DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

BUSINESS PROCESS REENGINEERING IN SUPPLY CHAIN MANAGEMENT AND SUPPLIER SELECTION WITH DEA-AHP AT VESTEL ELECTRONICS

by
Onur METE

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İZMİR

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We have read the thesis entitled **BUSINESS PROCESS REENGINEERING IN SUPPLY CHAIN MANAGEMENT AND SUPPLIER SELECTION WITH DEA-AHP AT VESTEL ELECTRONICS** completed by **ONUR METE** under supervision of **Doç.Dr. LATİF SALUM** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

Companies are developing to provide the desired products and services to customers faster, cheaper and qualified in order to survive in continuously improving competition conditions. Managers come to realize that supply chain management system has to succeed in order to be competitive in the market. Supplier selection process has gained importance at supply chain management, since the cost of raw materials and component parts constitute the main cost of a product and most of the firms have to spend considerable amount of their revenues on purchasing.

Selecting the right suppliers can significantly reduce the purchasing costs and improve competitiveness. As technological complexity increased, supplier selection has become more dynamic and complex to analyze, and to solve. Selection process involves considering a series of strategic variables, performance criteria and alternatives.

In this thesis, Data Envelopment Analysis (DEA) with Analytic Hierarchical Process (AHP) method is used for supplier selection at a television company. Screen label product group is selected. Business process reengineering is used before supplier selection analysis because supplier selection will be meaningful with setting up strong structure on that product group. The improvement opportunities are evaluated and successfully applied for screen label product group. The proposed algorithm has been designed for sequencing the multi input and output units. In the initial step of the algorithm, all inputs and outputs of the units are formulated using DEA. These models are, then, decomposed in the LINDO package program; later in the second step, pair-wise comparison values with the help of AHP, are sequenced using the affection of binary correlation matrix.

Keywords : Business Process Reengineering, Supplier Selection, Data Envelopment Analysis, Analytic Hierarchy Process

VESTEL ELEKTRONİK ŞİRKETİNDE TEDARİK ZİNCİRİ YÖNETİMİ DEĞİŞİM MÜHENDİSLİĞİ UYGULAMALARI VE VZA-AHS İLE TEDARİKÇİ SEÇİMİ

ÖZ

Artan rekabetçi koşullarda ayakta kalabilmek için firmalar müşterilere istedikleri ürün ve hizmeti daha hızlı daha ucuz ve daha kaliteli olarak sunmak için sürekli gelişme içerisindedirler. Yöneticiler, markette rekabetçi kalabilmenin tedarik zinciri yönetiminin başarılı olmasıyla ortaya çıkacağının farkına vardılar. Hammadde ve yarı mamul maliyeti ürün maliyetinin büyük bir bölümünü oluşturduğu için birçok firma elde ettiği kazancın büyük bir bölümünü malzeme maliyetine yatırmak zorunda kalmaktadır, dolayısıyla tedarikçi seçimi, tedarik zinciri yönetiminde büyük önem kazanmıştır.

Doğru tedarikçileri seçmek satın alma maliyetlerini düşürerek rekabet gücünü geliştirmektedir. Globalleşme arttıkça tedarikçi seçimi daha dinamik ve karmaşık hale gelmiştir ve bu problem birçok stratejik değişkeni ve performans kriterlerini birlikte değerlendirmeyi gerektirir.

Bu tezde, televizyon üreticisi bir şirkette tedarikçi seçimi yapılmış olup Veri Zarflama Analizi (VZA) ve Analitik Hiyerarşi Süreci (AHS) kullanılmıştır. Ekran etiketi ürün grubu seçilmiştir. Ürün grubunun sağlam temellerinin olması, tedarikçi seçimini anlamlı kılacağı için değişim mühendisliği çalışmaları tedarikçi seçiminden önce uygulanmıştır. Değişim mühendisliğinde ekran etiketi ürün grubu için iyileştirme fırsatları değerlendirilmiş ve başarılı şekilde uygulanmıştır. Önerilen algoritma, çok girişli ve çıkışlı birimleri tam sıralamak için tasarlanmıştır. Algoritmanın ilk aşamasında, birimlere ait tüm girdi ve çıktılar VZA tarafından formüle edilmiştir. Bu modeller LINDO paket programında çözülmüş, ikinci aşamada, ikili karşılaştırmalı değerler AHS yardımıyla, tedarikçiler arası ikili ilişkiler matrisinden etkilenerek sıralanmıştır.

Anahtar sözcükler: Değişim Mühendisliği, Tedarikçi Seçimi, Veri Zarflama Analizi, Analitik Hiyerarşi Süreci

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CHAPTER ONE
INTRODUCTION

Companies maintain their production operations in a very competitive environment. High quality and high service ability are requested by the market. On the other hand, cost is the main focus area for the companies. The tough competition and conditions in the market force companies to seek new ways to use their resources more effectively and increase the performance of their systems.

Many companies understand the importance of the relations between suppliers and customers in order to be more competitive in the market and maintain the market share. Companies can achieve lower purchasing costs, better quality, and flexibility in production and customer satisfaction by cooperative efforts with suppliers. Companies can satisfy exactly what customers need with the correct project scopes. For these reasons, companies opt for restructuring their systems for better supplier and customer integration, and creation of value.

Nowadays, customers have power to impose price and quality terms on suppliers. It is crucial for companies to meet market demands on the right conditions and at the right time in the global market. Exploiting its resources in the most effective way is the main target for a manufacturer company. In this manner, companies have to control their product sales prices, all types of costs, and their performances. It may not be always enough to improve internal operations for a company to reach the targets in terms of cost and performance. At the same time, companies have to improve their relations with their suppliers for a more effective supply chain. On average, manufacturers' purchases of goods and services constitute up to 70% of total product cost, and in high-technology firms, purchased materials and services represent up to 80% of total product cost (Ghodsypour and O'Brien, 2001). As a result, companies have to focus on supplier selection in order to maintain their operations in an effective way.

In order to be more effective, companies focus on their core functions, and outsource other functions. Supplier relations become significant when companies outsource most of their functions. These activities also increase a company's commitment to the suppliers and the importance of supplier selection systems.

On the other hand, if there is not a very serious capacity problem, companies prefer to choose limited number of suppliers depending on the requested quantities and capacities of the suppliers and improve relationship with them. This strategy also increases the importance of the supplier selection system. When companies work with limited number of suppliers, they are forced to give more business to suppliers. As a result, the impact of suppliers on the success of the business is increasing.

Supplier selection decision is one of the main decisions of companies. The aim of the supplier selection problem is the determination of the most suitable supplier in terms of a company's targets. Supplier selection decision requires making decisions subject to many controversial and numeric criterions.

The importance of the supplier selection decision is related with the share of the associated cost of the material that will be supplied from that supplier to the total cost of the product. Supplier selection decision will have more importance if the share of the cost of the material is high.

Because of the fact that purchases of goods and services constitute the biggest share of the total product costs, the purchasing and procurement activities have a big importance on the profitability of the company. The components will be supplied at right quantity and quality, and time with a good price with the effective management of purchasing and procurement activities (Dobler, 1996).

In recent years, companies are experiencing a rapidly increasing technological changes and market conditions that require the production of higher quality goods and services. Supplier selection and management systems have been forced to change in these circumstances (Nassimbeni, 2003). Now, companies do not evaluate their suppliers only as a supplier of components or services. They evaluate their suppliers as their business partners and include them in all internal processes starting with the new product development. This is another factor that increases the importance of supplier selection decision in a supply chain management system.

One of the supplier selection techniques is Data Envelopment Analysis (DEA). DEA is a linear mathematical model that allows measuring the efficiency of decision units. A DEA result shows (in)efficient suppliers with respect to the others. However, the result does not indicate which one is the most efficient. Thus, additional decision analysis should be done in order to rank all suppliers efficiency.

The most useful feature of DEA is to show inefficiency result of decision units (suppliers) and the reasons of this result. For example, a result table indicates that some amount should be decreased from inputs or increase outputs in order to get efficient suppliers.

Business Process Reengineering (BPR) helps organizations mainly rethink the whole business process to get operational cost down and quality improvement in order to become competitive in the market. BPR should be applied in order to set up a stronger structure in any kind of business.

The aim of this research is to analyze BPR application opportunities and support improved system with a supplier selection method. DEA-AHP analysis is chosen for this election action.

In this research, BPR analysis and applications are carried out in order to provide a strong basis for supplier selection analysis. Thus, in addition to Business Process Reengineering, Data Envelopment Analysis is applied. However, DEA alone does not give sufficient information for ranking suppliers. Thus, DEA-AHP application is carried out for ranking supplier efficiency that is explained in detail in chapter four.

CHAPTER TWO
BUSINESS PROCESS REENGINEERING (BPR)

In this chapter, Business Process Reengineering history, definition of BPR, how to implement a BPR project, tools to support BPR, BPR team and missions, BPR techniques, and success and failure factors of BPR are introduced. The aim of this chapter is to give theoretical information about BPR in order to provide a basis for supplier selection analysis. Supplier selection would be meaningless, if the sound basis (strong structure) were not set up for that product group.

2.1 History of Business Process Reengineering

In 18th century with industrial revolution, the first philosophy starts with Adam Smith. Production process is divided into pieces to improve the entire process. Later on, Henry Ford applied this philosophy to assembly/manufacturing and Alfred Sloan applied it to management. Mass production and division of labor build up on this philosophy. Reengineering tries to change all of these concepts and basic methodology and gives a new perspective.

As a result of the world's economic revolution, customers become the dominant power in their sector. Customers have lots of choice. Products and services lifetime get shorter, as well as development of new product duration. Thus, engineers decide that long term success can be gained by the whole process improvement. Adam Smith's division of labor model gives place to the whole process oriented approach. The important point is to work unconventional, rather than to work too much. The question one should ask is how to recreate this company with current knowledge and technology.

After 1980's, customers have lots of choice that brings about much more expectations. Moreover, without frontier sales causes trouble for firms. Thus, firms are obliged to think to review their business processes according to customer needs in order to survive. Also, experiences show that quality, cost, flexibility, innovativeness, speed and service oriented approach provides competitiveness in the market. In particular, Total Quality Management is chosen by Japanese Businessmen in 1980's. However, in 1990's, TQM alone was not satisfactory. Therefore, investigation of new techniques for a process was vitally important. As a result, BPR applications started after 1990's.

BPR (Business Process Reengineering) theory emerged in the United States in 1988, which was put forward firstly on Harvard Business Review by the famous management guru Michael Mr. Hammer, who was the former Massachusetts Institute of Technology professor. BPR was then used as some kind of emerging management thinking about innovation. BPR regards organizing the process as the starting point, thinks activity value of each contribution in the production process fundamentally, uses modern information technology, changes human and the course of its work completely, and re-establishes the relationship between organizations at all levels (Lui and Guo, 2010).

Recent conditions show that firms working conditions obey Adam Smith's division of labor principle. The main idea is to divide a project into simple and basic pieces in order to decrease total process time. Henry Ford applied this principle to production lines and moving assembly lines. Moreover, Alfred Sloan, who was the owner of General Motors, applied Smith's principle to management operations for the first time. After Second World War, new regulations showed up from Ford, ITT and GE. Firms divide top management's and inspector's role into pieces (Hammer and Champy, 1993).

Nowadays, almost every company has different functional departments. Similar processes are integrated into one department. So, different functional departments show up like marketing, financing, production etc. Departments help the whole mechanism for getting raw material to final product. This mechanism is called organizational process. Reengineering is the rethinking the whole organizational process with effectiveness and productivity point of view.

In 1990's, computer technology was in progress and helped radical improvements on business management. Speed was the key point for production sector in those days. Western business firms wanted to get fast reaction from all improvement action. Thus, Reengineering was the most famous and remarkable idea in trials. Especially US and European companies started to use Reengineering Methodology after 1990’s (Gadd and Oakland, 1995).

All in all, reengineering starts to be applied in business world in US and Europe after 1990’s. Reengineering applications prove that the idea gives fast and positive reaction. Thus, this makes it most popular. Today, reengineering applications increasingly continue.

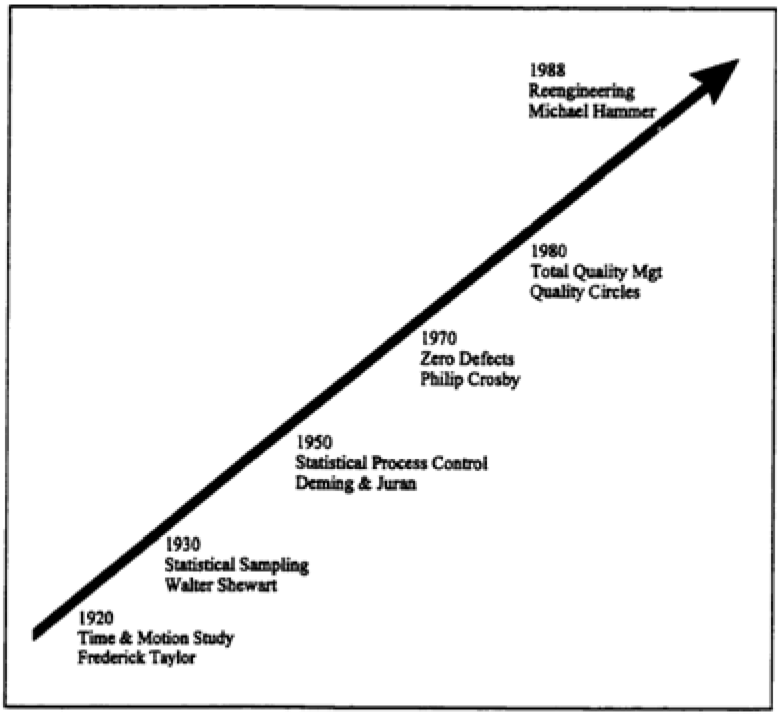

Figure 2.1 TQM and BPR in context of changing management philosophies (Metchick, 1999)

2.2 What is Reengineering?

Business Process Reengineering (BPR) is the way of analyzing processes and improves the business outcome. BPR helps organizations mainly rethink the whole business process to get operational costs down and quality improvement in order to become competitive in the market.

Many firms adopted Total Quality Management (TQM) in the 1980’s hoping to win back business lost to the Japanese (Hammer, 1990). Two of these firms, Ford and Xerox, realized that incremental improvement from the traditional TQM approach was not enough to overcome their high infrastructure costs. They started rethinking and making radical changes over their business processes. These changes and similar efforts by four other major firms were reported in two articles (Hammer, 1990; Davenport and Short 1990). The term Business Process Reengineering had been introduced to the world, and TQM was criticized as no longer adequate in the increasingly dynamic competitive environment (Harrington, 1991).

In today's ever-changing world, the only thing that does not change is 'change' itself. This world is increasingly driven by the three Cs; Customer, Competition and Change. Companies are on the lookout for new solutions for their business problems. Recently, some of the more successful business corporations in the world seem to have hit upon an incredible solution: BPR.

Some of the recent headlines in the popular press read, Wal-Mart reduces restocking time from six weeks to thirty-six hours. Hewlett Packard's assembly time for server computers decreases to four minutes. Taco Bell's sale soars from $500 million to $3 billion. The reason behind these success stories: Business Process Reengineering (Muthu and Whitman, 1999).

2.3 How to Implement a BPR project

The best way to map and improve an organization's procedures is to take a top down approach, and not undertake a project in isolation. That means:

✓ Starting with mission statements that define the purpose of the organization and describe what sets it apart from others in its sector or industry.

✓ Producing vision statements which define where the organization is going, to provide a clear picture of the desired future position.

✓ Building these into a clear business strategy thereby deriving the project objectives.

✓ Defining behaviors that will enable the organization to achieve its' aims.

- ✓ Producing key performance measures to track progress.
- ✓ Relating efficiency improvements to the culture of the organization.
- ✓ Identifying initiatives that will improve performance.

Once these building blocks are in place, the BPR exercise can begin (Carter, 2005).

2.3.1 A Five Step Approach to Business Process Reengineering

Davenport (1990) prescribes a five-step approach to the Business Process Reengineering model:

1. Develop the business vision and process objectives: The BPR method is driven by a business vision which implies specific business objectives such as cost reduction, time reduction, output quality improvement.

2. Identify the business processes to be redesigned: most firms use the 'high-impact' approach which focuses on the most important processes or those that conflict most with the business vision. A lesser number of firms use the 'exhaustive approach' that attempts to identify all the processes within an organization and then prioritize them in order of redesign urgency.

3. Understand and measure the existing processes: identify and observe the whole business steps to avoid the repeating of old mistakes and to provide a baseline for future improvements with comparing Scientific Management.

4. Identify IT levers: awareness of IT capabilities can and should influence BPR.

5. Design and build a prototype of the new process: the actual design should not be viewed as the end of the BPR process. Rather, it should be viewed as a prototype, with successive iterations. The metaphor of prototype aligns the Business Process Reengineering approach with quick delivery of results, and the involvement and satisfaction of customers.

As an additional 6th step of the BPR method, sometimes you adapt the organizational structure and the governance model, towards the newly designed primary process.

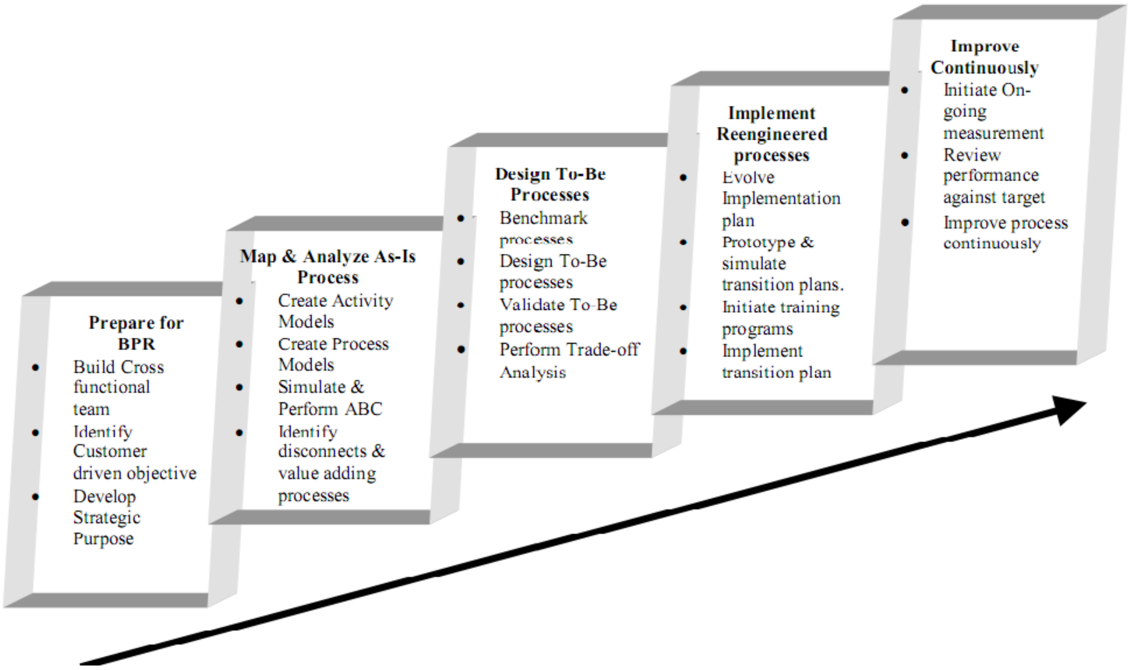

Figure 2.2 Business process reengineering application map (Muthu and Whitman, 1999)

2.4 Tools to Support BPR

When a BPR project is undertaken across the organization, it can require managing a massive amount of information about the processes, data and systems. If you do not have an excellent tool to support BPR, the management of this information can become an impossible task. The use of a good BPR documentation tool is vital in any BPR project.

Carter (2005) mentions on the types of attributes that should look for in BPR software:

✓ Graphical interface for fast documentation

✓ Object oriented technology, so that changes to data (e.g. job titles) only need to be made in one place, and the change automatically appears throughout all the organization's procedures and documentation

- ✓ Drag and drop facility so you can easily relate organizational and data objects to each step in the process

- ✓ Customizable meta data fields, so that you can include information relating to your industry, business sector or organization in your documentation

- ✓ Analysis, e.g., swim-lanes, to show visually how responsibilities in a process are transferred between different roles, or where data items or computer applications are used.

- ✓ Support for Value Stream mapping

- ✓ CRUD or RACI reports, to provide evidence for process improvement

- ✓ The ability to assess the processes against agreed international standards

- ✓ Simulation software to support 'what-if' analyses during the design phase of the project to develop LEAN processes

- ✓ The production of word documents or web site versions of the procedures at the touch of a single button, so that the information can be easily maintained and updated

2.5 BPR Team and Missions

The BPR project team is appointed at the start of a BPR project at the start of the project by the steering group.

BPR team should consist of the following;

1. Senior User
2. Lead BPR Consultant
3. Other BPR Consultants
4. Practice Representative(s) (service users)
5. Human Resource Expert

6. IT Expert

The various members of the team will bring different skills to the project but will act as a true team, sharing tasks and responsibilities.

The senior user and lead BPR consultant will usually be the link between this team and the steering group, and will also be instrumental in setting up the team, e.g., by 'seconding' service users to the team.

The BPR consultants will have expertise in BPR work, whilst the practice representatives will know their business area. This sharing of knowledge and experience is essential to a successful BPR project. Whilst the BPR experts can train users, the users' experience of the business area will be critical. For example, they can start questioning the processes common to their day-to-day work by training users in how to draw up Role Activity Diagrams.

Other members of the project team, such as IT and HR, may also be given a central role, or be called in as necessary.

The BPR project team will tend to meet informally, but regularly (Sprint, 2009).

2.6 BPR Techniques

The goal of BPR is radical improvement of a process. Specific tools and techniques are used to reach a total improvement. Best reengineering applications show that authors and consultants prefer many different tools under the name of BPR. These tools and techniques include the following;

✓ **Process visualization.** While many authors refer to the need to develop an ideal end state for processes to be re-engineered, Barrett (1994) suggests that the key to successful reengineering lies in the development of a vision of the process.

Achieving organizational transformation depends on the creation of a powerful vision of what the future should be like. An effective vision derives its power from its ability to guide and motivate the BPR team and the organization at large. A compelling vision must be clearly defined and then communicated if there is to be

BPR success. The right BPR vision is difficult to create. The challenge is to be able not only to mentally formulate a compelling future end-state image, but to create one that can be communicated to the various members of the reengineering team and then shared and tested with other key stakeholders as well. To accomplish this, process visualization should be described in two different ways: first a narrative description, and second a graphical depiction. BPR offers unprecedented opportunities for organizations to reconfigure themselves to be more effective, efficient, and humane. Organizations have the capacity to create entirely new ways of working that are wholly committed to customer satisfaction and that eliminate needless bureaucracy and non-value-adding activities from the organization (Barrett, 1994).

✓ **Process mapping/operational method study.** Cypress (1994) suggests that the tools of operational method studies are ideally suited to the reengineering task, but that they are often neglected. Recent evidence suggests that these concepts have been incorporated into tools such as IDEF0 (Integrated Definition Method), DFD (Data Flow Diagrams), and OOA(Object Oriented Analysis) (Yu and Wright, 1997).

✓ **Change management.** Several authors concentrate on the need to take account of the human side of reengineering, in particular the management of organizational change. Some authors (e.g. Mumford and Beekma, 1994; Bruss and Roos, 1993) suggest that the management of change is the largest task in reengineering.

✓ **Benchmarking.** Several authors suggest that benchmarking forms an integral part of reengineering, since it allows the visualization and development of processes which are known to be in operation in other organizations (Harrison and Pratt, 1992; Chang, 1994; Furey, 1993).

✓ **Process and customer focus.** The primary aim of BPR, according to some authors, is to redesign processes with regard to improving performance from the customer's perspective (Chang, 1994). This provides a strong link with the process improvement methodologies suggested by authors from the quality field, such as Harrington (1991). In some cases, notably in Chang (1994), the terminology is almost identical to that used by quality practitioners in the improvement of processes.

The major difference, as outlined earlier, appears to be one of scale (Neill and Sohal, 1998).

Most consultants prefer to apply mixture of these techniques for improvement. Strategic and cross functional activities have to be integrated with other aspects of management. Furthermore, Total Quality Management tools and techniques should be run parallel to BPR for the long term improvement. Most consultants consider TQM while revising the whole process and integrate to BPR process maps.

In summary, therefore, BPR can be seen to represent a range of activities concerned with the improvement of processes. While some authors appear to suggest that tools and techniques are the key, most authors suggest that a strategic approach to BPR and the development of a BPR strategy is the key to success (Guha et al. 1993).

2.7 Success Factors of BPR

Among the main success factors are support from top management, ambitious objectives, deployment of a creative team in problem solving, and a process approach and integration of electronic data processing (EDP) (Peppard and Fitzgerald, 1997). Ascari et al. (1995) have discussed four other elements leading to successful BPR:

- Culture
- Processes
- Structure
- Technology

A study by Ascari (1995) found that the companies that implemented BPR agreed that its impact on the change of their culture was related to the organization’s rethinking of its fundamental business processes. The focus was also on identifying and improving the core processes.

Success factors that lead to successful outcomes for reengineering projects are introduced below.

2.7.1 Top Management Sponsorship (strong and consistent involvement)

Significant changes to even one of these areas require resources, money, and leadership. Implementation efforts can be strongly resisted and ineffective if top management does not support.

Reengineering managers or consultants give much attention to this topic because every staff can be affected directly or indirectly. So, no one accept to snooping someone to their work. Therefore, someone has to warn and tell the benefit of BPR to workers in order to get success.

2.7.2 *Strategic Alignment (with company strategic direction)*

Strategic structure represents mission and vision of the company. Change has to start from main structure of company. Strategic directions of the organization should be connected with reengineering project goals. This connection should thread from top to down. So, every personnel should remember the company's strategic vision while doing his work.

2.7.3 Business Case for Change (with measurable objectives)

Reengineering idea should be summarized to top management as one page or less. This page should include only the aim of change and advantage/gain of this change.

2.7.4 Proven Methodology (that includes a vision process)

Several BPR mythologies and applied cases should be investigated and proven. Implementation of BPR can proceed under the leadership of proven cases.

2.7.5 Effective Change Management (address cultural transformation)

Cultural transformation cannot be forced. The meetings should be arranged that includes all related workers. The important point is that meetings should be honest and frequent communication.

One of the most overlooked obstacles to successful project implementation is resistance from those whom implementers believe will benefit the most. Thus, meetings should be done for breaking taboos.

The better you manage the change, the less pain you will have during the transition, and your impact on work productivity will be minimized.

2.7.6 Line Ownership (pair ownership with accountability)

The terms of engagement and accountability must be clear. The ownership must ultimately rest with the line operation, whether it is manufacturing, customer service, logistics, or sales.

BPR is a rescue operation. Unfortunately, the ability of external consultants to implement significant change in an organization is small. The chances are only slightly better for staff groups. Ultimately, the solution and results come back to those accountable for day to day execution.

2.7.7 Reengineering Team Composition (in both breadth and knowledge)

The reengineering team composition should be a mixed bag. For example,

- Some members who do not know the process at all,
- Some members that know the process inside-out,
- Include customers if you can,
- Some members representing impacted organizations,

- One or two technology gurus,
- Each person your best and brightest, passionate and committed, and
- Some members from outside of your company.

Moreover, keep the team under 10 players. If you are finding this difficult, give back some of the representative members. Not every organization should or needs to be represented on the initial core team. If you fail to keep the team a manageable size, you will find the entire process much more difficult to execute effectively.

Seven reengineering success factors have been introduced in this module. Subsequent modules in this series written by industry experts will take several of these success factors into greater depth (Prosci, 1996).

2.8 Failure Factors of BPR

Beside the success factors, many authors also highlighted some failure factors in implementing BPR. Aggarwal (1998) highlighted failures of BPR implementation, which were related to managers' arrogance, resistance, crisis, cost, vision, etc. Hammer and Champy (1993) highlighted some failure factors like failure to have a process perspective, a fixed process which is not flexible enough to be responsive to the needs and requirements, not involving employees (i.e., bottom-up) in decision making, assigning someone who does not understand BPR, technology limitations, designing a project but with focus on cost reduction and downsizing, having a weak team, and problems with communication.

Lack of financial and human resources, lack of strategic vision and mission, inflexible organizational structure, inadequate information technology capabilities and lack of champion for BPR efforts are the main problems that can be tackled.

2.9 Reengineering Recommendations

- BPR must be accompanied by strategic planning, which must address leveraging IT as a competitive tool.

- Place the customer at the center of the reengineering effort and concentrate on reengineering fragmented processes that lead to delays or other negative impacts on customer service.

- BPR must be owned throughout the organization, not driven by a group of outside consultants.

- Case teams must be comprised of both managers as well as those who actually do the work.

- IT group should be an integral part of the reengineering team from the start.

- BPR must be sponsored by top executives, who are not about to leave or retire.

- BPR projects must have a timetable, ideally between three to six months, so that the organization is not in a state of limbo.

- BPR must not ignore corporate culture and must emphasize constant communication and feedback (Weicher et al., 1995).

CHAPTER THREE
SUPPLIER SELECTION AND DATA ENVELOPMENT ANALYSIS

In this chapter, definition of supply and supply chain management, the role of purchasing within the supply chain, stages of supplier selection process, information about Data Envelopment Analysis (DEA), weighted linear DEA model, and DEA based ranking applications and information about DEA-AHP will be introduced. The aim of this chapter is to give detailed theoretical information about supplier selection and data envelopment analysis.

3.1 Supplier Selection

Many factors in today's global markets have influenced companies to search for a competitive advantage by focusing on their entire supply chain. Various activities involve in supply chain management. Purchasing is one of the most strategic points because it provides opportunities to reduce costs, and consequently, increase profits. An essential task within the purchasing function is supplier selection.

Companies need to work with different suppliers to maintain their activities. In manufacturing industries, raw materials and component parts can equal up to 70% of product cost. In such circumstances, the purchasing department can play a key role in cost reduction, and supplier selection is one of the most important functions of purchasing management (Ghodsypour and O’Brien, 1998).

For many years, the traditional approach to supplier selection has been to select suppliers solely on the basis of price (Degraeve and Roodhooft, 1996). However, as companies have learned that the sole emphasis on price as a single criterion for supplier selection is not efficient, they have turned into a more comprehensive multi-criteria approach. Recently, these criteria have become increasingly complex as environmental, social, political, and customer satisfaction concerns have been added to the traditional factors of quality, delivery, cost, and service (Mendoza, 2007).

Several factors may affect a suppliers' performance. Dickson (1966) identified 23 different criteria for vendor selection including quality, delivery, performance history, warranties, price, and technical capability and financial position. Hence, supplier selection is a multi-criteria problem which includes both tangible and intangible criteria, some of which may conflict.

Basically there are two kinds of supplier selection problem. In the first kind of supplier selection, one supplier can satisfy all the buyer's needs (single sourcing). The management needs to make only one decision about which supplier is the best. In the second type (multiple sourcing), no supplier can satisfy all the buyer's requirements. In such circumstances management wants to split order quantities among suppliers for a variety of reasons including creating a constant environment of competitiveness (Huo and Wei, 2008).

3.2 What is Supply?

The total amount of a product (goods or service) available for purchase at any specified price is called supply.

Supply is determined by price, cost and price of other goods:

✓ Price: The amount that customers are willing to pay for getting products or services. Producers will try to obtain the highest price whereas the buyers will try to pay the lowest possible price, both settling at the equilibrium price where supply equals demand.

✓ Cost of inputs: The lower the input price, the higher the profit at a price level. Then, more products will be offered at that price.

✓ Price of other goods: Lower prices of competing goods will reduce the price and the supplier may switch to more profitable products to reduce the supply cost.

3.3 Supply Chain Management

Supply chain is the network of organizations that are involved, through upstream and downstream linkages, in different processes and activities that produce value in the form of products and services delivered to the ultimate consumer (Christopher, 1992). In other words, a supply chain consists of multiple firms, both upstream (i.e., supply) and downstream (i.e., distribution), and the ultimate consumer.

Over the last years there has been an increasing interest in supply chain related issues. Chopra and Meindl (2007) formally define a supply chain as all the stages that are directly or indirectly involved in satisfying customer demand. These stages include customers, retailers, wholesalers/distributors, manufacturers, and suppliers. From an organizational standpoint, the supply chain comprises all functions involved in fulfilling customer's requirements and needs. These functions include purchasing, product development, marketing, operations, finance, and customer service. Figure 3.1 illustrates Supply Chain Management Cycle.

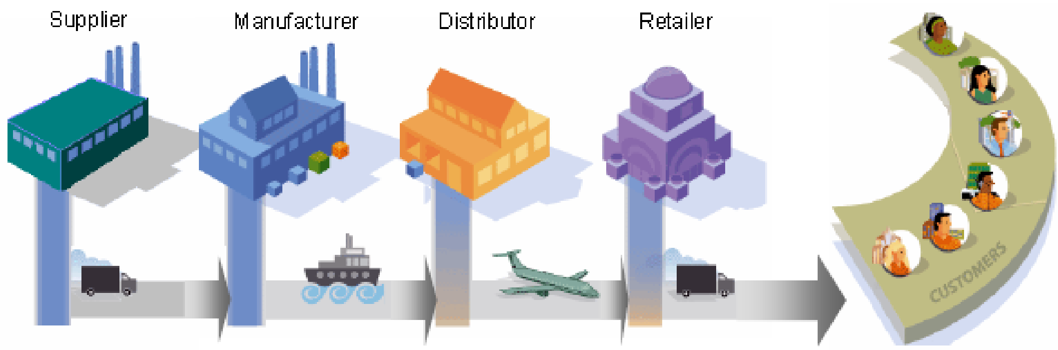

Figure 3.1 Supply chain management (Mendoza, 2007)

According to Panayides and Lun (2009), Supply Chain Management (SCM) is defined as encompassing the planning and management of all activities involved in sourcing and procurement, conversion, and all logistics management activities. Importantly, it also includes coordination and collaboration with channel partners, which can be suppliers, intermediaries, third party service providers, and customers. In essence, Supply Chain Management integrates supply and demand management within and across companies.

SCM is a concept that is gaining popularity and importance. From a practitioner point of view, SCM is critically important or very important to 89% of the surveyed executives. Furthermore, 51% of the executives in the report stated that their investments in SCM have increased significantly over the last three years (Naslund, 2010). SCM has also been frequently discussed and researched by practitioners and academics over the last two decades. Stock and Boyer (2009) describe how the number of SCM articles continues to grow on a yearly basis after the rapid surge that started in the middle of the 1990s. Additionally, the number of academic dissertations dealing with SCM-related topics has steadily increased since the early 1990s.

One reason for the increased interest in SCM is that organizations progressively find themselves reliant upon having effective supply chains, or networks, to successfully compete in the global market economy (Lambert, 2008). In the competitive global environment, performance can no longer solely be determined by the decisions and actions that occur within a firm as the execution of all members involved contributes to the overall results of the supply chain. Similarly, Wen et al. (2007) mean that competition has changed from being between individual enterprises to increasingly being between supply chains. As organizations form global alliances, it is imperative that they understand how SCM can be successfully applied especially as organizations face challenges including mitigating risks and disruptions in the supply chain (Neureuther, 2009). For these reasons, there is a need for companies to manage not only their own organizations but also their relationships with other companies in the same supply chain (Stock et al. 2010).

Naturally, another reason for the increased interest is the potential benefits of SCM. Benefits include improvement in returns on investments (ROI) and returns on assets (ROA). Ultimately, the goal of SCM is to achieve greater profitability by adding value and creating efficiencies, thereby increasing customer satisfaction (Stock and Boyer 2009). Costs decrease as a result of reduced redundancies, lower inventory levels, shorter lead time and lessened demand uncertainties. Improved process performance results in enhanced product quality, customer service, market responsiveness, and target market access (Tummala et al. 2006). Performance is thus improved through better use of internal and external capabilities creating a seamlessly coordinated supply chain, elevating inter-company competition to inter-supply chain competition (Lambert, 2008).

The three major flows that occur in a supply chain are physical, information, and money (Lee, 2000). Figure 3.2 shows a pictorial representation of these flows.

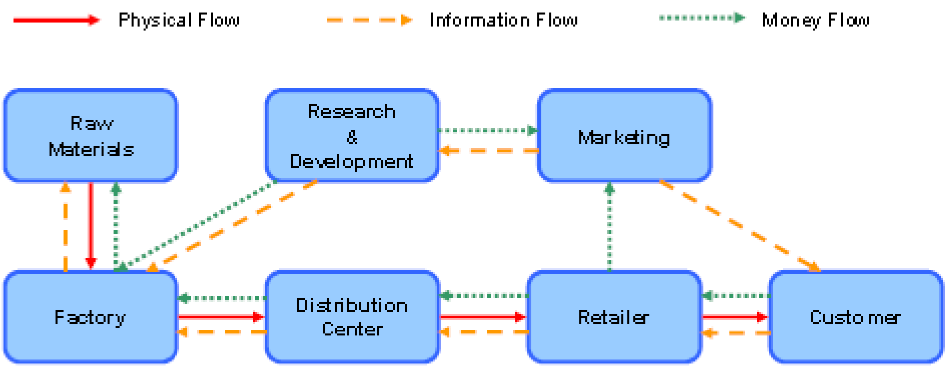

Figure 3.2 Supply chain flows (Mendoza, 2007)

To satisfy customer demand in a typical supply chain, raw materials are procured from diverse companies. These raw materials flow through a series of production and distribution stages until the final customer is reached with a finished product. This is what typically represents the physical flow. Next, in order to efficiently coordinate the physical flows in a supply chain, information flow plays an important role. Information flow involves transmitting orders and updating the status of delivery. For instance, information about customer demand must be available at each stage involved in the production and distribution process. Last, money flows from the customer upstream to each one of the stages involved in the supply chain. For example, customers transfer money to retailers and retailers transfer money to the distributors. Similarly, different transactions involving money take place across all the stages of the supply chain (Mendoza, 2007).

The goal of any supply chain is to maximize supply chain profitability. According to Chopra and Meindl (2007) profitability is defined as the difference between revenue generated from the customer and the overall cost across the supply chain.

Hence, the profitability in a supply chain is represented by the total profit shared across all members of the supply chain. This implies that the success of any supply chain should be measured as a whole and not as the success of each separate member involved. Consequently, in order to increase profitability, reduce costs, and improve customer satisfaction, effective supply chain strategies must take into account the individual stages of the supply chain as well as the interaction among them (Mendoza, 2007).

3.4 Role of Purchasing within the Supply Chain

Purchasing within an organization usually encompasses all activities related to the buying process. According to Van Weele (2005) these activities are: determining the need, selecting the supplier, arriving at a proper price, specifying terms and conditions, issuing the contract or order, and ensuring proper delivery. The increasing importance of supply chain management is motivating companies to fit purchasing and sourcing strategies into their supply chain objectives. Figure 3.3 illustrates the main activities within the purchasing function.

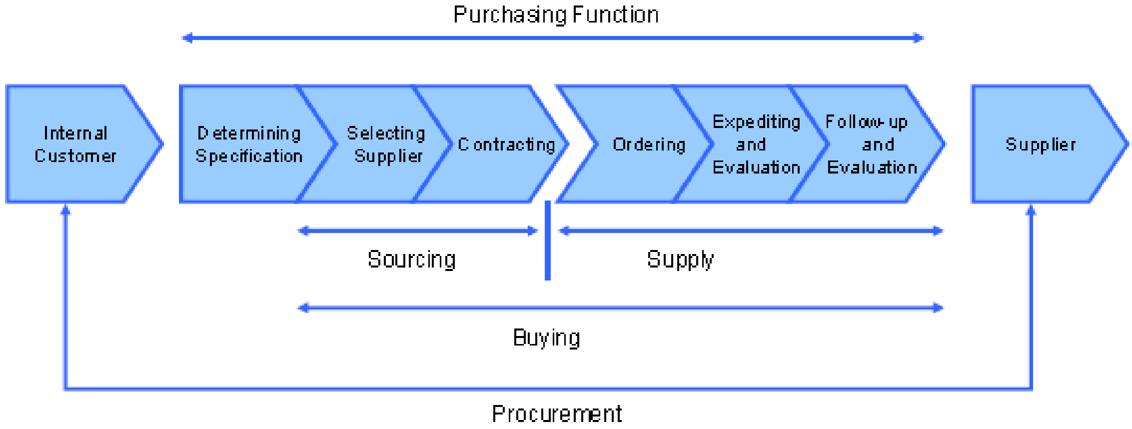

Figure 3.3 Purchasing function flow (Weele, 2005)

One of the issues in purchasing functions is to select suppliers capable of procuring the demanded items that meet the required specifications. Monczka et al. (2005) defined supplier selection as an essential task of purchasing. Moreover, Ellram and Carr (1994) concluded that purchasing plays a key role in corporate strategic success through the appropriate selection of suppliers supporting the company's long term strategy and competitive positioning.

As mentioned before, the cost of raw materials and component parts represents the largest percentage of the total product's cost in most industries. Figure 3.4 shows the impact of supplier’s cost of goods sold on a company's total cost.

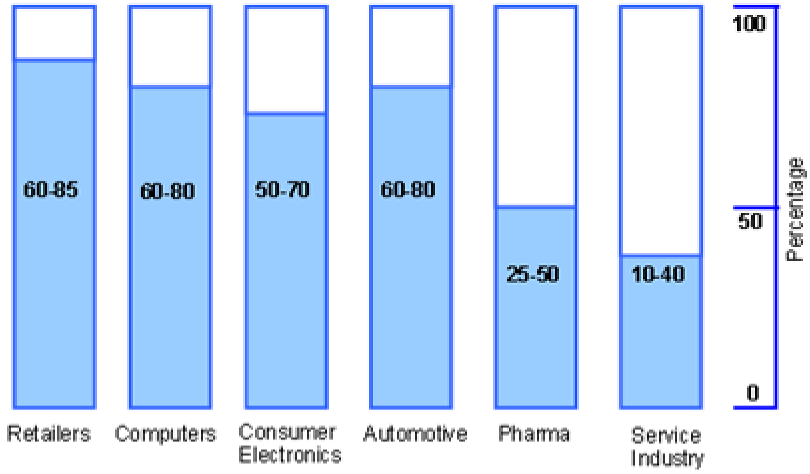

Figure 3.4 Purchased materials and services as a percentage of cost of goods sold (Weele, 2005)

According to Van Weele (2005), the cost of product for consumer electronics is 50% to 70% (Figure 3.4). For television sector which is involved in consumer electronics; this is true for Brand and OEM companies like Vestel(OEM), TPV(OEM), Samsung(brand), LG(brand), etc.

3.5 Stages of Supplier Selection Process

Supplier selection includes 7 steps. The quality of suppliers depends on the quality of these steps. These steps are presented in Figure 3.5.

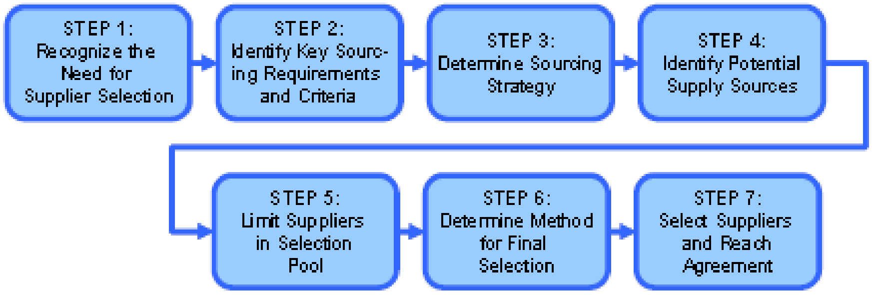

Figure 3.5 Supplier selection steps (Mendoza, 2007)

Step1 : Recognize the need for supplier selection

The first step in supplier selection usually implies the identification of the need for a specific product or service. Different situations may trigger the need for supplier selection. For example, new product development, modifications to a set of existing suppliers due to a bad performance, the end of a contract, expansion to different markets, and current suppliers' capacity that is not sufficient to meet forecast of demand. These situations are particular to every company (Mendoza, 2007).

Step 2: Identify Key Sourcing Requirements and Criteria

Supplier selection is complicated because of the multiple criteria involved in the decision process. Additionally, many times these criteria may conflict each other. Therefore, defining the proper criteria becomes critical.

Some of the most widely used criteria in supplier selection are supplier's capacity, quality, and purchasing price. However, the set of criteria to be chosen largely depends on the company's objectives and the type of industry in which the company competes.

Step 3: Determine Sourcing Strategy

Sourcing requires that companies clearly define the strategy approach to be taken during the supplier selection process. Examples of sourcing strategies are: single versus multiple suppliers, domestic versus international and short term versus long term supplier contracts.

This research assumes that single sourcing may not be an appropriate strategy in most purchasing situations. Single sourcing tends to minimize total costs by determining the best supplier for each purchased part or product. However, dependency on a single supplier exposes the buying company to a greater risk of supply interruption. An example of realized supply risk resulting from a single sourcing strategy is the case of Toyota's 1977 brake valve crisis. Toyota's assembly plants in Japan were forced to shut down for several days after a fire at its only supplier's (Aisin Seiki) main plant. This facility was the only source for valves that were used in all Toyota vehicles (Nishiguchi and Beaudet, 1998).The estimated cost of this single event was $195 million (70,000 units of production). Thereafter, Toyota sought at least two suppliers for each part (Treece, 1997).

Multiple sourcing strategies provide a greater flexibility due to the diversification of the firm's total requirements. In addition to ensuring product availability, working with multiple suppliers is important because suppliers are motivated to be competitive in factors such as price and quality (Jayaraman et al. 1999).

Step 4: Identify Potential Supply Sources

The importance of the item under consideration influences the resources spent on identifying potential suppliers. For example, major resources are spent when potential suppliers are needed for an item of high strategic importance (Monczka et al. 2005).

Step 5: Limit Suppliers in Selection Pool

Given the limited resources of a company, a purchaser needs to pre-screen the potential suppliers to reduce their number before proceeding with a more detailed analysis and evaluation. The supplier selection criteria determined in step two plays a key role in this reduction process. Howard (1998) defined this reduction process as selection of which suppliers satisfy expectations before further analysis.

Step 6: Determine a Method for Final Selection

There exist many different ways to evaluate and select suppliers. The important point is finding effective decision-making methodologies that capture important aspects of the supplier selection problem.

Step 7: Select Suppliers and Reach Agreement

The final step of the supplier evaluation and selection process is to clearly select those suppliers that best meet the company's sourcing strategy. This decision is often accompanied with determining the order quantity allocation to selected suppliers.

3.6 Data Envelopment Analysis

Data Envelopment analysis (DEA) is a linear programming based technique that converts multiple input and output measures into a single comprehensive measure of productive efficiency (Charnes et al. 1978).

Charnes, Cooper, and Rhodes (1978) have initiated a new nonparametric method, referred to as Data Envelopment Analysis (DEA), in order to measure the empirically derived relative efficiency of decision-making units (DMUs). Since the development of DEA, the estimation technique has been widely applied to various efficiency analyses in profit and nonprofit organizations.

DEA is an operations research-based method for measuring the performance efficiency of decision units that are defined by multiple inputs and outputs. DEA uses multiple inputs and outputs of a decision unit into a single measure of performance, generally referred to as relative efficiency. It is suggested that DEA may be used to assess retail/supplier productivity/efficiency and to address some of the problems with existing retail/supplier productivity measures. While traditional approaches are more appropriate for macro-level analysis, DEA is a micro-level or store-level productivity measurement tool that may have more managerial relevance (Yoo and Donthu, 1998).

Charnes, Cooper, and Rhodes (1978) were the first to propose the DEA methodology as an evaluation tool for decision units. Since then, DEA has been applied successfully as a performance evaluation tool in many fields including manufacturing, schools, banks, pharmacies, small business development centers, nursing home chains, maintenance units of the US Air Force, and hospitals.

In the marketing literature, Charnes (1985) first discussed potential applications of DEA. However, it has not been extensively applied in marketing. Kamakura, Ratchford, and Agrawal (1988) used DEA to measure market efficiency and welfare loss. Mahajan (1991) examined operations of insurance companies. Parsons (1990) and Boles, Donthu, and Lohtia (1995) studied performance of salespersons using DEA (Yoo and Donthu, 1998).

3.6.1 DEA Publication Statistic

Distribution of DEA publications is shown in Figure 3.6. DEA publications have a tendency to increase year by year.

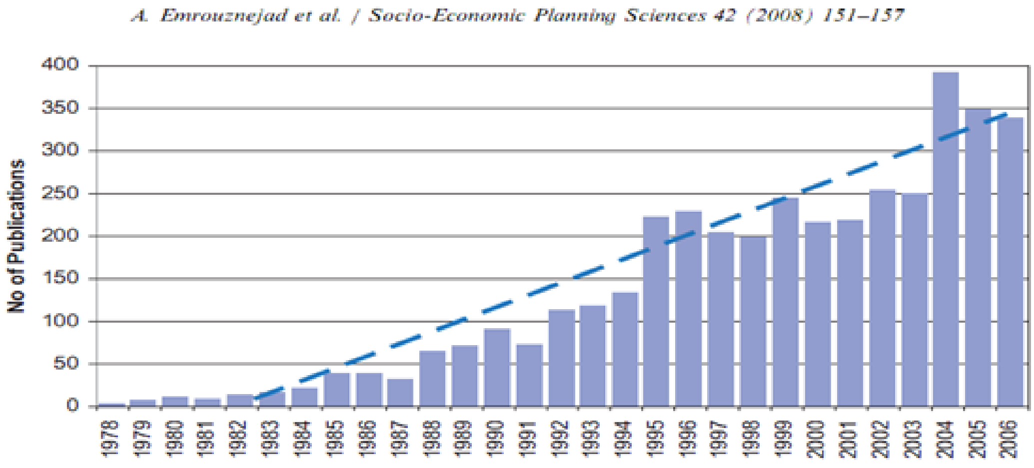

Figure 3.6 Distribution of DEA publications by year

Figure 3.7 shows well-known authors in Data Envelopment Analysis research.

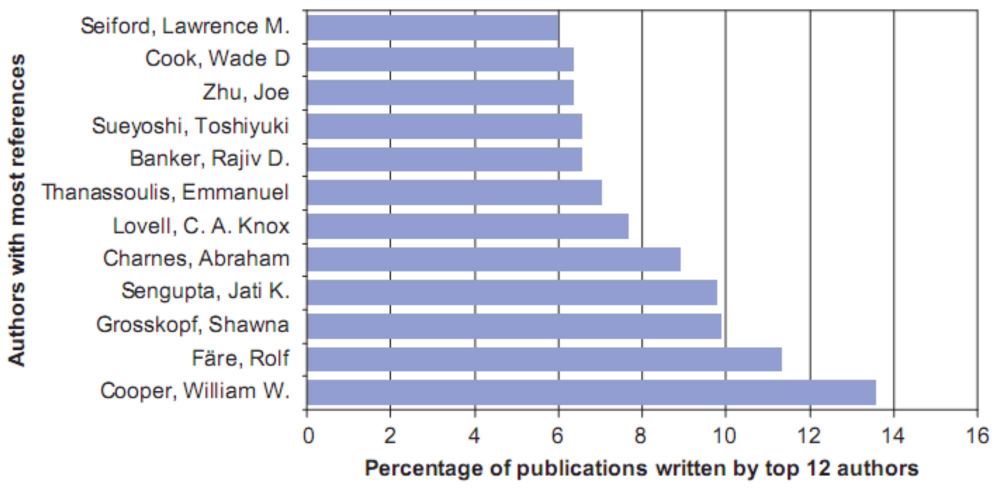

Figure 3.7 Percentage of publications written by top 12 authors

Table 3.1 shows the research area types and how many publications it has.

Table 3.1 List of the most popular keywords by number of publication

Keywords	No. of publications	Keywords	No. of publications
DEA or Data Envelopment Analysis	1637	Mathematical programming	118
Efficiency	558	Optimization	112
Decision making unit(s)	392	Health care or hospital	103
Linear programming	341	Multivariate analysis	89
Decision theory	269	Production	84
Mathematical models	216	Parametric	80
Productivity	215	Benchmarking	78
Operations research	215	Regression analysis	76
Economics	192	Production control	73
Management	181	Statistical models or methods	72
Performance (management, or evaluation)	176	Humans resource allocation	61
Bank or banking	135	Statistical analysis	58
Nonparametric	120	Education	44
Technical efficiency	120	Nonparametric statistics	40

3.7 Supplier Selection with DEA

DEA method aids the buyer in classifying the suppliers (or their initial bids) into two categories: the efficient suppliers and the inefficient suppliers. For example, Weber (1996) suggested the use of DEA for measuring vendor performance. He used DEA to evaluate vendors by considering unit price, percentage of rejects, percentage of late deliveries, business allocation units, etc. He rightfully claimed that DEA is an objective method, contrasted with the usual subjective methods available for vendor selection (Ramanathan, 2007).

The reason for supplier selection with DEA is to measure production efficiency of suppliers and compare compatible suppliers in order to choose more effective ones. Data Envelopment Analysis uses inputs and outputs conceptions considering supplier selection for production sector. The choice of the input and output variables is critical to the successful application of this technique. Inputs are the source of suppliers for generating products such as labor source and machine capacity. Outputs are the final result of the companies profile while using inputs. In other words, output variables often reflect the goal's or objective's results of the company such as failure rate, lead time, unit price, etc.

By considering all comparable inputs and outputs, DEA can measure the supplier's performance efficiency. The most distinguishing feature of DEA is that in computing the relative performance efficiency, the best performing suppliers are used as the bases for comparison. Comparing the supplier's performance with that of the best performing suppliers (often referred to as benchmarking) is an important step towards achieving a supplier selection operation oriented towards excellence. Suppliers can use internal (own company) or external (outside suppliers) standards as their benchmark (Yoo and Donthu, 1998).

Productivity or efficiency can be measured with DEA by two ways;

1. Producing the maximum value of outputs for any given amount of inputs
2. The minimum use of inputs for any given amount of outputs.

The first task of DEA is to find the most efficient suppliers, which produce a so-called efficient frontier, analogous to isoquants (equal-product curves) of production functions in microeconomics. The efficient frontier is a series of points, a line, or a surface connecting the most efficient suppliers, which are determined from a comparison of inputs and outputs of all retail outlets under consideration. Thus DEA produces the relative efficiency boundaries, which are called envelopes (Yoo and Donthu, 1998).

Efficiency boundaries, i.e., envelopes, can be analyzed with efficiency scores. Suppliers whose efficiency is less than one are placed inside the frontier. Thus, DEA efficiency less than one shows inefficiency based on comparable suppliers. A supplier is deemed efficient (efficiency = 1), if its output is optimal (maximum possible) in comparison with the inputs and outputs of all comparable suppliers.

Efficiency is the ratio of the weighted sum of outputs to the weighted sum of inputs. For example, if a retail outlet uses two input variables X_1 and X_2 and two output variables Y_1 and Y_2, its efficiency is given by;

$$h_1 = \frac{U_1 Y_1 + U_2 Y_2}{V_1 X_1 + V_2 X_2}$$

In using DEA, the weights are estimated separately for each supplier such that its efficiency is the maximum attainable. DEA estimates the weights U_1, U_2, V_1, and V_2 for supplier 1 such that its estimated efficiency h_1 will be the maximum possible. However, the weights U_1, U_2, V_1, and V_2 estimated for supplier 1 should be such that when they are applied to the inputs X_i and outputs Y_i of all other units in the analysis, their ratio of the weighted outputs to the weighted inputs should be less than or equal to one. Similarly, DEA will estimate a separate set of weights for each supplier such that the estimated weights will lead to a maximum attainable efficiency for that supplier. The estimated *U*'s and *V*'s for all suppliers should be greater than zero (Yoo and Donthu, 1998).

The efficiency of any supplier is computed as the maximum of a ratio of the weighted outputs to the weighted inputs, subject to the condition that similar ratios, using the same weights, for all other suppliers under consideration are less than or equal to one. Hence the maximum efficiency, h_o, for supplier *o* is:

$$Max\ h_0 = \frac{\sum_{r=1}^{s} U_r Y_{ro}}{\sum_{i=1}^{m} V_i X_{io}}$$

$$subject\ to \quad \frac{\sum_{r=1}^{s} U_r Y_{rj}}{\sum_{i=1}^{m} V_i X_{ij}} \leq for\ all\ j = 1, \ldots n$$

$$U_r\ V_i > 0;\ r = 1, \ldots, s; i = 1, \ldots, m$$

Y_{rj} and X_{ij} are the r^{th} output and i^{th} input observations for the j^{th} supplier. U_r and V_i are the variable weights to be estimated by the data of all comparable suppliers that are being used to arrive at the relative efficiency for the o^{th} supplier. The above formulation has s output variables, m input variables, and n suppliers. In practice, the above formulation is first linearized and then solved using the methods of linear programming. The dual of the linear program is usually estimated easier to solve (Mahajan, 1991).

In summary, the main advantages of DEA-based supplier productivity evaluations are as follows (Yoo and Donthu, 1998);

1. Utilizes both output and input observations,

2. Accommodates multiple inputs and outputs,

3. Accommodates both controllable and uncontrollable factors,

4. Computes a single index of productivity,

5. Develops a relative measure of performance for each retail outlet using best performers as the bases.

6. Does not force one functional form relating the inputs and outputs of all observations.

3.8 Data Envelopment Analysis Steps (Daneshvar, 2009)

1. Choosing observation product/service group
2. Choosing input and output key product indicators
3. Evaluation of general result of observation product/service group
4. Additional supportive analysis such as Analytic Network Process, Analytic Network Process etc. (If needed)

- ✓ **Choosing observation product/service group**

Some parameters (input and output) should have the same conditions in order to evaluate decision units (supplier, hospitals etc.). In other words, the units of inputs and outputs should be identical for comparison. Moreover, decision unit's market conditions should be the same for accurate evaluation (Daneshvar, 2009).

Decision units can be suppliers, departments, machines etc. The first step starts with investigating decision units where efficiency can be measurable. Decision units should have homogeneous input/output data because DEA cannot be applied if one of the inputs or outputs is missing. If the number of inputs is k and the number of output is t, then the number of the decision units should be at least $k+t$ for the accurate solution (Golany et al. 1985).

- ✓ **Choosing input and output key product indicators**

At first, all possible key product indicators (KPI) should be listed. The list should be as much as exhaustive. KPI list should be investigated for choosing input and output data. Elimination of KPIs should be carried out for getting an absolute list. The absolute list includes all important independent criteria for comparing decision units (suppliers).

- ✓ **Evaluation of general result of observation product/service group**

DEA results are investigated at this part. Each product/service group has a DEA rank. The results are sorted such that the larger come first. The product/service group which has one point from DEA analysis is an efficient group considering other decision units.

- ✓ **Additional supportive analysis**

If DEA analysis results show lots of optimum/efficient decision units, this decision units can be reinvestigated with supportive tools such as Analytic Hierarchy Process or Analytic Network Process. These methods provide reassortment of decision units' efficiency which has already efficient ranks.

3.9 Strong and Weak Side of DEA

3.9.1 Strong Side of DEA (Daneshvar, 2009):

1. It is possible to analyze all input and output data at the same time. The effect of all factors plays a role to reach the final table.

2. Output/input data units (parameters) can be dissimilar when using in a DEA mathematical model. Therefore, different criteria parameters can be analyzed thanks to DEA.

3. It finds a most suitable solution group while measuring the efficiency of each group.

4. It is a management tool that can be used for general overview of the subject group. It does not need any detail about production. Only inputs and outputs are necessary.

5. It divides decision units as efficient and inefficient for easy evaluation.

6. It is an effective tool for management of strategic planning, source and supply chain.

7. Requested input/output and DEA result can be saved as archives into a database. This database can be helpful for new employees who are eager to learn later.

3.9.2 Weak Side of DEA (Daneshvar, 2009):

1. DEA results may show more than one effective decision unit. Additional analysis such as ANP and AHP has to be carried out in order to find which one is more effective.

2. The ranks/weights of decision variables are accepted as equal. There is not such an option on DEA to give weights to decision variables.

3. The whole system should be reflected as input or output. Otherwise, DEA results are wrong.

4. DEA uses only physical input/output so it is limited to measure technical productivity.

5. Insufficient input/output data causes unreliable results. If critical input/output is not used for DEA, the result of DEA will be prejudiced.

6. Exorbitant inputs/outputs data may cause conflicts while giving final judgment. There are many decision changes for each decision unit, and commenting on the final judgment can be complex.

7. DEA is a static analysis that uses limited periodic data.

8. DEA is a nonparametric technique. So it is hard to apply hypothesis tests.

9. Each decision unit needs additional mathematical model for solving DEA. So, complicated structure can be time-consuming.

3.10 Performance Measurement with DEA

DEA measures decision unit's efficiency by proportional form of inputs/outputs. In general, each decision unit's efficiency can be found as the biggest value of the weighted total outputs over the weighted total inputs while all constraints are equal to or lower than one (Charnes et al. 1978).

DEA model measures the efficiency of any decision units that obtained as the maximum of a ratio of weighted outputs to weighted inputs subject to the condition that the ratios for every decision units are less than or equal to one.

Model's objective function tries to get maximum outputs over inputs value (efficiency value) while forcing the other decision unit's efficiency value (1) less than or equal to one. In other words, constraint (1) forces outputs over inputs value less than or equal to 1 while objective function measures decision unit's efficiency (outputs over inputs).

Let k be decision units (k=1,2,..n). Each decision unit has m units inputs and t units outputs. Decision units have inputs as X_{ik} (i=1,2,..,m) and outputs as Y_{rk} (r=1,2,…,t) which are used in DEA modeling.

DEA efficiency measurement model is;

$$e_k = max \sum_{r=1}^{t} U_r Y_{rk} \Big/ \sum_{i=1}^{m} V_i X_{ik}$$

Subject to

(1)

$$\sum_{r=1}^{t} U_r Y_{rk} \Big/ \sum_{i=1}^{m} V_i X_{ik} \leq 1 \quad k = 1,2,\dots,n$$

$$U_r \geq 0 \qquad r = 1,2,\dots,t$$

$$V_i \geq 0 \qquad i = 1,2,\dots,m$$

e_k: k^{th} decision unit's efficiency

U_r: the weight of r^{th} units of output

V_i: the weight of i^{th} units input

Y_{rk}: r^{th} output related with k^{th} decision unit

X_{ik}: i^{th} input related with k^{th} decision unit

n: Total decision units

t: Total output units

m:Total input units

All the constraints above generate the DEA model structure. Each decision unit's input/output data can be compared by the DEA model in order to determine the efficiency. It is hard to solve this model as it is not a linear model. Thus, Charnes and

Cooper (1978) suggested weighted linear DEA model which is provided with the transformed variables.

3.10.1 Weighted Linear DEA Model (WLDEA)

DEA model is a nonconvex non-linear formulation. Weighted DEA is a linear form of DEA.

Constraint (2) is added to WLDEA model for linearization of DEA's objective function. ($\sum_{i=1}^{m} V_i\, X_{ik} = 1$)

$$e_k = max \sum_{r=1}^{t} U_r\, Y_{rk} \Big/ \sum_{i=1}^{m} V_i\, X_{ik} \longrightarrow\longrightarrow\longrightarrow\longrightarrow e_k = max \sum_{r=1}^{t} U_r Y_{ik}$$

DEA model constraint changes to linear form which is shown at second constraint (3) of WLDEA.

$$\sum_{r=1}^{t} U_r\, Y_{rk} \Big/ \sum_{i=1}^{m} V_i\, X_{ik} \leq 1 \quad \longrightarrow\longrightarrow\longrightarrow\longrightarrow \sum_{r=1}^{t} U_r\, Y_{rk} - \sum_{i=1}^{m} V_i\, X_{ik} \geq 0 \quad k = 1,2,\dots,n$$

WLDEA Model:

$$e_k = max \sum_{r=1}^{t} U_r Y_{ik}$$

Subject to:

(2)

$$\sum_{i=1}^{m} V_i\, X_{ik} = 1$$

(3)

$$\sum_{r=1}^{t} U_r\, Y_{rk} - \sum_{i=1}^{m} V_i\, X_{ik} \geq 0 \quad k = 1,2,\dots,n$$

$U_r \geq 0$ $\quad\quad r = 1,2, \dots, t$

$V_i \geq 0$ $\quad\quad i = 1,2, \dots, m$

e_k: k^{th} decision unit's efficiency

U_r: the weight of r^{th} units of output

V_i: the weight of i^{th} units input

Y_{rk}: r^{th} output related with k^{th} decision unit

X_{ik}: i^{th} input related with k^{th} decision unit

n: Total decision units

t: Total output units

m:Total input units

The weighted linear DEA model is easier to use thanks to its linear structure. If k^{th} decision unit is effective, the objective function will be one, otherwise it will have positive value which is less than one.

Ineffective decision units are compared with all similar decision units at the same time. Considering trial and error method, if a small cluster is chosen rather than all decision units, ineffective decision unit's WLDEA result can be one (effective). In addition, inputs or outputs can be changeable till the WLDEA result shows effective. So, which input/output variable will be changed should be clarified in order to make a decision unit effective.

DEA models can measure effectiveness of decision units. It cannot rank decision units properly. This is the weak side of DEA.

3.11 DEA-Based Ranking Applications

DEA method divides decision units into efficient and inefficient ones. In general, if a decision unit's DEA result is one or less than one, the decision unit is efficient or inefficient, respectively (Talluri, 2000). DEA does not allow ranking efficient decision units but this situation does not mean that all efficient decision units are at the same level. Thus, Analysts should support DEA with an additional method in order to find which one is the most effective. The followings are the most encountered methods in the literature:

✓ Crosswise effective matrix is used for DEA efficiency ranking by Sexton (1986).

✓ Basic cross functions and data were used in matrix by Doyle and Green (1993). However they noted that this method is not absolutely consistent.

✓ The study of Mardia and Kent and Bibby (1979) shows that Fisher's linear discriminant function can be used as supportive tool of DEA. Fisher's linear discriminant is a classification method that projects high-dimensional data onto a line and performs classification in this one-dimensional space. In general, Fisher's linear discrimination method is used for disjoint two or more groups. A linear function finds out the maximum total square of intergroup over Total Square of intra-group. Variables should be multiunit normal distribution and each group's covariance matrix should be equal.

✓ The study of Cook and Kress (1990) shows that the subjective analysis method can be used as a supportive tool of DEA for ranking decision units.

✓ The study of Ganley and Cubbin (1992) developed common weights for maximizing efficiency rates of all decision units in DEA (Ganley and Cubbin 1992).

✓ Andersen and Petersen (1993) propose a model which allows giving weights related with management decision. This method is used for getting a result from DEA. However, decision constraint set and test units are eliminated for this model. New linear model which has reduced constraints allows technical efficiency result greater than one. This result provides ranking decision units which have already efficient.

✓ Retzlaff and Roberts (1996) use discrimination analysis. The study compares DEA models. This statistical method is used for separating groups into minor topics. Discrimination analysis investigates correlation of independent variables. Independent variables shows separation score for getting right linear combination.

✓ Sinuany and Stern Friedman (1998) use proportional discrimination analysis (DR/DEA). This method allows variation of DEA's input and output depending on the decision unit. In addition, cross functional efficiency matrix is used for ranking.

To sum up, each additional model has a drawback for decision unit ranking. Some of them are used for ranking of ineffective decision units and the others for ranking effective decision units. Moreover, multiple linkages between variables may cause improper parameters. Thus, before the analysis, there should be an investigation about correlation between independent variables. All in all, mistakes are observed by human fault or lack of management judgment reflection into models.

3.12 DEA –AHP Hybrid Method Application

Combination of DEA and Analytic Hierarchy Process (AHP) is not a new idea. This method is used many times in order to investigate the whole system for understanding missing points. The followings show DEA-AHP applications in the literature:

✓ The study of Bowen (1990) compares the AHP and DEA methods for a site selection problem and discussed their similarities in both structure and results. They suggested a two step process of integrating them in site selection. The first step is to apply the DEA to exclude numerically inefficient sites and the second step is to apply the AHP for DEA efficient sites. They claimed that this combination would have the dual advantage of utilizing both objective and subjective data as well as reducing the number of paired comparison judgments and would provide a means of discriminating between DEA efficient sites (Wang et al. 2007).

- ✓ Shang and Sueyoshi (1995) proposed a unified framework for the selection of the most appropriate flexible manufacturing system (FMS) for a manufacturing organization. The recommended framework took advantage of the AHP method to quantify the intangible (nonmonetary) benefits associated with corporate goals and long-term objectives, the simulation model to analyze the tangible benefits, the accounting procedure to determine the required inputs such as expenditures and resources for realizing the potential benefits, and the DEA method to identify the most efficient FMS system (Daneshvar, 2009).
- ✓ Seifert and Zhu (1998) investigated excesses and deficits in Chinese industrial productivity for the years (1953–1990) by combining the DEA with other management science approaches such as Delphi, AHP and assurance region (AR) techniques. The additive DEA model was modified to weighted constant returns to scale (CRS) additive DEA model, where the weights were obtained through expert opinion by the Delphi and AHP approaches. Various multiple input and multiple output sets were selected to study overall performance, industrial development and product-related efficiency of Chinese industry. Their study demonstrated that DEA could be combined with other methods to yield more valid results, insights and recommendations (Wang et al. 2007).
- ✓ Zhang and Cui (1999) developed a project evaluation system for the State Information Center of China to manage investments in the various parts (sub-systems) of the State Economic Information System (SEIS) of China, where the total investment was divided into five sub-funds: capital construction, system maintenance, application development, training and facility purchase. Each of the five sub-funds was further divided and allocated to the sub-systems according to the four factors: efficiency rating, predicted requirement, local constraints (regional development in social and economic aspects) and the development strategy of the SEIS (the sub-system’s position in the strategy of the SEIS). In this decision support system, the DEA method was used to evaluate the relative efficiencies of the sub-systems of the SEIS and to determine their weights for the factors ‘efficiency rating’ and ‘requirement’, and the AHP was constructed to determine a reasonable

investment allocation ratio and the weights of the sub-systems for the factors 'local constraints' and 'strategies', respectively (Wang et al. 2007).

- ✓ Sinuany-Stern, Mehrez, and Hadad (2000) presented an AHP/DEA methodology for fully ranking organizational units with multiple inputs and multiple outputs. The suggested AHP/DEA methodology was composed of two main stages. In the first stage, the DEA was run for each pair of units separately to create a pairwise comparison matrix. In the second stage, the pairwise comparison matrix created in the first stage was utilized for fully ranking the units via the AHP. The advantage of the AHP/DEA methodology was that the AHP pairwise comparisons were derived mathematically from the input/output data by running pairwise DEA models and there was no subjective evaluation involved in the methodology (Daneshvar, 2009).
- ✓ A hierarchical DEA/AHP methodology (Yang and Kuo, 2003) is used for facility design. Alternative designs are evaluated with quantitative data. Performance data are gained thanks to AHP, performance edges are determined with the DEA analysis.
- ✓ Takamura and Tone (2003) conducted a comparative site evaluation study for relocating Japanese government agencies out of Tokyo, putting emphasis on the methodological aspects. In their study, the AHP method was used to determine the weights of criteria or the assurance region of DEA model. It was claimed that the results obtained by the DEA assurance region model had particular merits for both candidates and evaluators (Wang et al. 2007).
- ✓ Saen, Memariani, and Lotfi (2005) proposed a method of determining relative efficiency of slightly nonhomogeneous decision making units (DMUs) by using the DEA. Homogeneous DMUs are assumed to consume common inputs to produce common outputs. However, this assumption may not always be true in many real applications. For instance, not all bank branches offer automatic teller machine (ATM) services. If simply a zero value were allocated to the branches that do not offer ATM service, the resultant efficiencies would be unfair and unrealistic. In Saen's (2005) approach, those DMUs lacking one or more features (inputs and/or outputs) were considered as the units with missing value(s). The missing values were inserted by series mean multiplied by adjusted factors determined by the AHP. The

relative efficiency of DMUs is then computed by a chance constrained DEA model. It was believed that the two-dimensional estimation of missing values took into account both the interpolation and the potential of the DMU in the function which it lacks and would decrease the bias in the final efficiency score (Daneshvar, 2009).

- ✓ Liu and Hai (2005) presented a voting AHP method for supplier selection. The voting AHP determines the weights of criteria not by pairwise comparisons but by voting. The DEA method was used to aggregate votes each criterion received in different ranking places into an overall score of each criterion. The overall scores were then normalized as the relative weights of criteria (Wang et al. 2007).
- ✓ The study of Ramanathan (2007) showed that the supplier selection problem was solved with DEA with the approaches of total cost of ownership (TCO) and AHP. Interesting way of combining the objective and subjective information is provided by the results of the TCO and AHP approaches through the application of data envelopment analysis (DEA). The proposed integrated DEA-TCO-AHP model is the first of its kind to be applied to the supplier selection problem.
- ✓ The study of Kazançoğlu (2008) showed that Supplier selection and performance evaluation is done with DEA/AHP. AHP evaluation is made before DEA. AHP results are supported with the DEA result. Inputs and outputs are chosen from all evaluation criteria. All input/output criteria are normalized in order to adjust all the units. DEA gives effective suppliers list and checkout with AHP result. Efficient two suppliers are chosen from DEA result list related with AHP ranking.
- ✓ The study of Daneshvar (2009) showed that Performance evaluation between IRAN University departments is done with DEA/ANP. All inputs/outputs are chosen and used in DEA with their own units. In this research, it can be seen that many different units can be solved in a DEA models. In other words, questionnaire results, teacher quantity and department budget can be analyzed at the same mathematical model. Pair-wise comparison with AHP is done in order to get the efficiency matrix. DEA-AHP cannot give sufficient solution. Departments relation related with teacher exchange are investigated with ANP. Each department has efficiency rate. DEA/AHP and ANP provides ranking of all university departments.

DEA-AHP Sequential Hybrid Algorithm Flowchart is shown in Figure 3.8. The flowchart is common for all kind of DEA-AHP system.

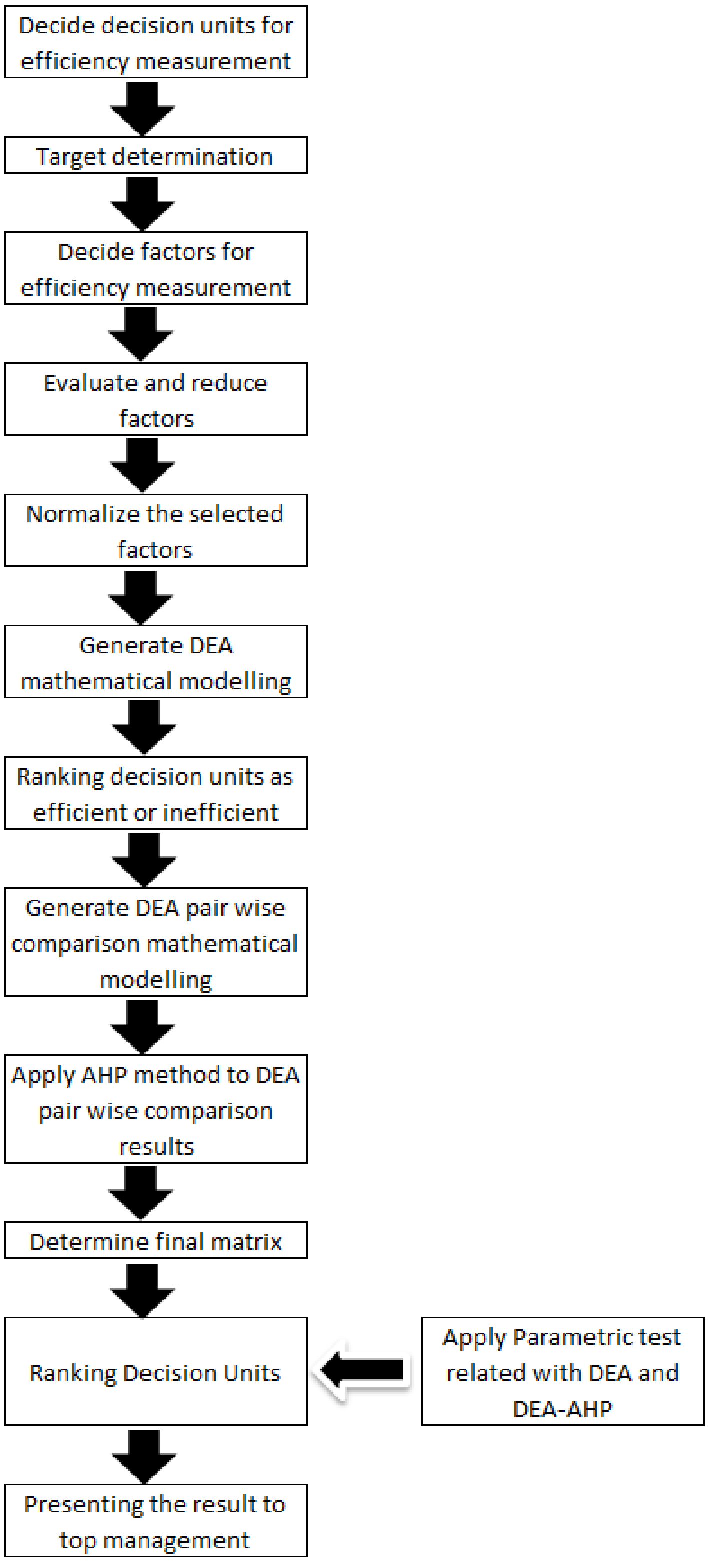

Figure 3.8 DEA-AHP sequential hybrid algorithm flowchart

3.13 DEA-AHP Application Steps

3.13.1 First Phase: Weighted Linear DEA Model

Model:

$$e_k = max \sum_{r=1}^{t} U_r Y_{ik}$$

Subject to:

$$\sum_{i=1}^{m} V_i X_{ik} = 1$$

$$\sum_{r=1}^{t} U_r Y_{rk} - \sum_{i=1}^{m} V_i X_{ik} \geq 0 \quad k = 1,2,\dots,n$$

$$U_r \geq 0 \qquad r = 1,2,\dots,t$$

$$V_i \geq 0 \qquad i = 1,2,\dots,m$$

e_k: k^{th} decision unit's efficiency

U_r: the weight of r^{th} units of output

V_i: the weight of i^{th} units input

Y_{rk}: r^{th} output related with k^{th} decision unit

X_{ik}: i^{th} input related with k^{th} decision unit

n: Total decision units

t: Total output units

m:Total input units

In general, each decision unit will be compared with all the other decision units at the same time. DEA measures efficiency of each decision unit. Moreover, each decision unit takes a part into group which is efficient or inefficient. If the k^{th} decision unit is effective, then e_k = 1 and the related constraint is zero. If the k^{th} decision unit is not effective, then $0<e_k<1$.

Finally, an efficiency result matrix is created thanks to WLDEA. The final matrix result can exactly show which of the decision units are efficient compared with other decision units. Additional analysis should be carried out in order to find out which one is more effective (ranking effectives).

3.13.2 Second Phase: DEA Pair-wise Comparison

In general, the concept is the same as DEA modeling. However, pair-wise comparison is used in this phase differently from DEA. Pair-wise results will help to generate an AHP starting matrix.

Comparison of the couple of decision units shows which one is more effective. If k decision unit is effective compared to k', objective function will be one and related constraint is set to zero. Otherwise, objective function will be positive and less than one.

Select two of decision units and compare them with DEA pair-wise comparison model. First constraint is used for linearization of DEA. Second constraint represents the first decision units. Also, first decision unit's outputs are used in objective function. Third constraint represents the second decision unit.

Objective function

$$E_{k,k'} = max \sum_{r=1}^{s} U_r Y_{rk}$$

Constraints:

$$\sum_{t=1}^{m} V_i X_{ik} = 1$$

$$\sum_{r=1}^{s} U_r\, Y_{rk} - \sum_{i=1}^{m} V_i\, X_{ik} \leq 0$$

$$\sum_{r=1}^{s} U_r\, Y_{rk'} - \sum_{i=1}^{m} V_i\, X_{ik'} \leq 0$$

$U_r \geq 0 \qquad\qquad r = 1,2, \dots, s$

$V_i \geq 0 \qquad\qquad i = 1,2, \dots, m$

$k' = 1, \dots, n \quad k = 1, \dots, n \; and \quad k \neq k'$

$E_{k,k'}$: k decision units efficiency compared with k'

U_r: the weight of r^{th} units output related with k decision unit

V_i: the weight of i^{th} units input related with k decision unit

Y_{rk}: r^{th} units output related with k decision unit

X_{ik}: i^{th} units input related with k decision unit

3.13.3 Third Phase: DEA- AHP Matrix Determination

The pair-wise comparison matrix will be used in a simple AHP application. This application is not a subjective approach. Weighted DEA and AHP application is applied consecutively. A decision unit which has the first priority level from ranking results is defined as the best (most effective) decision unit.

Step 1:

$e_{kk'}$ results are gathered from the second phase. Matrix E is constituted thanks to generating pair wise comparison from DEA model.

Table 3.2 DEA/AHP application, the E matrix

	1	2	3	…	n
1	1	$e_{1,2}$	$e_{1,3}$	…	$e_{1,n}$
2	…	1	…	…	$e_{2,n}$
3	…	…	1	…	$e_{3,n}$
…	...	...	...	1	…
n	$e_{n,1}$	$e_{n,2}$	$e_{n,3}$	…	1

Step 2:

AHP pair-wise comparison matrix is attained from $a_{k,k'}$ formulation. $a_{k,k'}$ Formula shows productivity of k decision unit compared with k' decision unit.

$$a_{k,k'} = \frac{e_{k,k'}}{e_{k',k}}$$

Table 3.3 DEA/AHP application, the A matrix

	1	2	3	…	n
1	1	$a_{1,2}$	$a_{1,3}$	…	$a_{1,n}$
2	…	1	..	…	$a_{2,n}$
3	…	…	1	…	$a_{3,n}$
..	…	…	...	1	...
N	$a_{n,1}$	$a_{n,2}$	$a_{n,3}$	…	1

Step 3:

DEA/AHP rules follow up from stage 2, A matrix. Each unit is divided into total related columns that composed N matrix (Table 3.6). In other words, N matrix is called normalized matrix of A.

$$a'_{k,k'} = \frac{a_{k,k'}}{\sum_{k=1}^{n} a_{k',k}}$$

Table 3.4 DEA/AHP application sum of the A matrix

	1	2	3	…	n
1	1	$a_{1,2}$	$a_{1,3}$	…	$a_{1,n}$
2	…	1	..	…	$a_{2,n}$
3	…	…	1	…	$a_{3,n}$
..	…	…	...	1	...
N	$a_{n,1}$	$a_{n,2}$	$a_{n,3}$	…	1
	$\boldsymbol{T_{n,1}}$	$\boldsymbol{T_{n,2}}$	$\boldsymbol{T_{n,3}}$	**…**	$\boldsymbol{T_{n,N}}$

$T_{n,1} = a_{1,1} + a_{1,2} + a_{1,3} + \cdots + a_{n,1}$

Table 3.5 The N matrix of DEA/AHP

	1	2	3	…	n
1	$1/T_{n,1}$	$a_{1,2}/T_{n,2}$	$a_{1,3}/T_{n,3}$	…	$a_{1,n}/T_{n,N}$
2	…	$1/T_{n,2}$	..	…	$a_{2,n}/T_{n,N}$
3	…	…	$1/T_{n,3}$	…	$a_{3,n}/T_{n,N}$
..	…	…	...	…	...
N	$a_{n,1}/T_{n,1}$	$a_{n,2}/T_{n,2}$	$a_{n,3}/T_{n,3}$	…	$1/T_{n,N}$
	$\boldsymbol{T_{n,1}}$	$\boldsymbol{T_{n,2}}$	$\boldsymbol{T_{n,3}}$	**…**	$\boldsymbol{T_{n,N}}$

Step 4:

At this stage column vector elements are determined as sum up each row.

Table 3.6 Sumrow of the N matrix

	1	2	3	…	N	**SUMROW**
1	$1/T_{n,1}$	$a_{1,2}/T_{n,2}$	$a_{1,3}/T_{n,3}$	…	$a_{1,n}/T_{n,N}$	$S_{1,N}$
2	…	$1/T_{n,2}$	..	…	$a_{2,n}/T_{n,N}$	$S_{2,N}$
3	…	…	$1/T_{n,3}$	…	$a_{3,n}/T_{n,N}$	$S_{3,N}$
..	…	…	...	…	...	$S_{..,N}$
N	$a_{n,1}/T_{n,1}$	$a_{n,2}/T_{n,2}$	$a_{n,3}/T_{n,3}$	…	$1/T_{n,N}$	$S_{n,N}$
	$\boldsymbol{T_{n,1}}$	$\boldsymbol{T_{n,2}}$	$\boldsymbol{T_{n,3}}$	**…**	$\boldsymbol{T_{n,N}}$	

Sum row is a column vector whose elements can be calculated as;

$$S_{1,N} = 1 + \left(a_{1,2}/T_{n,2}\right) + \left(a_{1,3}/T_{n,3}\right) + \cdots + \left(a_{1,n}/T_{nN}\right)$$

Step 5:

Column vector (stage4) will be normalized as below.

$$a_k''' = \frac{a_k''}{\sum_{k=1}^{n} a_k''}$$

At the end of the 5th step DEA/AHP result is gained. The final matrix shows ranking of all decision units which could not be attained from just DEA modeling. All decision units' efficiency has less than one point, but it can be said that the point which is closest to one is the most efficient decision unit.

All in all, DEA/AHP provides ranking all decision units. The top management can decide which decision unit is the most efficient one thanks to DEA/AHP.

CHAPTER FOUR
BUSINESS PROCESS REENGINEERING APPLICATIONS AT VESTEL ELECTRONICS

The main topics of this chapter are general information about Vestel Electronics, definition of screen label product group, information about screen label price analysis table and business process reengineering applications and savings. The aim of this chapter is to provide a basis for supplier selection analysis using BPR. Supplier selection will be meaningful with setting up strong structure on that product group thanks to BPR. The improvement opportunities are evaluated and successfully applied for screen label product group at Vestel Electronics.

4.1 Vestel Electronics

Vestel is not a brand company, Original Equipment Manufacturer (OEM). An original equipment manufacturer is a company which designs and manufactures a product which is specified and eventually branded by another firm for sale. Vestel produces TVs under the name of the some well known brands, e.g., Toshiba, Finlux, Augen, Telefunken, Medion, Sharp, Sanyo, and Quadro.

Vestel Electronics is a television producer in Europe. 15000 personnel work for getting Vestel much better position in the world. Over 7 million televisions are produced and sold in a year. Furthermore, over 179 foreign companies are Vestel’s customers. 95% of the total TV production rate produces for exportation. 5% of the total production allocated for domestic market.

Foreign trade department is the key factor to keep Vestel’s production sustainable. The department takes the orders from over 179 countries. Actual order ranges are determined and recorded by this way.

Vestel is a company which is called follower in the market. Follower means that if the world’s bestseller companies make a decision to change their production plan to new technological conditions, Vestel will fill a gap that occurs with old technology. For instance, Cathode Ray Tube (CRT) televisions give place to Liquid Crystal

Display (LCD) in 2007 - 2008. After this new player substitution, Vestel's production forecast's increase thanks to CRT space in the world.

Vestel plans production according to monthly forecasts formed with actual sales plus prediction. Prediction changes according to world's financial and social conditions. For example, 2008 world economical crises condition has affected TV production rates downwards. In contrast, Every World Cup which happens every four years affects TV production rates upwards. As a result, TV production has seasonal and non seasonal effects.

All in all, Vestel has extraordinary dynamic conditions. So, regular production cannot be expected. None of the personnel knows four months future's production type and exact quantity. The important thing is the management of this complexity.

4.2 BPR Application in the Purchasing Department

Business Production Reengineering is carried out at Central Purchasing Department. The purchasing department issues purchase orders for supplies, services, equipment, and raw materials to decrease the administrative costs associated with the repetitive ordering of basic consumable items.

4.3 Screen Label

Screen Labels are one of the product groups for TVs outsourced from suppliers. They are used for marketing of TV brands at show-rooms. Salesman takes into account screen labels in order to remember TV's property. Expected life time is maximum 30 days. One of the label samples is shown in Figure 4.1;

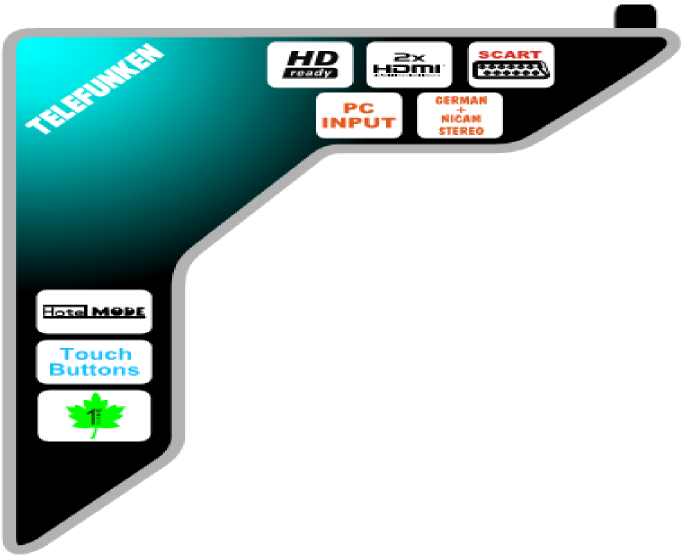

Figure 4.1 Screen label sample

Screen Labels stick on to TV edges. This label includes television's specific features that customers are curious about. They may have also a brand name that Vestel's working for.

Process visualization and Process/Customer focus techniques will be used for BPR in Screen Label production group.

Process and customer focus technique is used as BPR project in this study. BPR's general idea is to redesign the existing process for improvement. Thus, process oriented approach has to coordinate with customer needs. Inconsistent progress is meaningless. If customer needs are not considered, improvements are useless. Customer's expectations should be executed while costs are getting down.

Screen Label is a customer expectation that we have to add to the bill of material of television for special customers. Actually, most of the customers prefer designing template of the screen label and send to Vestel to fit on this template. Vestel's designers choose the features of the product and add them to template as small pictures.

Over 30 brands and 12 sizes of the television cause lots of variability in the system. Moreover, every small changes in the main board causes additional version of one of the inch. This means every inch of one brand can have lots of versions. As mentioned before, a screen label shows product features. Every version of the TV means different feature and different feature may cause different code of screen label.

80-100 new Screen Label codes are recorded to SAP for every month. This situation is expectable considering 240 new versions TV per 3 month. So, lots of screen label code means lots of partial order that causes much more setup time, which is undesirable for supplier’s production.

4.4 Necessity of Suppliers for Screen Label

Vestel purchases screen labels from suppliers. Make or buy analysis are repeated every year but make decision has not come out until now. Screen label make or buy analysis for 2011 will be considered at the next section.

The first consideration for choosing a supplier is supplier’s vision, mission, references and capacity. Suppliers have to be ready extraordinary work tempo since Vestel has high capacity production and additionally dynamic conditions like frequent changes at production plan. Experience is the key factor to understand this kind of complexity.

In addition, supplier’s organizational structure has to meet Vestel’s dynamic needs. If personnel system is not enough to support, there is not option to start business. Moreover, Quality certificates have to be received before for Vestel’s Research and Development (R&D) department determination because R&D department has to approve supplier for making purchasing.

After all requisitions are confirmed, commercial issues can be start.

Commercial issues start with making a request for price offer. Just product price is meaningless for Purchasing Specialist. Minor topics and sub-prices that constitute final price is the prerequisite for understanding the whole price structure. This structure is called price analysis table.

4.5 Screen Label Price Analysis Table

Screen labels can be printed by digital printing or offset printing. Digital printing has lots of opportunities such as low setup time, low waste raw material etc. but depreciation cost is five times higher than offset printing. In addition, digital printing provides constant quality and on time delivery. However the cost has to be under control and if it is possible, keep the price equal to offset printing.

If the price difference is really high, offset printing can be manageable but Vestel's dynamic condition makes this management much more painful. This management has some risk factors. Purchasing specialist has to consider these risks.

Considering Vestel today's conditions, digital printing is chosen for all suppliers in order to get constant quality. Thus, Price Analysis Tables are formed according to digital printing.

In simple terms, price analysis table includes all minor effects on price. The analysis table should be user-friendly and allow for getting feedback quickly when changes happen.

The screen label analysis table is prepared thanks to MS Excel program. The table consists of two parts. The first part which is commonly used while getting price includes the major variables. It is called easy view. The second part includes minor variables that do not change while changing an order's conditions. It is called background. The easy view table triggers the background table to get information.

4.5.1 Screen Label Price Analysis Table Easy View

The table allows changes on raw material type, adhesive qualification and dimension of screen label. Finally, unit prices are determined according to order quantity range. Quantity dependent prices are recorded to Enterprise Resource Planning program, SAP. SAP and procurement specialist uses these prices while opening new orders.

Table 4.1 shows easy view which is used for supplier's label price determination.

Table 4.1 Screen label analysis table

Screen Label Price Analysis Table								
	Firm Name	Nonadhesive part	SCRENN LABEL's					
			Widht mm	Height mm	Profit Rate %			
Row Material		No	130	13	12			
Metalize		Quantity Range	750	1.750	3.750	6.250	8.750	17.500
			0,1832 €	0,0908 €	0,0571 €	0,0454 €	0,0403 €	0,0340 €

1) Raw Material Types;

i) Metalized

ii) Opaque

Metalized and opaque raw materials have nearly the same quality, but surface colors of materials change.

2) Adhesive Qualification;

i) Adhesive

ii) Non-adhesive

Back of Screen label has to be adhesive in order to stick on television. However, the area that has to stick on glass surface (not plastic) has to be non-adhesive in order not to leave a mark on glass. Figure 4.2 shows adhesive/non-adhesive area on a screen label.

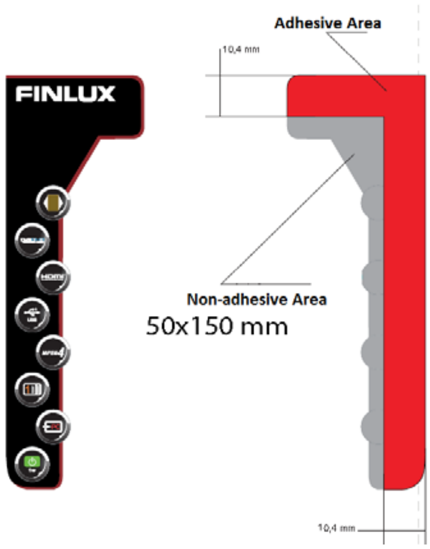

Figure 4.2 Adhesive / non-adhesive difference of screen label

3) Common Dimensions of Screen Labels (Width x Height)

i) 130x13

ii) 120x120

iii) 138,5x25

iv) 120x100

v) 150x150 (can be seen at Figure 4.2)

vi) 130x130 etc.

All the 3 major topics affect the price directly and trigger the background.

4.5.2 Screen Label Price Analysis Table Background

All minor topics that constitute product prices are located in the background table. Raw materials unit prices are shown. Moreover, Label production process has some changes depending on order quantity.

Key factors of Background progress;

✓ Order quantity

- ✓ How many layers are needed to reach order quantity?
- ✓ How long will raw materials be used?
- ✓ Click Cost for one layer
  - Payment to HP Digital Company for renting
- ✓ Lamination cost
  - Lamination provides more shiny label, and prevents overlap of colors
- ✓ Digital Printer speed (mt/min)
- ✓ General operating expense of supplier
- ✓ Maintenance Cost and Knife Cost
- ✓ Digital Printer Depreciation Value (machine purchasing price- scrap worth)
- ✓ Transportation Cost
- ✓ Profit Rate etc.

One of the sample look from Background table can be seen in Table 4.2.

Table 4.2 Background of screen label cost structure

Background of SCR. Label cost structure	
Label Width	138,5
Label Height	25
Order Quantity	2.500
Row Material Type	opak
Nonadhesive Part?	evet
mtlz	6
opak	5
How many label in one layer width?	3
How many label in one layer height?	10
Total Layer Necessity	30
Total Layer (cons. Waste)	96
REPEAT LENGHT	439,5
Mt. Quantity	68,26
Total Meter (inc. Setup)	82
Total m2 for row material	25
Row Material m2 Price	5,00
TotaL Raw Material Cost	**124,1 €**
Click Cost for 1 layer	0,02
Total Colour	6
Total Click cost	**11,5 €**
Lamination Cost	**19,9 €**
Digital Machine Speed mt./min.	9
Printing Time (min)	24
General Operating Expense	**65,7 €**
Depreciation (Digital Printer HP)	**13,1 €**
Maintenance cost	3,3 €
Depreciation PRIMING&FINISHING	**74,2 €**
Knife Cost	**4,1 €**
Transportation Cost	**15,0 €**
Packaging Cost	**5,0 €**
Total Cost	336 €
Unit Cost	0,1344 €
Product Unit Price (inc. Profit)	0,1505 €

DEPRECIATION	€/Min	Total working minutes in a year
Depreciation (Digital Printer HP) (€/Dk)	0,54 €	220.320
GOE+ Direct Labor Cost+Rent etc. (€/Dk)	2,72 €	
DEPRECIATION PRIMING&FINISING	0,18 €	

This background which is shown in Table 4.2 consists of the details mentioned above. All details are interconnected with a formulation. All costs are investigated and fall into background. Thus, every detail is under control. If one of the minor topics price's changes, purchase specialist knows how this change affects screen label's unit price.

4.6 Screen Label Make or Buy Analysis

Screen label production is not a core business activity for Vestel. However, general point of view for all product groups is that if the unit label cost is low enough to produce the product, its production is considerable in the company. Purchasing specialist should make this analysis every year for each product group.

Vestel's Screen Label production cost is compared with supplier's production cost. There has to be a Digital Printer investment cost. If the break even point is affordable, the product can be produced in the company, otherwise purchase spacialist continue to buy Screen Labels from suppliers. Table 4.3 shows internal operating cost details for using at Make or Buy anaysis.

Table 4.3 Internal operating cost table for screen label

	VESTEL Cost	Supplier Cost	Details
Labour Cost	13.500 TL	39.500 TL	5 Labor /shift
Engineer & Designer Cost		38.000 TL	
Rent	- TL	5.000 TL	
Spare Part	1.200 TL	1.200 TL	
Consumables	300 TL	300 TL	Office Supplies
R&D		4.000 TL	
Maintenance	3.750 TL	2.500 TL	
Electricity	3.000 TL	4.500 TL	KW/hour dif.
Packaging	600 TL	2.000 TL	
Food & Transfer Cost (Labor)	2.295 TL	4.500 TL	
Depreciation	1.400.000 TL		
Cargo	- TL	1.000 TL	
	20.895 TL	**100.000 TL**	
	10.447,5 €	**50.000,0 €**	

INDIGO (Digital)		
Machine Capacity	10.000	M2 /ay
App. Screen Label Capacity	1.944.444	Etiket /ay

Indigo Digital (HP) printing machine is used for Make or Buy analysis because lots of label suppliers choose this machine considering cost/efficiency rate. Also HP's technical service capability is very good. The other Digital Printing machines capacities are nearly the same with Indigo (HP) or above. So, Indigo is suitable for analysis.

Since Vestel has already engineers and designer's for Screen Labels, and there is not building renting cost, there should be a difference. Supplier's internal operating cost is 50.000 Euro/Month, higher than Vestel's internal operating cost, 10.447 Euro/Month. So, the important point is to find breakeven point of Make or Buy analysis.

On the other hand, Vestel's label raw material prices, click cost, depreciation cost and maintenance cost are higher than supplier's cost because supplier's core business is printing. They give printing services to lots of customers and their requirements are voluminous. Thus, suppliers bargaining power is higher than Vestel's.

All these data are recorded to Screen Label analysis table's background to determine the label unit price. Table 4.4 illustrates the result of Make or Buy analysis that shows break-even point of making investment.

Table 4.4 Make or Buy analysis result

Make or Buy Analysis RESULT		
Required Label Quantity/year	**4.524.182**	
Vestel Unit Price	€ 0,0558	1000-2500 Q
Supplier Unit Price	€ 0,0908	1000-2500 Q
Machine Cost	€ 984.839	Not inc.Screp Cost & cons. 5 year Compound interest(%5)
Break-Even Point	28.136.009	
Avg. Order Quantity/Month	377.015	
Berak-even Point	74,6	/month
	6,2	**/year**

Make or Buy Analysis report shows that even if Vestel's unit price is lower than supplier's, the break-even point is 6,2 year. This table was presented to Central Purchasing Managers. They make buy decision.

4.7 Major Topics of Business Process Reengineering Applications

4.7.1 Code Consolidation

As all know, customer's expectations increasingly continue. Therefore, lots of TV brands expectations are increasing too. Vestel is a TV subcontractor and has to shape production as customer driven.

Vestel has dynamic conditions. 240 new version of TV are placed in production plan in a quarter. Thus, 80-100 new Screen Label code/month is recorded to Enterprise Resource Planning (ERP) system in addition to regular codes. This means that label suppliers need to cope with extra 80-100 codes/month.

The first step of BPR is code consolidation. The issue is how we can readjust Screen Label design or procedure for giving a same service with minimum ERP code. Suppliers have to make adjustment while shifting the production for new code. Thus, if code consolidation can be applied, suppliers will struggle with less code.

Figure 4.3 shows a screen label sample in order to give an idea for understanding the variation of features.

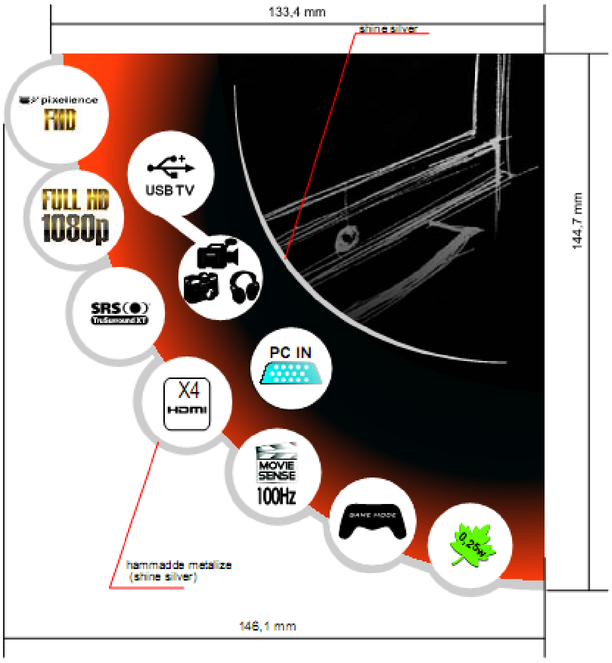

Figure 4.3 Screen label sample 2

Finding the right question is the key factor of BPR progress.

✓ Why does Vestel struggle for numerous ERP codes for screen labels?

Dimension of a label is the first reason. Dimensions and templates are determined by customer. So, Vestel cannot attempt to change the template. Persuasion of all customers is only one way to change dimension. This is the hard way but not impossible.

Changing TV brand causes extra ERP code. Persuasion of customers for not to use brand logo on screen label is the only solution for decreasing ERP codes.

There are 97 features that are used for screen label. Some of them are chosen to generate one label code. Customers try to show every detail of TV on Label Screen.

✓ Major Categories of 97 features;

o Shape

- o Entries
- o Power
- o Pixellence
- o Sound etc.

Is it necessary to see this variety on screen labels? How can we drop redundant features?

The result is that customers become convinced not to use brand name on screen labels and also agreed on determination of the redundant features for optimization.

Redundant Features eliminated:

1) No need to write any company name on a Screen Label anymore.

2) Ultra led slim, Led slim, led 22 led slim etc. are combined as LED SLIM.

Figure 4.4 Redundant features group 1

3) HD Ready 1080P, Full HD, FHD TV, FULL HD 1080P etc. are combined as FULL HD 1080p.

Figure 4.5 Redundant features group 2

4) 1 USB, 2USB, 4 USB, USB TV, Multimedia Player are combined as USB Multimedia Player.

Figure 4.6 Redundant features group 3

5) HDMI, X2 HDMI, X4 HDMI are combined as HDMI.

Figure 4.7 Redundant features group 4

6) PC IN, PC INPUT, PC Connectivity etc. are combined as PC Connectivity.

Figure 4.8 Redundant features group 5

7) Scart, 2 Scart, 1 Scart+ Bav In out etc. are combined as Scart Common.

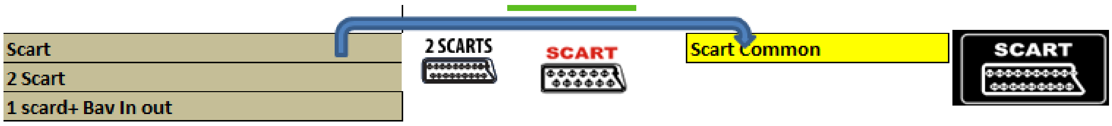

Figure 4.9 Redundant features group 6

8) 100HZ, Movie Sense 100Hz are common as 100 HZ.

Figure 4.10 Redundant features group 7

9) Digital Color Enhancement 100.000:1 and Dynamic Contrast 100.000 are common to 100000:1 DYNAMIC CONTRAST. Also, TNT INTEGREE, TNT HD etc. are common to TNT INTEGREE.

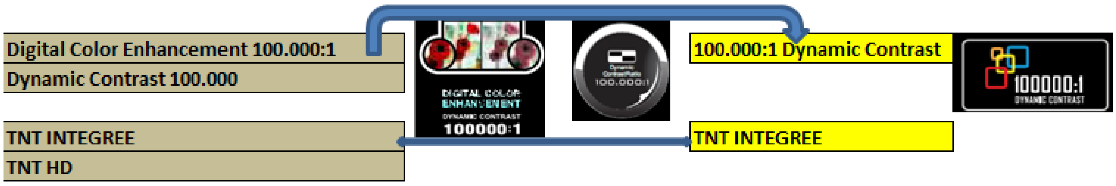

Figure 4.11 Redundant features group 8

10) Energy (1wt), Energy (0,25wt), Eco Energy are common as low standby power.

Figure 4.12 Redundant features group 9

Some of the features are deleted and some of the more explanatory features are added such as ***Fast Response Time, ***Crystal Clear Panel etc.

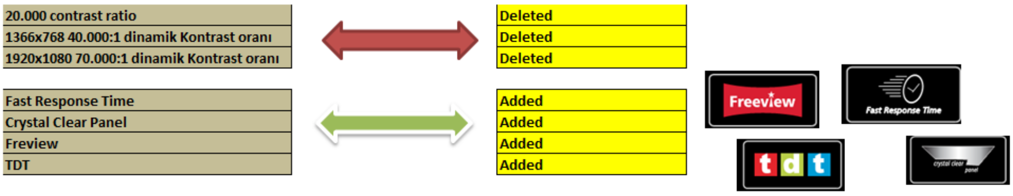

Figure 4.13 Deleted / Added features

All in all, 33 detailed features are shown in 10 pictures. In addition, new design labels get rid of one of the important complaints from Vestel's assembly line. When the unexpected production plan changes, screen labels obligated to get back to inventory. However, there is not any ERP code on labels. So, this situation causes loss at the inventory. New label designs have ERP codes.

Figure 4.14 shows new designs of screen labels after code consolidation.

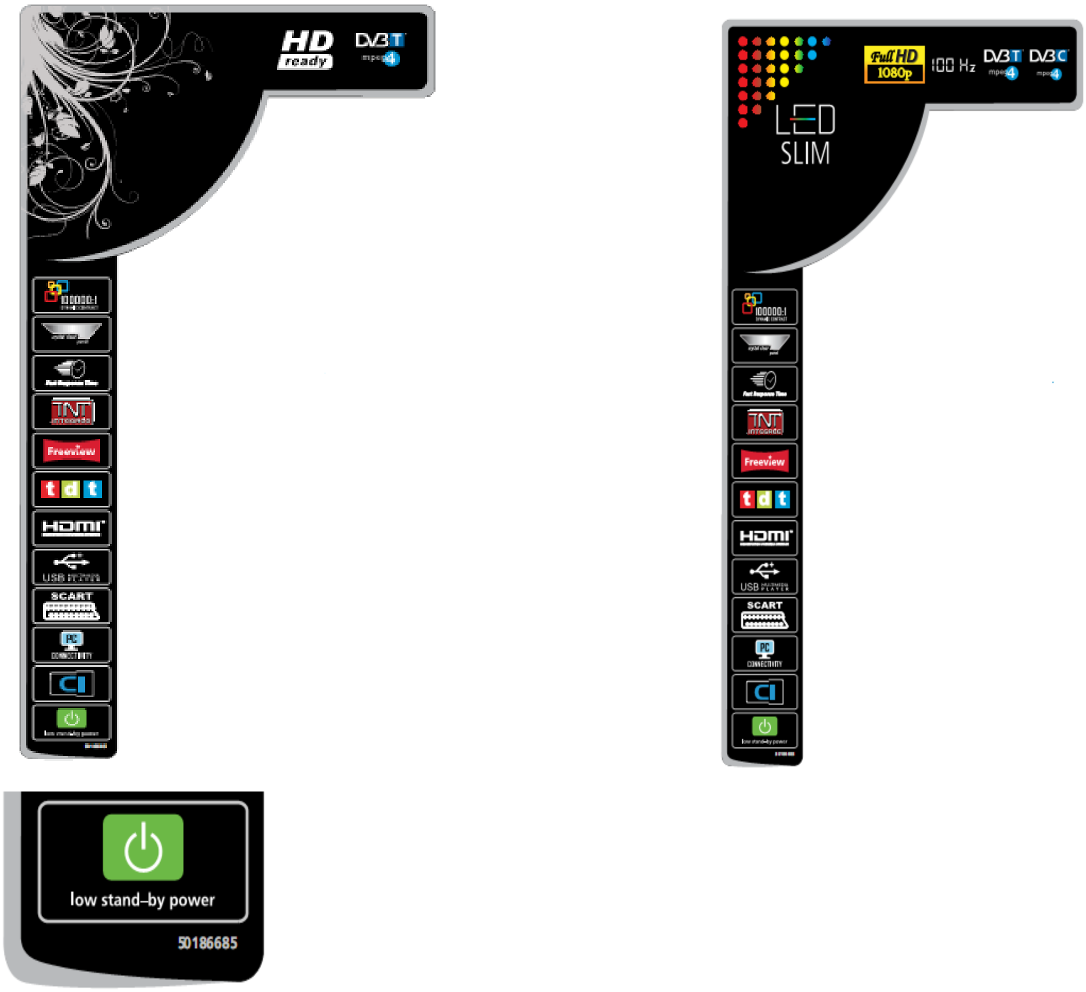

Figure 4.14 New design of screen labels

New design labels organized that one part has common features and the other part has varying features.

4.7.1.1 *Code Consolidation Savings*

All saving calculations related with code consolidation are done while taking into consideration of purchasing in November 2010.

Group 1 (HD Ready + Scart)

Table 4.5 is a sample view of consolidation of codes related with group 1 that includes HD ready and scart features at the same label.

Table 4.5 Code consolidation of HD Ready + Scart

Kod	Row material	Dimension 1	Dimension 2	Adhesi	Brand	USB Common	HDMI Common	Energy Common	Scart Common	PC Connectivity	HD Ready
50160782	opak	130	13	NO		1	1	1	1	1	1
50173682	opak	138,5	25	NO		1	1	1	1	1	1
50175439	metalize	120	100	NO	led tv	1	1	1	1	1	1
50175968	opak	130	13	NO		1	1	1	1	1	1
50176771	metalize	300	100	YES	Finlux	1	1	1	1	1	1
50176800	opak	138,5	25	NO		1	1	1	1	1	1
50176926	opak	130	13	NO		1	1	1	1	1	1
50176929	opak	130	13	NO		1	1	1	1	1	1
50176976	opak	130	13	NO		1	1	1	1	1	1

The first column of the table shows the ERP code of Screen Label which starts with 50. Raw material and dimension are not the important point because when code consolidation was successful, new design of label has been represented.

In group 1; all USB features, HDMI features, Energy, Scart, HD ready is combined under the name of an ERP code. 36 ERP codes are consolidated into 1 ERP code by this way.

Table 4.6 shows the saving calculations of group 1 that includes HD Ready and Scart features.

Table 4.6 Saving result of code consolidation (HD Ready + Scart)

	HD READY 1	Dim 1	Dim 2	Quantity	Total Cost TRY
1	50160782	130	13	500	269,75 TL
2	50173682	138,5	25	1800	393,96 TL
3	50175439	120	100	1500	1.057,05 TL
4	50175968	130	13	11500	905,65 TL
5	50176771	300	100	1000	1.303,60 TL
6	50176800	138,5	25	1220	348,02 TL
7	50176926	130	13	6070	595,44 TL
8	50176929	130	13	11700	915,46 TL
9	50176976	130	13	2200	393,70 TL
10	50177084	138,5	25	1000	217,90 TL
11	50177087	138,5	25	870	189,66 TL
12	50177135	130	13	500	269,75 TL
32	50177356	130	13	1000	178,80 TL
33	50177615	150	50	2400	1.197,00 TL
34	50177618	138,5	25	500	190,92 TL
35	50176932	130	13	8500	460,35 TL
36	50177185	130	13	3000	178,50 TL
	Total Wighted AVG.	135	19	105.850	17.820,58 TL

135x19		
0	€	0,1852
1.000	€	0,1012
2.500	€	0,0675
5.000	€	0,0558
7.500	€	0,0507
10.000	€	0,0444
100000+	€	0,0392
Payment	8.292,39 TL	
TOTAL	36 code	
Saving	- 9.528,19 TL	

36 ERP codes are combined as one code but all codes could not be shown in Table 4.6 due to a long list. All dimension's weighted average is calculated considering purchasing quantity. The 135x19 dimension is the key dimension for the calculation of group 1 saving. Actually, weighted average method is one of the ways

to know average dimension. However, actual dimension can be known after all designs are completed. All screen label designs have not been completed till now.

Combination provides to order huge quantity screen labels at a time. 100.000+ price is multiplied by total group 1 quantity (105.850) and new status cost is found as 8.292,39 TL.

Group 1's total cost was 17.820,58 TL in October 2010. So, this can be recorded as **9.528,19 TL saving** for a month in group 1.

Group 2 (FULL HD 1080P + SCART)

Table 4.7 is a sample view of consolidation of codes related with group 2 that includes FULL HD 1080P and scart features at the same label.

Table 4.7 Code consolidation of Full HD 1080P + Scart

Kod	Row material	Dimension 1	Dimension 2	Adhesi	Brand	USB Common	HDMI Common	Energy Common	Scart Common	PC Connectivity	HD Ready	Full HD 1080P
50170710	metalize	150	50	var	Finlux	1	1	**1**	1	1		1
50175570	metalize	120	100	yok	Telefunken	1	1	**1**	1	1		1
50176660	opak	130	13	yok		1	1	**1**	1	1		1
50176811	metalize	150	50	var	Finlux	1	1	**1**	1	1		1
50177336	metalize	157,3	22,15	yok	meredian	1	1	**1**	1	1		1
50177414	opak	130	13	YOK		1	1	**1**	1	1		1
50177417	opak	138,5	25	YOK		1	1	**1**	1	1		1
50177511	metalize	120	100	yok		1	1	**1**	1	1		1
50177554	metalize	120	100	var		1	1	**1**	1	1		1

In group 2; all USB features, HDMI features, Energy, Scart, FULL HD 1080P are combined under the name of an ERP code. 25 ERP codes are consolidated into one ERP code by this way.

Table 4.8 shows the saving calculations of group 2 that includes FULL HD 1080P and scart features.

Table 4.8 Saving result of code consolidation (Full HD 1080P + Scart)

	FULL HD 1080 P	Dim 1	Dim 2	Quantity	Total Cost
1	50170710	150	50	1000	500,00 TL
2	50175570	120	100	1000	704,50 TL
3	50176660	130	13	1000	178,00 TL
4	50176811	150	50	500	615,70 TL
5	50177336	157,3	22,15	500	273,70 TL
6	50177414	130	13	1000	178,00 TL
7	50177417	138,5	25	1000	218,20 TL
8	50177511	120	100	460	658,77 TL
9	50177554	120	100	2500	1.748,75 TL
10	50177649	125	100	500	272,90 TL
11	50177994	300	100	1000	1.150,45 TL
12	50178008	177	81	1352	1.072,88 TL
13	50178142	130	13	1000	179,30 TL
14	50178144	130	13	1000	177,90 TL
22	50177645	125	30	1000	396,40 TL
23	50177905	130	13	1000	179,04 TL
24	50177156	138,5	25	500	190,92 TL
25	50177763	138,5	25	1500	270,65 TL
	Total Wighted AVG.	143	36	35.212	12.694,45 TL

143x36		
0	€	0,2197
1.000	€	0,1353
2.500	€	0,1018
5.000	€	0,0899
7.500	€	0,0848
10.000	€	0,0785
30.000+	€	0,0759
Payment	5.343 YTL	
TOTAL	25 code	
Saving	- 7.352 TL	

25 ERP codes are combined as one code but all codes could not be shown in Table 4.8 due to a long list. All dimension's weighted average is calculated considering purchasing quantity. The 143x36 dimension is the key dimension for the calculation of group 2 saving. 30.000+ price is multiplied by total group 2 quantity (35.212) and new status cost is found as 5.343 TL.

Group 2's total cost was 12.694 TL in October 2010. So, this can be recorded as **7.352 TL saving** for a month in group 2.

Group 3 (FULL HD 1080P – SCART)

Table 4.9 is a sample view of consolidation of codes related with group 3 that includes Full HD 1080P without scart feature at the same label.

Table 4.9 Code consolidation of Full HD 1080P - Scart

Kod	Row material	Dimension 1	Dimension 2	Adhesi	Brand	USB Common	HDMI Common	Energy Common	Scart Common	PC Connectivity	HD Ready	Full HD 1080P
50158582	metalize	144,7	133,4	var		1	1	1		1		1
50174016	metalize	300	100	var	Finlux	1	1	1		1		1
50176750	metalize	300	100	var	Finlux	1	1	1		1		1
50176991	metalize	300	100	var	Finlux	1	1	1		1		1
50176994	metalize	300	100	var	Finlux	1	1	1		1		1
50177623	metalize	300	100	var	Finlux	1	1	1		1		1
50174016	metalize	300	100	yok	Finlux	1	1	1		1		1
50175165	metalize	300	100	yok	Finlux	1	1	1		1		1
50176994	metalize	300	100	var	Finlux	1	1	1		1		1
50177345	metalize	300	100	var	Finlux	1	1	1		1		1

In group 3; all USB features, HDMI features, Energy, No Scart, FULL HD 1080P are combined under the name of an ERP code. 10 ERP codes are consolidated into one ERP code by this way.

Table 4.10 shows the saving calculations of group 3 that includes Full HD 1080P without scart feature.

Table 4.10 Saving result of code consolidation (Full HD 1080P - Scart)

	FULL HD 1080 P - Scart	Dim 1	Dim 2	Quantity	Total Cost
1	50158582	144,7	133,4	284	576,26 TL
2	50174016	300	100	1000	1.312,80 TL
3	50176750	300	100	1000	1.320,95 TL
4	50176991	300	100	1000	1.303,60 TL
5	50176994	300	100	1000	1.314,80 TL
6	50177623	300	100	1000	1.313,20 TL
7	50174016	300	100	1000	1.312,80 TL
8	50175165	300	100	740	971,32 TL
9	50176994	300	100	1000	1.314,80 TL
10	50177345	300	100	1500	1.848,96 TL
	Total Wighted AVG.	295	101	9.524	12.589,49 TL

295x101		
0	€	0,6546
1.000	€	0,5700
2.500	€	0,5362
5.000	€	0,5243
7.500	€	0,5193
10.000	€	0,5129
Payment		9.890,81 TL
TOTAL	10 code	
Saving		-2.698,68 TL

10 ERP codes are combined as one code. All codes are shown in Table 4.10. All dimension's weighted average is calculated considering purchasing quantity. The 295x101 dimension is the key dimension for the calculation of group 3 saving. 7.500+ price is multiplied by total group 3 quantity (9.524) and new status cost is found as 9.891 TL.

Actually, dimensions can be determined after all designs are completed. Thus, this screen label group average dimension is expected much lower than weighted average. So, the saving can be higher than this calculation since this situation is an assumption.

Group 3's total cost was 12.589 TL in October 2010. So, this can be recorded as **2.699 TL saving** for a month in group3.

Group 4 (HD READY – SCART)

Table 4.11 is a sample view of consolidation of codes related with group 4 which includes HD READY without scart feature at the same label.

Table 4.11 Code consolidation of HD Ready - Scart

Kod	Row material	Dimension 1	Dimension 2	Adhesi	Brand	USB Common	HDMI Common	Energy Common	Scart Common	PC Connectivity	HD Ready	Full HD 1080P
50159727	opak	130	13	yok		1	1	1		1	1	
50165749	opak	130	13	yok		1	1	1		1	1	
50170686	opak	139	33			1	1	1		1	1	
50173793	opak	130	13	yok		1	1	1		1	1	
50175305	metalize	300	100	var	Finlux	1	1	1		1	1	
50176119	metalize	300	100	yok	Finlux	1	1	1		1	1	
50176290	opak	130	13	yok		1	1	1		1	1	
50177395	metalize	150	50	var	finlux	1	1	1		1	1	
50177512	opak	130	13	yok		1	1	1		1	1	
50178052	opak	130	13	yok		1	1	1			1	
50143565	metalize	144,7	133,4	var		1	1	1		1	1	

In group 4, all USB features, HDMI features, Energy, No Scart, HD ready are combined under the name of an ERP code. 18 ERP codes are consolidated into one ERP code by this way.

Table 4.12 shows the saving calculations of group 4 that includes HD Ready without scart feature at the same label.

Table 4.12 Saving result of code consolidation (HD READY - Scart)

	HD READY - SCART	Dim 1	Dim 2	Quantity	Total Cost
1	50159727	130	13	1000	179,30 TL
2	50165749	130	13	3700	527,78 TL
3	50170686	139	33	1000	257,60 TL
4	50173793	130	13	1000	178,00 TL
5	50175305	300	100	2060	1.702,98 TL
6	50176119	300	100	1000	1.313,20 TL
7	50176290	130	13	3000	338,10 TL
8	50177395	150	50	1700	1.041,43 TL
9	50177512	130	13	1000	179,30 TL
10	50178052	130	13	7500	597,00 TL
11	50143565	144,7	133,4	1000	1.305,50 TL
12	50162126	160	120	100	83,54 TL
13	50165749	130	13	3700	527,78 TL
14	50173793	130	13	1000	178,00 TL
15	50175305	300	100	2060	1.702,98 TL
16	50177395	150	50	1700	1.041,43 TL
17	50177485	130	13	2000	358,08 TL
18	50177518	130	13	1000	178,78 TL
	Total Wighted AVG.	157	33	35.520	11.690,78 TL

157x33		
0	€	0,2254
1.000	€	0,1410
2.500	€	0,1075
5.000	€	0,0956
7.500	€	0,0905
10.000	€	0,0842
30.000+	€	0,0815
Payment	5.792,07 TL	
TOTAL	18 code	
Saving	-5.898,71 TL	

18 ERP codes are combined as one code, and all codes are shown in Table 4.12. All dimension’s weighted average is calculated considering purchasing quantity. The 157x33 dimension is the key dimension for the calculation of group 4 saving.

Combination provides to order huge quantity screen labels at a time. 30.000+ price is multiplied by total group 4 quantity (35.520) and new status cost is found as 5.792 TL.

Group 1's total cost was 11.691 TL in October 2010. So, this can be recorded as **5.899 TL saving** for a month in group 4.

4.7.1.2 Code Consolidation Result

All in all, all calculations are done for November 2010. The aim was finding unnecessary codes. So, 262 ERP codes are proved to be manageable with 89 codes thanks to new designs (34%). All brands are convinced to switch to new designs. The new designs have been used since January 2011, but all designs have not been completed.

Saving ERP codes provides losing less effort for placing a contract in the ERP system. Moreover, the new system opens huge amount order, so suppliers' coordination has been easy since January 2011.

All calculations are done considering one month plan. However, Vestel's real condition is setup on 3-week plans. Thus, -13% cost saving is not realistic. The plan has to be 3 weeks period, so the order quantity will be lower than saving calculation. 25.477 TL saving is decreasing to 19.108 TL that considers 21-day plans.

To sum up, there is a 165.325 TL saving in addition to 34% ERP codes saving thanks to the code consolidation. This is exactly 9,6% of total Screen Label cost. Table 4.13 shows the saving results of code consolidation.

Table 4.13 Total saving result of code consolidation

NOVEMBER RESULT		TURNOVER	TOTAL CODE
Paid	54.795 YTL		
After Cons. Payment	29.318 YTL		
CON. RESULT SAVİNG/month		- 25.477 YTL	89
NOVEMBER 2010		200.077 YTL	262
		-13%	34%
Real Saving (cons. 21 days plan)		- 19.108 YTL	
Real Saving %		-9,6%	Saving Asmp.
Yearly		1.731.090 YTL	- 165.325 YTL

4.7.2 Order Balance for Less Than 1000 Quantity

Vestel's screen label order range is hyper variable. Taking account of over 7 million television production in a year, this situation is expectable. Screen label orders have to divide into pieces according to ERP codes, raw material types, adhesive features, and dimension differences.

Table 4.14 shows a sample of calculations and quantity discount.

Table 4.14 Quantity discount sample view

Screen Label Price Analysis Chart								
	Firm Name	Nonadhesive part	SCRENN LABEL's					
			Widht mm	Height mm	Profit Rate %			
Row Material		No	130	13	12			
Metalize		Quantity Range	750	1.750	3.750	6.250	8.750	17.500
			0,1832 €	0,0908 €	0,0571 €	0,0454 €	0,0403 €	0,0340 €

Quantity \ Dimension	M130x13	M130x16,3	M130x10	M120X100	M120x25	M120x29	M146,1x144,7
0	0,1832	0,1837	0,1826	0,3546	0,1889	0,1990	0,4045
1.000	0,0908	0,0947	0,0855	0,2705	0,1050	0,1149	0,3205
2.500	0,0571	0,0611	0,0516	0,2366	0,0709	0,0813	0,2866
5.000	0,0454	0,0494	0,0398	0,2248	0,0591	0,0695	0,2746
7.500	0,0403	0,0443	0,0348	0,2198	0,0541	0,0645	0,2696
10.000	0,0340	0,0380	0,0285	0,2135	0,0477	0,0581	0,2633

Code requirements are seen after production forecast releases. Procurement specialist role is ordering this requirement before internal production. In addition, procurement specialist may take a risk to get excess order in order to get quantity discount.

There are five quantity ranges that are agreed on;

- 0 to 1000
- 1000 to 2500
- 2500 to 5000
- 5000 to 7500
- 7500 to 10.000

0-1000 unit's prices are always calculated but never used. Minimum order quantity is 1000 units up to now. Suppliers exceptionally make a production under 1000 units.

In general, Vestel has additional 80 new television versions in a month. Thus, requirements may be lower than 1000 units. This causes excess inventory. When the stock control is carried out in a quarter, excess inventory causes underperformance. The Chief Executive Officer (CEO) sensitively pays attention on this subject.

Procurement specialist may depart from the rule according to requirements and convince the suppliers. However, he or she has to consider payment amount while coordinating stock. If he or she gives the highest priority to stock control, the total cost may become immense.

Thus, a new regulation comes out as necessity. Actual requirements are investigated and minimum order quantity negotiations have started. Screen label suppliers do not want to produce below 1000 unit because every code changes cause extra setup. 500 unit productions take 2-3 minutes but setup time is 15 minutes. They complain about not to gain enough below 1000 units. This request seems fair but they have to consider yearly 4.500.000 screen label production. Vestel is a huge market for this kind of suppliers. They do not take a risk to lose Vestel. So, they need to adjust their production to Vestel's conditions if they think that under 1000 unit causes loss.

Figure 4.15 shows the quantity fluctuations which is lower than 1000 units.

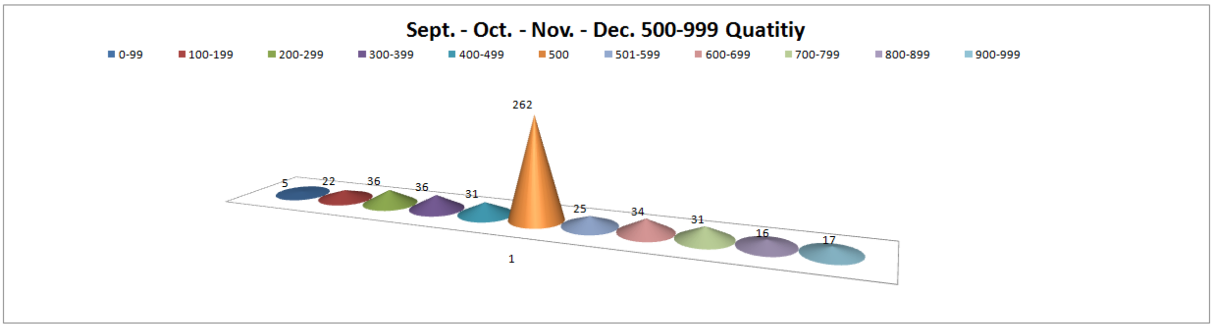

Figure 4.15 Under 1000 qty requirements of screen label

So, minimum order quantity is set to 500 units according to the requirements range. Suppliers are convinced to fit new regulation.

Table 4.15 shows saving which comes from new stock control regulation.

Table 4.15 Saving result with min order quantity 500

							Ağırlıklı Ort. 164x49				
	Sept.	Oct.	Nov.	Dec.	TOTAL	Order Quantity	164x49 Ebat fiyatları		Saving		
0-99	0	0	2	3	5	99	0,00	€ 0,4594	-€ 161,1		
100-199	1	1	7	13	22	199	1.000,00	€ 0,2619	-€ 709,0	Below 500 is fixed to 500; above 500 is fixed to 1000 as a Supllier Order	
200-299	2	4	9	21	36	299	2.500,00	€ 0,1829	-€ 1.160,2		
300-399	2	10	8	16	36	399	5.000,00	€ 0,1552	-€ 1.160,2		
400-499	1	7	13	10	31	499	7.500,00	€ 0,1434	-€ 999,1		
500	0	77	131	54	262	500	10.000,00	€ 0,1286	-€ 8.443,6		
501-599	4	7	8	6	25	599			-€ 331,4		
600-699	2	9	20	3	34	699			-€ 2.012,7		
700-799	4	8	10	9	31	799			-€ 3.259,3		
800-899	5	7	1	3	16	899			-€ 2.417,3		
900-999	0	5	5	7	17	999			-€ 3.349,4	**Yearly Saving**	Yearly Label Turnover
								TOTAL	**-€ 24.003,4**	**-€ 72.010**	**€ 865.545**
										Saving %	**-8,3%**

Saving is calculated with the price of the 164x49 dimension. The dimension is the result of weighted average of below 1000 units.

Fewer than 500 units are fixed to 500 and above 500 units are fixed to 1000 units. The result of the new regulation under the name of stock control is 72.010 Euro/year saving. This amounts to 8,3% of total screen label turnover. Moreover, stock control affects stock performance rate positively which is controlled every quarter in a year.

4.7.3 Order Balance for 10.000 Unit +

Screen label quantity discount ends at 10.000 units as mentioned before. This situation brings back a question. What about the over 10.000 unit's requirements?

The investigation shows that there is limited number of orders that can be reorganized.

Table 4.16 shows requirements which are higher than 10.000 quantities in order to get a new order balance regulation.

Table 4.16 ERP codes of 10.000+requirements

OCTOBER

Total Requirements	
ERP Codes	**201010**
50176131	123.510
50136096	31.910
50154675	18.200
50154411	12.700
50176929	11.700

These code prices should be lower than 7.500 to 10.000 range price. Thus, new calculations are done and entered into an agreement with suppliers.

Table 4.17 shows saving which comes from new 10.000+ requirement's regulation.

Table 4.17 Saving result with 10.000+ requirement's regulations

	M146,1x144,7					
0,00	0,6040		1.Order	2.Order		
1.000,00	0,4071	50176131	60k	63,5k		
2.500,00	0,3281	50136096	20k	12k		
5.000,00	0,3003	Saving			**TOTAL/month**	**TOTAL LABEL TURNOVER**
7.500,00	0,2885	50176131	-€ 683	-€ 723	**-€ 1.405**	**€ 865.545**
10.000,00	0,2738	50136096	€ -	€ -	Yearly Saving %	**-2%**
25000+	0,2676					
50.000+	0,2624					
100.000+	0,2610					

The saving for 10.000 + units is calculated. ERP codes show all dimension's as the 146,1x144,7. Since Vestel has 21 days plan, 50176131 and 50136096 (ERP codes) requirements is divided into two pieces. Total yearly saving is 16.864 Euro. This is 2% of yearly Screen Label's turnover.

4.7.4 Total Reengineering Result

Table 4.18 is a saving summary of Business Process Reengineering application on Screen Label Product group.

Table 4.18 Total saving result of business process reengineering (BPR)

	Yearly Saving		%
Code Consolidation	-€	82.663	-9,6%
Order Balance under 1000 Q	-€	72.010	-8,3%
Order Balance 10.000 +	-€	16.860	-1,9%
Total Saving	**-€**	**171.533**	**-19,8%**
Total Label Turnover	€	865.545	

Code consolidation provides 9,6%, stock control (Under 1000 units) provides 8,3% and Order balance for 10.000+ provides 1,9% saving. Every saving action over 10% is considered Reengineering. Vestel's screen label reengineering applications are successful with 19,8% cost down.

CHAPTER FIVE
APPLICATION OF DEA-HYBRID ALGORITHM

In general, the DEA application for screen label product group is explained in this chapter. Selecting the variables for the DEA model, DEA-AHP application, and sensitivity analysis are also detailed.

5.1 The Purpose and Scope

This is a research related with supplier selection in Vestel Electronics Company. There are various product groups for external sourcing such as capacitor, resistor, tuner, memory, metal frame, plastic frame, screen label, cartoon box, and artwork. External sources are purchased from 80% East Asia, 10% Europe, and 10% Turkey.

Central Purchasing Department has a direct relation and responsibility with suppliers. In addition, purchasing specialist mission is to source right product with adequate quality while paying bottom price at the market. Each product group is extremely important for sustainability at the electronic market while considering 2.500.000.000 TL yearly turnover.

Screen labels are one of the important product groups in the system because almost every television has a screen label. Briefly stated as numerically, 4.500.000 Screen labels are sourced in 2010.

Supplier selection is applied to screen label product group. 11 label suppliers have been declared to the Vestel’s Enterprise Resource Planning (ERP) system. Vestel proceeds a supplier selection progress two times in a year for screen label product group. Two or three suppliers are chosen for a period according to Vestel’s capacity requirement forecast.

Two or three suppliers can make a production as requested quantity. However, quality, delivery, Non-ROHS problems may occur in a 6 month period. The important thing is to choose the right supplier in order to avoid these kinds of problems as much as possible. If these problems cause delay/downtime in Vestel, supplier's resource approval is canceled. So, new supplier replacement may be needed in the middle of the period. This progress is really painful for both sides because supplier had scheduled a production planning regardless of Vestel. Thus, new term unit price level is most likely higher than agreed prices at the beginning of a year. Obviously, this is a negative effect for Vestel. All in all, selecting right supplier at the beginning has high importance level for Central Purchasing Department.

Supplier's information is gathered from their company's database and Vestel's enterprise resource planning system (ERP) for the DEA data preparations. However, the company's confidential policy sets bound to reach demanded data. The data are collected as much as possible in permitted region.

5.2 Selecting Variables for DEA

Input and output quantities should be excessive in order to make decomposition of decision units. Thus, inputs and outputs should be selected as much as possible at DEA-AHP. 12 decision units will be analyzed in this research. The total of inputs and outputs should be lower than 12 for the research reliability (Boussofiane, 1991).

Supplier selection is a nontrivial problem for central purchasing department. As supplier selection with DEA-AHP has been a new methodology since 2005, it is used in this research.

Manisa, İzmir and İstanbul are the based regions for screen label product group's supplier selection. Screen label suppliers (decision units) still have a business relation with Vestel or had a business in the past.

Vestel's Screen Label Suppliers;

1. Turkuaz (Manisa)
2. SETAG (İzmir)
3. Damla Baskı (İstanbul)

4. Doğuş Etiket (İzmir)

5. Modi (İstanbul)

6. Faruk Aydın Boyser (Manisa)

7. İstanbul Etiket (İstanbul)

8. Yılpar (İzmir)

9. Rotec (İzmir)

10. Sembol Barkod & Etiket (İstanbul)

Screen label suppliers can be all around Turkey but the procurement lead time is a significant selective criterion. Thus, the lead time has to be considered in the DEA-AHP modeling.

The key point is to choose the right input and output selection. The person with comprehensive knowledge about screen label product group should assign input and output variables. Thus, Central Purchasing Department's Region Manager has determined decision variables as below.

Input variables;

1. Monthly Available Capacity

2. Number of workers

3. Total Digital Printer Quantity

4. Raw Material Stocking Volume (days)

5. Vehicle Fleet (qty)

Output Variables;

1. Lead Time (day)

2. Yearly Refusal Product Rate

3. Unit cost

4. Delivery on Time

5. Financial Turnover

6. Internal Rejection Rate (directly effects unit cost, so no need to evaluate)

7. Missed faults caught by customer shipment base (insufficient data - eliminated)

8. Cycle Time for 1000 units (directly effects unit cost, so no need to evaluate)

9. Order quantities sent less than needed (insufficient data - eliminated)

All above input and output variables are evaluated. Variables which have unnecessary and imperfect data are eliminated for the DEA-AHP hybrid algorithm.

Table 5.1 shows the absolute list of inputs

Table 5.1 Vestel Electronics screen label product group inputs

INPUTS					
	Available Capacity /month	*Number of Workers*	*Total Digital Printer Mechine*	*Raw material stocking volume (days)*	*Vehicle Fleet (qty)*
TURKUAZ	500.000	23	2	30	2
SETAG	300.000	24	2	75	1
ATLI	180.000	26	1	30	1
DAMLA BASKI	200.000	24	5	30	2
DOĞUŞ ETİKET	220.000	28	1	45	2
Modi	400.000	52	3	90	2
Faruk Aydın Boyser	140.000	60	1	30	1
İstanbul Etiket	400.000	48	5	60	3
YILPAR	200.000	70	1	30	2
ROTEC	800.000	45	2	90	2
SEMBOL BARKOD & ETİKET	320.000	50	4	60	3
Unit	***Quantity***	***Worker***		***Days***	

Table 5.2 shows the absolute list of outputs.

Table 5.2 Vestel Electronics screen label product group outputs

OUTPUTS					
	Lead Time (day)	*Refusal Product Rate (1 year)*	*Unit cost*	*Delivery on Time*	*Financial Turnover*
TURKUAZ	2	0,10%	€ 0,285	99,7%	€ 5.000.000
SETAG	3	0,13%	€ 0,433	99,5%	€ 5.000.000
ATLI	3	5,20%	€ 0,455	93,8%	€ 600.000
DAMLA BASKI	4	0,53%	€ 0,402	96,2%	€ 3.000.000
DOĞUŞ ETİKET	4	0,15%	€ 0,440	97,4%	€ 3.000.000
Modi	6	1,00%	€ 0,725	94,3%	€ 1.500.000
Faruk Aydın Boyser	3	3,20%	€ 0,505	80,2%	€ 1.200.000
İstanbul Etiket	5	1,00%	€ 0,433	80,1%	€ 4.500.000
YILPAR	4	5,00%	€ 0,815	89,0%	€ 750.000
ROTEC	4	6,50%	€ 0,754	90,0%	€ 1.740.000
SEMBOL BARKOD & ETİKET	3	8,00%	€ 0,608	81,5%	€ 5.200.000
Unit	***Days***		***Euro***	***Shipment #***	***Euro***

DEA hybrid algorithm can analyze multi-unit variables with two ways;

1. The units such as dollar and quantity can be used at the same DEA algorithm without any modification.

2. Multi-units are categorized as the same unit in the same DEA algorithm. This method is called normalization.

Normalization is applied in this research in order to control multi-unit variables. The next step is to categorize all inputs and outputs for normalization. Table 5.3 shows categorization of all inputs and outputs.

Table 5.3 Categorization of inputs and outputs

Category	Minor Topic
output negative	Lead Time (day)
output negative	Refusal Product Rate (1 year)
output negative	Unit cost
output positive	Delivery on Time
output positive	Financial Turnover
input positive	Available Capacity /month
input positive	Number of Workers
input positive	Total Digital Printer Mechine
input positive	Raw material stocking volume (days)
input positive	Vehicle Fleet (qty)

All negative category shows that minimum value is better for related decision variables. For example, a minimum lead time is a desirable result, so the supplier with the minimum lead time should get maximum normalization point.

All positive category shows that a maximum value is better for related decision variables. For instance, maximum delivery on time rate is a desirable result, so the supplier with the maximum delivery on time rate should get maximum normalization point.

Table 5.4 shows normalization table for DEA-AHP. Positive category maximum value and negative category minimum value get maximum normalization point.

Table 5.4 The normalization table for DEA-AHP

Normalize Table		TURKUAZ	SETAG	ATLI	DAMLA BASKI	DOĞUŞ ETİKET	Modi	Faruk Aydın Boyser	İstanbul Etiket	YILPAR	ROTEC	SEMBOL BARKOD & ETİKET
output	Lead Time (day)	100,00	66,67	66,67	50,00	50,00	33,33	66,67	40,00	50,00	50,00	66,67
	Refusal Product Rate (last 1 year)	100,00	75,00	1,92	19,00	66,67	10,00	3,13	10,00	2,00	1,54	1,25
	Unit cost	100,00	65,82	62,64	70,90	64,77	39,31	56,44	65,82	34,97	37,80	46,88
	Delivery on Time Rate	100,00	99,80	94,08	96,49	97,64	94,58	80,44	80,34	89,27	90,27	81,75
	Financial Turnover	96,15	96,15	11,54	57,69	57,69	28,85	23,08	86,54	14,42	33,46	100,00
input	Available Capacity /month	62,50	37,50	22,50	25,00	27,50	50,00	17,50	50,00	25,00	100,00	40,00
	Number of Workers	32,86	34,29	37,14	34,29	40,00	74,29	85,71	68,57	100,00	64,29	71,43
	Total Digital Printer Mechine	40,00	40,00	20,00	100,00	20,00	60,00	20,00	100,00	20,00	40,00	80,00
	Raw material stocking volume (days)	33,33	83,33	33,33	33,33	50,00	100,00	33,33	66,67	33,33	100,00	66,67
	Vehicle Fleet (qty)	66,67	33,33	33,33	66,67	66,67	66,67	33,33	100,00	66,67	66,67	100,00

5.3 First Phase: Weighted Linear DEA Model Application for Screen Label Supplier Selection Model:

WLDEA model is a linear form of DEA.

DEA Model's objective function tries to get maximum outputs over inputs value (efficiency value) while forcing the other decision unit's efficiency value less than or equal to one which is shown as the first constraint (1).

The DEA efficiency measurement model is;

$$e_k = max \sum_{r=1}^{t} U_r\, Y_{rk} \Big/ \sum_{i=1}^{m} V_i\, X_{ik}$$

Subject to

(1)

$$\sum_{r=1}^{t} U_r\, Y_{rk} \Big/ \sum_{i=1}^{m} V_i\, X_{ik} \leq 1 \quad k = 1,2,\dots,n$$

$$U_r \geq 0 \qquad r = 1,2,\dots,t$$

$$V_i \geq 0 \qquad i = 1,2,\dots,m$$

e_k: k^{th} decision unit's efficiency

U_r: the weight of rth units of output

V_i: the weight of i^{th} units input

Y_{rk}: r^{th} output related with k^{th} decision unit

X_{ik}: i^{th} input related with k^{th} decision unit

n: Total decision units

t: Total output units

m:Total input units

WLDEA (Linear form of DEA):

$$e_k = max \sum_{r=1}^{t} U_r Y_{ik}$$

Subject to:

(2)

$$\sum_{i=1}^{m} V_i X_{ik} = 1$$

(3)

$$\sum_{r=1}^{t} U_r Y_{rk} - \sum_{i=1}^{m} V_i X_{ik} \geq 0 \quad k = 1,2, \dots, n$$

$$U_r \geq 0 \qquad r = 1,2, \dots, t$$

$$V_i \geq 0 \qquad i = 1,2, \dots, m$$

e_k: k^{th} decision unit's efficiency

U_r: the weight of r^{th} units of output

V_i: the weight of i^{th} units input

Y_{rk}: r^{th} output related with k^{th} decision unit

X_{ik}: i^{th} input related with k^{th} decision unit

n: Total decision units

t: Total output units

m:Total input units

WLDEA application for screen label supplier selection:

$Max\ 100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5$

Subject to

$62{,}5v_1 + 32{,}86v_2 + 40v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \qquad v_1 \geq 0$

$u_2 \geq 0 \qquad v_2 \geq 0$

$u_3 \geq 0 \qquad v_3 \geq 0$

$u_4 \geq 0 \qquad v_4 \geq 0$

$u_5 \geq 0 \qquad v_5 \geq 0$

The Model's objective function tries to get maximum outputs value (efficiency value) while forcing all supplier's outputs minus inputs value less than or equal to 0.

The WLDEA phase 1 model is used for evaluating one of the suppliers within all the others. The above model is an example for Turkuaz's evaluation. The weighted linear DEA model is easier to use thanks to its linear structure. Considering above WLDEA model, if Turkuaz is effective, the objective function will be one, otherwise it will have positive value which is less than one.

The objective function can be switched with other supplier's data in order to see the other supplier's efficiency. The model should be applied for each supplier. Thus, there are 11 models for the evaluation. (Appendix 1) Lingo is used to solve these models.

5.4 DEA Supplier Efficiency Result

Table 5.5 The DEA efficiency result

Label Supplier	DEA
TURKUAZ	1,000
SETAG	1,000
ATLI	1,000
DAMLA BASKI	1,000
DOĞUŞ ETİKET	1,000
Faruk Aydın Boyser	1,000
SEMBOL BARKOD & ETİKET	1,000
YILPAR	0,966
İstanbul Etiket	0,786
ROTEC	0,558
Modi	0,505

DEA Phase 1 model result shows the efficient suppliers with ranking 1. However, this evaluation shows that 7 of the 11 suppliers are efficient. Vestel Electronics cannot prefer to use 7 suppliers for one product group. Thus, additional analysis should be done for understanding which one is the most efficient Label supplier. DEA-AHP is a preferred method, and the next step, phase 2, starts.

5.5 Second Phase: DEA Pair Wise Comparison Application for Screen Label Supplier Selection

Comparison of the couple of screen label suppliers shows which one is more effective. If the kth supplier is effective compared to the k′th, the objective function will be one and the related constraint is set to zero. Otherwise, the objective function will be positive and less than one.

Model:

$$\boldsymbol{E_{1,2}} = \textbf{Max}\ 100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5$$

Subject to

$$62{,}5v_1 + 32{,}86v_2 + 40v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$u_1 \geq 0 \quad u_2 \geq 0 \quad u_3 \geq 0 \quad u_4 \geq 0 \quad u_5 \geq 0 \quad v_1 \geq 0 \quad v_2 \geq 0 \quad v_3 \geq 0 \quad v_4 \geq 0 \quad v_5 \geq 0$$

$$\boldsymbol{E_{1,3}} = \boldsymbol{Max}\ 100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5$$

Subject to

$$62{,}5v_1 + 32{,}86v_2 + 40v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$u_1 \geq 0 \quad u_2 \geq 0 \quad u_3 \geq 0 \quad u_4 \geq 0 \quad u_5 \geq 0 \quad v_1 \geq 0 \quad v_2 \geq 0 \quad v_3 \geq 0 \quad v_4 \geq 0 \quad v_5 \geq 0$$

The other pair-wise comparison models ($E_{1,4}$, $E_{1,5}$, $E_{1,6}$ etc.) are shown in Appendix 2.

Two of decision units are selected and compared with the DEA pair-wise comparison model. Constraint one is used for linearization of DEA. Second constraint represents the first decision unit's outputs minus inputs data which is forced to less than or equal to zero. Considering non-linear form of DEA, this constraint tries to get outputs over inputs (efficiency value) less than or equal to one.

In general, above formulation is a linear form of DEA. However, pair-wise comparison application's DEA model can be explained with non-linear form of DEA. Non-linear model of DEA shows that the model's objective function tries to get maximum outputs over inputs value (efficiency value), while forcing the other decision unit's efficiency value (1) less than or equal to one.

Since there are 11 suppliers, and 10 input and output variables are evaluated, 110 models will be created. (Appendix 2) The results are gathered from Lindo package program and shown in Table 5.6 which is E matrix of DEA-AHP.

5.6 Third Phase: DEA- AHP Application for Screen Label Supplier Selection

First Step:

Table 5.6 shows the results of 110 DEA-AHP models. The E Matrix (Table 5.6) contains pair wise comparisons of the suppliers.

Table 5.6 The E Matrix of DEA-AHP application

E Matrix	TURKUAZ	SETAG	ATLI	DAMLA BASKI	DOĞUŞ ETİKET	Modi	Faruk Aydın Boyser	İstanbul Etiket	YILPAR	ROTEC	SEMBOL BARKOD & ETİKET
TURKUAZ	1	1	1	1	1	1	1	1	1	1	1
SETAG	1	1	1	1	1	1	1	1	1	1	1
ATLI	1	1	1	1	1	1	1	1	1	1	1
DAMLA BASKI	1	1	1	1	1	1	1	1	1	1	1
DOĞUŞ ETİKET	1	1	1	1	1	1	1	1	1	1	1
Modi	1	0,78971	1	1	0,96866	1	1	1	1	1	1
Faruk Aydın Boyser	1	1	1	1	1	1	1	1	1	1	1
İstanbul Etiket	1	1	1	1	1	1	1	1	1	1	1
YILPAR	1	1	1	1	1	1	1	1	1	1	1
ROTEC	0,90270	0,90451	1	1	1	1	1	1	1	1	1
SEMBOL BARKOD & ETİKET	1	1	1	1	1	1	1	1	1	1	1

Step 2: ***A Matrix***

$$a_{k,k'} = \frac{e_{k,k'}}{e_{k',k}}$$

Table 5.7 The A Matrix of DEA-AHP application

A Matrix	TURKUAZ	SETAG	ATLI	DAMLA BASKI	DOĞUŞ ETİKET	Modi	Faruk Aydın Boyser	İstanbul Etiket	YILPAR	ROTEC	SEMBOL BARKOD & ETİKET
TURKUAZ	1	1	1	1	1	1	1	1	1	1,10779	1
SETAG	1	1	1	1	1	1,26628	1	1	1	1,10557	1
ATLI	1	1	1	1	1	1	1	1	1	1	1
DAMLA BASKI	1	1	1	1	1	1	1	1	1	1	1
DOĞUŞ ETİKET	1	1	1	1	1	1,03235	1	1	1	1	1
Modi	1	0,78971	1	1	0,96866	1	1	1	1	1	1
Faruk Aydın Boyser	1	1	1	1	1	1	1	1	1	1	1
İstanbul Etiket	1	1	1	1	1	1	1	1	1	1	1
YILPAR	1	1	1	1	1	1	1	1	1	1	1
ROTEC	0,90270	0,90451	1	1	1	1	1	1	1	1	1
SEMBOL BARKOD & ETİKET	1	1	1	1	1	1	1	1	1	1	1
Sum of A	10,90270	10,69422	11	11	10,96866	11,29863	11	11	11	11,21336	11

$$Sum\ of\ A,\ e.g\ \rightarrow T_{n,1} = a_{1,1} + a_{1,2} + a_{1,3} + \cdots + a_{n,1}$$

Step 3:

$$N\ matrix\ e.g \rightarrow N_{1,1} = \frac{A_{1,1}}{T_{n,1}}$$

Table 5.8 The N matrix of DEA-AHP application

N Matrix	TURKUAZ	SETAG	ATLI	DAMLA BASKI	DOĞUŞ ETİKET	Modi	Faruk Aydın Boyser	İstanbul Etiket	YILPAR	ROTEC	SEMBOL BARKOD & ETİKET
TURKUAZ	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,098792	0,090909
SETAG	0,091720	0,093508	0,090909	0,090909	0,091169	0,112074	0,090909	0,090909	0,090909	0,098594	0,090909
ATLI	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909
DAMLA BASKI	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909
DOĞUŞ ETİKET	0,091720	0,093508	0,090909	0,090909	0,091169	0,091370	0,090909	0,090909	0,090909	0,089179	0,090909
Modi	0,091720	0,073845	0,090909	0,090909	0,088312	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909
Faruk Aydın Boyser	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909
İstanbul Etiket	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909
YILPAR	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909
ROTEC	0,082796	0,084579	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909
SEMBOL BARKOD & ETİKET	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909

Step 4:

$$S_{1,N} = 1 + \left({a_{1,2}}/{T_{n,2}}\right) + \left({a_{1,3}}/{T_{n,3}}\right) + \cdots + \left({a_{1,n}}/{T_{nN}}\right)$$

Table 5.9 Sum of the N matrix of DEA-AHP application

N Matrix	TURKUAZ	SETAG	ATLI	DAMLA BASKI	DOĞUŞ ETİKET	Modi	Faruk Aydın Boyser	İstanbul Etiket	YILPAR	ROTEC	SEMBOL BARKOD & ETİKET	Sum of row
TURKUAZ	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,098792	0,090909	1,009150
SETAG	0,091720	0,093508	0,090909	0,090909	0,091169	0,112074	0,090909	0,090909	0,090909	0,098594	0,090909	1,032520
ATLI	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909	0,999538
DAMLA BASKI	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909	0,999538
DOĞUŞ ETİKET	0,091720	0,093508	0,090909	0,090909	0,091169	0,091370	0,090909	0,090909	0,090909	0,089179	0,090909	1,002401
Modi	0,091720	0,073845	0,090909	0,090909	0,088312	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909	0,977017
Faruk Aydın Boyser	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909	0,999538
İstanbul Etiket	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909	0,999538
YILPAR	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909	0,999538
ROTEC	0,082796	0,084579	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909	0,981684
SEMBOL BARKOD & ETİKET	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909	0,999538

Step 5: Normalization of DEA-AHP result

$$a_k''' = \frac{a_k''}{\sum_{k=1}^{n} a_k''}$$

Table 5.10 The N matrix normalization of DEA-AHP application

N Matrix	TURKUAZ	SETAG	ATLI	DAMLA BASKI	DOĞUŞ ETİKET	Modi	Faruk Aydın Boyser	İstanbul Etiket	YILPAR	ROTEC	SEMBOL BARKOD & ETİKET	Sum of row	DEA-AHP Result (Normalize)
TURKUAZ	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,098792	0,090909	1,009150	0,091741
SETAG	0,091720	0,093508	0,090909	0,090909	0,091169	0,112074	0,090909	0,090909	0,090909	0,098594	0,090909	1,032520	0,093865
ATLI	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909	0,999538	0,090867
DAMLA BASKI	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909	0,999538	0,090867
DOĞUŞ ETİKET	0,091720	0,093508	0,090909	0,090909	0,091169	0,091370	0,090909	0,090909	0,090909	0,089179	0,090909	1,002401	0,091127
Modi	0,091720	0,073845	0,090909	0,090909	0,088312	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909	0,977017	0,088820
Faruk Aydın Boyser	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909	0,999538	0,090867
İstanbul Etiket	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909	0,999538	0,090867
YILPAR	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909	0,999538	0,090867
ROTEC	0,082796	0,084579	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909	0,981684	0,089244
SEMBOL BARKOD & ETİKET	0,091720	0,093508	0,090909	0,090909	0,091169	0,088506	0,090909	0,090909	0,090909	0,089179	0,090909	0,999538	0,090867

5.7 DEA and DEA-AHP Results

Table 5.11 DEA and DEA-AHP result

Label Suppliers	DEA Results	DEA-AHP Results
SETAG	1,0	0,0939
TURKUAZ	1,0	0,0917
DOĞUŞ ETİKET	1,0	0,0911
ATLI	1,0	0,0909
DAMLA BASKI	1,0	0,0909
Faruk Aydın Boyser	1,0	0,0909
SEMBOL BARKOD &	1,0	0,0909
YILPAR	0,9659	0,0909
İstanbul Etiket	0,7865	0,0909
ROTEC	0,5576	0,0892
Modi	0,5051	0,0888

The DEA results show that SETAG, Turkuaz, Doğuş Etiket, Atlı, Damla Baskı and Faruk Aydın Boyser companies are efficient suppliers. However, DEA alone is not enough for Vestel. There has to be additional elimination. So, DEA-AHP analysis is applied. The DEA-AHP result is enough to see which one is more effective (Table 5.11).

The supplier which is closest to one is efficient. So, the suppliers are sorted in descending order. All in all, central purchasing specialist can make a selection depending on DEA-AHP results. Hence, supplier relationship got harmed with Turkuaz in 2010, SETAG and Doğuş Etiket is selected as efficient supplier for the second half of 2011.

5.8 Sensitivity Analysis

Sensitivity analysis is an investigation into how projected performance varies along with changes in the key assumptions. Sensitivity analysis is very useful when attempting to determine the impact of the actual outcome of a particular variable. By creating a given set of scenarios, the analyst can determine how changes in one variable(s) will impact the target variable.

As a purchasing specialist, one of the most important performance indicators is the unit price. The amount of cost down or price-rise directly affects the specialist's yearly performance. Thus, sensitivity analysis is done for unit price which is one of the outputs (decision variable) in DEA. The aim is to understand the effects of unit price change on the top three suppliers. When we investigate the top three suppliers, there is no need to evaluate Turkuaz because of supplier business relation. So, SETAG's and Doğuş Etiket's unit price based sensitivity analysis will be carried out.

The supplier's unit cost/price table is shown in Table 5.12.

Table 5.12 Screen label firm's unit prices

	TURKUAZ	SETAG	ATLI	DAMLA BASKI	DOĞUŞ ETİKET	Modi	Faruk Aydın Boyser	İstanbul Etiket	YILPAR	ROTEC	SEMBOL BARKOD & ETİKET
Unit cost	€ 0,285	**€ 0,433**	€ 0,455	€ 0,402	**€ 0,440**	€ 0,725	€ 0,505	€ 0,433	€ 0,815	€ 0,754	€ 0,608

The purpose of the sensitivity analysis is to find out how much price-rise can be tolerable in order not to lose the top three efficiency grades at DEA-AHP. In other words, the breaking point of unit prices will be determined for SETAG and Doğuş Etiket screen label suppliers.

5.8.1 Sensitivity Analysis on Unit Prices for SETAG

Unit Price breaking point for SETAG is 0,875€. Every point at the range between 0,433€ to 0,875€ is tested. Even 0,870€ is not enough to cause to fall in fourth grade at efficiency list.

Table 5.13 shows the SETAG's limit unit price in order to lose SETAG's efficiency rank.

Table 5.13 Screen label firm's unit prices (SETAG unit price updated)

	TURKUAZ	SETAG	ATLI	DAMLA BASKI	DOĞUŞ ETİKET	Modi	Faruk Aydın Boyser	İstanbul Etiket	YILPAR	ROTEC	SEMBOL BARKOD & ETİKET
Unit cost	€ 0,285	€ 0,875	€ 0,455	€ 0,402	€ 0,440	€ 0,725	€ 0,505	€ 0,433	€ 0,815	€ 0,754	€ 0,608

Updated SETAG's unit price triggers normalization value which is shown in Table 5.14.

Table 5.14 Screen label firm's normalization table (SETAG unit price normalized value updated)

	Normalize Table	TURKUAZ	SETAG	ATLI	DAMLA BASKI	DOĞUŞ ETİKET	Modi	Faruk Aydın Boyser	İstanbul Etiket	YILPAR	ROTEC	SEMBOL BARKOD & ETİKET
output	Lead Time (day)	100,00	66,67	66,67	50,00	50,00	33,33	66,67	40,00	50,00	50,00	66,67
	Refusal Product Rate (last 1 year)	100,00	75,00	1,92	19,00	66,67	10,00	3,13	10,00	2,00	1,54	1,25
	Unit cost	100,00	32,57	62,64	70,90	64,77	39,31	56,44	65,82	34,97	37,80	46,88
	Delivery on Time Rate	100,00	99,80	94,08	96,49	97,64	94,58	80,44	80,34	89,27	90,27	81,75
	Financial Turnover	96,15	96,15	11,54	57,69	57,69	28,85	23,08	86,54	14,42	33,46	100,00
input	Available Capacity /month	62,50	37,50	22,50	25,00	27,50	50,00	17,50	50,00	25,00	100,00	40,00
	Number of Workers	32,86	34,29	37,14	34,29	40,00	74,29	85,71	68,57	100,00	64,29	71,43
	Total Digital Printer Mechine	40,00	40,00	20,00	100,00	20,00	60,00	20,00	100,00	20,00	40,00	80,00
	Raw material stocking volume (days)	33,33	83,33	33,33	33,33	50,00	100,00	33,33	66,67	33,33	100,00	66,67
	Vehicle Fleet (qty)	66,67	33,33	33,33	66,67	66,67	66,67	33,33	100,00	66,67	66,67	100,00

DEA-AHP model for sensitivity analysis;

$\boldsymbol{E_{6,2}} = \textbf{Max}\ 33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5$

Subject to

$50v_1 + 74{,}29v_2 + 60v_3 + 100v_4 + 66{,}67v_5 = 1$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 75u_2 + 32{,}57u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$u_1 \geq 0 \quad u_2 \geq 0 \quad u_3 \geq 0 \quad u_4 \geq 0 \quad u_5 \geq 0 \quad v_1 \geq 0 \quad v_2 \geq 0 \quad v_3 \geq 0 \quad v_4 \geq 0 \quad v_5 \geq 0$

$\boldsymbol{E_{7,2}} = \textbf{Max}\ 66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5$

$17{,}5v_1 + 85{,}71v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$66{,}67u_1 + 75u_2 + 32{,}57u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$u_1 \geq 0 \quad u_2 \geq 0 \quad u_3 \geq 0 \quad u_4 \geq 0 \quad u_5 \geq 0 \quad v_1 \geq 0 \quad v_2 \geq 0 \quad v_3 \geq 0 \quad v_4 \geq 0 \quad v_5 \geq 0$

The other pair-wise comparison models that include change conditions are shown in Appendix 3.

Table 5.15 The E matrix of DEA-AHP (sensitivity analysis of SETAG)

E Matrix *(Sensistivity analysis of SETAG)*	TURKUAZ	SETAG	ATLI	DAMLA BASKI	DOĞUŞ ETİKET	Modi	Faruk Aydın Boyser	İstanbul Etiket	YILPAR	ROTEC	SEMBOL BARKOD & ETİKET
TURKUAZ	1	1	1	1	1	1	1	1	1	1	1
SETAG	1	1	1	1	1	1	1	1	1	1	1
ATLI	1	1	1	1	1	1	1	1	1	1	1
DAMLA BASKI	1	1	1	1	1	1	1	1	1	1	1
DOĞUŞ ETİKET	1	1	1	1	1	1	1	1	1	1	1
Modi	1	**1**	1	1	0,9686604	1	1	1	1	1	1
Faruk Aydın Boyser	1	1	1	1	1	1	1	1	1	1	1
İstanbul Etiket	1	1	1	1	1	1	1	1	1	1	1
YILPAR	1	1	1	1	1	1	1	1	1	1	1
ROTEC	0,9027	**1**	1	1	1	1	1	1	1	1	1
SEMBOL BARKOD & ETİKET	1	1	1	1	1	1	1	1	1	1	1

Table 5.16 The A matrix of DEA-AHP (sensitivity analysis of SETAG)

A Matrix *(Sensitivity analysis of Setag)*	TURKUAZ	SETAG	ATLI	DAMLA BASKI	DOĞUŞ ETİKET	Modi	Faruk Aydın Boyser	İstanbul Etiket	YILPAR	ROTEC	SEMBOL BARKOD & ETİKET
TURKUAZ	1	1	1	1	1	1	1	1	1	1,107788	1
SETAG	1	1	1	1	1	1	1	1	1	1	1
ATLI	1	1	1	1	1	1	1	1	1	1	1
DAMLA BASKI	1	1	1	1	1	1	1	1	1	1	1
DOĞUŞ ETİKET	1	1	1	1	1	1,032354	1	1	1	1	1
Modi	1	1	1	1	0,9686604	1	1	1	1	1	1
Faruk Aydın Boyser	1	1	1	1	1	1	1	1	1	1	1
İstanbul Etiket	1	1	1	1	1	1	1	1	1	1	1
YILPAR	1	1	1	1	1	1	1	1	1	1	1
ROTEC	0,9027	1	1	1	1	1	1	1	1	1	1
SEMBOL BARKOD & ETİKET	1	1	1	1	1	1	1	1	1	1	1
Sum of A	10,90270	11,00000	11,00000	11,00000	10,96866	11,03235	11,00000	11,00000	11,00000	11,10779	11,00000

Table 5.17 The N matrix of DEA-AHP (sensitivity analysis of SETAG)

N Matrix (Sensitivity analysis of Setag)	TURKUAZ	SETAG	ATLI	DAMLA BASKI	DOĞUŞ ETİKET	Modi	Faruk Aydın Boyser	İstanbul Etiket	YILPAR	ROTEC	SEMBOL BARKOD & ETİKET	Sum of row	DEA-AHP Result (Normalize)
TURKUAZ	0,091720	0,090909	0,090909	0,090909	0,091169	0,090642	0,090909	0,090909	0,090909	0,099731	0,090909	1,009626	**0,091784**
SETAG	0,091720	0,090909	0,090909	0,090909	0,091169	0,090642	0,090909	0,090909	0,090909	0,090027	0,090909	0,999922	**0,090902**
ATLI	0,091720	0,090909	0,090909	0,090909	0,091169	0,090642	0,090909	0,090909	0,090909	0,090027	0,090909	0,999922	**0,090902**
DAMLA BASKI	0,091720	0,090909	0,090909	0,090909	0,091169	0,090642	0,090909	0,090909	0,090909	0,090027	0,090909	0,999922	**0,090902**
DOĞUŞ ETİKET	0,091720	0,090909	0,090909	0,090909	0,091169	0,093575	0,090909	0,090909	0,090909	0,090027	0,090909	1,002855	**0,091169**
Modi	0,091720	0,090909	0,090909	0,090909	0,088312	0,090642	0,090909	0,090909	0,090909	0,090027	0,090909	0,997065	**0,090642**
Faruk Aydın Boyser	0,091720	0,090909	0,090909	0,090909	0,091169	0,090642	0,090909	0,090909	0,090909	0,090027	0,090909	0,999922	**0,090902**
İstanbul Etiket	0,091720	0,090909	0,090909	0,090909	0,091169	0,090642	0,090909	0,090909	0,090909	0,090027	0,090909	0,999922	**0,090902**
YILPAR	0,091720	0,090909	0,090909	0,090909	0,091169	0,090642	0,090909	0,090909	0,090909	0,090027	0,090909	0,999922	**0,090902**
ROTEC	0,082796	0,090909	0,090909	0,090909	0,091169	0,090642	0,090909	0,090909	0,090909	0,090027	0,090909	0,990998	**0,090091**
SEMBOL BARKOD & ETİKET	0,091720	0,090909	0,090909	0,090909	0,091169	0,090642	0,090909	0,090909	0,090909	0,090027	0,090909	0,999922	**0,090902**

Efficiency results are shown in Table 5.18 according to unit price 0,875€ for SETAG.

Table 5.18 DEA-AHP result (sensitivity analysis of SETAG)

Rank	Label Suppliers	DEA Results	DEA-AHP Results
1	TURKUAZ	1,0	0,0918
2	DOĞUŞ ETİKET	1,0	0,0912
3	ATLI	1,0	0,0909
4	SETAG	1,0	0,0909
5	DAMLA BASKI	1,0	0,0909
6	Faruk Aydın Boyser	1,0	0,0909
7	SEMBOL BARKOD &	1,0	0,0909
8	YILPAR	0,9659	0,0909
9	İstanbul Etiket	0,7865	0,0909
10	Modi	0,5051	0,0906
11	ROTEC	0,5576	0,0901

Assuming all other factors are constant, SETAG's unit price range tolerance is up to 0,875€ in order not to lose position.

5.8.2 Sensitivity Analysis on Unit Prices for Doğuş Etiket

Unit Price breaking point for Doğuş Etiket is 0,73€. Every point at the range between 0,44€ to 0,73€ is tested. Even 0,72€ is not enough to cause to fall in fourth grade at efficiency list.

Table 5.19 shows the Doğuş Etiket's limit unit price in order to lose Doğuş Etiket's efficiency rank.

Table 5.19 Screen label firm's unit prices (Doğuş Etiket unit price updated)

	TURKUAZ	SETAG	ATLI	DAMLA BASKI	DOĞUŞ ETİKET	Modi	Faruk Aydın Boyser	İstanbul Etiket	YILPAR	ROTEC	SEMBOL BARKOD & ETİKET
Unit cost	€ 0,285	€ 0,433	€ 0,455	€ 0,402	€ 0,730	€ 0,725	€ 0,505	€ 0,433	€ 0,815	€ 0,754	€ 0,608

Updated Doğuş Etiket's unit price triggers normalization value which is shown in Table 5.20.

Table 5.20 Screen label firm's normalization table (Doğuş Etiket unit price normalization value updated)

	Normalize Table	TURKUAZ	SETAG	ATLI	DAMLA BASKI	DOĞUŞ ETİKET	Modi	Faruk Aydın Boyser	İstanbul Etiket	YILPAR	ROTEC	SEMBOL BARKOD & ETİKET
output	Lead Time (day)	100,00	66,67	66,67	50,00	50,00	33,33	66,67	40,00	50,00	50,00	66,67
	Refusal Product Rate (last 1 year)	100,00	75,00	1,92	19,00	66,67	10,00	3,13	10,00	2,00	1,54	1,25
	Unit cost	100,00	65,82	62,64	70,90	39,04	39,31	56,44	65,82	34,97	37,80	46,88
	Delivery on Time Rate	100,00	99,80	94,08	96,49	97,64	94,58	80,44	80,34	89,27	90,27	81,75
	Financial Turnover	96,15	96,15	11,54	57,69	57,69	28,85	23,08	86,54	14,42	33,46	100,00
input	Available Capacity /month	62,50	37,50	22,50	25,00	27,50	50,00	17,50	50,00	25,00	100,00	40,00
	Number of Workers	32,86	34,29	37,14	34,29	40,00	74,29	85,71	68,57	100,00	64,29	71,43
	Total Digital Printer Mechine	40,00	40,00	20,00	100,00	20,00	60,00	20,00	100,00	20,00	40,00	80,00
	Raw material stocking volume (days)	33,33	83,33	33,33	33,33	50,00	100,00	33,33	66,67	33,33	100,00	66,67
	Vehicle Fleet (qty)	66,67	33,33	33,33	66,67	66,67	66,67	33,33	100,00	66,67	66,67	100,00

DEA-AHP model for sensitivity analysis;

$$\boldsymbol{E_{6,5}} = \mathbf{Max}\ 33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5$$

Subject to

$$50v_1 + 74{,}29v_2 + 60v_3 + 100v_4 + 66{,}67v_5 = 1$$

$$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$$

$$u_1 \geq 0 \quad u_2 \geq 0 \quad u_3 \geq 0 \quad u_4 \geq 0 \quad u_5 \geq 0 \quad v_1 \geq 0 \quad v_2 \geq 0 \quad v_3 \geq 0 \quad v_4 \geq 0 \quad v_5 \geq 0$$

$\boldsymbol{E_{7,5}} = \textbf{Max}\ 66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5$

Subject to

$17{,}5v_1 + 85{,}71v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad u_2 \geq 0 \quad u_3 \geq 0 \quad u_4 \geq 0 \quad u_5 \geq 0 \quad v_1 \geq 0 \quad v_2 \geq 0 \quad v_3 \geq 0 \quad v_4 \geq 0 \quad v_5 \geq 0$

The other pair-wise comparison models that include change conditions are shown in Appendix 4.

Table 5.21 The E matrix of DEA-AHP (sensitivity analysis of Doğuş Etiket)

E Matrix (Sensistivity analysis of DOĞUŞ)	TURKUAZ	SETAG	ATLI	DAMLA BASKI	DOĞUŞ ETİKET	Modi	Faruk Aydın Boyser	İstanbul Etiket	YILPAR	ROTEC	SEMBOL BARKOD & ETİKET
TURKUAZ	1	1	1	1	1	1	1	1	1	1	1
SETAG	1	1	1	1	1	1	1	1	1	1	1
ATLI	1	1	1	1	1	1	1	1	1	1	1
DAMLA BASKI	1	1	1	1	1	1	1	1	1	1	1
DOĞUŞ ETİKET	1	1	1	1	1	1	1	1	1	1	1
Modi	1	0,78971	1	1	**1,00000**	1	1	1	1	1	1
Faruk Aydın Boyser	1	1	1	1	1	1	1	1	1	1	1
İstanbul Etiket	1	1	1	1	1	1	1	1	1	1	1
YILPAR	1	1	1	1	1	1	1	1	1	1	1
ROTEC	0,90270	0,90451	1	1	1	1	1	1	1	1	1
SEMBOL BARKOD & ETİKET	1	1	1	1	1	1	1	1	1	1	1

Table 5.22 The A matrix of DEA-AHP (sensitivity analysis of Doğuş Etiket)

A Matrix (Sensitivity analysis of DOĞUŞ)	TURKUAZ	SETAG	ATLI	DAMLA BASKI	DOĞUŞ ETİKET	Modi	Faruk Aydın Boyser	İstanbul Etiket	YILPAR	ROTEC	SEMBOL BARKOD & ETİKET
TURKUAZ	1	1	1	1	1	1	1	1	1	1,107788	1
SETAG	1	1	1	1	1	1,26628	1	1	1	1,105572	1
ATLI	1	1	1	1	1	1	1	1	1	1	1
DAMLA BASKI	1	1	1	1	1	1	1	1	1	1	1
DOĞUŞ ETİKET	1	1	1	1	1	1	1	1	1	1	1
Modi	1	0,7897146	1	1	1	1	1	1	1	1	1
Faruk Aydın Boyser	1	1	1	1	1	1	1	1	1	1	1
İstanbul Etiket	1	1	1	1	1	1	1	1	1	1	1
YILPAR	1	1	1	1	1	1	1	1	1	1	1
ROTEC	0,9027	0,904509	1	1	1	1	1	1	1	1	1
SEMBOL BARKOD & ETİKET	1	1	1	1	1	1	1	1	1	1	1
Sum of A	10,90270	10,69422	11,00000	11,00000	11,00000	11,26628	11,00000	11,00000	11,00000	11,21336	11,00000

Table 5.23 The N matrix of DEA-AHP (sensitivity analysis of Doğuş Etiket)

N Matrix (Sensitivity analysis of DOĞUŞ)	TURKUAZ	SETAG	ATLI	DAMLA BASKI	DOĞUŞ ETİKET	Modi	Faruk Aydın Boyser	İstanbul Etiket	YILPAR	ROTEC	SEMBOL BARKOD & ETİKET	Sum of row	DEA-AHP Result (Normalize)
TURKUAZ	0,091720	0,093508	0,090909	0,090909	0,090909	0,088760	0,090909	0,090909	0,090909	0,098792	0,090909	1,009145	**0,091740**
SETAG	0,091720	0,093508	0,090909	0,090909	0,090909	0,112396	0,090909	0,090909	0,090909	0,098594	0,090909	1,032582	**0,093871**
ATLI	0,091720	0,093508	0,090909	0,090909	0,090909	0,088760	0,090909	0,090909	0,090909	0,089179	0,090909	0,999532	**0,090867**
DAMLA BASKI	0,091720	0,093508	0,090909	0,090909	0,090909	0,088760	0,090909	0,090909	0,090909	0,089179	0,090909	0,999532	**0,090867**
DOĞUŞ ETİKET	0,091720	0,093508	0,090909	0,090909	0,090909	0,088760	0,090909	0,090909	0,090909	0,089179	0,090909	0,999532	**0,090867**
Modi	0,091720	0,073845	0,090909	0,090909	0,090909	0,088760	0,090909	0,090909	0,090909	0,089179	0,090909	0,979869	**0,089079**
Faruk Aydın Boyser	0,091720	0,093508	0,090909	0,090909	0,090909	0,088760	0,090909	0,090909	0,090909	0,089179	0,090909	0,999532	**0,090867**
İstanbul Etiket	0,091720	0,093508	0,090909	0,090909	0,090909	0,088760	0,090909	0,090909	0,090909	0,089179	0,090909	0,999532	**0,090867**
YILPAR	0,091720	0,093508	0,090909	0,090909	0,090909	0,088760	0,090909	0,090909	0,090909	0,089179	0,090909	0,999532	**0,090867**
ROTEC	0,082796	0,084579	0,090909	0,090909	0,090909	0,088760	0,090909	0,090909	0,090909	0,089179	0,090909	0,981679	**0,089244**
SEMBOL BARKOD & ETİKET	0,091720	0,093508	0,090909	0,090909	0,090909	0,088760	0,090909	0,090909	0,090909	0,089179	0,090909	0,999532	**0,090867**

Efficiency results are shown in Table 5.24 according to unit price 0,73€ for Doğuş Etiket .

Table 5.24 DEA-AHP result (sensitivity analysis of Doğuş Etiket)

Rank	Label Suppliers	DEA Results	DEA-AHP Results
1	SETAG	1,0	0,0939
2	TURKUAZ	1,0	0,0917
3	DAMLA BASKI	1,0	0,0909
4	DOĞUŞ ETİKET	1,0	0,0909
5	ATLI	1,0	0,0909
6	Faruk Aydın Boyser	1,0	0,0909
7	SEMBOL BARKOD &	1,0	0,0909
8	YILPAR	0,9659	0,0909
9	İstanbul Etiket	0,7865	0,0909
10	ROTEC	0,5576	0,0892
11	Modi	0,5051	0,0891

Doğuş Etiket's unit price tolerance is up to 0,73€ in order not to lose efficient position considering all the other factors are constant.

To sum up, Unit price 0,875€ which is the bound level for SETAG is the highest price among the other supplier's. Also, 0,73€, which is the bound level for Doğuş Etiket, is a high price too. This situation shows the weak side of the DEA.

DEA-AHP sensitivity analysis shows the exact bound value according to the DEA rules. However, the rules do not include weighted value between inputs and outputs (decision variables) in the DEA algorithm. The DEA algorithm has an acceptance that all decision variables have the same weight. For example, the unit price has the same importance level with the lead time in this research.

If the unit price is more important than the other decision units, the unit price should get more weighted value considering the other decision variable's. So, if the unit price gets high weighted value, the sensitivity analysis bound values will be lower than 0,875€ for SETAG or 0,73€ for Doğuş Etiket. However, there is not such an option on DEA as giving weight on decision variables.

CHAPTER SIX
CONCLUSION

This research was carried out in Vestel Electronics Central Purchasing Department. Screen Label product group's processes were reviewed. Reorganization and improvement of processes were done via Business Process Reengineering. Finally, supplier selection process was done thanks to DEA-AHP hybrid algorithm.

2010 screen label turnover is €865.545 and total purchasing quantity is 4.424.182. In addition, Monthly new product code quantity was 180-200. The management of this enormous capacity requirement and product variety is extremely difficult. The aim of this research was to determine improvement opportunities and make supplier selection analysis while controlling this difficulty.

Code consolidation, order balance for fewer than 1000 quantities and order balance for excess of 1000 quantities were the major topics of BPR. Total savings from these topics was €171.533, i.e., 19,8% of total screen label turnover.

The DEA-AHP analysis result showed that SETAG, Turkuaz and Doğuş Etiket were the most efficient suppliers which have had product quality approval in VESTEL. Vestel's screen label product group's long term strategy is to work with two suppliers (multiple-sourcing) for six month period while considering capacity constraints.

Supplier relationship got harmed with Turkuaz in 2010. Thus, Vestel's central purchasing department's top management decision is that Turkuaz is not a workable firm for the forthcoming year. To sum up, SETAG and Doğuş Etiket have been selected as efficient suppliers for the second half of 2011.

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APPENDIX 1

Model 1:

$Max\ 100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5$

Subject to

$62{,}5v_1 + 32{,}86v_2 + 40v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \qquad v_1 \geq 0$

$u_2 \geq 0 \qquad v_2 \geq 0$

$u_3 \geq 0 \qquad v_3 \geq 0$

$u_4 \geq 0 \qquad v_4 \geq 0$

$u_5 \geq 0 \qquad v_5 \geq 0$

Model 2:

$Max\ 66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5$

Subject to

$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \qquad v_1 \geq 0$

$u_2 \geq 0 \qquad v_2 \geq 0$

$u_3 \geq 0 \qquad v_3 \geq 0$

$u_4 \geq 0 \qquad v_4 \geq 0$

$u_5 \geq 0 \qquad v_5 \geq 0$

Model 3:

$Max\ 66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5$

Subject to

$22{,}5v_1 + 37{,}14v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \qquad v_1 \geq 0$

$u_2 \geq 0 \qquad v_2 \geq 0$

$u_3 \geq 0 \qquad v_3 \geq 0$

$u_4 \geq 0 \qquad v_4 \geq 0$

$u_5 \geq 0 \qquad v_5 \geq 0$

Model 4:

$Max\ 50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5$

Subject to

$$25v_1 + 34{,}29v_2 + 100v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$$

$$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0 \qquad v_1 \geq 0$

$u_2 \geq 0 \qquad v_2 \geq 0$

$u_3 \geq 0 \qquad v_3 \geq 0$

$u_4 \geq 0 \qquad v_4 \geq 0$

$u_5 \geq 0 \qquad v_5 \geq 0$

Model 5:

$Max\ 50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0$

$u_2 \geq 0 \quad v_2 \geq 0$

$u_3 \geq 0 \quad v_3 \geq 0$

$u_4 \geq 0 \quad v_4 \geq 0$

$u_5 \geq 0 \quad v_5 \geq 0$

Model 6:

$Max\ 33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5$

Subject to

$$50v_1 + 74{,}29v_2 + 60v_3 + 100v_4 + 66{,}67v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$$

$$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0 \qquad v_1 \geq 0$

$u_2 \geq 0 \qquad v_2 \geq 0$

$u_3 \geq 0 \qquad v_3 \geq 0$

$u_4 \geq 0 \qquad v_4 \geq 0$

$u_5 \geq 0 \qquad v_5 \geq 0$

Model 7:

$Max\ 66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5$

Subject to

$$17{,}5v_1 + 85{,}71v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$$

$$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0 \qquad v_1 \geq 0$

$u_2 \geq 0 \qquad v_2 \geq 0$

$u_3 \geq 0 \qquad v_3 \geq 0$

$u_4 \geq 0 \qquad v_4 \geq 0$

$u_5 \geq 0 \qquad v_5 \geq 0$

Model 8:

$$Max\ 40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5$$

Subject to

$$50v_1 + 68{,}57v_2 + 100v_3 + 66{,}67v_4 + 100v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$$

$$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$40u_1 + 10u_2 + 65,82u_3 + 80,34u_4 + 86,54u_5 - 50v_1 - 68,57v_2 - 100v_3 - 66,67v_4 - 100v_5 \leq 0$$

$$50u_1 + 2u_2 + 34,97u_3 + 89,27u_4 + 14,42u_5 - 25v_1 - 100v_2 - 20v_3 - 33,33v_4 - 66,67v_5 \leq 0$$

$$50u_1 + 1,54u_2 + 37,8u_3 + 90,27u_4 + 33,46u_5 - 100v_1 - 64,29v_2 - 40v_3 - 100v_4 - 66,67v_5 \leq 0$$

$$66,67u_1 + 1,25u_2 + 46,88u_3 + 81,75u_4 + 100u_5 - 40v_1 - 71,43v_2 - 80v_3 - 66,67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0 \qquad v_1 \geq 0$

$u_2 \geq 0 \qquad v_2 \geq 0$

$u_3 \geq 0 \qquad v_3 \geq 0$

$u_4 \geq 0 \qquad v_4 \geq 0$

$u_5 \geq 0 \qquad v_5 \geq 0$

Model 9:

$Max\ 50u_1 + 2u_2 + 34,97u_3 + 89,27u_4 + 14,42u_5$

Subject to

$$25v_1 + 100v_2 + 20v_3 + 33,33v_4 + 66,67v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96,15u_5 - 62,5v_1 - 32,86v_2 - 40v_3 - 33,33v_4 - 66,67v_5 \leq 0$$

$$66,67u_1 + 75u_2 + 65,82u_3 + 99,8u_4 + 96,15u_5 - 37,5v_1 - 34,29v_2 - 40v_3 - 83,33v_4 - 33,33v_5 \leq 0$$

$$66,67u_1 + 1,92u_2 + 62,64u_3 + 94,08u_4 + 11,54u_5 - 22,5v_1 - 37,14v_2 - 20v_3 - 33,33v_4 - 33,33v_5 \leq 0$$

$$50u_1 + 19u_2 + 70,9u_3 + 96,49u_4 + 57,69u_5 - 25v_1 - 34,29v_2 - 100v_3 - 33,33v_4 - 66,67v_5 \leq 0$$

$$50u_1 + 66,67u_2 + 64,77u_3 + 97,64u_4 + 57,69u_5 - 27,5v_1 - 40v_2 - 20v_3 - 50v_4 - 66,67v_5 \leq 0$$

$$33,33u_1 + 10u_2 + 39,31u_3 + 94,58u_4 + 28,85u_5 - 50v_1 - 74,29v_2 - 60v_3 - 100v_4 - 66,67v_5 \leq 0$$

$66,67u_1 + 3,13u_2 + 56,44u_3 + 80,44u_4 + 23,08u_5 - 17,5v_1 - 85,71v_2 - 20v_3 - 33,33v_4 - 33,33v_5 \leq 0$

$40u_1 + 10u_2 + 65,82u_3 + 80,34u_4 + 86,54u_5 - 50v_1 - 68,57v_2 - 100v_3 - 66,67v_4 - 100v_5 \leq 0$

$50u_1 + 2u_2 + 34,97u_3 + 89,27u_4 + 14,42u_5 - 25v_1 - 100v_2 - 20v_3 - 33,33v_4 - 66,67v_5 \leq 0$

$50u_1 + 1,54u_2 + 37,8u_3 + 90,27u_4 + 33,46u_5 - 100v_1 - 64,29v_2 - 40v_3 - 100v_4 - 66,67v_5 \leq 0$

$66,67u_1 + 1,25u_2 + 46,88u_3 + 81,75u_4 + 100u_5 - 40v_1 - 71,43v_2 - 80v_3 - 66,67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \qquad v_1 \geq 0$

$u_2 \geq 0 \qquad v_2 \geq 0$

$u_3 \geq 0 \qquad v_3 \geq 0$

$u_4 \geq 0 \qquad v_4 \geq 0$

$u_5 \geq 0 \qquad v_5 \geq 0$

Model 10:

$Max\ 50u_1 + 1,54u_2 + 37,8u_3 + 90,27u_4 + 33,46u_5$

Subject to

$100v_1 + 64,29v_2 + 40v_3 + 100v_4 + 66,67v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96,15u_5 - 62,5v_1 - 32,86v_2 - 40v_3 - 33,33v_4 - 66,67v_5 \leq 0$

$66,67u_1 + 75u_2 + 65,82u_3 + 99,8u_4 + 96,15u_5 - 37,5v_1 - 34,29v_2 - 40v_3 - 83,33v_4 - 33,33v_5 \leq 0$

$66,67u_1 + 1,92u_2 + 62,64u_3 + 94,08u_4 + 11,54u_5 - 22,5v_1 - 37,14v_2 - 20v_3 - 33,33v_4 - 33,33v_5 \leq 0$

$50u_1 + 19u_2 + 70,9u_3 + 96,49u_4 + 57,69u_5 - 25v_1 - 34,29v_2 - 100v_3 - 33,33v_4 - 66,67v_5 \leq 0$

$50u_1 + 66,67u_2 + 64,77u_3 + 97,64u_4 + 57,69u_5 - 27,5v_1 - 40v_2 - 20v_3 - 50v_4 - 66,67v_5 \leq 0$

$$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0 \qquad v_1 \geq 0$

$u_2 \geq 0 \qquad v_2 \geq 0$

$u_3 \geq 0 \qquad v_3 \geq 0$

$u_4 \geq 0 \qquad v_4 \geq 0$

$u_5 \geq 0 \qquad v_5 \geq 0$

Model 11:

$Max\ 66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5$

Subject to

$$40v_1 + 71{,}43v_2 + 80v_3 + 66{,}67v_4 + 100v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$$

$$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0 \qquad v_1 \geq 0$

$u_2 \geq 0 \qquad v_2 \geq 0$

$u_3 \geq 0 \qquad v_3 \geq 0$

$u_4 \geq 0 \qquad v_4 \geq 0$

$u_5 \geq 0 \qquad v_5 \geq 0$

APPENDIX 2

Model $E_{1,2}$:

$Max\ 100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5$

Subject to

$62{,}5v_1 + 32{,}86v_2 + 40v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{1,3}$:

$Max\ 100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5$

Subject to

$62{,}5v_1 + 32{,}86v_2 + 40v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{1,4}$:

$Max\ 100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5$

Subject to

$62{,}5v_1 + 32{,}86v_2 + 40v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$$50u_1 + 19u_2 + 70,9u_3 + 96,49u_4 + 57,69u_5 - 25v_1 - 34,29v_2 - 100v_3 - 33,33v_4 - 66,67v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{1,5}$:

$Max\ 100u_1 + 100u_2 + 100u_3 + 100u_4 + 96,15u_5$

Subject to

$$62,5v_1 + 32,86v_2 + 40v_3 + 33,33v_4 + 66,67v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96,15u_5 - 62,5v_1 - 32,86v_2 - 40v_3 - 33,33v_4 - 66,67v_5 \leq 0$$

$$50u_1 + 66,67u_2 + 64,77u_3 + 97,64u_4 + 57,69u_5 - 27,5v_1 - 40v_2 - 20v_3 - 50v_4 - 66,67v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{1,6}$:

$Max\ 100u_1 + 100u_2 + 100u_3 + 100u_4 + 96,15u_5$

Subject to

$$62,5v_1 + 32,86v_2 + 40v_3 + 33,33v_4 + 66,67v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96,15u_5 - 62,5v_1 - 32,86v_2 - 40v_3 - 33,33v_4 - 66,67v_5 \leq 0$$

$$33,33u_1 + 10u_2 + 39,31u_3 + 94,58u_4 + 28,85u_5 - 50v_1 - 74,29v_2 - 60v_3 - 100v_4 - 66,67v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{1,7}$:

$Max\ 100u_1 + 100u_2 + 100u_3 + 100u_4 + 96,15u_5$

Subject to

$$62,5v_1 + 32,86v_2 + 40v_3 + 33,33v_4 + 66,67v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{1,8}$:

$Max\ 100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5$

Subject to

$$62{,}5v_1 + 32{,}86v_2 + 40v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{1,9}$:

$Max\ 100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5$

Subject to

$$62{,}5v_1 + 32{,}86v_2 + 40v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{1,10}$:

$Max\ 100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5$

Subject to

$62{,}5v_1 + 32{,}86v_2 + 40v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{1,11}$:

$Max\ 100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5$

Subject to

$62{,}5v_1 + 32{,}86v_2 + 40v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{2,1}$:

$Max\ 66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5$

Subject to

$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{2,3}$:

$Max\ 66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5$

Subject to

$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{2,4}$:

$Max\ 66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5$

Subject to

$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{2,5}$:

$Max\ 66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5$

Subject to

$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{2,6}$:

$Max\ 66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5$

Subject to

$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{2,7}$:

$Max\ 66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5$

Subject to

$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{2,8}$:

$Max\ 66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5$

Subject to

$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$

$66,67u_1 + 75u_2 + 65,82u_3 + 99,8u_4 + 96,15u_5 - 37,5v_1 - 34,29v_2 - 40v_3 - 83,33v_4 - 33,33v_5 \leq 0$

$40u_1 + 10u_2 + 65,82u_3 + 80,34u_4 + 86,54u_5 - 50v_1 - 68,57v_2 - 100v_3 - 66,67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{2,9}$:

$Max\ 66,67u_1 + 75u_2 + 65,82u_3 + 99,8u_4 + 96,15u_5$

Subject to

$37,5v_1 + 34,29v_2 + 40v_3 + 83,33v_4 + 33,33v_5 = 1$

$66,67u_1 + 75u_2 + 65,82u_3 + 99,8u_4 + 96,15u_5 - 37,5v_1 - 34,29v_2 - 40v_3 - 83,33v_4 - 33,33v_5 \leq 0$

$50u_1 + 2u_2 + 34,97u_3 + 89,27u_4 + 14,42u_5 - 25v_1 - 100v_2 - 20v_3 - 33,33v_4 - 66,67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{2,10}$:

$Max\ 66,67u_1 + 75u_2 + 65,82u_3 + 99,8u_4 + 96,15u_5$

Subject to

$37,5v_1 + 34,29v_2 + 40v_3 + 83,33v_4 + 33,33v_5 = 1$

$66,67u_1 + 75u_2 + 65,82u_3 + 99,8u_4 + 96,15u_5 - 37,5v_1 - 34,29v_2 - 40v_3 - 83,33v_4 - 33,33v_5 \leq 0$

$50u_1 + 1,54u_2 + 37,8u_3 + 90,27u_4 + 33,46u_5 - 100v_1 - 64,29v_2 - 40v_3 - 100v_4 - 66,67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{2,11}$:

$Max\ 66,67u_1 + 75u_2 + 65,82u_3 + 99,8u_4 + 96,15u_5$

Subject to

$$37,5v_1 + 34,29v_2 + 40v_3 + 83,33v_4 + 33,33v_5 = 1$$

$$66,67u_1 + 75u_2 + 65,82u_3 + 99,8u_4 + 96,15u_5 - 37,5v_1 - 34,29v_2 - 40v_3 - 83,33v_4 - 33,33v_5 \leq 0$$

$$66,67u_1 + 1,25u_2 + 46,88u_3 + 81,75u_4 + 100u_5 - 40v_1 - 71,43v_2 - 80v_3 - 66,67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{3,1}$:

$Max\ 66,67u_1 + 1,92u_2 + 62,64u_3 + 94,08u_4 + 11,54u_5$

Subject to

$$22,5v_1 + 37,14v_2 + 20v_3 + 33,33v_4 + 33,33v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96,15u_5 - 62,5v_1 - 32,86v_2 - 40v_3 - 33,33v_4 - 66,67v_5 \leq 0$$

$$66,67u_1 + 1,92u_2 + 62,64u_3 + 94,08u_4 + 11,54u_5 - 22,5v_1 - 37,14v_2 - 20v_3 - 33,33v_4 - 33,33v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{3,2}$:

$Max\ 66,67u_1 + 1,92u_2 + 62,64u_3 + 94,08u_4 + 11,54u_5$

Subject to

$$22,5v_1 + 37,14v_2 + 20v_3 + 33,33v_4 + 33,33v_5 = 1$$

$$66,67u_1 + 75u_2 + 65,82u_3 + 99,8u_4 + 96,15u_5 - 37,5v_1 - 34,29v_2 - 40v_3 - 83,33v_4 - 33,33v_5 \leq 0$$

$$66,67u_1 + 1,92u_2 + 62,64u_3 + 94,08u_4 + 11,54u_5 - 22,5v_1 - 37,14v_2 - 20v_3 - 33,33v_4 - 33,33v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model E$_{3,4}$:

$Max\ 66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5$

Subject to

$22{,}5v_1 + 37{,}14v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{3,5}$:

$Max\ 66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5$

Subject to

$22{,}5v_1 + 37{,}14v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{3,6}$:

$Max\ 66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5$

Subject to

$22{,}5v_1 + 37{,}14v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{3,7}$:

$Max\ 66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5$

Subject to

$22{,}5v_1 + 37{,}14v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{3,8}$:

$Max\ 66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5$

Subject to

$22{,}5v_1 + 37{,}14v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{3,9}$:

$Max\ 66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5$

Subject to

$22{,}5v_1 + 37{,}14v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{3,10}$:

$Max\ 66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5$

Subject to

$$22{,}5v_1 + 37{,}14v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$$

$$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{3,11}$:

$Max\ 66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5$

Subject to

$$22{,}5v_1 + 37{,}14v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$$

$$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{4,1}$:

$Max\ 50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5$

Subject to

$$25v_1 + 34{,}29v_2 + 100v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{4,2}$:

$Max\ 50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5$

Subject to

$$25v_1 + 34{,}29v_2 + 100v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$$

$$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{4,3}$:

$Max\ 50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5$

Subject to

$$25v_1 + 34{,}29v_2 + 100v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$$

$$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{4,5}$:

$Max\ 50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5$

Subject to

$25v_1 + 34{,}29v_2 + 100v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{4,6}$:

$Max\ 50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5$

Subject to

$25v_1 + 34{,}29v_2 + 100v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{4,7}$:

$Max\ 50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5$

Subject to

$25v_1 + 34{,}29v_2 + 100v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model E$_{4,8}$:

$Max\ 50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5$

Subject to

$25v_1 + 34{,}29v_2 + 100v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model E$_{4,9}$:

$Max\ 50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5$

Subject to

$25v_1 + 34{,}29v_2 + 100v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model E$_{4,10}$:

$Max\ 50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5$

Subject to

$25v_1 + 34,29v_2 + 100v_3 + 33,33v_4 + 66,67v_5 = 1$

$50u_1 + 19u_2 + 70,9u_3 + 96,49u_4 + 57,69u_5 - 25v_1 - 34,29v_2 - 100v_3 - 33,33v_4 - 66,67v_5 \leq 0$

$50u_1 + 1,54u_2 + 37,8u_3 + 90,27u_4 + 33,46u_5 - 100v_1 - 64,29v_2 - 40v_3 - 100v_4 - 66,67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{4,11}$:

$Max\ 50u_1 + 19u_2 + 70,9u_3 + 96,49u_4 + 57,69u_5$

Subject to

$25v_1 + 34,29v_2 + 100v_3 + 33,33v_4 + 66,67v_5 = 1$

$50u_1 + 19u_2 + 70,9u_3 + 96,49u_4 + 57,69u_5 - 25v_1 - 34,29v_2 - 100v_3 - 33,33v_4 - 66,67v_5 \leq 0$

$66,67u_1 + 1,25u_2 + 46,88u_3 + 81,75u_4 + 100u_5 - 40v_1 - 71,43v_2 - 80v_3 - 66,67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{5,1}$:

$Max\ 50u_1 + 66,67u_2 + 64,77u_3 + 97,64u_4 + 57,69u_5$

Subject to

$27,5v_1 + 40v_2 + 20v_3 + 50v_4 + 66,67v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96,15u_5 - 62,5v_1 - 32,86v_2 - 40v_3 - 33,33v_4 - 66,67v_5 \leq 0$

$50u_1 + 66,67u_2 + 64,77u_3 + 97,64u_4 + 57,69u_5 - 27,5v_1 - 40v_2 - 20v_3 - 50v_4 - 66,67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{5,2}$:

$Max\ 50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{5,3}$:

$Max\ 50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{5,4}$:

$Max\ 50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{5,6}$:

$Max\ 50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{5,7}$:

$Max\ 50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{5,8}$:

$Max\ 50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{5,9}$:

$Max\ 50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{5,10}$:

$Max\ 50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{5,11}$:

$Max\ 50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{6,1}$:

$Max\ 33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5$

Subject to

$50v_1 + 74{,}29v_2 + 60v_3 + 100v_4 + 66{,}67v_5 = 1$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{6,2}$:

$Max\ 33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5$

Subject to

$50v_1 + 74{,}29v_2 + 60v_3 + 100v_4 + 66{,}67v_5 = 1$

$$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{6,3}$:

$Max\ 33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5$

Subject to

$50v_1 + 74{,}29v_2 + 60v_3 + 100v_4 + 66{,}67v_5 = 1$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{6,4}$:

$Max\ 33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5$

Subject to

$50v_1 + 74{,}29v_2 + 60v_3 + 100v_4 + 66{,}67v_5 = 1$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{6,5}$:

$Max\ 33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5$

Subject to

$50v_1 + 74{,}29v_2 + 60v_3 + 100v_4 + 66{,}67v_5 = 1$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{6,7}$:

$Max\ 33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5$

Subject to

$50v_1 + 74{,}29v_2 + 60v_3 + 100v_4 + 66{,}67v_5 = 1$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{6,7}$:

$Max\ 33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5$

Subject to

$50v_1 + 74{,}29v_2 + 60v_3 + 100v_4 + 66{,}67v_5 = 1$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{6,8}$:

$Max\ 33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5$

Subject to

$50v_1 + 74{,}29v_2 + 60v_3 + 100v_4 + 66{,}67v_5 = 1$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{6,9}$:

$$Max\ 33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5$$

Subject to

$$50v_1 + 74{,}29v_2 + 60v_3 + 100v_4 + 66{,}67v_5 = 1$$

$$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{6,10}$:

$$Max\ 33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5$$

Subject to

$$50v_1 + 74{,}29v_2 + 60v_3 + 100v_4 + 66{,}67v_5 = 1$$

$$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{6,11}$:

$$Max\ 33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5$$

Subject to

$$50v_1 + 74{,}29v_2 + 60v_3 + 100v_4 + 66{,}67v_5 = 1$$

$$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $\mathbf{E_{7,1}}$:

$Max\ 66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5$

Subject to

$$17{,}5v_1 + 85{,}71v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $\mathbf{E_{7,2}}$:

$Max\ 66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5$

Subject to

$$17{,}5v_1 + 85{,}71v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$$

$$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $\mathbf{E_{7,3}}$:

$Max\ 66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5$

Subject to

$$17{,}5v_1 + 85{,}71v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$$

$$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{7,4}$:

$Max\ 66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5$

Subject to

$$17{,}5v_1 + 85{,}71v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$$

$$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{7,5}$:

$Max\ 66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5$

Subject to

$$17{,}5v_1 + 85{,}71v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$$

$$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{7,6}$:

$Max\ 66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5$

Subject to

$17{,}5v_1 + 85{,}71v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{7,8}$:

$Max\ 66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5$

Subject to

$17{,}5v_1 + 85{,}71v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{7,9}$:

$Max\ 66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5$

Subject to

$17{,}5v_1 + 85{,}71v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{7,10}$:

$Max\ 66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5$

Subject to

$17{,}5v_1 + 85{,}71v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{7,11}$:

$Max\ 66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5$

Subject to

$17{,}5v_1 + 85{,}71v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{8,1}$:

$Max\ 40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5$

Subject to

$50v_1 + 68{,}57v_2 + 100v_3 + 66{,}67v_4 + 100v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{8,2}$:

$Max\ 40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5$

Subject to

$50v_1 + 68{,}57v_2 + 100v_3 + 66{,}67v_4 + 100v_5 = 1$

$$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{8,3}$:

$Max\ 40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5$

Subject to

$50v_1 + 68{,}57v_2 + 100v_3 + 66{,}67v_4 + 100v_5 = 1$

$$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{8,4}$:

$Max\ 40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5$

Subject to

$50v_1 + 68{,}57v_2 + 100v_3 + 66{,}67v_4 + 100v_5 = 1$

$$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $\mathbf{E_{8,5}}$:

$Max\ 40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5$

Subject to

$50v_1 + 68{,}57v_2 + 100v_3 + 66{,}67v_4 + 100v_5 = 1$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $\mathbf{E_{8,6}}$:

$Max\ 40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5$

Subject to

$50v_1 + 68{,}57v_2 + 100v_3 + 66{,}67v_4 + 100v_5 = 1$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $\mathbf{E_{8,7}}$:

$Max\ 40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5$

Subject to

$50v_1 + 68{,}57v_2 + 100v_3 + 66{,}67v_4 + 100v_5 = 1$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{8,9}$:

$Max\ 40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5$

Subject to

$50v_1 + 68{,}57v_2 + 100v_3 + 66{,}67v_4 + 100v_5 = 1$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{8,10}$:

$Max\ 40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5$

Subject to

$50v_1 + 68{,}57v_2 + 100v_3 + 66{,}67v_4 + 100v_5 = 1$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{8,11}$:

$Max\ 40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5$

Subject to

$50v_1 + 68{,}57v_2 + 100v_3 + 66{,}67v_4 + 100v_5 = 1$

$$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{9,1}$:

$Max\ 50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5$

Subject to

$$25v_1 + 100v_2 + 20v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{9,2}$:

$Max\ 50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5$

Subject to

$$25v_1 + 100v_2 + 20v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$$

$$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{9,3}$:

$Max\ 50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5$

Subject to

$$25v_1 + 100v_2 + 20v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{9,4}$:

$Max\ 50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5$

Subject to

$25v_1 + 100v_2 + 20v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{9,5}$:

$Max\ 50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5$

Subject to

$25v_1 + 100v_2 + 20v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{9,6}$:

$Max\ 50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5$

Subject to

$25v_1 + 100v_2 + 20v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{9,7}$:

$Max\ 50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5$

Subject to

$$25v_1 + 100v_2 + 20v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$$

$$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{9,8}$:

$Max\ 50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5$

Subject to

$$25v_1 + 100v_2 + 20v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$$

$$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{9,10}$:

$Max\ 50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5$

Subject to

$25v_1 + 100v_2 + 20v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{9,11}$:

$Max\ 50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5$

Subject to

$25v_1 + 100v_2 + 20v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{10,1}$:

$Max\ 50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5$

Subject to

$100v_1 + 64{,}29v_2 + 40v_3 + 100v_4 + 66{,}67v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{10,2}$:

$Max\ 50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5$

Subject to

$100v_1 + 64{,}29v_2 + 40v_3 + 100v_4 + 66{,}67v_5 = 1$

$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{10,3}$:

$Max\ 50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5$

Subject to

$100v_1 + 64{,}29v_2 + 40v_3 + 100v_4 + 66{,}67v_5 = 1$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{10,4}$:

$Max\ 50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5$

Subject to

$100v_1 + 64{,}29v_2 + 40v_3 + 100v_4 + 66{,}67v_5 = 1$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{10,5}$:

$Max\ 50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5$

Subject to

$100v_1 + 64{,}29v_2 + 40v_3 + 100v_4 + 66{,}67v_5 = 1$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{10,6}$:

$Max\ 50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5$

Subject to

$100v_1 + 64{,}29v_2 + 40v_3 + 100v_4 + 66{,}67v_5 = 1$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{10,7}$:

$Max\ 50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5$

Subject to

$100v_1 + 64{,}29v_2 + 40v_3 + 100v_4 + 66{,}67v_5 = 1$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{10,8}$:

$Max\ 50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5$

Subject to

$100v_1 + 64{,}29v_2 + 40v_3 + 100v_4 + 66{,}67v_5 = 1$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{10,9}$:

$Max\ 50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5$

Subject to

$100v_1 + 64{,}29v_2 + 40v_3 + 100v_4 + 66{,}67v_5 = 1$

$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{10,11}$:

$Max\ 50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5$

Subject to

$$100v_1 + 64{,}29v_2 + 40v_3 + 100v_4 + 66{,}67v_5 = 1$$

$$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{11,1}$:

$Max\ 66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5$

Subject to

$$40v_1 + 71{,}43v_2 + 80v_3 + 66{,}67v_4 + 100v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{11,2}$:

$Max\ 66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5$

Subject to

$$40v_1 + 71{,}43v_2 + 80v_3 + 66{,}67v_4 + 100v_5 = 1$$

$$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{11,3}$:

$Max\ 66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5$

Subject to

$40v_1 + 71{,}43v_2 + 80v_3 + 66{,}67v_4 + 100v_5 = 1$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{11,4}$:

$Max\ 66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5$

Subject to

$40v_1 + 71{,}43v_2 + 80v_3 + 66{,}67v_4 + 100v_5 = 1$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{11,5}$:

$Max\ 66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5$

Subject to

$40v_1 + 71{,}43v_2 + 80v_3 + 66{,}67v_4 + 100v_5 = 1$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model E$_{11,6}$:

$Max\ 66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5$

Subject to

$40v_1 + 71{,}43v_2 + 80v_3 + 66{,}67v_4 + 100v_5 = 1$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model E$_{11,7}$:

$Max\ 66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5$

Subject to

$40v_1 + 71{,}43v_2 + 80v_3 + 66{,}67v_4 + 100v_5 = 1$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model E$_{11,8}$:

$Max\ 66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5$

Subject to

$40v_1 + 71{,}43v_2 + 80v_3 + 66{,}67v_4 + 100v_5 = 1$

$$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model E$_{11,9}$:

$Max\ 66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5$

Subject to

$$40v_1 + 71{,}43v_2 + 80v_3 + 66{,}67v_4 + 100v_5 = 1$$

$$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model E$_{11,10}$:

$Max\ 66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5$

Subject to

$$40v_1 + 71{,}43v_2 + 80v_3 + 66{,}67v_4 + 100v_5 = 1$$

$$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

APPENDIX 3

Model $E_{1,2}$:

$Max\ 100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5$

Subject to

$62{,}5v_1 + 32{,}86v_2 + 40v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{2,1}$:

$Max\ 66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5$

Subject to

$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{2,3}$:

$Max\ 66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5$

Subject to

$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{2,4}$:

$$Max\ 66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5$$

Subject to

$$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$$

$$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{2,5}$:

$$Max\ 66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5$$

Subject to

$$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$$

$$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{2,6}$:

$$Max\ 66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5$$

Subject to

$$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$$

$$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model E$_{2,7}$:

$$Max\ 66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5$$

Subject to

$$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$$

$$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model E$_{2,8}$:

$$Max\ 66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5$$

Subject to

$$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$$

$$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model E$_{2,9}$:

$Max\ 66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5$

Subject to

$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$

$$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \le 0$$

$$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \le 0$$

$u_1 \ge 0 \quad v_1 \ge 0 \quad u_2 \ge 0 \quad v_2 \ge 0 \quad u_3 \ge 0 \quad v_3 \ge 0 \quad u_4 \ge 0 \quad v_4 \ge 0 \quad u_5 \ge 0 \quad v_5 \ge 0$

Model E$_{2,10}$:

$Max\ 66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5$

Subject to

$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$

$$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \le 0$$

$$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \le 0$$

$u_1 \ge 0 \quad v_1 \ge 0 \quad u_2 \ge 0 \quad v_2 \ge 0 \quad u_3 \ge 0 \quad v_3 \ge 0 \quad u_4 \ge 0 \quad v_4 \ge 0 \quad u_5 \ge 0 \quad v_5 \ge 0$

Model E$_{2,11}$:

$Max\ 66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5$

Subject to

$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$

$$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \le 0$$

$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{3,2}$:

$Max\ 66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5$

Subject to

$22{,}5v_1 + 37{,}14v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{4,2}$:

$Max\ 50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5$

Subject to

$25v_1 + 34{,}29v_2 + 100v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{5,2}$:

$Max\ 50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 66{,}67u_2 + 64{,}77u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{6,2}$:

$Max\ 33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5$

Subject to

$50v_1 + 74{,}29v_2 + 60v_3 + 100v_4 + 66{,}67v_5 = 1$

$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{7,2}$:

$Max\ 66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5$

Subject to

$17{,}5v_1 + 85{,}71v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{8,2}$:

$Max\ 40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5$

Subject to

$50v_1 + 68{,}57v_2 + 100v_3 + 66{,}67v_4 + 100v_5 = 1$

$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{9,2}$:

$Max\ 50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5$

Subject to

$25v_1 + 100v_2 + 20v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{10,2}$:

$Max\ 50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5$

Subject to

$100v_1 + 64{,}29v_2 + 40v_3 + 100v_4 + 66{,}67v_5 = 1$

$$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{11,2}$:

$Max\ 66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5$

Subject to

$$40v_1 + 71{,}43v_2 + 80v_3 + 66{,}67v_4 + 100v_5 = 1$$

$$66{,}67u_1 + 75u_2 + \mathbf{32,57}u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$$

$$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

APPENDIX 4

Model $E_{1,5}$:

$Max\ 100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5$

Subject to

$62{,}5v_1 + 32{,}86v_2 + 40v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 66{,}67u_2 + \mathbf{39{,}04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{2,5}$:

$Max\ 66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5$

Subject to

$37{,}5v_1 + 34{,}29v_2 + 40v_3 + 83{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 66{,}67u_2 + \mathbf{39{,}04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{3,5}$:

$Max\ 66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5$

Subject to

$22{,}5v_1 + 37{,}14v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$$50u_1 + 66{,}67u_2 + \mathbf{39,04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{4,5}$:

$Max\ 50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5$

Subject to

$$25v_1 + 34{,}29v_2 + 100v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$$

$$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 66{,}67u_2 + \mathbf{39,04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{5,1}$:

$Max\ 50u_1 + 66{,}67u_2 + \mathbf{39,04}u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$$

$$100u_1 + 100u_2 + 100u_3 + 100u_4 + 96{,}15u_5 - 62{,}5v_1 - 32{,}86v_2 - 40v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 66{,}67u_2 + \mathbf{39,04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model E$_{5,2}$:

$Max\ 50u_1 + 66{,}67u_2 + \mathbf{39{,}04}u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$66{,}67u_1 + 75u_2 + 65{,}82u_3 + 99{,}8u_4 + 96{,}15u_5 - 37{,}5v_1 - 34{,}29v_2 - 40v_3 - 83{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 66{,}67u_2 + \mathbf{39{,}04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{5,3}$:

$Max\ 50u_1 + 66{,}67u_2 + \mathbf{39{,}04}u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$66{,}67u_1 + 1{,}92u_2 + 62{,}64u_3 + 94{,}08u_4 + 11{,}54u_5 - 22{,}5v_1 - 37{,}14v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$50u_1 + 66{,}67u_2 + \mathbf{39{,}04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{5,4}$:

$Max\ 50u_1 + 66{,}67u_2 + \mathbf{39{,}04}u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$50u_1 + 19u_2 + 70{,}9u_3 + 96{,}49u_4 + 57{,}69u_5 - 25v_1 - 34{,}29v_2 - 100v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 66{,}67u_2 + \mathbf{39{,}04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{5,6}$:

$Max\ 50u_1 + 66{,}67u_2 + \mathbf{39{,}04}u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$50u_1 + 66{,}67u_2 + \mathbf{39{,}04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{5,7}$:

$Max\ 50u_1 + 66{,}67u_2 + \mathbf{39{,}04}u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$50u_1 + 66{,}67u_2 + \mathbf{39{,}04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model E$_{5,8}$:

$Max\ 50u_1 + 66{,}67u_2 + \mathbf{39{,}04}u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$$50u_1 + 66{,}67u_2 + \mathbf{39,04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$$

$$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{5,9}$:

$Max\ 50u_1 + 66{,}67u_2 + \mathbf{39,04}u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$$

$$50u_1 + 66{,}67u_2 + \mathbf{39,04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{5,10}$:

$Max\ 50u_1 + 66{,}67u_2 + \mathbf{39,04}u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$$

$$50u_1 + 66{,}67u_2 + \mathbf{39,04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$$

$$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$$

$u_1 \geq 0$ $v_1 \geq 0$ $u_2 \geq 0$ $v_2 \geq 0$ $u_3 \geq 0$ $v_3 \geq 0$ $u_4 \geq 0$ $v_4 \geq 0$ $u_5 \geq 0$ $v_5 \geq 0$

Model $E_{5,11}$:

$Max\ 50u_1 + 66{,}67u_2 + \mathbf{39,04}u_3 + 97{,}64u_4 + 57{,}69u_5$

Subject to

$27{,}5v_1 + 40v_2 + 20v_3 + 50v_4 + 66{,}67v_5 = 1$

$50u_1 + 66{,}67u_2 + \mathbf{39,04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{6,5}$:

$Max\ 33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5$

Subject to

$50v_1 + 74{,}29v_2 + 60v_3 + 100v_4 + 66{,}67v_5 = 1$

$50u_1 + 66{,}67u_2 + \mathbf{39,04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$33{,}33u_1 + 10u_2 + 39{,}31u_3 + 94{,}58u_4 + 28{,}85u_5 - 50v_1 - 74{,}29v_2 - 60v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{7,5}$:

$Max\ 66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5$

Subject to

$17{,}5v_1 + 85{,}71v_2 + 20v_3 + 33{,}33v_4 + 33{,}33v_5 = 1$

$50u_1 + 66{,}67u_2 + \mathbf{39,04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 3{,}13u_2 + 56{,}44u_3 + 80{,}44u_4 + 23{,}08u_5 - 17{,}5v_1 - 85{,}71v_2 - 20v_3 - 33{,}33v_4 - 33{,}33v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{8,5}$:

$Max\ 40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5$

Subject to

$50v_1 + 68{,}57v_2 + 100v_3 + 66{,}67v_4 + 100v_5 = 1$

$50u_1 + 66{,}67u_2 + \mathbf{39,04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$40u_1 + 10u_2 + 65{,}82u_3 + 80{,}34u_4 + 86{,}54u_5 - 50v_1 - 68{,}57v_2 - 100v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

$\geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{9,5}$:

$Max\ 50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5$

Subject to

$25v_1 + 100v_2 + 20v_3 + 33{,}33v_4 + 66{,}67v_5 = 1$

$50u_1 + 66{,}67u_2 + \mathbf{39,04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 2u_2 + 34{,}97u_3 + 89{,}27u_4 + 14{,}42u_5 - 25v_1 - 100v_2 - 20v_3 - 33{,}33v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{10,5}$:

$Max\ 50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5$

Subject to

$100v_1 + 64{,}29v_2 + 40v_3 + 100v_4 + 66{,}67v_5 = 1$

$50u_1 + 66{,}67u_2 + \mathbf{39,04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$50u_1 + 1{,}54u_2 + 37{,}8u_3 + 90{,}27u_4 + 33{,}46u_5 - 100v_1 - 64{,}29v_2 - 40v_3 - 100v_4 - 66{,}67v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$

Model $E_{11,5}$:

$Max\ 66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5$

Subject to

$40v_1 + 71{,}43v_2 + 80v_3 + 66{,}67v_4 + 100v_5 = 1$

$50u_1 + 66{,}67u_2 + \mathbf{39,04}u_3 + 97{,}64u_4 + 57{,}69u_5 - 27{,}5v_1 - 40v_2 - 20v_3 - 50v_4 - 66{,}67v_5 \leq 0$

$66{,}67u_1 + 1{,}25u_2 + 46{,}88u_3 + 81{,}75u_4 + 100u_5 - 40v_1 - 71{,}43v_2 - 80v_3 - 66{,}67v_4 - 100v_5 \leq 0$

$u_1 \geq 0 \quad v_1 \geq 0 \quad u_2 \geq 0 \quad v_2 \geq 0 \quad u_3 \geq 0 \quad v_3 \geq 0 \quad u_4 \geq 0 \quad v_4 \geq 0 \quad u_5 \geq 0 \quad v_5 \geq 0$