DOKUZ EYLÜL UNIVERSITY GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

COMPUTER AIDED ANALYSIS OF A PLATE SUBJECTED TO A CIRCULAR MOVING LOAD

by Elif Burcu YEĞEN

> **March, 2008 İZMİR**

COMPUTER AIDED ANALYSIS OF A PLATE SUBJECTED TO A CIRCULAR MOVING LOAD

A Thesis Submitted to the Graduate School of Natural and Applied Sciences of Dokuz Eylül University In Partial Fulfillment of the Requirements for the Degree of Master of Science

in Mechanical Engineering, Machine Theory and Dynamics Program

by

Elif Burcu YEĞEN

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M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"COMPUTER AIDED ANALYSIS OF A PLATE SUBJECTED TO A CIRCULAR MOVING LOAD"** completed by **ELİF BURCU YEĞEN** under supervision of **PROF.DR. HİRA KARAGÜLLE** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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COMPUTER AIDED ANALYSIS OF A PLATE SUBJECTED TO A CIRCULAR MOVING LOAD

ABSTRACT

Moving load problem is investigated by engineers in various engineering structures such as beams and plates. In this thesis, vibration analyses of plates subjected to a circular moving load are realized by using the finite element method. ANSYS parametric design language is used to create the finite element model of the plate considering circular trajectory. A single point load is moved over the circular trajectory. The amplitude of the circular moving load changes harmonically on the plate. The two excitation frequencies corresponding to the first two natural frequencies of the plate are used in the harmonic circular moving load. The dynamic time response is obtained from the middle point of the plate. Natural frequencies of the plate are found with the modal analysis. The results are compared with the reference study. The effects of the radius of the circular path, forcing frequency and rotating speed of the moving load are investigated.

Keywords: Moving load, finite element method, ANSYS, vibration of plate, rectangular plate, computer aided analysis.

DAİRESEL HAREKETLİ YÜK ALTINDAKİ PLAKANIN BİLGİSAYAR DESTEKLİ ANALİZİ

ÖZ

Hareketli yük problemi mühendisler tarafından kirişler ve plakalar gibi çeşitli mühendislik yapılarında incelenmektedir. Bu tezde, dairesel hareketli yük altındaki plakaların titreşim analizleri sonlu eleman yöntemi kullanılarak gerçekleştirilmiştir. Plakanın sonlu eleman modelini yaratmak için ANSYS parametrik dizayn dili dairesel yörünge dikkate alınarak kullanılmıştır. Bir tekil noktasal yük dairesel yörünge üstünde hareket ettirilmiştir. Dairesel hareketli yükün genliği plaka üstünde harmonik olarak değiştirilmiştir. Plakanın ilk iki doğal frekansı uyarım frekansı olarak harmonik dairesel hareketli yükte kulanılmıştır. Dinamik zaman cevabı, plakanın merkezinden elde edilmiştir. Plakanın doğal frekansları modal analiz ile elde edilmiştir. Sonuçlar referans çalışma ile karşılatırılmıştır. Dairesel yörüngenin yarıçapının, hareketli yükün zorlama frekansı ve dönüş hızı büyüklüğünün etkileri araştırılmıştır.

Anahtar sözcükler : Hareketli yük, sonlu eleman yöntemi, ANSYS, plakaların titreşimi, dikdörtgen plaka, bilgisayar destekli analiz.

CONTENTS

CHAPTER TWO - RECTANGULAR PLATE UNDER CIRCULAR

CHAPTER ONE INTRODUCTION

1.1 Introduction

Moving loads have important effects on the dynamic behavior of the engineering structures. Therefore, moving load problem has a large spectrum of applications in various engineering fields. The literature is extensive for vibration of structural system due to moving load. However, not much investigation was oriented toward the dynamic characteristics of a plate undergoing forces moving along a circular path. For this reason, we studied that topic in this thesis.

1.2 Literature Review

Hilal & Zibdeh (2000) studied fundamental problem of vibration of beams with general boundary conditions traversed by moving loads. The moving load is assumed to move with accelerating, decelerating and constant velocity type of motions. They applied analytical formulation to Euler-Bernoulli beams and also examined the effect of different boundary conditions and damping. Wu, Whittaker & Cartmell (2000) used equivalent nodal force technique to beam structure for analyzing the dynamic response of structures to time variant moving load. Later, they implemented same technique to calculate the effect of two-dimensional motion of the trolley on the response of the base of the structure of a mobile gantry crane model.

 Wu, Whittaker & Cartmell (2001) presented dynamic responses of the structures to moving bodies using combined finite element and analytical methods including inertia effects. Chen, Huang & Shih (2001) calculated the response of an infinite Timoshenko beam on a viscoelastic foundation to a harmonic moving load.

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Kıral & Karagülle (2002) studied the moving load problem numerically to analyze the dynamic behavior of a single span beam resting on a elastic foundation by using I-DEAS.

Pesterev, Bergman et al (2003) studied in depth the asymptotic of the solutions of the moving oscillator problem and found that in the limiting case the moving oscillator problem and the moving mass problem for a simply supported beam are equivalent in the sense of the beam displacements, but not in the sense of beam stresses. Also, it was shown that for small values of spring stiffness, the moving oscillator problem is equivalent to the moving load problem. Wu (2003) further extended this technique to plate element structure and presented one dimensional equivalent beam model to replace conventional 2-D plate under moving load. Pesterev, Yang et al. (2003) have considered the vibration of a beam subjected to a constant moving force. They formulate simple tools to calculate the maximum deflection of the beam for any given velocity of the moving force. It is shown that there exists a unique response-velocity dependence function, which satisfies a particular boundary function. A unique amplitude-velocity dependence function is formulated for simply supported and clamped-clamped beams. These unique functions are used to calculate the maximum beam response without complex computations. The response of the beam is approximated by means of the first natural mode. The response is also calculated by including higher modes. These responses are compared with each other and the error range is less than one percent. Therefore, it is concluded from this study that the first fundamental mode alone is sufficient for finding the maximum deflection of a beam when subjected to a moving force. Wu (2003) also probed a rectangular plate subjected to circular moving loads. Fig. 1.1 shows a rotating mechanism used in the this study. Oniszczuk (2003) analyzed undamped forced transverse vibrations of an elastically connected double beam system. The problem is formulated and solved in the case of simply supported beams and the classical modal expansion method is applied. Zibdeh & Hilal (2003) investigated the random vibration of simply - supported laminated composite coated beam traversed by a random moving load. The moving load is assumed to move with accelerating, decelerating and constant velocity type of motions.

Figure 1.1 (a) Sketch for the rotating mechanism and (b) its corresponding mathematical model for the dynamic analysis of the rectangular bottom plate.

De Faria (2004) proposed a new strategy that is based on an adaptive mesh scheme and on the use of perturbation technique for Mindlin elements structure under off-nodal moving load. Bilello & Bergman (2004) presented a theoretical and experimental study on the response of a damaged Euler – Bernoulli beam traversed by a moving mass. Damage is modeled through rotational springs whose compliance is evaluated using linear elastic fracture mechanics. Kargarnovin & Younesian (2004) studied the response of a Timoshenko beam with uniform cross – section and infinite length supported by a generalized Pasternak – type viscoelastic foundation subjected to an arbitrary – distributed harmonic moving load. Kim (2004) investigated the vibration and stability of an infinite Euler - Bernoulli beam resting on a Winkler foundation when the system is subjected to a static axial force and a moving load with either constant or harmonic amplitude variations. The effects of load speed, load frequency, damping on the deflected shape, maximum displacement

and critical values of the velocity, frequency and axial force are also studied. Law & Zhu (2004) studied the dynamic behavior of damaged reinforced concrete bridge structures under moving vehicular loads. The vehicle is modeled as a moving mass or by four - degree of freedom system with linear suspensions and tires flexibility, and the bridge is modeled as a continuous Euler-Bernoulli beam simply supported at both ends.

Wu (2005) presented a technique for predicting the dynamic responses of a two dimensional (2-D) full-size rectangular plate undergoing a transverse moving line load by using the one dimensional (1-D) equivalent beam model.

A lot of analytical and numerical methods were improved to study moving load problem. Especially, the finite element method has been one of the most important solution techniques.

The Finite Element Method (FEM) has become useful tool to find approximate solutions for the numerical analysis of a wide range of engineering problems. The finite element method makes it possible to build up complex geometrical shape easily. It is divided many small subdomain that is called finite elements. These elements are connected with the nodes. The equations of motion of the finite element model can be expressed in matrix form. Thus, it might be easier to develop a general purpose computer program that is able to produce accurate results for all kinds of parameters. The general purpose computer program allows not only change parameters of the analysis after the system is modeled, but also rerun analysis several times with minimal cost.

In this study, the ANSYS computer aided engineering (CAE) software is used to model the plate to obtain the finite element discretization and finally to perform the finite element vibration analysis based on the Newmark integration method. Two different boundary conditions are considered in beam and plate vibrations (clamped – clamped and hinged – hinged). The results obtained of plate in this study are compared with the results obtained of Wu's study (2003).

1.3 Thesis Overview

The solution of the moving load problem is performed by developing computer programs to calculate the dynamic displacements of the plate subjected to circular moving loads. The thesis is organized as follows:

Chapter 1 includes the literature review on the moving load problem and overview of the thesis. Chapter 2 vibration analyses of hinged-hinged and clamped-clamped plates subjected to circular moving load is presented. Vibration results are compared with the Jia-Jang Wu's study. Chapter 3 has the conclusions of the present study. A list of the computer programs is included in the Appendices.

CHAPTER TWO RECTANGULAR PLATE UNDER CIRCULAR MOVING LOAD

2.1 Introduction

Vibration of structural system due to moving load is an important problem in engineering. The finite element analysis is a computer aided numerical technique useful in solving for the response of a structure subjected to loading. Finite element model of the structure is created easily in many engineering programs. The model is divided into small elements. These elements are connected by nodes at which the finite element boundary conditions are applied. Mass and stiffness matrices are created for each element and combined simultaneous equations are solved. Finite element programs use graphic displays to review results.

This chapter includes two main parts. Modeling and vibration analyses of hingedhinged and clamped-clamped plates subjected to circular moving load is presented at first. Then, the vibration analysis of plates is studied with different parameters.

2.2 Modeling

ANSYS parametric design language is used to develop the finite element model of the plate considering circular trajectory. The parameters in the developing code l_x , l_y , h, r_0 , dthdeg and bcsel are length, width, thickness, radius of circular path, angle between nodes on circular path and boundary condition selection parameter, respectively.

The finite element model of the plate is constructed using SHELL63 elements by ANSYS. SHELL63 that is elastic shell has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. Stress stiffening and large deflection capabilities are included. The geometry, node locations and the coordinate system of SHELL63 are shown at Figure 2.1. The

element is defined by four nodes, four thicknesses, elastic foundation stiffness, and the orthotropic material properties.

 x_{II} = Element x-axis if ESYS is not supplied.

Figure 2.1 SHELL63 geometry (ANSYS, 2004)

First, nodes on circular path are created at the model. ANSYS uses random node numbers to generate mesh areas. Therefore, three circles are formed and generate elements between their nodes to order of node numbers. And then, keypoints on circular path and edge of plate are produced. Two areas are created between keypoints. First area is circle generate between centre keypoint of plate and keypoints on circular path. Second area is whole plate generate between centre keypoint of plate and edge keypoints of plate. First area is subtracted from second area. Finally, all areas are meshed and applied boundary conditions. Subtracting areas is realized by a developed ANSYS program which is below:

 $k1=kdc+1$ $*$ do,i,1,kdc1-2,1 $a,1,k1,k1+1$ $k1 = k1 + 1$ *if,k1,eq,kdc1,then $a,1,kdc1,kdc+1$ *endif *enddo aadd,all a,kdc2+1,kdc2+2,kdc2+3,kdc2+4,kdc2+5,kdc2+6,kdc2+7,kdc2+8 asba,1,kdc1,

All the translational DOF for the boundary nodes along width edge are constrained except that the DOF of rotations about the y-axis are free for the hingedhinged plate and all the DOF for the same as boundary nodes of hinged-hinged plate are constrained.

A uniform undamped clamped-clamped rectangular plate is shown in Figure 2.2. The dimensions of the plate are; length $l_x = 2m$, width $l_y = 1m$ and thickness $h = 0.01m$. The plate is modeled with 329 elements and 406 nodes and made of steel with density $a \rho = 7820 \frac{kg}{m^3}$, modulus of elasticity $E = 206.8$ *GN* / m^2 and Poisson's ratio $\nu = 0.29$.

Figure 2.2 Finite element model of plate.

2.3 Dynamic Response Study

2.3.1 Free Vibrations of the Plate

The usual first step in performing a dynamic analysis is determining the natural frequencies and mode shapes of the structure. These results characterize the basic dynamic behavior of the structure and are an indication of how the structure will respond to dynamic loading. Modal analysis is performed by ANSYS with Block Lancozs method to calculate the lowest 10 natural frequencies and the corresponding mode shapes.

The lowest 10 mode shapes of clamped and hinged plate are shown in Figure 2.3 and Figure 2.4.

The four natural frequencies ω_{p_i} (i=1, 3, 6, 9) as shown in Figure 2.3 are called the beamlike modes. ω_{p_i} (i=2, 4, 5) are called the torsional modes because each unconstrained node rotates about the longitudinal centre line of the plate. ω_{p_i} (i=7, 8, 10) are called the hybrid modes.

Figure 2.3 First 10 natural mode shapes of clamped plate.

The four natural frequencies ω_{p_i} (i=1, 3, 5, 9) as shown in Figure 2.4 are called the beamlike modes. ω_{p_i} (i=2, 4, 6) are called the torsional modes because each unconstrained node rotates about the longitudinal centre line of the plate. ω_{p_i} (i=7, 8, 10) are called the hybrid modes.

Figure 2.4 First 10 natural mode shapes of hinged plate.

The comparisons of natural frequencies of our model and Jia-Jang Wu's model are shown in Table 2.1. It shows that our model is most sensitive than Jia-Jang Wu's model. Our first 2 mode shapes are similar acting his model that presents in Figure 2.5 and Figure 2.6.

The lowest 10 natural frequencies of Clamped plate and Hinged plate						
	Natural Frequencies of			Natural Frequencies of		
Mode	Clamped Plate, ω (Hz)			Hinged Plate, ω (Hz)		
N ₀	Our model	Wu's model		Our model	Wu's model	
	13.6240	13.8201		5.8902	5.9015	
2	22.3510	20.5217		17.1080	15.7954	
3	37.6010	38.8017		23.8550	24.1024	
4	51.2120	47.1634		40.1450	36.6485	
5	67.6350	55.2698		54.0630	53.8316	
6	73.9250	75.7331		65.1470	55.4407	
7	90.0110	78.0695		72.6850	66.3103	
8	98.4230	84.0931		90.8580	70.9606	
9	122.45	104.7738		96.5130	95.3750	
10	139.96	127.9466		116.31	101.1105	

Table 2.1 The lowest 10 natural frequencies of Clamped and Hinged plate for our and Wu's study.

Figure 2.5 Mode shapes for the clamped–clamped plate: (a) 1st mode and (b) 2nd mode of Wu' model and (c) 1st mode and (d) 2nd mode of ours. (Wu, 2003, Fig. 6)

Figure 2.6 Mode shapes for the hinged–hinged plate: (a) 1st mode and (b) 2nd mode of Wu' model and (c) 1st mode and (d) 2nd mode of ours.(Wu, 2003, Fig. 5)

2.3.2 Vibration Response of the Plate

The plate subjected to a sinusoidal force $F_s = 10 \sin \omega t$ N moving along a circular path with radius $r = 0.3m$ is studied. $x_G = 1m$ and $y_g = 0.5m$ are coordinates of the center of the circular path. The sinusoidal force moves along the circular path counter clockwise with a constant rotating speed for 10s and then keep free vibration following time for 10s. In this study, first two natural frequencies were used to forcing frequency in Figure 2.7(a) when $\omega = 5.8902Hz$, in Figure 2.7(b) when $\omega = 17.1080Hz$, in Figure 2.8(a) when $\omega = 13.6240Hz$ and in Figure 2.8(b) when $\omega = 22.3510Hz$.

The time step is the time increment between consecutive time points. Natural frequencies are used to determine the time step. The time step is chosen as $\Delta t = 1/(20 * f_i)$, where f_i is the ith natural frequency to be considered at belonging the natural frequency numbers i=1, 2, 3 etc. The time step, Δt , is 0.008 s, 0.03 s, 0.004 s and 0.002 s respectively.

The comparison of the time histories for the vertical z displacements of centre of our plates and Wu's plates is presented in Figure 2.7 and Figure 2.8.

The response amplitude raises with the expansion of time t in the first 10s because of undamped forced vibrations and then stays unchanged after 10s caused by undamped free vibrations in Figure 2.7(a) and Figure 2.8(a). The centre of the plate is located at the top of the first mode shape as shown in Figure 2.5 and Figure 2.6 so that forced and free vibration responses for the centre of the plate are nearly symmetric with respect to the static equilibrium position of the centre. Our model has lower frequency than Wu's model as shown in Table 2.1. Hence a comparison between Figure 2.7(a) and (c) shows that our response amplitude is higher than Wu's response amplitude.

The centre of the plate is located on the line node of the second mode shape as shown Figure 2.5 and Figure 2.6. Thus the maximum central vertical z displacement of center is very small and any small responses of the plate will reach this maximum value. Therefore the response amplitude does not raise with the expansion of time t for the first 10s. The plate vibrates freely after 10 s. Whole these analysis responses are obtained the truth of our model.

Figure 2.7 Time histories for the vertical z displacements of the centre of hinged plate subjected to a single sinusoidal force, $F_s = 10 \sin \omega t$ N, moving along a circular path of radius $r_0 = 0.3$ m with a constant forcing frequency ω (a) $\omega = 5.8902Hz$, (b) $\omega = 17.1080Hz$, (c) and (d) are Wu's results (Wu, 2003, Fig. 7).

Figure 2.8 Time histories for the vertical z displacements of the centre of clamped plate subjected to a single sinusoidal force, $F_s = 10 \sin \omega t$ N, moving along a circular path of radius $r_0 = 0.3m$ with a constant forcing frequency ω (a) $\Omega = 13.6240Hz$, (b) $\omega = 22.3510Hz$, (c) and (d) are Wu's results. (Wu, 2003, Fig. 8)

2.2.2.1 Vibration Analysis of the Plate with Different Parameters

Both the rotating speed ω and the forcing frequency Ω are equal to the first two natural frequencies and each other at the previous subsection in this chapter. In this subsection, the moving load with various rotating speed and forcing frequency are studied.

The vertical z displacements of the centre of clamped plate subjected to a single sinusoidal force are presented in Figure 2.9, $F_s = 10$ Sinot N, moving along a circular path of radius $r = 0.3m$ with constant forcing frequency $\omega = 13.6240H_z$ for various

rotating speed. The vertical z displacements of the centre of clamped plate subjected to a single sinusoidal force are shown in Figure 2.10, $F_s = 10 \sin \omega t$ N, moving along a circular path of radius $r_0 = 0.3m$ with constant rotating speed $\omega = 13.6240H_z$ for various forcing frequency. Displacement is reached the maximum value at the first natural frequency in Figure 2.9 and Figure 2.10. When displacement is made a suddenly peak at first natural frequency value in Figure 2.9, distribution is made a regular increase in Figure 2.10. Therefore, when rotating speed equals to first natural frequency, is more important than when forcing frequency equals to first natural frequency.

Figure 2.9 The vertical z displacements of the centre of clamped plate subjected to a single sinusoidal force, $F_s = 10 \sin \omega t$ N, moving along a circular path of radius $r_0 = 0.3m$ with constant forcing frequency $\Omega = 13.6240Hz$ for various rotating speed.

Figure 2.10 The vertical z displacements of the centre of clamped plate subjected to a single sinusoidal force, $F_s = 10 \sin \omega t$ N, moving along a circular path of radius $r_0 = 0.3m$ with constant rotating speed $\omega = 13.6240Hz$ for various forcing frequency.

The FE model results of the plate subjected to moving load with different rotating speed and forcing frequency are shown in Figures 2.11 - 2.15. When radius decreases, displacements are increase. It is reason that center of plate has a maximum peak at first natural frequency. It is show that rotating speed is important. When rotating speed equals to first natural frequency, displacement values are higher than other results.

The computer codes developed by ANSYS parametric design language for the plate model and the whole analysis of hinged - hinged plate and clamped – clamped plate are given Appendix.

Figure 2.11 The vertical z displacements of the centre of clamped plate subjected to a single sinusoidal force, $F_s = 10 \sin \omega t$ N, moving along a circular path of various radius with constant rotating speed and forcing frequency $\omega = \Omega = \omega_1 = 13.6240Hz$.

Figure 2.12 The vertical z displacements of the centre of clamped plate subjected to a single sinusoidal force, $F_s = 10 \sin \omega t$ N, moving along a circular path of various radius with constant rotating speed $\omega = 10Hz$ and forcing frequency $\Omega = \omega_1 = 13.6240Hz$.

Figure 2.13 The vertical z displacements of the centre of clamped plate subjected to a single sinusoidal force, $F_s = 10 \sin \omega t$ N, moving along a circular path of various radius with constant rotating speed $\omega = 25Hz$ and forcing frequency $\Omega = \omega_1 = 13.6240Hz$.

Figure 2.14 The vertical z displacements of the centre of clamped plate subjected to a single sinusoidal force, $F_s = 10 \sin \omega t$ N, moving along a circular path of various radius with constant rotating speed $\omega = \omega_1 = 13.6240Hz$ and forcing frequency $\Omega = 10Hz$.

Figure 2.15 The vertical z displacements of the centre of clamped plate subjected to a single sinusoidal force, $F_s = 10 \sin \omega t$ N, moving along a circular path of various radius with constant rotating speed $\omega = \omega_1 = 13.6240Hz$ and forcing frequency $\Omega = 25Hz$.

CHAPTER FOUR CONCLUSIONS & FUTURE WORKS

A circular moving load on a rectangular plate is modeled using ANSYS. The code that is developed in ANSYS parametric design language (APDL) is used to create the finite element model of the plate considering circular trajectory. Forced vibration analysis of hinged-hinged and clamped-clamped plates under the effect of circular moving load using finite element method is successfully carried out with the help of general finite element program ANSYS. A single point load is moved over the circular trajectory. The amplitude of the circular moving load changes harmonically on the plate. The two excitation frequencies corresponding to the first two natural frequencies of the plate are used in the harmonic circular moving load. The dynamic time response is obtained from the middle point of the plate. Natural frequencies of the plate are found with the modal analysis. The results are compared with the reference study. The effects of the radius of the circular path, forcing frequency and rotating speed of the moving load are investigated.

For free response analysis, the lowest mode shapes of plate can be represented as beamlike, torsional and hybrid modes. For vertical displacement of plate under single moving load, the beamlike modes are dominant and very similar to mode shapes in beam element.

When the excitation frequency selected as the first natural frequency of the plate, resonance is observed. The response amplitude increases at each cycle. When the excitation frequency selected as the second natural frequency of the plate, resonance is not observed because of second mode shape is not excited.

Circular moving load problem can be investigated under the effect of damping ratio in the future work, and also it can be analyzed for different frequencies and different structures.

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APPENDIX

THE COMPUTER CODES DEVELOPED BY APDL

```
!========================================================== 
! This program is used to create finite element model of plate 
!========================================================== 
/config,nres,100000 
/prep7 
/title,Circular Moving Force of Rectangular Plate Model 
!----***plate model parameters***-------- 
r0=0.3dr=r0/5dthdeg=12 !enter degree
lx=21y=1h=10e-3dsmesh=r0/3 
bcsel=2 ! 1-Clamped 
            ! 2-Hinged 
!---------------------------------------- 
pi=4*atan(1)dth=dthdeg*pi/180 
thson=2*pi 
mp,ex,1,206.8e9 ! Elasticity modulus for metal
mp, dens, 1,7820 ! Density
mp,nuxy, 1,0.29 ! Posisson's ratio
et,1,shell63 
r,1,h,h,h,h 
nd=1n,nd,0,0 
*do,th,0,thson-dth,dth
```
 $x=r0*cos(th)$

 $y = r0*sin(th)$

nd=nd+1

n,nd,x,y

*enddo

ndc=nd

 $r01=r0+dr$

nd=ndc

*do,th,0,thson-dth,dth

 $x = r01 * cos(th)$

 $y=$ r01 $*$ sin(th)

nd=nd+1

n,nd,x,y

*enddo

ndc1=nd

r02=r0-dr

nd=ndc1

*do,th,0,thson-dth,dth

 $x=r02*cos(th)$

 $y = r02*sin(th)$

nd=nd+1

n,nd,x,y

*enddo

ndc2=nd

eind=1

nel=ndc-1-1

 en ,eind,2,ndc+1,ndc+2,3

egen,nel,1,1,1

en,ndc-1,ndc,ndc1,ndc+1,2

eind=ndc

en,eind, 2 ,ndc $1+1$,ndc $1+2$, 3

egen,nel,1,ndc,2*ndc

*enddo kdc=kd $r01=r0+dr$ kd=kdc *do,th,0,thson-dth,dth $x = r01 * cos(th)$ $y = r01*sin(th)$ $kd=kd+1$ k, kd, x, y *enddo kdc1=kd $r02=r0-dr$ kd=kdc1

 $en, 2*(ndc-1), ndc, ndc2, ndc1+1, 2$

: $line 5$

 $kd=1$

 $k, kd, 0, 0$

 $x = r0*cos(th)$

 $y = r0$ *sin(th)

 $kd=kd+1$

 k, kd, x, y

*do,th,0,thson-dth,dth

```
*do,th,0,thson-dth,dth
```
 $x = r02 \cdot cos(th)$

 $y = r02*sin(th)$

 $kd=kd+1$

 k, kd, x, y

*enddo

kdc2=kd

*if,kon,eq,1,then

*go,:line10

*endif

 $d, all, all, 0$ $nsel, A, loc, x, (lx/2)$ $\rm d, \rm all, \rm all, \rm 0$ *elseif,bcsel,eq,2 !Hinged BC $nsel,s,loc,x,(-lx/2)$ d , all, ux , 0 $d,$ all,uy, 0 d , all, uz, 0 d,all,rotx,0 d,all,rotz,0 $nsel, A, loc, x, (lx/2)$ d, all, ux, 0 $\rm d, \rm all, \rm uy, \rm 0$ $d,$ all,uz, 0 $\rm d, all, rotx, 0$ d,all,rotz,0 *endif nsel, all

! This program is used to perform the analysis of modal and transient

!==

!==

/input,pmnew,txt $f0=10$ $excel=2$!1-f1 !2-f2 ansel=1 !-- !******* Select Analysis ******* ! 1- Modal analysis ! 2- Transient analysis of sinusoidal force !--- *if,ansel,eq,1,then /solu ! Modal Analysis antype,modal,new modopt,lanb,10 solve *get,f1,mode,1,freq *get,f2,mode,2,freq finish /POST1 SET,LIST finish *elseif,ansel,eq,2 /solu ! Modal Analysis antype,modal,new modopt,lanb,10 solve *get,f1,mode,1,freq *get,f2,mode,2,freq finish

```
*if, excel, eq, 1, then
          f=f1*elseif, excel, eq, 2
          f=f2*endif
dt=1/f/20w=2*pi*ft0=1/fdto=t0/(ndc-1)
tson=1nloop = nint(tson/t0)/solu! Transient analysis for moving load problem
antype, trans, new
outres, all, all
kbc,1tintp,,0.25,0.5,0.5
timint, on, ALL
trnopt, FULL
deltim,dt
nind=0*do,i,0,t0,dtonind=nind+1
*enddo
ny=nind
*DIM, fs1, ny
                       ! DEFINE ARRAYS WITH DIMENSION
*DIM, fs2, ny
*DIM, fs3,, ny
*DIM, fs4,, ny
*VFILL,fs1(1),RAMP,0,dto! ARRAY A(N) : TIME IN SECOND
                 ! MULTIPLYING FACTOR : FREQUENCY = (2*pi*f)*VFACT,w
*VFUN,fs2(1),COPY,fs1(1)! RESULT ARRAY fs2(N)=FREQUENCY*fs1(ny)
*VFUN, fs3(1), SIN, fs2(1)
                              ! ARRAY fs3(N): SIN(fs2(ny))
```

```
*VFACT,f0 ! MULTIPLYING FACTOR : AMPLITUDE A 
*VFUN,fs4(1),COPY,fs3(1) ! ARRAY fs4(ny) : f0*fs3(ny)
tlp=0j1=1j2=0*do,i1,1,nloop,1 
   f,2,fz,fs4(1) 
 time,tlp+j1*dt/100 solve 
 *do,nd,2,ndc-1,1
   f,nd,fz,0 
  f,nd+1,fz,fs4(j1+1) 
  time,tlp+j1*dto 
   solve 
 j1=j1+1 flist,2,ndc,cn 
  *enddo 
j1=1j2=j2+1tlp=j2*t0 f,ndc,fz,0 
 time,tlp+dt/1000 
 solve 
eplot 
*enddo 
ns=nd 
*if,excel,eq,2,then 
f,ns,fz,f0 
*endif 
time,tson+tson 
solve 
/post26
```
 $nsol, 2, 1, u, z$ $/axlab, x, time(sec)$ $/axlab,y,displacement(m)$ plvar,2 ${\rm finish}$ *endif