ENTROPY APPLICATIONS FOR CUSTOMER SATISFACTION SURVEY IN INFORMATION THEORY

A Thesis Submitted to the

Graduate School of Natural and Applied Sciences of Dokuz Eylül University In Partial Fulfillment of the Requirements for the Degree of Master of Science in Statistics Program

> **by ASLI GÜNDÜZ**

February, 2010 İ**ZM**İ**R**

M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"ENTROPY APPLICATIONS FOR CUSTOMER SATISFACTION SURVEY IN INFORMATION THEORY"** completed by **ASLI GÜNDÜZ** under supervision of **ASSIST. PROF. DR. ÖZLEM EGE ORUÇ** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assist. Prof. Dr. ÖZLEM EGE ORUÇ

Supervisor

Prof.Dr. Şennur SOMALI Asssist. Prof.Dr. Emel KURUOĞLU

(Jury Member) (Jury Member)

Prof.Dr. Mustafa SABUNCU Director Graduate School of Natural and Applied Sciences

ACKNOWLEDGMENTS

 I would like to thank my advisor Assist. Prof. Dr. Özlem EGE ORUÇ, who has guided me with her close interest in all phases of this study.

 I would also like to thank Prof. Dr. Serdar KURT and other academic staff in Dokuz Eylül University Statistics Department.

And special thank to my employer for material support.

Aslı GÜNDÜZ

ENTROPY APPLICATIONS FOR CUSTOMER SATISFACTION SURVEY IN INFORMATION THEORY

ABSTRACT

 In this study, first of all, definitions of entropy and information terms, which express the measurement of uncertainty existing in a probabilistic system are given. The properties and types of these terms are shown in details and the relation and differences between them are mentioned. Applications are performed by using these two terms in order to determine the number of questions to be included in the scale that will be prepared to obtain information about customer satifaction.

 In the study, customer satisfaction scale, whose Cronbach' s alpha coefficient is found as 0,77, is applied on the BSS Boray San. Tic. A.Ş. company customers. Probability distribution tables are formed according to the results of the subscales of the survey. Shannon entropy, joint entropy, relative entropy and mutual information values are calculated using these tables. With the result of the applied survey it is found out that according to the calculated entropy values, it is possible to reach the aimed information through fewer questions. In brief, the possibility of reaching the same information through fewer questions is shown.

Key Words: Probability Theory, Entropy, Relative Entropy (Kullback – Leibler Divergence), Joint Entropy, Conditional Entropy, Mutual Information, Survey, Scale, Customer Satisfaction.

Bİ**LG**İ **TEOR**İ**S**İ**NDE MÜ**Ş**TER**İ **MEMNUN**İ**YET**İ **ANKET**İ İ**Ç**İ**N ENTROP**İ **UYGULAMASI**

ÖZ

Bu çalışmada, öncelikle olasılıksal bir sistemde var olan belirsizliğin ölçümünü ifade eden entropi ve bilgi kavramı tanımlarına yer verilmiştir. Bu iki kavramın özellikleri ve çeşitleri ayrıntılarıyla gösterilmiş, aralarındaki ilişki ve farklılıklara değinilmiştir. Müşteri memnuniyeti hakkında bilgi sahibi olabilmek için hazırlanacak olan ölçeğin optimum kaç soru içermesi gerektiği belirlemek için bu iki kavram kullanılarak uygulama yapılmıştır.

 Çalışmada Cronbach's alpha coefficient'i 0.77 olarak bulunan müşteri memnuniyeti ölçeği BSS Boray San. Tic. A.Ş. firması müşterilerine uygulanmıştır. Anketin alt ölçeklerine ait sonuçlardan elde edilen puanlardan olasılık dağılım tabloları oluşturulmuştur. Bu tablolar kullanılarak Shannon entropi, bileşik entropi, göreli entropi ve karşılıklı bilgi değerleri hesaplanmıştır. Uygulanan anket sonucu ile ulaşılmak istenen bilgiye, hesaplanan entropi değerlerine göre daha az soru ile ulaşılabilecegi bulunmuştur.

Anahtar Kelimeler: Olasılık Teorisi, Entropi, Göreli Entropi (Kullback-Leibler Uzaklığı), Koşullu Entropi, Bileşik Entropi, Karşılıklı Bilgi, Anket, Ölçek, Müşteri Memnuniyeti.

CONTENTS

Page

CHAPTER THREE - BASIC CONCEPTS OF INFORMATION THEORY . 13

CHAPTER ONE INTRODUCTION

 Information theory is the branch of mathematics that describes how uncertainty should be quantified, manipulated and represented. In information theory, entropy is a measure of the uncertainty associated with a random variable. The term by itself in this context usually refers to the Shannon entropy**,** which quantifies, in the sense of an expected value, the information contained in a message, usually in units such as bits. Equivalently, the Shannon entropy is a measure of the average information content one is missing when one does not know the value of the random variable. Ever since the fundamental premises of information theory were laid down by Cladue Shannon (1948), it has had far reaching implications for almost every field of science and technology. Information theory has also had an important role in survey scale studies(Oruç Ege, Kuruoğlu&Vupa, 2009).

 Surveys are used to collect quantitative information about items in a population. Developing a survey is as much an art as it is a science. In addition, just as an artist has a variety of different colors to choose from the palette, you have a variety of different question formats with which to question an accurate picture of your customers, clients and issues that are important to them. A good survey question should be short and straightforward. Further it should not be too long. The scale used in survey is defined by a set of two or more survey items that cohere in terms of individual's responses. A scale combines an individual's responses to a number of survey items into one score. Survey studies are common in political polling and government, health, social science and marketing research.

 A scale that measures customer satisfaction is dealt with in this study. By how many questions the intended information would be reached using this scale with the calculated entropy values was investigated. This study applies also entropy in information theory.

 The organization of this thesis is as follows: In chapter one, introduction was presented. Chapter two, basic information on the main concepts of survey methodology was explained. In chapter three, entropy in information theory and its properties are introduced. In chapter four, the application of information theory for the customer satisfaction survey which comprised the main topic of the study.

CHAPTER TWO BASIC CONCEPTS OF SURVEY METHODOLOGY

 In this chapter, basic information on the main concepts of survey methodology was explained.

2.1 Survey

Surveys of human populations and institutions are common in political polling and government, health, social science and marketing research. Surveys provide information on organizational performance. A good survey question should be short and straightforward. A survey should not be too long. When the questions are administered by a researcher, the survey is called a structured interview. When the questions are administered by the respondent, the survey is referred to as a questionnaire. The questions are usually structured and standardized. Surveys are standardized to ensure reliability, generalizability and validity. Every respondent should be presented with the same questions and in the same order as other respondents. A customer survey identifies the factors that enhance customer relationships, customer loyalty, and increase sales. Customer satisfaction surveys provide information for improving performance and profitability by identifying the root causes of customer satisfaction.

 Market research systematically identifies the current and forecasted characteristics and demographics of an area or industry in which a product or service may be offered. This will help determine the cost of doing business in that area, including potential sales, price point, competition, distribution, and other factors related to successful business implementation. A market research survey and its related analysis are imperative to a successful product or service launch.

2.2 Survey Design

 Knowing what the client wants is the key factor to success in any type of business. News media, government agencies and political candidates need to know what the public thinks. Associations need to know what their members want. Large companies

need to measure the attitudes of their employees. The best way to find this information is to conduct a survey.

The steps in a survey project are;

- Establish the goals of the project (What you want to learn),
- Determine the sample (Whom you will interview),
- Choose interviewing methoodology (How you will interview),
- Create the questionnaire (What you will ask),
- Pre test the questionnaire (If practical test the questions),
- Conduct interviews and enter data (Ask the questions),
- Analyze the data (Produce the reports).

2.2.1 Establishing Goals

The first step in any survey is deciding what we want to learn. The goals of the project determine whom we will survey and what we will ask them. If our goals are unclear, the results will probably be unclear. Some typical goals include learning more about:

- The potential market for a new product or service,
- Ratings of current products or services,
- Employee attitudes,
- Customer/patient satisfaction levels,
- Reader/viewer/listener opinions,
- Association member opinions,
- Opinions about political candidates or issues,
- Corporate images.

These sample goals represent general areas. The more specific we can make our goals, the easier it will be to get usable answers.

2.2.2 Selecting Sample

There are two main components in determining whom we will interview. The first is deciding what kind of people to interview. Researchers often call this group the target population. If we conduct an employee attitude survey or an association membership survey, the population is obvious. If we are trying to determine the likely success of a product, the target population may be less obvious. Correctly determining the target population is critical. If we don' t interview the right kinds of people, we will not successfully meet your goals.

 The next thing to decide is how many people we need to interview. Statisticians know that a small, representative sample will reflect the group from which it is drawn. The larger the sample, the more precisely it reflects the target group. However, the rate of improvement in the precision decreases as our sample size increases. For example, to increase a sample from 250 to 1000 only doubles the precision. We must make a decision about our sample size based on factors such as: time available, budget and necessary degree of precision.

2.3 Questionnaire Design

 Questionnaires are an inexpensive way to gather data from a potentially large number of respondents. Often they are the only feasible way to reach a number of reviewers large enough to allow statistically analysis of the results. A well-designed questionnaire that is used effectively can gather information on both the overall performance of the test system as well as information on specific components of the system.

 It is important to remember that a questionnaire should be viewed as a multi-stage process beginning with definition of the aspects to be examined and ending with interpretation of the results. Every step needs to be designed carefully because the final results are only as good as the weakest link in the questionnaire process. Although questionnaires may be cheap to administer compared to other data collection methods, they are every bit as expensive in terms of design time and interpretation.

The steps required to design and administer a questionnaire include:

- 1. Defining the Objectives of the Survey,
- 2. Determining the Sampling Group,
- 3. Writing the Questionnaire,
- 4. Administering the Questionnaire,
- 5. Interpretation of the Results.

 Questionnaire design is a long process that demands careful attention. A questionnaire is a powerful evaluation tool and should not be taken lightly. Design begins with an understanding of the capabilities of a questionnaire and how they can help your research. If it is determined that a questionnaire is to be used, the greatest care goes into the planning of the objectives. Questionnaires are like any scientific experiment. One does not collect data and then see if they found something interesting. One forms a hypothesis and an experiment that will help prove or disprove the hypothesis.

2.4 Question Types

 Questionnaires are quite flexible in what they can measure, however they are not equally suited to measuring all types of data. Questions may be designed to gather either qualitative or quantitative data. By their very nature, quantitative questions are more exact then qualitative. For example, the word "easy" and "difficult" can mean radically different things to different people. Any question must be carefully crafted, but in particular questions that assess a qualitative measure must be phrased to avoid ambiguity. Qualitative questions may also require more thought on the part of the participant and may cause them to become bored with the questionnaire sooner. In general, we can say that questionnaires can measure both qualitative and quantitative data well, but that qualitative questions require more care in design, administration, and interpretation.

 In general, there are two types of questions one will ask, open format or closed format.

 Open format questions are those that ask for unprompted opinions. In other words, there are no predetermined set of responses, and the participant is free to answer however he chooses. Open format questions are good for soliciting subjective data or when the range of responses is not tightly defined. An obvious advantage is that the variety of responses should be wider and more truly reflect the opinions of the respondents. This increases the likelihood of you receiving unexpected and insightful suggestions, for it is impossible to predict the full range of opinion. It is common for a questionnaire to end with and open format question asking the respondent for her unabashed ideas for changes or improvements.

 Open format questions have several disadvantages. First, their very nature requires them to be read individually. There is no way to automatically tabulate or perform statistical analysis on them. This is obviously more costly in both time and money, and may not be practical for lower budget or time sensitive evaluations. They are also open to the influence of the reader, for no two people will interpret an answer in precisely the same way. This conflict can be eliminated by using a single reader, but a large number of responses can make this impossible. Finally, open format questions require more thought and time on the part of the respondent. Whenever more is asked of the respondent, the chance of tiring or boring the respondent increases.

 Closed format questions usually take the form of a multiple-choice question. There is no clear consensus on the number of options that should be given in an closed format question. Obviously, there needs to be sufficient choices to fully cover the range of answers but not so many that the distinction between them becomes blurred. Usually this translates into five to ten possible answers per questions. For questions that measure a single variable or opinion, such as ease of use or liability, over a complete range (easy to difficult, like to dislike), conventional wisdom says that there should be an odd number of alternatives. This allows a neutral or no opinion response. Other schools of thought contend that an even number of choices is best because it forces the respondent to get off the fence. This may induce the some inaccuracies for often the respondent may actually have no opinion. However, it is equally arguable that the neutral answer is over utilized, especially by bored questionnaire takers. For larger questionnaires that test opinions on a very large number of items, such as a music test, it may be best to use an even number of choices to prevent large numbers of no-thought neutral answers.

 Closed format questions offer many advantages in time and money. By restricting the answer set, it is easy to calculate percentages and other hard statistical data over the whole group or over any subgroup of participants. Modern scanners and computers make it possible to administer, tabulate, and perform preliminary analysis in a matter of days. Closed format questions also make it easier to track opinion over time by administering the same questionnaire to different but similar participant groups at regular intervals. Finally closed format questions allow the researcher to filter out useless or extreme answers that might occur in an open format question.

Whether questions are open or closed format, there are several points that must by considered when writing and interpreting questionnaires:

Clarity: This is probably the area that causes the greatest source of mistakes in questionnaires. Questions must be clear, succinct, and unambiguous. The goal is to eliminate the chance that the question will mean different things to different people. If the designers fails to do this, then essentially participants will be answering different questions.

Leading Questions: A leading question is one that forces or implies a certain type of answer. It is easy to make this mistake not in the question, but in the choice of answers. A closed format question must supply answers that not only cover the whole range of responses, but that are also equally distributed throughout the range. All answers should be equally likely.

Embarrassing Questions: Embarrassing questions dealing with personal or private matters should be avoided. Your data is only as good as the trust and care that your respondents give you. If you make them feel uncomfortable, you will lose their trust. Do not ask embarrassing questions.

Hypothetical Questions: Hypothetical are based, at best, on conjecture and, at worst, on fantasy. This forces the respondent to give thought to something he may have never considered. This does not produce clear and consistent data representing real opinion. Do not ask hypothetical questions.

Prestige Bias: Prestige bias is the tendency for respondents to answer in a way that make them feel better. People may not lie directly, but may try to put a better light on themselves.

 There is little that can be done to prevent prestige bias. Sometimes there just is no way to phrase a question so that all the answers are noble. The best means to deal with prestige bias is to make the questionnaire as private as possible.

2.5 Meaning of Customer Satisfaction

Customer satisfaction, a business term, is a measure of how products and services supplied by a company meet or surpass customer expectation. Customer satisfaction is tied directly to profitability. If the customers are happy, they tend to be loyal. If they are loyal they not only buy more, they refer other customer.

 In a competitive marketplace where businesses compete for customers, customer satisfaction is seen as a key differentiator and increasingly has become a key element of business strategy.

 Customer satisfaction is an ambiguous and abstract concept and the actual manifestation of the state of satisfaction will vary from person to person and product / service to product / service. The state of satisfaction depends on a number of both psychological and physical variables which correlate with satisfaction behaviors such as return and recommend rate. The level of satisfaction can also vary depending on other options the customer may have and other products against which the customer can compare the organization' s products.

2.6 Measuring Customer Satisfaction

 Organizations are increasingly interested in retaining existing customers while targeting non-customers; measuring customer satisfaction provides an indication of how successful the organization is at providing products and/or services to the marketplace. The satisfaction loop below shows the steps to a successful and accurate customer satisfaction mesurement in Figure 2.1.

Figure 2.1 Steps of designing a survey

 Customer satisfaction measures how well a company's products or services meet or exceed customer expectations. These expectations often reflect many aspects of the company's business activities including the actual product, service, company, and how the company operates in the global environment. Customer satisfaction measures are an overall psychological evaluation that is based on the customer's lifetime of product and service experience.

 If we want to measure customer satisfaction, we need a very clear definition of exactly what we are measuring. The simple definition of customer satisfaction is a measure of how our organisation's total product performs in relation to a set of customer requirements. It's relative to what the customer expected in the first place. For to measure satisfaction, we have to measure both sides of equation. The expectation part – called importance ratings, and the satisfaction part – called satisfaction ratings or performance ratings.

 First we have to identify customers' requirements. Second, we have to measure satisfaction. Having generated importance and satisfaction measures, we can use them to produce some survey outcomes (Hill, Brierley & MacDougall, 2003).

 Effective marketing focuses on two activities: retaining existing customers and adding new customers. Customer satisfaction measures are critical to any product or service company because customer satisfaction is a strong predictor of customer retention, customer loyalty and product repurchase.

 Satisfaction measures involve three psychological elements for evaluation of the product or service experience: cognitive (thinking/evaluation), affective (emotionalfeeling/like-dislike) and behavioral (current/future actions). Customer satisfaction usually leads to customer loyalty and product repurchase. But measuring satisfaction is not the same as measuring loyalty.

 A consumer's attitude (liking/disliking) towards a product can result from any product information or experience whether perceived or real. It is meaningful to measure attitudes towards a product or service that a consumer has never used, but not satisfaction.

 A cognitive element is defined as an appraisal or conclusion that the product was useful (or not useful), fit the situation (or did not fit), exceeded the requirements of the problem/situation (or did not exceed). Cognitive responses are specific to the situation for which the product was purchased and specific to the consumer's intended use of the product, regardless if that use is correct or incorrect.

2.7 Expectations Measures

 Many different approaches to measuring satisfaction exist in the consumer behavior literature. Leonard Berry in 2002 expanded previous research to define ten dimensions of satisfaction, including: Quality, Value, Timeliness, Efficiency, Ease of Access, Environment, Inter-Departmental Teamwork, Front Line Service Behaviors, Commitment to the Customer and Innovation. Berry's dimensions are often used to develop an evaluative set of satisfaction measurement questions that focus on each of the dimensions of customer satisfaction in a service environment.

Customer satisfaction surveys often include multiple measures of satisfaction, including:

- Overall measures of customer satisfaction
- Affective measures of customer satisfaction
- Cognitive measures of customer satisfaction
- Behavioral measures of customer satisfaction
- Expectancy value measures of customer satisfaction

 A diagnostic approach to satisfaction measurement is to examine the gap between the customer's expectation of performance and their perceived experience of performance. This "satisfaction gap" involves measuring both perception of performance and expectation of performance along specific product or service attributes dimensions.

 Customer satisfaction is largely a reflection of the expectations and experiences that the customer has with a product or service. However expectations also reflect that influences the evaluation of the product or service. When we make major purchases, we research the product or service and gain information from the advertising, salespersons, and word-of-mouth from friends and associates. This information influences our expectations and ability to evaluate quality, value, and the ability of the product or service to meet our needs.

CHAPTER THREE BASIC CONCEPTS OF INFORMATION THEORY

 Information theory is a branch of applied mathematics and electrical engineering involving the quantification of information. Historically, information theory was developed by Claude E. Shannon(1948) to find fundamental limits on compressing and reliably storing and communicating data. Since its inception it has broadened to find applications in many other areas, including statistical inference, natural language processing, cryptography, engineering, biology, medical science, sociology, and psychology. A key measure of information in the theory is known as entropy, which is usually expressed by the average number of bits needed for storage or communication. Intuitively, entropy quantifies the uncertainty involved when encountering a random variable. This chapter briefly defines Shannon entropy, relative entropy (Kullback-Leibler divergence), joint entropy, conditional entropy and mutual information.

3.1 Entropy

 In information theory, entropy is a measure of the uncertainty associated with a random variable. This section briefly defines Shannon Entropy.

 Let X be a discrete random variable, taking a finite number of possible values x_1, x_2, \dots, x_n with probabilities p_1, p_2, \dots, p_n respectively such as $\sum\limits_{}$ = $\geq 0, i = 1, 2, 3, ..., n$ and $\sum_{i=1}^{n} p_i =$ n $i = 1$ $p_i \ge 0, i = 1,2,3,..., n$ and $\sum p_i = 1$.

 The information received from X when it produces a symbol i is given in formula $(3.1.1).$

$$
I_X = -\log p_i \tag{3.1.1}
$$

The Shannon entropy of X is defined by (3.1.2).

$$
H(X) = E(I_X) = -\sum_{i=1}^{n} p(x_i) \log p(x_i)
$$
 (3.1.2)

The notation $E(I_X)$ denotes the expected value of I_X , which is a more specific term in statistics than "average". Entropy can also be called average uncertainty. The entropy function $H(X)$ is maximum when $p_i =1/n$ for all i. This makes intuitive sense, because uncertainty is greatest when all outcomes are equally likely. The entropy graph of an event with two possible equal results $(p_1 = p_2 = 0.5)$ is given in Figure 3.1 (Cover & Thomas,2006).

Figure 3.1 Maximum entropy in case of equal probability

3.2 Joint, Conditional and Relative Entropy

 Various entropies can be calculated for the states under interest. In this section some explanations on some entropy types will be given.

3.2.1 Joint Entropy

X and Y be two discrete random variables taking values $\{x_1, \dots, x_n\}$ and $\{y_1, \ldots, y_m\}$ respectively: If $P(X=x_i, Y=y_i)$ denote the joint probability mass function of X and then the joint entropy of these random variables is defined by $(3.2.1.1).$

$$
H(X,Y)=\sum_{j}\sum_{i} p(X=x_{i},Y=y_{j}) \log P(X=x_{i},Y=y)
$$
 (3.2.1.1)

Joint entropy is also called the common information measure.

If X and Y are independent, then the joint entropy equals to the sum of the entropies of each random variable and the formula is given (3.2.1.2).

$$
H(X, Y) = H(X) + H(Y)
$$
\n(3.2.1.2)

3.2.2 Conditional Entropy

 X and Y are random variables that have joint probability distributions. When the values of the random variable Y are given, the measurement of the uncertainty in the random variable x is the conditional entropy of X dependent on Y. Knowing Y always decreases the uncertainty of X . It is shown as $H(X|Y)$ and can be calculated as (3.2.2.1) (Cover & Thomas, 2006).

$$
H(X|Y) = \sum_{i} \sum_{j} p(x_i, y_j) \log(p(x_i / y_j))
$$
\n(3.2.2.1)

If the variables X and Y are independent of each other, the chain rule that shows the combination of the joint entropy and conditional entropy explained above is given (3.2.2.2).

$$
H(X, Y) = H(X) + H(Y | X)
$$
\n(3.2.2.2)

3.2.3 Relative Entropy

 The Kullback – Liebler divergence is a way of comparing two distributions; a "true" probability distribution $p(X)$, and an arbitrary probability distribution $q(X)$. If we compress data in a manner that assumes $q(X)$ is the distribution underlying some data, when, in reality, $p(X)$ is the correct distribution, the Kullback - Leibler divergence is the number of average additional bits per datum necessary for compression. It is thus defined in equation (3.2.3.1) and (3.2.3.2).

For discrete random variables;

$$
D(p||q) = \sum_{x \in X} p(x) \log \frac{p(x)}{q(x)}
$$
\n(3.2.3.1)

For continous random variables;

$$
D(p||q) = \int_{x \in X} f(x) \log \frac{f(x)}{g(x)} dx
$$
\n(3.2.3.2)

3.3 Concept of Information and Features

 Information is a complex concept. Therefore, it is hard to provide a general definition. Researchers come to common grounds in this point and develop mathematical terms for information systems analysis. Information is storable, visible, transferable, re – obtainable, observable and interpretable.

 In this part of study, the concept of information used in the framework of information theory and its features will be briefly explained. Information is the processed state of data and facts related to objects, event or people.

The value of;

$$
I(x_i) = -\log_2 (p) = \log_2 (1/p)
$$
 (3.3.1)

calculated for the $\{x_1, x_2, \ldots, x_i\}$ values of the discrete random variable X in the state of $i = 1, 2, \ldots, n$, is called the information content of x_i state (3.3.1). The information value of the random variable X is calculated given in (3.3.2).

$$
I(x_i) = \sum_{i} p_i * I(x_i)
$$
 (3.3.2)

 This value is the weighted average of the information contents of the values that X has taken, and the probability of taking these values; and at the same time it is called entropy. The information content that the random variable takes is only dependent on the random variable' s probability of the taking that value. As lower this probability is so bigger is the information content.

There are four basic axioms for information:

- Information is a value that is not negative. I(p) ≥ 0
- The information value of an accurate event is zero. $I(1) = 0$
- For two independent event, the information is obtained from observations equals to the sum of two informations. $I(p_1 * p_2) = I(p_1) + I(p_2)$
- I(p) is monotonous and constant.

3.4 Mutually Information

The mutual information $I(X;Y)$ of two random variables X and Y is the KL divergence between their joint distribution and the product of their marginals.

$$
I(X;Y) = \sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}
$$
(3.4.1)

 By (3.4.1) definition the mutual information provides some measure of the dependence between the variables.

Let X and Y be two discrete random variables, then $I(X;Y) \ge 0$ and equality if X and Y are independent.

 Let X and Y be two discrete random variables, then their mutual information $I(X;Y)$ obeys

- Symmetry $I(X;Y) = I(Y;X)$
- $I(X;Y) = H(X) H(X | Y)$
- \bullet $I(X;X) = H(X)$
- Chain rule: $I(X_1, X_2, ..., X_n; Y) = \sum_{i=1}^{n} I(X_i, X_i; Y_i)$ $\prod_{i=1}^{n} I(X_i;Y|X_1,...,X_{i-1})$
- Data processing inequality:

If (X, Y, Z) form a Markov chain, then $I(X;Y) \geq I(X;Z)$. As a consequence, $I(X;Y) \geq I(X; f(Y))$ for any function *f* of Y.

3.5 Relationship between Entropy and Information

 The definition of information is done by taking the definition of entropy, which masures the randomness in a system, as a model. Therefore, information and entropy can be considered as intertwined concepts. Answering the question yes or no which is assumed to be that is uncertainty, there is uncertainty for answers. Here the question carries an information value. If the answer is known accurately, asking the question will be unnecessary. Increase of information causes entropy to decrease by decreasing uncertainty. Thus, minimum uncertainty is obtained by maximum information.

$$
I(X; Y) = \log (P(X | Y)) + I(X) = H(X) - H(X | Y)
$$

H(X,Y) = H(X) + H(Y | X)
= H(Y) + H(X | Y) (3.5.1)

 The relations between the conditional entropies and joint of the random variables X and Y can be defined as equation (3.5.1) (Cover & Thomas, 2006).

CHAPTER FOUR APPLICATIONS

 In this chapter, measuring satisfaction and building a satisfaction survey requires at least a basic knowledge of the satisfaction measurement literature, combined with your own customer satisfaction experience. Customer satisfaction is the most common of all marketing surveys and is part of the "big three" research studies in marketing that include market segmentation and concept testing.

 Effective marketing focuses on two activities; retaining existing customers and adding new customers. Customer satisfaction measures are critical to any product or service company because customer satisfaction is a strong predictor of customer retention, customer loyalty and product repurchase.

 A scale that measures customer satisfaction is dealt with in this study. The number of questions for intended information to be reached using this scale together with entropy values was investigated. The usual measures of customer satisfaction involve a survey with a set of statements using a likert technique or scale. The customer is asked to evaluate each statement and in term of their perception and expectation of performance of the organization being measured. This study applies also entropy in information theory.

 Scaling in this survey was examined under 4 subscales titles and the subscales were named "Marketing Services Assessment (MSA)", "Operation Services Assessment (OSA)", "Accounting Service Assessment (ASA)" and "General Assessment (GA)". First, the probability distribution tables were constructed by use of the answers given by the customers concerning the subscales of MSA, OSA, ASA and GA. By using these tables, the Shannon entropy, joint entropy, relative entropy and mutual information values were calculated.

 The survey was applied to 60 customers in order to measure customer satisfaction. It was composed of 18 questions and its Cronbach' s coefficient α was determined from formula (4.1)

$$
\alpha = \frac{n}{n-1} \left(1 - \frac{\sum_{i=1}^{n} \sigma_{Y_i}^2}{\sigma_X^2} \right)
$$
(4.1)

as $\alpha = 0.77$ where *n* is the number of components, σ_x^2 is the variance of the observed total test scores and $\sigma_{y_i}^2$ is the variance of component *i*. Each question was evalueted with 1 to 3 scores in such a way that it would be one of the scales of "bad, not bad – not good, good". The attitude or information scores of the respondents of the survey was added separately and ordered. In addition, several subscales were determined for these 18 questions. The scaling included in the survey was examined under 4 subscale titles. The first subscale was mentioned in the literature as "Marketing Service Assessments"(MSA). The second is "Operation Service Assessments"(OSA), third is "Accounting Service Assessments" (ASA) and the final scale as "General Assesstment"(GA). The subscale MSA was composed of a total of 5 questions, OSA was composed of a total of 7 questions whereas the subscale ASA had 3 questions and finally the subscale GA was composed of 3 questions.

 The questions representing the subscales in the survey were determined separately and probability distribution tables were constructed separately for each subscale from the frequency values calculated considering the scores of the questions representing each subscale. Using these probabilty distribution tables, Shannon entropy values were computed for MSA, OSA, ASA and GA. With a view to examining what kind of entropy values the subscales of MSA, OSA, ASA and GA had with gender, joint probability distribution tables were constructed separately from the frequencies obtained from Gender – MSA, Gender – OSA, Gender – ASA and Gender – GA scores. The joint entropy values of all subscales and gender were calculated separately from the joint probability distribution tables constructed by means formula (3.2.1.1). Mutual information values were computed separately for all subscales and gender using the same joint probability distribution tables. These values were calculated using formula (3.4.1). The relative entropy values were computed by means of formula (3.2.3.1) in order to examine whether the probability distribution of genders, calculated separately depending on all subscales, were similar or not. All values were interpreted within the scope of the information theory.

 15 out of 60 customers undertaking the questionnaire were females and 45 of them were males. The subscales of MSA, OSA, ASA and GA were regarded as random variables in the study in order to compute the entropy values. The Shannon entropy values were calculated by using the probability distributions constructed for the random variables of MSA, OSA, ASA and GA. The frequencies, probabilities and entropy values of these random variables are given in Table 4.1.

Score	MSA		OSA		ASA		GA	
		P		P		\boldsymbol{P}		P
1	17	0.06	39	0.10	5	0.03	16	0.09
$\mathbf{2}$	145	0.48	211	0.55	65	0.36	91	0.51
3	138	0.46	132	0.35	110	0.61	73	0.41
Total	300	1.00	382	1.00	180	1.00	180	1.00
Entropy		1.26		1.34		1.11		1.34

Table 4.1 Frequency (*f*) and probability (*P*) table for the MSA, OSA, ASA and GA variables

Moreover, the histogram of the entropy values of four random variables is observed in Figure 4.1.

Figure 4.1 Entropy values for MSA, OSA, ASA and GA random variables

 The entropy value 1.26 of MSA indicates that it is enough to ask two question for MSA. Likewise, the entropy values found for OSA (1.34), ASA (1.11) and GA (1.31) also indicate that it would be sufficient to ask two question in order to be informed to this end. In the scale applied, 5 question were asked in order to be informed about MSA, 7 questions were asked in order to be informed about OSA, 3 questions were asked in order to be informed about ASA and 3 questions were asked in order to be informed about GA. As a consequence this part, it was sufficient to ask two questions so as to be informed about each of these variables.

 To investigate what kind of entropy values the variables of MSA, OSA, ASA and GA had with gender, joint probability distribution tables were constructed separately from the frequencies obtained from Gender – MSA, Gender – OSA, Gender – ASA and Gender – GA scores. Table 4.2 demonstrates joint probability distribution of Gender – MSA, Gender – OSA Gender – ASA and Gender – GA .

Gender	MSA					
	$\mathbf{1}$	$\boldsymbol{2}$	3	Total		
Male(0)	0.043	0.360	0.347	0.750		
Female (1)	0.014	0.123	0.113	0.250		
Total	0.057	0.483	0.460	1.000		
Gender			OSA			
	$\mathbf{1}$	$\overline{2}$	3	Total		
Male(0)	0.084	0.413	0.278	0.775		
Female (1)	0.010	0.115	0.100	0.225		
Total	0.094	0.528	0.378	1.000		
Gender	ASA					
	$\mathbf{1}$	$\overline{2}$	3	Total		
Male(0)	0.022	0.283	0.444	0.749		
Female (1)	0.006	0.078	0.167	0.251		
Total	0.028	0.361	0.611	1.000		
Gender	GA					
	1	$\overline{2}$	$\mathbf{3}$	Total		
Male(0)	0.083	0.372	0.295	0.750		
Female (1)	0.006	0.133	0.111	0.250		
Total	0.089	0.505	0.406	1.000		

Table 4.2 Joint probability distribution for gender and MSA, OSA, ASA and GA

Table 4.3 gives joint entropy values of all subscales and gender.

Variables	Joint Entropy
MSA	2.0644
OSA	2.0998
ASA	1.9192
GA	2.1314

Table 4.3 Joint entropy for variables

The result in the joint entropy $H(X, Y) = 2.0644$ with $X =$ Gender and $Y = MSA$ means that on average it would require two questions to guess the level of both variables. The same result is also valid for OSA, ASA and GA.

 29 out of 60 customers undertaking the questionnaire were working to management and 31 of them were working to organisation. To investigate what kind of entropy values the variables of MSA, OSA, ASA and GA had with working position, joint probability distribution tables were constructed separately from the frequencies

Position	MSA					
	1	$\mathbf{2}$	3	Total		
Management (0)	0.037	0.227	0.220	0.484		
Organisation (1)	0.020	0.256	0.240	0.516		
Total	0.057	0.483	0.460	1.000		
Position			OSA			
	$\mathbf{1}$	$\overline{2}$	3	Total		
Management (0)	0.038	0.267	0.179	0.484		
Organisation (1)	0.055	0.273	0.188	0.516		
Total	0.093	0.540	0.367	1.000		
Position	ASA					
	1	$\overline{2}$	3	Total		
Management (0)	0.011	0.172	0.300	0.483		
Organisation (1)	0.017	0.189	0.311	0.517		
Total	0.028	0.361	0.611	1.000		
Position	GA					
	1	$\overline{2}$	3	Total		
Management (0)	0.028	0.239	0.217	0.484		
Organisation (1)	0.061	0.267	0.188	0.516		
Total	0.089	0.506	0.405	1.000		

Table 4.4 Joint probability distribution for position and MSA, OSA, ASA and GA

Table 4.5 gives joint entropy values of all subscales and working position.

Table 4.5 Joint entropy for variables to position

The result in the joint entropy $H(X, Y) = 2.2529$ with $X =$ Position and $Y = MSA$ means that on average it would require two questions to guess the level of both variables. The same result is also valid for OSA, ASA and GA.

 Mutual information values were computed using the joint probability distribution calculated for gender and all subscales. The entropy value was found as 0.811 bits for gender and mutual information value calculated for Gender – MSA, Gender – OSA, Gender – ASA and Gender – GA . Mutual information values are given for all variables in Table 4.6.

	$I(MSA, Gender) = H(MSA) + H(Gender) - H(MSA, Gender)$
	$I(MSA, Gender) = 1.26 + 0.811 - 2.0644 = 0.0066$ bits.
	$I(OSA, Gender) = H(OSA) + H(Gender) - H(OSA, Gender)$
	$I(OSA, Gender) = 1.34 + 0.811 - 2.0998 = 0.0512$ bits.
	$I(ASA, Gender) = H(ASA) + H(Gender) - H(ASA, Gender)$
	$I(ASA, Gender) = 1.11 + 0.811 - 1.9192 = 0.0018$ bits.
$I(GA, \;{Gender})$	$= H(GA) + H(Gender) - H(GA, Gender)$
	$I(GA, Gender)$ = 1.34 + 0.811 - 2.1314 = 0.0196 bits.

Table 4.6 Mutual information for variables and gender

 Mutual information values were computed using the joint probability distribution calculated for position and all subscales. The entropy value was found as 0.9992 bits for position and mutual information value calculated for Position-MSA, Position-OSA, Position-ASA and Position-GA. Mutual information values are given for all variables in Table 4.7.

	$I(MSA, Position) = H(MSA) + H(Position) - H(MSA, Position)$
	$I(MSA, Position) = 1.26 + 0.9992 - 2.2529 = 0.0063$ bits.
<i>I</i> (<i>OSA,Position</i>)	$= H(OSA) + H(Position) - H(OSA, Position)$
	$I(OSA, Position) = 1.34 + 0.9992 - 2.3274 = 0.0118$ bits.
	$I(ASA, Position) = H(ASA) + H(Position) - H(ASA, Position)$
	$I(ASA, Position) = 1.11 + 0.9992 - 2.1077 = 0.0015$ bits.
I(GA, Position)	$= H(GA) + H(Position) - H(GA, Position)$
I(GA, Position)	$= 1.34 + 0.9992 - 2.3253 = 0.0739$ bits.

Table 4.7. Mutual information for variables and position

 In probability theory and information theory, the mutual information or transformation, of two random variables is a quantity that measures the mutual dependence of the two variables. If X and Y are independent, then knowing X does not give any information about Y and vice versa, so their mutual information is zero. The mutual information value calculated for the MSA – Gender variables, which are not independent, can be interpreted as follows. The variables MSA and Gender seem not to have a lot of information in common, only 0.0066 bits of information. The mutual information values also found for OSA – Gender, ASA – Gender and GA – Gender are interpreted in the same way. Table 6 exhibits shared information between pairs of variables. The pair sharing the most information is OSA – Gender, while the least is ASA – Gender. The mutual information value calculated for the MSA – Position variables, which are not independent, can be interpreted as follows. The variables MSA and Gender seem not to have a lot of information in common, only 0.0063 bits of information. The mutual information values also found for OSA – Position, ASA – Position and GA – Position are interpreted in the same way. Table 4.7 exhibits shared information between pairs of variables. The pair sharing the most information is GA – Position, while the least is ASA – Position.

 The relative entropy is an appropriate measure of the similarity of the underlying distribution. If the distribution *f* and *g* are similar, the difference between D(*f*||*g*) and $D(g||f)$ is small. In this study, the marginal probability distributions of both genders were found depending on each subscale. The marginal probability distribution of both genders for the subscale of MSA is given in Table 4.8.

Table 4.8 Marginal probability distribution of male and female for MSA

Male (Y)				Total
P(Y	0.058	0.480	0.462	1.000
Female (X)				Total
P(X)	0.053	0.493	0.454	.000

 In order to investigate whether these distributions are similar or not, the relative entropy (Kullback – Liebler distance) values are computed as follows:

 $D(f_M \parallel f_F) = 0.058 \ln(0.058/0.053) + 0.480 \ln(0.480/0.493) + 0.462 \ln(0.462/0.454)$ $D(f_M || f_F) = 0.00065$

 $D(f_F || f_M) = 0.053 \ln(0.053/0.058) + 0.493 \ln(0.493/0.480) + 0.454 \ln(0.454/0.462)$ $D(f_F || f_M) = 0.00064$

 The fact that these values are found to be close demonstrates that both genders show a similar distribution. Likewise, the relative entropy values found for genders according to OSA, ASA and GA are found in the following way.

 Again in this study, the marginal probability distributions of both working position were found depending on each subscale. The marginal probability distribution of both position for the subscale of MSA is given in Table 4.9.

Table 4.9 Marginal probability distribution of organization and management for MSA

Organization (Y)				Total
	0.039	0.496	0.465	000.l
Management (X)				Total
	0.076	0.469	0.455	000 .

 In order to investigate whether these distributions are similar or not, the relative entropy (Kullback – Liebler distance) values are computed as follows:

 $D(f_M \parallel f_O) = 0.076 \ln(0.076/0.039) + 0.469 \ln(0.469/0.497) + 0.455 \ln(0.455/0.465)$ $D(f_M || f_O) = 0.01733$

 $D(f_o \parallel f_M) = 0.039 \ln(0.039/0.076) + 0.497 \ln(0.497/0.469) + 0.465 \ln(0.465/0.455)$ $D(f_0 || f_M) = 0.02132$

 The fact that these values are found to be close demonstrates that both working positions show a similar distribution. Likewise, the relative entropy values found for positions according to OSA, ASA and GA are found in the following way.

CHAPTER FIVE CONCLUSION

 The analyses performed in this study prove useful to find the degree of uncertainty and to determine the number of questions in a selected scale with entropy method. It was found out that if we only want to be informed about the level of customer satisfaction, the number of questions in the scale to be designed has to be fewer while the number of questions concerned has to be increased if it is desired to determine the level of customer satisfaction together with gender.

 For other studies, the survey can be reorganized by desingning the scale with a new number of questions determined by the entropy method and reliability analyzes can be made again and information on customer satisfaction can be accessed in a shorter period of time.

 In addition to all these, the entropy values were interpreted within the scope of the information theory and various recommendations were made for the researchers, who may apply such a study in the future, pertaining to the number of questions of the new scales to be designed sa as to rapidly access information about customer satisfaction.

REFERENCES

- Cover, T. M., & Thomas, J.A. (2006). *Elements of Information Theory (2nd ed.).* John Wiley & Sons: New Jersey.
- Kullback S. (1987). The Kullback Leibler distance. *The American Statistician (41), 340 – 341.*

Shannon, E. C. (1948). A Mathematical Theory of Communication. *The Bell System Technical Journal (27), 379 – 423.*

Ege Oruç, Ö., Kuruoğlu, E., & Vupa, Ö. (2009). An application of entropy in survey scale. *Entropy, 2009 (11), 598 – 605; DO*İ*: 10. 3990 / E11040598.*

Rea, L. M., & Parker, R. A. (2005). *Designing and Conducting Survey Research: A Comprehensive Guide (3rd ed.).*John Wiley & Sons.

Reh, J.F. (2009). *Customer Satisfaction Survey.* Retrieved January 10, 2010, from http://management.about.com/od/competitiveinfo/a/CustomerSatSurv.htm.

- Hill, N., Brierley, J., & Mac Dougall, R. (2003). *How to Measure Customer Satisfaction (2nd ed.).* Gower Pub Co.
- Von Baeyer, H.C. (2004). *Information The New Language of Science.* Harvard University Press.

APPENDIX

Appendix A Results of Reliabilty Analysis (SPSS 13.0)

Case Processing Summary

a. Listwise deletion based on all variables in the procedure.

Reliability Statistics

Item Statistics

Item-Total Statistics

Scale Statistics

Appendix A Continue

Grand Mean $= 4.40$

a. Tukey's estimate of power to which observations must be raised to achieve additivity = 3.124.

b. The covariance matrix is calculated and used in the analysis.

Hotelling's T-Squared Test

The covariance matrix is calculated and used in the analysis.

Case Processing Summary

a. Listwise deletion based on all variables in the procedure.

Reliability Statistics

Item Statistics

Appendix A Continue

Item-Total Statistics

Scale Statistics

ANOVA with Friedman's Test and Tukey's Test for Nonadditivity

Grand Mean $= 4.40$

a. Tukey's estimate of power to which observations must be raised to achieve additivity = 3.124 b. The covariance matrix is calculated and used in the analysis.

Hotelling's T-Squared Test

The covariance matrix is calculated and used in the analysis.

Case Processing Summary

a. Listwise deletion based on all variables in the procedure.

Appendix A Continued

Reliability Statistics

Item Statistics

Item-Total Statistics

Scale Statistics

ANOVA with Friedman's Test and Tukey's Test for Nonadditivity ^b

Grand Mean $= 4.58$

a. Tukey's estimate of power to which observations must be raised to achieve additivity = 16.201.

b. The covariance matrix is calculated and used in the analysis.

Hotelling's T-Squared Test

Hotelling's T-Squared		ᆦ	ਮਿΩ	
.093	.537			

The covariance matrix is calculated and used in the analysis.

Appendix B Custumer Satisfaction Scale

