modes (310mmx 310mmx120 mm)	C3 Coupled Modes (310mmx 310mmx400mm)	R1 Ratio C1/S	R2 Ratio C2/S	R3 Ratio C3/S
46.164	91.987	74.403	56.227	1.9926	1.6117	1.2180
94.160	93.070	93.531	93.760	0.9884	0.9933	0.9957
94.160	93.149	93.651	93.842	0.9893	0.9946	0.9966
138.33	137.77	138.05	138.12	0.9959	0.9979	0.9985
169.01	168.51	168.76	169.01	0.9970	0.9985	1.0000
169.84	179.85	174.17	170.05	1.0589	1.0255	1.0012
210.71	210.06	210.35	210.55	0.9969	0.9983	0.9993
210.71	210.09	210.40	210.58	0.9971	0.9985	0.9994
270.90	269.71	270.26	270.50	0.9956	0.9976	0.9985
270.90	269.85	270.37	270.57	0.9961	0.9980	0.9988

Table 5.8 Comparison of the mode frequencies of plate with 310mmx310mmx1mm dimensions backed by different cavity depths

S Structural Modes (Ideas)	C1 Coupled Modes (310mmx 310mmx60 mm)	C2 Coupled Modes (310mmx 310mmx120 mm)	C3 Coupled Modes (310mmx 310mmx400mm)	R1 Ratio C1/S	R2 Ratio C2/S	R3 Ratio C3/S
92.321	109.96	101.52	94.863	1.1911	1.0996	1.0275
188.29	187.10	187.61	187.86	0.9937	0.9964	0.9977
188.29	187.18	187.73	187.95	0.9941	0.9970	0.9982
276.61	275.98	276.30	276.38	0.9977	0.9989	0.9992
337.94	337.40	337.67	337.94	0.9984	0.9992	1.0000
339.61	341.48	340.45	338.76	1.0055	1.0025	0.9975
421.30	420.22	420.72	421.04	0.9974	0.9986	0.9994
421.30	420.24	420.77	421.08	0.9974	0.9987	0.9995

Tables 5.1-5.3 show the modal frequencies of the three models with a flexible plate of thickness 0.5 mm. For all of these models the fundamental frequency of the plate is less than the first cavity mode (transverse mode). Therefore cavity shows a positive stiffness effect and the coupled fundamental mode increases almost 100 %. Table 5.1 for the smallest cavity shows that coupling also has a considerable effect on the subsequent two structural modes (94.160) and on the fifth mode (169.184). Table 5.2 for the medium cavity shows the considerable effect only on the fifth mode besides the fundamental mode, of course less than that of the smallest cavity. As shown in Table 5.3 two cavity modes arise for the largest cavity. Only the fundamental is affected by the cavity, in the least level among the considered cavities. However, some minor incompatibilities are seen in tabulated values due to the numerical solution. Another common characteristic of all cavities are the increase of the cavity mode frequency in the coupled solution. The effect of the cavity depth on the structural modal frequencies of the 0.5 mm plate is shown in Table 5.7 as an overall result.

Tables 5.4-5.6 show the modal frequencies of the same three models with a plate thickness of 1mm. Since the plate thickness is increased, structural mode frequencies also increase in this case. However, this increase is not sufficient for the cavity mode to appear before the first structural mode. Therefore all fundamental modes in Tables 5.4-5.6 are affected positively from the stiffness effect of the cavity. However, since the flexible plate is thicker now, the coupling effect of the cavity is less. Even for the smallest cavity only the fundamental mode is considerably affected as shown in Table 5.4. Table 5.6 shows that two new cavity modes appear depending on the increase of the cavity depth. All cavity modes increase again in the coupled solution as shown in Tables 5.4-5.6. The cavity effect on the plate of 1 mm thickness is combined in Table 5.8.

General free vibration solutions of the six models used in this thesis give the following results:

1. The fundamental mode of the flexible plate is the most affected mode of the plate-cavity system due to coupling. This effect is in the form of “positive stiffness”.

2. The effect of cavity on the flexible plate increases as the cavity depth gets smaller and the plate thickness gets thinner.
3. The effect of the coupled solution on the first cavity mode (transverse mode) increases as the cavity depth gets smaller and the plate thickness gets thinner as the 2^{nd} result.

CHAPTER SIX

FORCED VIBRO-ACOUSTIC RESPONSE OF THE FLEXIBLE PLATE BACKED BY A CAVITY

In this thesis, uncoupled and coupled frequency response analyses of a box model that had the smallest cavity depth (60 mm) and thinner plate thickness (0.5 mm) among the models presented in Chapter 5, was performed. This model is called "Box Model" and shown in Figure 6.1. The flexible plate was assembled to the cavity by clamped boundary conditions. Different types of excitations were applied to Box Model and their response characteristics were examined. The following analyses were performed subsequently:

1. A structural harmonic force with an amplitude of 10N was applied to the center of the plate as shown in Figure 6.1.
2. A structural force with two harmonics (almost periodic), each has an amplitude of 10N was applied to the center of the plate.
3. Random excitations were applied.

   a) A structural random force with a constant 10N amplitude through the considered frequency range (0 – 600 Hz) was applied to the center of the plate.

   b) An acoustical source, a monopole with a source strength Q= 10 m^3/s shown in Figure 6.1, was applied with the excitation in case a). Acoustical source has the same spectral characteristics with mechanical excitation.

   c) A structural random force with a constant 1N amplitude through the considered frequency range (40-600 Hz) was applied to the center of the plate.

As stated by Frendi and Robinson (1993), the coupling effect increases with the level of the excitation. Therefore for the first four cases high level of excitations were used to show and examine the coupling effect clearly. For this reason, high response values were obtained and these are presented in terms of decibel (dB). In the calculations of dB values, a_{ref}=9.81 m/s^2 and $p_{ref} = 2\times 10^{-5}$ Pa were used.

4. A brief application was performed to examine the effect of the excitation level on the response characteristics. Therefore the mechanical excitation level in case 3.a) was 10 times decreased and the results were discussed.

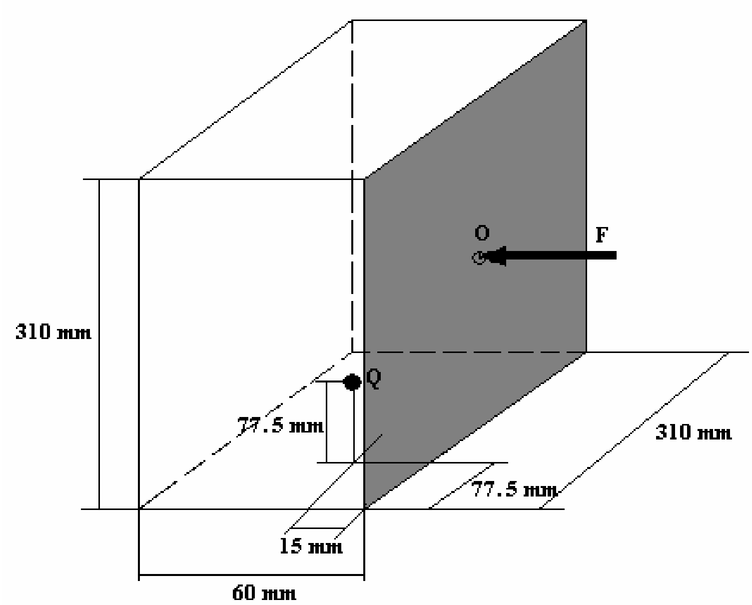


Figure 6.1Mechanical and acoustical excitations acting the Box Model, F: Excitation force, Q: Monopole source, O: Center point of the flexible plate.

6.1 A Structural Harmonic Excitation

Response accelerations and response sound pressures to the structural harmonic excitation of two points on the plate, labeled as Point0 and Point2 as shown in Figure 6.2, are given in Figures 6.3-6.10.

In order to find the response characteristics, flexible plate was excited separately by two harmonic forces, each has an amplitude of 10 N applied at 46 Hz and 92 Hz ($F_s = 10\sin 2\pi 46t$ and $F_s = 10\sin 2\pi 92t$). 46.164 Hz is the first structural mode frequency and 92.987 Hz is the first coupled mode frequency of this model as presented in Table 5.1. The acceleration and sound pressure responses were computed in the frequency range which contains the first modal frequencies. Therefore 40-50 Hz frequency range was used for uncoupled case and 85-95 Hz range for coupled case, as shown in Figures 6.3-6.10.

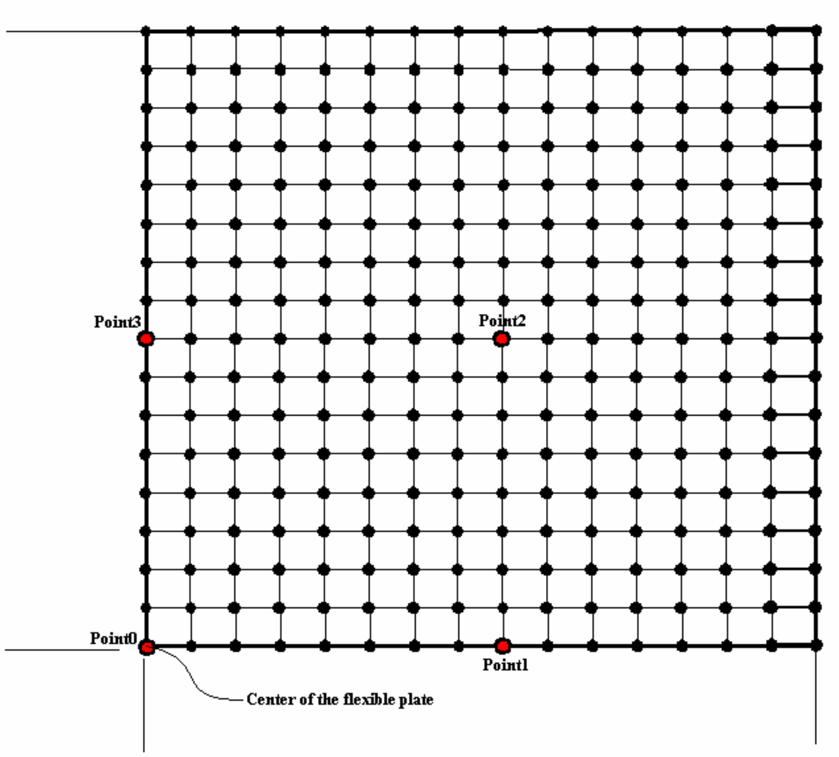


Figure 6.2 Locations of response points on the quarter part of the flexible plate

When Figures 6.3-6.10 are examined; the decrease of response levels both for coupled acceleration and coupled sound pressures are seen. This situation can be explained as damping effect of the cavity on the plate vibration. In Chapter 5, the stiffness effect of the cavity on the plate vibration was shown by free vibration analyses. Due to stiffness effect of coupling, the first modal frequency increases. In respect to this situation, stiffness effect and damping effect must be considered for low frequency analyses especially at the first structural mode frequency. Lyon (1963) classifies the frequency ranges of plate-cavity systems into three categories. The low frequency range is up to the fundamental mode of the plate, mid frequency range is between the fundamental mode and the cavity mode, and high frequency range is after the first cavity mode. The acceleration response is higher at Point0, since it is also a force excitation point. However, Point0 and Point2 have almost the same level of sound pressures.

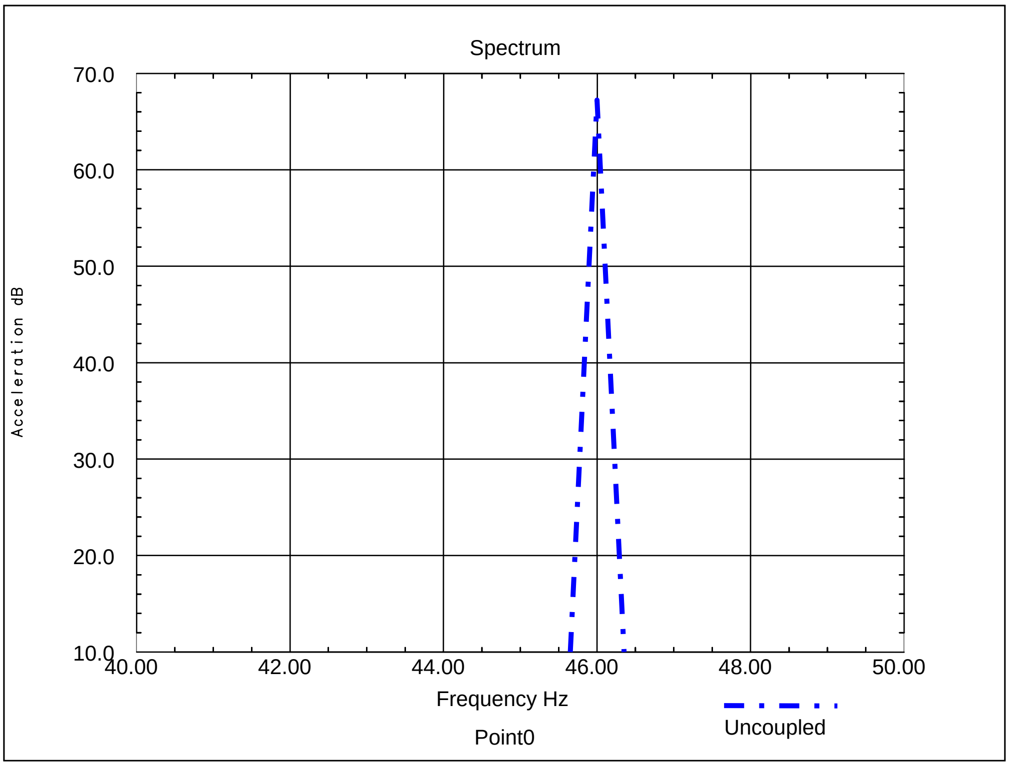


Figure 6.3 The uncoupled acceleration response of Point0 on the plate subjected to harmonic force.

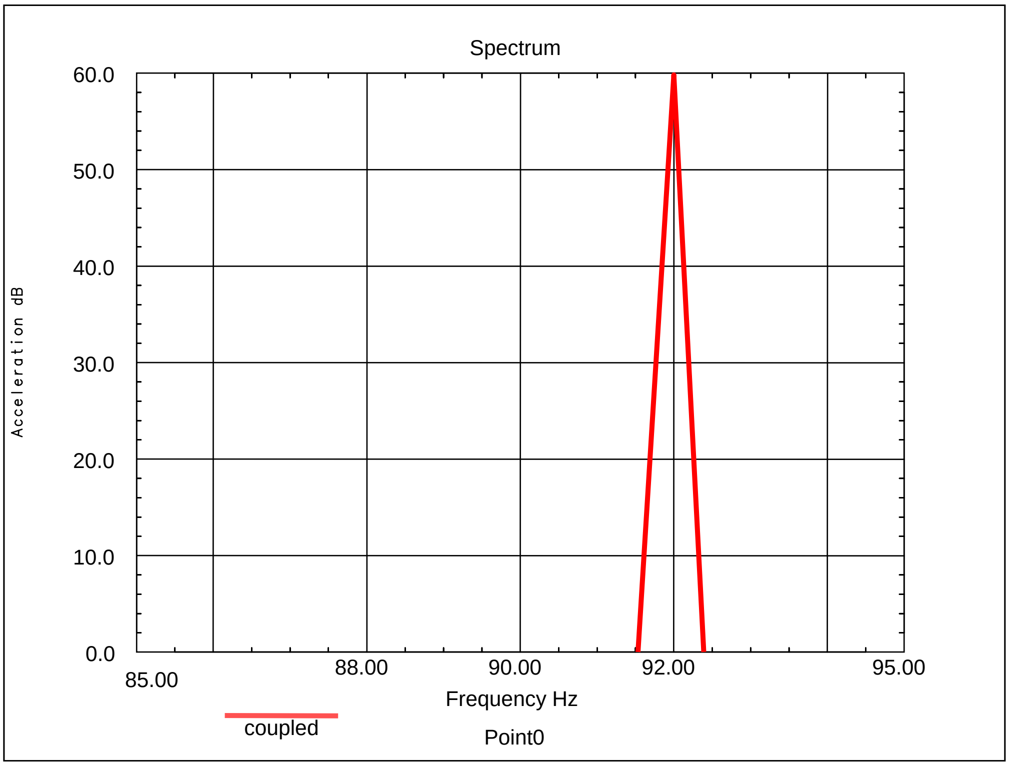


Figure 6.4 The coupled acceleration response of Point0 on the plate subjected to harmonic force.

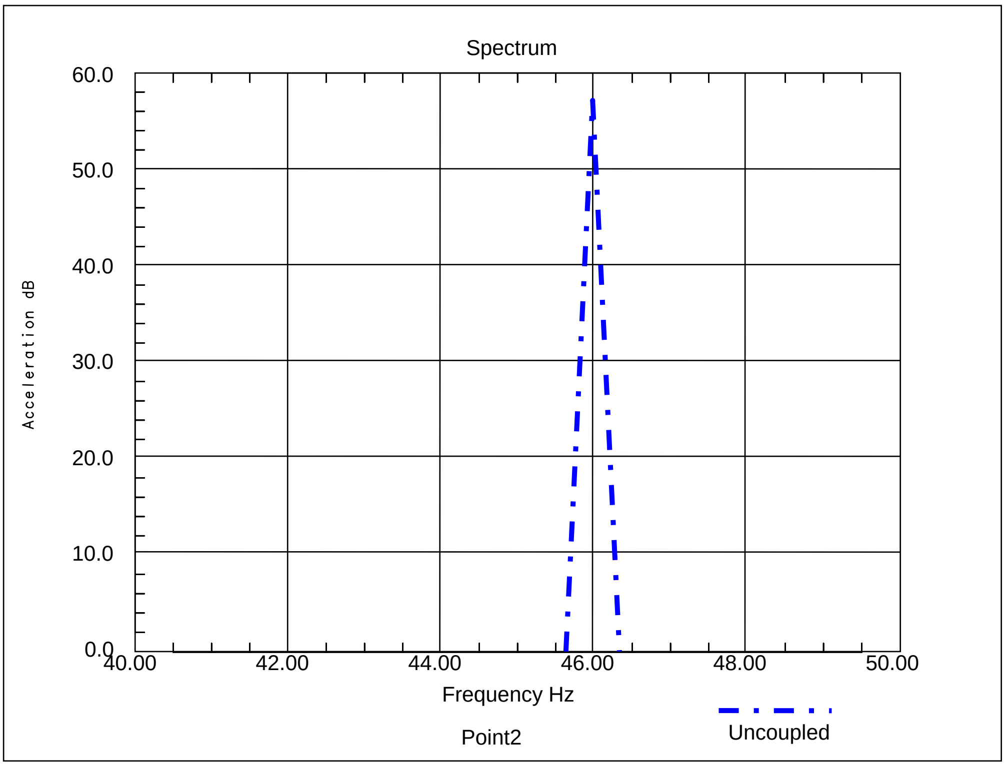


Figure 6.5 The uncoupled acceleration response of Point2 on the plate subjected to harmonic force.

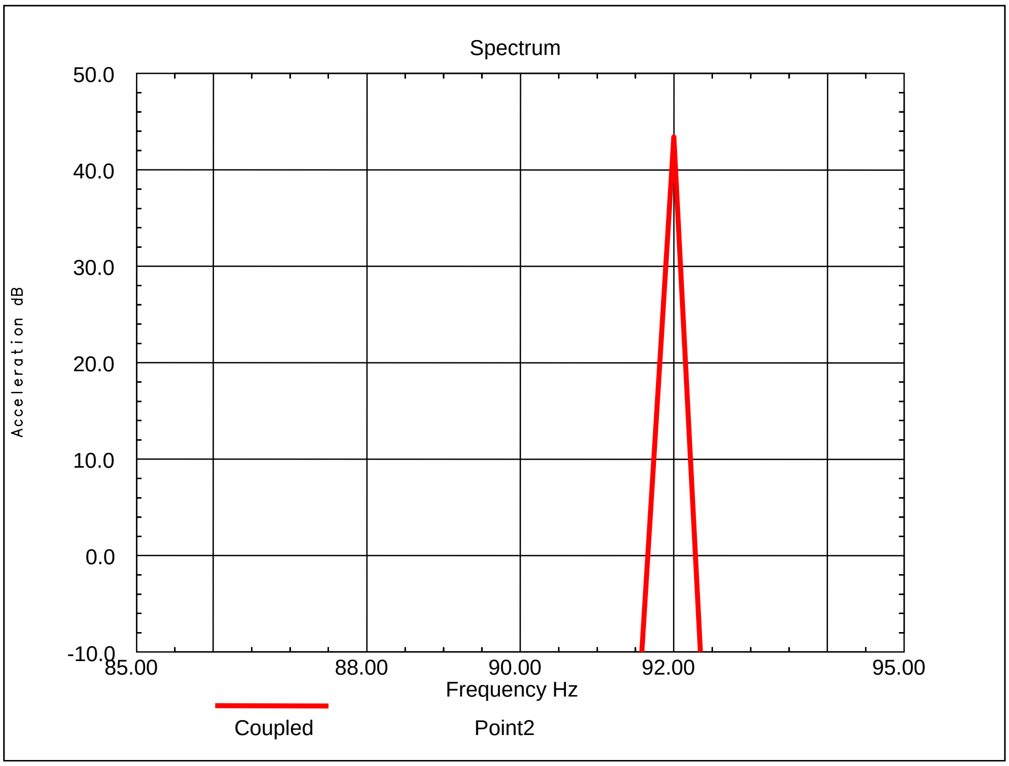


Figure 6.6 The coupled acceleration response of Point2 on the plate subjected to harmonic force.

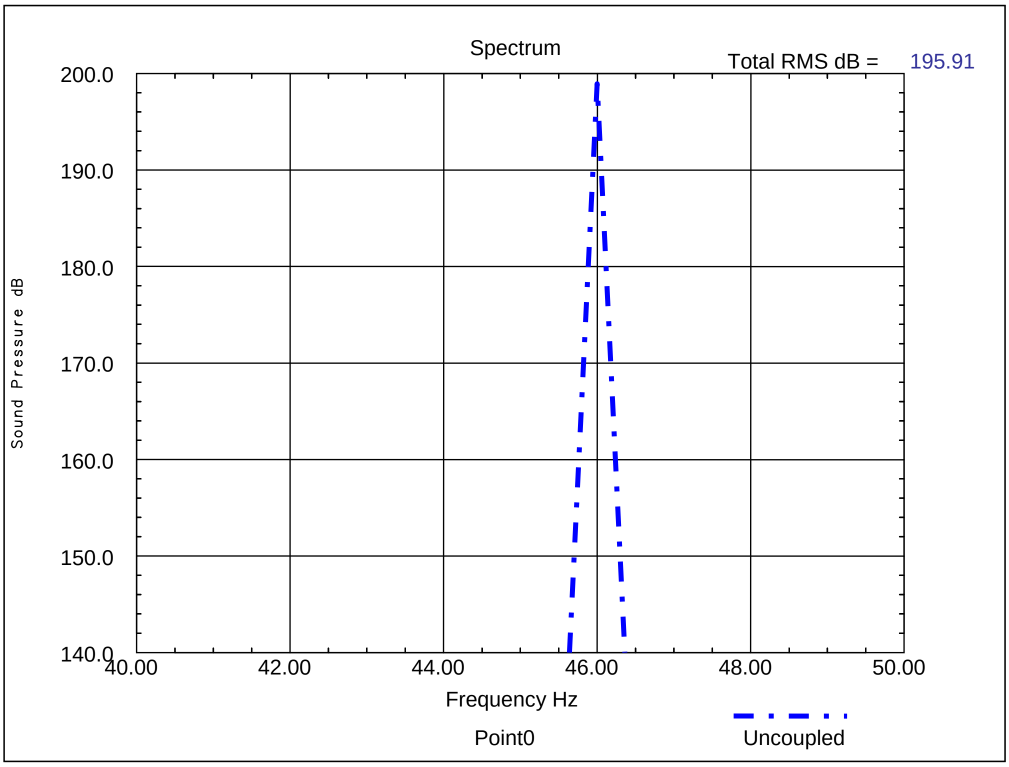


Figure 6.7 The uncoupled sound pressure response of Point0 on the plate subjected to harmonic force.

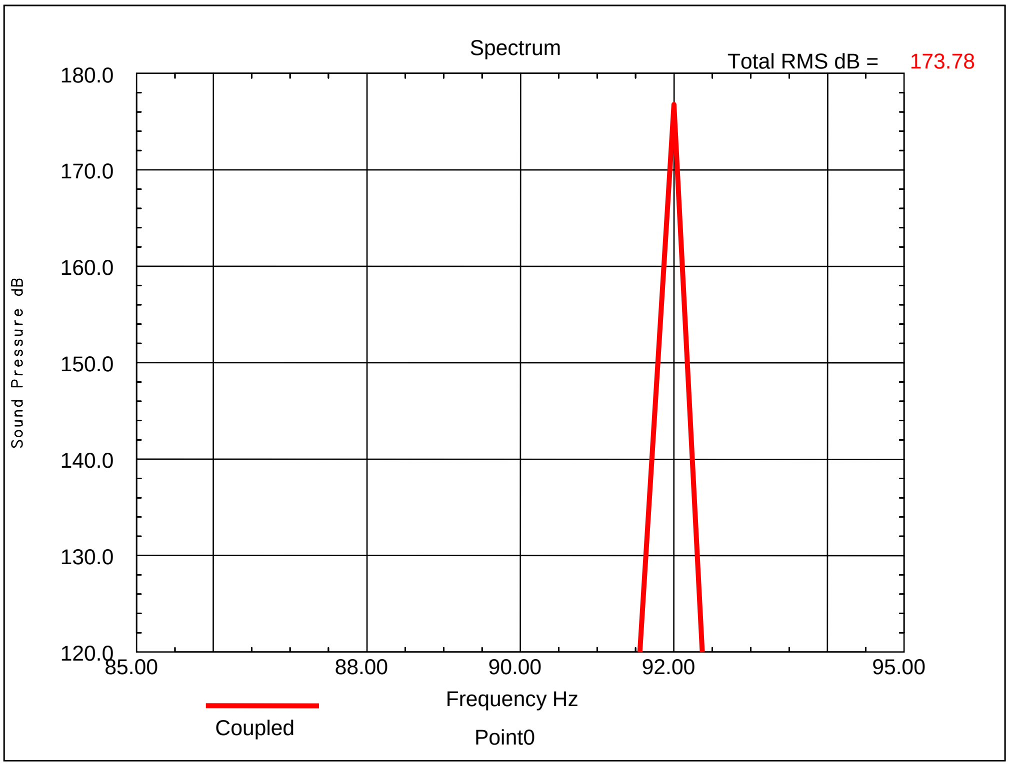


Figure 6.8 The coupled sound pressure response of Point0 on the plate subjected to harmonic force.

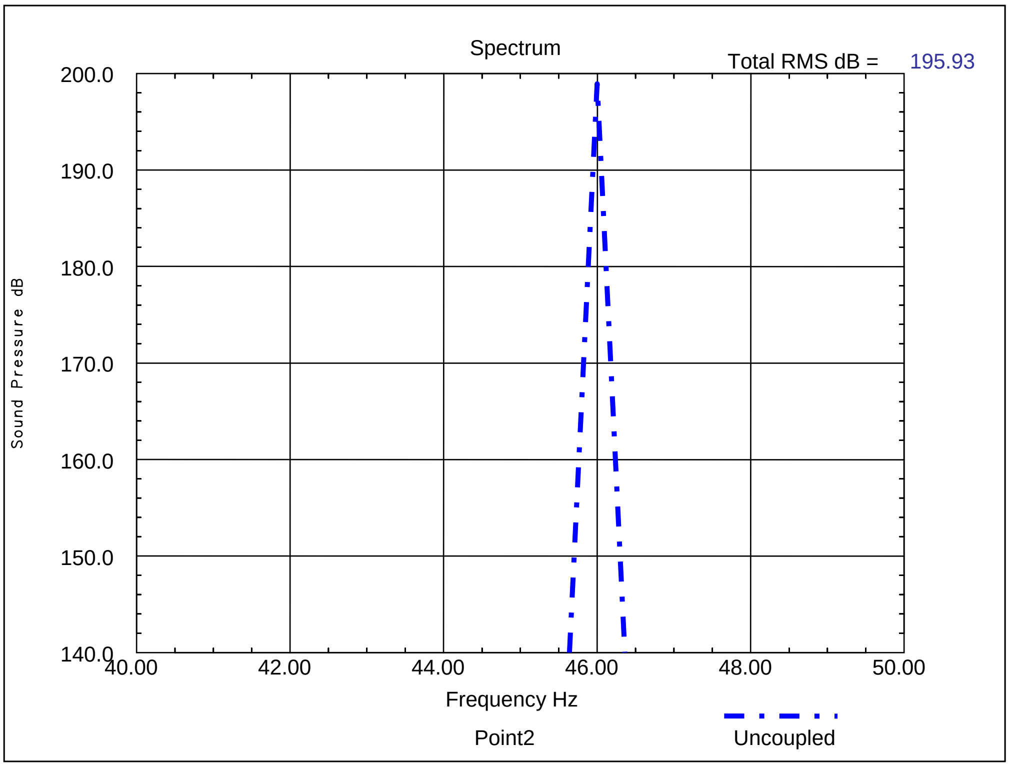


Figure 6.9 The uncoupled sound pressure response of Point2 on the plate subjected to harmonic force.

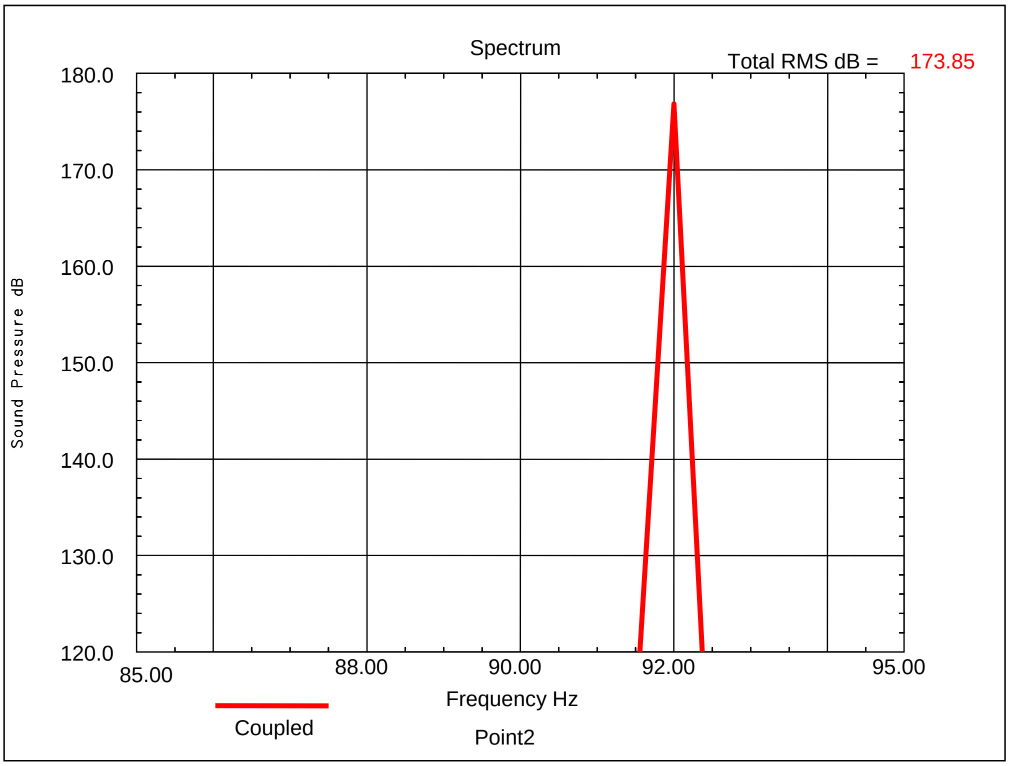
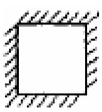
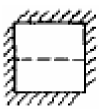
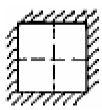
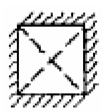
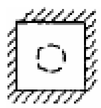
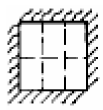


Figure 6.10 The coupled sound pressure response of Point2 on the plate subjected to harmonic force.

6.2 A Structural Excitation with Two Harmonics

In this analysis, a structural force was formed by using the mode shapes and modal frequencies of the square clamped plates in Table 6.1 and Table 2, respectively. As shown in Table 6.1, the first and fifth structural mode shapes will be more affected by the structural force applied at the center of the plate. Because the other mode shapes have nodal lines passing through the center. Therefore the force was composed of two harmonics. Those were around the first and fifth structural mode frequencies in the uncoupled case (46 Hz and 169 Hz). Consequently the force in coupled case was composed of the two harmonics corresponding to the uncoupled ones. These are 91.987 Hz and 179.85 Hz as seen in Table 5.1. The rounded values, 92 Hz and 180 Hz, were used in the combination of the force for the coupled case. Each of these harmonics had 10N amplitude. The time behaviours of superimposed forces are shown in Figures 6.11 and 6.12. As seen from Figure 6.11 a periodic force is formed from the superposition of two harmonic forces with a period of 0.02 second, corresponding to 50 Hz frequency. The superposition of forces with coupled modal frequencies gives an almost periodic force with a period of 0.011 second, corresponding to approximately 91 Hz, as seen in Figure 6.12. 50 Hz and 91 Hz resultant frequencies are around the first uncoupled and coupled mode frequencies of the plate, respectively. The uncoupled and coupled acceleration and sound pressure responses of Point0 and Point2 were computed and presented in Figures 6.13-6.20.

Table 6.1 Mode shapes and frequencies of clamped square plates (Harris, C.M., & Piersol, A.G. (Eds.), 2002)

$\omega_n \big/ \sqrt{Dg/\gamma h a^4}$	35.99	73.41	108.27	131.64	132.25	165.15
NODAL LINES						

$\omega_n = 2\pi f_n \quad D = Eh^3/12(1-\mu^2)$

h: PLATE THICKNESS

a: PLATE LENGTH

γ : WEIGHT DENSITY

Table 6.2 Comparison of modal frequencies of clamped square plates computed by I-DEAS and formulations in Table 6.1

	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode	6td Mode
I-DEAS	46.16437	94.16025	138.3344	169.0098	169.8431	210.7078
Formulation In Table 6.1	46.2281	94.2929	139.0696	169.0876	169.8712	212.1302

When the present acceleration responses (Figures 6.13-6.16) and the acceleration responses to the harmonic force of Point0 and Point2 (Figures 6.3-6.6) are compared, a decrease in the first peak level of the present uncoupled acceleration is seen. Also the damping effect on the first coupled structural mode can not be seen. The decrease on the first structural uncoupled mode may be explained by the frequency characteristics of the structural force which is used in this analysis. By the superposition of two harmonic forces, the frequency of exciting force is changed from 46Hz to 50 Hz and therefore the first modal frequency is exceeded. When the sound pressure responses in Figures 6.17-6.20 are examined, the damping effect of coupling on the peak levels is observed, as expected.

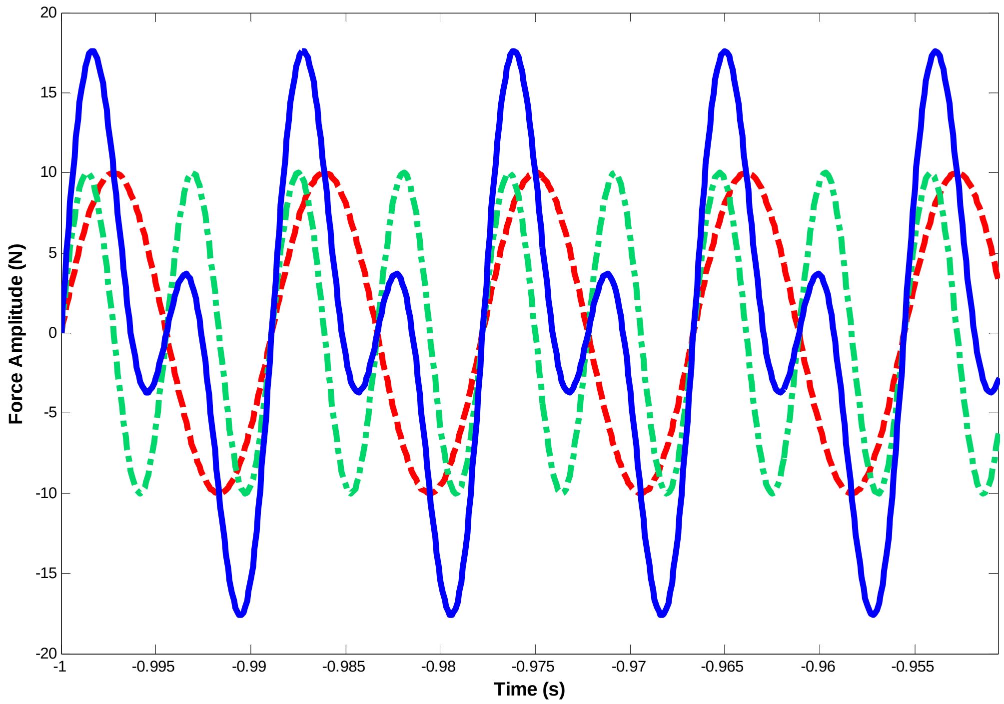


Figure 6.11 Periodic behaviour of superimposed force composed of 46 Hz and 169 Hz.

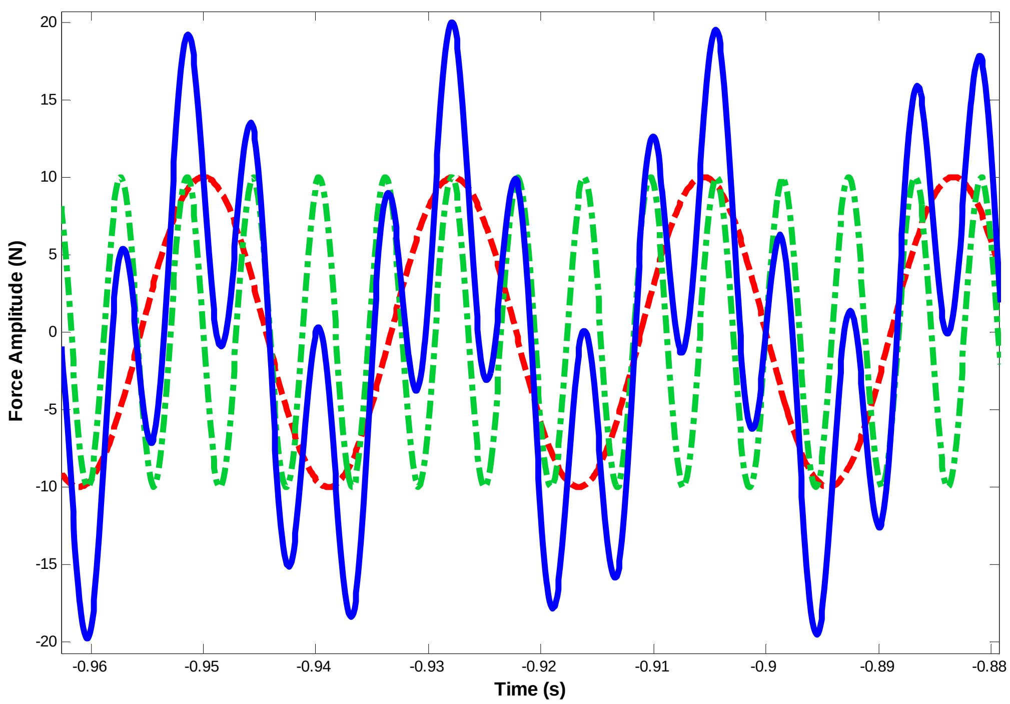


Figure 6.12 Periodic behaviour of superimposed force composed of 92 Hz and 180 Hz.

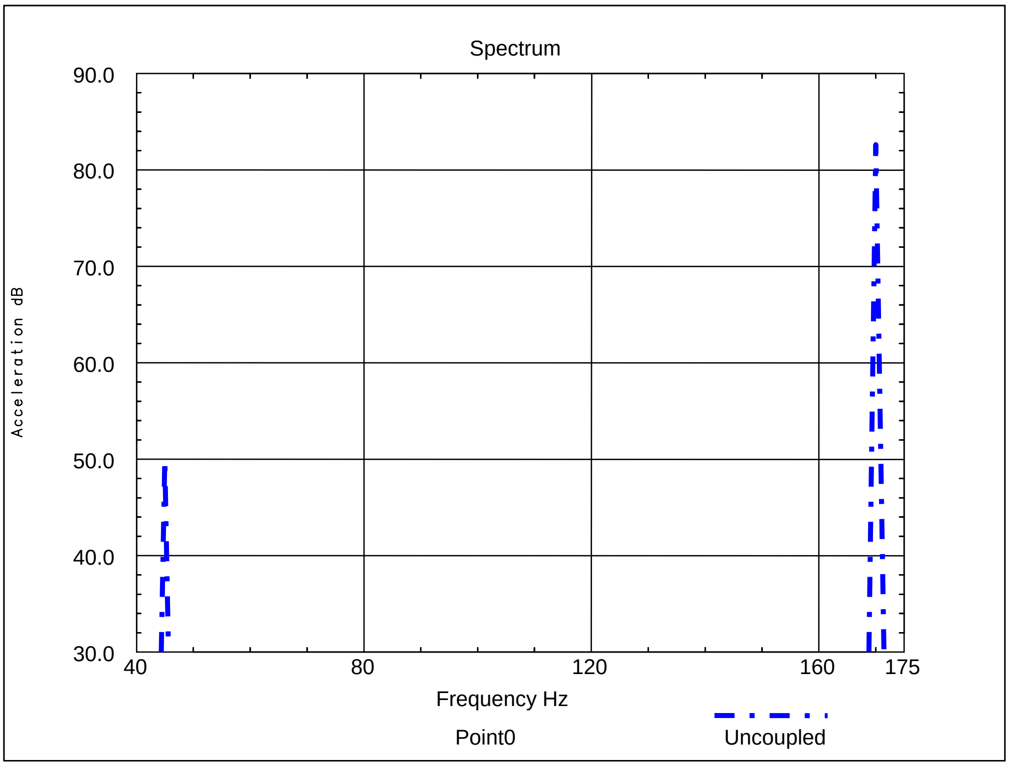


Figure 6.13 The uncoupled acceleration response of Point0 on the plate subjected to structural excitation with two harmonics.

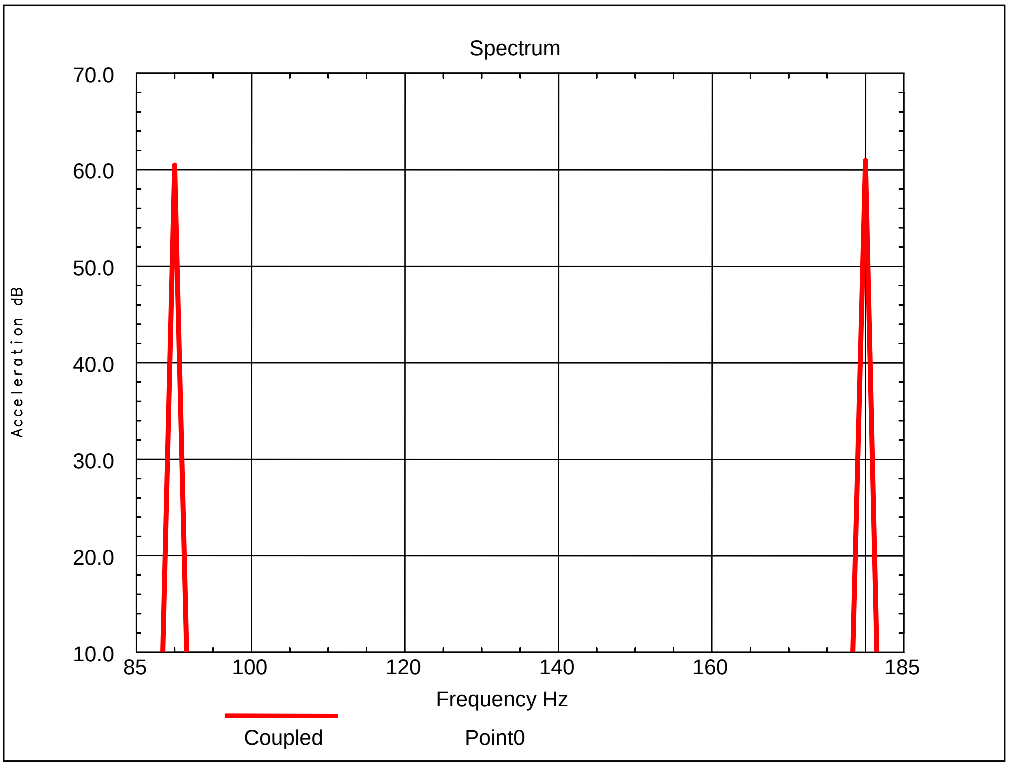


Figure 6.14 The coupled acceleration response of Point0 on the plate subjected to structural excitation with two harmonics.

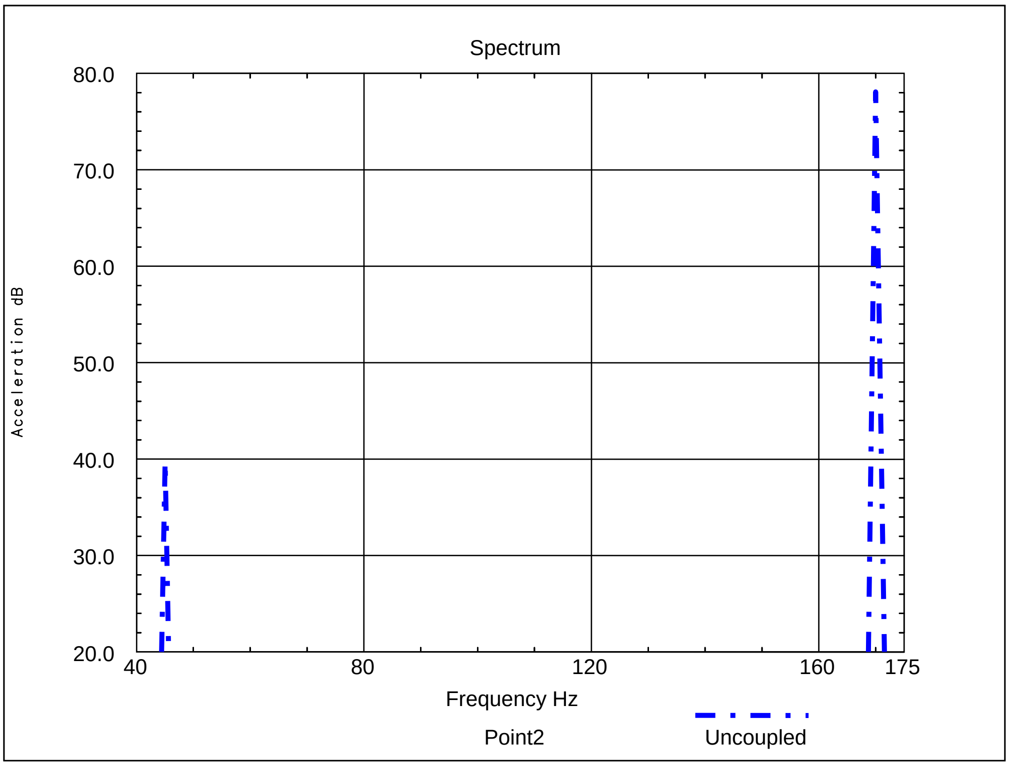


Figure 6.15 The uncoupled acceleration response of Point2 on the plate subjected to structural excitation with two harmonics.

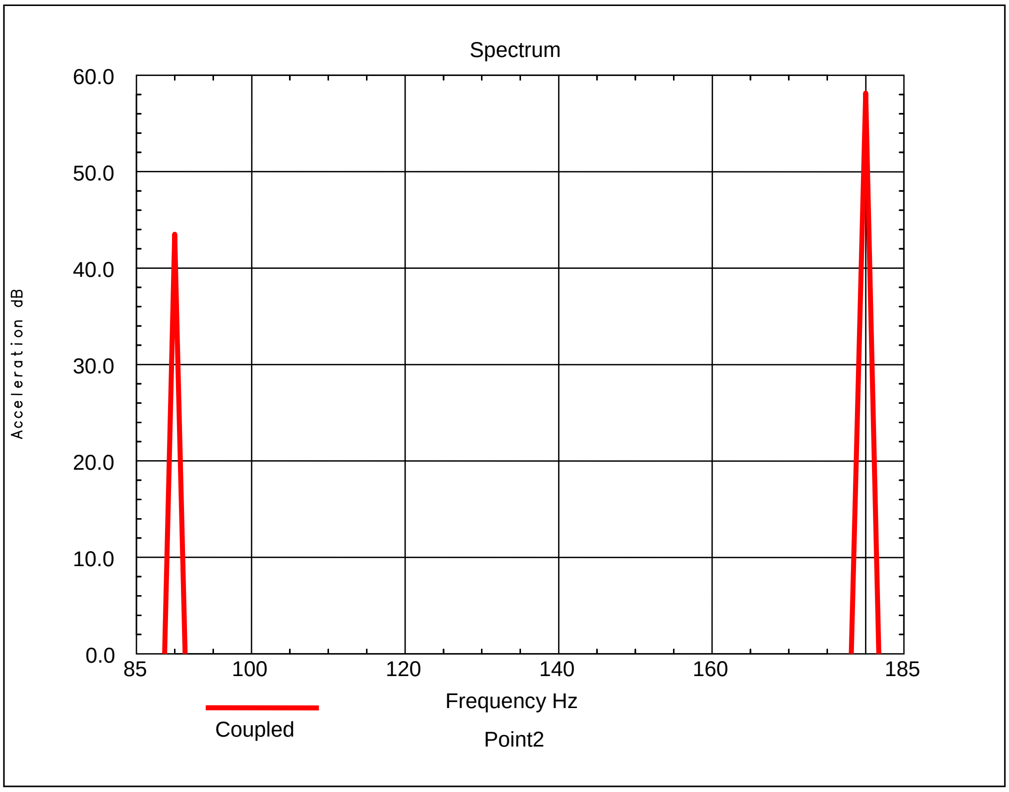


Figure 6.16 The coupled acceleration response of Point2 on the plate subjected to structural excitation with two harmonics.

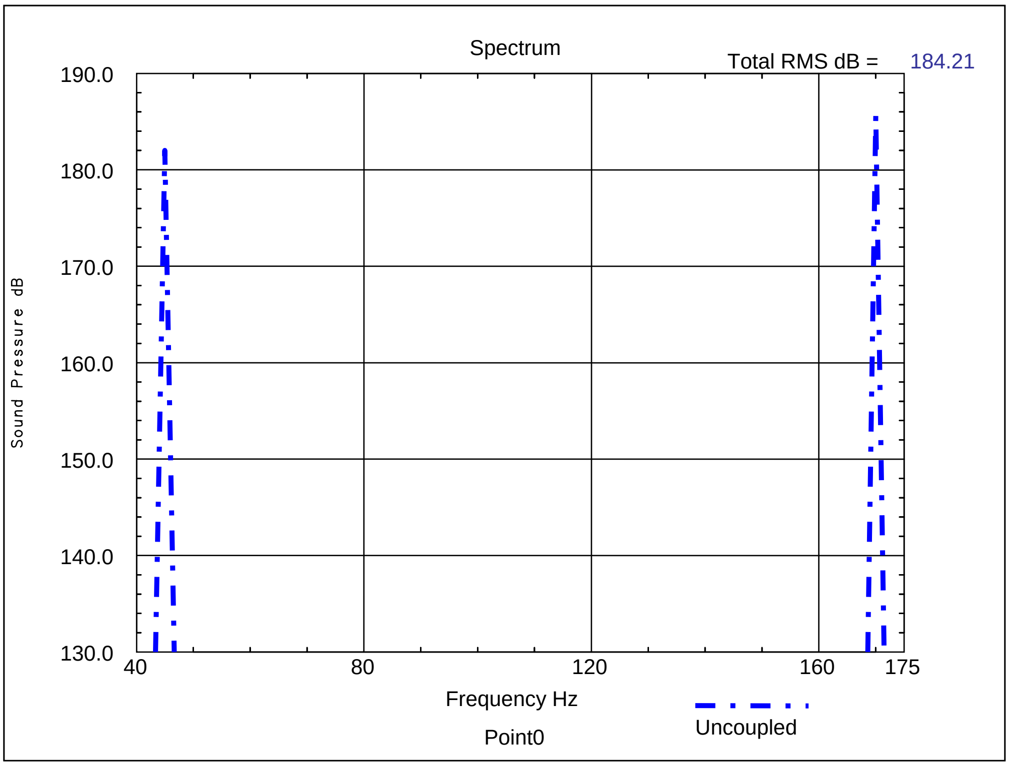


Figure 6.17 The uncoupled sound pressure response of Point0 on the plate subjected to structural excitation with two harmonics.

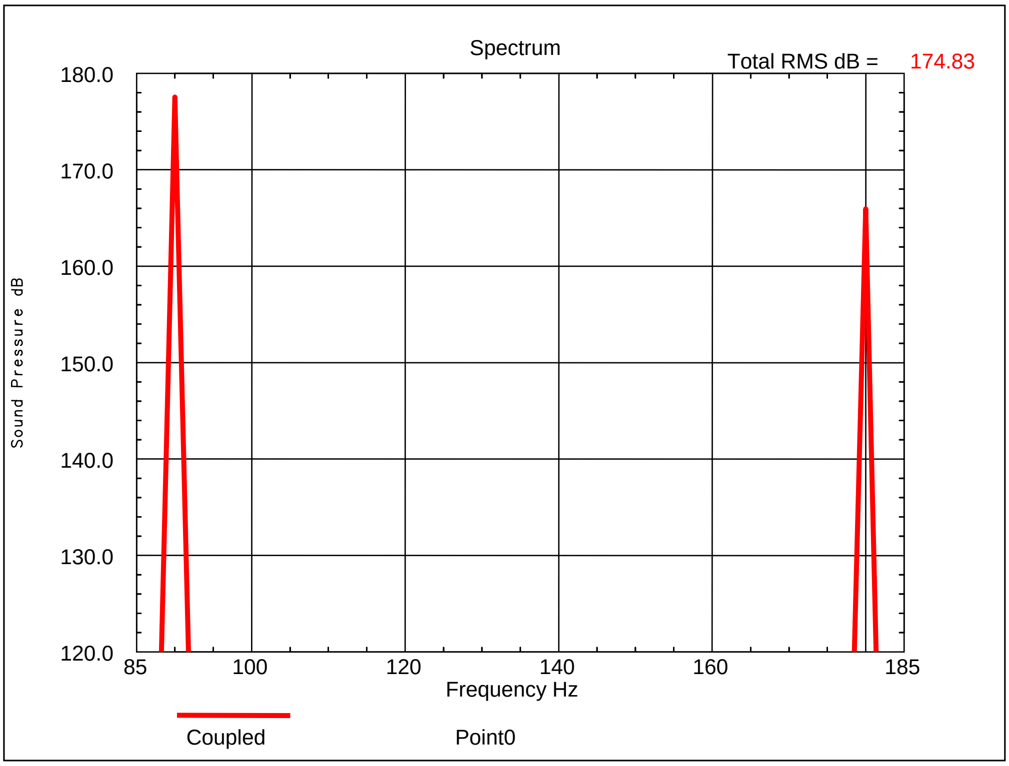


Figure 6.18 The coupled sound pressure response of Point0 on the plate subjected to structural excitation with two harmonics.

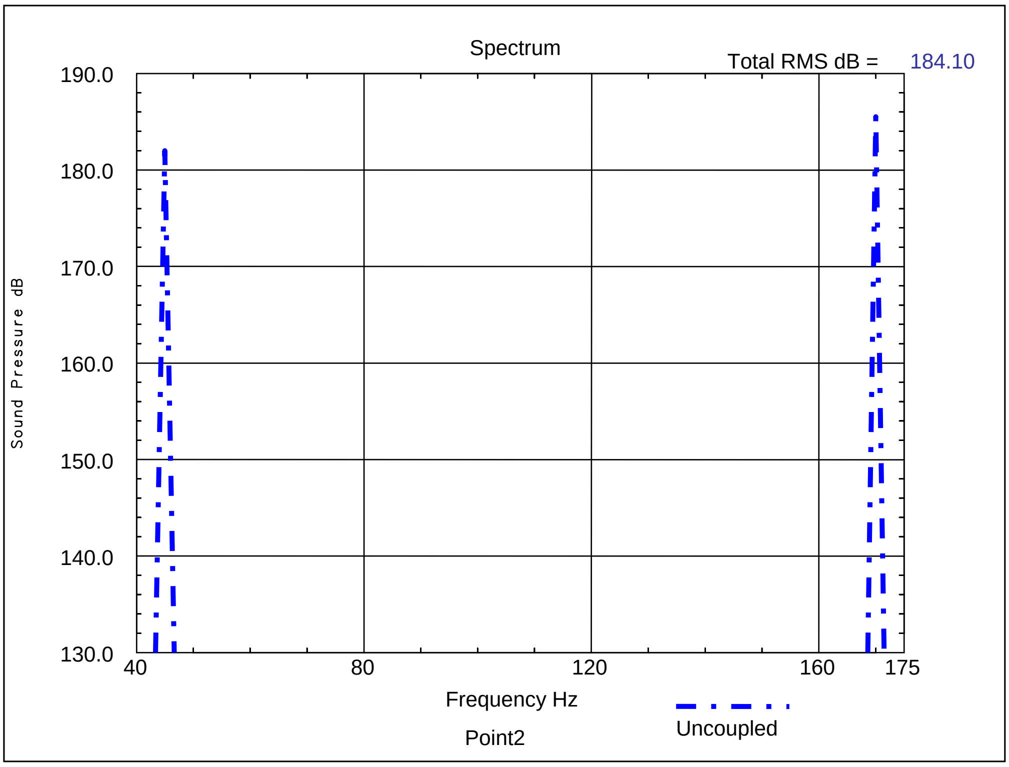


Figure 6.19 The uncoupled sound pressure response of Point2 on the plate subjected to structural excitation with two harmonics.

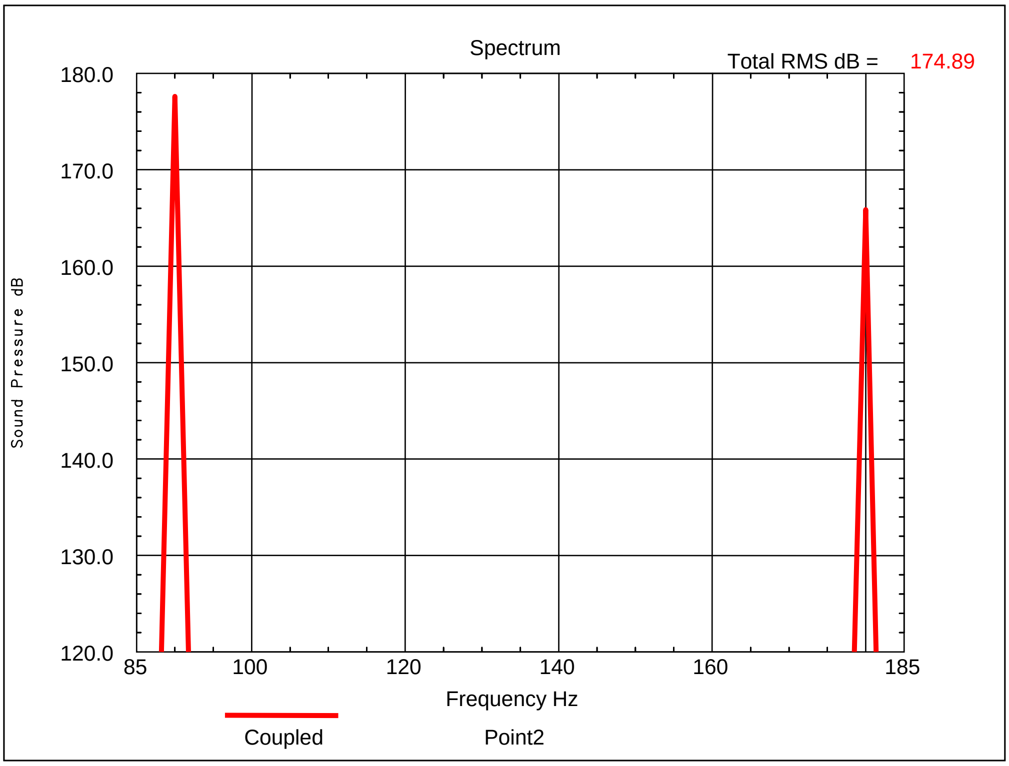


Figure 6.20 The coupled sound pressure response of Point2 on the plate subjected to structural excitation with two harmonics.

6.3 Random Excitations

As the random excitation case three different forces were applied. The first analysis was performed for showing the system behaviour excited by a random structural force in the range of 40-600 Hz. The second analysis was performed for showing the added effect of the monopole source, located inside the box besides the previous random force in the frequency range of 0-600 Hz. With the third and the last brief analysis it was intended to examine the effect of the excitation level on the response characteristics.

6.3.1 A Structural Random Excitation

In this analysis, a structural random force with a constant amplitude of 10N through the considered frequency range (40-600 Hz) was used. The excitation in this case may be considered as narrow-band type. Uncoupled and coupled acceleration response spectra of various points on the plate labeled as Point0, Point1, Point2, and Point3 shown in Figure 6.2 are presented in Figures 6.21-6.24. Uncoupled and coupled acoustic pressure responses of these points are given in Figures 6.25-6.28. All of these responses are composed of structural modes of the flexible plate. As can be found from Table 5.1 they are all odd (1^{st}, 5^{th}, 7^{th}, 13^{th} and 17^{th}) modes, respectively. Also as presented in Table 5.1, the first cavity mode frequency of the Box Model is 548.73 Hz and no response peak arises around this frequency in Figures 6.21-6.28. This situation implies that structural modes should have priority for engineering design when only a structural force is applied to such a plate-cavity system.

As shown in Figure 6.21 which illustrates the response of excitation point, Point0, the coupling effect is obviously seen from the first two peaks encountered at uncoupled (46.164 Hz, 169.84Hz) and coupled (91.987 Hz, 179.85 Hz) modes. The stiffness effect of the cavity is seen again on the first two response peaks. Amplitude of the first response peak increases whereas the second one decreases when coupled analysis is performed as in the case of force with two harmonics (Figures 6.13 and 6.14). Figures 6.22-6.24 show that the same coupling effects are obtained for Point1, Point2 and Point3. The damping effect of coupling arises at some of the mid modal frequencies whereas stiffness effect disappears. It may be stated that the first modal response to random excitation does not have damping behaviour in coupled analysis.

A similar behaviour of surface sound pressures and structural accelerations, due to coupling is observed in Figures 6.25-6.28 except the first mode. The sound reduction effect of the cavity on the coupled system is perceived at the beginning of the first response peak. The sound pressure level reduction due to coupling is also seen on the RMS records at the upper right corners of Figures 6.25-6.28.

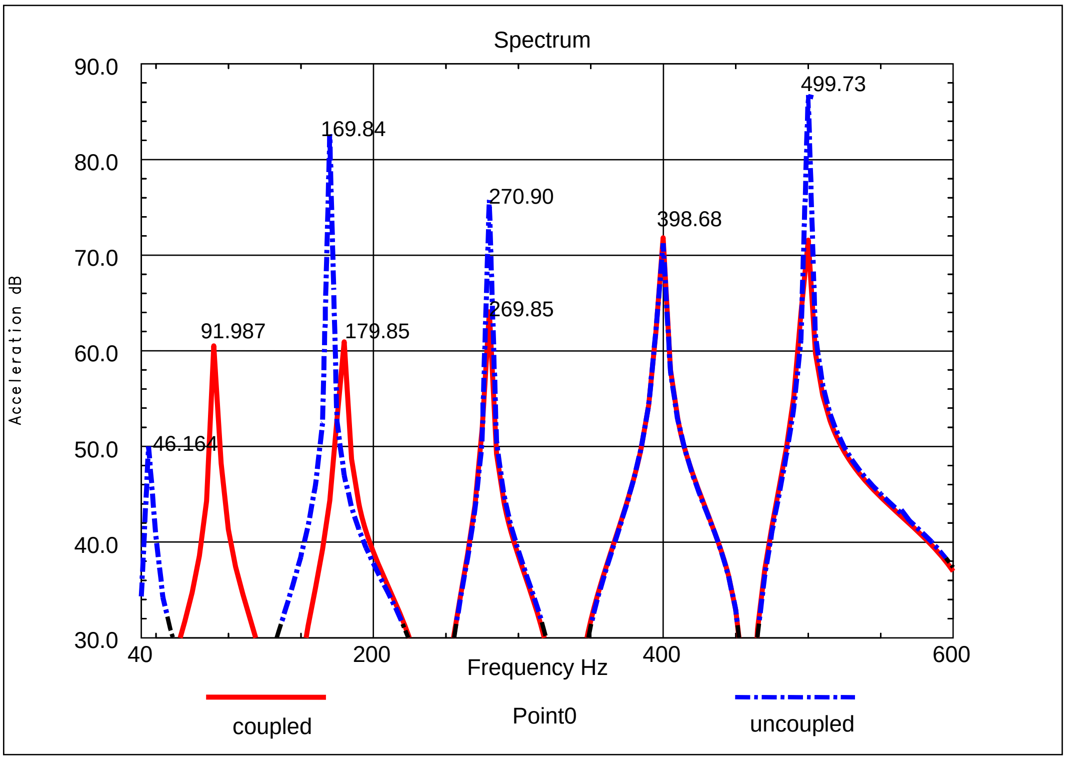


Figure 6.21 Acceleration response of Point0 on the plate subjected to a structural random excitation.

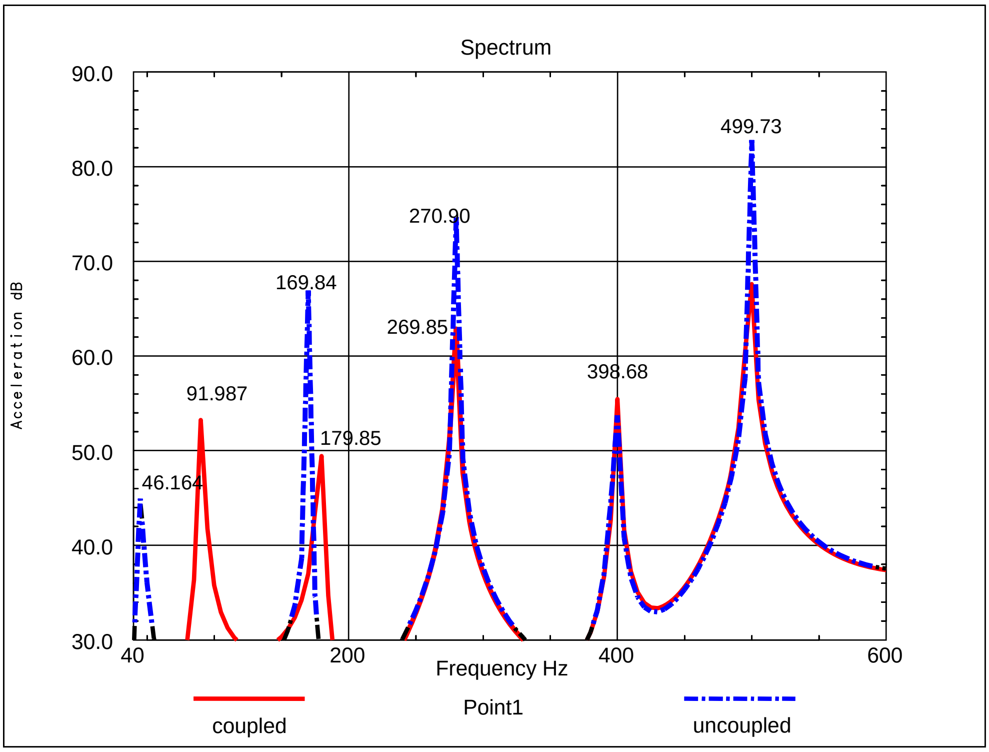


Figure 6.22 Acceleration response of Point1 on the plate subjected to a structural random excitation .

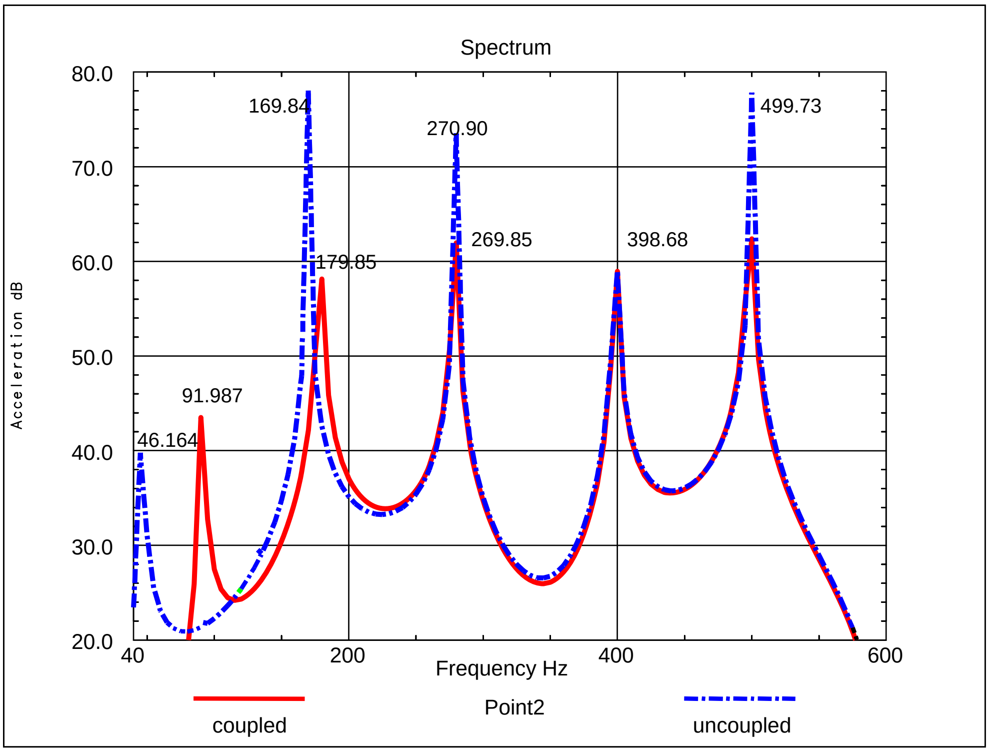


Figure 6.23 Acceleration response of Point2 on the plate subjected to a structural random excitation.

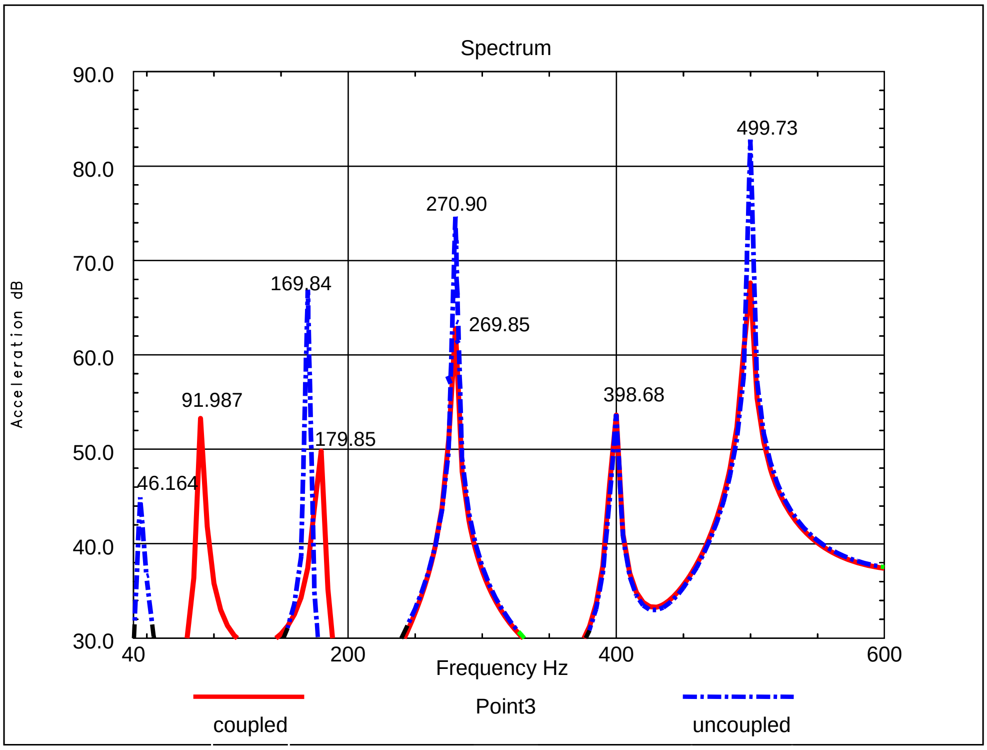


Figure 6.24 Acceleration response of Point3 on the plate subjected to a structural random excitation.

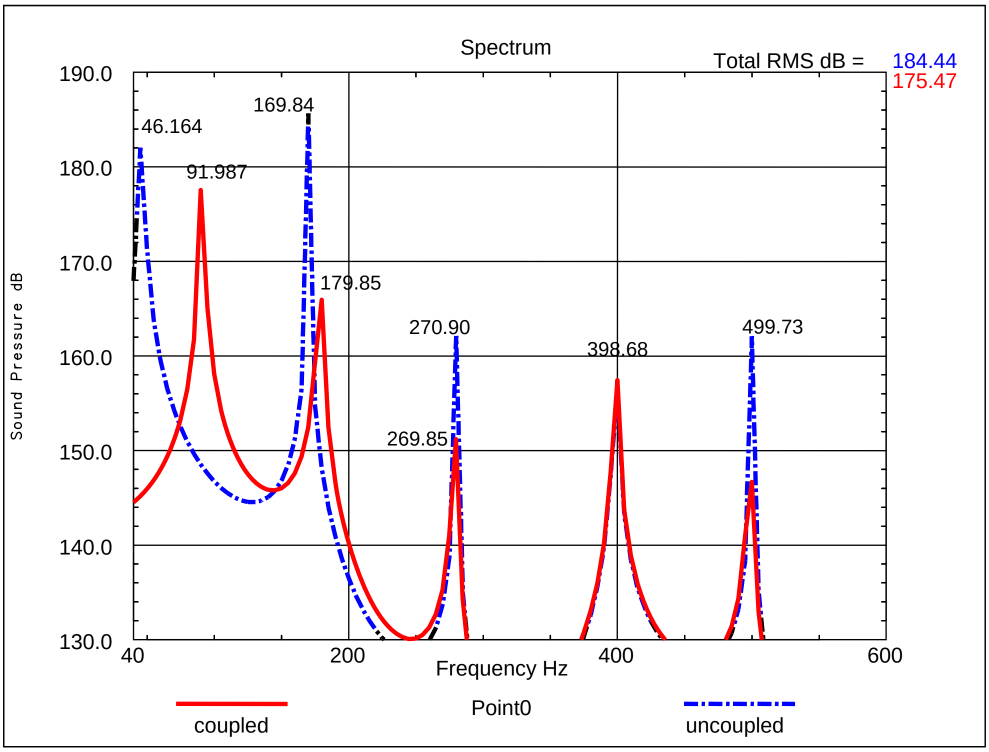


Figure 6.25 Sound pressure response of Point0 on the plate subjected to a structural random excitation.

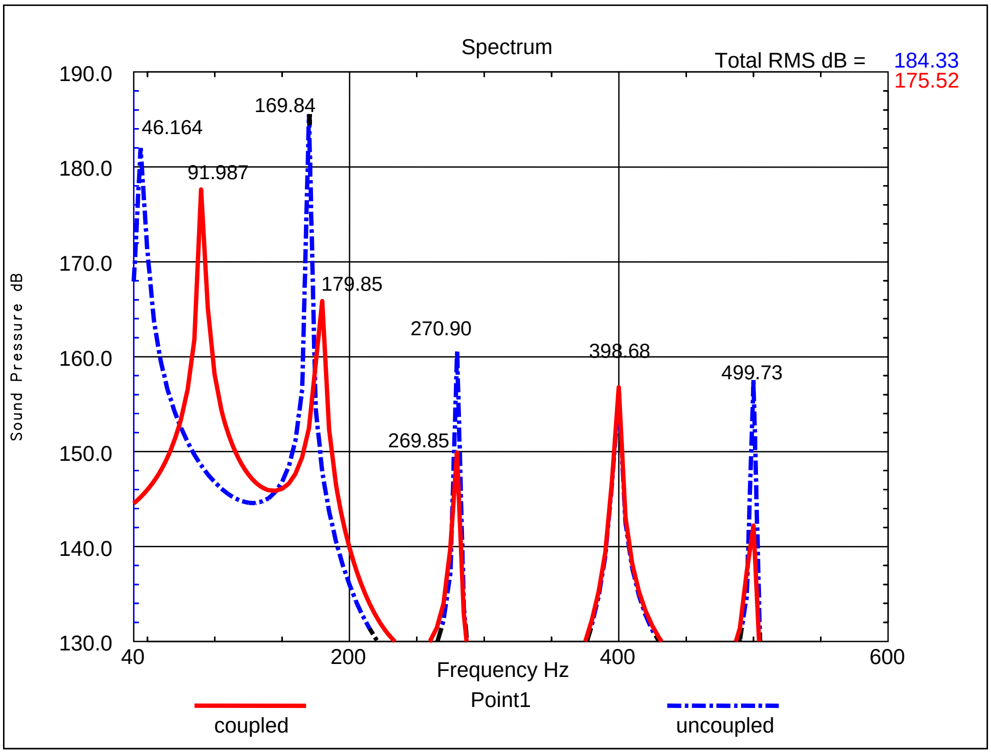


Figure 6.26 Sound pressure response of Point1 on the plate subjected to a structural random excitation.

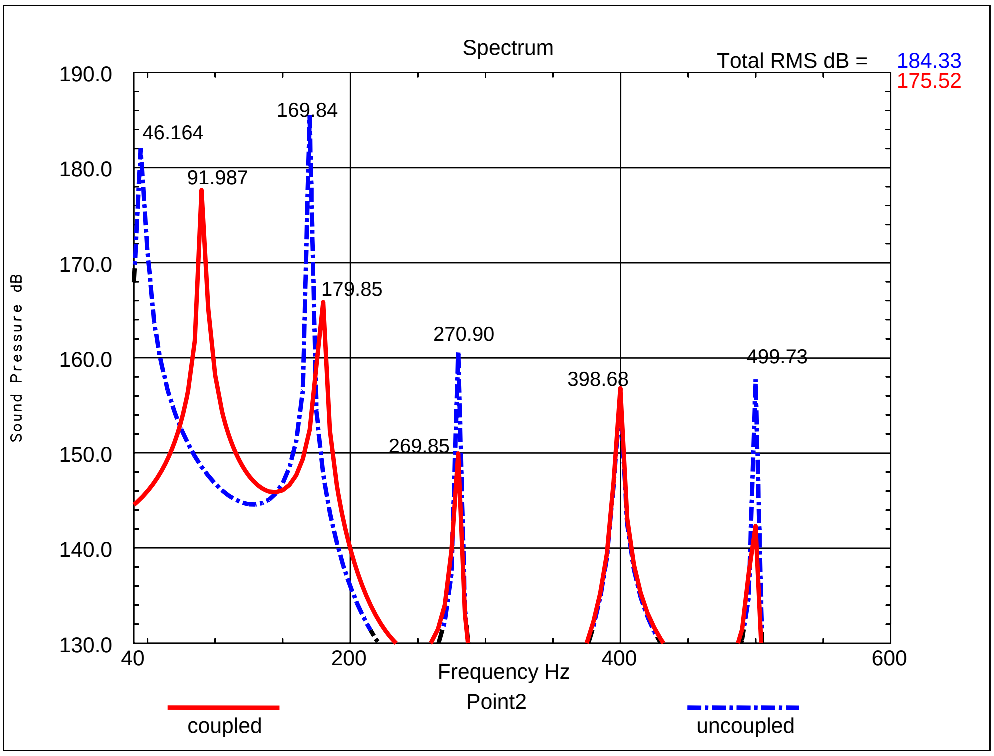


Figure 6.27 Sound pressure response of Point2 on the plate subjected to a structural random excitation.

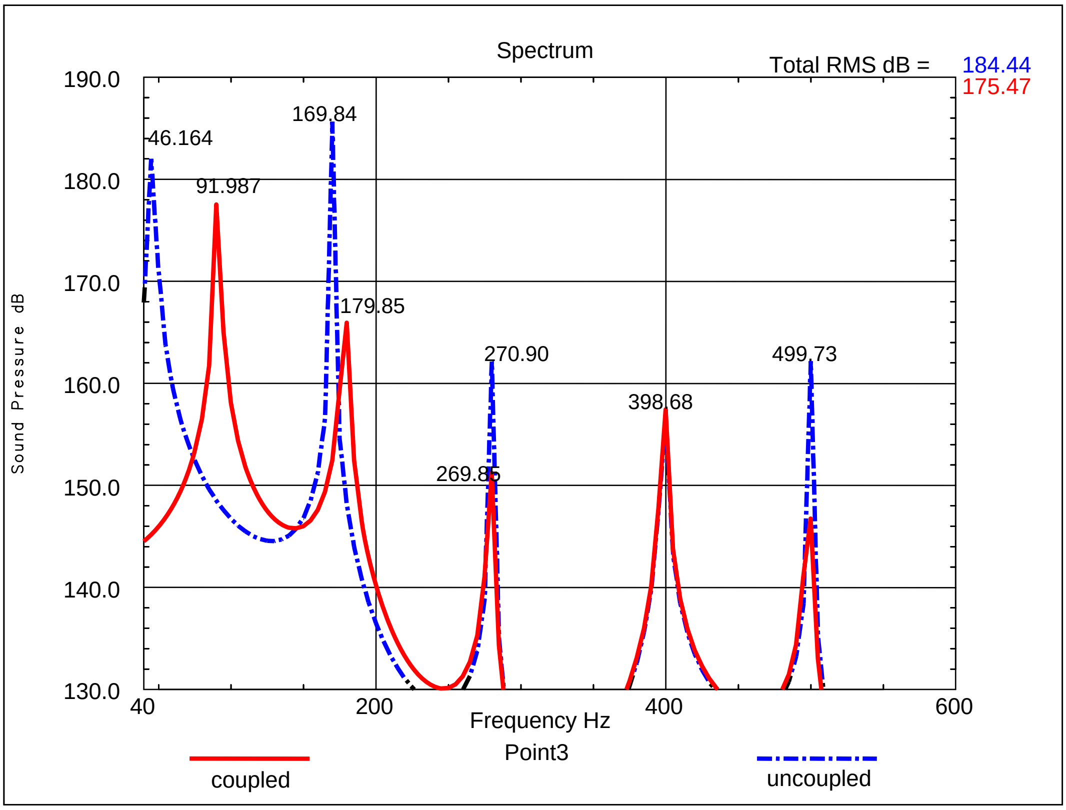


Figure 6.28 Sound pressure response of Point3 on the plate subjected to a structural random excitation.

6.3.2 A Structural Random Excitation and An Acoustical Random Excitation

In this analysis, there is a combination of different types of excitations: A mechanical force with a constant amplitude of 10N through the considered frequency range (0-600 Hz), and a monopole source with a source strength of 10m^3/s through the same range. Response spectra are divided into two frequency ranges and they are presented separately. The first range is 0-40 Hz (until the first structural mode frequency-low frequencies) and the second range is 40-600 Hz (a little higher than the first cavity mode frequency-mid frequencies). The uncoupled and coupled response spectra for Point0 and Point2 in 0-40 Hz frequency range are presented in Figures 6.29-6.36.

When Figures 6.29-6.36 are examined, it is seen that in this low frequency range coupled acceleration responses have larger values than uncoupled responses. This shows that cavity acts as a soft spring and taking into account its effect increases the motion of the plate. However coupled sound pressures have lower values than uncoupled pressures. Therefore a sound reduction effect of coupling arises in this low frequency range.

Uncoupled and coupled acceleration responses at Point0, Point1, Point2, and Point3 of the Box Model subjected to both structural and acoustical excitations for 40-600 Hz frequency range are presented in Figures 6.37-6.40. Acoustic pressure responses of these points in the same frequency range are given in Figures 6.41-6.44.

In Figures 6.37-6.40 the stiffness effect of the coupling is seen for the first two structural modes as in all the other excitation cases. However, in coupled analyses, depending on the location of the response point on the plate, some differences or irregularities are observed in the frequency spectra. These arise due to the effect of monopole source in coupled analysis. The regular behaviour in Figure 6.37 obtained for the excitation point, Point0, is identical to the results for the previous excitation case. However, an increase in the first mode's acceleration level due to the effect of monopole is observed. The other effects of monopole source to coupling; that is irregularities at response spectra and formation of new modal peaks towards the high frequencies shown in Figures 6.38-6.40, arise for the other observation points. These figures show that, with the inclusion of acoustical excitation both uncoupled and coupled cavity modes are excited in addition to structural coupled modes. In strictly speaking, coupled modes should not be named as "coupled structural" or "coupled cavity". However, these terms imply that these coupled modes are formed by the transformation of which type of modes; structural or cavity. The uncoupled and coupled modes arise together on the same response curve and form peak families. In Figures 6.38-6.40 peak families including response peaks at a cavity mode (548.73 Hz) and a coupled mode (569.6 Hz) are clearly observed for Point1, Point2 and Point3. A new coupled mode appears at 587 Hz for the acceleration response of Point2 to the combined excitation.

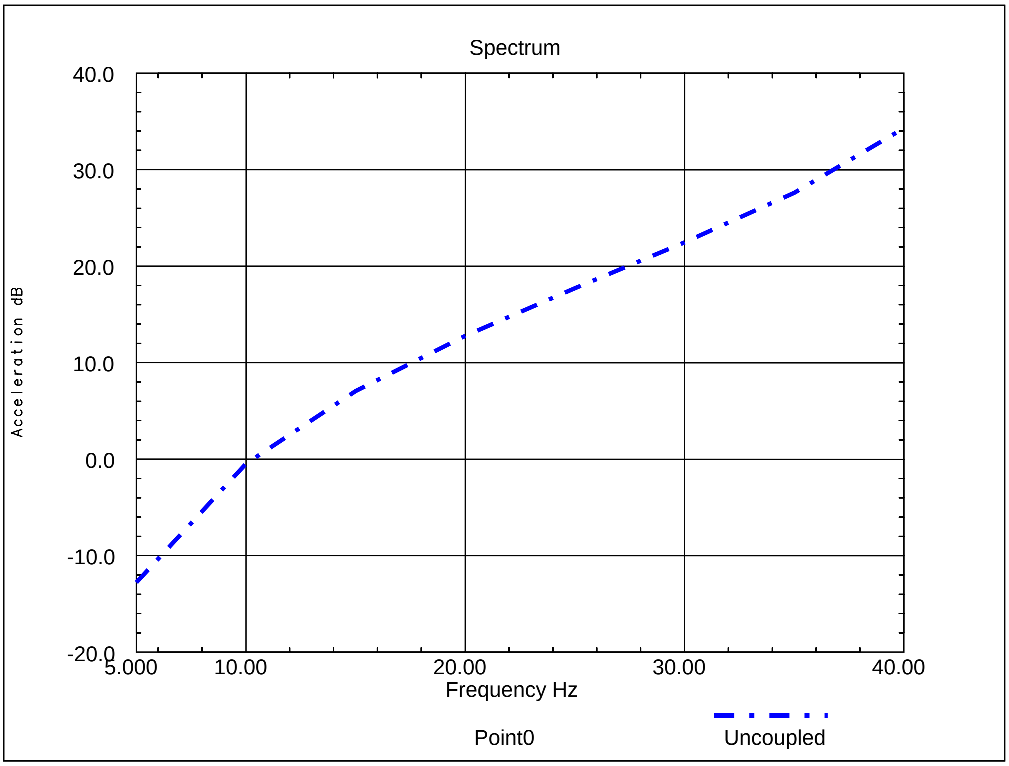


Figure 6.29 Uncoupled acceleration response of Point0 on the plate subjected to both structural and acoustical excitation.

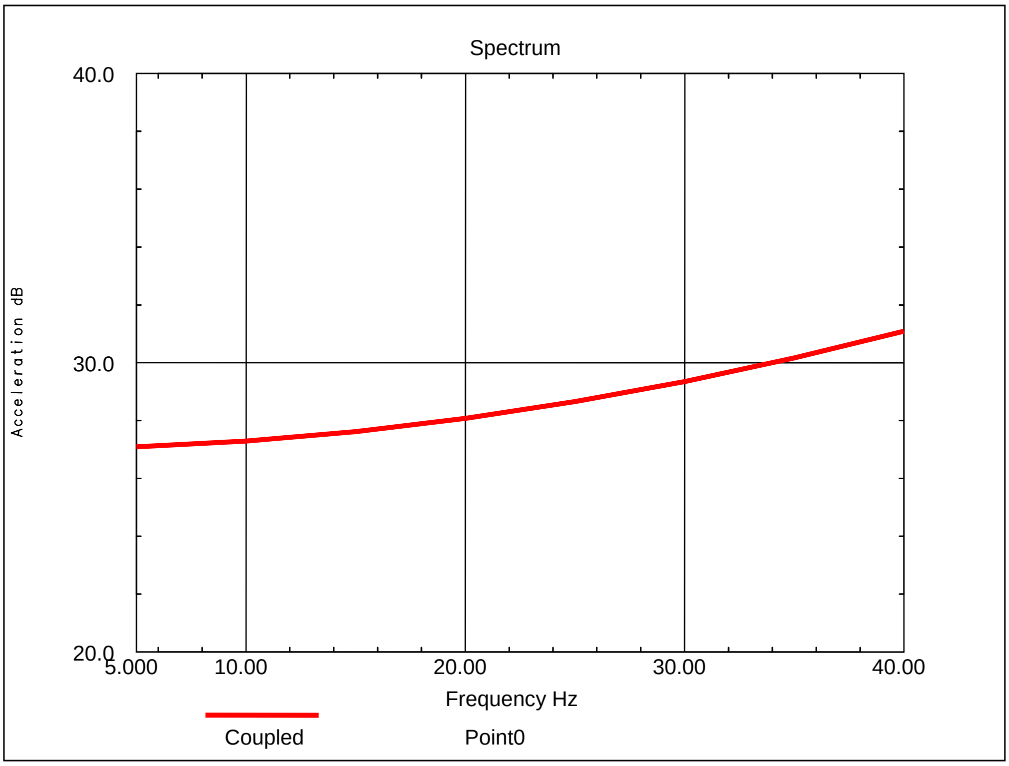


Figure 6.30 Coupled acceleration response of Point0 on the plate subjected to both structural and acoustical excitation.

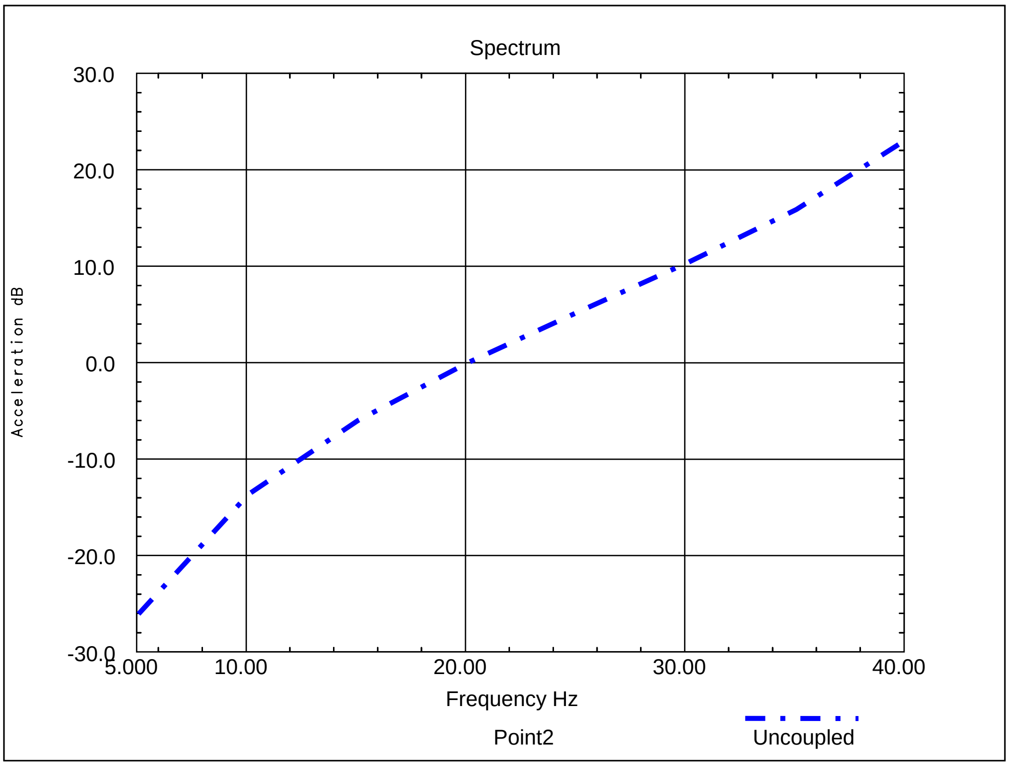


Figure 6.31 Uncoupled acceleration response of Point2 on the plate subjected to both structural and acoustical excitation.

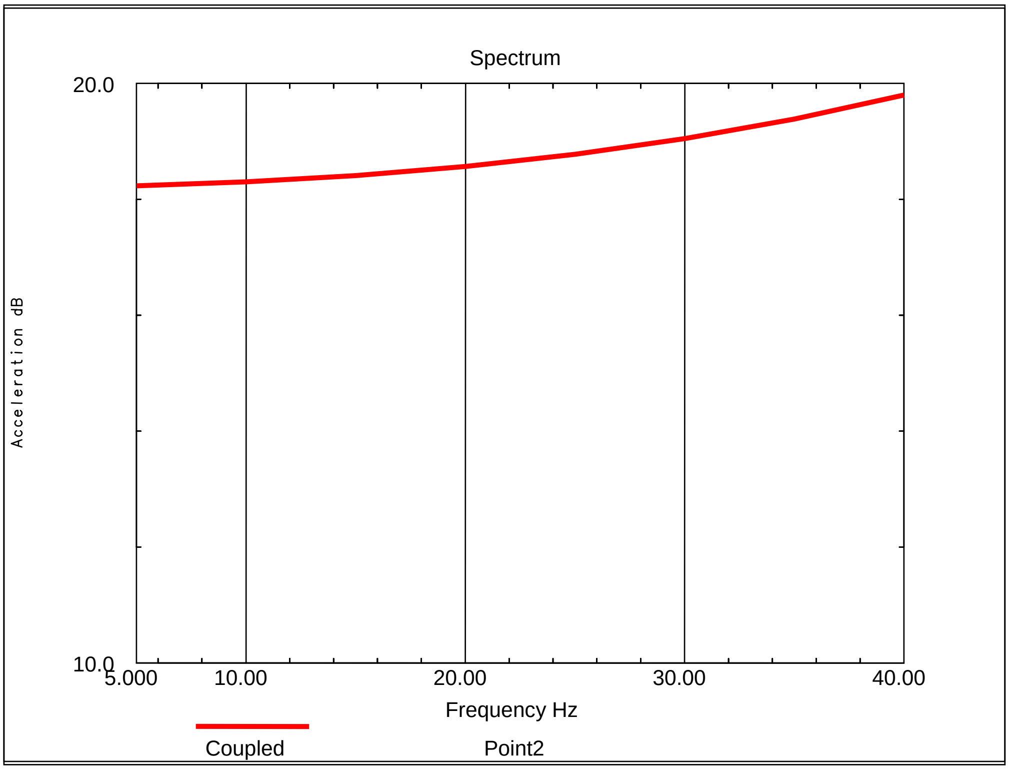


Figure 6.32 Coupled acceleration response of Point2 on the plate subjected to both structural and acoustical excitation.

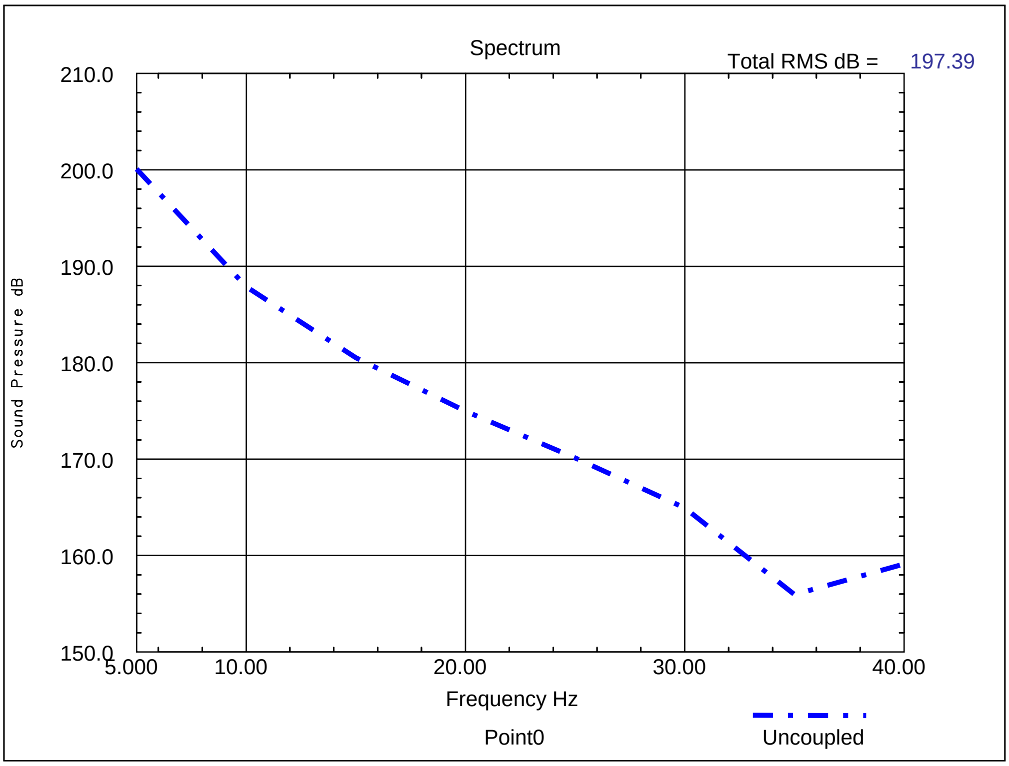


Figure 6.33 Uncoupled sound pressure response of Point0 on the plate subjected to both structural and acoustical excitation.

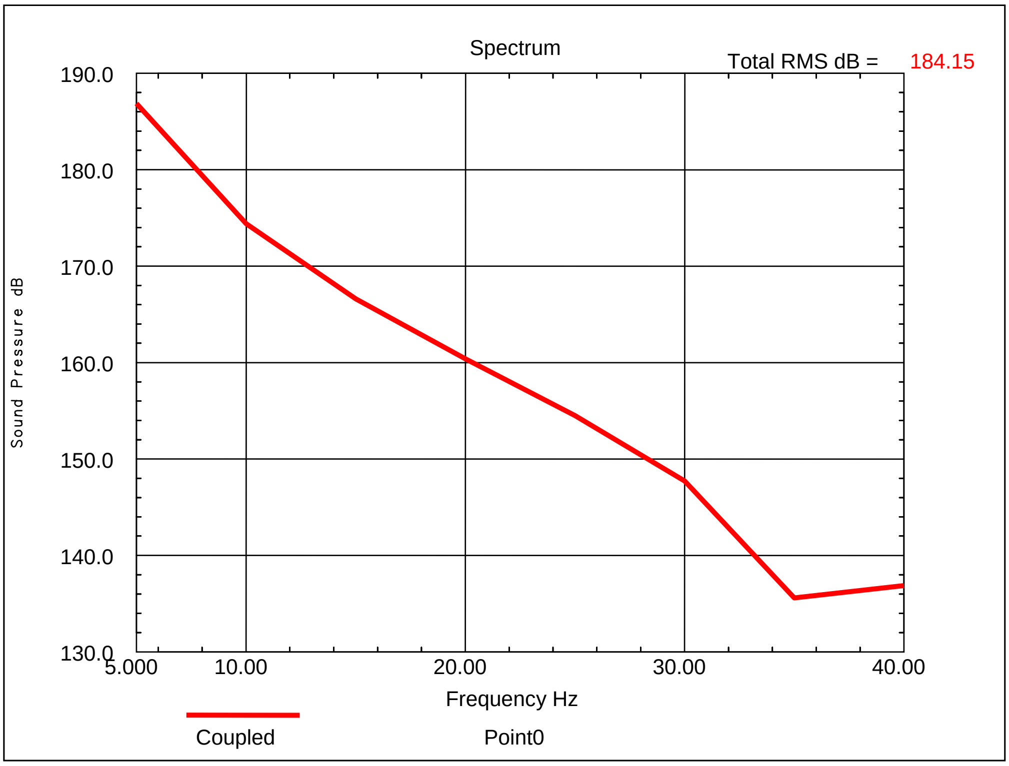


Figure 6.34 Coupled sound pressure response of Point0 on the plate subjected to both structural and acoustical excitation.

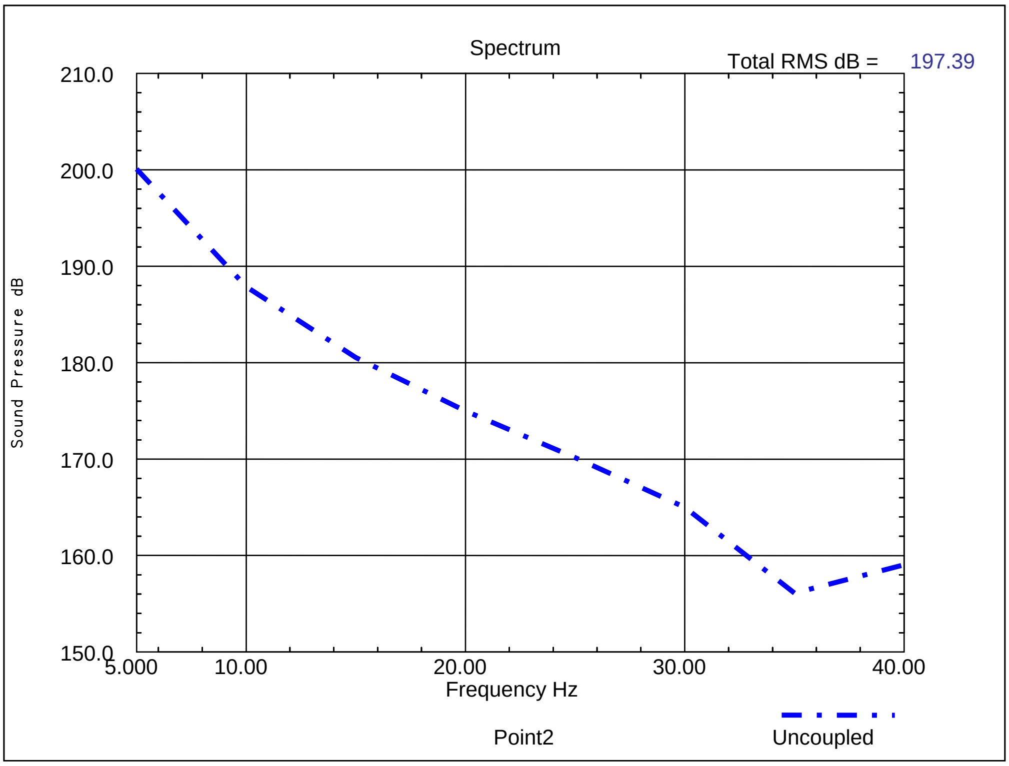


Figure 6.35 Uncoupled sound pressure response of Point2 on the plate subjected to both structural and acoustical excitation.

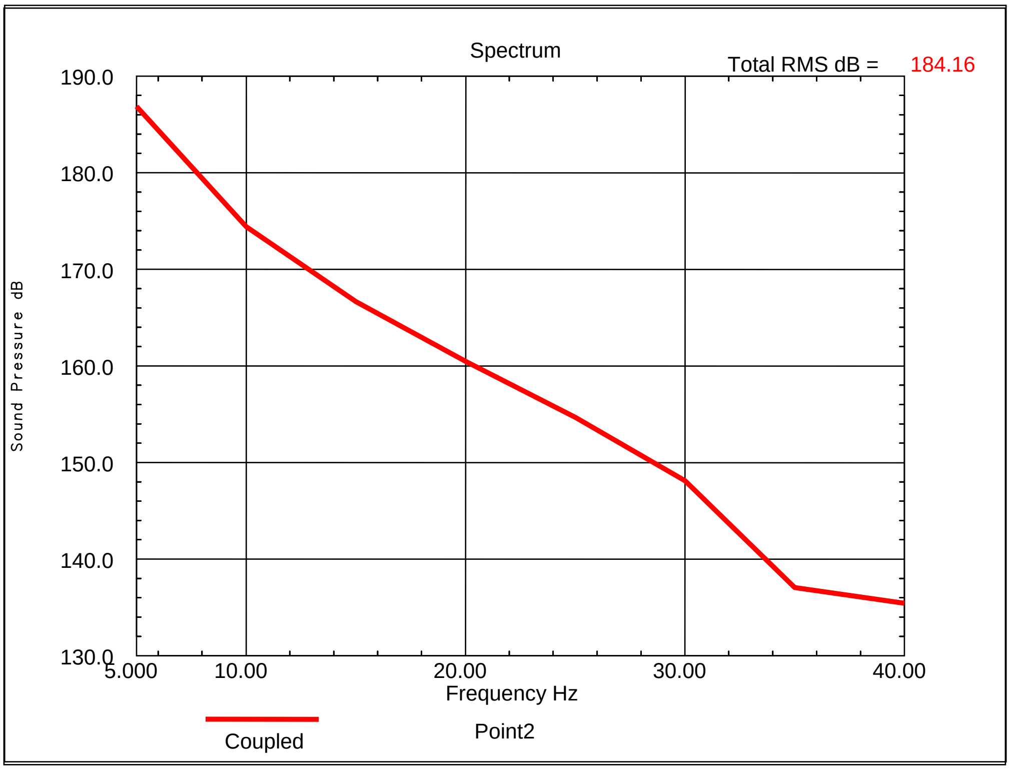


Figure 6.36 Coupled sound pressure response of Point2 on the plate subjected to both structural and acoustical excitation.

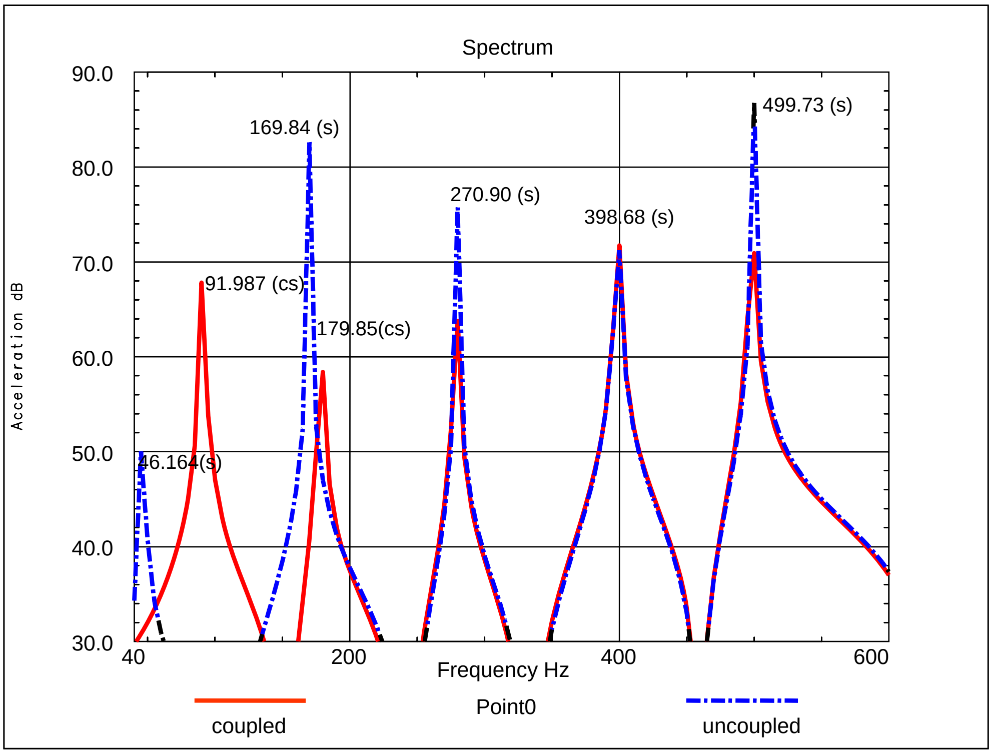


Figure 6.37 Acceleration response of Point0 on the plate subjected to both structural and acoustical excitation, s: structure mode, cs: coupled structure mode.

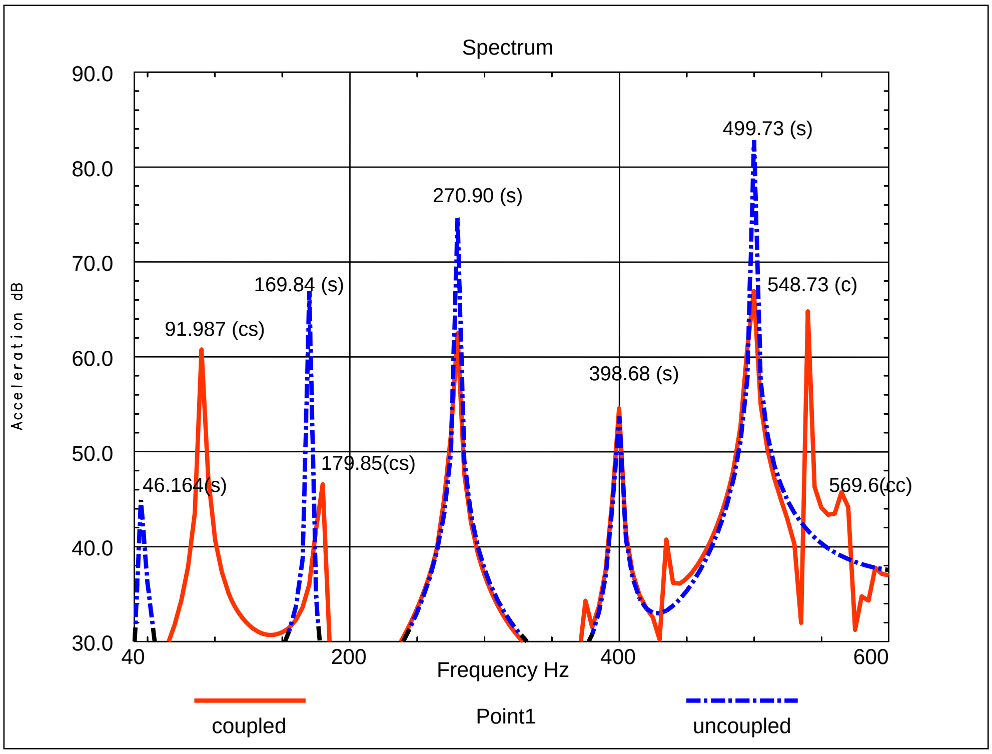


Figure 6.38 Acceleration response of Point1 on the plate subjected to both structural and acoustical excitation, s: structure mode, c: cavity mode, cs: coupled structure mode, cc: coupled cavity mode.

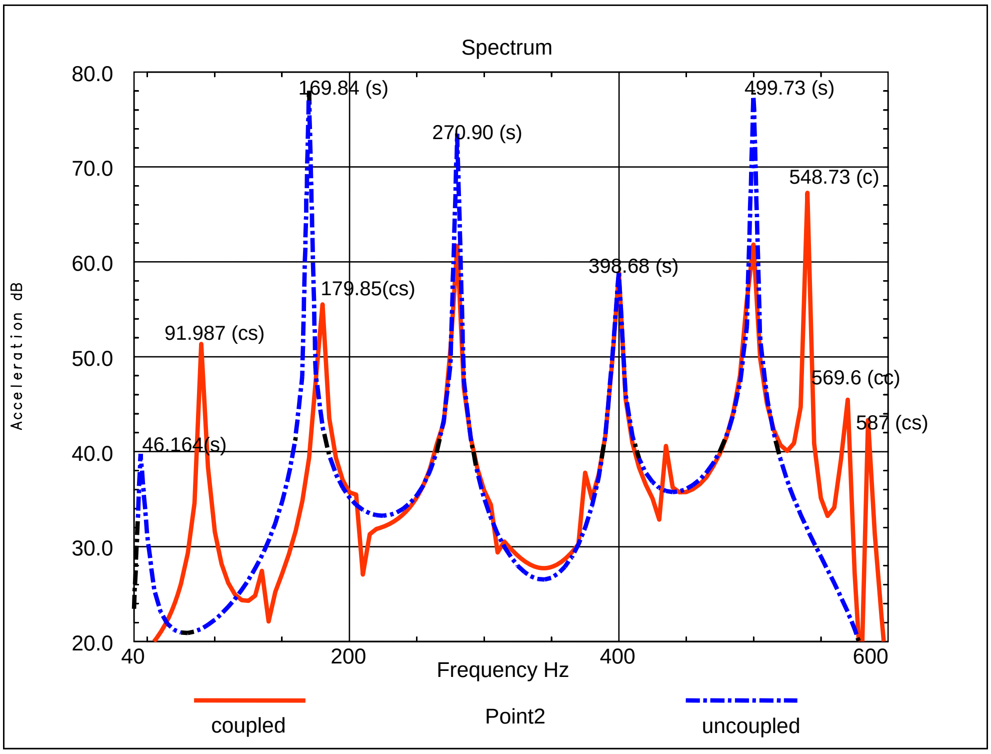


Figure 6.39 Acceleration frequency response of Point2 on the plate subjected to both structural and acoustical excitation, s: structure mode, c: cavity mode, cs: coupled structure mode, cc: coupled cavity mode.

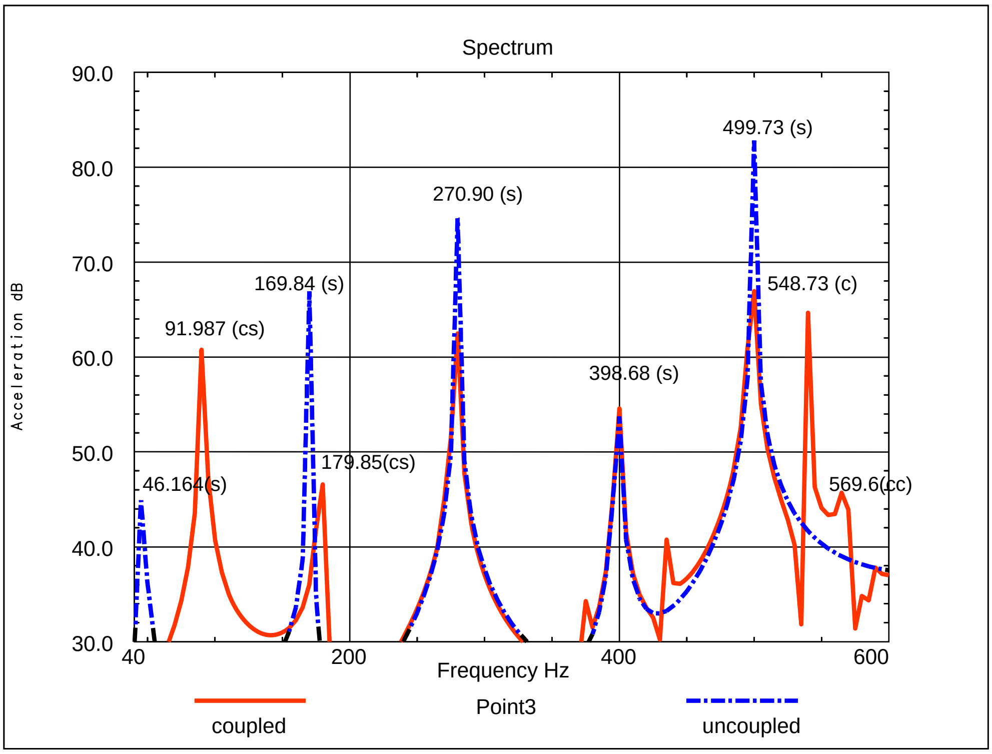


Figure 6.40 Acceleration frequency response of Point3 on the plate subjected to both structural and acoustical excitation, s: structure mode, c: cavity mode, cs: coupled structure mode, cc: coupled cavity mode.

The effect of monopole source both for coupled and uncoupled analyses is more clearly observed in sound pressure spectra (Figures 6.41-6.44). Due to coupling of the monopole, the first sound pressure peak increases in the coupled analysis and a few small irregularities are seen for Point1, Point2 and Point3. As shown in Figures 6.42-6.44, the uncoupled structural mode arises at 552.41 Hz due to the uncoupled effect of monopole source. This mode shifts left to 540.6 Hz in coupled solution. This behaviour slightly resembles the virtual mass effect which would be faced in coupled acceleration analysis. Also 588.6 Hz mode is slightly triggered in the coupled solution in addition to 569.6 Hz mode whereas 548.73 Hz mode forming peak family disappears.

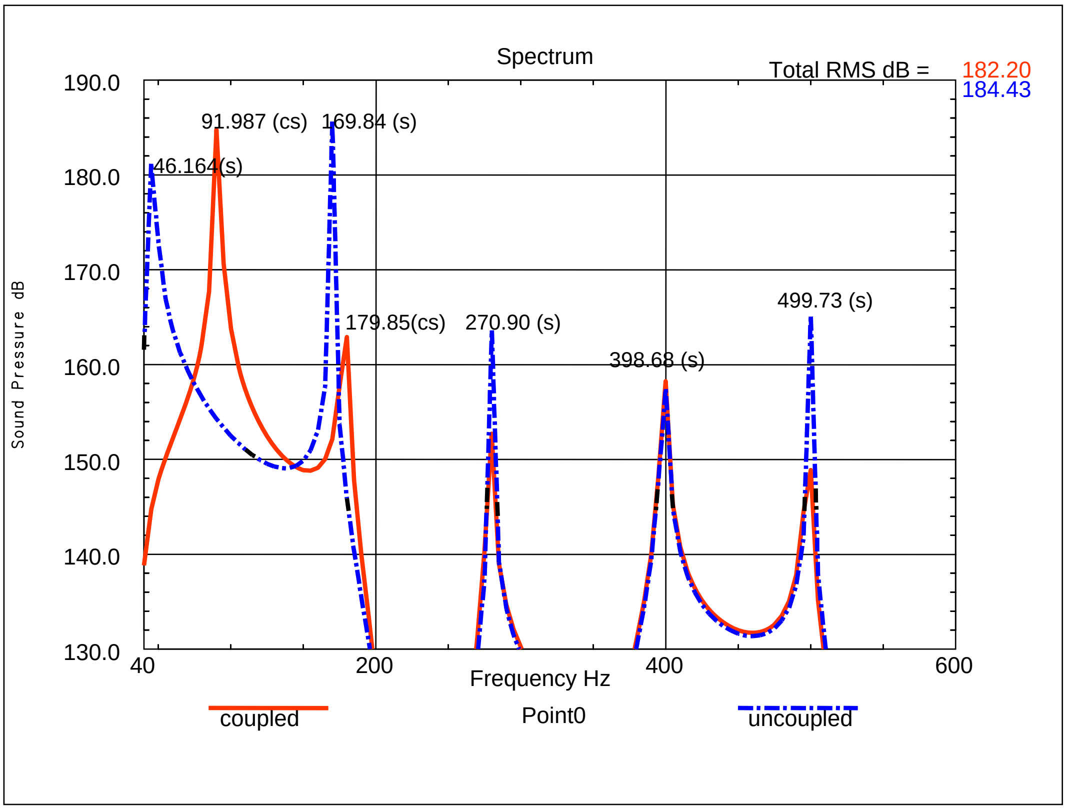


Figure 6.41 Sound pressure response of Point0 on the plate subjected to both structural and acoustical excitation, s: structure mode, c: cavity mode, cs: coupled structure mode, cc: coupled cavity mode.

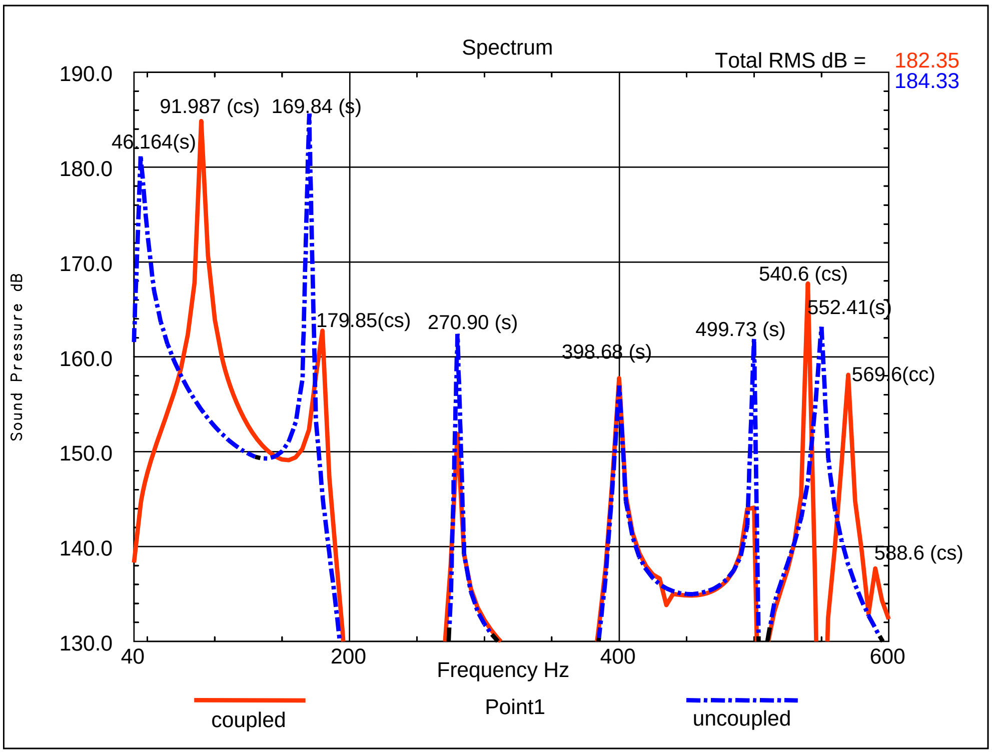


Figure 6.42 Sound pressure response of Point1 on the plate subjected to both structural and acoustical excitation, s: structure mode, c: cavity mode, cs: coupled structure mode, cc: coupled cavity mode.

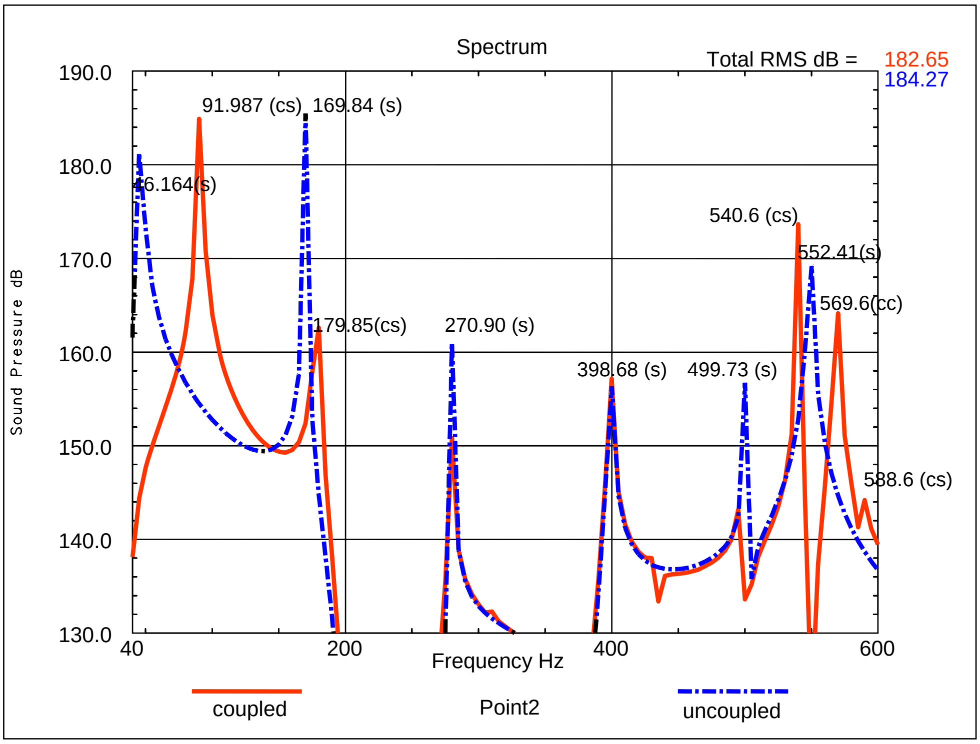


Figure 6.43 Sound pressure response of Point2 on the plate subjected to both structural and acoustical excitation, s: structure mode, c: cavity mode, cs: coupled structure mode, cc: coupled cavity mode.

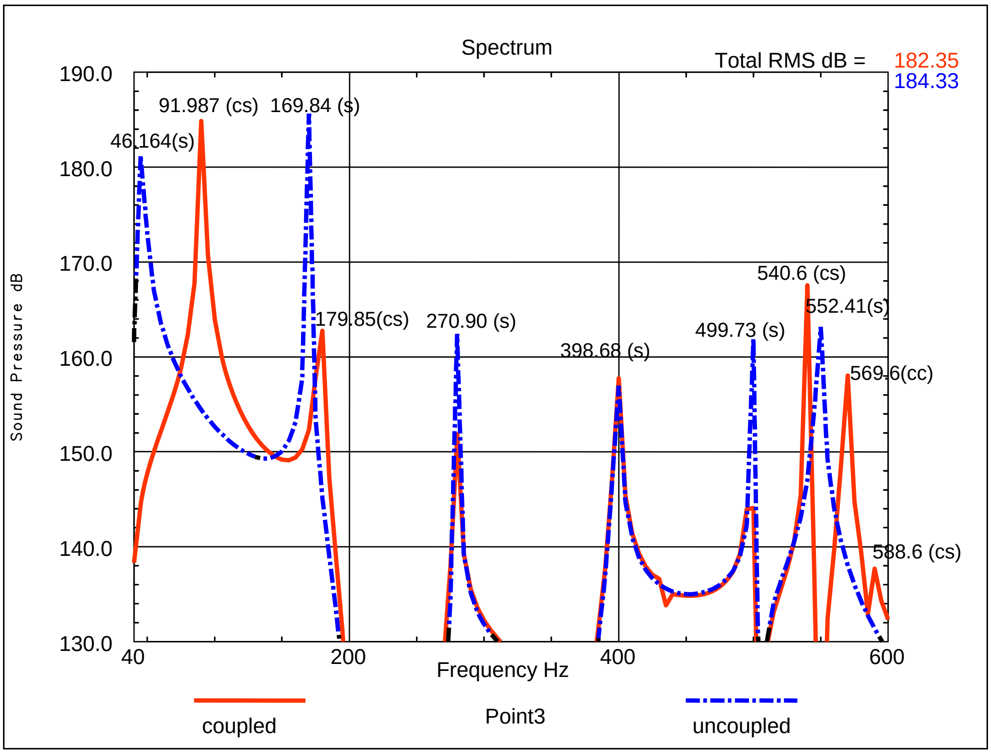


Figure 6.44 Sound pressure response of Point3 on the plate subjected to both structural and acoustical excitation, s: structure mode, c: cavity mode, cs: coupled structure mode, cc: coupled cavity mode.

Table 6.3 shows the total “RMS” sound pressure values both for coupled and uncoupled analyses. The total “RMS” values of coupled responses are smaller than the uncoupled responses for both of the two excitation cases. The difference between these total “RMS” values, whose square value (mean square) is an energetic parameter, may be taken as an indicator for the damping effect. This difference can be expressed as a loss of energy stored in the system due to sound attenuation caused by the cavity. Table 6.3 also indicates that the total energy for the last excitation case is higher than that for the previous case for coupled analysis whereas they are approximately equal for uncoupled analysis. This information implies that only coupling analysis takes into account the sound attenuation effect caused by the cavity. Therefore, uncoupled analysis does not yield reliable results because of overlooking energy (sound and vibration) reduction effect.

Table 6.3 Uncoupled and coupled total "RMS" values (dB) of sound pressures at response points

	Total "RMS" values (dB)			
	Excited by structural force (coupled)	Excited by structural force (uncoupled)	Excited by structural force and acoustical source (coupled)	Excited by structural force and acoustical source (uncoupled)
Point0	175.47	184.44	182.20	184.43
Point1	175.52	184.33	182.35	184.33
Point2	175.52	184.33	182.65	184.27
Point3	175.47	184.44	182.35	184.33

6.3.3 A Structural Random Excitation with Lower Amplitude

A structural random force with a constant 1N amplitude through the considered frequency range (40-600 Hz) was applied to the center of the plate. In Figure 6.45, coupled acceleration response of Point0 for the range of 40-600 Hz is given to show the characteristics of the system under the random force with a decreased amplitude. When Figure 6.45 is compared with the corresponding Figure 6.21 for 10N excitation case, it is found that a logarithmic decrease in acceleration levels in dB scale appears with the reduction in excitation level. That is, a 100% reduction in the level of the force results in 20 dB decrease in the response dB levels. The acceleration and sound pressure responses of Point0 and Point2 are given in Figures 6.46-6.47 for the range of 40-200 Hz which contains the most affected modal frequencies from the coupling. The comparison of Figures 6.46 a, b and 6.47 a, b with the corresponding Figures 6.21, 6.25, 6.23 and 6.27 respectively show that except the decrease on the amplitude of the excitation level does not affect the response characteristic of the system.

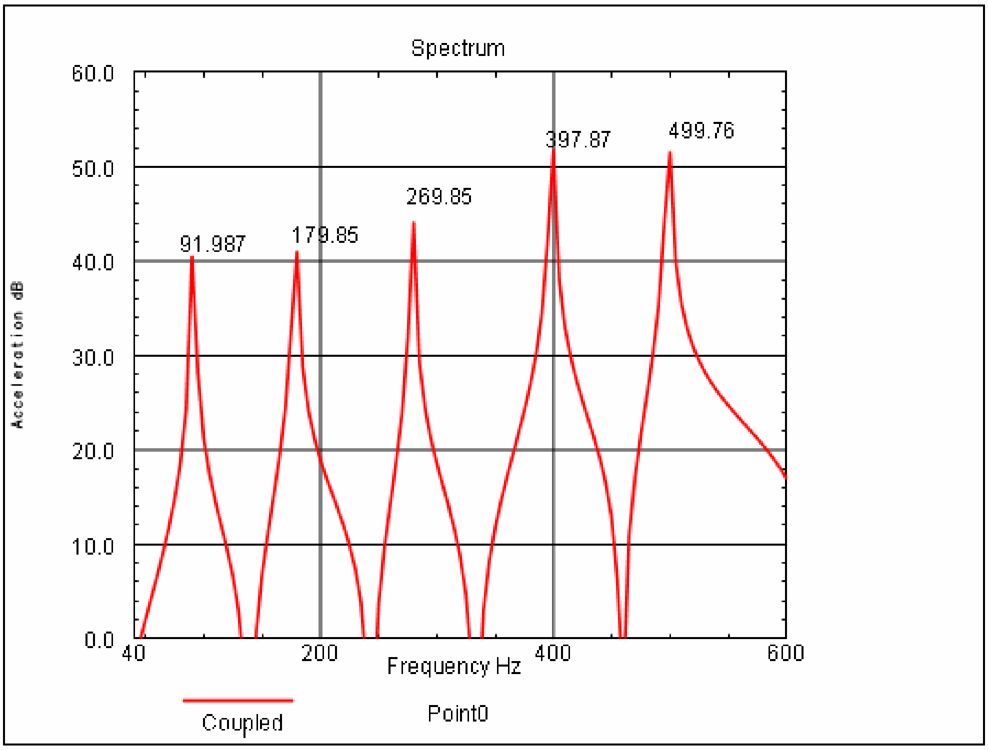
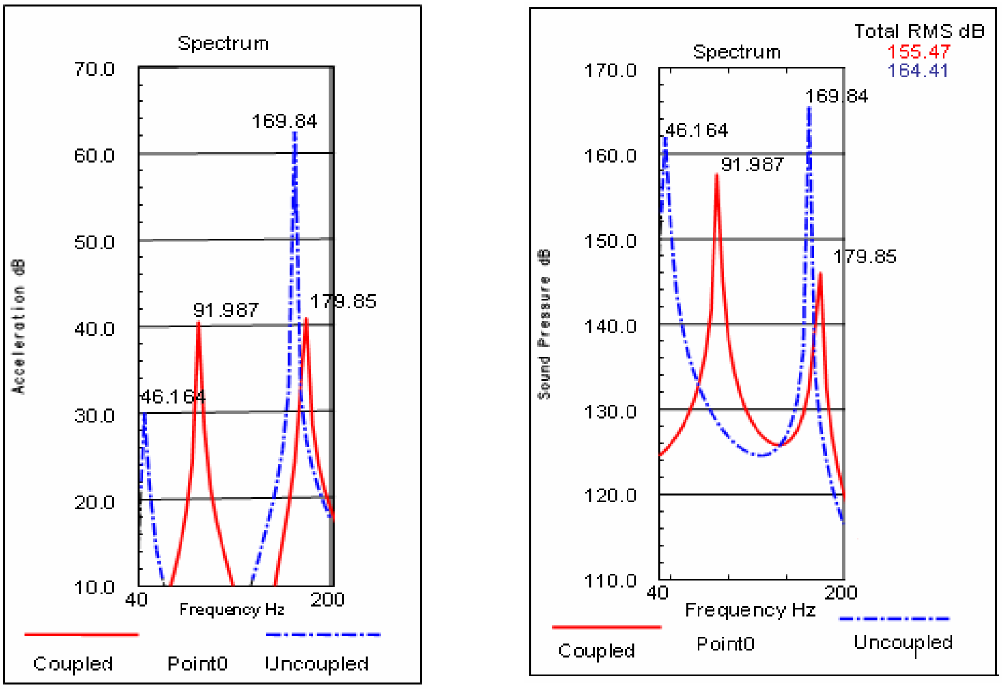
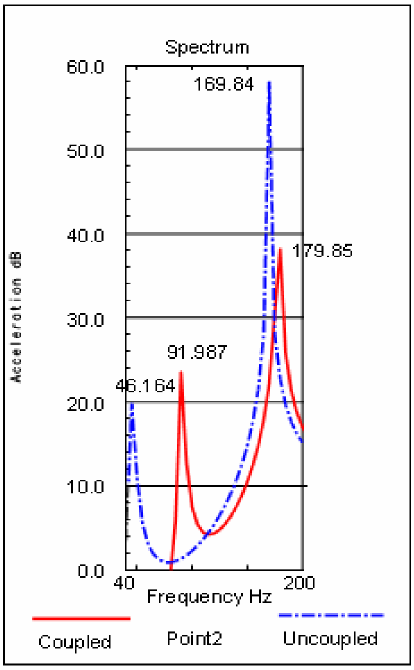


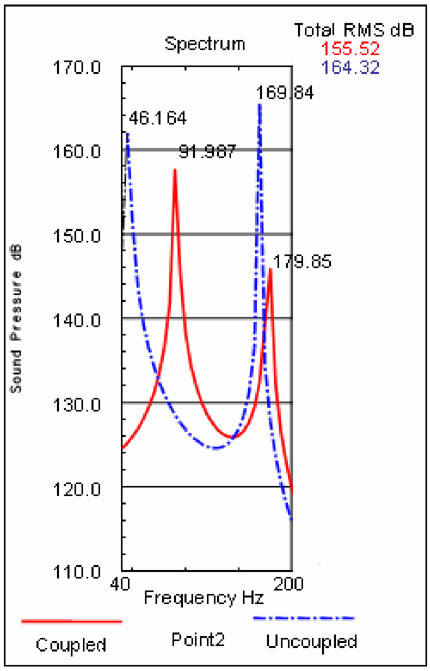
Figure 6.45 Coupled acceleration response of Point0 on the plate subjected to structural random excitation with a constant amplitude of 1N.



(a) (b)

Figure 6.46 (a) Coupled and uncoupled acceleration response of Point0 on the plate subjected to structural random excitation with a constant amplitude of 1N, (b) Coupled and uncoupled sound pressure response of Point0 on the plate subjected to structural random excitation with a constant amplitude of 1N.





(a) (b)

Figure 6.47 (a) Coupled and uncoupled acceleration response of Point2 on the plate subjected to structural random excitation with a constant amplitude of 1N, (b) Coupled and uncoupled sound pressure response of Point2 on the plate subjected to structural random excitation with a constant amplitude of 1N.

The response characteristics presented in this chapter may be summarized as:

1. The coupling effect must be considered especially for the first few modal frequencies regarding structural, acoustical and energy point of view.
2. The coupling effects are encountered as stiffness and damping (sound attenuation) at low and low-mid frequencies, respectively.
3. Structural forces can not excite cavity modes. However, acoustical forces excite both structural and cavity modes.
4. At the beginning of high frequency region, uncoupled and the corresponding coupled modes may appear together in the coupled solution. These modes form "peak families"
5. The reduction in the excitation level does not affect the response characteristic of the plate-cavity systems.
6. Forced response analyses imply the importance of coupling both low and mid frequencies defined by Lyon (1963). Due to the numerical limitations, only

the lower range of high frequencies could be examined. However, high frequency analysis requires completely different solution techniques.

CHAPTER SEVEN

CONCLUSION

In noise and vibration control engineering, vibration of a thin plate backed by a cavity, sound in this cavity and their mutual effects are of considerable importance for accurate and realistic design of various machines and vehicles. The natural behaviour of mutual interaction of sound and vibration requires a simultaneous coupled mathematical analysis. Therefore, especially for low frequency problems, a hybrid approach called as "Coupled FEM/BEM" is commonly used as a numerical technique.

This work is mainly focused on a parametric study based on a simple coupled vibro-acoustic model including a plate-cavity system in order to examine the coupling effects. This box model, in fact, may be a simple subsystem for complicated cavity-plate systems, because every complicated system may be modelled by using many box models by defining a relationship between these models. All analyses were performed by using I-DEAS Vibro-Acoustic module which uses coupled FEM/BEM approach. The parametric study is composed of six cases by changing the thickness of the plate and the depth of the cavity. In this regard, free and forced uncoupled and coupled analyses were performed and the results were evaluated by structural, acoustical and energy point of views.

Free vibro-acoustic analyses have shown that the first structural mode (fundamental mode) and the first cavity mode (transverse mode) are the most affected modes. If the fundamental mode is higher than transverse mode the coupling clearly increases the frequency of the fundamental mode by introducing a stiffness effect of the cavity. The increase in the thickness of the plate makes the system stiffer and, thus, increases the coupled mode frequencies. Besides, while the depth of the cavity becomes smaller, the coupling effect becomes higher.

In order to obtain frequency response characteristics of the coupled plate-cavity system various types of forces were used for excitation. These are harmonic; almost periodic; random structural forces; random structural and acoustical sources; random structural force with low amplitude. Forced vibro-acoustic analysis have showed that coupling response behaviour resembles the free vibration behaviour. However, acoustic modes may be excited only by acoustical excitations. The decrease in the coupled peaks and in total “RMS” values of sound pressures illustrate that there is a loss of energy therefore sound reduction in the cavity. However, damping effect of the cavity on the plate accelerations may be seen sometimes towards the mid frequencies. Acoustical excitations affect the frequency spectra and cause the formation of peak families and new modal responses.

Since uncoupled analysis neglects the mutual interaction of sound and vibration, it yields unrealistic behaviour and unacceptable results in box like structures even at lower frequencies. A system consists of a thinner plate backed by a smaller cavity is more sensitive to the coupling effect. Therefore, engineers or designers should consider dynamical nature of Plate-cavity systems and must perform coupled analysis in order to make accurate prediction for realistic vibro-acoustic analyses.

REFERENCES

Chen, Z.S, Hofstetter, G., & Mang. H.,A. (1998). A Galerkin-type BE-FE formulation for elasto-acoustic coupling*, Comput. Methods Appl. Mech. Eng., 152, 147-155.*

Coyette, J. P., (1999). The use of finite-element and boundary-element models for predicting the vibro-acoustic behaviour of layered structures, *Advances in Engineering Software, 30, 133-139.*

Ding, W.P., & Chen, H. L. (2001). A symmetrical finite element model for structure-acoustic coupling analysis of an elastic, thin-walled cavity, *Academic Press, 547-559.*

Evertsine, G.C. (1997). Finite element formulations of structural acoustics problems, *Computers & Structures . 65(3), 307-321.*

Frendi, A., & Robinson, J. (1993). Effect of acoustic coupling on random and harmonic plate vibrations, *Journal of the Acoustical Society of America 31, No. 11, 1992-1997.*

Graff, K.F. (1973). *Wave motion in elastic solids*, Ohio State University Press, 274-275.

Guy, R. W., & Bhattacharya, M. C. (1973). The transmission of sound through a cavity-backed finite plate, *Journal of Sound and Vibration 27(2), 207-223.*

Harris, C.M., & Piersol, A.G. (Eds.) (2002). *Harris' Shock and Vibration Handbook* (5th ed.), New York: McGraw-Hill.

Kopuz, Ş. (1995). An integrated FEM/BEM approach to the prediction of interior noise levels of vehicle body structures. *Doctorate Thesis, The Middle East Technical University, The Graduate School of Natural and Applied Sciences, Ankara .*

Lee, Y. Y. (2002). Structural-acoustic coupling effect on the nonlinear natural frequency of a rectangular box with one flexible wall, *Applied Acoustics (63), 1157-1175.*

Li, Y.Y., & Cheng, L. (2006). Vibro-acoustic analysis of a rectangular-like cavity with a tilted wall, *Applied Acoustics, avaliable online at www.sciencedirect.com.*

Lyon, R. H. (1963). Noise reduction of rectangular enclosures with one flexible wall, *Journal of the Acoustical Society of America 35, 1791-1797*.

Mariem, J. B., & Hamdi, A. (1987). A new boundary finite element method for fluid-structure interaction problems, *International Journal for Numerical Methods in Engineering 24, 1251-1267.*

Pretlove, A. J. (1965). Free vibrations of a rectangular panel backed by a closed rectangular cavity, *Journal of Sound and Vibration 2, 197-209.*

Pretlove, A. J. (1966). Forced vibrations of a rectangular panel backed by a closed rectangular cavity, *Journal of Sound Vibration 3 (3*), 252-261.

Sarıgül, A.S. (1990). Çok Cisimli Ses Alanlarinin Helmholtz Integral Denklemi ile Hesaplanmasi, *Doktora Tezi, DEÜ, Fen Bilimleri Enstitüsü, İzmir*.

Vlahopoulos, N., Raveendra, S.T., Vallance, C., & Messer, S. (1999). Numerical implementation and applications of a coupling algorithm for structural acoustic models with unequal discretization and partially interfacing surfaces, *Finite Elements in Analysis and Design 32 257-277.*