DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND
APPLIED SCIENCES

# MULTIRESOLUTION IMAGE REGISTRATION BASED ON WAVELET TRANSFORM

by
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July, 2009
İZMİR

# MULTIRESOLUTION IMAGE REGISTRATION BASED ON WAVELET TRANSFORM

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**by**
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## M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"MULTIRESOLUTION IMAGE REGISTRATION BASED ON WAVELET TRANSFORM"** completed by **BARBAROS YAMAN** under supervision of **ASST. PROF. DR. HALDUN SARNEL** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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## MULTIRESOLUTION IMAGE REGISTRATION BASED ON WAVELET TRANSFORM

## ABSTRACT

Image registration is the process of estimating an optimal geometrical transformation between images. The main goal of image registration methods is to find transformation parameters between references and rotated and/or translated and/or scaled input images, assuming that affine geometrical transformation exists. In this thesis, a multi-resolution approach of wavelet transform (WT) has been used to perform image registration operation. Wavelet transform based image registration methods are iterative so optimization methods are needed to decrease computational time and increase robustness. Of the optimization methods, genetic algorithm (GA) and simultaneous perturbation stochastic approximation (SPSA) have been combined with multi-resolution scheme to overcome drawbacks of method. Normalized cross correlation and normalized mutual information have been used to measure similarity of images.

Image registration implementations and performance tests have been performed in Matlab. In this thesis, five different application of wavelet transform based image registration have been implemented. These are full search algorithm with using normalized cross-correlation, simultaneous perturbation stochastic approximation based registration with using normalized cross-correlation and normalized mutual information, and genetic algorithm based image registration with using normalized cross-correlation and mutual information. Image registration methods have been tested with both noiseless images and noisy images. SPSA based method with using mutual information achieves high accuracy and gives the best results in terms of computational time.

**Keywords:** Image registration, wavelet transform (WT), genetic algorithm (GA), simultaneous perturbation stochastic approximation (SPSA), normalized cross-correlation, mutual information.

# DALGACIK DÖNÜŞÜMÜNE DAYALI ÇOK ÇÖZÜNÜRLÜKLÜ GÖRÜNTÜ ÇAKIŞTIRMA

## ÖZ

Görüntü çakıştırma, görüntüler arasındaki en iyi geometrik dönüşümü tahmin etme işlemidir. Görüntü çakıştırma metotlarının ana amacı, affine geometrik dönüşüm olduğunu kabul edersek referans ve döndürülmüş ve/veya ötelenmiş ve/veya ölçeklendirilmiş giriş görüntüsü arasındaki geometrik dönüşüm değişkenlerinin bulunmasıdır. Bu tezde, dalgacık dönüşümünün (DD) çok çözünürlüklü yaklaşımı görüntü çakıştırma işlemini gerçekleştirmek için kullanıldı. Dalgacık dönüşümüne dayalı görüntü çakıştırma metotları döngülüdür; bundan dolayı hesaplama zamanını azaltmak ve dayanıklılığı artırmak için eniyileme metotlarına ihtiyaç duyulmaktadır. Metodun eksikliklerinin üstesinden gelmek için, eniyileme yöntemlerinden genetik algoritma (GA) ve eşzamanlı sarsım stokastik yaklaştırması (ESSY) çok çözünürlüklü yapı ile birleştirilmiştir. Normalize çapraz korelasyon ve karşılıklı bilgi miktarı görüntülerin benzerliğini ölçmek için kullanıldı.

Görüntü çakıştırma uyarlaması ve performans testleri Matlab'da gerçekleştirildi. Bu tezde beş farklı dalgacık dönüşümüne dayalı görüntü çakıştırma uygulaması gerçekleştirilmiştir. Bunlar, normalize çapraz korelasyon kullanılarak tam arama algoritması, normalize çapraz korelasyon ve karşılıklı bilgi kullanarak eşzamanlı sarsım stokastik yaklaştırmasına dayalı çakıştırma, ve normalize çapraz korelasyon ve karşılıklı bilgi kullanarak genetik algoritmaya dayalı çakıştırma. Görüntü çakıştırma metotları hem gürültüsüz hem de gürültülü görüntülerle test edildi. Karşılıklı bilgi kullanarak eşzamanlı sarsım stokastik yaklaştırmasına (ESSY) dayalı çakıştırma yöntemi yüksek doğruluk elde etmekte ve hesaplama süresi bakımından en iyi sonuçları vermektedir.

**Anahtar sözcükler:** Görüntü çakıştırma, dalgacık dönüşümü (DD), genetik algoritma (GA), eşzamanlı sarsım stokastik yaklaştırması (ESSY), normalize çapraz korelasyon ve karşılıklı bilgi miktarı.

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# CHAPTER ONE

# INTRODUCTION

## 1.1 Introduction

Image registration is a fundamental image processing technique in remote sensing and medical applications. Image registration is alignment of two or more images which are taken at different times by different sensors. Application areas of image registration are remote sensing, computer vision and medical image analysis. Major image registration applications are target localization, motion tracking, change detection, image mosaicing, weather forecasting, tumor detection, monitoring tumor growth and disease localization.

Image registration methods consist of the following steps:

- Feature extraction: edges, contours, and line intersections, corners, significant features of images are extracted by using manual or automatic operations.
- Feature matching: correspondence between input and reference image is established. Different operations can be used for this purpose.
- Transformation estimation: mapping function is used to align reference and input image. In this step, mapping function is computed.
- Image transformation: Input image is transformed according to mapping function parameters.

The image registration can be implemented by using different methods. Many image registration methods are iterative and computational time makes them unsuitable for many applications. Fourier domain methods are non-iterative but they are less robust to noise than spatial domain based iterative methods. To overcome this drawback of the iterative methods, some transforms and optimization methods are used. These techniques make iterative methods more robust and faster.

Wavelet multi-resolution scheme is an effective method to generate low resolution images keeping significant features. This property is used to overcome the drawback of iterative registration methods. This type of registration is called “coarse to fine” methods. This coarse-to-fine hierarchical search strategy applies the usual registration methods, but it starts at a coarse resolution level of the input and reference images. While image resolution goes up to higher level (fine resolution), the accuracy of estimated transformation parameters gradually increase. The search space of the registration operation are decreased by “coarse to fine” algorithm. So, computational time of the operation decreases and more robust results are obtained. Many registration applications were realized with wavelet multi-resolutions scheme by using azimuth estimation (Zheng & Chellappa,1993), by using control points (Corvi & Nicchiotti, 1995), by using “coarse to fine” strategy (Le Moigne, Campbell & Cromp, 2002).

To obtain more effective results, the optimization methods are combined with a multi-resolution scheme of the wavelets. The main goal of these optimization methods is to maximize a similarity measure such as correlation or mutual information between the reference image and the transformed input image. In this work, genetic algorithm and simultaneous perturbation stochastic approximation are used as two optimization methods to obtain more robust registration results in lower computational time. Genetic algorithm is based on finding possible solutions at first, and then the best solution giving a registration result. Simultaneous perturbation stochastic approximation is a simple optimization method for complex problems. It is based on calculation of two gradient functions.

Some examples to the multi-resolution image registration work, one with using genetic algorithm (Chalermwat & El-Ghazawi, 1999), and two with simultaneous perturbation stochastic approximation (Cole-Rhodes, Johnson, Le Moigne & Zavorin,2003 ) (Li, Sato & Murakami, 2006) can be given.

In this work, some multi-resolution based image registration methods with using genetic algorithm and simultaneous perturbation stochastic approximation are implemented. In these implementations, five transformation parameters (rotation, translation in x direction, translation in y direction, scale in x direction and scale in y direction) are searched to register images. The normalized cross correlation and normalized mutual information are used to measure the similarity between the reference image and a rotated or/and translated or/and scaled input image.

### 1.2 Outline

This thesis has eight chapters. Chapter 1 presents introduction to image registration and related literature review.

In chapter 2, image registration theory is given. Image registration methods are also mentioned.

Chapter 3 examines wavelet theory and multi-resolution scheme of wavelets. In this thesis, wavelet decomposition of images is used in the implementation so theory of decomposition process is given in this chapter.

In chapter 4, theory of simultaneous perturbation stochastic approximation (SPSA) is examined. SPSA implementation is also given step by step.

Genetic algorithm is presented in chapter 5. All genetic algorithm operations and examples are examined. Finally, image registration with genetic algorithm is presented.

Implementations of the methods are given in chapter 6. In this thesis, image registration is realized with three different optimization methods, which are full search algorithm, SPSA and GA. These methods are also implemented with two different similarity measures that are normalized cross correlation and normalized mutual information.

In chapter 7, the test results and characteristics of the methods are given. Finally, Conclusion is presented in chapter 8.

# CHAPTER TWO

# IMAGE REGISTRATION THEORY

## 2.1 Definition

Image registration is alignment of two or more images of the same scene taken at different times, from different viewpoints, by different sensors. Image registration is used to find optimal spatial transformation parameters between two images.

If images are defined as two 2-dimensional arrays of a given size denoted by $I_1$ and $I_2$ where $I_1(x, y)$ and $I_2(x, y)$ each map to their respective intensity values, then the mapping between images can be expressed as:

$$I_2(x, y) = g(I_1(f(x, y))) \quad 2.1$$

Where $f$ is a 2D spatial coordinate transformation, i.e.,

$$(x', y') = f(x, y) \quad 2.2$$

And $g$ is 1D intensity or radiometric transformation function.

The main goal of image registration is to find an optimal transformation between two images which are called reference image and input image. Registration problem can be defined as finding the optimal transformation between images. It is frequently expressed parametrically as two single-valued functions, $f_X$ and $f_Y$

$$I_2(x, y) = I_1(f_X(x, y), f_Y(x, y)) \quad 2.3$$

Digital image registration is used in many applications of image processing, such as remote sense, robotics, medical applications. Generally image registration is used for obtaining accurate match between images. Image registration is usually motivated by such goals as object recognition, model matching, robotics, pose estimation, or change detection.

## 2.2 Distortion

An important consideration for selecting the registration method to be employed for a given problem is the source of misregistration. (Brown, 1992). The source of misregistration is the cause of the misalignment between images, the misalignment that must be found in order to properly register the two images. If distortion type can be estimated, more accurate registration operation can be realized. When sufficient information about the misregistration source is available, image registration can be implemented.

Distortion can be created with many factors such as:

- Sensor noise
- Perspective changes from sensor viewpoint
- Object changes such as movements or deformations
- Nonuniform illumination
- In satellite images, clouds and shadows
- Different sensors

## 2.3 Image Registration Components

Image registration can be described as the combination of four components: feature space, search space, search strategy and similarity metric.

### *2.3.1 Feature Space*

Feature space is related with set of information of images for registration application. Feature space determines the extracted information of input and reference image. This component is the first step of image registration application. There are many different choices of this parameter such as raw pixel intensities, edges, contours, corners, line intersections, moment invariants, and Fourier descriptors. Choosing best feature space improves performance of the registration algorithm and eliminates the uncorrected variations which may otherwise, make matching unreliable.

Selection of feature space parameter is an important step because each one of them has some advantages and disadvantages.

Using raw intensity values of the image has the advantage of using all image information. But using all intensity values cause high computational time for optimization algorithms. Raw intensity values can be preferred if image does not contain easily detectable objects. Raw intensity values are generally used in medical image registration.

Using geometric features (salient poles, landmarks, contours, corners, etc.) reduces the computational time of the registration algorithm but image deformations can strongly affect the accuracy of registration algorithm.

### *2.3.2 Search Space*

Search space represents the class of the transformation that established between reference and input images. This determines fundamental characteristics of the registration process. Transformation model can be determined according to application. The most common general transformations are rigid, affine, projective, and curved. In 2D case, these can be represented as shown in figure 2.1.

A transformation T is linear if for every constant $c$

$$T(x_1 + x_2) = T(x_1) + T(x_2) \qquad 2.4$$

And

$$cT(x) = T(cx) \tag{2.5}$$

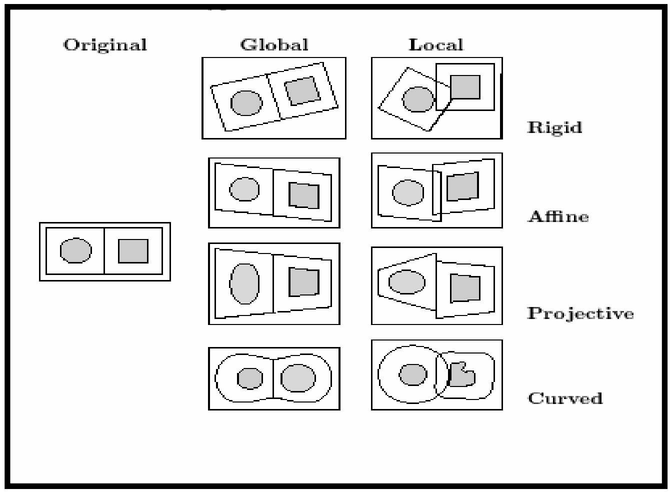



Figure 2.1 Transformation models

The rigid transformation is combination of rotation, translation and scale change. Since rotation matrix is orthogonal (rows and columns are perpendicular to each other), angle and length are preserved after registration process. Rigid transformation expression is given in equation 2.6.

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} t_x \\ t_y \end{pmatrix} + s * \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \tag{2.6}$$

Affine transformation has some differences from the rigid transformation such as affine transform doesn't have orthogonal rotation matrix so angles and lengths are not preserved after registration process. But parallel lines in reference image remain as parallel after registration process. The most commonly used registration transformation is the affine transformation (equation 2.7) which is sufficient to match two images of a scene taken from the same viewing angle but from a different position.

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} a_{13} \\ a_{23} \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \qquad 2.7$$

A more complex transformation than affine transformation is projective transformation which is given by equation 2.8.

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \frac{\begin{bmatrix} m_0 & m_1 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} m_2 \\ m_5 \end{bmatrix}}{\begin{bmatrix} m_6 & m_7 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + 1} \qquad 2.8$$

There are two choices of mapping models, namely, global and local mapping. Global transformation model applies to the whole image with the same parameters while for local transformations parameters vary across the image.

#### *2.3.3 Similarity Metric*

Similarity metric is a measure of matching features between images. Registration transformation can be calculated or evaluated according to this parameter.

Similarity metric has a very important role during the operation of the optimization algorithm. It is used to evaluate the alignment of the images. The choice of similarity metric is one of the most important components of image registration process. Accuracy and robustness against noise are two important criteria in selection of a similarity metric and efficient registration results can be obtained with selection of a proper similarity metric. The similarity metric can be used to find the parameters of the final registration transformation. Generally, optimization algorithms minimize or maximize the similarity metric to find transformation parameters.

Noise can reduce the performance of registration algorithms, even sometimes causing failure of image matching. Therefore, a similarity metric robust to noise must be preferred for optimization operations.

If cross correlation or the sum of the absolute differences is used as similarity metric, the transformation is found at the peak value. The peak value determines the best match for the searched transformation parameters. However, noise can easily move the peak value to a wrong transformation parameter set in the search space. Phase correlation and mutual information are two other similarity metrics.

### *2.3.4 Search Strategy*

Optimization of the similarity metric is the main problem of image registration. Computational time is an important parameter for registration algorithms. Search strategy determines transformations to be computed and evaluated through the search space, therefore, strongly affects the overall computational time. Multi-resolution analysis, decision sequencing and optimization techniques are common search strategies.

## 2.4 Image Registration Methods

Registration methods can be classified with respect to various criteria such as application area, dimensionality of data, type and complexity of assumed image deformations, computational cost or feature matching. Generally image registration methods can be classified according to image registration components. Figure 2.2 shows image registration methods according to Zitova's image registration survey.

### *2.4.1 Feature Detection*

There are two main approaches for automatic registration with feature detection.

#### *2.4.1.1 Area Based Methods*

Area-based methods give importance to the feature matching step more than on their detection. There is no feature detection in these approaches so the first step of image registration is omitted. (Zitova & Flusser, 2003).

*2.4.1.2 Feature Based Methods*

These methods require extraction of features such as regions (lake, forest), lines and line intersections. For an efficient algorithm, features must be detectable on both reference and input images. Feature based methods do not use image intensity values so these methods are more suitable than area based methods when illumination changes are expected.

Feature based methods can be used if image contain easily detectable features or objects. So, in remote sensing applications, feature based methods can be preferable but, in medical registration, area based methods can be more efficient.

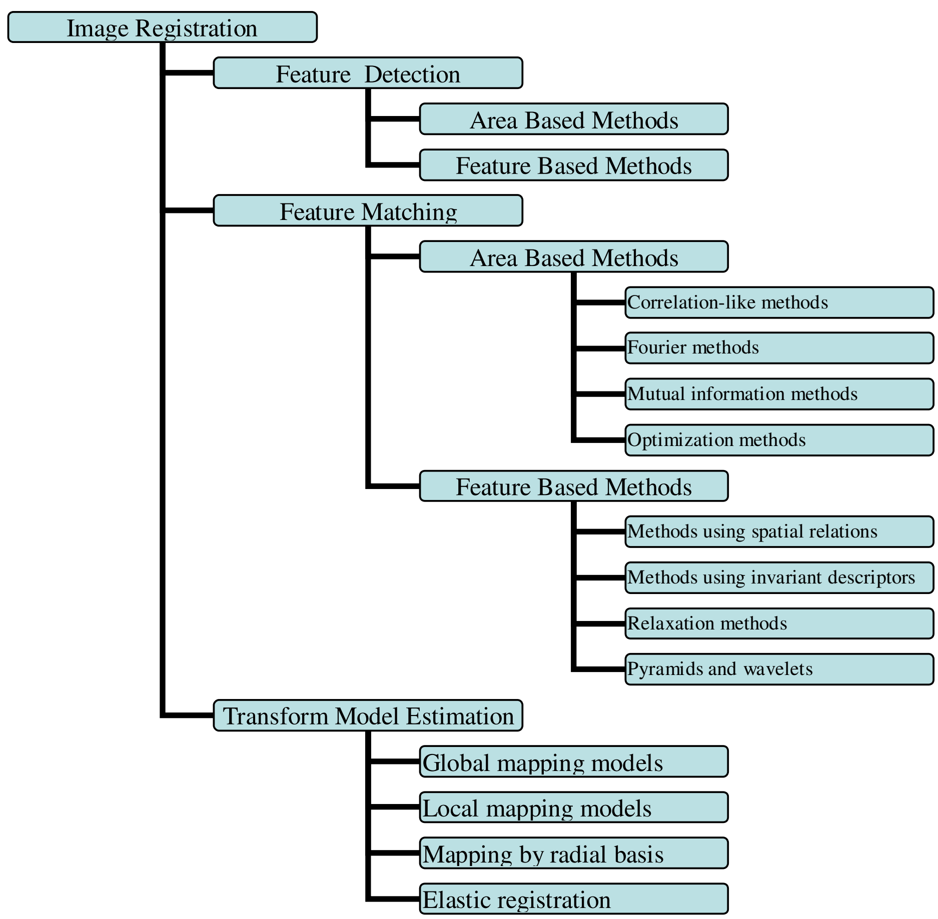



Figure 2.2 Image registration methods

### *2.4.2 Feature Matching*

Some registration methods are classified according to matching technique of features on reference and input images.

#### *2.4.2.1 Area Based Methods*

These methods are similar to correlation based methods. Predefined sizes of windows or entire images are used for matching process of images. Area based methods have some disadvantages. The contents of a window is not independent of local deformation of image. So, a window in the reference image can't match exactly to the corresponding window in an input image. These methods are also sensitive to noise, illumination variation and sensor type.

*2.4.2.1.1 Correlation Like Methods.* In these methods, normalized or classical cross correlation (equation 2.9) is used to measure similarity of reference and input images.

$$CC(i,j) = \frac{\sum_W (W - E(W))(I_{(i,j)} - E(I_{(i,j)}))}{\sqrt{\sum_W (W - E(W))^2}\sqrt{\sum_{I(i,j)} (I_{(i,j)} - E(I_{(i,j)}))^2}} \qquad 2.9$$

Correlation is calculated for window pairs or entire images to search maximum value of correlation. If the subpixel accuracy of registration is needed, interpolation of cross correlation must be calculated.

Main drawback of correlation based method is the flatness of similarity measure maxima. Generally, correlation process can't create sharp peaks at maximum point. So, registration process can fail or give inaccurate results. Edge correlation or image filtering can improve registration performance. Another drawback of these methods is the computation complexity.

Correlation based methods can be more successful with translated and scaled images.

*2.4.2.1.2 Fourier Methods.* The Fourier transform is another choice for registration applications. Transformation parameters can also be represented in Fourier domain. Translation, scaling and rotation have their counterpart in Fourier domain.

In Fourier domain registration methods, it is assumed that images are band limited and appropriate for Nyquist sampling theorem. A preprocessing operation is applied to ensure these criterions. For example, Gaussian smoothing can be applied to limit the bandwidth of images.

Fourier domain methods achieve robustness against frequency dependent and correlated noise. Phase correlation and power cepstrum are two well known Fourier domain methods.

Different properties of Fourier transform are used to achieve alignment operation. Phase correlation is related with shift theorem of Fourier transform. Translation in spatial domain is represented as a phase shift in Fourier domain. $I_1$ and $I_2$ are images with $(x_0, y_0)$ displacement:

$$I_2(x, y) = I_1(x - x_0, y - y_0) \qquad 2.10$$

Their corresponding Fourier transforms will be related with;

$$F_2(w_X, w_Y) = e^{-j(w_X x_0 + w_Y y_0)} F_1(w_X, w_Y) \qquad 2.11$$

Phase difference is equal to cross power spectrum

$$\frac{F_1(w_X, w_Y)F_2^*(w_X, w_Y)}{\left|F_1(w_X, w_Y)F_2^*(w_X, w_Y)\right|} = e^{(w_X x_0 + w_Y y_0)} \quad 2.12$$

Inverse Fourier transform of the phase difference is an impulse function that is zero everywhere except displacement.

This technique is insensitive to change of spectral energy. So, it is relatively scene independent so it can be successful with images that are taken from different sensors.

Determination of rotation and translation on same image has more complex calculations. Firstly, rotation is determined and after that translation is calculated. Rotation is invariant to Fourier transform. Rotation of an image cause rotation of its Fourier transform with same angle. By using cross power spectrum; rotation and then translation can be calculated.

Fourier transform methods have some advantages such as less sensitivity to noise and low computational complexity.

*2.4.2.1.3 Mutual Information Methods.* Mutual information is a very popular similarity metric in image processing. It is based on information theory. The mutual information measures the statistical dependence between two scalar features and has therefore become a popular measure for registration.

Mutual information needs sufficiently large image overlap for reliable estimate of image statistics. With $A$ and $B$ being random variables of two images to be registered it can be expressed by

$$MI(A,B) = H(A) + H(B) - H(A,B) \quad 2.13$$

where $H(A)$ and $H(B)$ are the marginal entropies of $A$ and $B$ and $H(A,B)$ is their joint entropy. All entropies can be computed from the probability distributions of $A$ and $B$, and their joint probability distribution. Mutual information reaches maximum when the two images are geometrically aligned.

Mutual information has some different mathematical definitions. Equation 2.14 gives another definition where $P_A(a)$ and $P_B(b)$ denote the marginal probability distributions, and $P_{AB}(a,b)$ is the joint probability distribution of $A$ and $B$, then mutual information is defined as :

$$MI(A,B) = \sum_{a,b} p_{AB}(a,b).\log \frac{p_{AB}(a,b)}{p_A(a).p_B(b)} \qquad 2.14$$

Marginal entropy and joint entropy can be calculated as:

$$H(A) = \sum_{a} - p_A(a) \log p_A(a) \qquad 2.15$$

$$H(B) = \sum_{b} - p_B(b) \log p_B(b) \qquad 2.16$$

$$H(A,B) = \sum_{a} \sum_{b} - p_{A,B}(a,b) \log p_{A,B}(a,b) \qquad 2.17$$

Where $P_A(a)$ and $P_B(b)$ are marginal probability mass functions, and $P_{AB}(a,b)$ joint probability mass function. These probability functions can be calculated as:

$$P_A(a) = \sum_{b} P_{A,B}(a,b) \qquad 2.18$$

$$P_B(b) = \sum_{a} P_{A,B}(a,b) \qquad 2.19$$

$$P_{A,B}(s,b) = \frac{h(a,b)}{\sum_{a} \sum_{b} h(a,b)} \qquad 2.20$$

Where *h* is the joint histogram of images**.**

*2.4.2.1.4 Optimization Methods.* Optimization methods are used for finding maximum of similarity metric or minimum of dissimilarity measure. The only method yielding global extreme solution is an exhaustive search over the entire image. Although it is computationally demanding, it is often used if only translations are to be estimated (Zitova & Flusser, 2003). Levenberg–Marquardt method and Genetic algorithm are the most preferable methods.

An issue must be considered regarding these optimization methods. Next to the dissimilarity measure term, sometimes the formula to be minimized contains too, so-called regularization or penalty term, which interconnects the transformation and data to be transformed. These two terms together form the cost function (energy) associated with the registration. The aim of the optimization methods is to minimize the cost function and such methods are referred to as energy minimization methods. The regularization term is usually omitted in case of rigid body transforms, but exists in elastic registration.

*2.4.2.2 Feature-Based Methods*

Main goal of registration is to find a correspondence between reference and input images. Feature based methods uses different control points to achieve this aim. For example, center of lines, gravity center of regions can be used as control points.

*2.4.2.2.1 Methods Using Spatial Relations.* These methods are based on spatial relations of features. Information about spatial distribution of control points and distance between each other help methods to find mapping function. Clustering technique and chamfer matching are among the methods using spatial relations. In clustering technique, control points pair of reference and input images are computed and represented as a point in space of transformation parameters.

The parameters of transformations that closely map the highest number of features tend to form a cluster, while mismatches fill the parameter space randomly. The cluster is detected and its centroid is assumed to represent the most probable vector of matching parameters. Mapping function parameters are thus found simultaneously with the feature correspondence. Local errors do not influence globally the registration process.

Chamfer matching is the process of finding the position of an object (subimage) in an image where both the subimage and the image are binary. The sum of distances between corresponding object and image points is used as the match-rating. When an object is an entire image, the process will find the amount of shift that is needed to align the two images. The amount of shift where the images best align is determined using the sum of distances between closest points in the images. (Goshtasby, 2005).

Chamfer matching can be used to determine the positions of regions in the input image with respect to the reference image when local geometric differences between the images are small, although global geometric difference between the images can be large. Given a template and an image, the process of locating the template within the image involves shifting the template within the image and at each shift position determining the sum of distances of closest object points in the template and the image. The smaller the sum, the closer the template is considered to be from the true match position in the image. The process, therefore, involves starting from an initial position in the image, determining the sum of distances of points in the template to points in the image closest to them and shifting the template in the image in the gradient-descent direction of the sum until a minimum is reached in the sum. If the template perfectly matches the image at a shift position, the sum of distances obtained as a result will be zero. Due to noise and segmentation errors, however, this rarely occurs even when a perfect match is possible.

*2.4.2.2.2 Methods Using Invariant Descriptors.* Invariant descriptors can be effective if they can tolerate the local image deformations. Invariant descriptors must satisfy some conditions as invariance, uniqueness and stability. The choice of type of invariant descriptors can be changed according to geometric deformation and feature characteristic. Minimum distance rule can be used while searching best matching pairs. The simplest feature descriptor is image intensity values. With these descriptors, iterative search algorithm and cross correlation can be used.

*2.4.2.2.3 Relaxation Methods.* A large group of the registration methods is based on the relaxation approach, as one of the solutions to the consistent labeling problem (CLP): to label each feature from the sensed image with the label of a feature from the reference image, so it is consistent with the labeling given to the other feature pairs. The process of recalculating the pair figures of merit, considering the match quality of the feature pairs and of matching their neighbors, is iteratively repeated until a stable situation is reached.

*2.4.2.2.4 Pyramids and Wavelets.* These kinds of methods are based on reducing the test locations in search area and amount of data used in the tests. Pyramidal structure or wavelet-like methods are useful to reduce the image size without loss of image characteristic. Generally, these methods are called as coarse to fine methods. In these methods, usual registration search algorithms are used. Registration process starts with coarse resolution (generated by wavelets or Gaussian pyramids) and then gradually correspondence between input and reference images is improved while going to fine resolution. Coarse resolution reduces search space and computational time of registration operation is decreased. In these methods, coarse level operation is very important because false match in coarse level cause fail of registration algorithm.

The coarse-to-fine approach is particularly useful when high-resolution images with large local geometric differences need to be registered. At a coarse resolution (small scale), the global geometric difference between the images is determined and the sensed image is resampled to globally align the reference image. As the scale is increased, more landmarks are selected to better represent the local geometric differences between the images. The process is repeated until images at full scale are registered. (Goshtasby, 2005)

Multi-resolution decomposition of wavelet is the widely used structure in image registration applications. Generally, image is filtered with two filter, one low pass and one high pass filter both working along the image rows and columns. Then image decomposition is achieved recursively, giving four sets of wavelet coefficients that are used for finding the correspondences between two images in registration applications.

### *2.4.3 Transform Model Estimation*

The correspondence of the control points from the input and reference images together with the fact that the corresponding control point pairs should be as close as possible after the sensed image transformation are employed in the mapping function design. The type of the mapping function must comply with the assumed geometric deformation of registration operation. Generally, mapping functions can be divided into two main categories as global mapping and local mapping. Global mapping model uses all control points to estimate mapping function parameters for entire image. In general, number of control points must be higher than required number of control points to determine mapping function. Global mapping models are, for example, convenient for satellite images.

In medical imaging, local deformations exist in many cases. Global mapping functions can’t model local deformations successfully. In local mapping model, several patches constitute the images and each patch has separate mapping function. So, local mapping models can be used for this application.

Elastic based registration is an approach that differs from those using any parametric mapping function determined by control points. The images are considered pieces of a rubber sheet, on which external forces stretching the image and internal forces defined by stiffness or smoothness constraints are applied to bring them into alignment with the minimal amount of bending and stretching. The feature matching and mapping function design steps of the registration are done simultaneously.

Geometrically, the image transformation is initially modeled with local affine transformation leading to a global elastic registration. The advantage of local transformations is their capability of modeling highly nonlinear global transformations without the numerical instability of high order nonlinear global mapping models. Unfortunately the disadvantage is the computational inefficiency due to significantly larger number of model parameters needed for estimation. Furthermore, global consistency constraint is another problem that needs to be tackled.

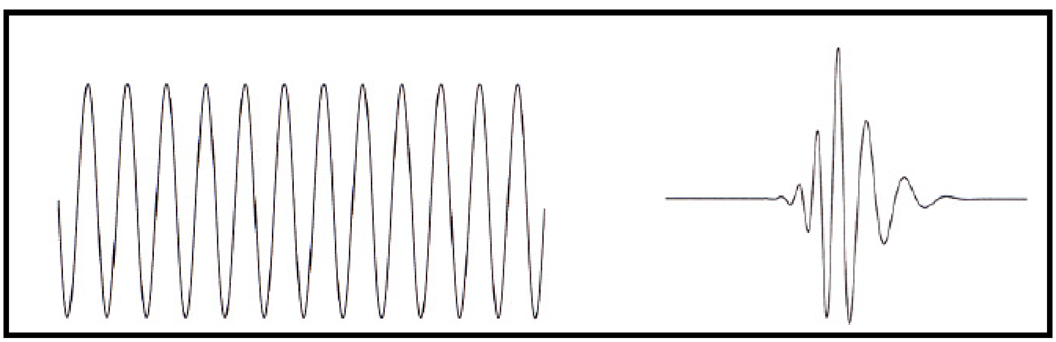
In addition to geometrical transformation, intensity variations between the images are explicitly modeled with local changes in brightness and contrast. Modeling intensity variations allows for effective registration in the presence of noise, causing local intensity variations, as well as registration of certain multi-modal images.

## CHAPTER THREE

## WAVELETS

### 3.1 Introduction

Wavelet transform is a method to transform a function into more useful another form. Information content of signal doesn't change with this transformation. The wavelet transform is used to overcome shortcomings of Short Time Fourier Transform (STFT). This decomposition is to project signal f($x$) onto a family of functions which are the dilations and translations of a unique function $\psi(x)$. The function $\psi(x)$ is called as wavelet. Signals can be locally characterized in both time and frequency domain. According to this, wavelet can analyze non stationary signals.



(a) (b)

Figure 3.1 (a) Wave, (b) wavelet

A wave is a periodic oscillating function of time or space. Wavelets can be defined as localized waves. They have their energy concentrated in time or space and are suited to analysis of transient signals. While Fourier transform and STFT use waves to analyze signals, the wavelet transforms uses wavelets of finite energy.

The Fourier transform is not satisfactory for analyzing signals whose spectra vary with time. The Fourier transform depends on sinus and cosine waves so it can't represent irregularities very well. The Fourier bases are perfectly local in frequency domain but they are global in time domain. There are two drawback of Fourier transform such as:

- Fourier analysis can't characterize signal locally in time domain
- Fourier expansion can approximate stationary signals well but it can' succeed of non-stationary signals.

Short time Fourier transform is used to overcome of Fourier transform drawbacks. In short-time Fourier transform, the signal is multiplied by a sliding window that localizes the signal in time domain, but results in a convolution between the signal spectrum and the window spectrum, that is, a blurring of the signal in frequency domain. The narrower the window, the better we localize the signal and the poorer we localize its spectrum. (Sheng, 2000)

Spatial versus frequency resolution of different transforms are summarized in figure 3.2. Fixed time-frequency resolution of STFT causes serious problems in many applications. Variable Heisenberg boxes represents compatibility of wavelet transform for many cases.

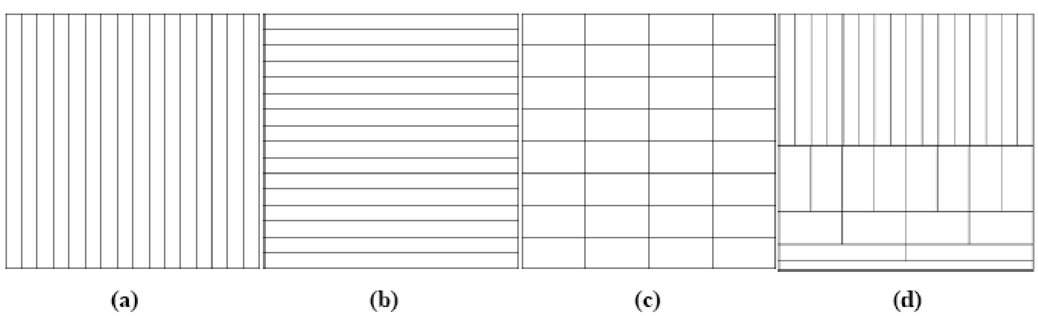



Figure 3.2: X-axis: spatial resolution. y-axis: frequency resolution, (a) discrete sampling is not localized on frequency axis, (b) Fourier transforms are not localized on spatial axis, (c) windowed Fourier transforms with constant Heisenberg boxes, (d) wavelet transforms with variable Heisenberg boxes.

Wavelet transform has many types as discrete wavelet transform, continuous wavelet transform, wavelet series or wavelet packets. Each of them is used for different applications.

### 3.2 Wavelet Conditions

A function must ensure some conditions to be classified a wavelet.

1) A wavelet must have finite energy

$$E = \int_{-\infty}^{\infty} \left|\psi(t)\right|^2 dt < \infty \qquad 3.1$$

E is the energy of function.

2) A wavelet must satisfy admissibility condition

Fourier transform of wavelet is:

$$\hat{\psi}(f) = \int_{-\infty}^{\infty} \psi(t) e^{-i(2\Pi f)f} dt \qquad 3.2$$

Admissibility condition can be represented as:

$$C_g = \int_{0}^{\infty} \frac{\left|\hat{\psi}(f)\right|^2}{f} df < \infty \qquad 3.3$$

This equation means wavelet has no zero frequency components. In other words, wavelet must have a zero mean.

3) An addition criterion that must hold for complex wavelets is that the Fourier transform must both be real and vanish for negative frequencies.

### 3.3 Continuous Wavelet Transform

The wavelet transform of a continuous signal x(t) can be represented as;

$$W_X(b,a) = |a|^{-\frac{1}{2}} \int_{-\infty}^{\infty} x(t)\psi^*(\frac{t-b}{a})dt \qquad 3.4$$

The continuous wavelet transform is calculated as the product of continuous signal x(t) and translated and scaled version of $\psi(t)$. $\psi(t)$ is called as mother wavelet or basis function. The mother wavelet used to generate all the basis functions is designed based on some desired characteristics associated with that function. All the wavelet functions used in the transformation are derived from the mother wavelet through translation (shifting) and scaling (dilation or compression). The parameters b and a are called translation and dilation parameters respectively. The asterisk indicates that the complex conjugate of wavelet function is used in the transform. So equation can be represented as convolution of x(t) with time reversed and scaled wavelet:

$$W_X(t,a) = |a|^{\frac{-1}{2}} x(t) * \psi_a(t) \qquad 3.5$$

where,

$$\psi_a(t) = \psi^*(\frac{-t}{a}) \qquad 3.6$$

Constant expression $|a|^{\frac{-1}{2}}$ is used to ensure all scaled functions have same energy.

### 3.4 Discrete Wavelet Transform

In signal and image processing data are represented with finite number of values so discrete version of continuous wavelet transform is needed. The continuous wavelet transform generates redundant information about signal on time–scale plane. The discrete wavelet transform keeps enough information of signal that it reconstructs the signal perfectly from its wavelet coefficients. In this situation, only the discrete orthonormal basis of wavelets are considered and filtered by a high-pass and a low-pass filter in multi-resolution fashion. An orthonormal basis of wavelets can be defined by a scaling function and its corresponding conjugate filter. It is easy to implement and reduces the computation time and resources required.

The continuous wavelet transform analyzes signals with using a set of basis functions which relate to each other by simple scaling and translation. In the discrete wavelet transform, digital filtering techniques are used to obtain time-scale representation of signals. The signal to be analyzed is passed through filters with different cutoff frequencies at different scales.

Let f(t) be a continuous time signal that is uniformly sampled at intervals $N^{-1}$ over [0,1]. Its wavelet transform can only be calculated at scales $N^{-1}<s<1$. In discrete computations, it is easier to normalize the sampling distance to 1 and thus consider the dilated signal

$$f(t) = f(N^{-1}t) \tag{3.7}$$

So wavelet transform of function can be expressed as:

$$Wf(u,s) = N^{-1/2}Wf(Nu,Ns) \tag{3.8}$$

To simplify notation denote $f[n] = f(n)$ is the discrete signal of size N. Its discrete wavelet transform is computed at scales $s=a^j$ with $a=2^{1/v}$, which provides v intermediate scales in each $[2^j,2^{j+1}]$. (Mallat, 1998)

Let $\psi(t)$ be a wavelet whose support is included in $[-K/2, K/2]$. A discrete wavelet scaled by $a^j$ is defined by;

$$\psi_j[n] = \frac{1}{\sqrt{a^j}} \psi\left(\frac{n}{a^j}\right) \tag{3.9}$$

$$2 \le a^j \le NK^{-1} \tag{3.10}$$

## 3.5 Multi-Resolution Analysis

Multi resolution analysis is used to divide a complicated function into simpler versions. The discrete wavelet transform of images can be realized by iteration of filters with rescaling. In image processing, orthonormal basis of wavelets are used and filtered by using low and high pass filter to realize multi-resolution analysis.

The multi-resolution algorithm is used to reduce computation time of the registration process. The DWT is computed by successive low pass and high pass filtering of the discrete time-domain signal or image. In image processing operations 2D discrete wavelet transform is used to realize wavelet decomposition of image. Figure 3.3 indicates first level multi resolution decomposition of images. This structure produces three detailed subimage (HL, LH and HH) and a lower resolution subimage (LL).

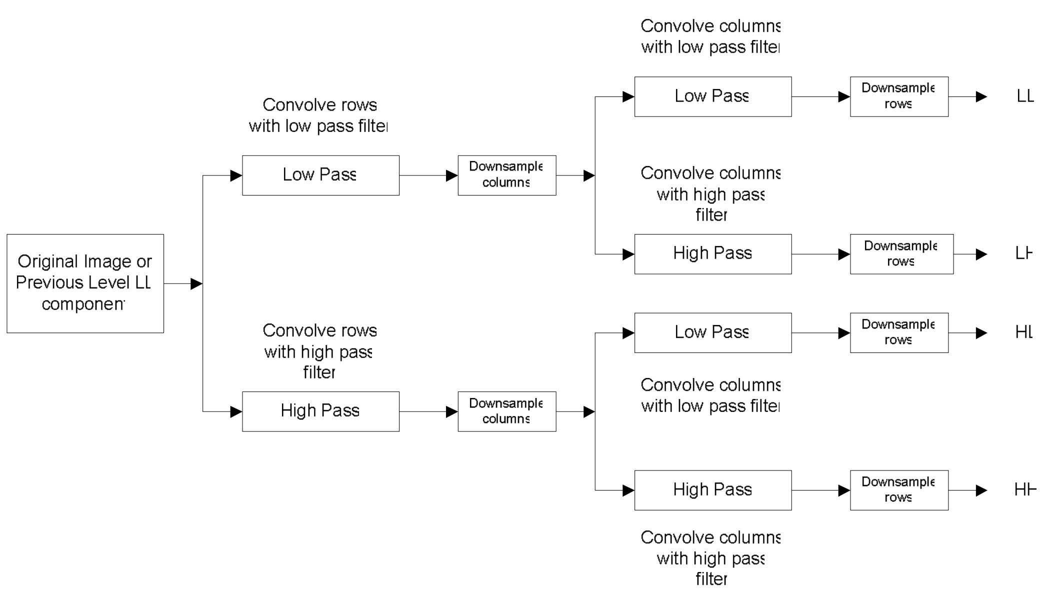



Figure 3.3 Wavelet decomposition of image

The wavelet decomposition produces four subimages. Each subimage contain different characteristic of image. HL image contains vertical edge features of image. LH image contain horizontal edges of images. LL is the low resolution version of original image. And HH contains high frequency information of image so it is noisy.

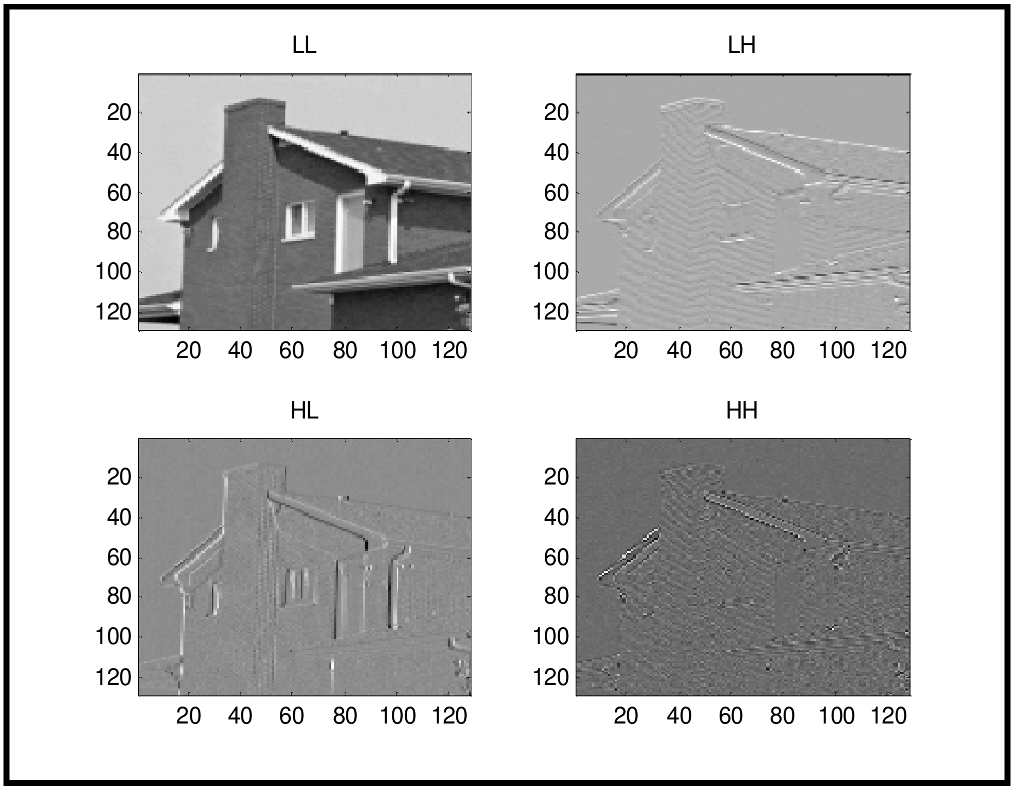



Figure 3.4 First level of wavelet decomposition of image

Figure 3.4 indicates one level decomposition of image. Original image is 256x256 RGB image and subimages are generated as 128x128 images. Each subimage contains different characteristic of original image. In this thesis, four level wavelet decomposition processes is used for registration.

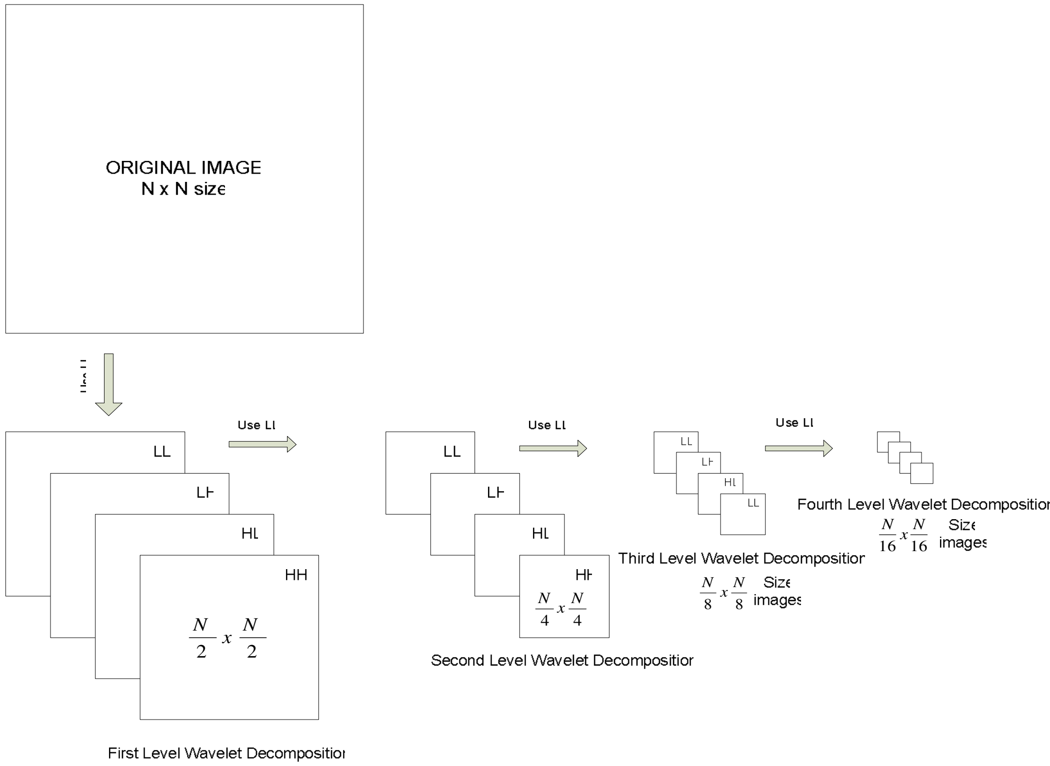



Figure 3.5 Four level wavelet decomposition of image

In image registration applications three or four level of wavelet decomposition is used. Figure 3.5 indicates four level of decomposition. In Figure 3.6, 4x4 image is used to calculate wavelet decomposition with using Daubechies wavelet filters in Matlab environment. Daubechies wavelet filters ("db1" in Matlab) are used to decomposition operation.

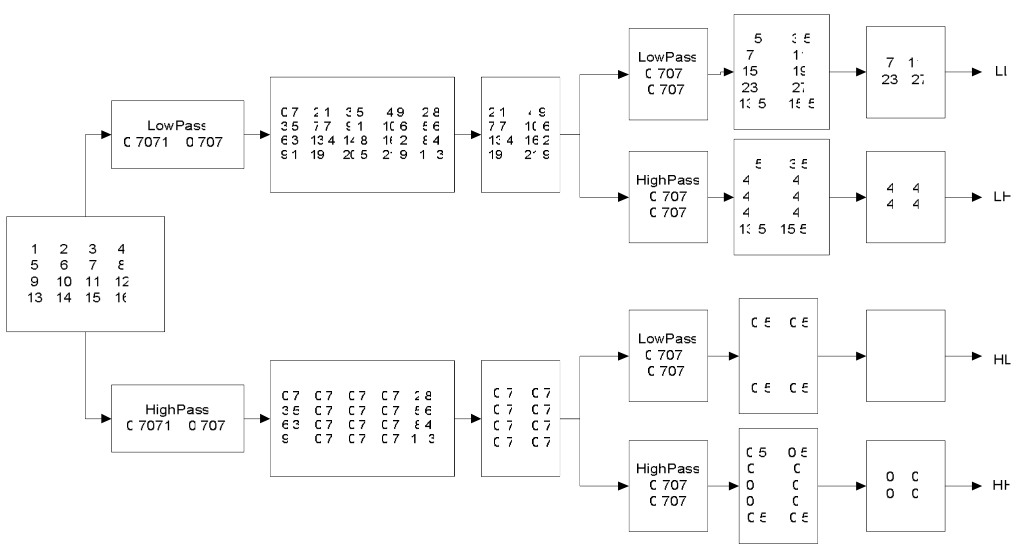



Figure 3.6 Wavelet decomposition of 4x4 image with using Daubechies wavelets

### 3.6 Image Registration with Using Wavelet Transform

The wavelet transform is used to extract the information of image with using multi-resolution of image. Multi-resolution technique preserves important features of image when image size gets lower and eliminates weak features of image. So, multi-resolution method avoids loosing any local or global feature of image.

The advantage of using wavelet for image registration can be summarized as:

1) Multi-resolution wavelet decomposition highlights strong features at low resolution and eliminates weak features at high resolution.

2) Wavelet based image registration methods are generally iterative. In this iterative method, search interval decrease and accuracy of operation increases at each iteration. So, registration algorithm achieves higher accuracy with lower computational time.

3) Multi-resolution approach of wavelet is used to bring image to same resolution without significant information loss and without blurring higher resolution data.

4) Multi-resolution wavelet transforms can be implemented very easily on a parallel computer. (Le Moigne, Campbell & Cromp, 2002).

Wavelet based registration method generally have these properties:

- Wavelets are used to extract feature points
- Low pass filter causes smoothing the image
- Wavelet are used to decompose the images
- The extracted feature points are matched
- Wavelet based methods generally are used in a coarse to fine search strategy with a wavelet pyramid. These methods are iterative and accuracy of estimation increases at each iteration.

# CHAPTER FOUR

# SIMULTANEOUS PERTURBATION STOCHASTIC APPROXIMATION (SPSA)

## 4.1 Definition

An image registration method has two important performance parameters such as accuracy and computation time. The computation time is the most important disadvantage of iterative registration methods. SPSA (Spall, 1998) is an effective optimization method to drawback this disadvantage. It can be defined as the optimization process of finding the best input parameters without evaluating all possibilities. This goal of SPSA reduces computation time while maximizing calculation of a similarity metric.

SPSA is based on a highly efficient and easily implemented simultaneous perturbation approximation to the gradient. SPSA calculates an approximation of the objective function gradient at each iteration and then adjusts the current solution estimate according to the gradient estimate. Finite differences are used in generating the gradient estimation. The SPSA optimization technique only requires two evaluations of the objective function to generate a gradient estimation regardless of the number of parameters being optimized.

SPSA is a descent method capable of finding global minima or maxima of similarity metric objective function. Its main feature is the gradient approximation that requires only two measurements of the similarity metric function, regardless of the number of transformation parameters. This help to reduce computation time and increase the accuracy of operation.

The goal of the SPSA algorithm is to minimize or maximize a loss function $L(\theta)$, where the loss function is a performance measure and $\theta$ is m dimensional vector of parameters to be adjusted. It is assumed that $L(\theta)$ is a differentiable function and minimum point $\theta^*$ corresponds to a point of zero gradient (equation 4.1).

$$g(\theta^*) = \left.\frac{\partial L(\theta)}{\partial \theta}\right|_{\theta=\theta^*} = 0 \qquad 4.1$$

This problem is solved by simultaneous annealing (SA) methods referred to also as "stochastic gradient algorithms", as they require knowledge of a direct noisy measurement of the (unknown) true gradient $g(\theta^*)$.

There are, however, a large number of problems where the direct measurement of gradient knowledge is difficult or impossible to obtain. For this reason, SA algorithms that do not depend on direct gradient measurements have been developed. Rather, these algorithms are based on an approximation to the gradient and are therefore referred to as "gradient approximation algorithms". The older of these "gradient-free" methods is the finite difference stochastic approximation (FDSA). It employs small one-at-a-time changes to each of the individual elements of $\theta$. FDSA was very costly to calculate and this led to the development of SPSA algorithm.

**4.2 SPSA Algorithm**

The SPSA algorithm is a gradient-free basic optimization method for image registration applications. The algorithm doesn't need gradient calculation. It is assumed that no direct measurement of $g(\theta)$ are available.

The basic SPSA algorithm in the general recursive SA form is:

$$\hat{\theta}_{k+1} = \hat{\theta}_k - a_k g_k(\hat{\theta}_k) \qquad 4.2$$

Sign of the equation determines whether the SPSA algorithm is to minimize or maximize the loss function. Subtraction indicates that the loss function is to be minimized while addition indicates that it is to be maximized by the algorithm. In this equation $g_k(\hat{\theta}_k)$ is the simultaneous perturbation of the gradient $g(\theta)$.

Under appropriate conditions, algorithm converges to optimum $\theta^*$ value. To realize the algorithm, $g_k(\hat{\theta}_k)$ must be calculated. This requires two loss function measurements.

$$\hat{g}_k(\hat{\theta}_k) = \begin{bmatrix} \dfrac{y(\hat{\theta}_k + c_k\Delta_k) - y(\hat{\theta}_k - c_k\Delta_k)}{2c_k\Delta_{k1}} \\ \vdots \\ \dfrac{y(\hat{\theta}_k + c_k\Delta_k) - y(\hat{\theta}_k - c_k\Delta_k)}{2c_k\Delta_{kp}} \end{bmatrix} \qquad 4.3$$

Where,

$$\Delta_k = \lfloor \Delta_{k1}, \Delta_{k2}, \dots, \Delta_{kp} \rfloor \qquad 4.4$$

$\Delta_k$ is a p dimensional random perturbation vector. The gradient approximation is based on the difference between the values of the loss function measured in two different points.

In case of the SPSA algorithm, the j*th* component of gradient of objective function at the $i^{th}$ iteration is calculated as follows:

$$\hat{g}_{ij}(\hat{\theta}_i) = \frac{y(\hat{\theta}_i + c_i\Delta_i) - y(\hat{\theta}_i - c_i\Delta_i)}{2.c_j.\Delta_{ij}} \qquad 4.5$$

Then the next estimate of parameter vector is calculated with using equation 4.2. This update process continues until a termination condition is satisfied.

### 4.3 SPSA Implementation

SPSA is used in many application areas such as control systems, pattern classification, and image registration algorithms. It generates successful results with basic implementation.

*Step 1*: Initialization and Coefficient Selection

In first step, $\hat{\theta}$ and nonnegative coefficients ( $a, c, A, \alpha, \gamma$ ) must be set to an initial guess. Different applications require different parameters. These parameters are used in SPSA gain sequence in iteration.

$$a_k = \frac{a}{(A + k + 1)^{\alpha}} \tag{4.6}$$

$$c_k = \frac{c}{(k + 1)^{\gamma}} \tag{4.7}$$

Generally, $\alpha$ is set to 0.602 and $\gamma$ is set to 0.101. These values are optimum ones. In equation 4.6 and 4.7 $a_k$ and $c_k$ represents the values of a and c in $k^{th}$ iteration. There are no strict rules for selection of these parameters. In this project, these parameters have selected with experimentally. Section 6.3 summaries which parameters are used in this thesis.

*Step 2*: Generation of Simultaneous Perturbation Vector

Generate a p-dimensional random perturbation vector $\Delta_k$, where each of components of $\Delta_k$ is independently generated from a zero mean probability distribution. Generally, very simple option of Bernoulli distribution with values ±1 with a probability of 0.5 for each random value is preferred.

*Step 3*: Loss Function Evaluations

Loss function is evaluated in two different forms at each iteration. In registration application, similarity metric defines the loss function. Evaluations of functions are:

$$y(\hat{\theta}_i + c_i \Delta_i) \tag{4.8}$$

$$y(\hat{\theta}_i - c_i \Delta_i) \tag{4.9}$$

*Step 4*: Gradient Approximation

Simultaneous gradient approximation is generated with using Bernoulli distribution. Generally Bernoulli distribution is used for $\Delta_k$ values. Equation 4.3 indicates that how gradient approximation is generated.

*Step 5*: Update $\theta$

In SPSA algorithm parameters are updated at each of iterations.

$$\hat{\theta}_{k+1} = \hat{\theta}_k - a_k g_k(\hat{\theta}_k) \tag{4.10}$$

Termination of the process can be realized in two different ways. Either, a small change in loss function in successive iterations causes the termination, or alternatively, a maximum allowable number of iterations are completed before terminating the algorithm.

Determination of the gain sequence of $a, c, A, \alpha, \gamma$ is a very important step of SPSA method. These variables determine the performance of the algorithm. For different applications, variables can be changed. Figure 4.1 gives only basic relations of the variables. In each application variables must be determined one by one.

$$
\begin{aligned}
&a, c > 0 \\
&A \geq 0 \\
&0 < \gamma < \alpha < 1 \\
&\alpha - 2\gamma > 0 \\
&3\gamma - \frac{\alpha}{2} \geq 0
\end{aligned}
$$

Figure 4.1 Basic relation of spsa gain sequence

Alpha and gamma are critical parameters since they determine the gain sequence. The recommended values for alpha and gamma are around 0.602 and 0.101, respectively, according to the theoretical convergence conditions provided in (Spall 1998). These values were also found to be good in practice from the convergence point of view.

### 4.4 Sample Matlab Code of SPSA

Implementation step of the SPSA algorithm is given in the previous section. A simple SPSA code can be written in Matlab as shown in figure 4.2. Detailed information can be obtained from (Spall, 1998). In this program a, c, alpha and gamma are predefined constants and they are determined depending on the application. Dimension of theta changes depending on the number of transformation parameters being searched.

```
for k=1:n
ak=a/(k+A)^alpha;
ck=c/k^gamma;
delta=2*round(rand(p,1))-1;
thetaplus=theta+ck*delta;
thetaminus=theta-ck*delta;
yplus=loss(thetaplus);
yminus=loss(thetaminus);
ghat=(yplus-yminus)./(2*ck*delta);
theta=theta-ak*ghat;
end
theta
```

Figure 4.2 Simple program code for spsa algorithm

# CHAPTER FIVE

# GENETIC ALGORITHM

## 5.1 Introduction

A genetic algorithm (GA) is a global search technique used in computing to find optimum solutions for optimization and search problems. Genetic algorithms were formally introduced in the 1970s by John Holland at University of Michigan.

GAs are computational models of natural evolution in which stronger individuals are more likely to be the winners in a competitive environment. Besides their intrinsic parallelism, GAs are simple and efficient techniques for optimization and search. The main advantage of the GA approach for image registration is that pre-alignment between views is not necessary to guarantee a good result. However, the GA is a stochastic method and generally time-consuming (Silva , Bellon & Boyer, 2005).

GAs is a simple and robust method for optimization and search and has intrinsic parallelism. GAs evaluates all possible solutions. GAs is iterative procedures that maintain a population of candidate solutions encoded in form of chromosome string. The initial population can be selected heuristically or randomly. For each generation, each candidate is evaluated and is assigned the fitness value that is generally a function of the decoded bits contained in each candidate's chromosome. These candidates will be selected for the reproduction in the next generation based on their fitness values. The selected candidates are combined using the genetic recombination operation "crossover". The crossover operator exchanges portions of bit string hopefully to produce better candidates with higher fitness for the next generation. The "mutation" is then applied to perturb the string of chromosome as to guarantee that the probability of searching a particular subspace of the problem space is never zero. It also prevents the algorithm from becoming trapped on local optima. Then, the whole population is evaluated again in the next generation and the process continues until it reaches the termination criteria. The termination criteria may be triggered by finding an acceptable approximate solution, reaching a specific number of generations, or until the solution converges.

.

```
k=0
Create random initial population (0)
Evaluate population (0)
Repeat
k=k + 1
Select parents for reproduction
Crossover if condition is met
Mutate if condition is met
Create the new population P(k)
Evaluate P(k)
Until termination condition is met
Return best individual of P(k)
```

Figure 5.1 Simple genetic algorithms

## 5.2 Genetic Algorithm

Genetic Algorithms are search and optimization techniques inspired by biological principles. Genetic algorithm is based on natural selection. Genetic algorithm is realized with combination of some operations.

### *5.2.1 Selection*

Selection is the step of GA to select the individuals from the population for generation of new chromosomes. Selection has many different types but all of them are based on fitness function. The individual genes with higher fitness values are chosen from a population for later generation.

There are many different approaches for selection operations such as roulette wheel, tournament selection and rank selection. In roulette wheel selection, the simulated roulette wheel generates a random number that is some fraction of the total fitness of the parent population; then it counts around the wheel until it finds the selected chromosome. In other words, each individual in the population is allocated a section of a roulette wheel. The size of the section is proportional to the fitness of the individual. A pointer is spun and the individual to whom it points is selected. This continues until the selection criterion has been met. The probability of an individual being selected is thus related to its fitness, ensuring that fitter individuals are more likely to leave child.

In the rank-based selection method, selection probabilities are based on a chromosome's relative rank or position in the population, rather than absolute fitness.

In image registration applications, the roulette wheel method is used because of the easiness of its calculation.

.

### *5.2.2 Crossover*

The crossover operation is another step for genetic algorithm. Crossover is used to swap certain parts of the two selected strings to capture the good parts of old chromosomes and create better new ones. This means exchange of some chromosomes of parents at the crossover point to generate new individuals.

There are different types of crossover in genetic algorithm applications. Such as one point crossover, two point crossover, uniform crossover, arithmetic crossover and heuristic crossover.

One point crossover**:** Randomly selects a crossover point within a chromosome then interchanges segments of the two parent chromosomes at this point to produce two new children (Silva, Bellon & Boyer, 2005). Figure 5.2 shows one point crossover, the genes in two 8-bit chromosomes A and B are exchanged at the cross point, second bit from the left. This produces two new chromosomes C and D. In general, the single-point crossover is suitable for most applications.

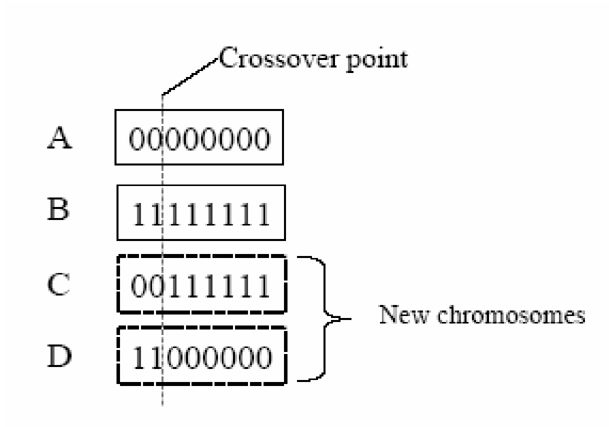



Figure 5.2 One point crossover

Two point crossovers: Randomly selects two crossover points within a chromosome then changes segments of the two parent chromosomes between these points to produce two new child as shown in figure 5.3.

Parent1: 1 2 3 4 5 6
Parent2: 7 8 9 1 2 3
Child 1 : 1 2| 9 1| 5 6
Child 2 : 7 8| 3 4| 2 3

Figure 5.3 Two point crossover

Uniform crossover: A crossover that decides (with some probability – known as the mixing ratio) which parent will contribute each of the gene values in the child chromosomes. This allows the parent chromosomes to be mixed at the gene level rather than the segment level (as with one and two point crossover). Figure 5.4 represents a uniform crossover example for 0.5 crossover probability. For each position random number is determined such as 0.1, 0.6, 0.7, 0.2, 0.9, 0.3. If the numbers greater than 0.5, child1 gets gene from parent1, child2 gets gene from parent2.

Parent1: 7 *3 *7 6 *1 3
Parent2: 1 *7 *4 5 *2 2

Child 1 : 7 7* 4* 6 2* 3
Child 2 : 1 3* 7* 5 1* 2

Figure 5.4 Uniform crossover examples

Arithmetic crossover: A crossover operator is used to combine two parent chromosome vectors to produce two new children according to the following equations. A random weighting factor "a" is determined and crossover process is completed according to equations in figure 5.5.

child1 = a * parent1 + (1-a) * parent2

child2 = (1-a) * parent1 + a * parent2

Figure 5.5 Arithmetic crossover

Heuristic crossover: In this method, result of fitness function is combined with a crossover operator and new child is generated with using equations given in figure 5.6. The value of "a" must lie between 0 and 1.

child1 = BestParent + a*(BestParent – WorstParent)

child 2 = BestParent

Figure 5.6 Heuristic crossover

### *5.2.3 Mutation*

Mutation is another operation of genetic algorithms. Mutation is needed to ensure that all possible chromosomes are reachable. The purpose of mutation in GAs is to allow the algorithm to avoid local minima by preventing the population of chromosomes from becoming too similar to each other, thus slowing or even stopping evolution. This reasoning also explains the fact that most GA systems avoid only taking the fittest of the population in generating the next but rather a random (or semi-random) selection with a weighting toward those that are fitter.

### *5.2.4 Fitness Function*

Fitness function is used to measure of correctness of possible solutions. In image registration applications, for example, cross correlation or mutual information can be used as fitness function.

### *5.2.5 Genetic Algorithm Parameters*

GAs have many parameters that need to be adjusted depending on application areas. Generally, these parameters are selected by experimental methods. Genetic algorithm parameters can be defined and summarized in table 5.1.

Table 5.1 Genetic algorithm parameters

| | |
|---|---|
| Population | Number of individuals to be evaluated for each generation |
| Chromosome | Individual encoded as a string of genes (bits, integer, etc.) |
| Mutation Probability | Says how often will be parts of chromosome mutated If mutation probability is 1, whole chromosome is changed, if it is 0, nothing is changed. |
| Selection | Selection of parent and child to generate new generation |
| Crossover Probability | Says how often crossover will be performed. If crossover probability is 1 then all children is made by crossover. If it is 0, whole new generation is made from exact copies of chromosomes from old population. |
| Fitness value | Measure the performance of each individual. In registration application, it is similarity metric. |

## 5.3 Image Registration with Genetic Algorithm

The accuracy is the main goal for most registration applications. So, optimization and search space reduction methods are used to increase the accuracy and reduce the computation time. Genetic algorithm is an effective method for reduction of search space but it is time consuming. In registration applications, population size is the most important parameter of all process. If the optimal population size is chosen, genetic algorithm can achieve high accuracy with low computational time.

Image registration methods combine genetic algorithm with wavelet multi-resolution methods to take advantages both of them. Multi-resolution scheme reduces the search space and genetic algorithm evaluates all possible values for registration parameters. In this thesis, genetic algorithm evaluates all possible values in fourth level of wavelet decomposition (low resolution image) and evaluates possible values in an interval for higher level of wavelet decomposition.

# CHAPTER SIX

# IMAGE REGISTRATION IMPLEMENTATION

## 6.1 Introduction

In this thesis, wavelet based image registration methods are realized. Multi-resolution scheme of Daubechies wavelets is used to extract information of images at all methods. Wavelet based image registration methods are iterative so, Simultaneous Perturbation Stochastic Approximation (SPSA) method and genetic algorithm are used to optimize iterative method. In this thesis, wavelet based image registration is realized as:

1. Full search algorithm with cross correlation as similarity metric
2. Search algorithm with using SPSA and
    - mutual information as similarity metric
    - cross correlation as similarity metric
3. Search algorithm with using genetic algorithm and
    - mutual information as similarity metric
    - cross correlation as similarity metric

In this thesis, all methods are realized to search for five parameters such as rotation angle, translation in x direction, translation in y direction, scale in x direction and scale in y direction. The parameters are limited between $-90^0$ and $90^0$ for rotation, -80 and 80 pixels for translation and 0.7 to 1.5 for scale parameters. Image registration implementation was realized using MATLAB on a 1.6 GHz Centrino processor, 512MB ram and Microsoft Windows XP.

## 6.2 Full Search Algorithm

Full search algorithm is completely iterative method for image registration operation. In its implementation in the thesis, similarity metric is limited to only cross correlation for which a maximum value is searched.

Algorithm starts with a large search interval for all parameters. At the highest level of decomposition, search is exhaustive over all search space for all parameters. The best transformation parameters values are chosen over all search space. Then these best parameters became center of new search interval. This process is repeated until first level of wavelet decomposition. Table 6.1 summarizes the full search algorithm.

Table 6.1 Wavelet search algorithm for four level decomposition

| **Wavelet level** | **Search space rotation** | **Search space translation** | **Search space scale** | **Result** |
|---|---|---|---|---|
| **4** | -90 to 90 with 2 degrees step | -5 to 5 for with 1 pixel step | 0.7 to 1.5 with 0.1 step | (θ4,Tx4,Ty4,Scx4,Scy4) |
| **3** | [θ4-2, θ4+2] with 1 degree step | [2*Tx4-2,2*Tx4+2]x[2*Ty4-2,2*Ty4+2] with 0.5 step | [Scx4-0.1,Scx4+0.1]x[Scy4-0.1,Scy4+0.1] with 0.02 step | (θ3,Tx3,Ty3,Scx3,Scy3) |
| **2** | [θ3-0.5, θ3+0.5] with 0.2 degree step | [2*Tx3-1,2*Tx3+1]x[2*Ty3-1,2*Ty3+1] with 0.2 step | [Scx3-0.03,Scx3+0.03]x[Scy3-0.03,Scy3+0.03] with 0.01 step | (θ2,Tx2,Ty2,Scx2,Scy2) |
| **1** | (θ2) | [2*Tx2-0.5,2*Tx2+0.5]x[2*Ty2-0.5,2*Ty2+0.5] with 0.1 step | (Scx2,Scy2) | (θ2,Tx1,Ty1,Scx1,Scy1) |
| | [θ2-0.2, θ2+0.2] with 0.1 degree step | (Tx1,Ty1) | (Scx1,Scy1) | (θ1,Tx1,Ty1,Scx1,Scy1) |

In the full search algorithm, step size in the search space is the main parameter. Too high a value causes wrong decision in $4^{th}$ level of wavelet. And, this causes a failure of the registration algorithm. Too low a step size on the other hand causes computational time to raise. So, in this thesis, optimum step sizes in a predefined search interval for all parameters were determined experimentally.

This method can be summarized as follows;

1. Wavelet decomposition of reference and input images are calculated; HL and LH components of wavelet decomposition are used in all algorithms. Then, only those points whose intensities belong to the top %15 of decomposition image

histogram are kept. Le Moigne recommends between %13 and %15 values. In my tests, I obtain the best result with using top %15 of histogram. This ensures a reduced computation time and an increased accuracy. (Le Moigne & Campbell & Cromp, 2002)

2. Start the searching algorithm with last level of decomposition (4. level for this application) and try to find the best transformation parameters values. Find the best match between input and reference images by iterative search algorithm. Reference image can be rotated and /or translated and/or scaled. Normalized cross correlation as similarity metric is used to decide which transformation parameters give the best match. Table 6.1 summarizes all iterative procedure of the algorithm. The best point is selected as the new center of search in the next wavelet level and the same operation is repeated in all other wavelet levels. To generate a transformed image from the reference image, bicubic or bilinear interpolation method can be preferred for accurate results with this algorithm.

### 6.3 Image Registration with SPSA

Full search image registration methods generate accurate results but they are too slow. Of the fast search algorithms, SPSA is an effective optimization method for accurate registration result with low computational time. In the thesis, SPSA based registration has been realized with two different similarity metrics; mutual information and cross correlation. Cross correlation measurement has been the most widely used similarity metric in image processing applications, although it is computationally expensive and noise sensitive. Mutual information is an alternative similarity metric for registration application.

Multi-resolution search strategy has been used to reduce the computational time and increase the robustness of registration algorithm.

Wavelet based image registration method is summarized in figure 6.1. In this method, wavelet decomposition is applied to reference and input images. This method is referred to as a “coarse to fine” method. Approximations to parameters are achieved at low resolution of image, so computational time of registration process is reduced.

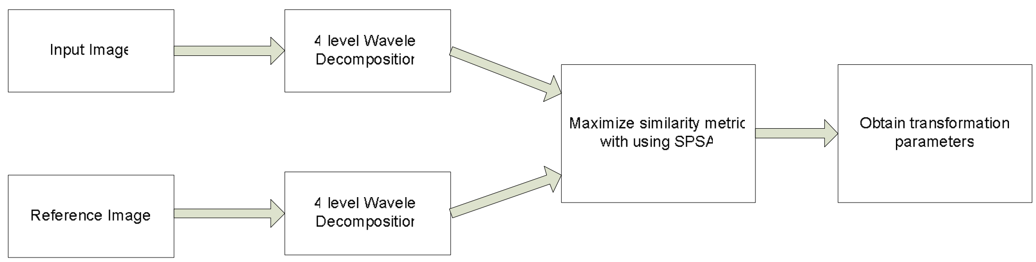



Figure 6.1 Wavelet based image registration method with spsa

SPSA algorithm achieves the approximations to all transformation parameters in the same iteration. At each iteration, SPSA needs an approximation to the gradient via simultaneous perturbations. The gradient approximation is based on only two function measurements regardless of the dimensions of the search parameter space. Approximated gradient is used to maximize the similarity metric.

Our image registration algorithm with SPSA can be implemented as follows;

1) Apply wavelet decomposition to reference and input images. Use the LL component of the multi resolution scheme.

2) Upper and lower limits of the transformation parameters are determined.

3) Determine values of the constant parameters and iteration numbers of SPSA. These values adjust the search step size and speed of the algorithm. The search intervals for rotation, translation and scale have different limits in this work, so I use different constants for them. Iteration number is selected high only at the lower image resolutions. This is very important to reduce the process time. Table 6.2 shows the values of the constants used in this work. These have been determined experimentally.

Table 6.2 SPSA algorithm use different constants for each decomposition level

| | 4th level of decomposition | 3rd level of decomposition | 2nd level of decomposition | 1st level of decomposition |
|---|---|---|---|---|
| Iteration Number | 3000 | 1000 | 500 | 200 |
| c_rotation | 100 | 20 | 10 | 1 |
| c_translation | 2 | 1 | 2 | 1 |
| c_scale | 0.2 | 0.2 | 0.1 | 0.01 |
| a_rotation | 20000 | 1000 | 2000 | 25 |
| a_translation | 10 | 10 | 100 | 100 |
| a_scale | 0.5 | 0.5 | 0.1 | 0.01 |
| A | 0 | 0 | 0 | 0 |

4) There are five parameters to be optimized, and these can be put in a vector form.

$$\theta = [\text{rotation\_angle; translation\_x; translation\_y; scale\_x; scale\_y}] \qquad 6.1$$

These parameters are updated during iterations by

$$\theta_{k+1} = \theta_k + a_k g_k \qquad 6.2$$

where gradient approximation vector is

$$g_k=[(g_k)^1\ (g_k)^2\ (g_k)^3\ (g_k)^4\ (g_k)^5] \qquad 6.3$$

For the five parameters, gradient vector can be written as;

$$(g_k)^i = \frac{y(\theta_k + c_k\Delta_k) - y(\theta_k - c_k\Delta_k)}{2c_k(\Delta_{k1})^i} \quad \text{for i=1,2,3,4,5} \qquad 6.4$$

5) With using these equations SPSA algorithm is realized and five transformation parameters are updated at each iteration. A simple SPSA code given in figure 4.2 can be adapted easily for the five parameters. This code can be used only for a single level of the wavelet decomposition. In this work, a four level of wavelet decomposition has been used.

The normalized mutual information and normalized cross correlation are used to measure similarity of reference and input images. All of the pixels in the image are used to calculate the normalized cross correlation. It can be expressed as;

$$\text{NormalizedCrossCorr}=\frac{\sum\sum(A(x,y).B(x,y))}{\sqrt{\sum\sum A(x,y)}\sqrt{\sum\sum B(x,y)}} \qquad 6.5$$

Mutual information is another similarity metric used in this thesis. A histogram with 64 bins is used in the mutual information calculations, since it produces a significantly smoother mutual information surface than the 256-bin histogram. The smoother surface works better with an optimization algorithm, and the reduced number of bins dramatically improves the runtime for mutual information registration. (Cole-Rhodes A. A., Johnson K. L., Le Moigne J. & Zavorin I,.December 2003).

The joint histogram is obtained by pairing the (*a*,*b*) gray levels at the same location in both image and incrementing the corresponding element of an array for the joint histogram as indicated by

$$h_{AB}\left(\left[\frac{a}{4}\right],\left[\frac{b}{4}\right]\right)\rightarrow h_{AB}\left(\left[\frac{a}{4}\right],\left[\frac{b}{4}\right]\right)+1 \qquad 6.6$$

In this project normalized mutual information is calculated by

$$MutualInformation=\frac{H(A)+H(B)}{H(A,B)} \qquad 6.7$$

where $H(A)$ and $H(B)$ are the marginal entropies of $A$ and $B$ and $H(A,B)$ is their joint entropy. All entropies are computed from the histograms of $A$ and $B$, and their joint histogram. Normalized mutual information reaches maximum when the two images are geometrically aligned.

Because of its high speed, nearest neighbor interpolation has been used in the computations of the geometrical transformations. A flowchart for the implemented SPSA based registration method is given in figure 6.2.

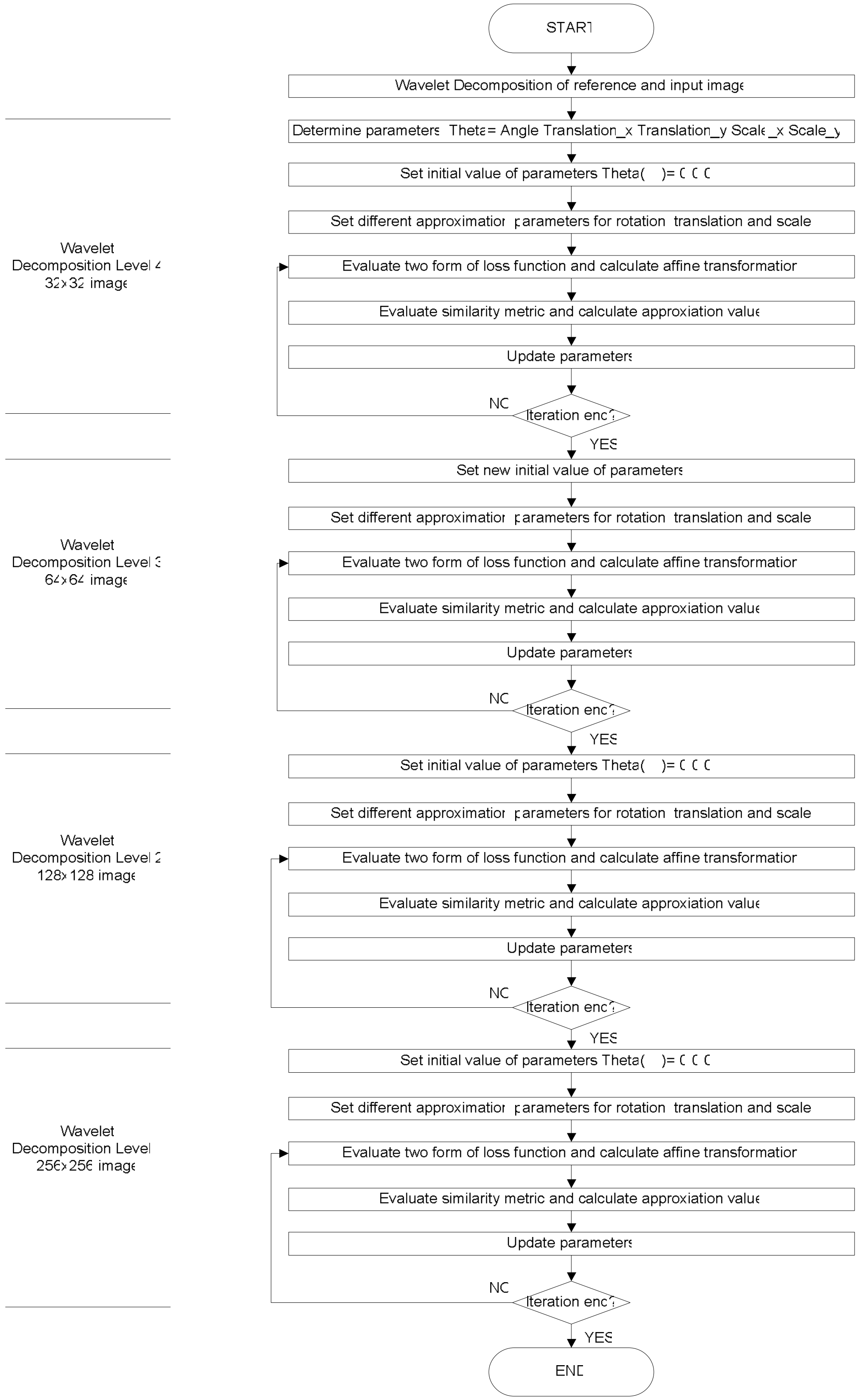



Figure 6.2 Flowchart for spsa algorithm

### 6.4 Image Registration with Genetic Algorithm

Genetic algorithm is another optimization method for registration. It is a robust method but suffers from high computational time. In this thesis, a multi resolution scheme integrated to the registration method overcomes this drawback. Our registration method can be summarized as follows:

1) Apply wavelet decomposition to reference and input images. Use LL component of the multi resolution scheme.

2) Upper and lower limits of the transformation parameters are determined. For example; [-90,90] for rotation, [-80,80] for translations and [0.7,1.5] for scales.

3) Determine values of the constant parameters for genetic algorithm. These parameters depend on the application in hand. High numbers of populations are generated at the lower image resolutions to increase the speed of the algorithm. Table 6.3 shows the values of the constants used in this work. These have been determined experimentally.

Table 6.3 Genetic algorithm parameter values

| | $4^{th}$ level of decomposition | $3^{rd}$ level of decomposition | $2^{nd}$ level of decomposition | $1^{st}$ level of decomposition |
|---|---|---|---|---|
| Number of generations | 5 | 5 | 10 | 10 |
| Population | 5000 | 2500 | 1000 | 100 |
| Crossover probability | 0.95 | 0.95 | 0.95 | 0.95 |
| Mutation probability | 0.01 | 0.01 | 0.01 | 0.01 |

4) A random initial population is created. Population members are represented by decimal numbers. An example for random population with two members is given in figure 6.3.

| | 1.member | 2.member |
|---|---|---|
| Rotation | 81.0233 | 47.1774 |
| Translation in x direction | -2.6886 | -0.4353 |
| Translation in y direction | 1.0684 | -4.8150 |
| Scale in x direction | 1.0888 | 1.3571 |
| Scale in y direction | 1.4130 | 1.0558 |

Figure 6.3 Example of population with two members

5) Evaluate fitness or similarity measure function. Then, select the best match and apply crossover and mutation if they are needed. Crossover operation takes the better parameter of one population member to another, to create more successful new generation. Mutation operation does some small changes in member to increase accuracy. For example; Figure 6.4 shows the population members in figure 6.3 after crossover and mutation operations. Crossover operation adjusts rotation values to 81.02, and then mutation operation adjusts it to 81.01 to obtain more effective result. In this example, we can see that these operations are realized for all transformation parameters. The roulette wheel selection method is used in our registration implementation because of its simple calculation. Then, new generation is created. This process is repeated until termination condition is satisfied. At the end of this process, some values are obtained for rotation, translation and scale parameters.

| | | |
|---|---|---|
| Rotation | 81.0100 | 81.0100 |
| Translation in x direction | -0.1000 | -0.1000 |
| Translation in y direction | -4.7000 | -4.7000 |
| Scale in x direction | 1.3000 | 1.3000 |
| Scale in y direction | 1.3000 | 1.3000 |

Figure 6.4 Population with two members

6) Step five generates a new center to continue searching. According to this new search center, limit of a new interval is determined. In a determined interval, new random population is created for next level of the wavelet decomposition. Then step 5 and step 6 is repeated to the first level of wavelet decomposition. Nearest neighbor interpolation has been used in the computations of the geometrical transformations. A flowchart for the implemented GA based registration method is given in figure 6.5. It also indicates the search intervals for the registration parameters used in this work.

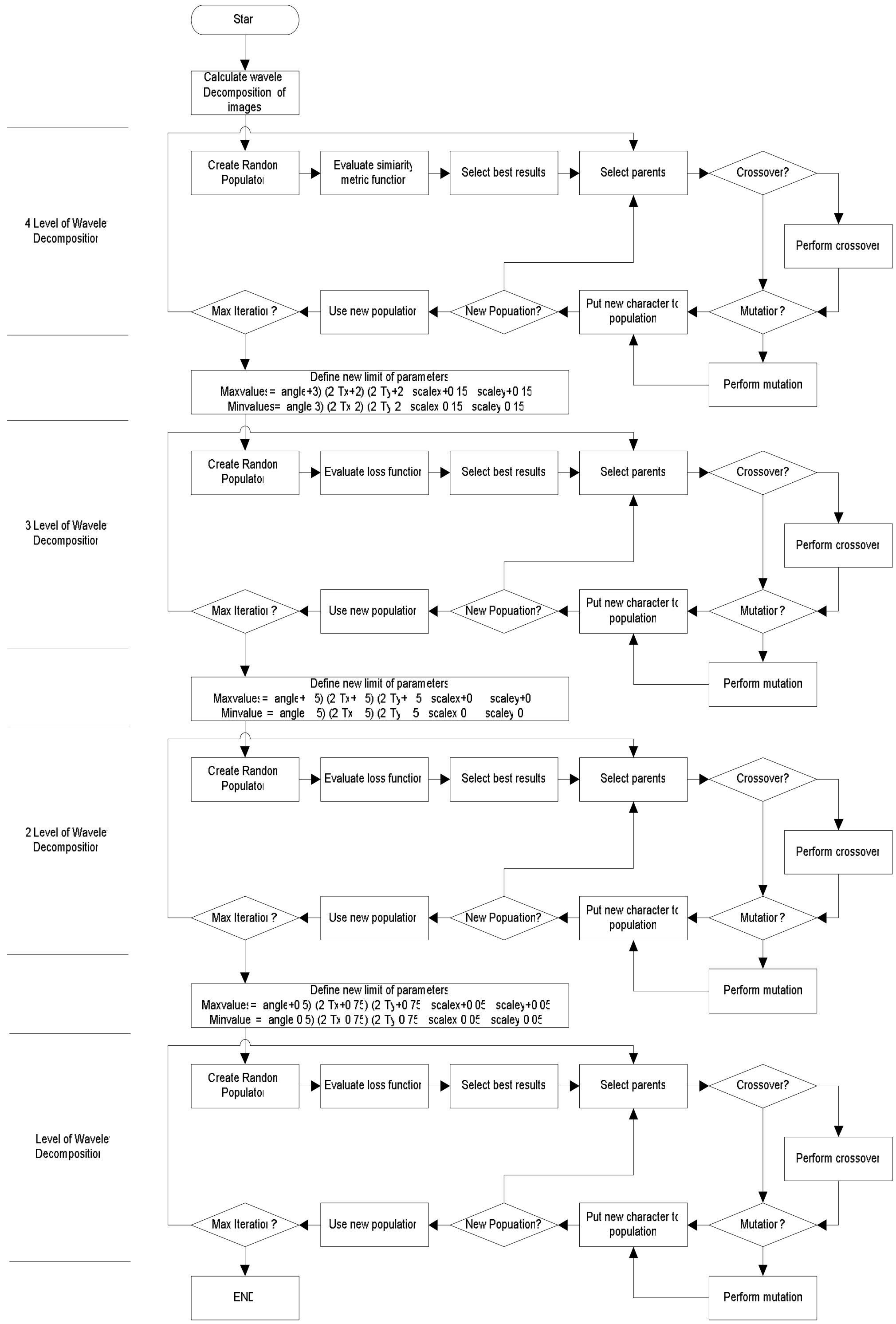



Figure 6.5 Flowchart of registration with using genetic algorithm

# CHAPTER SEVEN

# EXPERIMENTS AND RESULTS

## 7.1 Experimental Data Set

In this thesis, I used two different test images which are of 512x512 pixels. To measure the performances of the registration methods, I generated a geometrically transformed test data set from each test image by randomly rotating, translating and scaling it. Each test data set has 100 images with the known random transformation values. All software development and the performance tests are implemented in Matlab 7.1 environment. Some examples of the transformation values for the parameters from the data test are given in table 7.1.

Table 7.1 Example of test parameters

| Rotation | Translation in x dir. | Translation in y dir. | Scale in x dir. | Scale in y dir. |
|---|---|---|---|---|
| 0 | 46.2 | -27.5 | 1.1 | 1.13 |
| 3.5 | -21.6 | 34.6 | 1.06 | 1.11 |
| -5.6 | 43.5 | -12.561 | 0.95 | 1.09 |
| 8 | -34.12 | 71.5 | 0.95 | 1.05 |
| 9.9 | -23.46 | 43.2 | 1.1 | 1.13 |
| -11.54 | -23.6 | -47.70 | 1 | 1 |
| 14.4 | 56.3 | 6.75 | 1.19 | 1.3 |
| 16.4 | -15.4 | 59.7 | 0.79 | 1.31 |
| 22.2 | -24.65 | 35.63 | 0.8 | 1.35 |
| 22.24 | 24.65 | 35.6 | 1.44 | 1.37 |
| 23.4 | 54.3 | 45.3 | 1.01 | 1.22 |
| 23.4 | 54.37 | 45.3 | 0.86 | 0.92 |
| -28.5 | 65.8 | 75.5 | 1.35 | 1.28 |
| -32.57 | 24.63 | 15.78 | 1.02 | 0.96 |
| 35.2 | -24.5 | 65.3 | 1 | 1 |
| -35.2 | 24.5 | 65.3 | 1.24 | 1.28 |
| -40.2 | -24.65 | 35.6 | 0.99 | 1.05 |
| 43.5 | 45.3 | 23.5 | 1.16 | 0.94 |
| 51.5 | 46.87 | 34.64 | 0.92 | 1.09 |
| 58.34 | 65.3 | 34.6 | 1.27 | 1.13 |
| -58.5 | -50.76 | -42.5 | 1 | 1 |
| 62.98 | -56.2 | -5 | 0.99 | 1.28 |
| -64.5 | 0 | 72.5 | 1.09 | 1.26 |
| 79.47 | 45.7 | 13.83 | 1.24 | 1.28 |
| 81.565 | 45.3 | 79.1 | 1.23 | 0.823 |

The test images are a satellite image and well known lena image as shown in figure 7.1.

To measure the effect of noise on the performances of the methods implemented, I added 5db and 15 db zero mean Gaussian noise to the test images and repeated the tests. All tests were performed in the same system with Intel Centrino 1.6GHz Processor and 512MB memory.

(a) (b)

Figure 7.1 Original Test images, (a) test image 1, (b) test image 2

## 7.2 Results of Registration Methods

Table 7.1 and table 7.2 show the results of all methods. In the tables, average errors of each method for the transformation parameters are given. Time duration of any method is the same for both noiseless and noisy images. Average error of translation is in pixel, and rotation error in degree.

Table 7.2 Average errors of parameters for image 1

| TEST RESULTS FOR IMAGE 1 without NOISE | | | | | |
|---|---|---|---|---|---|
| | **Full Search Algorithm** | **SPSA** | | **Genetic Algorithm** | |
| | Norm. Cross Correlation | Mutual Information | Norm. Cross Correlation | Mutual Information | Norm. Cross Correlation |
| Rotation(deg) | 0.04 | 0.032 | 0.028 | 0.11 | 0.272 |
| Translation in x Direction(pixel) | 0.154 | 0.234 | 0.378 | 0.314 | 0.647 |
| Translation in y Direction(pixel) | 0.174 | 0.364 | 1.43 | 0.52 | 0.745 |
| Scale in x Direction | 0.003 | 0.0008 | 0.0031 | 0.0036 | 0.003 |
| Scale in y Direction | 0.005 | 0.001 | 0.0041 | 0.0001 | 0.001 |
| TEST RESULTS FOR IMAGE 1 With SNR=5db | | | | | |
| | **Full Search Algorithm** | **SPSA** | | **Genetic Algorithm** | |
| Parameter | Norm. Cross Correlation | Mutual Information | Norm. Cross Correlation | Mutual Information | Norm. Cross Correlation |
| Rotation(deg) | 0.179 | 0.076 | 0.129 | 0.110 | 0.435 |
| Translation in x Direction(pixel) | 0.246 | 0.297 | 2.105 | 0.468 | 0.95 |
| Translation in y Direction(pixel) | 0.312 | 0.446 | 2.682 | 0.763 | 1.12 |
| Scale in x Direction | 0.0054 | 0.0012 | 0.008 | 0.0037 | 0.005 |
| Scale in y Direction | 0.0062 | 0.0032 | 0.0079 | 0.001 | 0.007 |
| TEST RESULTS FOR IMAGE 1 With SNR=15db | | | | | |
| | **Full Search Algorithm** | **SPSA** | | **Genetic Algorithm** | |
| Parameter | Norm. Cross Correlation | Mutual Information | Norm. Cross Correlation | Mutual Information | Norm. Cross Correlation |
| Rotation(deg) | 0.105 | 0.0157 | 0.101 | 0.11 | 0.384 |
| Translation in x Direction(pixel) | 0.189 | 0.248 | 0.893 | 0.38 | 0.870 |
| Translation in y Direction(pixel) | 0.231 | 0.394 | 1.67 | 0.67 | 0.946 |
| Scale in x Direction | 0.0036 | 0.001 | 0.0035 | 0.003 | 0.0035 |
| Scale in y Direction | 0.0055 | 0.001 | 0.0063 | 0.001 | 0.0063 |
| Time Duration | 10000sec | 250sec | 250sec | 1600sec | 1600sec |

Table 7.3 Average errors of parameters for image 2

| TEST RESULTS FOR ORIGINAL IMAGE 2 without NOISE | | | | | |
|---|---|---|---|---|---|
| | **Full Search Algorithm** | **SPSA** | | **Genetic Algorithm** | |
| Parameter | Norm. Cross Correlation | Mutual Information | Norm. Cross Correlation | Mutual Information | Norm. Cross Correlation |
| Rotation(deg) | 0.079 | 0.018 | 0.076 | 0.172 | 0.397 |
| Translation in x Direction(pixel) | 0.236 | 0.189 | 0.598 | 0.420 | 0.743 |
| Translation in y Direction(pixel) | 0.13 | 0.246 | 1.548 | 0.64 | 0.844 |
| Scale in x Direction | 0.01 | 0.0006 | 0.0026 | 0.004 | 0.003 |
| Scale in y Direction | 0 | 0.0006 | 0.005 | 0.0001 | 0.001 |
| TES T RESULTS FOR IMAGE 2 With SNR=5db | | | | | |
| | **Full Search Algorithm** | **SPSA** | | **Genetic Algorithm** | |
| Parameter | Norm. Cross Correlation | Mutual Information | Norm. Cross Correlation | Mutual Information | Norm. Cross Correlation |
| Rotation(deg) | 0.14 | 0.051 | 0.204 | 0.22 | 0.45 |
| Translation in x Direction(pixel) | 0.438 | 0.388 | 1.319 | 0.67 | 0.839 |
| Translation in y Direction(pixel) | 0.354 | 0.682 | 2.035 | 1.2 | 1.37 |
| Scale in x Direction | 0.02 | 0.0016 | 0.0101 | 0.0046 | 0.005 |
| Scale in y Direction | 0.01 | 0.002 | 0.0115 | 0.002 | 0.002 |
| TEST RESULTS FOR IMAGE 2 With SNR=15db | | | | | |
| | **Full Search Algorithm** | **SPSA** | | **Genetic Algorithm** | |
| Parameter | Norm. Cross Correlation | Mutual Information | Norm. Cross Correlation | Mutual Information | Norm. Cross Correlation |
| Rotation(deg) | 0.098 | 0.0508 | 0.128 | 0.194 | 0.435 |
| Translation in x Direction(pixel) | 0.3 | 0.359 | 0.941 | 0.581 | 0.81 |
| Translation in y Direction(pixel) | 0.21 | 0.731 | 2.74 | 0.919 | 1.05 |
| Scale in x Direction | 0.014 | 0.0017 | 0.0095 | 0.004 | 0.0045 |
| Scale in y Direction | 0.004 | 0.0026 | 0.0089 | 0.002 | 0.0018 |
| Time Duration | 10000sec | 250sec | 250sec | 1600sec | 1600sec |

### *7.2.1 Full Search Algorithm Results*

In this section, we can examine the performance of the method for each parameter. The algorithm generates effective results for all parameters as expected.

Figures between 7.2 and 7.5 give the average error versus signal to noise (SNR) ratio for all parameters, obtained from the test images image 1 and image 2. Average error of translation is in pixel, and rotation error in degree.

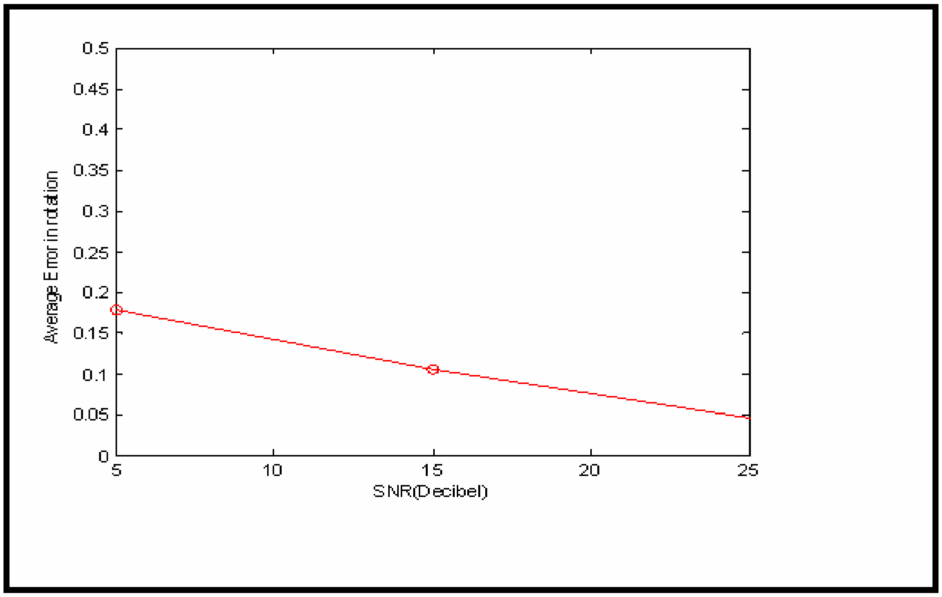



Figure 7.2 Average errors in rotation for full search algorithm of image 1.

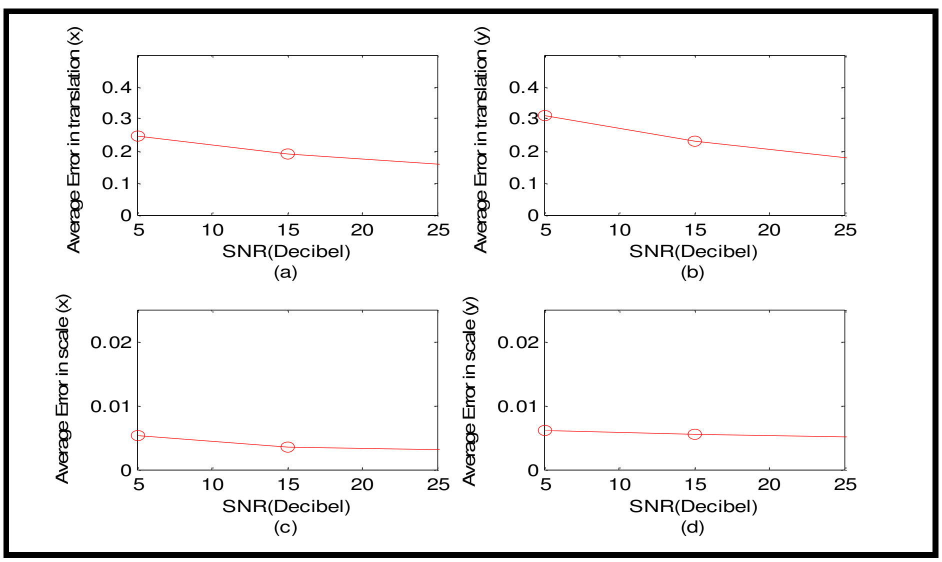



Figure 7.3 Average errors in translation and scales for full search algorithm of image 1, (a) for translation in x, (b) translation in y, (c) scale in x, (d) scale in y.

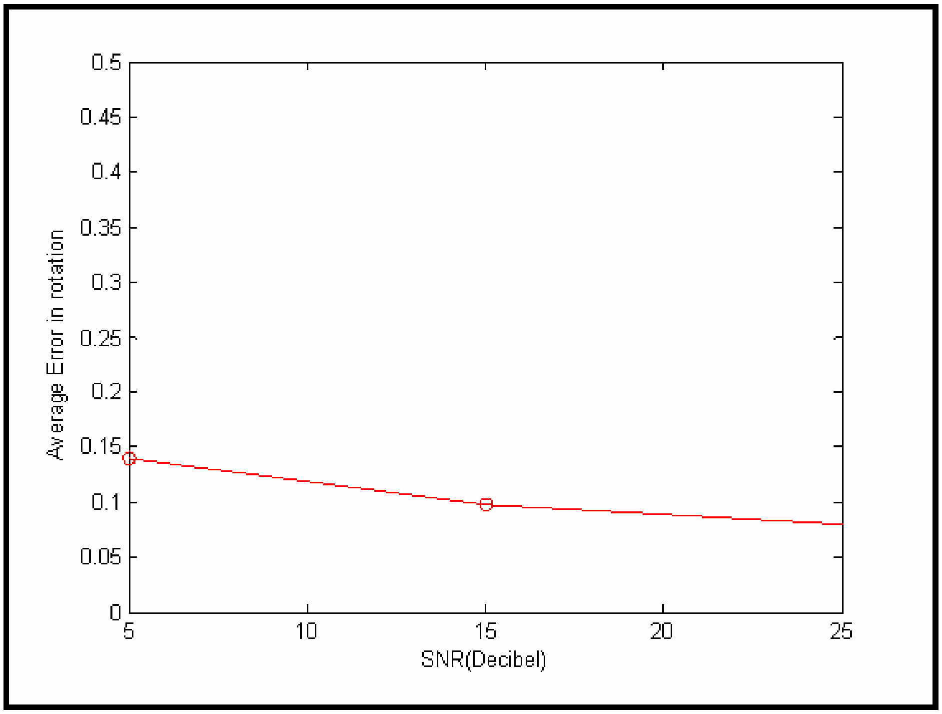



Figure 7.4 Average errors in rotation for full search algorithm of image 2.

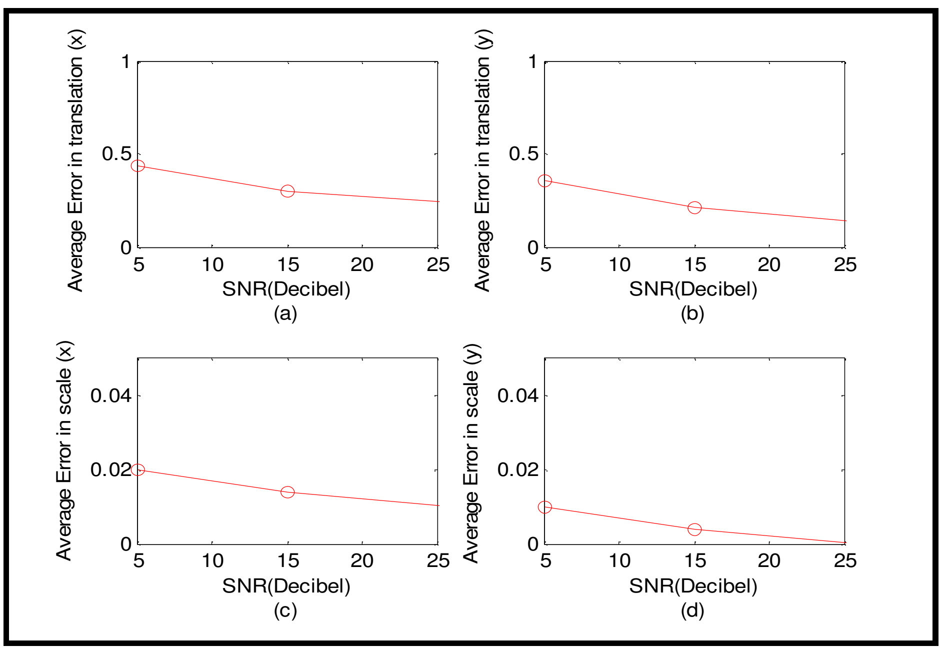



Figure 7.5 Average errors in translation and scales for full search algorithm of image 2, (a) for translation in x, (b) translation in y, (c) scale in x, (d) scale in y.

### *7.2.2 SPSA Results*

In this section, SPSA results are given for both normalized mutual information and normalized cross-correlation. When we examine the results, we can see that SPSA with using mutual information gives more accurate and robust results than the correlation based method. The correlation based method can estimate, especially, rotation and scale changes between images with a lower accuracy due to a wide correlation peak at the best match locations. This causes ambiguity in locating the maximum point of the peak and therefore increases the average error. Noise reduces the performance of correlation based method clearly, but mutual information gives better results in the presence of noise. Figures between 7.6 and 7.15 give the average error versus signal to noise (SNR) ratio for all parameters. Average error of translation is in pixel, and rotation error in degree. Figure 7.16 and figure 7.17 shows mutual information and cross-correlation values for different wavelet decomposition levels. In these figures, blue and red plot is seen. Blue represents high value of loss function and red represent low value of loss function in same iteration. Difference between these two parameters generates update values for transformation parameters.

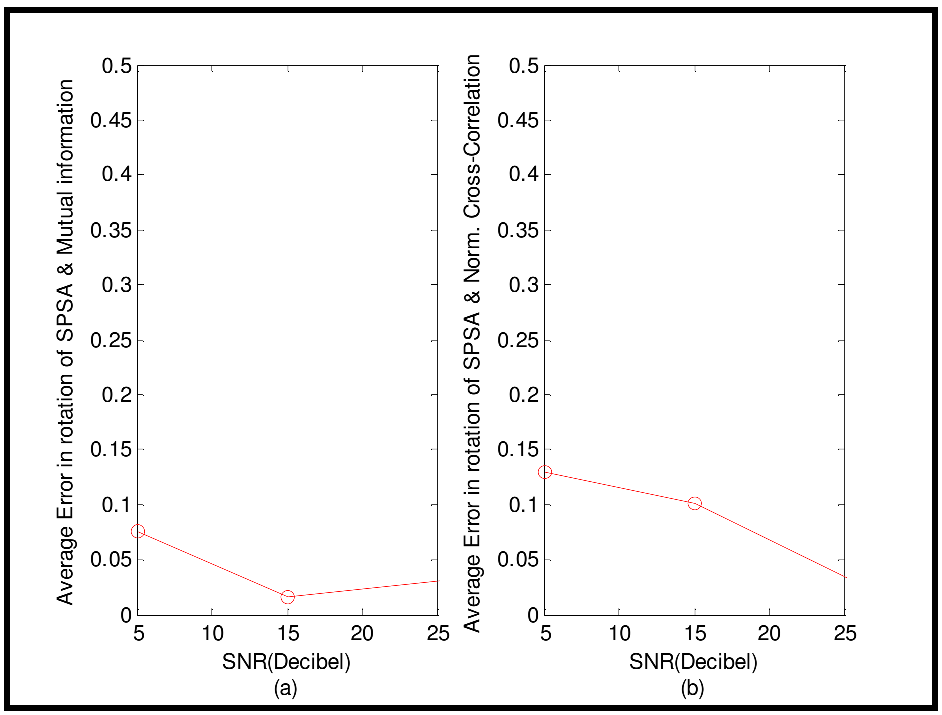



Figure 7.6 Average errors in rotation of spsa algorithm for image 1, (a) with using mutual information, (b) with using normalized cross-correlation

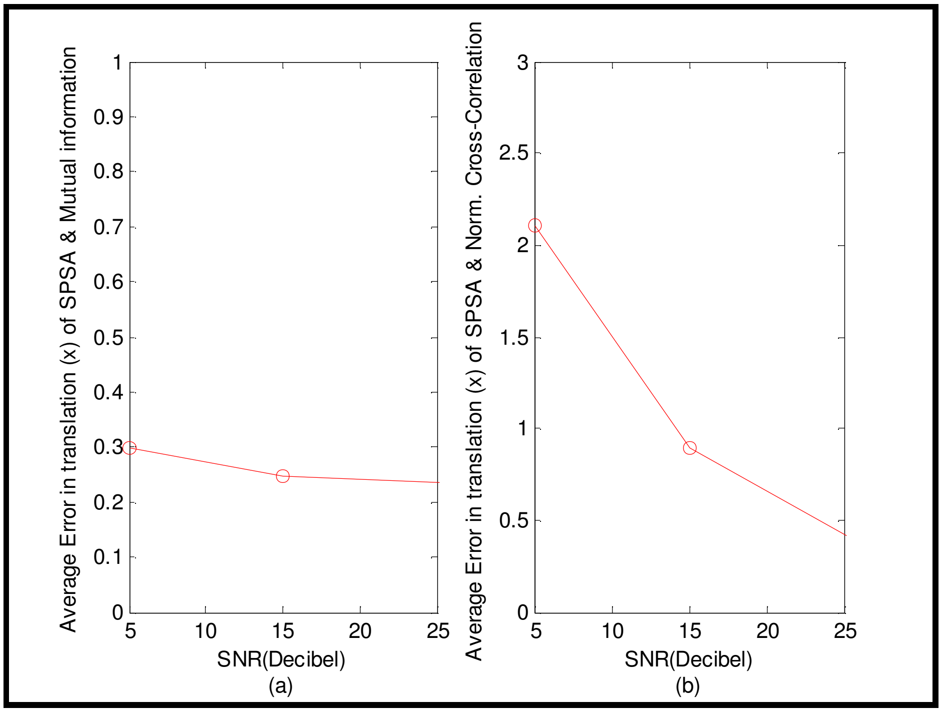



Figure 7.7 Average errors in translation x direction of spsa algorithm for image 1, (a) with using mutual information, (b) with using normalized cross-correlation

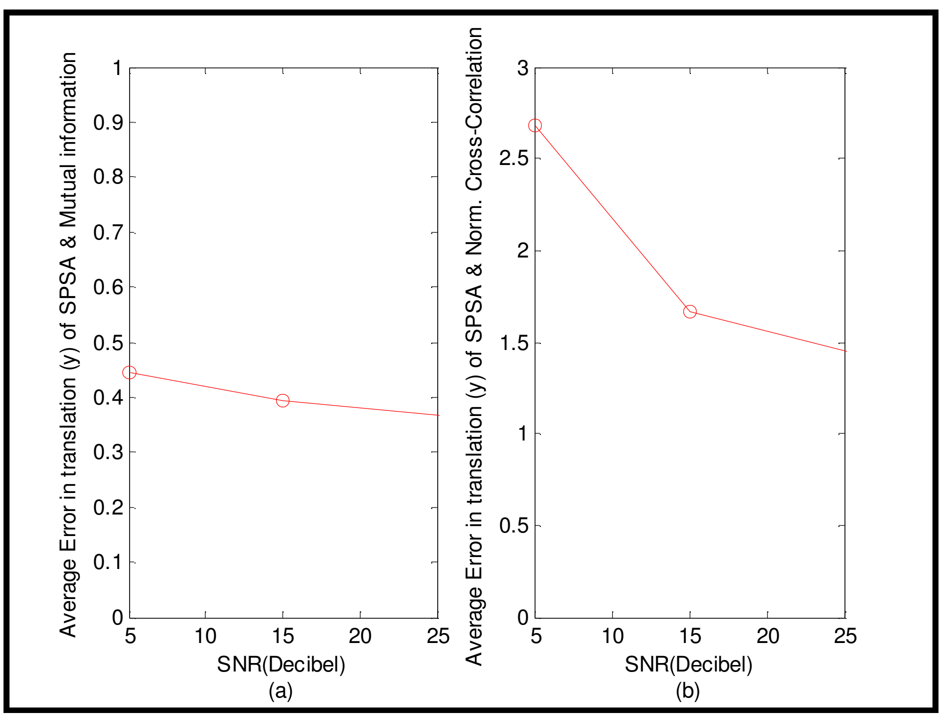



Figure 7.8 Average errors in translation y direction of spsa algorithm for image 1, (a) with Using mutual information, (b) with using normalized cross-correlation

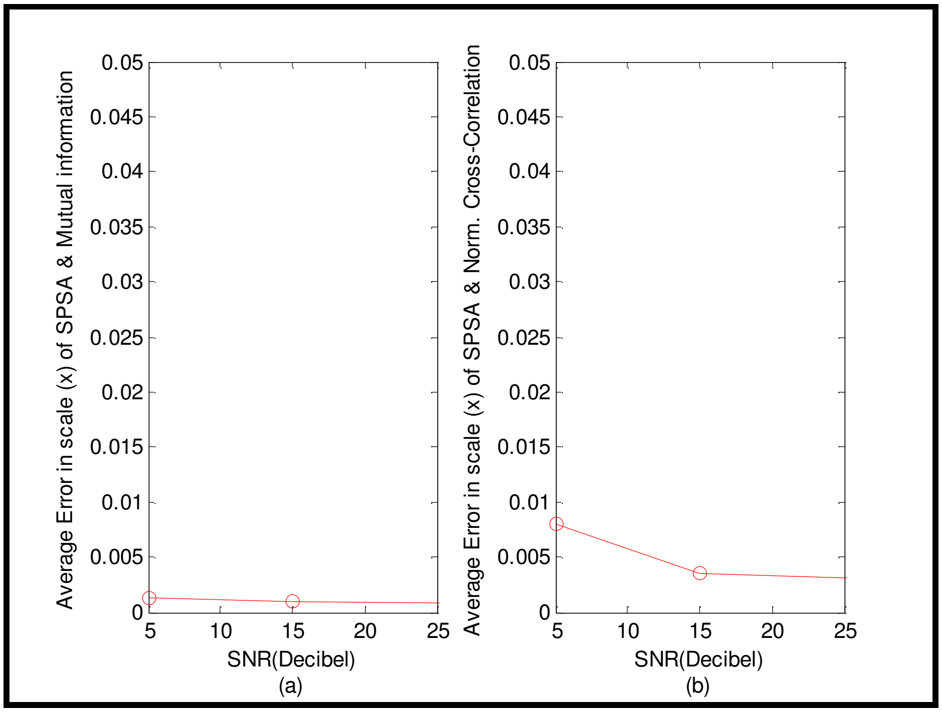



Figure 7.9 Average errors in scale x direction of spsa algorithm for image 1,(a) with using mutual information, (b) with using normalized cross-correlation

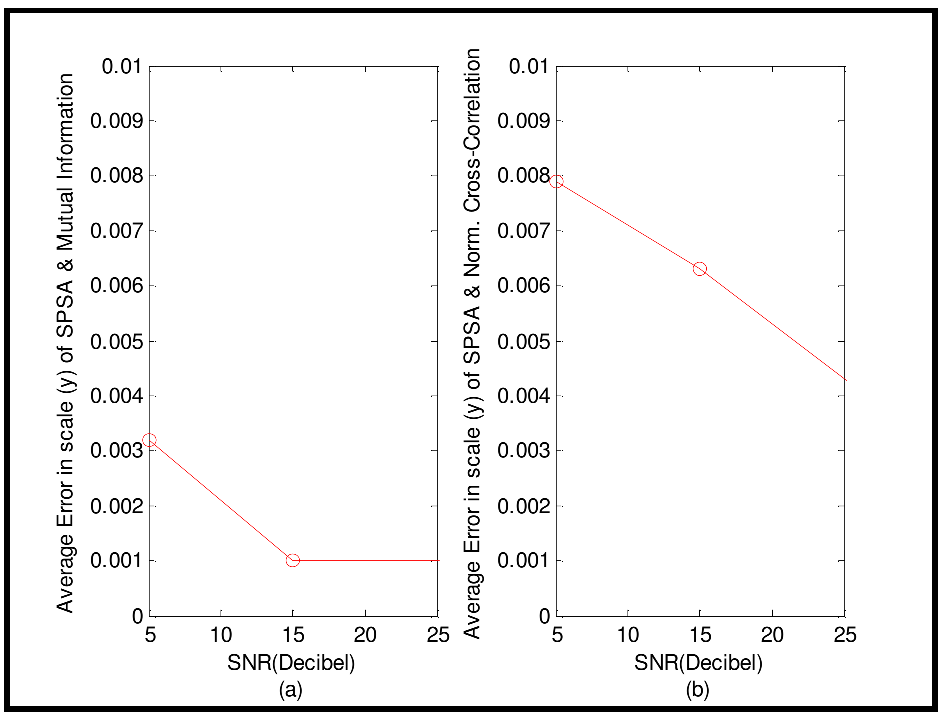



Figure 7.10 Average errors in scale y direction of spsa algorithm for image 1, (a) with using mutual information, (b) with using normalized cross-correlation

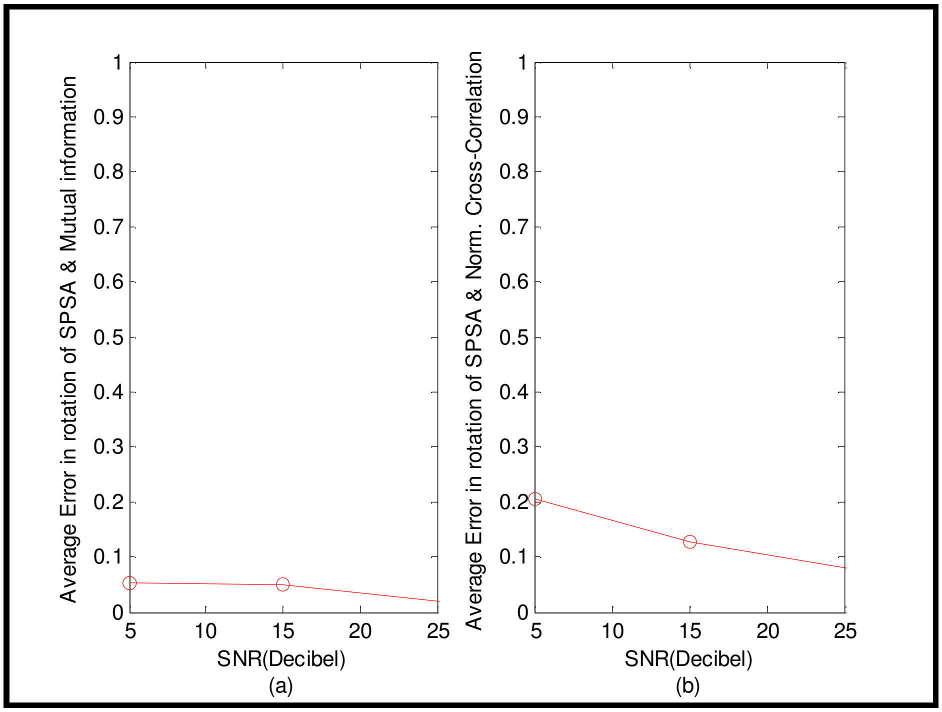



Figure 7.11 Average errors in rotation of spsa algorithm for image 2, (a) with using mutual Information, (b) with using normalized cross-correlation

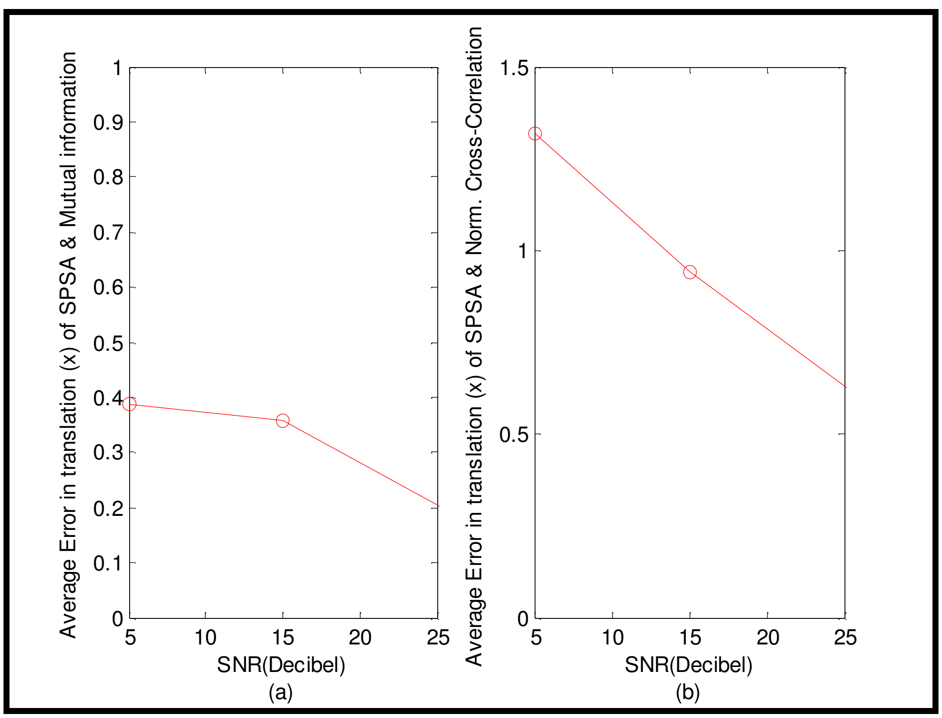



Figure 7.12 Average errors in translation x direction of spsa algorithm for image 2, (a) with using mutual information, (b) with using normalized cross-correlation

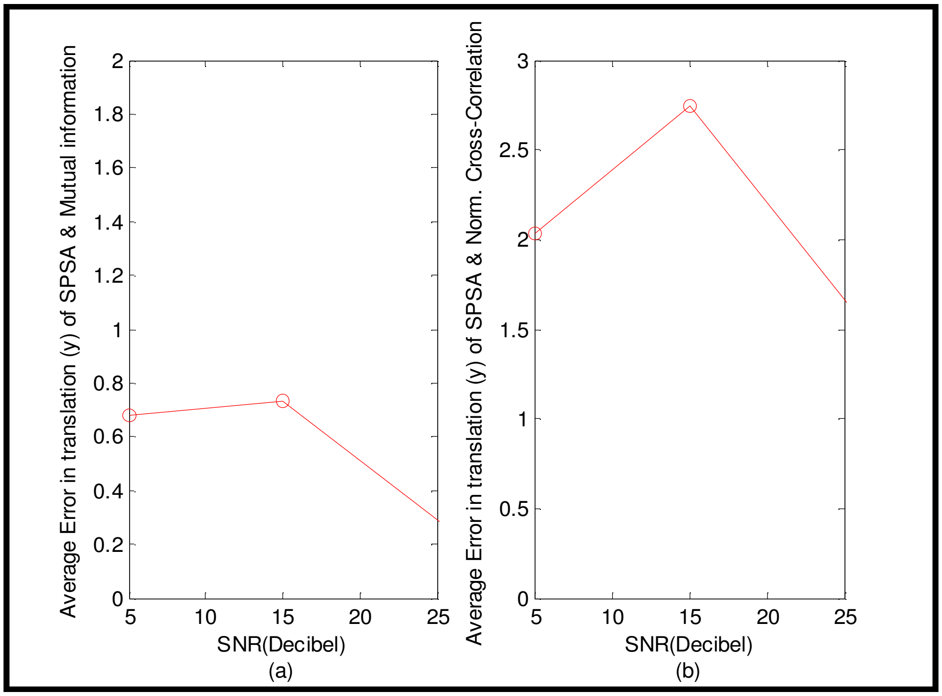



Figure 7.13 Average errors in translation y direction of spsa algorithm for image 2, (a) with using mutual information, (b) with using normalized cross-correlation

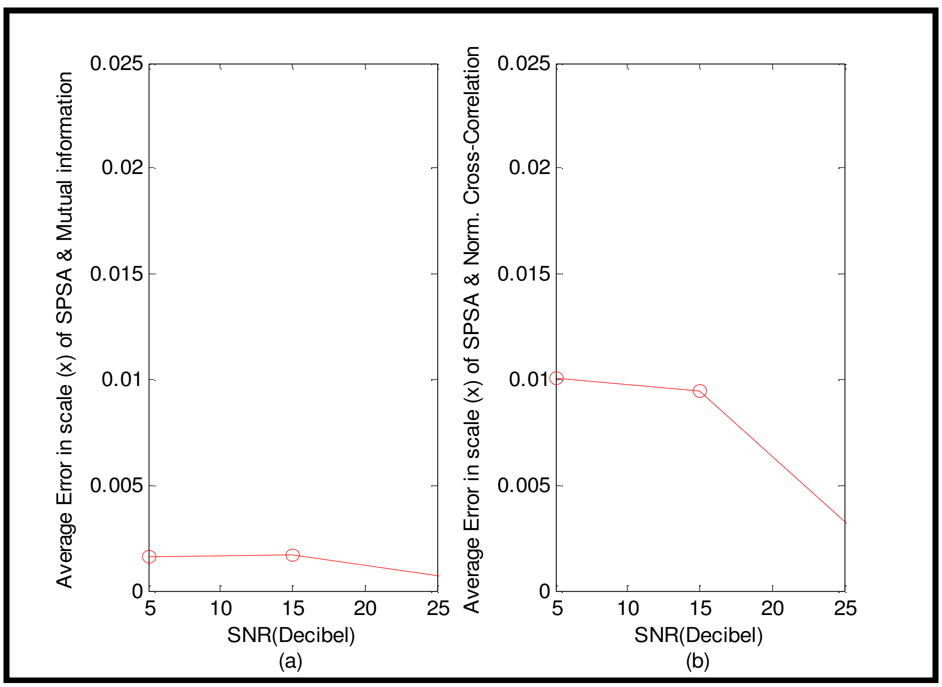



Figure 7.14 Average errors in scale x direction of spsa algorithm for image 2, (a) with using mutual information, (b) with using normalized cross-correlation

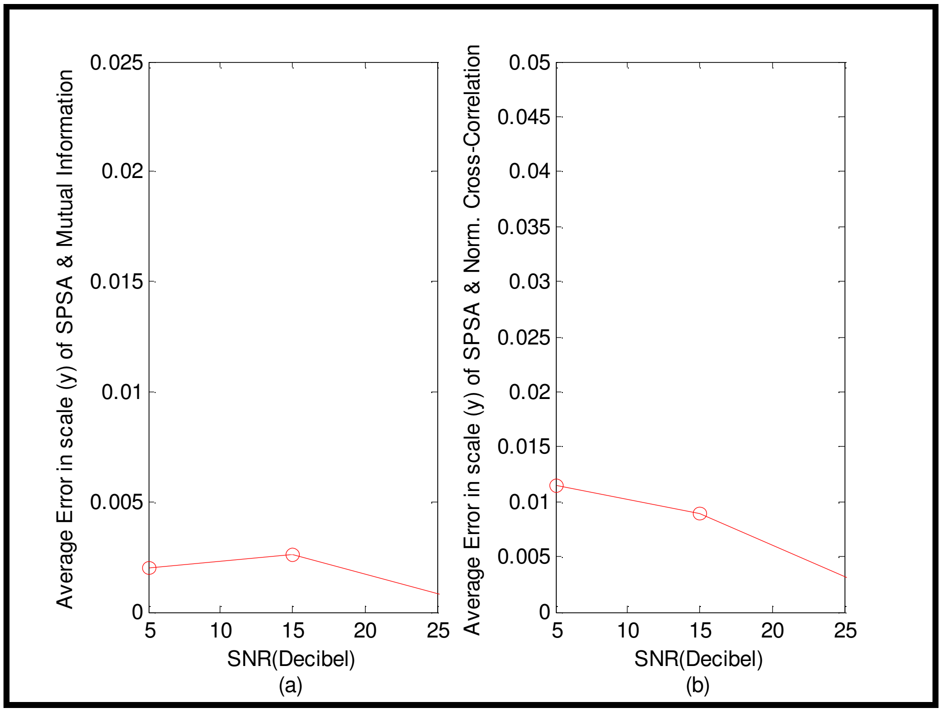



Figure 7.15 Average errors in scale y direction of spsa algorithm for image 2, (a) with using mutual information, (b) with using normalized cross-correlation

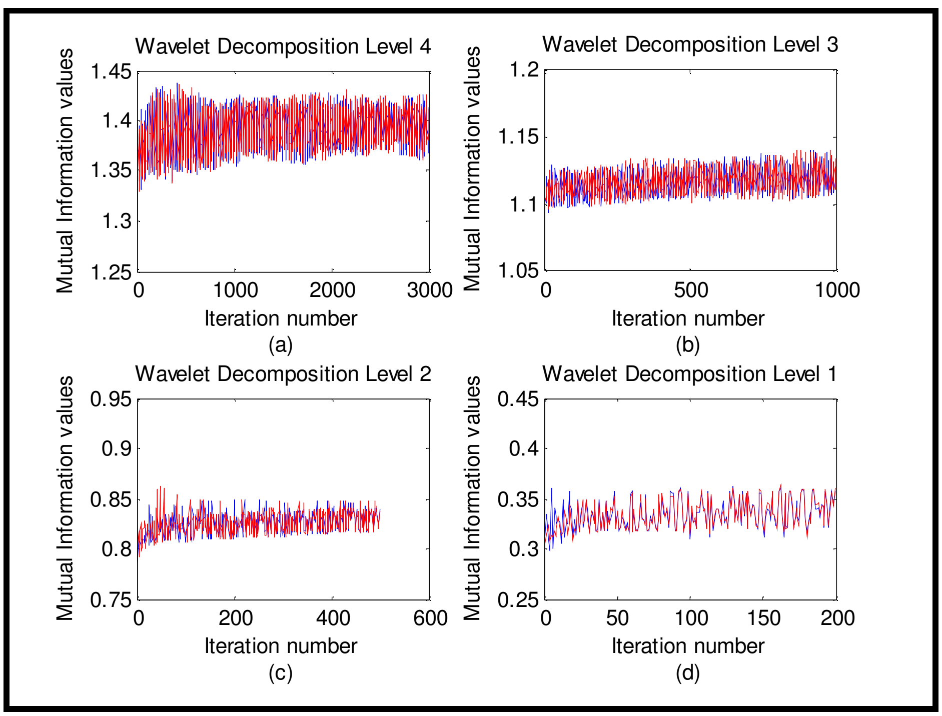



Figure 7.16 Mutual information values of spsa algorithm for wavelet decomposition levels (a) for level 4, (b) for level 3, (c) for level 2, (d) for level 1

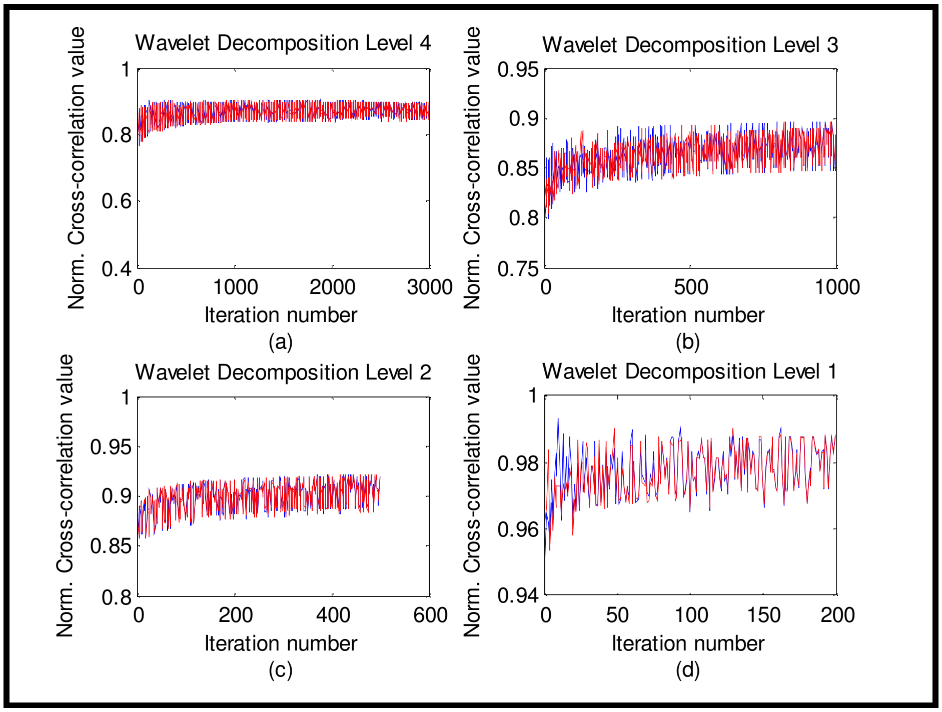



Figure 7.17 Normalized cross-correlation values of spsa algorithm for wavelet decomposition levels (a) for level 4 (b) for level 3 (c) for level 2 (d) for level 1

SPSA exhibit different behaviors depending on, the used similarity metric, whether it is mutual information or cross correlation. We can examine the responses of SPSA while finding transformation parameters by an example. In the example, transformation parameters for a test image are rotation=-40$^{0}$, translation in x direction= -30 pixels, translation in y direction=50 pixels, scale in x direction=0.9 and scale in y direction=1.15. Figures between 7.18 and 7.27, describe how the SPSA algorithm converges to the parameters. The algorithm converges to estimated rotation and scale values in a similar way for both mutual information and correlation, but, to estimated translations in a dissimilar way. Rotation and scale converges with monotonically but translation values have different characteristic.

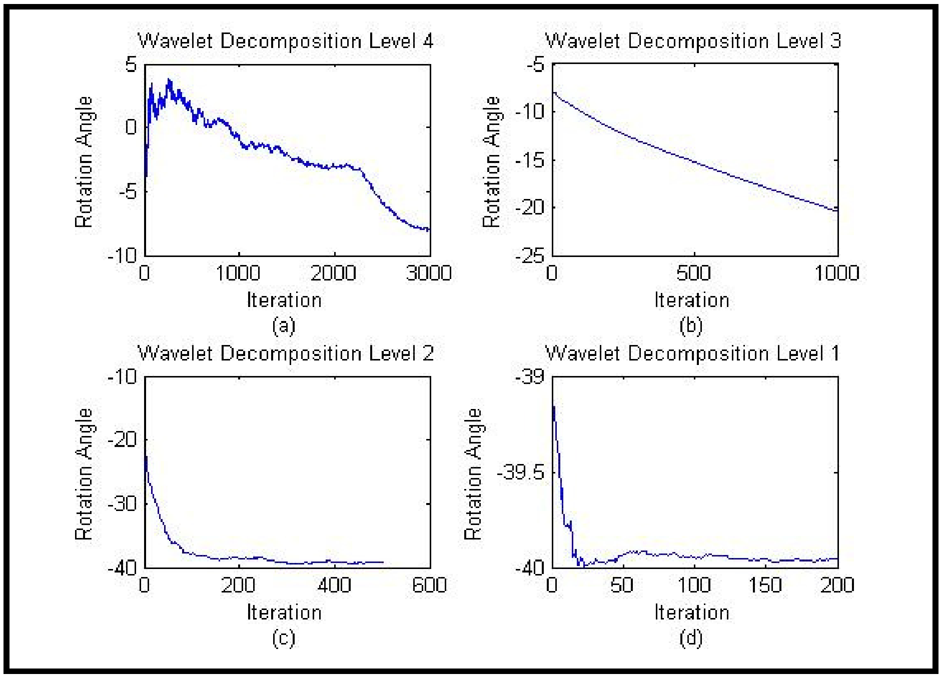



Figure 7.18 SPSA approximations for rotation with using mutual information rotation angle versus iteration numbers, (a) at wavelet decomposition level 4, (b) at wavelet decomposition level 3, (c) at wavelet decomposition level 2 and (d) level 1

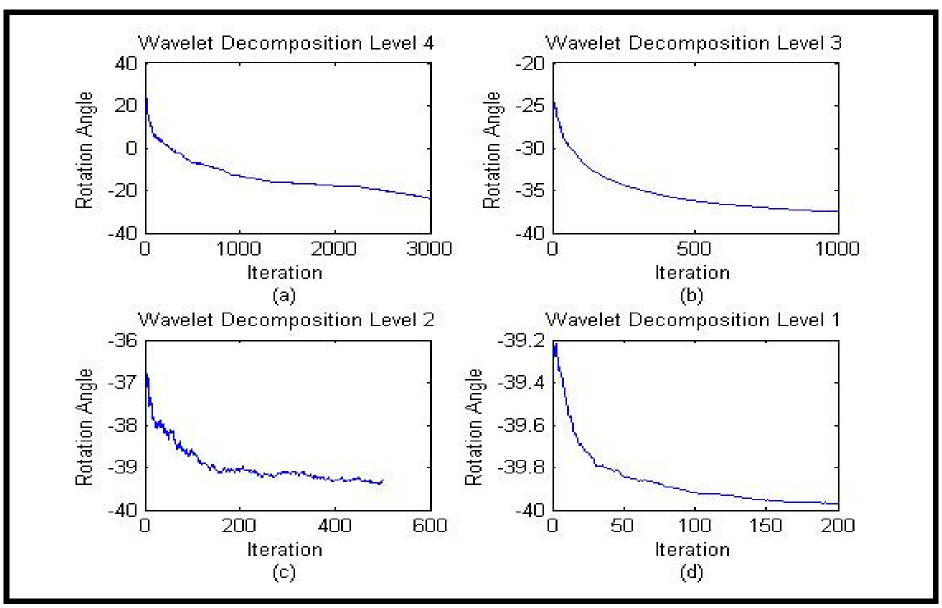



Figure 7.19 SPSA approximations for rotation with using normalized cross correlation, rotation angle versus iteration numbers, (a) at wavelet decomposition level 4, (b) at wavelet decomposition level 3, (c) at wavelet decomposition level 2 and (d) level 1

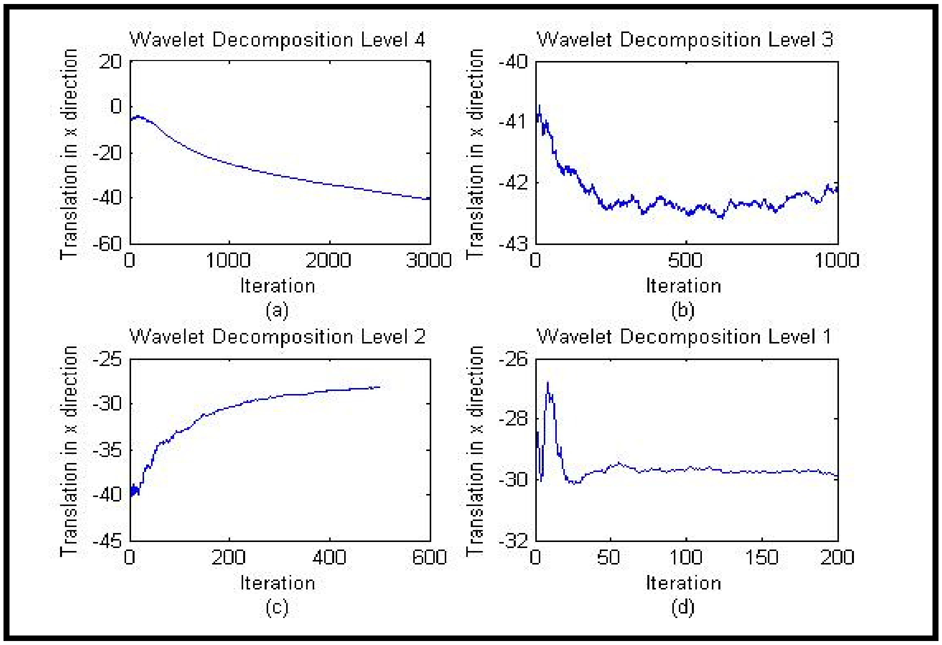



Figure 7.20 SPSA approximations for translation in x direction with using mutual information, translation versus iteration numbers, (a) at wavelet decomposition level 4, (b) at wavelet decomposition level 3, (c) at wavelet decomposition level 2 and (d) level 1

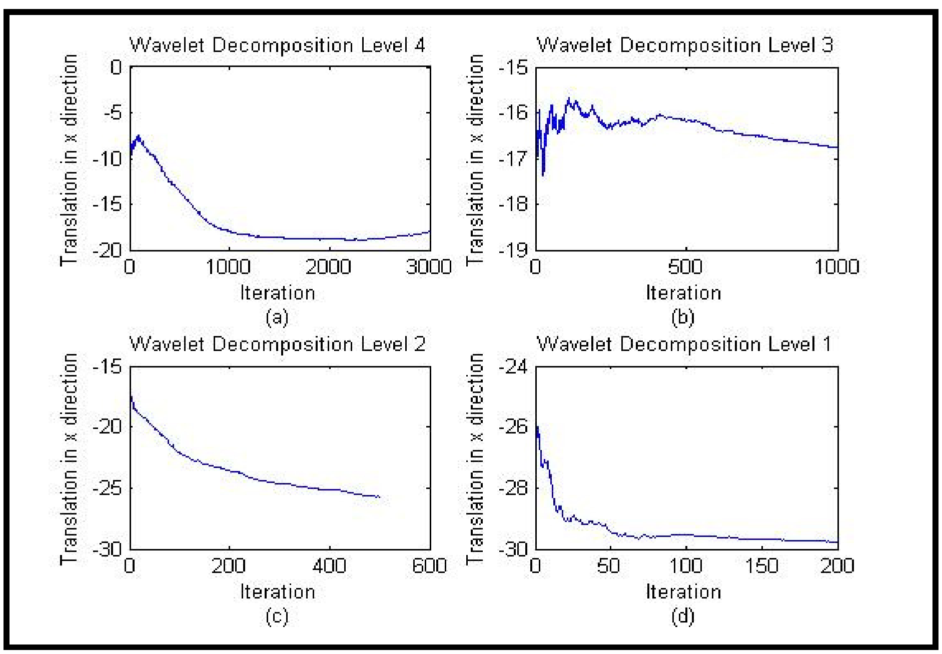



Figure 7.21 SPSA approximations for translation in x direction with using normalized cross correlation, translation versus iteration numbers, (a) at wavelet decomposition level 4, (b) at wavelet decomposition level 3, (c) at wavelet decomposition level 2 and (d) level 1

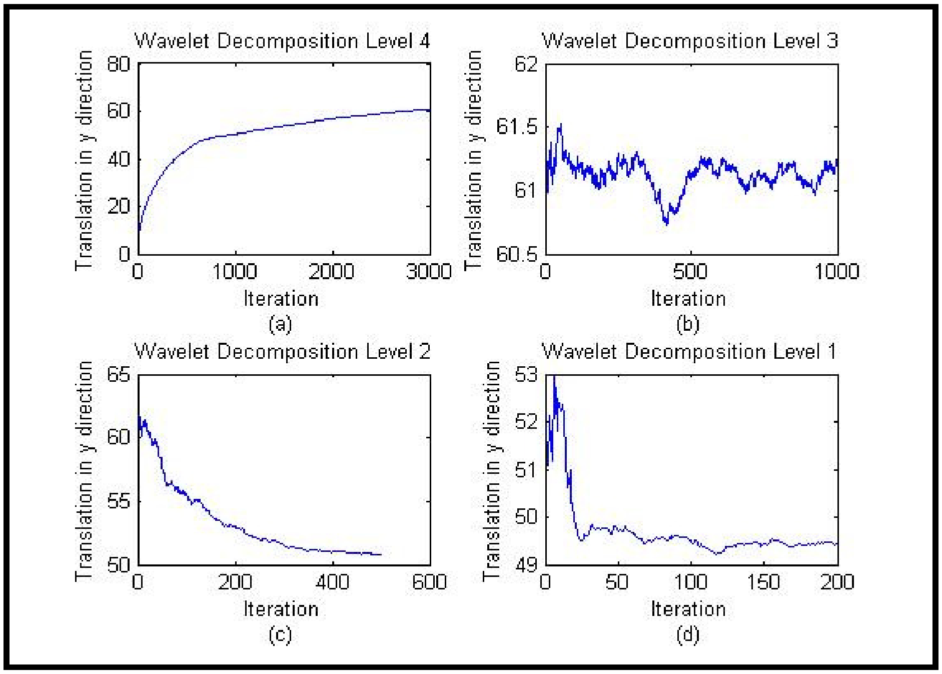



Figure 7.22 SPSA approximations for translation in y direction with using mutual information, translation versus iteration numbers, (a) at wavelet decomposition level 4, (b) at wavelet decomposition level 3, (c) at wavelet decomposition level 2 and (d) level 1

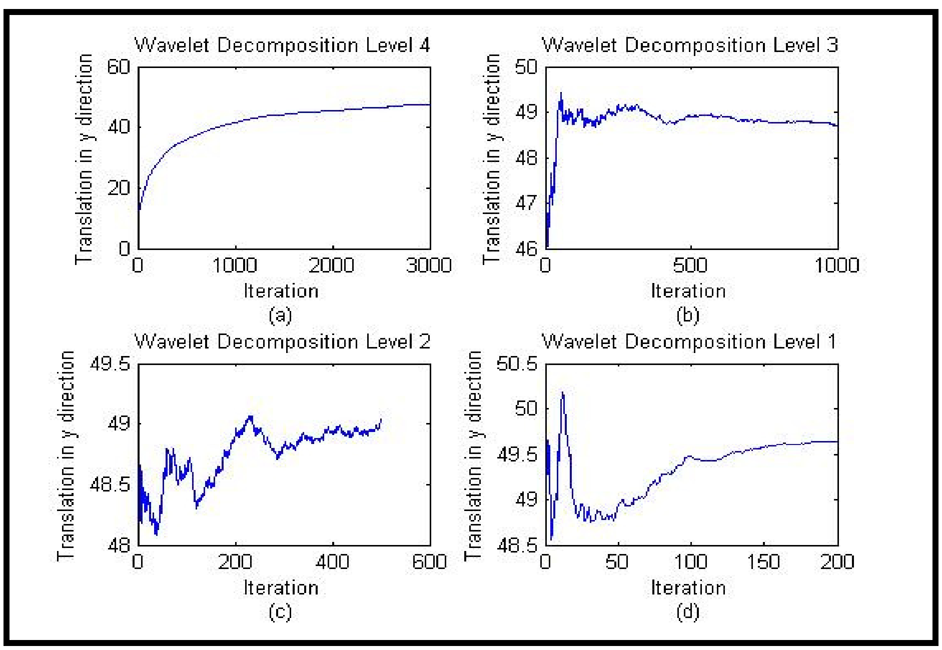



Figure 7.23 SPSA approximations for translation in y direction with using normalized cross correlation, translation versus iteration numbers, (a) at wavelet decomposition level 4, (b) at wavelet decomposition level 3, (c) at wavelet decomposition level 2 and (d) level 1

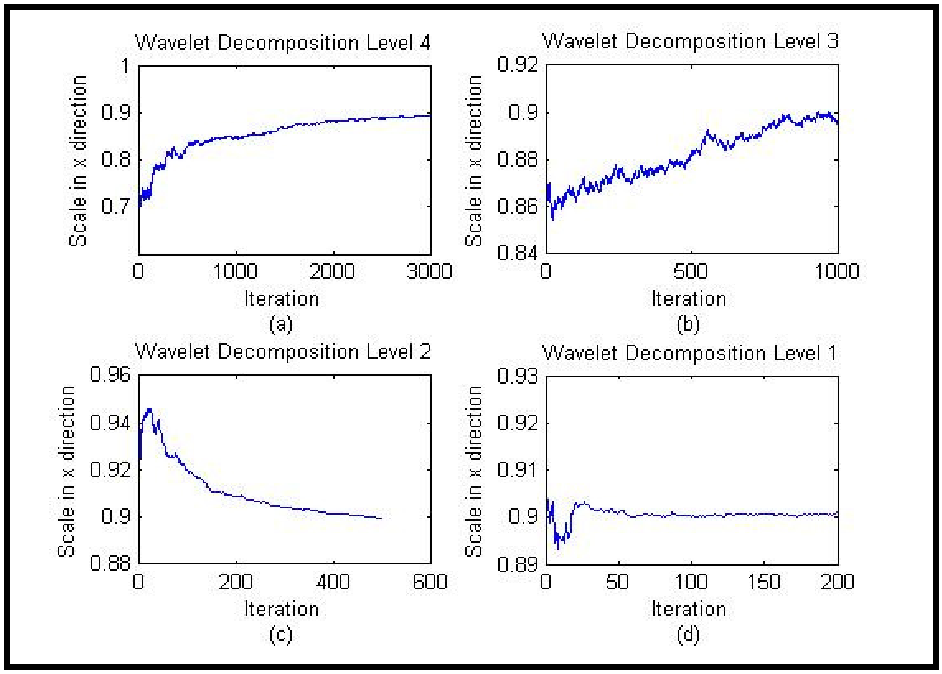



Figure 7.24 SPSA approximations for scale in x direction with using mutual information, scale versus iteration numbers,(a)at wavelet decomposition level 4,(b)at wavelet decomposition level 3, (c) at wavelet decomposition level 2, (d) at wavelet decomposition level 1

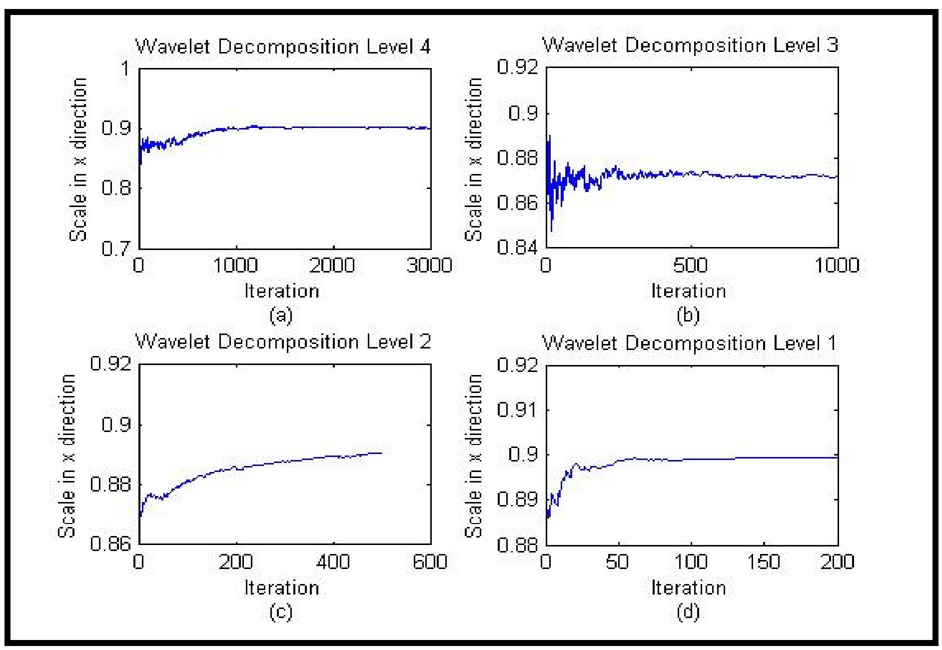



Figure 7.25 SPSA approximations for scale in x direction with using normalized cross correlation, scale versus iteration numbers (a) at wavelet decomposition level 4, (b) at wavelet decomposition level 3, (c) at wavelet decomposition level 2, (d) at wavelet decomposition level 1

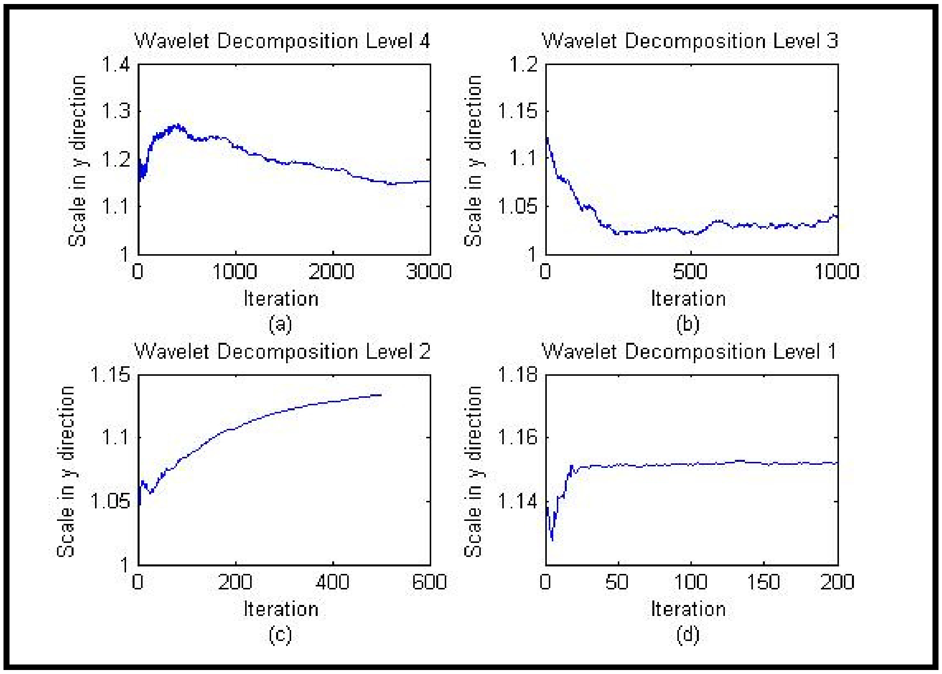



Figure 7.26 SPSA approximations for scale in y direction with using mutual information, scale versus iteration numbers, (a) at wavelet decomposition level 4, (b) at wavelet decomposition level 3, (c) at wavelet decomposition level 2, (d) at wavelet decomposition level 1

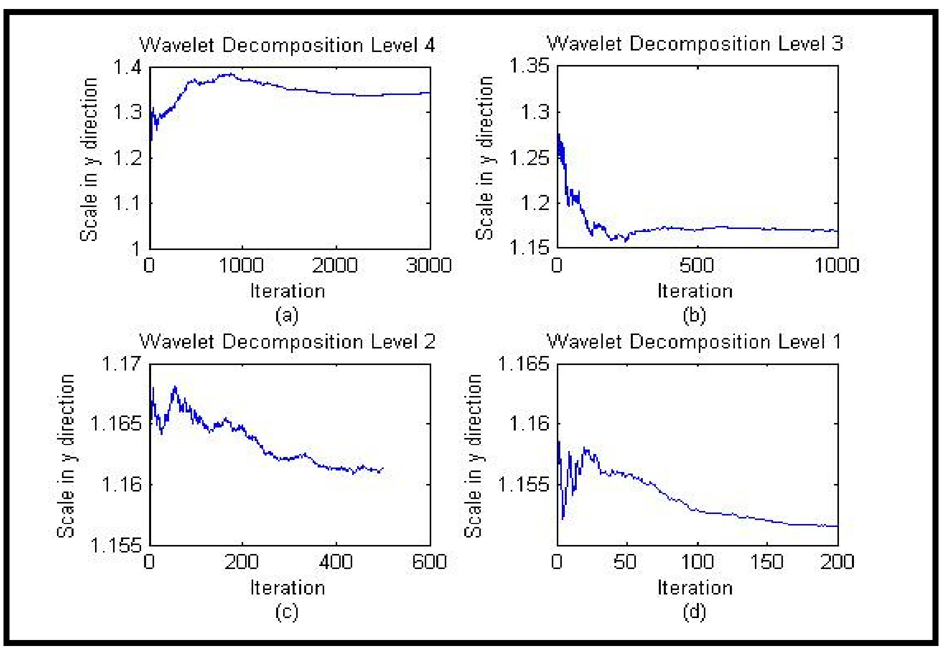



Figure 7.27 SPSA approximations for scale in y direction with using normalized cross correlation, scale versus iteration numbers, (a) at wavelet decomposition level 4,
(b) at wavelet decomposition level 3, (c) at wavelet decomposition level 2,
(d) at wavelet decomposition level 1

### *7.2.3 GA Results*

In this section, genetic algorithm (GA) results are given for both normalized mutual information and normalized cross-correlation. When we examine the results, we can see that GA with using mutual information gives more accurate and robust results than correlation based method. We can notice that noise reduces the performance of the correlation based method considerably, and mutual information based method gives better results in the presence of noise.

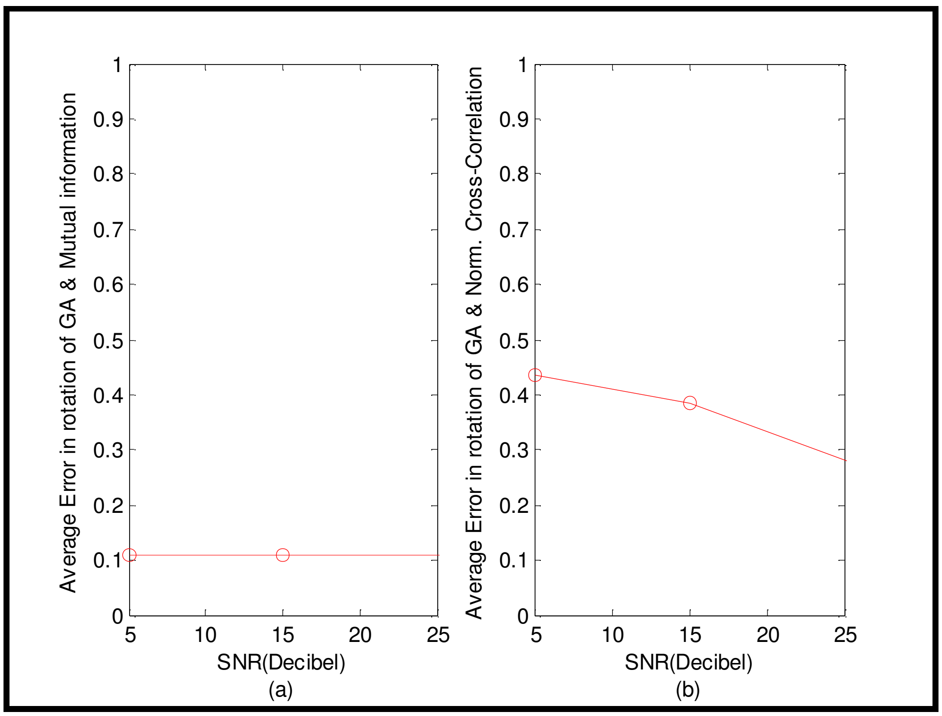



Figure 7.28 Average errors in rotation of genetic algorithm for image 1, (a) with using mutual Information, (b) with using normalized cross-correlation

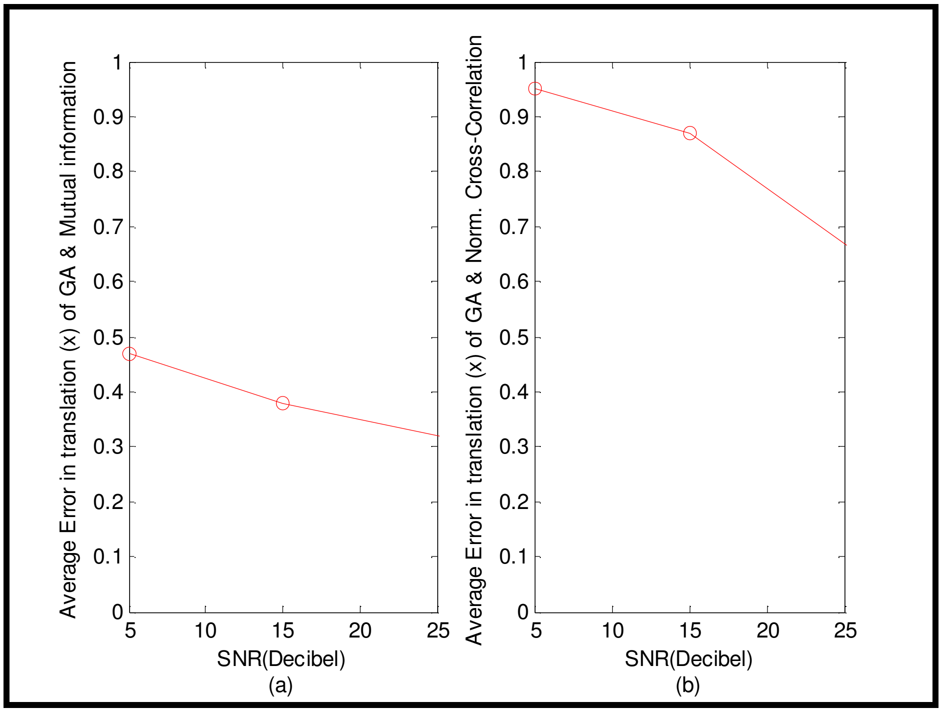



Figure 7.29 Average errors in translation x direction of genetic algorithm for image 1, (a)with using mutual information, (b) with using normalized cross-correlation

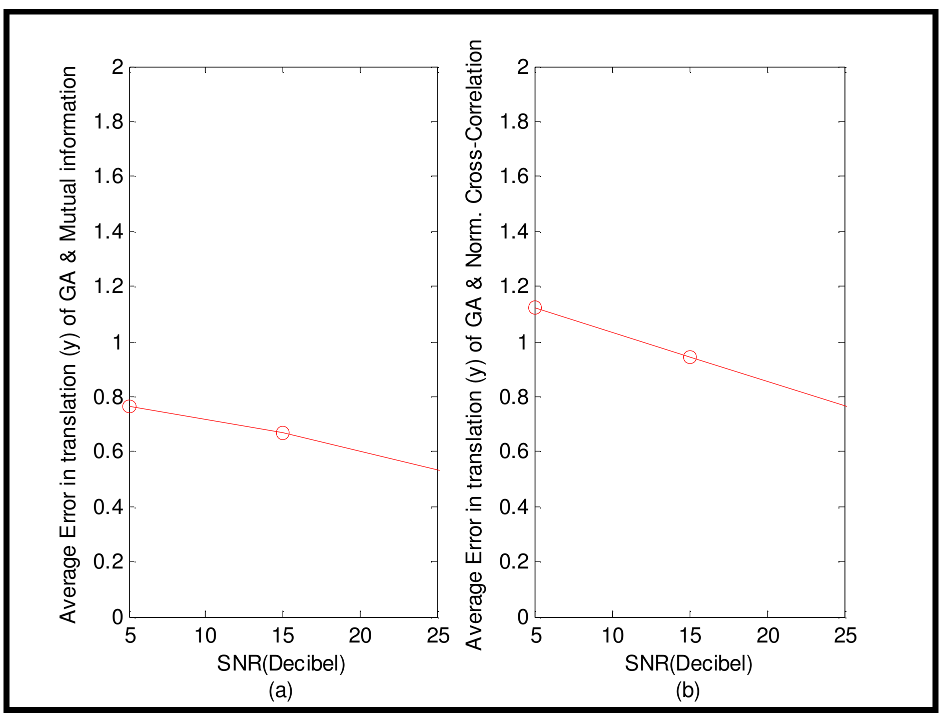



Figure 7.30 Average errors in translation y direction of genetic algorithm for image 1,(a)with using mutual information, (b) with using normalized cross-correlation

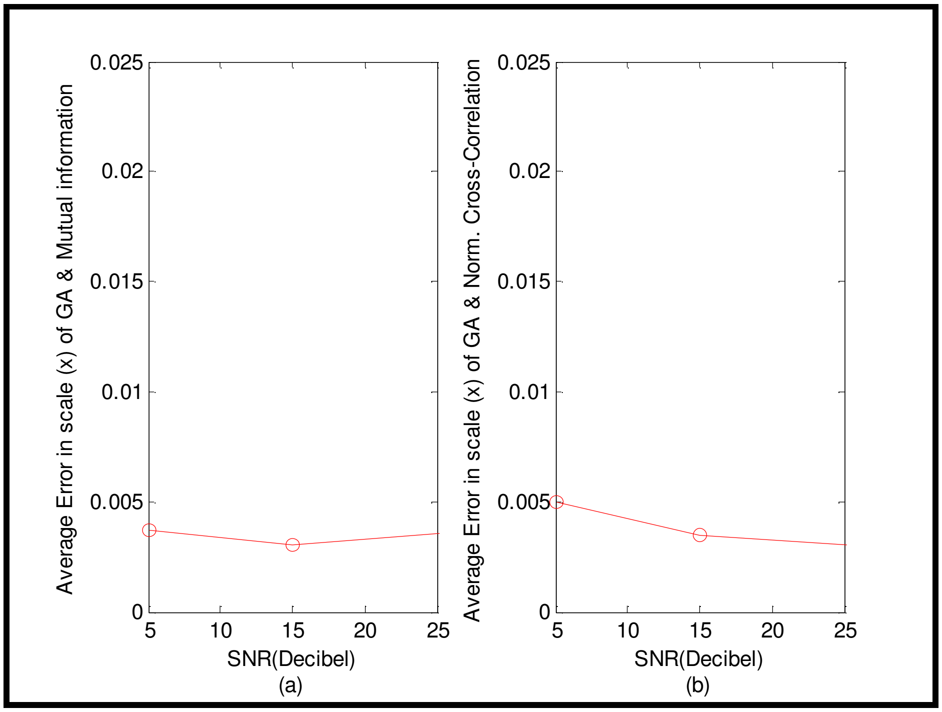



Figure 7.31 Average errors in scale x direction of genetic algorithm for image 1,(a)with using mutual information, (b) with using normalized cross-correlation

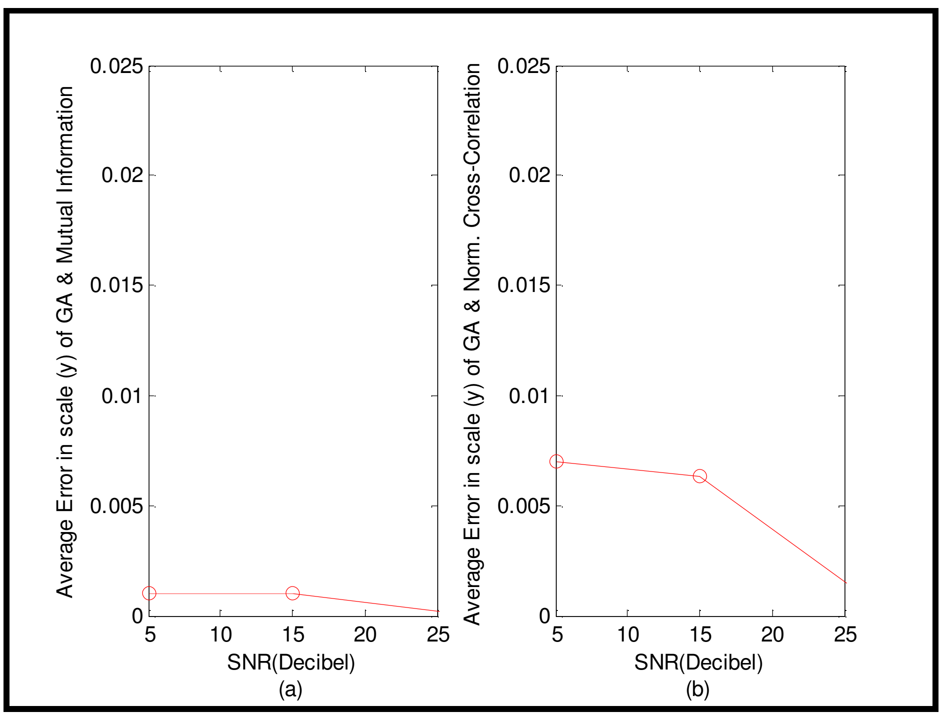



Figure 7.32 Average errors in scale y direction of genetic algorithm for image 1,(a)with using mutual information, (b) with using normalized cross-correlation

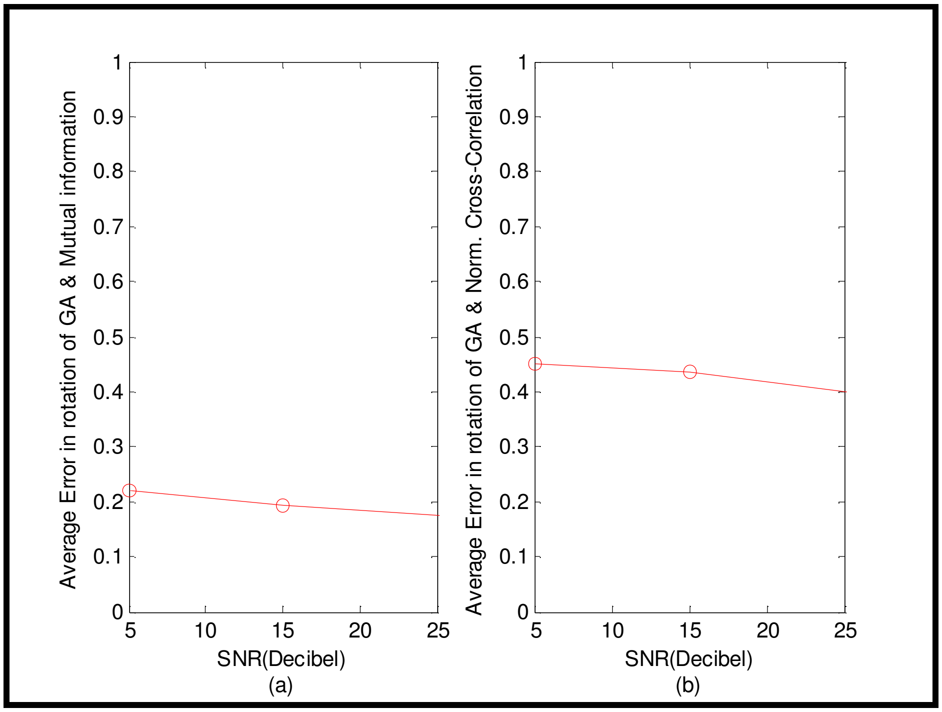



Figure 7.33 Average errors in rotation of genetic algorithm for image 2, (a) with using mutual information, (b) with using normalized cross-correlation

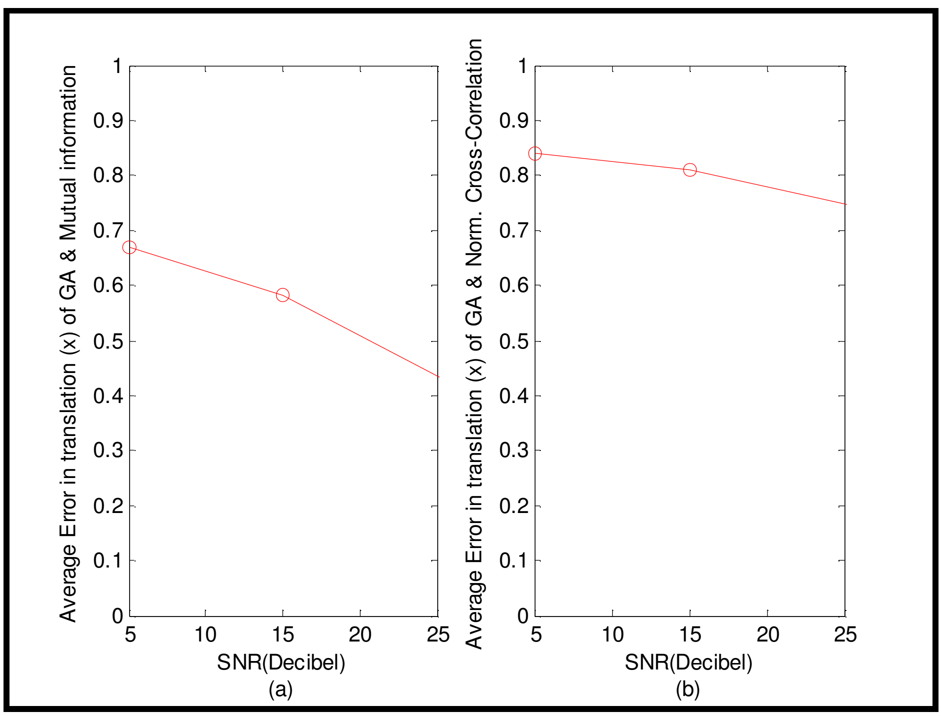



Figure 7.34 Average errors in translation x direction of genetic algorithm for image 2,(a)with using mutual information, (b) with using normalized cross-correlation

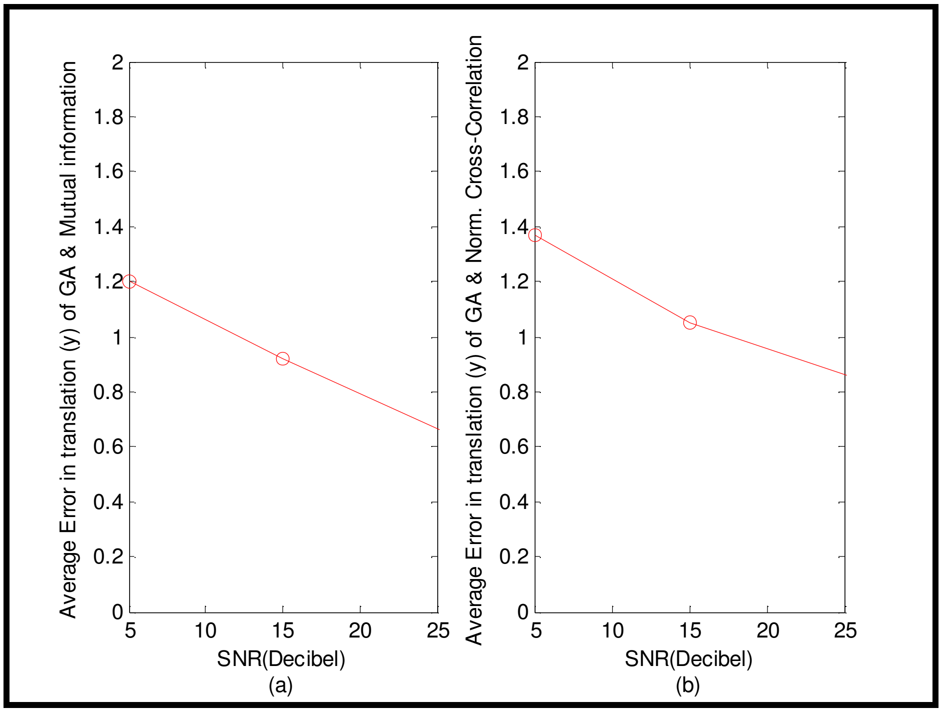



Figure 7.35 Average errors in translation y direction of genetic algorithm for image 2,(a) with using mutual information, (b) with using normalized cross-correlation

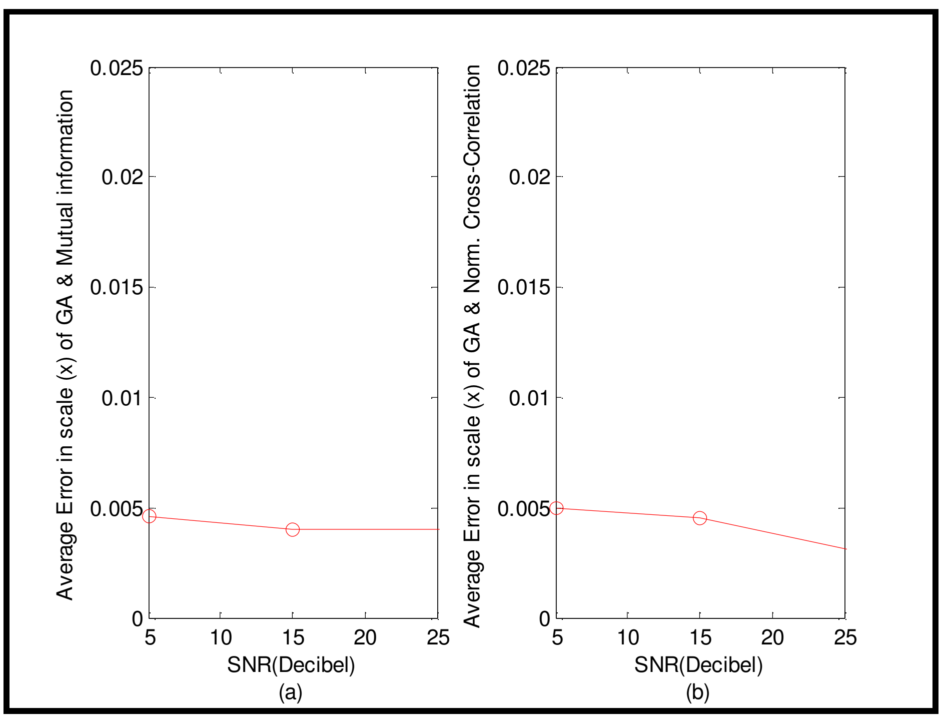



Figure 7.36 Average errors in scale x direction of genetic algorithm for image 2,(a)with using mutual information, (b) with using normalized cross-correlation

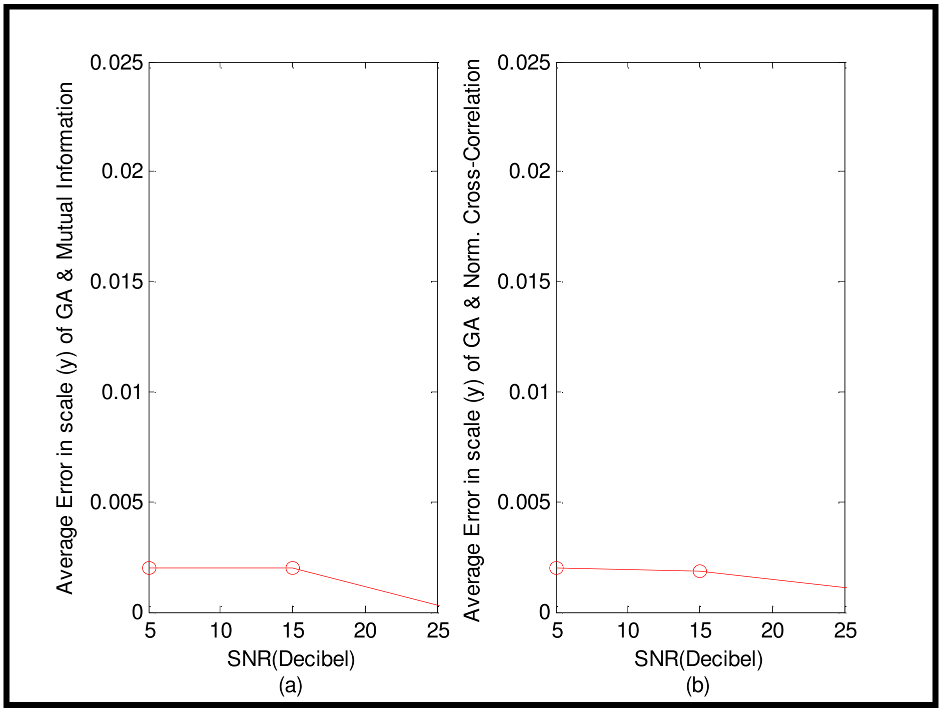



Figure 7.37 Average errors in scale y direction of genetic algorithm for image 2,(a)with using mutual information, (b) with using normalized cross-correlation

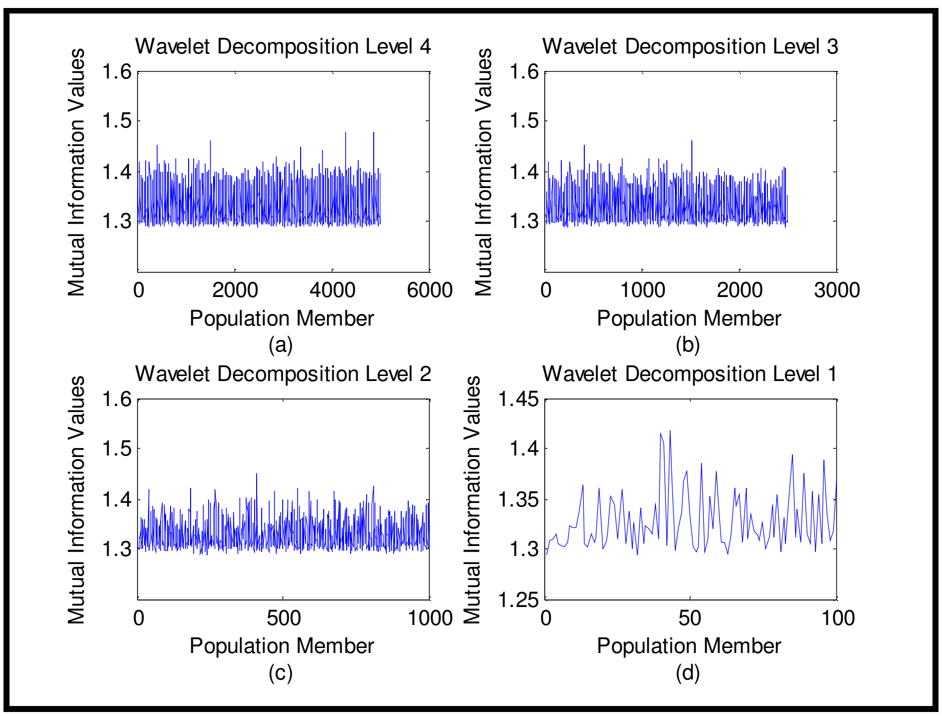



Figure 7.38 Mutual information values of genetic algorithm for wavelet decomposition levels (a) for level 4, (b) for level 3, (c) for level 2, (d) for level 1

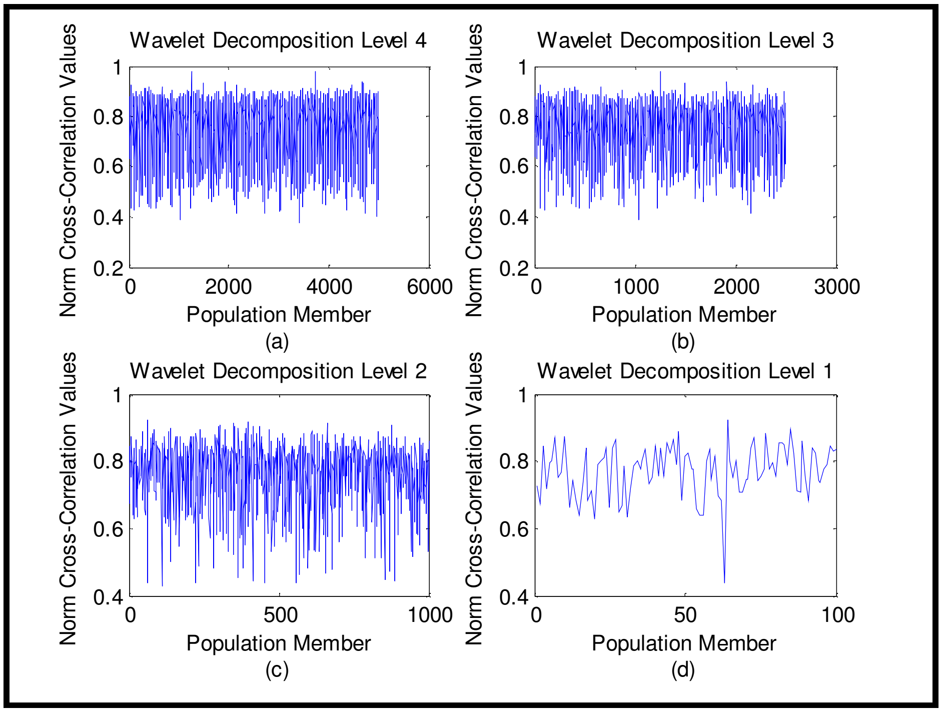



Figure 7.39 Correlation values of genetic algorithm for wavelet decomposition levels, (a)for level 4, (b) for level 3, (c) for level 2, (d) for level 1.

# CHAPTER EIGHT

# CONCLUSION

The main goal of a registration method is to find geometrical transformation parameters between two images. In this thesis, wavelet based image registration methods are implemented and tested. The affine geometrical transformation model with five parameters is used in this work. These are rotation angle, translations in both x and y directions and scales in both x and y directions. The implemented methods estimate them by searching through a search space. Five different configuration of optimization methods and similarity metrics are implemented and tested. Implementations start with the simple full search algorithm and then it is developed by two optimization methods. Simultaneous perturbation stochastic approximation algorithm (SPSA) and genetic algorithm (GA) are used to maximize the similarity measure between images.

Wavelet decomposition of images is used to speed up the process. Filtering operations of decomposition process removes high frequency noise from the images. So, the registration methods give reasonably good results in the presence of noise.

The methods are tested with both noisy and noiseless images. SPSA with mutual information gives the best results for both noisy and noiseless images. Indeed, the full search algorithm must find the best results, but a predefined fairly small step size of the algorithm restricts it. However, SPSA has descending step sizes determined recursively. So, it can approximate the best results. Step sizes of the full search algorithm can be reduced but this increases the computational time enormously. So, a manageable step size had to be determined. SPSA cannot give accurate results with the normalized cross correlation due to the characteristic wide correlation peak of this similarity metric around the best match locations. In contrast to this, SPSA with mutual information has a high accuracy owing to the narrow and sharp peak of this similarity metric at the best match locations. In chapter 7, we see that mutual information values decrease as the used wavelet decomposition level gets lower. On the other hand, correlation values are similar in all decomposition levels. This affects the results of the algorithm. Because, parameter update rule of SPSA is related to similarity metric values.

GA is another optimization method used in this thesis. When we examine the results, we can see that GA results are worse than SPSA results. Population size and iteration number are critical in this optimization method. If these parameters are selected high, the algorithm can give better results. But, this increases computational time, so optimum values must be chosen.

In implementing SPSA and GA based registration methods, there are some difficulties. For SPSA, its constant parameters are important in practice, because they depend on application and affect the speed and accuracy of the operation. In my application, I determined different parameters for each wavelet decomposition level. There is no strict rule for determination of parameters so I had to decide on them experimentally. In GA implementation, I tried different population sizes and iteration numbers for each wavelet decomposition level, and chose the optimum ones for the tests presented in the thesis.

In all methods, registration error in translation is higher than those in rotation and scale. This indicates that both similarity metrics are less sensitive to translation. Mutual information is more successful in determining translations.

In conclusion, as the experimental results indicate the full search algorithm and SPSA with mutual information are accurate and robust methods, while GA with mutual information is fairly accurate. Taking the computational time into account, doubtlessly, SPSA with mutual information outperforms the others.

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# APPENDIXS

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