DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

APPLICATION OF CHAIN LADDER METHOD FOR TRAFFIC INSURANCE IN TURKEY

by
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August, 2011
İZMİR

APPLICATION OF CHAIN LADDER METHOD FOR TRAFFIC INSURANCE IN TURKEY

A Thesis Submitted to the
Graduate School of Natural and Applied Sciences of Dokuz Eylül University
In Partial Fulfillment of the Requirements for the Degree of Master of Science
in Statistics Program

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August, 2011
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M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"APPLICATION OF CHAIN LADDER METHOD FOR TRAFFIC INSURANCE IN TURKEY"** completed by **SEHER VATANSEVER** under supervision of **ASSOC. PROF. DR. GÜÇKAN YAPAR** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ACKNOWLEDGMENTS

I would like to state my sincere gratitude to my supervisor Assoc. Prof. Dr. Güçkan YAPAR for his guidance, support and help throughout the course of this study. I am very thankful to my family and friends for their encouragement and support during my studies.

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APPLICATION OF CHAIN LADDER METHOD FOR TRAFFIC INSURANCE IN TURKEY

ABSTRACT

In this thesis, firstly, the history of chain ladder method which is one of the reserving methods is mentioned. Then, the concepts used in claims reserving are described. Finally, chain ladder method is studied and the reserves for the outstanding claims are estimated with respect to the data of claim payments which are taken from TRAMER (Motor TPL Insurance Information Center) using Chain ladder method, Inflation-adjusted chain ladder method and Loss ratio method. In addition, other reserving methods are described like Separation technique, Average cost per claim method, The loss ratio and Bornheutter-Ferguson method, Operational time model and The bootstrap method.

In application, firstly, the produced premium amounts for traffic insurance between 2003 and 2009 years are investigated according to the data taken from TRAMER on 14.03.2010. Then, a reserving method which is calculable easier is developed with respect to the figure of claim payments in 2003 and 2004 against to the chain ladder method to estimate reserve for outstanding claims. Finally, the estimates obtained from both chain ladder method and the developed method are compared with the actual values published in TRAMER on 07.08.2011.

Keywords: Reserving methods, Chain ladder method, Outstanding claim, Reserve, Run-off triangle.

TÜRKİYE'DEKİ TRAFİK SİGORTALARI İÇİN ZİNCİRLEME METODUNUN UYGULANMASI

ÖZ

Bu çalışmada, öncelikle rezerv metotlarından biri olan zincirleme metodunun tarihçesinden bahsedilmiştir. Sonrasında hasar karşılıklarında kullanılan kavramlar tanımlanmıştır. Son olarak zincirleme metodu üzerinde durulmuş ve TRAMER'den (Trafik Sigortaları Bilgi Merkezi) alınan hasar ödemeleri verilerine zincirleme metodu, enflasyon eklenmiş zincirleme metodu ve hasar oranı metodu uygulanarak muallak hasar karşılığı kestirilmiştir. Ek olarak, Ayrıştırma metodu, Hasar başına ortalama maliyet metodu, Hasar oranı ve Bornheutter-Ferguson metodu, İşlemsel zaman modeli, Bootstrap metodu gibi hasar karşılık metotları da tanımlanmıştır.

Uygulamada öncelikle 14.03.2010 tarihinde TRAMER'den alınan verilere göre 2003 ve 2009 yılları arasında üretilen prim miktarları trafik sigortası için incelenmiştir. Sonrasında, rezerv kestirimi yapmak için 2003 ve 2004 yıllarındaki hasar ödeme şekline göre zincirleme metoduna karşılık hesaplaması daha kolay bir rezerv metodu geliştirilmiştir. Son olarak hem zincirleme metodundan hem de geliştirilen metottan elde edilen kestirimler 07.08.2011 tarihinde TRAMER'de yayınlanan gerçek değerlerle karşılaştırılmıştır.

Anahtar sözcükler: Hasar karşılığı metotları, Zincirleme metodu, Muallak hasar, Rezerv, Hesap üçgeni.

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CHAPTER ONE
INTRODUCTION

Up to just a few decades ago, non-life insurance portfolios were financed through a pay-as-you-go system. All claims in a particular year were paid from the premium income of that same year, irrespective of the year in which the claim originated. The financial balance in the portfolio was realized by ensuring that there was an equivalence between the premiums collected and the claims paid in a particular financial year. Technical gains and losses arose because of the difference between the premium income in a year and the claims paid during the year.

The claims originating in a particular year often cannot be finalized in that year. For instance, long legal procedures are the rule with liability insurance claims, but there may also be other causes for delay, such as the fact that the exact size of the claim is hard to assess. Also, the claim may be filed only later, or more payments than one have to be made, as in disability insurance. All these factors will lead to delay of the actual payment of the claims. The claims that have already occurred, but are not sufficiently known, are foreseeable in the sense that one knows that payment will have to be made, but not how much the total payment is going to be. Also, there are losses that have to be reimbursed in future years.

As seems proper and logical, such claims are now connected to the years for which the premiums were actually paid. This means that reserves have to be kept regarding claims which are known to exist, but for which the eventual size is unknown at the time the reserves have to be set. For claims like these, several acronyms are in use. One has IBNR claims (Incurred But Not Reported) for claims that have occurred but have not been filed. Hence the name IBNR methods, IBNR claims and IBNR reserves for all quantities of this type (Kaas, Goovaerts, Dhaene & Denuit, 2001). The oldest IBNR method and by and large still the most often used one is a straightforward extrapolation called the Chain Ladder method (Straub, 1988).

The chain ladder method is one of the most famous methods used in reserving. It exploits all data from the run-off triangle and provides simple estimates of the expected ultimate cumulative claims. The chain ladder estimators of the expected ultimate cumulative claims result from classical statistical estimation principles (Schmidt & Wünsche, 1998). 'Ultimate' is used in the sense implied by the chain ladder method, and does not include any claims beyond the latest development year to have been observed (Verrall, 1991(b)).

In literature, the chain ladder method was used in a lot of study. These studies are given as follows:

Kremer (1982) formed the normal equations for the chain ladder linear model and also investigated the relationship between the linear model and the basic chain ladder method (Verrall, n.d.). However, Kremer (1982) noticed that the chain ladder included a log-linear cross-classification structure. A number of parametric stochastic versions of the chain ladder developed from this, e.g. Hertig (1985), Renshaw (1989), Verrall (1989, 1990, 1991) (Taylor, 2002).

Kremer (1982) proved that the chain ladder and multiplicative models are equivalent (Verrall, 1991(b)). However, Kremer (1985) proved that one obtains the same predictions with maximum likelihood (ML) method like with the most appealing link ratio method, the so called chain ladder method (Kremer, 1997).

Zehnwirth (1985) have been proposed other models as convenient for claims data contain a gamma curve, apart from the chain ladder linear model. Ajne (1989) have been recommended other models apart from the chain ladder linear model as appropriate for claims data include an exponential tail in which the first few delay years follow the chain ladder model and the later delay years follow an exponential curve (Verrall, n.d.).

Verrall (1990) approaches the subject of estimating outstanding claims using hierarchical Bayesian linear models, taking into account the fact that the chain ladder method is based on a linear model: the two-way analysis of variance model (ANOVA). He essentially applies a Bayesian analysis of the two-way ANOVA model to get Bayes and empirical Bayes estimates (de Alba, 2002).

In the literature, it is usually the total uncertainty in the claims reserves until the claims are finally settled that has been studied. For the distribution-free chain ladder method, this has first been made by Mack (1993) (Wüthrich, Merz & Lysenko, 2009). In Mack (1993), a distribution-free method was improved in order to estimate the prediction error of chain ladder reserve estimates (Braun, 2004).

Whereas the cross-classified models generally suppose stochastic independence of all cells in the data set, the chain ladder (in Mack's formulation) does not. It was represented by Mack (1993) that the algorithm of the classical chain ladder generated unbiased predictions of liability under its own assumptions. However, Mack (1994) specify that these stochastic models gave mean estimates of liability that differed from the "classical" chain ladder estimate. While the form of stochastic model underlying the classical chain ladder was speculative, due to the latter's heuristic nature, Mack proposed one. It is distribution free. Mack also defined the differences between this and the other stochastic models (Taylor, 2002).

The sequential chain ladder model is due to Schnaus and was suggested by Schmidt and Schnaus (1996). The sequential chain ladder model is a slight but suitable extension of the chain ladder model of Mack (1993). Hess and Schmidt (2002) give a systematic comparison of several models for the chain ladder method (Schmidt, 2006).

One of the models verifying the univariate chain ladder method is the model of Schnaus, represented in Schmidt and Schnaus (1996), which develops the model of Mack (1993). In Schmidt and Schnaus (1996) it is represented that the chain ladder predictors for the cumulative claim sizes of the first non-observable calendar year $n+1$ are indeed optimal under the assumptions of the model of Schnaus and according to an essentially classical optimality criterion. (Pröhl & Schmidt, 2005).

Verrall (2000) discovers the relationship between the chain ladder method and some stochastic models. He supposes a Poisson distribution for claim amounts and denotes that this model should not necessarily be used for all data (de Alba, 2002).

England and Verrall (2002a, 2002b) make with an exhaustive dissertation on the state of current claim reserving methodologies. The methodologies they define are primarily non-Bayesian, but they also discuss the Bayesian analysis of an over-dispersed Poisson chain ladder model (Scollnik, n.d.).

The multivariate chain ladder method is based on a stochastic model which is a multivariate version of the model of Schnaus and developes the univariate model of Mack and the bivariate model of Braun. Braun (2004) used his model as a foundation for the construction of estimators of the prediction errors of the univariate chain ladder predictors, but he did not use his model to replace the univariate chain ladder predictors by bivariate ones reflecting the correlation structure. Braun (2004) used his bivariate model to find estimators for the prediction errors of the univariate chain ladder predictors of two correlated portfolios which consider correlation between the portfolios and which are designed as to develop the estimators suggested by Mack (1993) neglecting correlation. As it is the case for the estimators suggested by Mack, the estimators suggested by Braun are generated in a reasonable but heuristic way; in particular, in both cases it is not known whether these estimators have any particular statistical features like, e.g., unbiasedness (Pröhl & Schmidt, 2005).

Another bivariate model of claim reserving, which is loosely related to the multivariate model of Schnaus, is the model of Quarg and Mack (2004). Under the assumptions of their model, Quarg and Mack suggest bivariate chain ladder predictors for the paid and incurred cumulative claims of the same portfolio with the goal of decreasing the difference between the univariate chain ladder predictors for the paid and incurred cumulative claims of the same portfolio. The model of Quarg and Mack is not contained in the multivariate model of Schnaus since it supposes a conditional correlation structure within the accident years instead of a completely specified conditional correlation structure between the paid and incurred cumulative claims (Pröhl & Schmidt, 2005).

Buchwalder et al. (2006) have explained that there are different approaches for the derivation of an estimate for the parameter estimation error in the distribution-free chain ladder method (Wütrich, Merz & Bühlmann, 2008).

The time series version of the chain ladder model, which uses stronger assumptions than the classical distribution-free chain ladder model considered in Mack (1993) is studied by Murphy (1994), Barnett & Zehnwirth (2000) or Buchwalder et al. (2006) (Wütrich, Merz & Bühlmann, 2008).

Braun (2004), Pröhl and Schmidt (2005), Schmidt (2006), and Merz and Wüthrich (2008) have interpreted a multivariate version for the distribution-free chain ladder model. Their study differs on the point of view of how the multidimensional chain ladder parameters are estimated. Braun (2004) and Merz and Wüthrich (2008) use the classical (univariate) estimators, whereas Pröhl and Schmidt (2005) and Schmidt (2006) use multivariate estimators considering the dependence structure between the coordinates and that are optimal in terms of a classical optimality criterion. On the one hand Braun (2004) and Merz and Wüthrich (2008) ensure an estimator of the mean square error of prediction (MSEP) for several correlated runoff portfolios. On the other hand, the studies of Pröhl and Schmidt (2005) and Scmidt (2006) for the multivariate estimators do not go beyond the study of first moments.

CHAPTER TWO
CLAIMS RESERVING DEFINITIONS

2.1 Introduction to Claims Reserving Definitions

Claim reserve estimation is approached by the actuary from a much different perspective than that of the claim adjuster. The claim reserve model is very close to the claims operations view of the financial cumulative claim reserves. The analyst should understand the claims and accounting perspectives of the total loss reserve, but will most often deal with issues inherent in the actuarial approach to the claim reserve aggregate.

First it is essential to define basic claim reserve terminology that can be used to standardize discussions of the claim reserve estimation process.

2.2 The Definition of Liabilities

Liabilities are claims on the resources of the company, to satisfy obligations of the company. Liabilities could be mortgages, bank debt, bonds issued, premiums received from clients but not yet earned, or benefits payable on behalf of clients due to contractual obligations, for example. Any change in a liability account, such as loss reserves, has a direct impact on insurer's income.

An obligation satisfies the accounting definition of a liability if it possesses three essential characteristics:

1) The obligation involves a probable future sacrifice of resources at a specified or determinable date,
2) The company has little or no discretion to avoid the transfer, and
3) The transaction or event giving rise to the obligation has already occurred.

A claim liability of a property and casualty insurer satisfies the second and third characteristics above. The first requirement is not generally satisfied in property and casualty claim situations. For instance, in a workers' compensation claim, payments must be made periodically at specified times, often weekly. However, in a third-party liability situation it is not possible to specify the date on which settlement will be made.

2.3 An Actuarial Model of Loss Development

Both the accounting model and the claims model of the reserving process deal with aggregates over a certain time period. Further, the claim department is concerned with individual file actions. An actuarial model can be constructed that supplies a structure behind the aggregate financial descriptions of claims activity. This can serve as a conceptual starting point for the analysis of reserves from the actuary's viewpoint.

Let $v(x)$ be the amount of loss arising from instant x. The function $v(x)$ can be thought of as the loss density. Then the amount of ultimate loss in the time period (a,b) can be calculated as

$$\int_a^b v(x)dx. \tag{2.1}$$

All observations of loss reserve situations are observations of various aggregate amounts, hence the form of $v(x)$ cannot be observed directly.

Since most observations of the loss amounts are at periods short of ultimate development, the development of loss statistics are needed to recognize over time. This can be done by introducing a development function $D(t)$, where D is a continuous function with

$$D(t) = 0, \text{ for } t < 0, \tag{2.2}$$

$$D(t) = 1, \text{ for } t > T,$$

where loss development continues for a duration of T.

Then aggregate losses from period (a,b) developed through time c are given by

$$\int_a^b v(x)D(c-x)dx\,. \tag{2.3}$$

The actuarial model requires that a proper form and parameters for the functions v and D be found that fit the observed aggregate calendar period loss data.

For instance if $v(x)=k$, a constant volume of losses, then

$$\int_a^b v(x)D(c-x)dx = k\int_a^b D(c-x)dx\,. \tag{2.4}$$

2.4 Accounting Date

A loss reserve is an estimate of the liability for unpaid claims as of a given date, called the accounting date. An accounting date may be any date. However it is generally a date for which a financial statement is prepared. This is most often a month end, quarter end, or year end.

2.5 Valuation Date

A loss reserve inventory as of a fixed accounting date may be evaluated at a date different than the accounting date. The valuation date of a reserve liability is the date as of which the evaluation of the reserve liability is made. Thus the reserve liabilities are needed to evaluate as of the close of a financial period. The valuation and accounting date would be identical.

The loss reserve liability is always an estimate, and the amount of the estimate will change as of successive valuation dates (Wiser, 1990).

2.6 Types of Reserve

In insurance and reinsurance there are many different types of reserves, such as premium reserves, claims reserves, catastrophe reserves, contingency reserves, currency fluctuation reserves, IBNR reserves, additional case reserves and of course all kinds of pretty well undefined ‘special’ reserves. Some reserve types are defined as follows:

Premium reserves have to be put aside as so-called unearned premiums, if, for example, a one-year policy is concluded on July 1, 1987 and the whole of its premium is collected at the beginning, when closing the books at the end of the year 1987 only half of this premium has been earned, the other half has to be allocated to the following year’s profit and loss account, as the policy only expires at June 30, 1988. On the other hand, if a claim occurs -think of a road accident, for example- not all necessary payments can be immediately made by the insurance company but, according to the specific circumstances, a certain amount of individual claims reserve has to be put aside for future payments on this case. The total of premium reserves and claims reserves of a company or portfolio is usually referred to as its technical reserves (Lorenz & Schmidt, 1999).

An insurance company’s technical reserve -the amounts set aside to meet its insurance liabilities- represent the principal liabilities of an insurance company. Reserves are required in respect of business written, both earned and unearned. Technical reserves are established to enable the company to meet and administer its contractual obligations to policyholders. Specific reserves are required to meet indemnity or other compensatory payments to policyholders, plus the associated administration costs. In addition, reserves of a contingent nature might be carried (for example, claims equalization or catastrophe reserves) in order to provide a further buffer against adverse development of claims and to smooth the emergence of profit (Booth, Chadburn, Cooper, Haberman & James, 1999).

The total amount of technical reserves is mostly higher than a one-year premium production, it can even be twice the yearly gross net premium income or more. All this money -which can be well invested, of course- does not belong to the company but to the clients -those insured and/or reinsured by the company- to pay their past or future claims. Contrary to the company's own capital, its equity, the technical reserves may be called foreign capital.

In reinsurance, particularly in non-proportional casualty business, the claims reserves as reported by the ceding companies are insufficient as a rule and have to be -sometimes substantially- reinforced, either individually by so-called additional case reserves or on a global basis by IBNR reserves, or both (Lorenz & Schmidt, 1999).

Because reserves constitute the largest liabilities carried by casualty insurers, one of the main activities of a practicing actuary is loss reserving (de Alba, 2002). Loss reserving is the term used to denote the actuarial process of estimating the needed amount of loss reserves. A loss reserve is a provision for an insurer's liability for claims (Wiser, 1990). The many uncertainties involved in the payment of losses make the estimation of the required reserves more difficult. Yet, some of the existing methods are simple to apply and have been in use for many years. However, it has become evident that there is a need for better ways, not only to estimate the reserves, but also to obtain some measures of their variability as well as information on their overall probable future behavior (de Alba, 2002).

2.7 Required Loss Reserve

The required loss reserve as of a given accounting date is the amount that must ultimately be paid to settle all claim liabilities. The value of the required loss reserve can only be known when all claims have been finally settled. Thus, the required loss reserve as of a given accounting date is a fixed number that does not change at different valuation dates. However, the value of the required loss reserve is generally unknown for an extremely long period of time.

2.8 Indicated Loss Reserve

The indicated loss reserve is the result of the actuarial analysis of a reserve inventory as of a given accounting date conducted as of a certain valuation date. This indicated loss reserve is the analyst's opinion of the amount of the required loss reserve. This estimate will change with successive valuation dates and will converge to the required loss reserve as the time between valuation date and the accounting date of the inventory increases.

2.9 Carried Loss Reserve

The carried loss reserve is the amount of unpaid claim liability shown on external or internal financial statements. The carried loss reserve for any subgroup of business is the result of the method of generating carried reserves used by the reporting entity for financial reporting reasons.

2.10 Loss Reserve Margin

The loss reserve margin is the difference between the carried reserve and the required reserve. Since the required reserve is an unknown quantity, the indicated margin only is found. The indicated loss reserve margin is defined to be the carried loss reserve minus the indicated loss reserve. One should not generally expect the margin to be zero, since for any subset of an entity's business it is unlikely that the carried loss reserve will be identical to either the indicated or required loss reserve. Even further, when the loss reserve is split into components the carried reserve for any component will most often not be identical to the indicated loss reserve.

2.11 Loss Reserve

The loss reserve can be considered to consist of two major subdivisions, the reserve for known claims and the reserve for unknown claims. Each of these major divisions can then be further broken into subdivisions. Known claims are those claims for which the entity has actually recorded some liability at some point in time. Thus a known claim may have been considered closed at one point, but later need to be reopened for further adjustment.

Through common usage the term ‘loss reserve’ has come to denote the property and casualty company’s provision for its liability for claims by or against policyholders. Loss reserving is the process of estimating the amount of the company’s liabilities for such claims which the company has contracted to settle for its policyholders.

2.12 Reserve for Known Claims

The reserve for known claims may be considered to consist of case reserves, a reserve for future development on case reserves, and a reserve for reopened claims. The total required reserve for known reserves is estimated by the indicated reserve for known claims. The indicated reserve for known claims is the sum of the carried case reserves for known, the indicated provision for future development on known claims, and the indicated provision for reopened claims.

2.13 Case Reserve

The case reserve is defined as the sum of the values assigned to specific claims by the entity’s case reserving procedure. Most often a claims file is valued by an estimate placed on the file by the claims examiner. The term adjusters’ estimates is used to refer to the aggregate of the estimates made by claims personel on individual claims, based on the facts of those particular claims. Formula reserves may be placed on reported cases. Formula reserves are reserves established by formulas for groups or classes of claims. The formulas may be based on any of a number of factors such as coverage, state, age, limits, severity of injury, or other variables.

2.14 Total Reserve

The total reserve for unreported claims consists of a reserve for claims incurred but not recorded (IBNR). This reserve can be further subdivided into a reserve for claims incurred but not yet reported to the company, and a reserve for those claims reported to the company but not yet recorded on the company’s books. This reserve may sometimes be referred to as a pipeline reserve. This distinction is important under claims made coverages, when the pipeline reserve is the only IBNR reserve needed. Most data used for estimation measures the lags from the time a loss is incurred to the time the claim is recorded on the insurer's books and records. If such data is used for the estimation process, then the estimated liability for both 'pipeline claims in transit' and unreported claims will result.

A total loss reserve for an insurer is composed of five elements:

1) Case reserves assigned to specific claims,
2) A provision for future development on known claims,
3) A provision for claims that re-open after they have been closed,
4) A provision for claims that have occurred but have not yet been reported to the insurer, and
5) A provision for claims that have been reported to the insurer but have not yet been recorded.

2.15 Loss Adjustment Expense Reserve

The loss adjustment expense reserve for a particular exposure period is the amount required to cover all future expenses required to investigate and settle claims incurred in the exposure period. This covers claims yet to be reported as well as claims already known.

Loss adjustment reserves may be charged to specific claims or may be general claims expense not directly attributable to any one file. This distinction leads to separate consideration of allocated loss adjustment expense and unallocated loss adjustment expense.

2.15.1 Allocated Loss Adjustment Expenses

Allocated loss adjustment expenses are those expenses such as attorneys' fees and legal expense which are incurred with and are assigned to specific claims.

2.15.2 Unallocated Loss Adjustment Expenses

Unallocated loss adjustment expenses are all other claim adjustment expenses, such as salaries, heat, light and rent, which are associated with the claim adjustment function but are not readily assignable to specific claims.

2.16 Development

Development is defined as the difference, on successive valuation dates, between observed values of certain fundamental quantities that may be used in the loss reserve estimation process. These changes can be changes in paid and carried amounts. Development on reported claims as of two valuation dates consists of the additional paid on case reserves plus the change in case reserves from the first valuation date. This is also the definition of incurred loss in a calendar period.

Another type of development relates to IBNR (incurred but not reported) claims. The development of IBNR claims is often referred to as emergence of IBNR. In reviewing the development on the prior year end reserve, it is useful to divide the total development into its case development and IBNR emergence components (Wiser, 1990).

The implicit assumption is that future development is independent of prior development (Murphy, 1993).

2.17 Data Availability and Organization

The availability of proper data is essential to the task of estimating loss and loss adjustment expense reserve needs. The actuary is responsible for informing management of the need for sufficiently detailed and quality data to obtain reliable reserve estimates.

Data must be presented that clearly displays development of losses by accident period, policy period, or report period, to enable the actuary to project the ultimate level of losses.

The effectiveness of the method depends very much on the organization of the historical data. One of the most common ways to organize such data is the loss development triangle. For a given accident year, which is the year the claim occurred, all payments on claims from that accident year are displayed in the same row. Each column indicates a subsequent year of payments on claims of that accident year. This data organization greatly facilitates comparison of the development history expected of an accident year.

Some rules for relevant data to be used for reserve analysis are given as follows:

1) Data may be provided by accident year, report year, policy year, underwriting year, or calendar year (in descending order of preference), by development year.
2) The number of years of development should be great enough so that further developments will be negligible.
3) Allocated loss expenses should be included with losses or shown separately; and clearly labelled.

2.18 Exploratory Data Analysis

Before the actuary begins his attempts to project immature loss data to ultimate loss estimates, it is important to review the data. The objective of this review is to understand the data in terms of

- rate of development,
- smoothness of development,
- presence of large losses,
- volume of data.

Review of the data will allow the analyst to form conclusions about:

- appropriate projection methodologies,
- anomalies in the data,
- appropriate questions to ask management concerning issues that manifest themselves in the data, that will further the analyst's understanding of the book of business that generated the data (Wiser, 1990).

2.19 Reserve Estimation Strategy

The apparent profitability and solvency of a business is highly dependent upon the reserve level and the reserving philosophy. Most of the key financial performance statistics used by insurance company analysts depend in some way upon the reserve level. Reserving is therefore a fundamental aspect of business management (Booth, Chadburn, Cooper, Haberman & James, 1999).

The overall approach to a reserve valuation problem can be broken into four phases:

1) Review of the data to identify its key characteristics and possible anomalies. Balancing of data to other verified sources should be undertaken at this point.
2) Application of appropriate reserve estimation techniques.

3) Evaluation of the conflicting results of the various reserve methods used, with an attempt to reconcile or explain the bases for different projections. At this point the proposed reserving ultimates are evaluated in contexts outside their original frame of analysis.
4) Prepare projections of reserve development that can be monitored over the subsequent calendar periods. Deviations of actual from projected developments of counts or amounts is one of the most useful diagnostic tools in evaluating accuracy of reserve estimates (Wiser, 1990).

2.20 Claim Settlement Process

The reserving methods examine different approaches to estimating reserves required in respect of outstanding claims. It is not sufficient to carry out a reserving method systematically: data complications require a comprehensive information of the underlying claims process (Booth, Chadburn, Cooper, Haberman & James, 1999).

The claim settlement process has a pattern:

- Claim event occurs,
- Claim is reported,
- Claim payment is made,
- Claim file is closed.

2.21 Delays in Claim Reporting and Claim Settlement

The settlement of claims is usually subject to delay, and it is necessary for the insurer to set up reserve provisions for claims corresponding to losses that have been incurred by the insured during the covered period but have not yet been settled. It is very important to estimate outstanding claims as accurately as possible to have a correct view of an insurer's financial situation (de Alba, 2002).

The delays can be at the claim settlement process. There are two types of delay:

- delay in claim reporting (have been incurred but not yet reported-IBNR)
- delay in claim settlement (have been reported but not yet settled-RBNS)

Goovaerts et. al. (1990) use the term "incurred but not reported" (IBNR) reserves both "for claims as yet unreported, and a reserve for claims known to the company but not completely paid". This is in agreement with what Brown (1993) calls "gross IBNR." Hence this study refers indistinctly either to outstanding claims, IBNR reserves, or loss reserving (de Alba, 2002).

There are also RBNS claims (Reported But Not Settled), for claims which are known but not completely paid. Other acronyms are IBNFR, IBNER and RBNFS, where the F is fully, the E for Enough. Large claims which are known to the insurer are often handled on a case-by-case basis (Kaas, Goovaerts, Dhaene & Denuit, 2001).

These 'delays' do not refer to any deliberate delaying on the part of the insurer, but to delays in notification of the claim by the insured and further delays caused by litigation, etc (Taylor, 1977).

The terms of the delays change extremely according to the type of business. In the case of damage-only business for heavy commercial vehicles, claims are reported almost immediately and settled soon after. A delay of several months is unusual. On the other hand, many employer liability claims are not reported until years have elapsed, and the time to settlement of some which are reported almost immediately may be 15 years or more (Hossack, Pollard & Zehnwirth, 1983).

The insurer does not know the exact total claim amount from the policies written in each origin year (Huerta, 2004). The method of estimating provisions for claims which have been reported but not yet settled is to be made in respect of all known outstanding claims at the accounting date. While these estimates are made, the four items are considered:

- the severity of the claim
- the likely time to settlement
- inflation between the accounting date and settlement
- trends in claim settlement

These are difficult factors to evaluate and combine in the estimate of outstanding claims. For example, in liability insurance, the severity of a claim may not occur for even years after the claim has been reported.

In recent years, the statistical approaches have been developed to estimate outstanding claims in recent years. The ways of estimating outstanding claims:

- attempt to find a consistent claim run-off pattern which has applied in the past
- apply that pattern (with adjustments for anticipated future claim inflation) to estimate the run-off of claims that have been incurred but are still outstanding (Hossack, Pollard & Zehnwirth, 1983).

2.22 The Run-off Triangle

An estimate is found from claim data considering the year that policies were started and the year they were settled. 'Year of origin' is the calendar year (financial year, business year) in which the event leading to a claim emerged (Hossack, Pollard & Zehnwirth, 1983). These data are summarised in a 'run-off triangle' (Huerta, 2004). The lower part of the run-off triangle can not be observed. The goal of claim reserving is to obtain predictions for this lower part and to determine the corresponding reserves. This leads to the run-off square. The run-off triangle is showed in Table 2.1.

Table 2.1 Run-off triangle

	Development year						
Year of origin	0	1	2	.	.	.	n
0	$S_{0,0}$	$S_{0,1}$	$S_{0,2}$	.	.	.	$S_{0,n}$
1	$S_{1,0}$	$S_{1,1}$	$S_{1,2}$	.	.	.	
2	$S_{2,0}$	$S_{2,1}$	$S_{2,2}$	.	.		
.	.			.			
.	.		.				
.	.	.					
n	$S_{n,0}$						

$S_{i,k}$ is the amount paid during development year k in respect of claims whose year of origin is i. The information relating to the area below and/or to the right of this triangle is unknown since it represents the future development of various cohorts of claims (Taylor, 1977).

Claims run-off data are generated when delay is incurred in settling insurance claims. Typically the format for such data is that of a triangle in which the rows (i) denote origin year or accident years and the columns (k) delay or development years. The settlement or payment year is $c = i + k - 1$. The entries in the body of the triangle are the cumulative claims (Renshaw, 1989).

The use of run-off triangles in loss reserving can be justified only if it is assumed that the development of the losses of every accident year follows a development pattern which is common to all accident years. This vague idea of a development pattern can be formalized in various ways (Schmidt, 2006).

The problem is to forecast outstanding claims on the basis of past experience. In other words to fill in the lower right hand triangle of claims. Sometimes it is also useful to extend the forecasts beyond the latest delay year (i.e. to the right of the claims run-off triangle). The standard actuarial technique does not attempt to do this (Verrall, n.d.).

2.23 Loss Development Data

We consider a portfolio of risks and we assume that each claim of the portfolio is settled either in the accident year or in the following n development years. The portfolio may be modelled either by incremental losses or by cumulative losses.

2.23.1 Incremental Losses

To model a portfolio by incremental losses, we consider a family of random variables $\{Z_{i,k}\}_{i,k\in\{0,1,\ldots,n\}}$ and we interpret the random variable $Z_{i,k}$ as the loss of accident year i which is settled with a delay of k years and hence in development year k and in calendar year $i+k$. We refer to $Z_{i,k}$ as the incremental loss of accident year i and development year k. The problem is to predict the non-observable incremental losses.

2.23.2 Cumulative Losses

To model a portfolio by cumulative losses, we consider a family of random variables $\{S_{i,k}\}_{i,k\in\{0,1,\ldots,n\}}$ and we interpret the random variable $S_{i,k}$ as the loss of accident year i which is settled with a delay of at most k years and hence not later than in development year k. We refer to $S_{i,k}$ as the cumulative loss of accident year i and development year k, to $S_{i,n-i}$ as a cumulative loss of the present calendar year n, and to $S_{i,n}$ as an ultimate cumulative loss. The problem is to predict the non-observable cumulative losses (Schmidt, 2006).

2.24 The Choice of Year of Origin or Claim Cohort

The basic groupings of claims into cohorts that define the period of origin are:

- reporting period,
- accident period,

- underwriting period.

Projection methods essentially extrapolate the loss development within a certain period of origin to an ultimate value. The results of the projections and, in particular, the meaning of the projected future development will vary as a consequence of the choice of year of origin.

2.24.1 Reporting Period

If the period of origin is defined as the reporting period, then claims are grouped according to the period (usually a year) in which they are reported to the insurer. By definition, therefore, once the period is over no new claims can be added to cohort. The projection achieved by completing the square represents the ultimate level of claims reported in the period of origin. The movement between the current level of claims and the projected ultimate level therefore represents the extent to which current case reserves have been over or under (IBNER) estimated, together with the cost of reopened claims, with reopened claims attributed to the period of origin in which they were originally reported. Projections based on this data will make no allowance for IBNR claims or for unexpired risks, so separate estimates would be needed for each of these to obtain a complete picture of the technical liabilities of the company. Special caution is also needed with the data since any cohort will be a mixture of claims from a variety of underwriting and calendar periods so that the cover provided and the specific perils included are unlikely to have remained the same over the exposure periods giving rise to the reported losses. Similarly, the environment (whether legal, social or economic) in which policies gave rise to claims might also changed.

Such a grouping of claims does not have a corresponding or straightforward premium or exposure measure. Statistics relating the claims to the original policy's premium or exposure details might be available but if a single policy gives rise to multiple claims with very different reporting dates then it is not clear how the policy exposure should be allocated to the different origin periods and care is needed in the interpretation of the resulting frequency and loss ratio statistics.

Grouping in relation to reporting year has a number of drawbacks. It is only appropriate for short tailed classes and business written on a claims made basis, or to provide information on IBNER and the quality of case reserves and the potential IBNER component (Booth, Chadburn, Cooper, Haberman & James, 1999).

2.24.2 Accident Period

The accident year is referred to as the year in which an event triggering insurance claims occurs (Wütrich, Merz & Lysenko, 2009). If the period of origin is the accident period, then claims are grouped by the period in which the accident occurred. This grouping is consistent with the usual one-year accounting basis and reflects the experience of all policies that were exposed (or earned) over the same period. Claims development within an accident period reflects IBNER developments on case reserves, reopened claims and delayed advice (IBNR) claims. It follows that the projected ultimate level for each accident year includes estimates of the future values for all these items.

The claims recorded within the accident period all stem from the same exposure period, so the broader economic and environmental influences on the propensity to make insurance claims is the same. However, the claims themselves could arise from policies issued over a period of several years, depending on the policies' duration. (For policies with a duration of one year, the period over which policies included in the exposure could have been issued is two years). The coverage offered might have changed over this period so the actual claims themselves might not be completely consistent (Booth, Chadburn, Cooper, Haberman & James, 1999).

2.24.3 Underwriting Period or Policy Period

The year in which the policy is written will be called the underwriting year, or year of business. In the years after the policy was written the company may receive claims related to that policy, and these claims are indexed by their business year and the delay (Verrall, n.d.). If the period of origin is the underwriting period, then claims are grouped in relation to the period in which the policy giving rise to the claim was underwritten. Claims reflect a consistent underlying policy structure: However, in contrast to the accident period approach, claims arise from policies exposed over a period of up to two calendar years, or longer, depending on the duration of the contracts. Thus the broader environment might not be as consistent as for the accident period definition. Furthermore, since the premium exposed in the first development period can be low (relative to the premium written), the claims reported by the end of the first development year might also be low and, possibly, not representative of the ultimate level of claims.

The claim development within the underwriting period includes all the liabilities arising from the business written, that is, IBNER, reopened claims, IBNR and unexpired risks. This approach is therefore useful in evaluating the ultimate result for a rating series. Since the ultimate level includes an element in respect of unexpired risks it is essential to test the implied cost of the unexpired risk (Booth, Chadburn, Cooper, Haberman & James, 1999).

CHAPTER THREE
RESERVING METHODS

3.1 Introduction to Reserving Methods

One of the major challenges to the casualty actuary is the estimation of the necessary financial provision for the unpaid liabilities of an insurer to claimants. The practical approaches devised by actuaries who have worked on providing these estimates include a wide range of methods that have not yet been formulated into a precise science. The intent of this chapter is to provide insight into the methods used by practicing actuaries in estimating claim liabilities (Wiser, 1990).

Methods to determine the reserves have been developed that each meet specific requirements, have different model assumptions, and produce different estimates. Each of the methods reflect the influence of a number of exogenous factors. In the direction of the year of origin, variation in the size of the portfolio will have an influence on the claim figures. On the other hand, for the factor development year changes in the claim handling procedure as well as in the speed of finalization of the claims will produce a change (Kaas, Goovaerts, Dhaene & Denuit, 2001).

Loss reserve estimation methods can only be properly applied to grouped data. A loss reserve inventory should deal with claim files arising from a time period with an explicit beginning and ending date. The start and end dates must relate to one of the distinctive dates in the life of a claim file. This could be the date of reporting, the date of loss, the date of policy inception, or the date of claim closing. The dates specified must be unambiguous and characteristic of an important event in the life of a claim (Wiser, 1990).

Methods used to estimate the necessary reserve provisions are usually classified as deterministic or nonstochastic and statistical or stochastic.

Different statistical approaches for dealing with the problem have been developed in recent years. These methods attempt to find a consistent claim runoff pattern that has applied in the past, assume that pattern is stable and that it will continue to hold, and then apply the pattern to estimate the claims that have been incurred but are still outstanding.

Even if the past settlement pattern is reasonably stable, the future runoff may be quite uncertain because of doubts that the pattern will continue, and because of claim inflation. The provision to be held also will be affected by assumed investment earnings. In practice, it is seldom possible to do any better than suggest a fairly wide range of not-unreasonable provisions based on different assumptions about future claim inflation, investment earnings, and so forth, and possibly on different statistical methodologies (de Alba, 2002).

3.2 The Mostly Used Reserving Methods

The mostly used reserving methods and general features of these methods are defined as follows:

- Chain ladder method
- Inflation-adjusted chain ladder method
- Separation technique
- Average cost per claim method
- The loss ratio and Bornheutter-Ferguson method
- Operational time model
- The bootstrap method

3.2.1 The Chain Ladder Method

The chain ladder method is one of the most famous methods used in reserving. The method is based on the assumption that proportionate relationships between values in sequential delay periods will, on average, repeat in the future. It exploits all data from the run-off triangle and provides simple estimates of the expected ultimate aggregate claims. The chain ladder estimators of the expected ultimate aggregate claims result from classical statistical estimation principles (Schmidt & Wünsche, 1998). 'Ultimate' is used in the sense implied by the chain ladder method, and does not include any claims beyond the latest development year to have been observed (Verrall, 1991(b)). The most extensively used reserving method is the chain ladder or link ratio method.

A chain ladder reserving method uses the observed data in order to estimate the missing single cell development factors in the lower triangle. Then these estimated development factors are used to develop estimates of the cumulative claim amounts in the lower triangle and, hence, of the missing incremental claim amounts and the loss reserve.

There are many possible ways in which to construct estimates of the missing single cell development factors in each column. Just to name a few possibilities, a practitioner might use the arithmetic (or a weighted) mean of the observed factors in each column, the most recent factor appearing in a column, or the average of some number (e.g., two or three) of the most recent factors appearing in a column in order to complete each column's missing entries. The popular set of estimates are known as the volume weighted development factors (chain ladder factors). The volume weighted development factors are weighted averages of the single cell development factors, with the cumulative claim amounts appearing in the denominator of the latter used as the weights involved in the calculation of the former (Scollnick, n.d.).

3.2.1.1 Chain Ladder Estimation

A family of random variables $\{S_{i,k}\}_{i,k\in\{0,1,\ldots,n\}}$ are considered. The random variable $S_{i,k}$ is interpreted as the aggregate claim size of all claims which occur in occurrence year i and which are settled before the end of calendar year $i+k$ (Schmidt & Wünsche, 1998). We assume that each of the random variables $S_{i,k}$ is strictly positive (Pröhl & Schmidt, 2005). The subscript k are considered as the development year (Schmidt & Wünsche, 1998). The enumeration of the development years represents delays with respect to the occurrence years (Schmidt, 2006). The numbers on the diagonal with $i+k-1=c$ denote the payments that were made in occurrence year c (Kaas, Goovaerts, Dhaene & Denuit, 2001).

It is assumed that all claims are settled before the end of development year n. The random variables $S_{i,n}$ will therefore be referred to as ultimate aggregate claims. The ultimate aggregate claims $S_{i,n}$ agree with the aggregate claims of occurrence year i. The observable aggregate claims can be represented by the run-off triangle (Schmidt & Wünsche, 1998):

Table 3.1 Run-off triangle

Occurrence year	**Development year**							
	0	1	…	k	…	n-i	…	n-1	n
0	$S_{0,0}$	$S_{0,1}$	…	$S_{0,k}$	…	$S_{0,n-i}$	…	$S_{0,n-1}$	$S_{0,n}$
1	$S_{1,0}$	$S_{1,1}$	…	$S_{1,k}$	…	$S_{1,n-i}$	…	$S_{1,n-1}$	
.	.	.		.		.			
.	.	.		.		.			
.	.	.		.		.			
i	$S_{i,0}$	$S_{i,1}$	…	$S_{i,k}$	…	$S_{i,n-i}$			
.	.	.		.					
.	.	.		.					
.	.	.		.					
n-k	$S_{n-k,0}$	$S_{n-k,1}$	…	$S_{n-k,k}$					
.	.	.							
.	.	.							
.	.	.							
n-1	$S_{n-1,0}$	$S_{n-1,1}$							
n	$S_{n,0}$								

The information relating to area below this triangle is unknown since it represents the future development of the various cohorts of claims.

A cumulative loss $S_{i,k}$ is said to be

- observable if $i+k \leq n$.
- non–observable or future if $i+k > n$.
- present if $i+k = n$.
- ultimate if $k = n$.

The purpose of loss reserving is to predict

- the ultimate cumulative losses $S_{i,n}$ and
- the accident year reserves $S_{i,n} - S_{i,n-i}$.

More generally: The aim is to predict

- the future cumulative losses $S_{i,k}$.
- the future incremental losses $Z_{i,k} = S_{i,k} - S_{i,k-1}$.
- the calendar year reserves $\sum_{j=p-n}^{n} Z_{j,p-j}$.
- the total reserve $\sum_{j=1}^{n} \sum_{l=n-j+1}^{n} Z_{j,l}$

with $i + k \geq n + 1$ and $p = n + 1, \ldots, 2n$.

Prediction refers to non-observable random variables whereas estimation refers to unknown parameters. For formal reasons, the case $i = 0$ is included in the discussion of prediction and estimation although the ultimate aggregate claim $S_{0,n}$ is observable (Lorenz & Schmidt, 1999).

The chain ladder method is based on the assumption that there exists a development pattern for factors. The chain ladder method relies completely on the observable cumulative losses of the run off triangle and involves no prior estimators at all. As estimators of the development factors, the chain ladder method uses the chain ladder factors (Schmidt, 2006).

For $i \in \{0,1,...,n\}$ and $k \in \{1,...,n\}$, we define the development factor

$$F_{i,k} = \frac{S_{i,k}}{S_{i,k-1}}. \tag{3.1}$$

For $k \in \{1,...,n\}$, we define the chain ladder factor

$$\hat{F}_k = \frac{\sum_{i=0}^{n-k} S_{i,k}}{\sum_{i=0}^{n-k} S_{i,k-1}}. \tag{3.2}$$

For development year k, the chain ladder factor $\hat{F}_k$ is the best approximation of the observable development factors when the approximation error are given the weight occurring in the representation of the chain ladder factor as a weighted mean (Schmidt & Schnaus, 1996). The chain ladder factors are weighted means and may be used to estimate the development factors (Schmidt, 2006).

Then the ultimate aggregate claims satisfy

$$S_{i,n} = S_{i,n-i} \prod_{k=n-i+1}^{n} F_{i,k}. \tag{3.3}$$

For $i \in \{1,...,n\}$, we define the chain ladder estimator

$$\hat{S}_{i,n} = S_{i,n-i} \prod_{k=n-i+1}^{n} \hat{F}_k. \tag{3.4}$$

We also consider the family $\{Z_{i,k}\}_{i,k\in\{0,1,...,n\}}$ of incremental claims which are defined:

$$Z_{i,k} = \begin{cases} S_{i,0} & \text{if } k = 0, \\ S_{i,k} - S_{i,k-1} & \text{if } k \geq 1. \end{cases} \tag{3.5}$$

The collection of all observable incremental claims contains the same information as the collection of all observable aggregate claims (Schmidt & Wünsche, 1998).

The chain ladder factors and the chain ladder predictors have particular properties:

- The chain ladder factor $\hat{F}_k$ is a weighed mean of the observable development factors from development year k such that the weights are determined by the observable aggregate claims from preceding development years.
- The chain ladder predictiors $\hat{S}_{i,k}$ and $\hat{Z}_{i,k}$ of the non-observable aggregate or incremental claims $S_{i,k}$ and $Z_{i,k}$, respectively, are determined by the aggregate claim $S_{i,n-i}$ of the last observable development year and the chain ladder factors $\hat{F}_{n-i+1},...,\hat{F}_k$ (Schmidt, 1999).

The enumeration of accident years and development years starting with 0 instead of 1 is widely but not yet generally accepted. It is useful for several reasons:

- For losses which are settled within the accident year, the delay of settlement is 0. It is therefore natural to start the enumeration of development years with 0.
- Using the enumeration of development years also for accident years implies that the incremental or cumulative loss of accident year i and development year k is observable if and only if $i+k \leq n$. In particular, the cumulative losses $S_{i,n-i}$ are those of the present calendar year n and are crucial in most methods of loss reserving.

The predictors and estimators we consider in the sequel are defined only under the condition that the realizations of certain sums of observable incremental claims are strictly positive; this condition is fulfilled when the realizations of all observable incremental claims are strictly positive. Although it would be convenient to assume that the incremental claims are strictly positive, we avoid this assumption since it is violated in the Poisson model (Lorenz & Schmidt, 1999).

3.2.1.2 One Example About Chain Ladder Method

Let give an example connected with the chain ladder method. The data were taken from TRAMER (Motor TPL Insurance Information Center) on 14.03.2010 and are in respect of paid claims between 2004 and 2009 years for traffic insurance. The paid claims are given in Table 3.2.

Table 3.2 Incremental paid claims

	Development period					
Year of origin	0	1	2	3	4	5
2004	140.189	353.581	71.598	18.944	16.270	14.134
2005	193.752	483.034	91.797	28.262	23.991	
2006	233.877	591.770	106.471	41.281		
2007	292.354	720.792	131.022			
2008	375.063	909.878				
2009	438.900					

In respect of claims originating in 2004, payments totalling 140.189 thousand TL were made that same year (development year 0), and payments totalling 353.581 thousand TL were made the following year 2005 (development year 1). The cumulative paid claims are given in Table 3.3.

Table 3.3 Cumulative paid claims

	Development period					
Year of origin	0	1	2	3	4	5
2004	140.189	493.770	565.368	584.312	600.582	614.716
2005	193.752	676.786	768.583	796.845	820.836	
2006	233.877	825.647	932.118	973.399		
2007	292.354	1.013.146	1.144.168			
2008	375.063	1.284.941				
2009	438.900					

The claim payments must be estimated after 2009 in respect of years of origin 2005, 2006, 2007, 2008 and 2009, in order to deduce the outstanding claim provision required in 2009 in respect of these years of origin.

It is necessary to compute the ratios between successive cumulative payments, within the year of origin. This shows the proportionate relationship between periods at different delay points. These ratios are illustrated in Table 3.4.

Table 3.4 Ratio of cumulative payments in successive development periods

	Development period				
Year of origin	0-1	1-2	2-3	3-4	4-5
2004	3.522	1.145	1.034	1.028	1.024
2005	3.493	1.136	1.037	1.030	
2006	3.530	1.129	1.044		
2007	3.465	1.129			
2008	3.426				
2009					

These ratios are calculated from cumulative paid claims in Table 3.3. For example,

$$3.522 = \frac{493.770}{140.189}$$

For 2004 origin year, cumulative payment to development year 2005 is 3.522 times that for 2004 (development years 0 to 1).

$$1.136 = \frac{768.583}{676.786}$$

For 2005 origin year, cumulative payment to development year 2007 is 1.136 times that for 2006 (development years 1 to 2).

$$1.044 = \frac{973.399}{932.118}$$

For 2006 origin year, cumulative payment to development year 2009 is 1.044 times that for 2008 (development years 2 to 3).

To complete the development triangle (Table 3.4) of year-to-year development ratios, development factors (m ratios) must be calculated. The development factors are calculated as:

$$m_{k+1/k} = \frac{\sum_{i=0}^{n-k-1} S_{i,k+1}}{\sum_{i=0}^{n-k-1} S_{i,k}}$$

The development factors are calculated for cumulative payments rather than the original yearly payments as they are generally more stable for the former. The development factors calculated from cumulative paid claims in Table 3.3 are given as follows:

$$m_{1/0} = \frac{493.770 + 676.786 + 825.647 + 1.013.146 + 1.284.941}{140.189 + 193.752 + 233.877 + 292.354 + 375.063} = 3.476$$

$$m_{2/1} = \frac{565.368 + 768.583 + 932.118 + 1.144.168}{493.770 + 676.786 + 825.647 + 1.013.146} = 1.133$$

$$m_{3/2} = \frac{584.312 + 796.845 + 973.399}{565.368 + 768.583 + 932.118} = 1.039$$

$$m_{4/3} = \frac{600.582 + 820.836}{584.312 + 796.845} = 1.029$$

$$m_{5/4} = \frac{614.716}{600.582} = 1.024$$

Table 3.5 Ratio of cumulative payments in successive development periods (Table 3.4 completed)

	Development period				
Year of origin	0-1	1-2	2-3	3-4	4-5
2004	3.522	1.145	1.034	1.028	1.024
2005	3.493	1.136	1.037	1.030	1.024
2006	3.530	1.129	1.044	1.029	1.024
2007	3.465	1.129	1.039	1.029	1.024
2008	3.426	1.133	1.039	1.029	1.024
2009	3.476	1.133	1.039	1.029	1.024

Table 3.5 leads to the completed cumulative period claims triangle. To complete Table 3.3, the calculated development factors are used.

Table 3.6 Cumulative paid claims (Table 3.3 completed)

	Development period					
Year of origin	0	1	2	3	4	5
2004	140.189	493.770	565.368	584.312	600.582	614.716
2005	193.752	676.786	768.583	796.845	820.836	840.536
2006	233.877	825.647	932.118	973.399	1.001.628	1.025.667
2007	292.354	1.013.146	1.144.168	1.188.791	1.223.266	1.252.624
2008	375.063	1.284.941	1.455.838	1.512.616	1.556.482	1.593.838
2009	438.900	1.525.616	1.728.523	1.795.935	1.848.017	1.892.369

For example;

1.525.616= 438.900* $m_{1/0}$

1.512.616= 1.284.941* $m_{2/1}$ * $m_{3/2}$

1.252.624= 1.144.168* $m_{3/2}$ * $m_{4/3}$ * $m_{5/4}$

1.593.838= 1.284.941* $m_{2/1}$ * $m_{3/2}$ * $m_{4/3}$ * $m_{5/4}$

The next step is to separate the constant cumulative payments into payments by development year. There is no need to complete the entries above the zig-zag line .

Table 3.7 Payments made in development year

	Development period					
Year of origin	0	1	2	3	4	5
2004						
2005						19.700
2006					28.299	24.039
2007				44.623	34.475	29.358
2008			170.897	56.778	43.866	37.356
2009		1.086.716	202.907	67.412	52.082	44.352

In Table 3.7, the entries under the triangle are calculated from the values in Table 3.6.

For example;

19.700= 840.536 - 820.836

24.039= 1.025.667 - 1.001.628

34.475= 1.223.266 – 1.188.791

56.778= 1.512.616 - 1.455.838

44.352= 1.892.369 – 1.848.017

In Table 3.7, totalling of the entries under the triangle give the estimated reserve which is showed in Table 3.8.

Table 3.8 Estimated reserve

Year of origin			**Reserve**
2005	19.700	=	19.700
2006	28.229 + 24.039	=	52.268
2007	44.623 + 34.475 + 29.358	=	108.456
2008	170.897 + 56.778 + 43.866 + 37.356	=	308.897
2009	1.086.716 + 202.907 + 67.412 + 52.082 + 44.352	=	1.453.469
Total			1.942.790

In respect of Table 3.8, reserve totalling 1.942.790 thousand TL is necessary to settle claims.

It can be checked whether the chain ladder model fits the data by comparing past payments with those predicted by the model. Even if an adequate or good fit is achieved, there is no guarantee that the model is valid for predicting future claim payments in respect of recent past years of origin.

Table 3.9 Cumulative claim payments: actual and estimated

		Development period					
Year of origin		0	1	2	3	4	5
2004	Actual	140.189	493.770	565.368	584.312	600.582	614.716
	Estimated	140.189	487.297	552.108	573.640	590.276	604.443
	Error	*	(-1%)	(-2%)	(-2%)	(-2%)	(-2%)
2005	Actual	193.752	676.786	768.583	796.845	820.836	
	Estimated	193.752	673.482	763.055	792.814	815.806	
	Error	*	(-0.5%)	(-0.7%)	(-0.5%)	(-0.6%)	
2006	Actual	233.877	825.647	932.118	973.399		
	Estimated	233.877	812.956	921.079	957.001		
	Error	*	(-2%)	(-1%)	(-2%)		
2007	Actual	292.354	1.013.146	1.144.168			
	Estimated	292.354	1.016.223	1.151.381			
	Error	*	(0.3%)	(0.6%)			
2008	Actual	375.063	1.284.941				
	Estimated	375.063	1.303.719				
	Error	*	(1%)				

In Table 3.9, the actual cumulative claim payments are the data in Table 3.3. The estimated cumulative claim payments are calculated by using development factors. For example; the estimated cumulative payment by the end of 2007 in respect of year of origin 2004 is

$$573.640 = 140.189 * m_{1/0} * m_{2/1} * m_{3/2} = 140.189*3.476*1.133*1.039$$

The actual cumulative payment is 584.312, so that the estimate includes an error of 2%. The other actual and estimated cumulative payments in Table 3.9 are computed in a similar manner. The percentage errors show that the model fits past cumulative payments adequately.

The claim payments in each of the years 2004-2009 in respect of the data in Table 3.3 are compared with the payments estimated by the chain ladder method.

3.2.1.3 General Expression on Chain Ladder Method

The chain ladder method is useful, giving a practitioner some feel for the historic development and indication of the future.

A number of simple variants to the claim weighted approach are possible. For example it could be that certain features of the data in Table 3.4 can be explained in terms of factors that are not representive of the 'normal' development pattern and as such might require manual adjustment (for example, an acceleration of claim settlement in one calendar period). In such circumstances the data might need to be improved, or the factors can be adjusted and the average development factors recalculated based on non-weighted averages. This is straightforward when the calculations are set up on a spreadsheet, and it is a useful method of allowing a range of results based on various 'what if' scenarios. Other variations include:

- using the latest factors only to test the impact of developments in the latest period on the ultimate claims;
- using all claim factors excluding the latest to examine the ultimate claims calculated using the previous year's projection basis;
- if there are trends in the development factors then the weights applied can be changed to give the more recent periods greater significance (Booth, Chadburn, Cooper, Haberman & James, 1999).

There is uncertainty in the chain ladder predictions for the ultimate claim. This uncertainty is estimation error. The estimation error, which comes from the fact that we have to estimate the true model parameters. Recent discussions have shown that it is very important to clearly define the understanding of the estimation error. Basically, it derives from the fact that the true parameters of the underlying model are not known and need to be estimated from the data in the upper left triangle (observations). One main task of the reserving actuary then is to quantify the appropriateness of these estimates (Wüthrich, Merz & Bühlmann, 2008).

The chain ladder is used as a benchmark in several of the references, because of its generalized use and ease of application. This facilitates comparison between methods (de Alba, 2002).

The chain ladder forecast of outstanding losses is known to be unbiased under suitable assumptions. According to these assumptions, claim payments in any cell of a payment triangle are dependent on those from preceding development years of the same accident year. If all cells are assumed stochastically independent, the forecast is no longer unbiased. However, it does not necessarily do so under the alternative assumption of independence between all cells. The bias has been studied in estimates of liability in the parametric cross-classified models, but little is known of the bias in the classical chain ladder forecast when all cells are independent (Taylor, 2002).

This all important chain ladder independence assumption says that the relationship between consecutive evaluations does not depend on the relationship between any other pair of consecutive evaluations. In mathematical terms, the random variable corresponding to losses evaluated at one point in time conditional on the previous evaluation is independent of any other evaluation conditional on its previous evaluation (Murphy, 1993).

Whether or not the chain ladder method should be applied in practice depends on two fundamental decisions: The first decision to take consists in the selection of a model which is believed to describe the generation of data in an appropriate way. Once the model has been selected, the next decision consists in the choice between the following alternatives:

- Only a single aggregate or incremental claim is to be predicted.
- The family of all aggregate or incremental claims from a fixed occurence year (or from all occurence years) is to be predicted (Schmidt, 1999).

It is shown that in many cases, but not always, the maximum likelihood estimators of the expected ultimate cumulative losses are identical with the chain ladder predictors of the ultimate cumulative losses (Schmidt, 2006).

Assumptions include that the time (number of periods) it takes for the claims to be completely paid is fixed and known, payments are made annually, and the development of partial payments follows a stable payoff pattern. This is in agreement with many existing models for claims reserving in nonlife (general) insurance that assume, explicitly or implicitly, that the proportion of claim payments, payable in the k -th development period, is the same for all periods of origin (de Alba, 2002).

The basic principle of the chain ladder method admits many variants. One may wonder if there is indeed proportionality between the columns. Undoubtedly, this is determined by effects that operate along the axis describing the year of origin of the claims. By the chain ladder method, only the run-off pattern can be captured, given that all other factors, at least the ones having an influence on the proportion of claims settled, remain unchanged over time (Kaas, Goovaerts, Dhaene & Denuit, 2001).

From the practical point of view, there are two arguments in favour of the chain ladder method: The method is simple, and it exploits all data from the run-off triangle. These arguments, however, do not settle the question whether or not the chain ladder method is preferable to other methods of loss reserving. The quality of the chain ladder method depends on the stochastic mechanism, or stochastical models generating the data.

It is important to the reason underlying the chain ladder method that the ‘exogeneous influences’ should not be too great. If this assumption is not valid, then the result, that the columns of the run-off triangle are proportional goes awry too, and the chain ladder method can give misleading results. One possible method of overcoming this weakness of the chain ladder is to recognise the variation (with i) of the ratios $S_{i,n+1} / S_{i,n}$, to seek trends in these rations and project these trends. This modification too has a serious drawback in that the trend may be almost entirely due to monetary inflation, and if rates of inflation have fluctuated in the past, there will not exist any smooth trend. Furthermore, if the rate of inflation is thought likely to fall during the next few years, then it is not clear how this trend should be reflected in the sequence (over i) of ratios $S_{i,n+1} / S_{i,n}$ (Taylor, 1977).

3.2.2 Inflation-adjusted Chain Ladder Method

The chain ladder method does not obviously enable for any calendar effect. The development factors calculated are based on payments from many different calendar periods. Although, inflation is implicit in the development factors, it is not definite what assumption for future inflation is actually made, other than that it is a weighted average of past values. The loss development form calculated, combines the true features of a class of business and external factors, which are independent of the pure claims process. If expectations for future inflation are very different from past trends, the implicit provision for inflation might inappropriate for estimation of future claims.

Making adjustments for calendar period influences such as inflation is considerably simple if the inflationary rates which the claims have experienced are present. However, in specifying the model it is necessary to examine incremental, rather than cumulative data, so that the calendar year influence is applied to the correct data.

The chain ladder model can be expanded into the form:

$$c_{ik} = S_i r_k \lambda_{i+k-1} + \varepsilon_{ik} . \tag{3.6}$$

S_i : the ultimate level for origin year i,

c_{ik} : the incremental payment made in development period k, due to year of origin i,

r_k : the proportion of S_i observed in development period k,

λ_{i+k-1} : is an inflation index for the calendar year.

The inflation adjusted chain ladder model adjust the chain ladder to contain an explicit allowance for inflation (Booth, Chadburn, Cooper, Haberman & James, 1999).

The various methods of estimating future aggregate claim amounts exploit the fact that, in the absence of exogeneous influences such as monetary inflation, changing rate of growth of a fund, changing mix of business in a fund etc., the distribution of delays between the incident giving rise to a claim and the payment of that claim remains relatively stable in time. In this case the columns (or rows) of the run-off triangle are, apart from random fluctuation, proportional to one another (Taylor, 1977).

3.2.2.1 One Example About Inflation-adjusted Chain Ladder Method

The inflation-adjusted chain ladder method is applied to data taken from TRAMER on 14.03.2010 to estimate outstanding claim provisions for traffic insurance. The past inflation rates are obtained as follows.

2004-2005	9.3%
2005-2006	7.7%
2006-2007	9.7%
2007-2008	8.4%
2008-2009	10.1%

To complete the projection there are two more stages. First, paid claims must have the impact of inflation removed by placing the incremental payments on a standard money basis. Then assumptions are required for future inflation rates, which are applied to the projected future incremental values. The incremental payments in Table 3.2 are rearranged using past inflation rates. The inflation adjusted incremental payments are given in Table 3.10.

Table 3.10 Incremental paid claims, having been adjusted for inflation

	Development period					
Year of origin	0	1	2	3	4	5
2004	216.059	498.572	93.740	22.609	17.913	14.134
2005	273.203	632.413	109.558	31.116	23.991	
2006	306.204	706.268	117.225	41.281		
2007	348.920	793.592	131.022			
2008	412.944	909.878				
2009	438.900					

Payments were assumed to have been made at the end of the calendar year. For example;

216.059= 140.189*1.093*1.077*1.097*1.084*1.101

632.413= 483.034*1.097*1.084*1.101

117.225= 106.471*1.101

After the incremental payments are adjusted for past inflation rates, the cumulative payments are calculated. The cumulative payments are showed in Table 3.11.

Table 3.11 Cumulative paid claims, having been adjusted for inflation

	Development period					
Year of origin	0	1	2	3	4	5
2004	216.059	714.631	808.371	830.980	848.893	863.027
2005	273.203	905.616	1.015.174	1.046.290	1.070.281	
2006	306.204	1.012.472	1.129.697	1.170.978		
2007	348.920	1.142.512	1.273.534			
2008	412.944	1.322.822				
2009	438.900					

To complete the entries below the triangle, the development factors must be calculated. The method of their calculation is the same as for the chain ladder method. The development factors are denoted in Table 3.12.

Table 3.12 Development factors

Development period	0-1	1-2	2-3	3-4	4-5
Development factors	3.274	1.120	1.032	1.022	1.017

These development factors calculated here are lower than calculated using the chain ladder method. Because, the chain ladder factors make an implicit allowance for future inflation based on past trends, whereas the factors calculated in Table 3.12 have no built-in element in respect of future inflation.

The calculated development factors are applied to the current money cumulative values given in Table 3.11 to give the future cumulative payments and the reserve requirement. The future cumulative payments are showed in Table 3.13.

Table 3.13 Cumulative paid claims, having been adjusted for inflation (Table 3.11 completed)

	Development period					
Year of origin	0	1	2	3	4	5
2004	216.059	714.631	808.371	830.980	848.893	863.027
2005	273.203	905.616	1.015.174	1.046.290	1.070.281	1.088.476
2006	306.204	1.012.472	1.129.697	1.170.978	1.196.740	1.217.085
2007	348.920	1.142.512	1.273.534	1.314.287	1.343.201	1.366.035
2008	412.944	1.322.822	1.481.561	1.528.971	1.562.608	1.589.172
2009	438.900	1.436.959	1.609.394	1.660.895	1.697.435	1.726.291

For example;

1.436.959= 438.900* $m_{1/0}$ = 438.900*3.274

1.528.971= 1.322.822* $m_{2/1}$ * $m_{3/2}$ = 1.322.822*1.120*1.032

The next step is to separate the constant cumulative payments into payments by development year. There is no need to complete the entries above the zig-zag line.

Table 3.14 Future incremental payments (current money)

	Development period					
Year of origin	0	1	2	3	4	5
2004						
2005						18.195
2006					25.762	20.345
2007				40.753	28.914	22.834
2008			158.739	47.410	33.637	26.564
2009		998.059	172.435	51.501	36.540	28.856

The entries under the triangle in Table 3.14 are calculated from Table 3.13.

For example;

18.195=1.088.476 - 1.070.281

28.914=1.343.201 - 1.314.287

28.856=1.726.291 - 1.697.435

The total of the entries under the triangle in Table 3.14 gives the estimated reserve. The estimated reserve is given in Table 3.15.

Table 3.15 Estimated reserve in current money

Year of origin		**Reserve**
2005	18.195	18.195
2006	25.762 + 20.345	46.107
2007	40.753 + 28.914 + 22.834	92.501
2008	158.739 + 47.410 + 33.637 + 26.564	266.350
2009	998.059 + 172.435 + 51.501 + 36.540 + 28.856	1.287.391
Total		1.710.544

In respect of Table 3.15, reserve totalling 1.710.544 is necessary to settle claims.

In order to calculate the nominal reserve, the incremental payments must be inflated at an appropriate rate. If we assume that, following an analysis of past inflation trends and a review of future expectations for the particular class of business, future claims inflation is expected to be 10% per annum. The nominal reserve is showed in Table 3.16.

Table 3.16 Incremental payments, with future inflation of 10% per annum

	Development period					
Year of origin	0	1	2	3	4	5
2004						
2005						20.015
2006					28.338	24.617
2007				44.828	34.986	30.392
2008			174.613	57.366	44.771	38.892
2009		1.097.865	208.646	68.548	53.498	46.473

The incremental payments, with future inflation of 10% per annum in Table 3.16 are calculated from the values in Table 3.14. For example;

20.015= 18.195*1.10

24.617= 20.345*$(1.10)^2$

44.771= 33.637*$(1.10)^3$

53.498=36.540*$(1.10)^4$

46.473=28.856*$(1.10)^5$

The net present value of future payments can also be calculated. For this, the discount rate must be calculated. The discount rate:

$$d = \frac{i}{1+i} = \frac{0.10}{1.1} = 0.09 = 9\% \qquad i: \text{future inflation rate=10\%}$$

The net present value of future payments is illustrated in Table 3.17.

Table 3.17 Incremental payments, with future inflation of 10% per annum, discounted at 9% per annum

	Development period					
Year of origin	0	1	2	3	4	5
2004						
2005						18.362
2006					25.998	20.720
2007				41.127	29.447	23.468
2008			160.195	48.284	34.571	27.552
2009		1.007.216	175.613	52.932	37.899	30.204

The incremental payments, with future inflation of 10% per annum, discounted at 9% per annum are calculated from Table 3.16. For example;

25.998= 28.338*$(1.09)^{-1}$

29.447= 34.986*$(1.09)^{-2}$

34.571= 44.771*$(1.09)^{-3}$

27.552= 38.892*$(1.09)^{-4}$

In respect of current money, nominal and net present value, the reserves are compared in Table 3.18. The current money, nominal, and net present value reserves for each year of origination can be obtained by simply summing the constituent future incremental claim amounts over the appropriate cells.

Table 3.18 Comparison of reserves allowing for 10% future inflation and a rate of discount 9%

	Reserve		
Year of origin	Current money	Nominal	Net present value
2004			
2005	18.195	20.015	18.362
2006	46.107	52.955	46.718
2007	92.501	110.206	94.042
2008	266.350	315.642	270.602
2009	1.287.391	1.475.030	1.303.864
Total	1.710.544	1.973.848	1.733.588

The reserves in Table 3.18 are calculated by summing the incremental claims in Table 3.14, 3.16 and 3.17 in respect of the year of origin, respectively. According to Table 3.18, the current money reserve is 1.710.544 thousand TL, nominal reserve is 1.973.848 thousand TL and the net present value is 1.733.588 thousand TL.

3.2.2.2 General Expression on Inflation-adjusted Chain Ladder Method

Although the method of adjusting for calendar year influences has been straightforward, their effects can be complex. It is important to remember that the various causes of inflation are not the only factors which have a calendar year effect on the data: changes in claims processing or in claim settlement philosophies, for example, can also appear as calendar year influences. Inflation adjusted projections are even more complex if they are based on incurred claims, since consideration must be given to the way inflation has been reflected in case estimates throughout the claims triangle. A computer spreadsheet package would facilitate an investigation of the impact on the reserves of different inflation and investment assumptions.

In the calculations for the inflation adjusted chain ladder method, it was assumed that payments in the period reflected the inflationary factors in the same period. In longer-tailed classes of business, and even for particular types of claim within a generally short-tailed class of business, this is not necessarily so. Payments can be extended over many years: for example, the amount of a claim in respect of a motor bodily injury accident might be set by a court in one time period but, following appeals and other delays, a different amount might be paid in a different time period. The court might award interest on any payments delayed due to the appeal process, but this will not necessarily match inflation (Booth, Chadburn, Cooper, Haberman & James, 1999).

3.2.3 Separation Technique

The separation technique is very identical to the chain ladder method but focuses on the derivation of calendar year factors from the data.

The model is similar to the inflation-adjusted chain ladder, thus,

$$c_{ik} = S_i r_k \lambda_{i+k-1} + \varepsilon_{ik} . \tag{3.7}$$

The year of origin effect is eliminated by scaling the data using an appropriate exposure measured, L_i. Thus,

$$\frac{c_{ik}}{L_i} = \frac{S_i}{L_i} r_k \lambda_{i+k-1} + \frac{\varepsilon_{ik}}{L_i} . \tag{3.8}$$

The ultimate aggregate claim amount per unit of exposure is supposed to be constant, so S_i / L_i is constant for all i and can be associated with the price index factor λ_{i+k-1}.

The separation technique is not extensively used, but it can be useful in a high-volume, stable, account. The choice of L_i must be coherent from year to year and will based on the type of business and the basis on which it is written. Premium income might be a suitable measure for certain classes, for example, but, where premium rates fluctuate due to the influence of the insurance pricing cycle they will need to be adjusted to eliminate this extraneous factor; for an employer's liability account, which has been written on a coherent basis in the past, the salary roll or number of employees might be consistent measures of exposure; for stable, high-volume, classes of business, such as private motor, the number of claims expected in a year could be used (Booth, Chadburn, Cooper, Haberman & James, 1999).

3.2.4 Average Cost per Claim

The combination of the claim numbers and the claim amounts leads to the consideration of average claim amounts and estimations (projections) based on average costs per claim. The method involves three steps:

1. the average cost per claim is estimated in each development period;
2. the number of claims settled in the next years is estimated using the chain ladder technique;
3. the average claim amount is increased to make an appropriate allocation for inflation.

The stability of the computation of claim numbers between delay periods and between origin periods is crucial. Similarly, the appropriateness between the period in which a payment is made and the period in which the paid claim counts is recorded is fundamental.

A number of operational factors need to be borne in mind when considering the appropriateness of this method. In particular:

- If nil claims are recorded in the numbers, then the average cost will fall. Ideally, nil claims should be excluded from the claim counts and/or analysed separately to assess their stability (for example, as a proportion of total claims; and their pattern in relation to the delay period). Similar consideration must be given to the administrative procedures for recording precautionary advices.
- Partial payments and, in particular, changes in the patterns of partial payments within periods of origin or between periods of origin may invalidate the assumption that past patterns of average claims will be maintained into the future. It might be preferable to accumulate partial

payments and record them as a single payment to correspond with the claim count.

- Changes in the mix of claim by claim type (for example, in a motor account a changing mix of bodily injury and physical damage claims) would distort averages. When more than one type of claim arises from a single event, the systems of the company might record a claim number for a loss under each section of the policy rather than record a single claim count for the original accident. In circumstances where there are distinct claim types, a separate projection by claim type might be preferable. This can reduce the distortions caused by a changing claim type mix and enable more appropriate allowances to be made for the effects of inflation, which might effect the various claims types differently.

In the later years of development, the numbers of claims settled might be very low and the average cost per claim can be highly variable from one year of origin to another. The assumed average cost in the tail is therefore largely a matter of judgement based on the types of claim that appear in the tail and an extrapolation of average costs in the preceding delay periods.

The average cost per claim approach is intuitively appealing and can be applied with a high degree of flexibility. In addition, the information it can provide on the severity of the individual claim types provides useful feedback to the rating process (Booth, Chadburn, Cooper, Haberman & James, 1999).

3.2.5 The Loss Ratio and Bornheutter-Ferguson Method

The loss ratio of a cohort of business is determined as the ratio of ultimate losses to the pertinent premiums. In the case of an accident year cohort, premiums will be gained; if underwriting year is used the appropriate premium will be that written over the period. A calculation of reserves could be made by multiplying premiums by the expected loss ratio, to get an estimate of the ultimate claim amount. The loss ratio can be estimated after consideration of past experience and market statistics. While loss ratio methods are generally applied to premiums, a more fundamental approach would be to relate expected losses to a more accurate measure of exposure. For instance, for motor vehicles this might be vehicle years or annual distance driven.

New companies might have no option other than to set reserves for outstanding claims based on loss ratios, since they might have too little actual claims data to support any other analysis. The choosed loss ratios might be used in the company's business planning exercises, if the expected conditions when the business plan was prepared have not varied markedly.

The actual loss ratio obviously is not known until the last claim has been paid and some method of adjusting the first estimate, as claims occur, is required. The Bornheutter-Ferguson method gives a way of combining the previous expectation of losses obtained by loss ratio estimates with the actual rate of occurance of claims. By doing so it reduces the effect that the most recent years of account have on the reserve estimate, can have an excessive effect on chain ladder estimates especially. Therefore, the Bornheutter-Ferguson method is especially useful when experience is temporary.

It must be investigated how the method works with the data set prepared for the chain ladder method. The key items of information required are:

- the initial expected ultimate loss ratio,
- the premium or exposure base to which the loss ratio is applied,
- the expected development pattern of claims.

As an example, the loss ratio method is applied to data taken from TRAMER on 14.03.2010 to estimate outstanding claim provisions for traffic insurance. The loss ratio is assumed 90% for each period of origin. The naive estimate of ultimate claims is calculated by multiplying the written premium by the initial expected loss ratio (IELR). These values are illustrated in Table 3.19. The loss ratio is the ratio of the expected undiscounted claims to premiums.

Table 3.19 Naive ultimate claim estimate

	Year of origin					
	2004	2005	2006	2007	2008	2009
Written premium	869.263	1.108.410	1.281.783	1.508.553	1.736.998	1.953.267
Naive ultimate claim estimate	782.337	997.569	1.153.605	1.357.698	1.563.298	1.757.940

The next step is to calculate development patterns using some method such as the chain ladder. To calculate the estimated reserve, the proportion of ultimate claims observed to the end of development period k must be calculated. The inverse of these factors represents the proportion of ultimate that is expected at each delay period. This is represented by R_k. The calculation of cumulative factors and development year factor R_k are given in Table 3.20.

Table 3.20 Calculation of development year factor R_k

Development period	0-1	1-2	2-3	3-4	4-5
Development factors	3.476	1.133	1.039	1.029	1.024
Cumulative factors	4.312	1.240	1.095	1.054	1.024
Inverse (R_k)	0.232	0.806	0.914	0.949	0.976

The cumulative factors in Table 3.20 are calculated by multiplying the development factors. For example;

4.312= 3.476*1.133*1.039*1.029*1.024

1.240= 1.133*1.039*1.029*1.024

1.095= 1.039*1.029*1.024

1.054= 1.029*1.024

The inverse R_k in Table 3.20 is calculated by proportion of the development factors to the cumulative factors. For example;

0.806= 3.476/4.312

0.914=1.133/1.240

0.949=1.039/1.095

0.976=1.029/1.054

0.232=1/4.312

By multiplying the estimate of ultimate claims in Table 3.19 by 1- R_k , the proportion of ultimate claims still outstanding at the end of development period k is estimated. The results of these calculations are represented in Table 3.21.

Table 3.21 Estimated reserve

Year of origin	**Naive estimated ultimate claims**	**Expected proportion observed,** R_k	**Expected proportion Outstanding, 1-** R_k	**Estimated Reserve**
2005	997.569	0.976	0.024	23.942
2006	1.153.605	0.949	0.051	58.834
2007	1.357.698	0.914	0.086	116.762
2008	1.563.298	0.806	0.194	303.280
2009	1.757.940	0.232	0.768	1.350.098
Total				1.852.916

In respect of Table 3.21 reserve totalling 1.852.916 TL is necessary to settle claims. The estimated ultimate claims for year of origin i and development period k are

$$P_i * (IELR)_i (1 - R_k) + C_{ik} , \tag{3.9}$$

where P_i is the premium income received and $(IELR)_i$ is the initial expected loss ratio, for year of origin i . The calculation of the estimated reserve in Table 3.21 is a weighted average of a prior expected value and the actual claims, similarly to credibility rating. As the year of origin matures, the prior weight $(1 - R_k)$ reduces so that, in effect, the actual experience is given greater credibility.

3.2.6 Operational Time Model

The chain ladder methods suppose that the claim amounts, the numbers of claims incurred and the rate of settlement remain stable from year to year. The method can be adapted to permit for the variations in the past, but not the next. Operational time models attempt to refer to variations in speed of settlement.

Operational time deal with the speed of settlement of a cohort of claims. Business in the cohort begins to be written at operational time 0, and the final claim is paid at operational time 1. A basic assumption of operational time reserving models is that claims paid at a particular time of operational time have similar features. For instance, claims settled within a short time of business being written are probably small, whereas those settled towards the end of the development period probably large. By using operational time, the difficulty with different categories of business having different development periods is evaded.

The methods need to data that should be easily accessible: claim amounts, numbers of claims settled and numbers of claims incurred. Although the next might be unknown, it would be logical to estimate the number using some proxy such as claims reported since the distribution of the number of claims incurred, given the number reported is fairly stable.

3.2.7 The Bootstrap Method

Bootstrapping is a strong method which allowes the calculation of a number of different estimates of a random variable, using empirical data as an approximation for the actual distribution of a related random variable. It provides a simple approach for calculating estimates of the error related to the claims reserves.

The chain ladder development factors can be used to estimate the ultimate claims at each development period and for each year of origin. The estimates of the past incremental claims, which were computed by multiplying the estimated ultimate claims by the appropriate development factors. It is assumed that the residual errors are random. This assumption should be tested to assure that there is no systematic component, but it is compatible with the chain ladder model. The importance of this assumption for the bootstrap method is that each error could equally well have arisen as the residual error from any other development period and year of origin.

To use the bootstrap method, a random sample (permitting for replacement) is drawn from the set of residual errors. The values from the random sample are added to the true incremental data to generate an alternative, but equally likely, outcome of the true claims, called pseudo data. The incremental ‘pseudo claims’ are then cumulated, the chain ladder method carried out as normal, and a second reserve estimate found.

A new set of residual errors is calculated using the second set of estimates and used to produce a new set of pseudo data and a new reserve estimate. After a sufficient number of estimates have been produced, the method assumes that, taken together, they provide a random sample of the distribution of the true reserve value, and can be used to calculate estimates of the moments of the distribution.

If the incremental claims have been affected by inflation then residual errors calculated on unadjusted data will not be random. Ideally, therefore, data should be adjusted for inflation before applying the bootstrap method, and the inflation adjusted chain ladder model used to calculate the reserve estimates. However, if the data are not inflation adjusted, it could be argued that the effect of inflation is amalgamated in the final result as an additional random component.

It is also possible to argue that, before being sampled, the residual errors should be adjusted for the scale of the year of origin with which they are associated. For example, the residual errors could be divided through by L_i and, once sampled, adjusted back up to the correct period of origin scale.

For simplicity the calculations have beeen carried out assuming that the error structure of the model is the same throughout the triangle. Thus, the pseudo data could be calculated by randomly sampling from the complete set of residuals for the whole triangle. In practice this is probably an oversimplification since the incremental payments and their associated errors are likely to depend on the development period. Incremental settlements for short-tailed classes of business are likely to be large in the first two development years and smaller later on; for longer-tailed classes of business there might be a peak of settlement activity around development years four or five, for example. Unless the residual errors are suitably scaled, the sampling for particular development years might have to be confined to the residual errors arising in those development years. This would lead to difficulties sampling for errors in the tail of the triangle, as the amount of data is limited, so that some judgement might be required to extend the sample or to standardize the sampling residuals to remove these distortions.

CHAPTER FOUR
APPLICATION

4.1 Introduction to Application

In this chapter, a reserving method was developed against to the chain ladder method. This method will be applied to the claim data taken from TRAMER to estimate the reserve for outstanding claims.

Firstly, premium amounts which were produced between 2003 and 2009 origin years were explained. The produced premium amounts were illustrated with the various graphs.

Next, a reserving method was developed with respect to the figure of the claim payments in 2003 and 2004. This developed reserving method was explained in detail. Then, it was used to estimate reserve for outstanding claims.

Finally, the comparable results of both chain ladder method and the developed method with the actual results published in TRAMER were explained.

4.2 About Application

Between 2004 and 2009 years paid claim data for traffic insurance were taken from the table of "year based cumulative claim report paid (Based on Policy Years)" published in TRAMER on 14.03.2010 to forecast the claim payments in future.

Motor TPL Insurance Information Center – TRAMER has been established since traffic insurance which have a significant and wide applicatian area in terms of premium volume in insurance sector in Turkey will own more healthy infrastructure. According to the data which were got at the end of May 2011, the number of policy in operation is 12.051.060. The number of vehicle is 15.382.908 at the end of March 2011. On the other hand, the number of uninsired vehicle is 3.331.848 and the proportion of uninsured vehicle is 21.66. At the end of January 2011, premium amount is 205.921 thousand TL and the number of claim file is 84.171.

4.3 The Premium Amounts

According to the data taken from TRAMER on 14.03.2010, between 2003 and 2009 years produced premium amounts for traffic insurance were given in Table 4.1.

Table 4.1 Premium amounts

Year	Policy Premiums (Thousand TL)
2003	513.084
2004	869.263
2005	1.108.410
2006	1.281.783
2007	1.508.553
2008	1.736.998
2009	1.953.267

When looking at this table, for example in 2006, premium amount is 1.281.783 thousand TL. The increasing trend was observed in the produced premium amounts in the basis of years. To see this increasing trend as visual, line and bar graphs were drawn for premium amounts. The line and bar graphs of the produced premium amounts were given in Figure 4.1 and Figure 4.2, respectively.

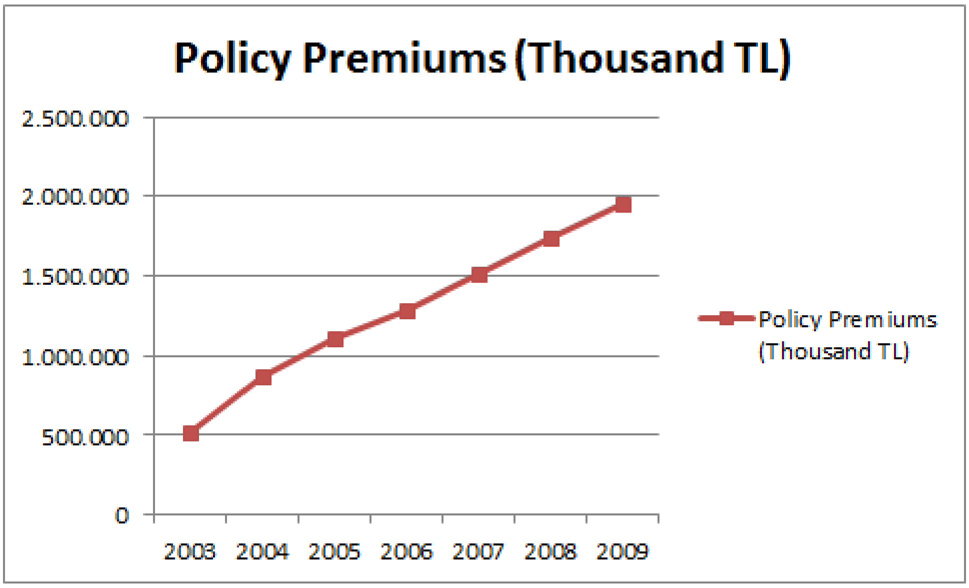

Figure 4.1 The line graph of premium amounts

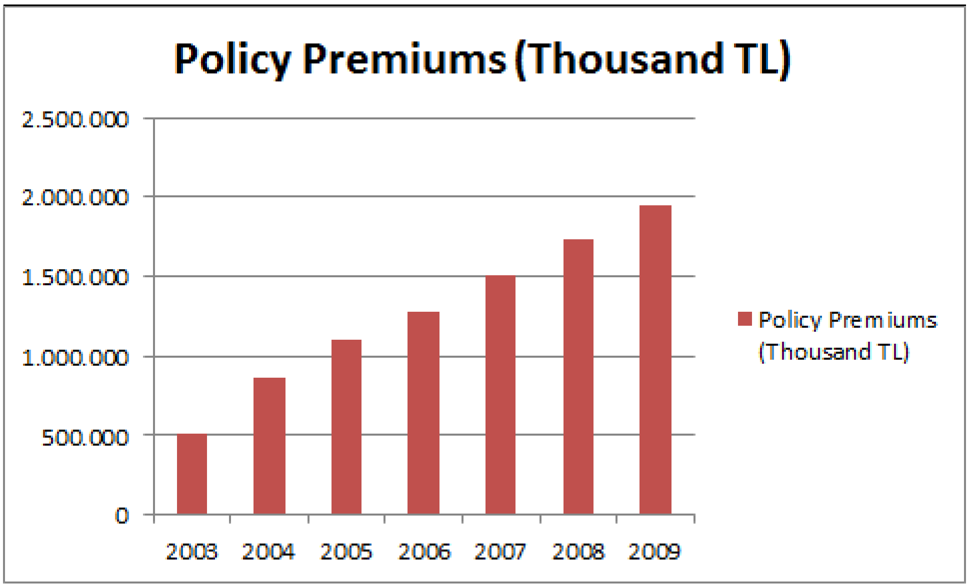

Figure 4.2 The bar graph of premium amounts

As seen in the line and bar graphs, the produced premium amounts have increasing trend between 2003 and 2009 years. Additionally, the rate of increase in premiums were illustrated in Table 4.2.

Table 4.2 The rate of increase in premiums

Year	The rate of increase in premiums
2003	
2004	69%
2005	28%
2006	16%
2007	18%
2008	15%
2009	12%

When looking at this table, the produced premium amounts between 2003 and 2009 have increased with decreasing ratio except 2007 year.

4.4 The Developed Reserving Method

Before the developed reserving method was explained, the cumulative paid claims between 2003 and 2009 years were studied. The figure of the claim payments will be obtained by drawing varied graphs. The cumulative claims were given in Table 4.3. The values in this table and following tables were given as thousand TL.

Table 4.3 Cumulative paid claims

	Development period					
Year of origin	0	1	2	3	4	5
2003	86.618	327.733	380.860	393.977	403.890	413.789
2004	140.189	493.770	565.368	584.312	600.582	614.716
2005	193.752	676.786	768.583	796.845	820.836	
2006	233.877	825.647	932.118	973.399		
2007	292.354	1.013.146	1.144.168			
2008	375.063	1.284.941				
2009	438.900					

According to this table, first year field (year of origin=0) shows the damage which occurs in the policy underwriting years. Second year field (year of origin=1) shows the damage which occurs in the following underwriting years. Also second year field is total of the first and second years claim amount. For example; 140.189 thousand TL shows claim amount which was occurred in 2004 and was paid in same year. Similarly, 565.368 thousand TL shows claim amount which was occurred in 2004 and was paid in 2006 (total of claim amounts in 2004, 2005 and 2006 years).

As seen in Table 4.3, the claim payments were completed for 2003 and 2004 origin years. For 2003 origin year, claim payment was completed in 2008. Similarly, claim payment was completed in 2009 for 2004 origin year. Therefore, claim payment figure was determined by cumulative paid claims for 2003 and 2004 origin years. The line and bar graphs of cumulative paid claims for 2003 and 2004 origin years were seen in Figure 4.3 and Figure 4.4, respectively.

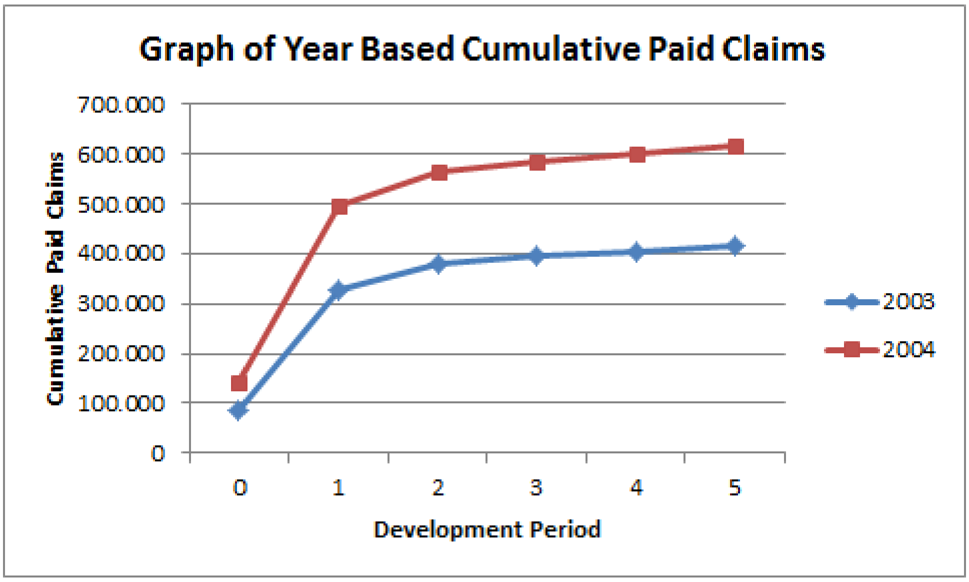

Figure 4.3 The line graph of cumulative paid claims for 2003 and 2004 origin years

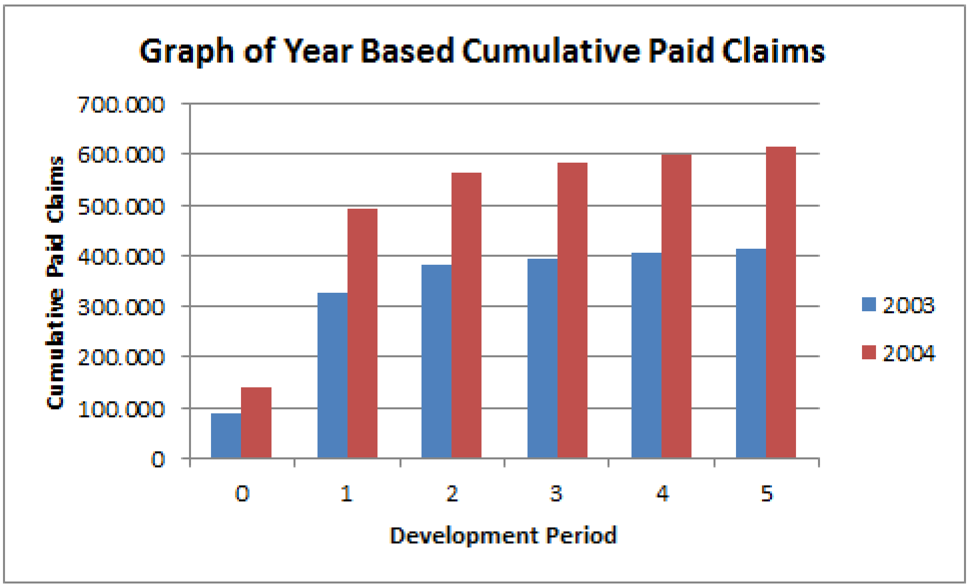

Figure 4.4 The bar graph of cumulative paid claims for 2003 and 2004 origin years

When looking at Figure 4.3 and Figure 4.4, it was observed that the claim payment figure for 2004 origin year was similar with claim payment figure for 2003 origin year. Furthermore, it can be assumed that the claim payment figure will be similar for the next origin years. The line and bar graphs of cumulative paid claims for the next origin years were seen in Figure 4.5 and Figure 4.6, respectively.

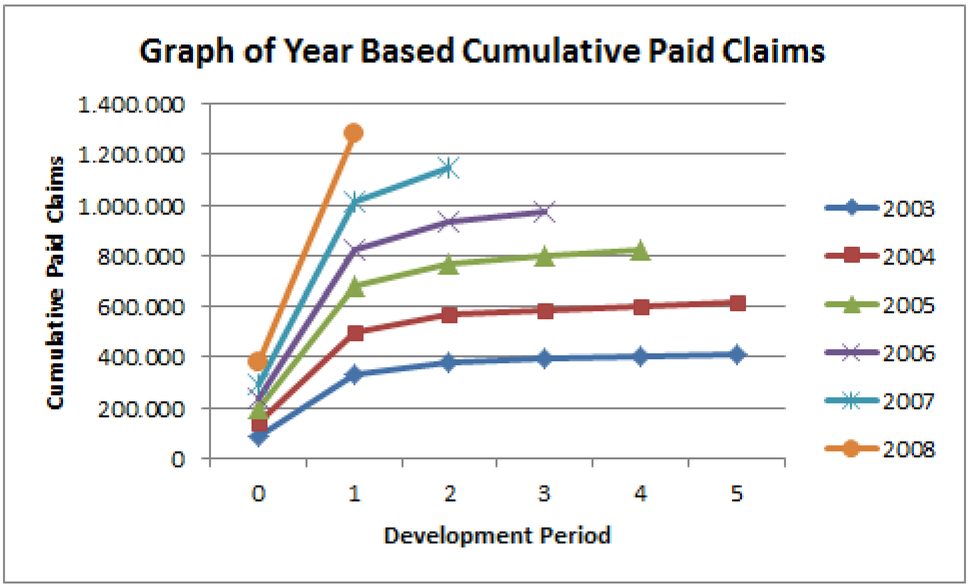

Figure 4.5 The line graph of cumulative paid claims between 2003 and 2009 years

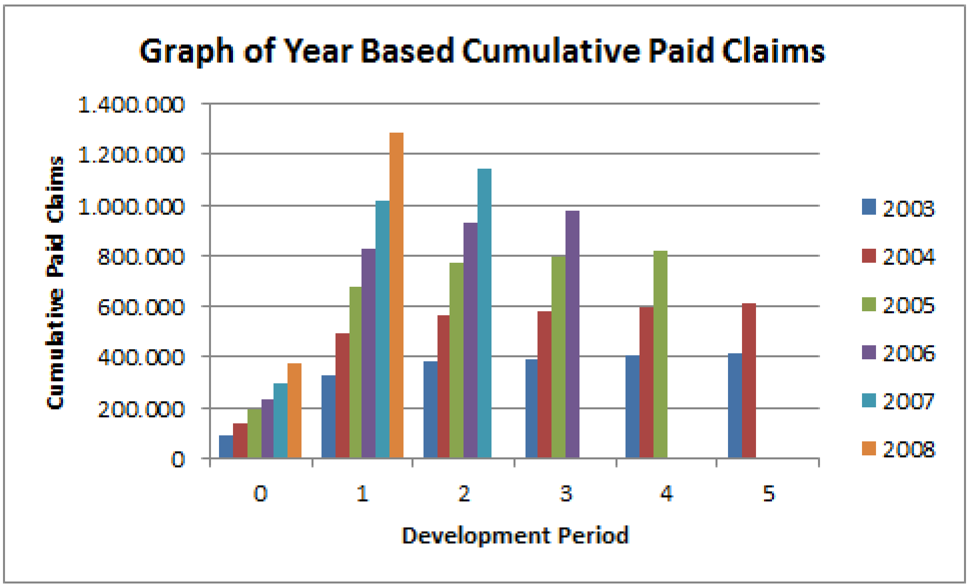

Figure 4.6 The bar graph of cumulative paid claims between 2003 and 2009 years

As seen in Figure 4.5 and Figure 4.6, claim payment figure is similar for the next origin years. The value of "0" at the horizontal axis displays that the year of claim occurrence is the same with the year of claim settlement. Namely, for example, the claim was occurred in 2004 origin year and was paid in 2004.

Additionally, the time series plot of cumulative paid claims was drawn by MINITAB statistical package program to examine whether cumulative paid claims on the basis of years have any specific trend. This plot can be seen in Figure 4.7.

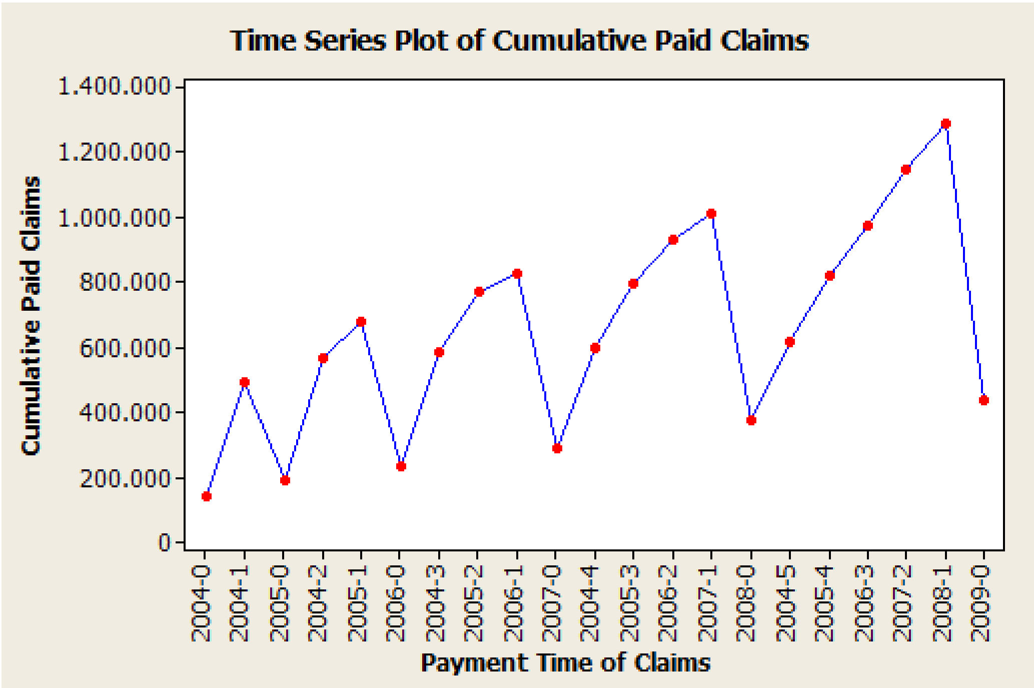

Figure 4.7 Time series plot of cumulative paid claims

As it is seen from the Figure 4.7, increases in paid claims on the basis of years were observed. Because of this, claim payments have increasing trend. The values in Figure 4.7 were given the order of claim occurrence and claim settlement.

Since claim payment figure was similar between 2003 and 2008 origin years, proportional relationship between cumulative paid claims was analyzed in successive development periods for 2004-2009 origin years. The ratios between cumulative paid claims in successive development periods can be seen in Table 4.4.

Table 4.4 Ratios of cumulative paid claims in successive development periods

	Development period				
Year of origin	**1**	**2**	**3**	**4**	**5**
2004	352.217%	114.500%	103.351%	102.784%	102.353%
2005	349.305%	113.564%	103.677%	103.011%	
2006	353.026%	112.895%	104.429%		
2007	346.548%	112.932%			
2008	342.593%				
Mean	348.738%	113.473%	103.819%	102.898%	102.353%

The values in Table 4.4 were calculated by using the values in Table 4.3. For example;

$$352.217\% = \frac{493.770}{140.189}$$

The value of 352.217% represents the ratio of cumulative paid claim in development period 1 to cumulative paid claim in development period 0 for 2004 origin year. Similarly,

$$103.011\% = \frac{820.836}{796.845}$$

The value of 103.011% represents the ratio of cumulative paid claim in development period 4 to cumulative paid claim in development period 3 for 2005 origin year.

As seen in Table 4.4, the ratios for each development period are similar. Therefore, averages of the ratios for each development period were calculated. These averages will be used as coefficient to estimate the claim payments in future. The calculated coefficients were given below.

$r_1 = 3.48738$

$r_2 = 1.13473$

$r_3 = 1.03819$

$r_4 = 1.02898$

$r_5 = 1.02353$

A model was developed against to the chain ladder method to estimate reserve for outstanding claim provisions. If the claim payments are known for the development period 0, the claim payments can be estimated for the next development periods. The developed model was denoted as:

y_0=paid claim in development period 0

$y_1 = y_0 r_1$

$y_2 = y_1 r_2$

$y_3 = y_2 r_3$

$y_4 = y_3 r_4$

$y_5 = y_4 r_5$

If we generalize the model, it can be obtained as below:

$y_k = y_{k-1} r_k$ k :development period

To estimate outstanding claim provisions in 2009, claim payments must be estimated after 2009 based on 2005, 2006, 2007, 2008 and 2009 years. Using the values of r_k and y_0, the lower right part of run-off triangle in Table 4.3 are got.

Table 4.5 Cumulative paid claims (Table 4.3 completed)

	Development period					
Year of origin	0	1	2	3	4	5
2004	140.189	493.770	565.368	584.312	600.582	614.716
2005	193.752	676.786	768.583	796.845	820.836	838.344
2006	233.877	825.647	932.118	973.399	988.697	1.011.961
2007	292.354	1.013.146	1.144.168	1.201.096	1.235.904	1.264.984
2008	375.063	1.284.941	1.484.212	1.540.894	1.585.550	1.622.857
2009	438.900	1.530.611	1.736.830	1.803.160	1.855.415	1.899.073

When looking at this table, for example, the claim payments can be estimated for the next development periods since the claim payment in development period 0 for 2009 origin year is known. The calculations were given below.

$y_0 = 438.900$

$y_1 = y_0 r_1 = 438.900 * 3.48738 = 1.530.611$

$y_2 = y_1 r_2 = 1.530.611 * 1.13473 = 1.736.830$

$y_3 = y_2 r_3 = 1.736.830 * 1.03819 = 1.803.160$

$y_4 = y_3 r_4 = 1.803.160 * 1.02898 = 1.855.415$

$y_5 = y_4 r_5 = 1.855.415 * 1.02353 = 1.899.073$

In Table 4.5, the value of 1.855.415 thousand TL shows claim amount that was occurred in 2009 and must be paid in 2013. The value of 1.264.984 thousand TL shows claim amount that was occurred in 2007 and must be paid in 2012.

At next step, estimated cumulative paid claims at the lower right part of the run-off triangle are converted to incremental paid claims. To find incremental paid claims, estimated values in Table 4.5 are used. Incremental paid claims were illustrated in Table 4.6.

Table 4.6 Payments made in development year

	Development period					
Year of origin	0	1	2	3	4	5
2004						
2005						17.508
2006					15.298	23.264
2007				56.928	34.808	29.080
2008			199.271	56.682	44.656	37.307
2009		1.091.711	206.219	66.330	52.255	43.658

For example;

23.264=1.011.961-988.697

44.656=1.585.550-1.540.894

206.219=1.736.830-1.530.611

The sum of the values at the lower right part of the run-off triangle in Table 4.6 give estimated reserve which can be seen in Table 4.7.

Table 4.7 Estimated reserve

Year of origin		**Reserve**
2005	17.508	17.508
2006	15.298 + 23.264	38.562
2007	56.928 + 34.808 + 29.080	120.816
2008	199.271 + 56.682 + 44.656 + 37.307	337.916
2009	1.091.711 + 206.219 + 66.330 + 52.255 + 43.658	1.460.173
Total		1.974.975

According to this table, total 1.974.975 thousand TL must be reserved as outstanding claim provisions for traffic insurance.

4.5 The Comparable Results with TRAMER

The some actual values of the estimated values at the lower right part of the run-off triangle have been published in TRAMER. The updated values on 07.08.2011 in TRAMER were given in Table 4.8.

Table 4.8 Results obtained from TRAMER

SECTOR TOTAL														
UW Year	Policy Premiums (Thousand TL)	Loss Status (paid; outstanding)	1st Year	%	2nd Year	%	3rd Year	%	4th Year	%	5th Year	%	5th Year	%
2003	513.084	Suspense	5.002		15.070		18.148		20.227		22.743		24.358	
		Paid	86.193	17.77	327.447	66.76	380.989	77.79	394.807	80.89	405.456	83.46	415.724	85.77
		Total	91.195		342.517		399.137		415.034		428.199		440.082	
2004	869.263	Suspense	8.489		27.620		32.994		37.523		40.287		42.801	
		Paid	140.072	17.09	491.996	59.78	562.909	68.55	582.782	71.36	600.032	73.66	614.544	75.62
		Total	148.561		519.616		595.903		620.305		640.319		657.345	
2005	1.108.410	Suspense	29.570		76.545		87.226		92.346		97.747			
		Paid	192.894	20.07	669.510	67.31	761.607	76.58	790.524	79.65	815.715	82.41	102.569	84.91
		Total	222.464		746.055		848.833		882.870		913.462			
2006	1.281.783	Suspense	35.585		85.431		102.273		110.630		117.079		120.935	
		Paid	231.871	20.87	825.669	71.08	932.983	80.77	975.561	84.74	1.006.966	87.69	1.027.070	89.56
		Total	267.456		911.100		1.035.256		1.086.191		1.124.045		1.148.005	
2007	1.508.553	Suspense	37.936		119.641		140.770		153.062		158.383			
		Paid	295.185	22.08	1.016.710	75.33	1.152.309	85.72	1.209.237	90.31	1.239.651	92.67		
		Total	333.121		1.136.351		1.293.079		1.362.299		1.398.034			
2008	1.736.998	Suspense	70.471		199.830		229.984		240.377					
		Paid	378.163	25.83	1.312.754	87.08	1.480.222	98.46	1.529.056	101.87				
		Total	448.634		1.512.584		1.710.206		1.769.433					
2009	1.953.338	Suspense	92.065		229.066		259.565							
		Paid	460.165	28.27	1.350.332	80.86	1.468.093	88.45						
		Total	552.230		1.579.398		1.727.658							
2010	2.306.652	Suspense	95.592		303.871									
		Paid	471.314	24.58	1.113.603	61.45								
		Total	566.906		1.417.474									
2011	1.611.912	Suspense	114.462											
		Paid	124.310	14.81										
		Total	238.772											

In Table 4.8, between 2003 and 2011 origin years, premium amounts, paid claims, unpaid claims (suspense), the total of paid claims and unpaid claims and the ratio of total payments to the premium amounts as cumulatively for each origin year have been observed. For example, the produced premium amount in 2008 is 1.736.998 thousand TL. The paid claim amount for 2010 origin year in second development year namely, in 2011 is 1.113.603 thousand TL. The outstanding claim amount for 2007 origin year in fourth development year namely, in 2010 is 153.062 thousand TL. The total payment for 2009 origin year in first development year namely, in 2009 is 552.230 thousand TL. For 2005 origin year and third development year, the ratio of total payments to the premium amount as cumulatively is 76.58%.

To examine which method (chain ladder method or the developed method) is more efficient, the estimated values obtained by chain ladder method and the developed model were compared with the actual values published in TRAMER, respectively. The estimated values obtained by chain ladder method and the actual values published in TRAMER were illustrated in Table 4.9 and Table 4.10, respectively.

Table 4.9 Results of chain ladder method

	Development period					
Year of origin	0	1	2	3	4	5
2005						840.536
2006					1.001.628	1.025.667
2007				1.188.791	1.223.266	1.252.624
2008			1.455.838	1.512.616	1.556.482	1.593.838
2009		1.525.616	1.728.523	1.795.935	1.848.017	1.892.369

Table 4.10 Results of TRAMER

	Development period					
Year of origin	0	1	2	3	4	5
2005						838.631
2006					1.006.966	1.027.070
2007				1.209.237	1.239.651	
2008			1.480.222	1.529.056		
2009		1.350.332	1.468.093			

When looking at Table 4.9 and Table 4.10, for 2006 origin year and fifth development period, the claim payment was estimated as 1.025.667 thousand TL against to the actual value of 1.027.070 thousand TL. The differences between corresponding values in these two tables were given in Table 4.11.

Table 4.11 Differences between table 4.10 and table 4.9

	Development period					
Year of origin	0	1	2	3	4	5
2005						-1.905
2006					5.338	1.403
2007				20.446	16.385	
2008			24.384	16.440		
2009		-175.284	-260.430			

For example;

1.403=1.027.070-1.025.667

16.385=1.239.651-1.223.266

As seen in Table 4.11, for 2005 and 2006 origin years and fifth development year, differences are smaller than others. Conversely, for 2009 origin year and first and second development years, differences are bigger than others. The reason of the wide difference in 2009 origin year and second development period that the year of 2011 is uncompleted. The sum of differences and the squares of the differences for each origin year were given in Table 4.12.

Table 4.12 Sum of differences and squares of differences

Year of origin	**Sum of Differences**	**Squares of differences**
2005	-1.905	3.629.025
2006	6.741	45.441.081
2007	36.831	1.356.522.561
2008	40.824	1.666.598.976
2009	-435.714	189.846.689.796
Total		192.918.881.439

According to the Table 4.12, the sum of squares of differences was obtained as 192.918.881.439.

The estimated values obtained by using the developed method and the actual values published in TRAMER were illustrated in Table 4.13 and Table 4.14, respectively.

Table 4.13 Results of $y_k = y_{k-1} r_k$ model

		Development period				
Year of origin	0	1	2	3	4	5
2005						838.344
2006					988.697	1.011.961
2007				1.201.096	1.235.904	1.264.984
2008			1.484.212	1.540.894	1.585.550	1.622.857
2009		1.530.611	1.736.830	1.803.160	1.855.415	1.899.073

Table 4.14 Results of TRAMER

	Development period					
Year of origin	0	1	2	3	4	5
2005						838.631
2006					1.006.966	1.027.070
2007				1.209.237	1.239.651	
2008			1.480.222	1.529.056		
2009		1.350.332	1.468.093			

When looking at Table 4.13 and Table 4.14, for 2006 origin year and fifth development period, the claim payment was estimated as 1.011.961 thousand TL against to the actual value of 1.027.070 thousand TL. The differences between corresponding values in these two tables were given in Table 4.15.

Table 4.15 Differences between table 4.14 and table 4.13

	Development period					
Year of origin	0	1	2	3	4	5
2005						287
2006					18.269	15.109
2007				8.141	3.747	
2008			-3.990	-11.838		
2009		-180.279	-268.737			

As seen in Table 4.15, for 2005 origin year and fifth development year, difference is smaller than others. Conversely, for 2009 origin year and first and second development years, differences are bigger than others. The reason of the wide difference in 2009 origin year and second development period that the year of 2011 is uncompleted. The sum of differences and the squares of the differences for each origin year were given in Table 4.16.

Table 4.16 Sum of differences and squares of differences

Year of origin	**Sum of Differences**	**Squares of differences**
2005	287	82.369
2006	33.378	1.114.090.884
2007	11.888	141.324.544
2008	-15.828	250.525.584
2009	-449.016	201.615.368.256
Total		203.121.391.637

According to the Table 4.12, the sum of squares of differences was obtained as 203.121.391.637.

The sum of squares of differences obtained for chain ladder method is 192.918.881.439. For the developed reserving method, the sum of squares of differences is 203.121.391.637. It was observed that it is not difference too much when the sum of squares of differences was compared.

CHAPTER FIVE
CONCLUSIONS

In this study, the reserve for outstanding claims was estimated with various reserving methods by using the data of cumulative paid claims taken from TRAMER on 14.03.2010. The model of $y_k = y_{k-1} r_k$ was developed against to the chain ladder method to estimate reserve. The comparable results that were founded from the same claim data are given in Table 5.1.

Table 5.1 Summary of reserve calculations

Reserving Methods	Estimated Reserve (Thousand TL)
Chain ladder method	1.942.790
Inflation-adjusted chain ladder method (Current money)	1.710.544
Inflation-adjusted chain ladder method (Nominal)	1.973.848
Inflation-adjusted chain ladder method (Net present value)	1.733.588
Loss ratio method	1.852.916
$y_k = y_{k-1} r_k$	1.974.975

As seen in Table 5.1, the reserving methods give approximate results. When the estimates compare with the actual values published in TRAMER, it is observed that the estimates obtained by using chain ladder method are closer to the actual values published in TRAMER. Nevertheless, it is not gap too much in the obtained estimates between chain ladder method and the model of $y_k = y_{k-1} r_k$. This developed model is computable easier than chain ladder method. It is sufficient to know the paid claim amount in origin years only. The lower right part of run-off triangle can be estimated with this method easily.

In recent years, loss has been significantly observed in traffic insurance. This study and the developed method will guide to insurance companies about outstanding claim provisions in future years.

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