FUZZY ENTROPY AND ITS APPLICATIONS

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We have read the thesis entitled **"FUZZY ENTROPY AND ITS APPLICATIONS"** completed by **YUSUF YENİYAYLA** under the supervision of **Assistant Professor EMEL KURUOĞLU** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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FUZZY ENTROPY AND ITS APPLICATIONS

ABSTRACT

Fuzzy logic is based on fuzzy sets. In the classical approach, an element either is or is not the element of the set. On the other hand, in the fuzzy approach, each element has a degree of membership to a set.

Fuzzy entropy is used to express the mathematical values of the fuzziness of fuzzy sets. The concept of entropy, the basic subject of information theory and telecommunications, is a measure of fuzziness in fuzzy sets.

This study encompasses two applications of fuzzy entropy in the field of image processing, which depend on Shannon's entropy and distance concept.

The first application is the enhancement of the cell count method. It is known that the cell count method of medical doctors requires extreme attention and takes too much time. In addition, it is known that this situation induces count errors due to doctors' workload and leads to loss of time. Consequently, some scientific studies are needed for cell counting. This study aims to use the segmentation method for clear vision in cell counting from histopathological images. The fuzzy entropy method is used in the segmentation process. Before the segmentation process, the images were cleared by removing the noise from the image and a better image for cell count was provided. In the segmentation process, a better threshold value is obtained compared to the previous works, by using generalized fuzzy entropy and Shannon's entropy. In this study, the results of fuzzy entropy and Shannon's entropy methods used in cell count application are compared as well.

The second application in this study uses the generalized fuzzy entropy method to remove the noise on an image. The noise on the human face image is reduced with the cost function obtained depending on the method of fuzzy entropy. It is aimed to contribute to the studies in the field of health by obtaining better results with this method in the clarification of images particularly such as MR, ECG and ultrasound.

Keywords: *Fuzzy logic, entropy, fuzzy entropy, generalized fuzzy entropy, image processing, image denoising.*

BULANIK ENTROPİ VE UYGULAMALARI

ÖZ

Bulanık mantığın temeli bulanık kümelere dayanır. Klasik yaklaşımda bir eleman ya kümenin elemanıdır ya da değildir. Bulanık yaklaşımda ise her bir elemanın bir kümeye üyelik derecesi söz konusudur.

Bulanık entropi bulanık kümelerin bulanıklığının matematiksel değerlerini ifade etmede kullanılır. Bilgi teorisinin ve telekomünikasyonun temel konusu olan entropi kavramı bulanık kümelerde bulanıklığın bir ölçümüdür.

Bu çalışmada, Shannon ve distance yöntemlerine dayandırılarak ifade edilen bulanık entropi çeşitleri ve görüntü işleme üzerine iki uygulama yer almaktadır.

Birinci uygulamada amaç hücre sayma metodu geliştirilmesidir. Tıp doktorlarının hücre sayma yöntemlerinin çok fazla dikkat gerektirdiği ve oldukça zaman aldığı bilinmektedir. Ayrıca bu durum doktorların iş yükünü artırmakta ve hatalı sayımlara neden olmaktadır. Bu nedenle hücre sayma işlemleri için yeni bilimsel çalışmalara ihtiyaç duyulmaktadır. Bu çalışmada, histapolojik görüntülerdeki hücre sayısını bulmak için bölütleme yöntemi kullanılmaktadır. Bölütleme işleminden önce, görüntü üzerindeki gürültünün giderilerek hücre sayımı için daha net bir görüntü elde edilmektedir. Shannon ve genelleştirilmiş bulanık entropi yöntemleri ile önceki çalışmalara göre daha iyi bölütleme değeri elde edilmektedir. Shannon ve genelleştirilimiş bulanık entropi yöntemleri ile hastalıklı dokulardaki hücre sayma işlemi sonuçları karşılaştırılmaktadır.

Tezdeki ikinci uygulama genelleştirilmiş bulanık entropinin görüntü üzerindeki gürültünün giderilmesi üzerine bir uygulamasıdır. Bulanık entropi yöntemine dayanarak elde edilen maliyet fonksiyonu ile insan yüzü görüntüsü üzerindeki gürültünün azaltılması gerçekleştirilmektedir. Özellikle MR, EKG, ultrason vb.

görüntülerin netleştirilmesinde bu yöntemle daha iyi sonuçlar elde edilerek sağlık alanındaki çalışmalara katkı sağlaması amaçlanmaktadır.

Anahtar kelimeler: *Bulanık mantık, entropi, bulanık entropi**,** genelleştirilmiş bulanık entropi, görüntü işleme, gürültü giderme.*

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CHAPTER ONE

INTRODUCTION

Fuzzy logic is based on fuzzy sets. In the classical approach, an element either is or is not the element of the set. However, in the fuzzy approach, each element has a degree of membership to the set.

Fuzzy entropy is used to express the mathematical values of the fuzziness of fuzzy sets. The concept of entropy, the basic subject of information theory and telecommunications, is a measure of fuzziness in fuzzy sets.

The examination of systems containing ambiguity gained a new dimension after fuzzy logic and the fuzzy set theory, which used the rules of this logic, had been developed by Lotfi A. Zadeh and published in his original manuscript dated 1965 (Zadeh, 1965). The use of Fuzzy Logic recently reached its peak after the intense use in Japanese products in the 1980s. Nowadays it is possible to run across fuzzy logic applications in almost every area.

The measurement of the degree of fuzziness in fuzzy sets is an important step for applied areas. Entropy methods are used to conduct goal-driven calculations by processing the data that have been transformed into a fuzzy structure. Fuzzy logic and the most important indicator of fuzziness, entropy, have recently been used in various scientific studies.

The concept of entropy was introduced by Boltzman at the end of the 19^{th} century as a measurement of the irregularity of an ideal gas in a closed container. Information Theory, on the other hand, emerged during the resolution of problems pertaining to telecommunications during the 1940s. The purpose of information theory is to investigate the rules related to the acquisition, transfer, processing and storage of information. The randomness phenomenon in information transfer has made the use of statistical methods inevitable in the investigation of these processes. Information

theory, further developed by Shannon (1965), tries to explain the amount of information in a data set and its regularity. Multitude and diversity of information indicate that the entropy of the data will be low (Klir and Yuan, 2000).

Entropy will be used together with the concept of information. It was previously mentioned that the Information Theory aimed to investigate the quantitative laws related to the acquisition, transfer, processing and storage of information. Before the definition of information, the definition of entropy, which is also called statistical entropy and which can be defined as the measure of diversity over the probability distributions, will be elaborated.

In order to transfer information, it should be appropriately encoded. This information should be transferred using the minimum number of symbols. If the condition of a material system is exactly known, the transferred information about this system is not of such great significance. However, there should be a condition to measure how valuable the information about a system under any random and unknown condition is. This is called the "instability degree" of a system. The instability degree of a system is dependent not only on the number of possible conditions but also on the probabilities of that system to be under one of those conditions. If all information in the system is at the same level and has the same probability, the entropy of the system is maximum.

In this study, the distance between fuzzy sets and the entropy method are used to calculate the degree of fuzziness of a system. The study discusses the most used entropy methods in the literature. Entropy methods based on fuzzy entropy, generalized fuzzy entropy and Shannon's entropy are elaborated in the study. The study, then, touches upon how fuzzy entropy is used in applications including different methods and techniques of image processing. Some of these applications are summarized below.

H.D. Cheng and Huijuan Xu (2001) argue that the fuzzy set theory yields better results in image processing than the other methods. It can be seen that a clearer image is obtained in mammography images by increasing the concentration and

segmenting the image into required areas using the fuzzy entropy method (Cheng and Xu, 2001).

Üstüntaş et al. defined an expert system, attempting on a new approach different from the previous statistical problem resolution methods in image mapping algorithms. In their study, they proposed a fuzzy logic based image mapping algorithm. The study is a photogrammetric application of fuzzy logic. A software program, together with a fuzzy algorithm, was devised to map an image and the control points defining the model on that image. The fuzzy logic algorithm defines the appropriateness of the topological and geometrical features of the target. The program seeks for the target near the approximate coordinates and makes the decision of the most appropriate point to start marking. In order for this method to work, the approximate orientation elements of the image should be known and the images evaluated should be overlapping (Üstüntaş et al., 2006).

Wen-Bing Tao et al. attain the best segmentation pixel values by segmenting the image into three areas using fuzzy entropy. They grouped the pixels of the image as the dark area, gray area and bright area and calculated the values that yielded the maximum entropy in total out of the entropies of these three areas (Tao et al., 2003).

Adel Fatemi uses a more efficient entropy method by making use of the membership degrees in fuzzy sets based on Shannon's entropy, instead of De Luca and Termini's entropy, which uses the probabilistic values of the pixels in image processing (Fatemi, 2011).

Chen et al. use the entropy method effectively in EMG signal analysis for feature extraction and classification of the signals (Chen et al., 2007).

E. Pasha et al. use the fuzzy entropy method to obtain the coefficients of linear equations. Instead of the most widely used coefficient obtaining methods in the literature, namely the least squares method and the approach which minimizes the

fuzzy coefficients matrix, Pasha et al. obtain the most appropriate coefficients by the fuzzy entropy method (Pasha et al., 2007).

In this study image analysis will be conducted using the fuzzy entropy methods. Firstly, the pixel matrices forming the image are made fuzzy through the S_function that transforms them into fuzzy sets and 8 neighbor pixel mean method. The new fuzzy matrix is segmented with the fuzzy entropy methods. Similarly, the cost function based on the fuzzy entropy methods is used to remove the noise on the image. While the examinations of cell images and cell counts in medicine are performed manually, the doctors declared that the process is exhaustive, error-prone and time-consuming. With the methods devised in this study, it is seen that the best segmentation of the cell images and thus a clearer view of the cells can be obtained.

Image segmentation has many application areas. In medicine, it is used to monitor atherosclerosis and ruptures in the vein by Gamma and X rays and to detect the diseased cells in tissue samples from patients; in industry, it is used to find missing elements and to detect disconnection between electrical circuit lines; in meteorology, it is used to interpret the satellite photographs and for weather forecasts and to monitor warming phenomenon due to the hole in the ozone layer by using special coloring processes (Kara et al., 2003).

Toroman and Türkoğlu worked on the cell count process using the extraction of the required patterns from the image with the image segmentation process. The most important feature of the process of counting the required cells in an image after transferring them into digital medium is its speed. The cell count procedure, which is normally conducted by experts in a couple of hours, takes a few minutes for a computer. Toroman and Türkoğlu argue that this time-saving enables the doctor to make decisions more rapidly and thus enable him/her to do more examinations (Toraman and Türkoğlu, 2006).

This thesis comprises six chapters, including the introduction. In Chapter Two, fuzzy logic is defined in general. Entropy is discussed in the next chapter, whereas Fuzzy Entropy is discussed in Chapter Four. As the main focus of this study is the application, the methods will be briefly reviewed.

In Chapter Five, two applications are presented. In the first application, cell count procedure is performed using the fuzzy entropy method in image processing on the tissue images obtained in an engineering laboratory. The second application performs an image denoising procedure using the cost function with the fuzzy entropy method. In both of the applications, the best segmentation value is obtained by transforming the pixels of the images into fuzzy values with S-function and appropriate entropy methods. In the cell count procedure, the processing of the images and the enhancement of cell images are performed at first hand. The removal of noise on the image, differentiation of the adherent cell and rounding of the cell walls are some of the preliminary stages. Likewise, denoising, the transformation of the pixels of the image into fuzzy sets and the removal of noise after forming a cost function with entropy methods are carried out in the other application.

In the last chapter, the results of the applications are presented and the fuzzy entropy applications in this study are discussed. Regarding the results obtained in the study, it is thought that these applications can be of help for the decision makers in ‘‘counting cells in a tissue image” and “denoising”.

CHAPTER TWO

FUZZY LOGIC

Fuzzy Logic can be defined as a function that relates the truth of a proposition to other propositions, to values in a set involving an infinite number of truth values between absolute true and absolute false, or to the [0, 1] real number range. This definition is an outcome of the first work by L. Zadeh on fuzzy sets. Fuzzy Logic is a way of approximate reasoning. Fuzzy Logic's having different truth values which are represented with different adjectival degrees (or numerically with values between [0,1]) has brought along the truth tables. However the certainty of these tables is not absolute. But they have approximate estimation rules as a distinctive feature (Baykal and Beyan, 2004).

The human brain interprets imprecise and incomplete sensory information provided by perceptive organs. Fuzzy set theory provides a systematic calculus to deal with such information linguistically, and it performs numerical computation by using linguistic labels stipulated by membership functions (Jang et al., 1997).

For example, A classical set in Eq.(2.1) is with a crisp boundary and classical set of real numbers greater than can 175 be expressed as

$$A=\{x>175\} \tag{2.1}$$

where the clear, unambiguous boundary is 175; such that, if x is greater than this number, then x belongs to the set A; otherwise x does not belong to the set. Although classical sets are suitable for various applications and have proven to be an important tool for mathematics and computer science, they do not reflect the nature of human concepts, and thoughts, which tend to be abstract and imprecise. As an illustration, a set of tall persons can be expressed numerically as a collection of persons whose height is more than 175 ft; this the set denoted in Eq. (2.1); if A= "tall person'' and x= "height''. Yet this is a natural and inadequate way of representing the usual concept of "tall person''. For one thing, the person, but not a person 174 ft tall. This

distinction is intuitively unreasonable. The flaw comes from the sharp transition between inclusion and exclusion in a set.

In contract to classical set, a fuzzy set, as the name implies, is a set without a crisp boundary. That is, the transition from "belong to set'' to "not belong to a set'' is gradual, and this smooth transition is characterized by membership functions that give fuzzy sets flexibility in modeling commonly used linguistic expression, such as "the water is hood'' or "the temperature is high''. As Zadeh pointed out in 1965 in his seminal paper entitled "Fuzzy Set'', such imprecisely defined sets or classes "play an important role in human thinking, particularly in the domains of pattern recognition, communication of information, and abstraction''. Note that the fuzziness does not come from the randomness of the constituent members of the sets, but from uncertain and imprecise nature of abstract thoughts and concepts (Jang et al., 1997).

2.1 Basic Definitions and Terminology

X is a space of objects and x is an element of X. A classical set A, $A \subseteq X$, is defined as a collection of element sort objects $x \in X$, such that each x can either belongs or not belongs to the set A. By defining a **characteristic function** for each element x in X, it can be represent a classical set A by a set of ordered pairs (x, 0) or (x, 1), which indicates $x \notin X$, respectively.

Unlike the aforementioned need conventional set, a fuzzy set expresses the degree to which an element belongs to a set. Hence the characteristic function of a fuzzy set is allowed to have values between 0 and 1, which denotes the degree of membership of an element in a given set.

2.1.1 Fuzzy sets and membership functions

If X is a collection of objects denoted generically by x, then a fuzzy set A in X is defined as a set of ordered pairs in Eq. (2.2),

$$A = \{(x, \mu_A(x) : x \in X\} \tag{2.2}$$

where $\mu_A(x)$ is called the **Membership Function** (**MF**) for the fuzzy set A. The MF maps each element of X to a membership grade (or membership value) between 0 and 1.

For simplicity of notation in Eq. (2.3), we now introduce an alternative way of denoting a fuzzy set A can be denoted as

$$A = \begin{cases} \sum_{x_i \in X} \mu_A(x_i) / x_{i,} & \text{if X is a collection of discrete objects} \\ \int_X \mu_A(x) / x, & \text{if X is a continuous space } (\text{usually the real line R}) \text{ .} \end{cases} \tag{2.3}$$

Example 2.2 *Fuzzy sets with a discrete ordered universe*

X = {0, 1, 2, 3, 4, 5, 6} be the set of numbers of children a family may choose to have. Then the fuzzy set A = "sensible number of children in a family" may be described as follows:

A= {(0,0.1), (1,0.3), (2,0.7), (3, 1), (4, 0.7), (5, 0.3), (6, 0.1)}

here we have a discrete ordered universe X; the MF fort he fuzzy set A is shown in Figure 2.1 (a). Again, the membership grades of this fuzzy set are obviously subjective measures.

Example 2.3 *Fuzzy sets with a continuous universe*

X = R be the set of possible ages for human beings. Then the fuzzy set B= "about 50 years old" may be expressed as

$$B = \{(x, \mu_B(x) : x \in X\}$$

where $\mu_B(x) = \dfrac{1}{1+(\dfrac{x-50}{10})^4}$. This is illustrated in Figure 2.1 (b).

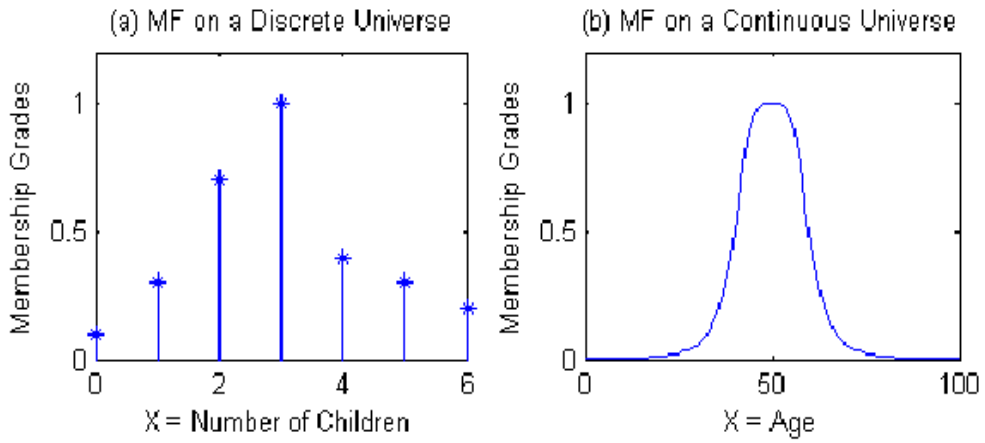


Figure 2.1 *(a)* A = "sensible number of children in a family"; (b) B = "about 50 years old."

Example 2.5 *Linguistic variables and linguistic values*

Suppose that X= "age." Then we can define fuzzy sets "young," "middle aged" and "old" that are characterized by MF's $\mu_{old}(x), \mu_{middleaged}(x)$ and $\mu_{old}(x)$ respectively. Just as a variable can assume various values, a linguistic variable "Age" can assume different linguistic values, such as "young" "middle aged" and "old" in this case. If "age" assumes the value of "young" then we have the expression "age is young" and so forth for other values. Typical *MF*'s for these linguistic values are displayed in Figure 2.2, where the universe of discourse X is totally covered by the *MF*'s and the transition from one *MF* to another is smooth and gradual.

Some of the most important concept of fuzzy sets are the concept of the support of a fuzzy set A of all points x in *X* such that $\mu_A(x) \rangle 0$ in Eq. (2. 4),

$$\text{support(A)} = \{x : \mu_A(x) \rangle 0\}, \tag{2.4}$$

the core of a fuzzy set A is the set of all points x in X such that $\mu_A(x) = 1$ in Eq.(2.5),

$$core(A) = \{x : \mu_A(x) = 1\}. \tag{2.5}$$

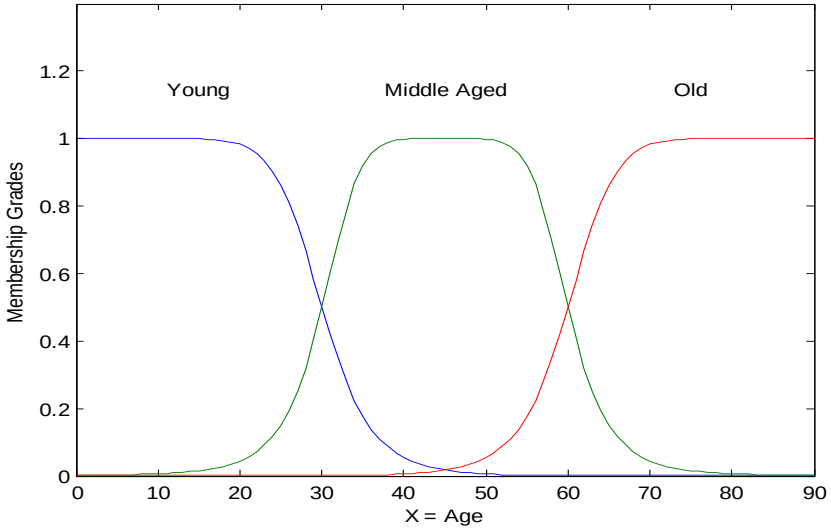


Figure 2.2 Typical MF's of linguistic values "young" "middle aged" and "old."

A fuzzy set A is normal if its core nonempty. In other words, we can always find a point $x \in X$ such that $\mu_A(x) = 1$.

A crossover point of a fuzzy set A is a point $x \in X$ at which $\mu_A(x) = 0.5$ in Eq. (2.6),

$$crossower(A) = \{x : \mu_A(x) = 0.5\}. \tag{2.6}$$

A fuzzy set whose support is a single point $x \in X$ with $\mu_A(x) = 1$ is called fuzzy singleton (Jang et al., 1997).

Other concepts of fuzzy set are concepts of support, the α-cut or α-level set of a fuzzy set A is a crisp set defined in Eq. (2.7).

$$A_\alpha = \{x : \mu_A(x) \geq \alpha\} \tag{2.7}$$

Strong α-cut or strong α-level set are defined similarly in Eq. (2. 8).

$$A'_{\alpha} = \{x : \mu_A(x) \rangle \alpha\} \tag{2.8}$$

Using the notation for a level set, we can express the support and core of a fuzzy set A as

$$support(A) = A'_0$$

$$core(A) = A_1$$

respectively.

2.2 Set -Theoretic Operations

Union, intersection and complement are the most basic operations on classical sets. Corresponding to the ordinary set operations of union, intersection and complement, fuzzy sets have similar operations, which were initially defined in Zadeh's seminal paper (Zadeh, 1965).

If set A is a subset of B in classical approach, it is denoted by $A \subseteq B$. Set A and B are fuzzy sets, and A is a subset of B if and only if $\mu_A(x) \leq \mu_B(x)$ in Eq. (2.9) or B is a subset of A in Eq. (2.10),

$$A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x) \tag{2.9}$$

$$A \supseteq B \Leftrightarrow \mu_A(x) \geq \mu_B(x) \tag{2.10}$$

The union and intersection of two fuzzy sets A and B are a fuzzy set C_1, C_2 respectively, written as $C_1 = A \cup B$, $C_2 = A \cap B$ whose MFs are related to those A and B in Eq. (2.1) and (2.2).

$$\mu_{C_1}(x) = \max(\mu_A(x), \mu_B(x)) = \mu_A(x) \vee \mu_B(x) \tag{2.11}$$

$$\mu_{C_2}(x) = \min(\mu_A(x), \mu_B(x)) = \mu_A(x) \wedge \mu_B(x) \tag{2.12}$$

The complement of fuzzy set *A*, denoted by $\overline{A}$ ($\overline{A}$, NOT A), is defined as below (Lee, 2005)

$$\mu_{\overline{A}}(x) = 1 - \mu_A(x) .$$

The Yager's complement function and operation. The well-known complement operators in Eq. (2. 13) (Baykal, 2004).

$$C_w(a) = (1 - a^w)^{\frac{1}{w}} \text{ and } \mu_{\overline{A}}(x) = (1 - \mu_A(x)^w)^{\frac{1}{w}} \tag{2.13}$$

A and B be fuzzy sets in X and Y, respectively. The Cartesian product of A and B, denoted by AxB, is a fuzzy set in the product space XxY with the membership function,

$$\mu_{AxB}(x, y) = \min(\mu_A(x), \mu_B(y)) .$$

2.3 Membership Function Formulation and Parametrization

A triangular MF and trapezoidal MF are best known and both of them have been used extensively, especially in real time implementation. They can be formulated in Eq. (2.14) and Eq. (2.15) as (Jang et al., 1997)

$$triangler(x, a, b, c) = \begin{cases} 0, & x\langle a \\ \dfrac{x - a}{b - a}, & a \le x \le b \\ \dfrac{c - x}{c - b}, & b \le x \le c \\ 0, & c\langle x \end{cases} \tag{2.14}$$

where the parameters a, b, c ($a < b < c$) determine the x coordinates of the three corners of the underlying triangular MF.

$$trapezoid(x,a,b,c,d) = \begin{cases} 0, & x\langle a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \dfrac{d-x}{d-c}, & c \le x \le d \\ 0, & d\langle x \end{cases} \tag{2.15}$$

where the parameters a, b, c, d ($a < b < c < d$) determine the x coordinates of the three corners of the underlying trapezoidal MF in Eq. (2.15). Both triangular and trapezoidal MFs are composed of straight line segments; they are not smooth at the corner points specified by the parameters. In the following, other types of MFs are defined by smooth and nonlinear functions such as Gaussian, Generalized bell etc. A Gaussian MF is specified by two parameters $\{c,\sigma\}$; c represent the MFs center and σ determines the MFs width as in Eq. (2. 16).

$$gaussian(x,c,\sigma) = e^{-\frac{1}{2}(\frac{x-c}{\sigma})^2} \tag{2. 16}$$

A generalized bell is specified by three parameters $\{a, b, c\}$ as in Eq. (2. 17)

$$bell(x,a,b,c) = \frac{1}{1+|\frac{x-c}{a}|^{2b}} \tag{2. 17}$$

where the parameter b is usually positive. Center is adjusted by c, width of the MF is changed by a, and b is used to control to slopes at the crossover points in Figure 2.4. Because of these two MFs, Gaussian and generalized bell, are becoming increasingly popular for specifying fuzzy sets. Gaussian functions are well known in probability and statistics (Jang et al., 1997).

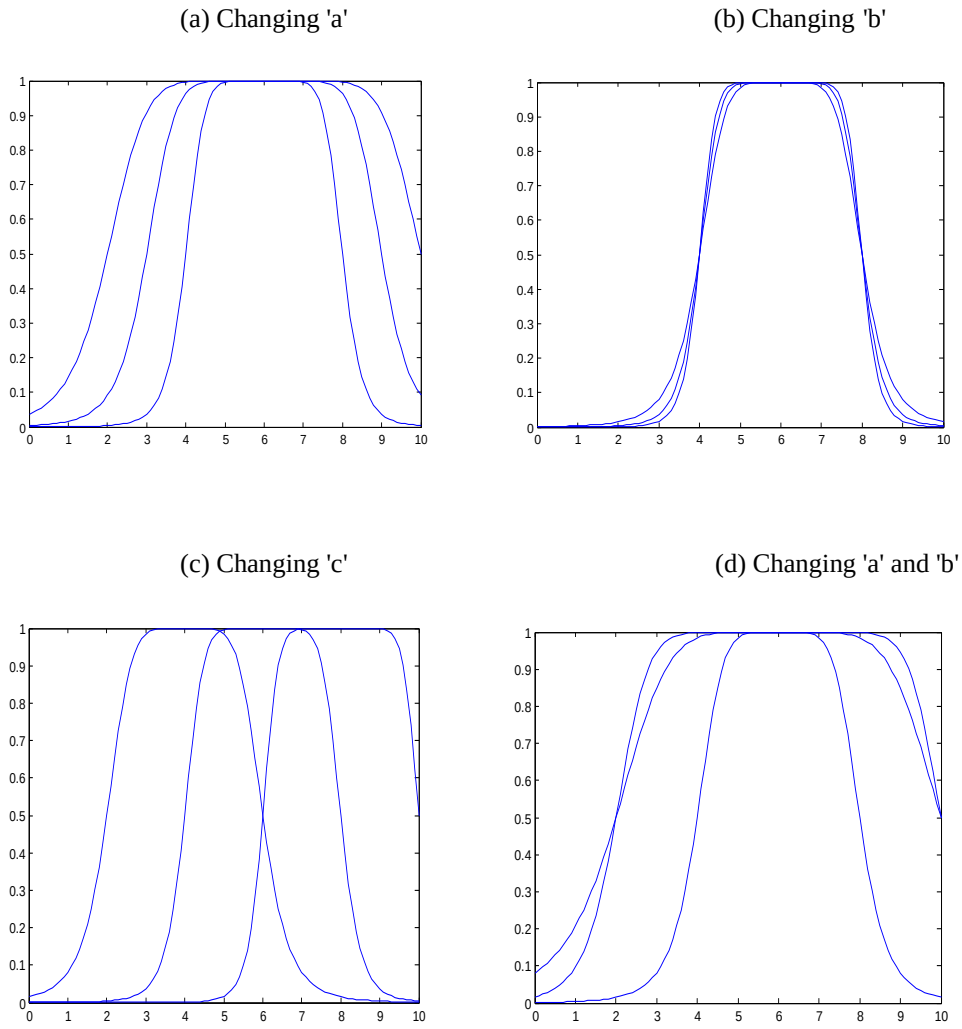


Figure 2.4 The effects of changing parameters in bell MFs: (a) changing parameters *a*; (b) changing parameters *b*; (c) changing parameters *c*; (d) changing *a* and *b* simultaneously but keeping their ratio constant.

Since the fuzzy entropy will be used in the study, firstly the fuzzy logic was summarized in this chapter. The next chapter will discuss the theoretical background of the concept of entropy.

CHAPTER THREE

ENTROPY

Entropy is the measure of the uncertainty of a system. In defining entropy, thermodynamics, statistical physics theory and the information theory definitions are the most widely used ones. In this study, the definitions other than the information theory definition are discarded since this study defines entropy in terms of information theory.

3.1 Defining Entropy

Entropy is a value which increases as the level of irregularity increases. Therefore it is directly proportional to irregularity. Let us explain entropy with an example: It can be said that the entropy of the earth increases every day. The earth has been losing its order since the first day of its formation and the possibility for the earth to return the order in the first day decreases day by day. Thus, since the direction of time is considered as the direction of the increase in entropy, we can replace information with entropy. We perceive the direction of time flow as the decrease direction of the information in the system. In other words, as the probability decreases or the irregularity increases the information will decrease.

The information required for all the possible states of a system to be known equals to the entropy of that system. The transition of a system from an ordered, organized and planned structure to an unordered, disorganized and unplanned state increases the entropy of that system. Therefore, information is considered as the reverse-entropy. One may wonder what kind of relation entropy, as a concept mostly encountered in thermodynamics, has with information theory. This relation is not an intuitional one, but it depends wholly on mathematical proofs. However, the two kinds of entropy have some differences. A chemist or a refrigeration engineer divides energy to temperature to express the thermodynamic entropy. However, for a communications engineer using the Shannon entropy, entropy is in the form of bits, and more importantly it is without any dimensions. The difference emerges from a

transformation. If, in a system, randomness is at the maximum or the probabilities of the messages at the information source for selection are equal, the entropy of that information source is at maximum.

In Shannon's theory information is equaled to fuzziness. Information theory deals with communication and is a completely statistical theory. C. E. Shannon's (1965) work titled "A Mathematical Theory of Communication" is considered as the origin of the theory. According to Shannon the information source is a person or a device that produces messages, with a statistical feature. Information is measured in terms of unpredictability or information value for the receiver. As it was mentioned before, information removes the uncertainty the receiver experiences. According to the theory, the emergence of a situation (creation of a message) of p probability at the source, is an information formation of $\log_2 \frac{1}{p}$. Here the reason for logarithmic base 2 is related to the selection of the unit. If we are to define entropy; it is the average amount of information (Cover and Thomas, 2006).

The information entropy of the X discrete random variable, which can obtain the values $\{x_1...x_n\}$ is calculated with the equation in Eq. (3. 1).

$$H(X) = -\sum_{i=1}^{n} p(x_i)\log_2 p(x_i) \tag{3.1}$$

If X random variable is a continuous variable, entropy is calculated with Eq. (3.2) (Shannon, 1948).

$$H(X) = -\int_{-\infty}^{+\infty} f(x_i)\log_2 f(x_i)dx \tag{3.2}$$

This examines the entropy calculations for discrete and continuous random variables with one example for each.

An individual is asked to think of a number between 1 and 16. The example tries to obtain the correct answer by asking yes/no questions (2-tuple answer) If the number of question attempts to reach the result is calculated with Shannon Eq. (3. 1);

$$H(X) = -\sum_{i=1}^{16} p(x_i)\log_2 p(x_i)$$

$$H(X) = -\sum_{i=1}^{16} \frac{1}{16}\log_2(1/16) + \frac{1}{16}\log_2(1/16).....+\frac{1}{16}\log_2(1/16)$$

$$= -16(\frac{1}{16}\log_2 2^{-4})$$

$$= 4 \text{ bit}.$$

This means that the answer can be found by asking an average of 4 questions.

The easiest logarithmic value in the calculations is generally the $\log_2$ (p) value. Therefore, this base is preferred mostly in calculating entropy and inofrmation value in the literature.

The four inferences below could be made about entropy as per the probability of different conditions;

First, any information cannot emerge with the realization of a condition with an emergence probability of 1. Under these circumstances entropy value is 0. For instance, when a loaded dice with 4 on each faces is tossed, it is known that number that 4 will come up each toss. This toss does not change our knowledge.

The second inference is that when the possible states of a system increase, its entropy value also increases.

Thirdly, emergence of a least probable state creates more information compared to the more probable one. For instance, instead of knowing that the sides of a coin

after a flip will be heads or tails, estimating the 6 numbers in a lottery draw will include vast amounts of information.

As the fourth inference, it can be concluded that the estimation of the results gets harder as the entropy of a system increases. The power of estimation will decrease as the fuzziness increases.

As another example, when X ~U(a,b),

$$H_{\Delta x}(X) = -\int_{D_x} f(x)\ln f(x)dx - \ln\Delta x = -\int_a^b \frac{1}{b-a}\ln\frac{1}{b-a}dx - \ln\Delta x = \ln\frac{b-a}{\Delta x}.$$

Similarly, if X ~N(0, σ_X^2) then,

$$H_{\Delta x}(X) = -\int_{D_x} f(x)\ln f(x)dx - \ln\Delta x = -\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_X} e^{-\frac{x^2}{2\sigma_X^2}} \ln\left(\frac{1}{\sqrt{2\pi}\sigma_X} e^{-\frac{x^2}{2\sigma_X^2}}\right)dx - \ln\Delta x$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma_X^2}} \frac{x^2}{2\sigma_X^2}dx + \ln\left(\sqrt{2\pi}\sigma_X\right) - \ln\Delta x$$

$$= \frac{1}{2} + \ln\left(\sqrt{2\pi}\sigma_X\right) - \ln\Delta x \quad .$$

Entropy has six features presented below:

1) Nonstorage
2) Static
3) Statistically independent
4) Continuous
5) Symmetrical
6) Summable.

3.2 Entropy Types

Different entropies can be calculated depending on the situations at hand. In this section some of the entropy types will be explained.

3.2.1 Joint Entropy

If the X discrete random variable with the values $\{x_1,.....,x_n\}$ and the Y discrete random variable with the values $\{y_1,.....,y_m\}$ have a jointed probability distribution $p(X=x_i, Y=y_j)$, the joint entropy of the X and Y random variables is calculated as in in Eq. (3.3).

$$H(X,Y) = -\sum_j \sum_i p(X=x_i, Y=y_j) \log p(X=x_i, Y=y) \quad (3.3)$$

When the random variables are continuous the joint entropy value is calculated as in Eq. (3. 4).

$$H(X, Y) = -\int_j \int_i f(x_i, y_j) \log f(x_i, y_j) dxdy \quad (3.4)$$

Joint entropy is also called the common information measure.

3.2.2 Conditional Entropy

Let X and Y are random variables with combined probability distribution. The measurement of fuzziness in X variable, when the values of Y are given, is the conditional entropy dependent on Y variable. Knowing Y always decreases the fuzziness of X. It is shown as H(X|Y). In calculating the conditional entropy of X depending on Y the Eq. (3.5) is used for discrete variables, and Eq. (3.6) is used for continuous variables (Oruç et al., 2009).

$$\mathbf{H(X|Y)=} \sum\sum p(x_i|y_j).\log\left(p(x_i|y_j)\right) \tag{3.5}$$

$$\mathbf{H(X|Y)=} \int\int f(x_i|y_j).\log\left(f(x_i|y_j)\right)dxdy \tag{3.6}$$

3.2.3 Relative Entropy

Relative entropy is a measure that shows the similarity between two probability distributions. In the literature it is also known as the Kullback-Leibler distance and is shown as D(p||q). It is calculated in Eq (3.7) for discrete variables and in Eq. (3. 8) for continuous variables.

$$D(p||q)= \sum_{x\in X} p(x)\log\frac{p(x)}{q(x)} \tag{3.7}$$

$$D(p||q)= \int_{x\in X} f(x)\log\frac{f(x)}{g(x)}dx \tag{3.8}$$

If the distributions examined are similar, the difference between D(p||q) and D(q||p) is smaller (Cover, 2006).

This chapter covered the theoretical framework for entropy, since fuzzy entropy will be used in the study. The next chapter will touch upon how fuzzy logic and entropy will be used together and it will explain the fuzzy entropy methods as a means to calculate the degree of fuzziness.

CHAPTER FOUR

FUZZY ENTROPY

Fuzziness is one of the universal attributes of human thinking and objective things. Fuzzy set theory is one of the efficient means of researching and processing fuzzy phenomena in real world. It is the fuzzy sets that may describe fuzzy objects effectively so that fuzzy sets play more and more important functions in system modeling and system design. Therefore, the quantitative analysis of fuzziness in a fuzzy set is an important problem. Entropy is an important concept in Shannon information, in which entropy is a kind of measurement for describing the degree of no restrain of stochastic vectors. Fuzzy set theory makes use of entropy to measure the degree of fuzziness in a fuzzy set, which is called fuzzy entropy. Fuzzy entropy is the measurement of fuzziness in a fuzzy set, and thus has especial important position in fuzzy systems, such as fuzzy pattern recognition systems, fuzzy neural network systems, fuzzy knowledge base systems, fuzzy decision making systems, fuzzy control systems and fuzzy management information systems (Min, 1998).

The Shannon's entropy and the distance between fuzzy sets methods are used to measure the fuzziness of the fuzzy sets. The most widely used methods Fuzzy entropy and generalized fuzzy entropy are elaborated below.

4.1 Fuzzy Entropy

According to information theory the entropy of a system is a measure of the amount of information of the system.

x_i, i= (1……N) be the possible outputs from source A with the probability $P(x_i)$. Entropy is defined as in Eq. (4.1),

$$H_{nonfuzzy}(A,P) = -\sum_{i=1}^{N} P(x_i)\log P(x_i) \tag{4.1}$$

where, $\sum_{i=1}^{N} P(x_i) = 1$.

The subscript "nonfuzzy" is used to distinguish from the fuzzy entropy. Large entropy implies a larger amount of information. Using the Lagrange multiplier method is defined as in Eq. (4.2).

$$F(A) = -\sum_{i=1}^{N} P(x_i) \log P(x_i) + \lambda(\sum_{i=1}^{N} P(x_i) - 1) \tag{4.2}$$

The maximum of $H_{nonfuzzy}(A, P)$ can be found in Eq. (4.3).

$$\frac{\partial F(A)}{\partial P(x_i)} = -\log P(x_i) - 1 + \lambda = 0 \tag{4.3}$$

$P(x_i) = e^{\lambda - 1}$ for $i = (1...N)$ is obtained From in Eq. (4.3). $H_{nonfuzzy}(A, P)$ will reach the maximum when $P(x_1) = P(x_2) = ... = P(x_N) = \frac{1}{N}$ of set an data can be viewed as information source *A* with the intensities as the possible outputs. The histogram distribution can be viewed as probability $P(x_i)$ in fact Eq. (4.1) describes the entropy in the ordinary domain. What measure of fuzziness need is entropy that can measure the amount of information for set in the fuzzy domain (Chen, 1997).

4.1.1 Fuzzy Entropy Based on Shannon Function

Many definitions of fuzzy entropy are studied. Most of them incorporate the membership μ_A with the probabilities *P* in the measure of fuzzy entropy. In order to measure the information amount of a fuzzy set. Zadeh suggests a definition about the entropy of a fuzzy set that takes both probabilities and memberships of elements into consideration. *A* be a fuzzy set with membership function μ_A are the possible outputs from source *A* with the probability $P(x_i)$ in Eq. (4.4).

$$H_{fuzzy}(A) = -\sum_{i=1}^{N} \mu_A(x_i) P(x_i) \log P(x_i) \tag{4.4}$$

The difference between in Eq. (4.1) and in Eq. (4.4) is the term $\mu_A(x_i)$ which serves as a weighted multiplier in Eq. (4.4). Therefore Zadeh's fuzzy entropy in Eq. (4.4) is also called weighted entropy.

Kaufman defined the entropy of a fuzzy set *A* as in Eq. (4.5),

$$H(\vartheta_A(x_1), \vartheta_A(x_2), ..., \vartheta_A(x_N)) = \frac{-1}{\ln(N)} \sum_{i=1}^{N} \vartheta_A(x_i) \ln(\mu_A(x_i)) \tag{4.5}$$

where, $\vartheta_A(x_i)$ is defined as $\vartheta_A(x_i) = \dfrac{\mu_A(x_i)}{\sum_{i=1}^{N} \mu_A(x_i)}$.

It took the probabilities of membership $\vartheta_A(x_i)$ in the fuzzy domain instead of the probabilities of elements $P(x_i)$ in ordinary domain for the calculation of fuzzy entropy (Chen, 1997).

Deluca and Termini proposed a quite different definition of the entropy of a fuzzy set A. This entropy based on Shannon function has a no probabilistic feature and is defined as in Eq. (4.6),

$$H(A) = \frac{1}{N \log 2} \sum_{i=1}^{N} S_n(\mu_A(x_i)) \tag{4.6}$$

where S_n is the Shannon's function for fuzzy set as in Eq. (4.7).

$$S_n(\mu_A(x_i)) = -\mu_A(x_i) \ln(\mu_A(x_i)) - (1 - \mu_A(x_i)) \ln(1 - \mu_A(x_i)) \tag{4.7}$$

4.1.2 Fuzzy Entropy Based on Distance

Support A is a fuzzy set on discourse $U = \{u_1, u_2, ..., u_n\}$, the membership vector of A is $A = (a_1, a_2, ..., a_n)^T$, here $a_i = \mu_A(u_i) \in [0,1]$, then the fuzzy entropy or classic fuzzy entropy based on distance of A as in Eq. (4. 8),

$$H_d(A) = \frac{2}{n}\sum_{i=1}^{n} | a_i - \mu_{A^0}(u_i) | \tag{4. 8}$$

where A^0 is a crisp set with minimal distance to A, the feature function of A^0 as in Eq. (4.9).

$$\mu_{A^0}(u_i) = \begin{cases} 0 & a_i \langle 0.5 \\ 1 & a_i \rangle 0.5 \end{cases} \tag{4. 9}$$

Fuzzy entropies have following properties:

1) Nonnegative; $H(A) \geq 0$
2) Certainty; if A crisp set, then $H(A) = 0$
3) Maximally; when $a_i = \frac{1}{2}$ $\forall$i $H(A)$ maximize and $H(A) = 1$
4) Symmetry; $H(A) = H(A^c)$, then here A^c is a complement set of A, and then $\mu_{A^c}(u) = 1 - \mu_A(u)$
5) If when $H(A) \geq \frac{1}{2}, \mu_B(u) \geq \mu_A(u)$, and when $H(A) \leq \frac{1}{2}, \mu_B(u) \leq \mu_A(u)$ for $\forall u$, then $H(A) \geq H(B)$ (Min and Sen, 1998).

4.2 Generalized Fuzzy Entropy

Although fuzzy entropy based on Shannon function and fuzzy entropy based on distance have different formats, they have some same features. In order to apply

fuzzy entropy effectively in fuzzy systems, a kind of generalized format of fuzzy entropy is presented as in Eq. (4.10),

$$H(A) = \sum_{i=1}^{n} w_i h(a_i) \tag{4.10}$$

where, $\sum_{i=1}^{n} w_i = 1$.

It can be seen that fuzzy entropy is practically the weighting sum of some nonlinear functions of membership vector of a fuzzy set. Different weighting coefficients w_i, and different function $h(a_i)$ may result in different computing formats for fuzzy entropy (Min and Sen, 1998).

4.2.1 Generalized Fuzzy Entropy Based on Shannon Function

Shannon entropy, among the fuzzy entropy types, is entropy defined dependently to the Shannon function. Generalized fuzzy entropy based Shannon function is defined as given in Eq. (4.11)

$$H(A) = w\sum_{i=1}^{n} h(\mu_A(x_i)) \tag{4.11}$$

where, Shannon function is $h(a_i) = -a_i \ln a_i - (1-a_i)\ln(1-a_i)$. Different weighting coefficients w, and different function $h(a_i)$ may result in different computing formats for fuzzy entropy.

4.2.2 Generalized Fuzzy Entropy Based on Distance

Support A is a fuzzy set on discourse $U = \{u_1, u_2, ..., u_n\}$, the membership vector of A is $A = (a_1, a_2, ..., a_n)^T$, here $a_i = \mu_A(u_i) \in [0,1]$, generalized fuzzy entropy based on distance of A as in Eq. (4.12)

$$H(A) = w\sum_{i=1}^{n} h(a_i) \tag{4.12}$$

where, $w = \frac{2}{n}$, A^0 is a common set with minimal distance to A in Eq. (4.9) and $h(a_i) = | a_i - \mu_{A^0}(u_i) |$.

Different weighting coefficients w, and different function $h(a_i)$ may result in different computing formats for fuzzy entropy.

It is necessary to note that from the generalized format of fuzzy entropy, in order to constructed practical and effective fuzzy system, the key task is to select appropriate weight coefficients and transform functions. In other words, the guidance of generalized fuzzy entropy, different entropy may be constructed for different problems, and thus better system characteristic may be obtained. For example, in constructing fuzzy learning system for forward neural networks and self-organizing feature mapping networks, kind of fuzzy entropy is presented and used to laming criterion, and thus obtain better learning effect (Min and Sen, 1998).

4.2.3 Generalized Fuzzy Entropy Based on Yager's Complementary Operator

$\mu_A(x)$, is the membership level of each x element of the fuzzy subset A of universal set X. In case the membership level of each x element in set A is 0.5, the entropy of A takes its highest value. In this study *F(X)* defines the fuzzy sets

designated on the universal sets *X; P(X)* defines the crisp sets designated on the universal sets *X*.

The complementary operator c(x) =1-x has only a transition point of value x=0.5. This operator is the most widely used operator in fuzzy sets and their applications. The rules of the generalized fuzzy entropy's complementary operator

c:[0,1] $\rightarrow$[0,1] can be listed as below:

1) c (0)=1 and c(1)=0,
2) for $\forall$ a, b $\in$[0, 1]; if a $\leq$ b then c(a) $\geq$c(b),
3) c is a continuous function,
4) c (c(x))=x.

The complementary function defined by Yager is given in Eq. (4.13) in case of 0<w<1.

$$c_w(x) = (1 - x^w)^{1/w} \tag{4.13}$$

In case of w=1, Yager's complementary operator becomes the fuzzy entropy operator.Yager's complementary operator, in case of m$\in$(0,1), has only one transition point $m = \frac{1}{\sqrt[w]{2}}$. The generalized fuzzy entropy function, $e_m = F(x) \rightarrow R^+$ has four conditions:

1) if $e_m(A) = 0$ then $A \in P(x)$.
2) if A=[m] then $e_m(A)$ takes the greatest value.
3) if A^*, is a sharpening set of set A, in case $\mu_A(x) \geq m$ it becomes $\mu_{A^*}(x) \geq \mu_A(x)$, and in case $\mu_A(x) \leq m$ it becomes $\mu_{A^*}(x) \leq \mu_A(x)$.
4) $e_m(A^{c_m}) = e_m(A)$.

For a fuzzy set A defined in a limited set X, Kohen entropy given in Eq. (4.14), Kasko entropy in Eq. (4.15), Tanimoto entropy in Eq. (4.16) and Yager's entropy in Eq. (4.17) is used in generalized fuzzy entropy computations (Kasko, 1986).

$$e(A^c) = \frac{M(A \cap A^{c_m})}{M(A \cup A^{c_m})} \tag{4.14}$$

$$e(A^c) = \sum_{i=1}^{n} \frac{\min(\mu_A(x), \mu_B(x))}{\max(\mu_A(x), \mu_B(x))} \tag{4.15}$$

$$e(A^c) = M(A \cap A^{c_m}) \tag{4.16}$$

$$e(A^c) = 1 - \frac{M(A \cup A^{c_m}) - M(A \cap A^{c_m})}{n} \tag{4.17}$$

Here it is defined as $M(A) = \sum_{i=1}^{n} \mu_A(x_i)$.

The next section, the applications of fuzzy entropy will be explained

CHAPTER FIVE

APPLICATIONS

In this chapter, image processing via fuzzy entropy methods will be primarily discussed. This chapter includes the cell count method and image denoising applications performed with these methods, and the results of these applications.

5.1 Image Processing With Fuzzy Entropy

Gonzales and Woods (2001) define image (or picture) as a representation of an object. As for image processing, it is the body of techniques to obtain an image out of a particular image. The image processing techniques are used to enable humans or computers to perceive or interpret an image. Image processing has been developed to solve three main problem pertaining images. These are;

- digitalizing, encoding and storing an image to ease data transfer,
- image enrichment and enhancement,
- and, image segmentation which is the first step of machine reading

(Gonzales and Woods, 2008).

Q is the set of gray pixels, q_{xy} is the pixel value of the terms (x, y) of set Q. The brightness value of term (x, y) in pixel set Q an example in Figure 5.1, is expressed as $\mu_Q(q_{xy})$ in fuzzy set.

$$Q = \left\{ q_{xy}, \mu_Q(q_{xy}) \right\} \quad , \; 0 \le q_{xy} \le L-1 \text{ and } \quad 0 \le \mu_Q(q_{xy}) \le 1 .$$

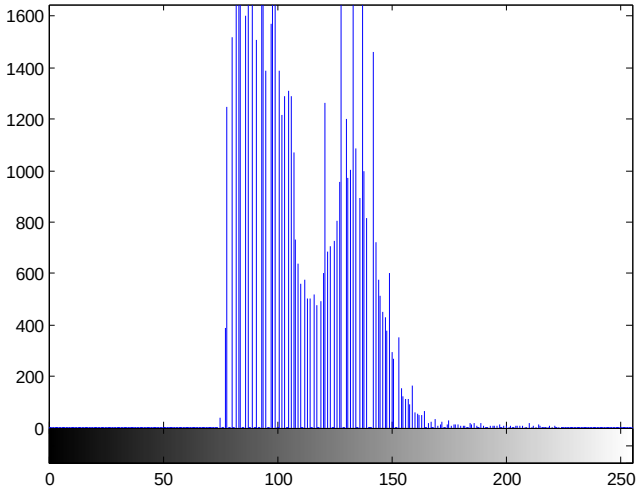


Figure 5.1 An example of set Q histogram

In this study, the fuzzy sets of the gray levels are generated using the S-function given in Eq. (5. 1).

$$S(x;a,b,c)=\begin{cases}0, & x \le a \\ \dfrac{(x-a)^2}{(b-a)(c-a)}, & a \le x \le b \\ 1-\dfrac{(x-c)^2}{(c-b)(c-a)}, & b \le x \le c \\ 1, & x \ge c\end{cases} \qquad (5.\ 1)$$

Here the values x are the elements of gray pixel values set. The shape parameters of the S-function is defined as a, b and c. b value is a point between a and c. The fuzzy set defined using the S-function, reaches its greatest value with the pixel with a value of 0.5. While in fuzzy entropy, the maximum entropy is achieved via the pixel with membership level of 0.5; in generalized entropy the maximum entropy can be achieved via different membership levels (Pal et al., 2000).

Using fuzzy entropy method, the segmentation pixel value is given as below in Eq. (5. 2) and Eq. (5.4).

$$g(x,y)=\begin{cases}0 & q(x,y)\le T\\ 1 & q(x,y)\ge T\end{cases} \tag{5.2}$$

Here, if $\mu_Q(t)=0.5$ then T=t.

The segmentation method depending on generalized fuzzy entropy is represented as m value depending on Yager complementary operator in Eq. (4.13).

In Eq.(5.2), if $\mu_Q(t)=m$ then *T=t*. *T=t* is the best segmentation pixel value. For the best segmentation, the complementary operator *w=1* of Yager's complementary function is computed as *c(x)=1-x*. Here, when *0<m<1* and *0<w<∞* , there is a cohesion of $c_m=(1-x^{-\frac{1}{\log_2 m}})^{-\log_2 m}$ in Eq. (4. 13) is found between the parameters *m* and *w* as $w=-\dfrac{1}{\log_2 m}$.

In image segmentation process using Shannon entropy depending on t pixel segmentation value is formulated in Eq. (5.3).

$$H(t)=-\sum_{i=0}^{t}\frac{p_i}{p(t)}\log_2\frac{p_i}{p(t)}-\sum_{i=t+1}^{L-1}\frac{p_i}{1-p(t)}\log_2\frac{p_i}{1-p(t)} \tag{5.3}$$

Here p_i value is defined as the probability of i. pixel to exist and, $p(t)=\sum_{i=1}^{t}p_i$. The best segmentation t value is found using the mathematical method defined in Eq. (5. 4).

$$t=Arg\{\max(H(t))\} \tag{5.4}$$

In this study, in the generalized fuzzy entropy computations, Yager's complementary operator is used. Kohen entropy given in Eq. (4. 14), Kasko entropy

in Eq. (4. 15), Tanimoto entropy in Eq. (4. 16) and Yager's entropy in Eq. (4. 17) is used in generalized fuzzy entropy computations for image processing (Yeniyayla and Kuruoğlu, 2010-I).

5.2 Application I: Cell Count

Obtaining the requested areas in the image more clearly can be administered via the best segmentation value obtained using the image processing techniques summarized above. The unwanted areas in the image are segmented as background and removed. Through this process working on the required areas of the image becomes easier. As the first application, the most appropriate segmentation values are obtained from the cell images processed by the Shannon entropy and generalized fuzzy entropy method, discussed above. The segmentation and cell count process for the segmented cell images are explained in steps. Cell count process using generalized fuzzy entropy and Shannon entropy methods are performed in six stages.

First one of these is obtaining good images of cells. Histopathology is an area that examines the microscopy of the lesions which develop in the natural structure of tissues and organs. The histopathologic images are taken from the Dokuz Eylül University, Department of Electronic Engineering laboratories. The resolution values of the images are 2288x1712 pixels at 72 dpi.'s. The resolution being high enables the segmentation process to be performed more successfully. All the operations during the application are performed with program in Appendix 1 using MATLAB 7.1.

As for the second phase, it is the conversion of color images to the grayscale mode. The color images are converted into grayscale levels using different image processing techniques.

In the third phase, the noise reduction process is applied, if there is noise on the image, thus providing a sharper image.

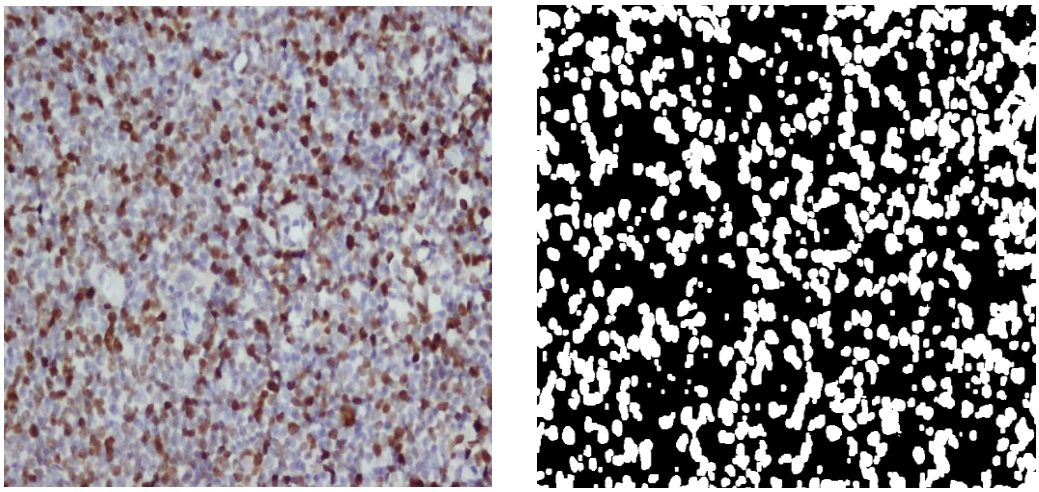
The fourth phase is about finding the best segmentation value. It is the most important and comprehensive phase in this study. First, the matrix of the pixel values of the grayscale image is obtained. Each term of the matrix showing the obtained pixel values is blurred using the S-function in Eq. (5. 1). In blurring process the values of (a, b, c), which are the parameters of S-function give the pixel values; a is the minimum pixel value, c is the maximum pixel value and b is the midpoint value between a and c. The complement set of the fuzzy set is obtained using Yager's complementary operator in Eq. (4. 13). In order to find the segmentation value in fuzzy entropy, the pixel with membership level of 0.5 is taken as the segmentation value. However, this method seems weak compared to generalized fuzzy entropy method. Instead, the m value, of Yager's complementary function, corresponding highest entropy level between 0 and 1 is computed. Later, the pixel value corresponding to the m value is found. This is the most appropriate segmentation value t in Eq. (5. 3). Thus, the images of cells are segmented and converted into black and white images (Yeniyayla and Kuruoğlu, 2010-II).

Similarly, in segmentation with Shannon entropy, first the probability values of the gray pixels are computed and the best t segmentation value is found using the formulas given in Eq. (5. 4) and (5. 5).

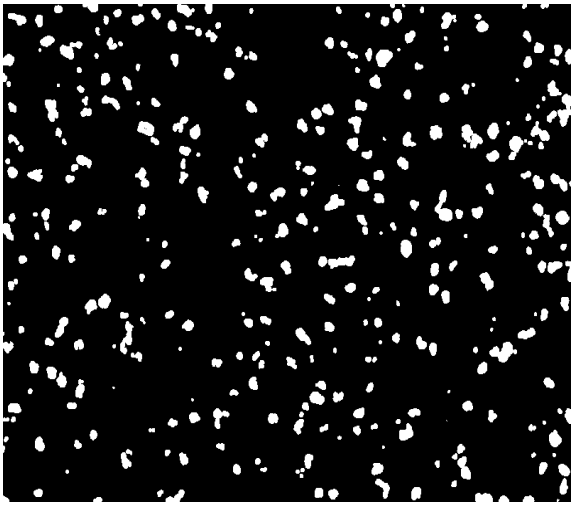
The fifth phase is the closing phase. With this process, the cells in the segmented images are made more prominent. However, there may be some unwanted traces in the images as well as distortions around the cell boundaries. With the closing phase, these unwanted traces are removed and the cell boundaries are made more oval.

The sixth and the last phase is the cell count phase. With the counting of independent segments, the number of the cells is found.

The shapes of the applications performed using generalized fuzzy entropy and Shannon entropy are given comparatively in Figures 2, 3 and 4 .

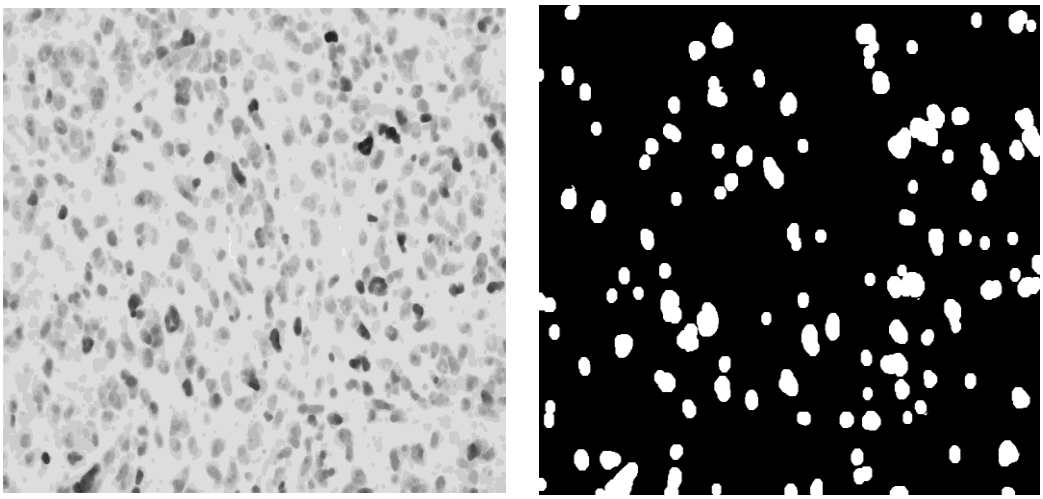


a) b)

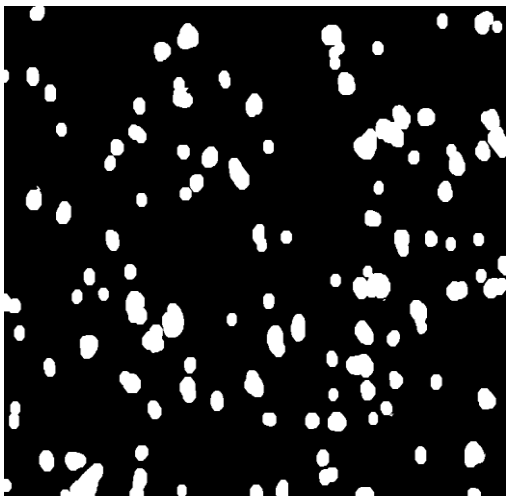


c)

Figure 5.2 a) Colored tissue cells, b) Cell image, segmented according to Shannon entropy, c) Cell image, segmented according to generalized fuzzy entropy (Image no: 1).

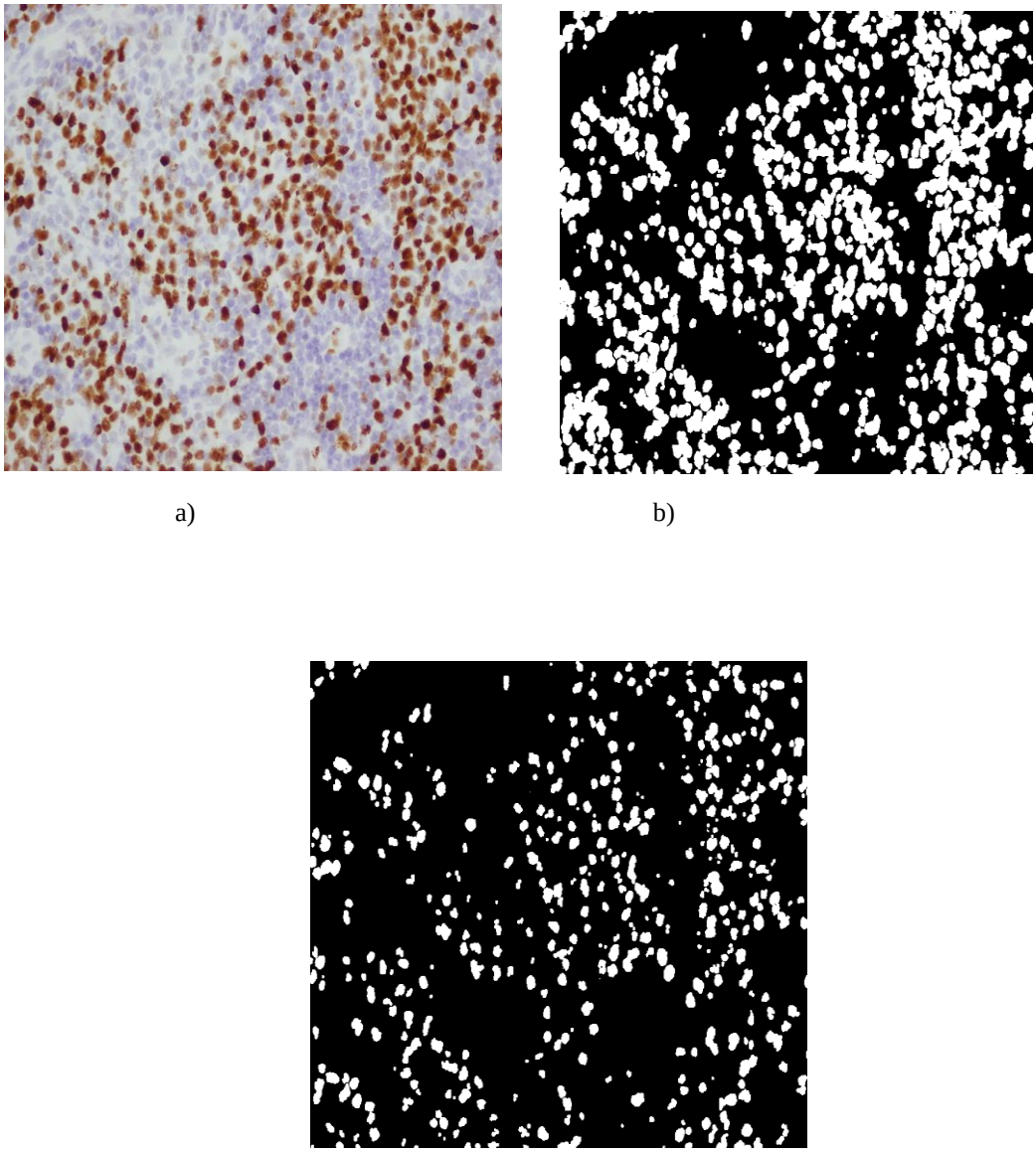


a) b)



c)

Figure 5.3 a) Grayscale bacteria image, b) Image segmented according to Shannon entropy, c) Image segmented according to generalized fuzzy entropy (Image no:2).



a) b)

c)

Figure 5.4 a) Color tissue cell, b) Cell image, segmented according to Shannon entropy, c) Cell image, segmented according to generalized fuzzy entropy (Image no: 3).

Table 5.1 Values computed using Shannon entropy and generalized fuzzy entropy.

Image no	Obtained by doctors	Shannon Entropy		Generalized Fuzzy Entropy	
		Segmentation pixel value	Cell count	Segmentation pixel value	Cell count
1	350	109	354	106	374
2	55	101	57	105	63
3	370	106	370	130	352

According to the values given in Table 5.1, the same values are obtained in both methods for image no:1. For other images, different values are obtained. Results 95% similar to the doctors' results, performed by manual count, are obtained. It is known that manual count processes are quite exhausting and time consuming. Also, the generalization of the results, obtained by counting a segment, to a cell image, which does not distribute homogenous, can yield erroneous outputs. While this study conducts the count process over the whole of the tissue, it is possible to obtain error-free results without generalizations. However, automatic count process, being unable to fully discriminate cells, which are adherent to each other in the images, can cause incomplete counting. In the following sections this problem will be tried to be avoided using image processing techniques and different algorithms.

5.2 Application II: Image Denoising

In this application a cost function is introduced by using the fuzzy entropy to choose a threshold value in image denoising problem. The results are explained with pilot this cost function on the some images. Images as fuzzy subsets of a plane with a membership degree of pixels are proportionate to their gray levels. The original, degraded, and restructured images as fuzzy sets *A*, *B*, and *C* are considered in

reference set *X*, where *X* is plan sheet. Image *B* was tried to be transformed into a denoised version of the image as *C* by an algorithm. This algorithm first finds the noised pixels then change them with mean of 8 neighbor pixels. However, there is a problem in choosing a threshold *t* as unexpected jumping of gray level in the algorithm to find the noised pixels. This threshold related to the image. A cost function is used to find the best threshold in every image (Yeniyayla and Kuruoğlu, 2011).

The cost function is based on distance of the denoised image *C* and original image *A.* In this study, fuzzy entropy of denoised image *C* is added to the cost function. This is necessary, because when the algorithm threshold value is decreased, denoised image is blurred, although its distance of original image is decreased.

A be a fuzzy version of original image and *C* be the denoised image, fuzzy entropy is intruded as the sum of distance between *A*, *C* and fuzzy entropy of *C*, because in addition to distance of original image and denoised image, it should be considered that entropy of denoised image as blurring due to change of noised pixels with mean of 8 neighbor pixels in Eq. (5.7) (Şen, 2004). Euclidian distance and Kaufmann's entropy are used to this goal.

C(A), the cost function of the fuzzy set A is given in Eq. (5.6); *D(A,C)*, the distance between the fuzzy sets a A and C depending on the Euclidian distance is given in Eq. (5.4); and the entropy, with respect to Yager entropy, of the fuzzy set *D(C)* is given in Eq. (5.5) (Lee, 2005).

$$C(A)=D(A,C)+D(C)$$

$$D(A,C)=\sqrt{\frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m}(\mu_A(x_i,x_j)-\mu_C(x_i,x_j))^2} \qquad (5.4)$$

$$D(C)=\sqrt{\frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m}\min(\mu_C(x_i,x_j),1-\mu_C(x_i,x_j))^2} \tag{5.5}$$

$$C(A)=\sqrt{\frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m}(\mu_A(x_i,x_j)-\mu_C(x_i,x_j))^2}+\sqrt{\frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m}\min(\mu_A(x_i,x_j),1-\mu_A(x_i,x_j))^2} \tag{5.6}$$

$$\mu_c(i,j)=1-\frac{1}{\dfrac{\sum_{m=i-1}^{i+1}\sum_{n=j-1}^{j+1}(\mu_B(m,n)-\mu_B(i,j))}{\max(\mu_B(m,n))}} \tag{5.7}$$

where,

$A=\{((x_i,x_j),\mu_A(x_i,x_j)):i=1,...,n \text{ and j=1,...,m}\}$ is the $m\times n$ pixels fuzzy original image,

$B=\{((x_i,x_j),\mu_B(x_i,x_j)):i=1,...,n \text{ and j=1,...,m}\}$ is the noised image, and

$C=\{((x_i,x_j),\mu_C(x_i,x_j)):i=1,...,n \text{ and j=1,...,m}\}$ is denoised image by *changed algorithm by mean of neighbors*.

$\mu_A(x_i,x_j)$ membership degrees is the elements of the fuzzy set A which was converted using S-function of the pixel matrices of the original image in Figure 5.5.

In this application it can be seen that the best segmented t value is obtained when the entropy value of the total cost function is minimum. In the application, first the noised B image is denoised by the mean of the 8 neighbor pixels in Eq. (5.7), and image C is obtained. Here the t value is used by 0.01 steps between 0.01 to 0.99 and 100 C images are obtained, and the entropy of the C(A) cost function for each image is calculated. The t segmentation value which corresponds to the lowest entropy of

the cost function is accepted as the best denoising value. And the C image corresponding to these values is obtained as the clearest image.

a). Original image b) Grayscale image

Figure 5.5 a) Original image b) Grayscale image

Figure 5.6 % 0.2 noised image

In figure 5.7 images, segmented for different t values, namely denoised images are presented

a) t=0.5 segmented image

b) t=0.4 segmented image

c) t=0.3 segmented image

d) t=0.2 segmented image

e) t=0.1 segmented image

f) t=0.07 segmented image

g) t=0.05 segmented image

h) t=0.03 segmented image

ı) t=0.02 segmented image

i) t=0.01 segmented image

j) t=0.0075 segmented image

k) t=0.007 segmented image

Figure 5.7 a) t=0.5 segmented image b) t=0.4 segmented image c) t=0.3 segmented image d) t=0.2 segmented image e) t=0.1 segmented image f) t=0.07 segmented image g) t=0.05 segmented image h) t=0.03 segmented image ı) t=0.02 segmented image i) t=0.01 segmented image j) t=0.0075 segmented image k) t=0.007 segmented image

According to the chart in Figure 5.8., the t segmentation value which corresponds to the minimum entropy value is approximately 0.03. The image corresponding to this value is presented in Figure 5.7-h. The denoised results are given for the images which correspond to the t values. It can be seen that there is a close correlation between these results and the change of entropy depending on the t value, in Figure 5.8. The minimum entropy value in the total cost function indicates the closest similarity between the original image and the denoised one. It can be seen that in Figure 5.7-a, which corresponds to the 0.5 t value with the highest entropy value in Figure 5.8, the noise is at the maximum.

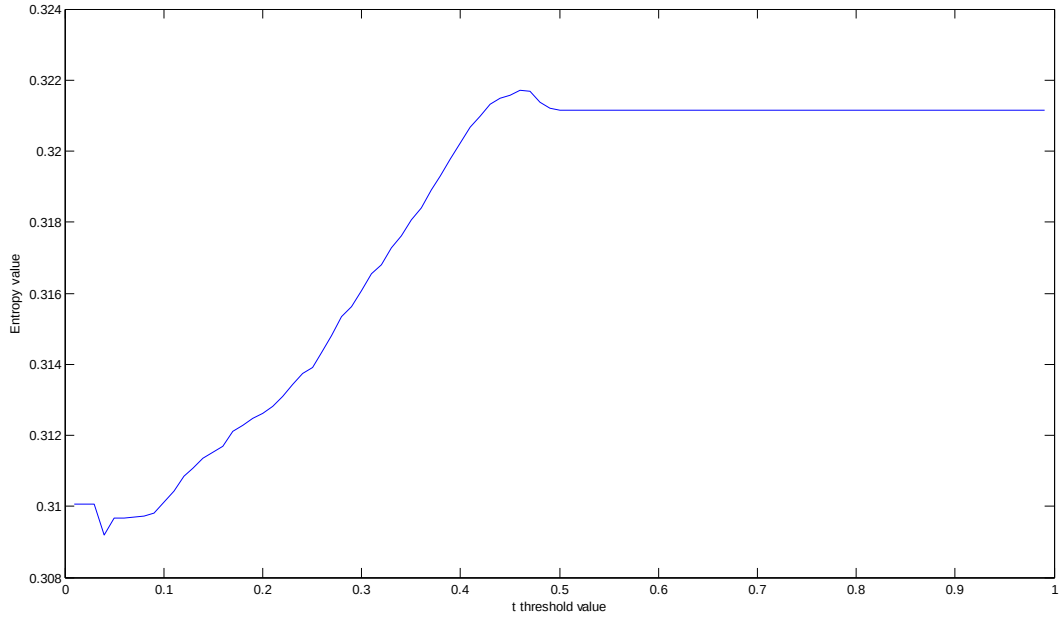


Figure 5.8 The chart of entropy depending on t value

There are numerous algorithms in denoising. In our study it can be seen that the cost function method, based on fuzzy entropy methods, yields better results. The use of fuzzy entropy method in various areas of research and its yielding better results compared to well known methods are referenced in scientific studies.

CHAPTER SIX

CONCLUSION

The results of an event or a scientific study cannot always be based on precise decisions in the universe we live in. Not everything we are aware of can be defined by mathematical concepts. The fuzzy logic approach, introduced by Zadeh (1965), scientifically expresses the ideas and perceptions of human beings and the events and objects in our universe. Calculation of the value, amount or, in other words, the weight of information about situations that do not have absolute boundaries or that cannot be mathematically categorized into any particular class makes the information scientifically significant. With this purpose, various fuzzy entropy methods have been developed. With the application of entropy in fuzzy sets, successful results are obtained in neural networks, machine learning, image processing and many other engineering areas. With the application in this study, cell count and image denoising processes related to image processing are performed.

In one of the applications in this study, the number of cells is counted using the generalized fuzzy entropy and Shannon's entropy. In one image, the same pixel values and cell number are obtained for both methods. The obtaining of different values in other images will be examined in the future. The types of images will be increased and the possibility of different results for different image types will be investigated.

Automatic cell count aims at providing a faster and error-free diagnosis process for physicians. However, in order to perform a more efficient segmentation, high resolution imaging devices should be used. It is considered that the cell count using generalized fuzzy entropy will make a contribution to the diagnosis of illnesses related to the cell number, such as anemia and hepatitis.

As the second application, the denoising process is performed using the generalized fuzzy entropy method. Cost function depending on the generalized fuzzy entropy is efficiently used in image denoising. In cases where the entropy value is lower, the best segmentation value is obtained. It is considered that the method developed here will yield better results in medicine and criminology.

It is aimed to contribute to the studies in the field of health by obtaining better results with this method in the clarification of images particularly such as MR, ECG and ultrasound.

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APPENDIX

Appendix 1. Program for Cell Count

```
clear all;
clc;

%Resmi Oku.
image=imread('yusuf2.tif');
figure,imshow(image)


se = strel('disk',5);
erodedBW = imerode(image,se);
 figure, imshow(erodedBW)

%Gray'e çevir.
gray_image=rgb2gray(erodedBW);

imshow(gray_image)
%pixval
impixelinfo
%Max,min pixeli bul.
max_pixel=max(max(gray_image));
min_pixel=min(min(gray_image));
mid_pixel=(max_pixel+min_pixel)/2;

%Resmin boyutları.
[row_number,column_number]=size(gray_image);
S_function=zeros(row_number,column_number);

%S fonksiyonu.
for row=1:row_number
    for column=1:column_number

        if gray_image(row,column)<=min_pixel
            S_function(row,column)=0;

        elseif min_pixel<gray_image(row,column) &&
gray_image(row,column)<=mid_pixel

S_function(row,column)=(((double(gray_image(row,column))-
double(min_pixel)))^2)/((double(mid_pixel-
min_pixel))*(double(max_pixel-min_pixel)));

        elseif mid_pixel<gray_image(row,column) &&
gray_image(row,column)<max_pixel
            S_function(row,column)=1-
((((double(gray_image(row,column))-
double(max_pixel)))^2)/((double(max_pixel-
mid_pixel))*(double(max_pixel-min_pixel))));

        elseif gray_image(row,column)>=max_pixel
```

```
            S_function(row,column)=1;

        end
    end
end

%Complement
complement=zeros(row_number,column_number);

%Entropi.
entropi=zeros(1,9);

%Min matrix ve Max matrix.
min_matrix=zeros(row_number,column_number);
max_matrix=zeros(row_number,column_number);

mum=1;
for m=0.1:0.1:0.9
    for row=1:row_number
        for column=1:column_number

            complement(row,column)=(1-(S_function(row,column))^(-
1/log2(m)))^(-log2(m));

min_matrix(row,column)=min(S_function(row,column),complement(row,c
olumn));

max_matrix(row,column)=max(S_function(row,column),complement(row,c
olumn));

        end
    end

     min_sum=sum(sum(min_matrix));
    max_sum=sum(sum(max_matrix));

    entropi(mum)=min_sum/2560000
    mum=mum+1
end

%Maximum entropi.
max_entropi=max(entropi)

pixel_degeri=zeros(row_number,column_number);

for row=1:row_number
    for column=1:column_number

        pixel_degeri(row,column)=abs(max_entropi-
S_function(row,column));

    end
end

en_yakin_pixel_degeri=min(min(pixel_degeri));
pixel_no=find(pixel_degeri==en_yakin_pixel_degeri);
```

```
%Threshold Değeri.
threshold_degeri=gray_image(pixel_no(1));

%Yeni Image.
new_image=zeros(row_number,column_number);

for row=1:row_number
    for column=1:column_number

        if gray_image(row,column)<=101
        new_image(row,column)=255;

        elseif gray_image(row,column)>101
        new_image(row,column)=0;
        end

    end
end


figure,
imshow(gray_image)
figure,
imshow(new_image)

[b,num]=bwlabel(new_image,4);
BW2 = bwareaopen(new_image, 20);
say=bwarea(BW2)
imshow(BW2)
top_piksel=sum(sum(BW2))
disp('number of counted cells');
disp(num);
```

Appendix 2. Program for Image Denoising

```
clear all;
clc;

%Resmi Oku.
image=(imread('zulal_2.tif'));
%dbg_print
%figure,imshow(image)

A = (rgb2gray(image));
A(A>200) = A(A>200)-20;
%dbg_print
%figure,imshow(A,[]);

B = imnoise(A,'salt & pepper',0.02);
%dbg_print
%figure,imshow(B,[]);

%%%%% calculate mu_a or S_function_B
max_pixel=max(max(B));
min_pixel=min(min(B));
mid_pixel=(max_pixel+min_pixel)/2;

%Resmin boyutları.
[row_number,column_number]=size(B);
S_function_B = zeros(row_number,column_number);

%B nin S fonksiyonu.
for row=1:row_number
    for column=1:column_number

        if B(row,column)<=min_pixel
            S_function_B(row,column)=0;

        elseif min_pixel<B(row,column) && B(row,column)<=mid_pixel
            S_function_B(row,column)=(((double(B(row,column))-
double(min_pixel)))^2)/((double(mid_pixel-
min_pixel))*(double(max_pixel-min_pixel)));

        elseif mid_pixel<B(row,column) && B(row,column)<max_pixel
            S_function_B(row,column)=1-((((double(B(row,column))-
double(max_pixel)))^2)/((double(max_pixel-
mid_pixel))*(double(max_pixel-min_pixel))));

        elseif B(row,column)>=max_pixel
            S_function_B(row,column)=1;

        end
    end
end

%%%%% calculate mu_a or S_function_A
max_pixel=max(max(A));
min_pixel=min(min(A));
```

```
mid_pixel=(max_pixel+min_pixel)/2;

%Resmin boyutları.
[row_number,column_number]=size(A);
S_function_A = zeros(row_number,column_number);

%B nin S fonksiyonu.
for row=1:row_number
    for column=1:column_number

        if A(row,column)<=min_pixel
            S_function_A(row,column)=0;

        elseif min_pixel<A(row,column) && A(row,column)<=mid_pixel
            S_function_A(row,column)=(((double(A(row,column))-
double(min_pixel)))^2)/((double(mid_pixel-
min_pixel))*(double(max_pixel-min_pixel)));

        elseif mid_pixel<A(row,column) && A(row,column)<max_pixel
            S_function_A(row,column)=1-((((double(A(row,column))-
double(max_pixel)))^2)/((double(max_pixel-
mid_pixel))*(double(max_pixel-min_pixel))));

        elseif B(row,column)>=max_pixel
            S_function_A(row,column)=1;

        end
    end
end

%%% TODO : find C by using h = 0:0.01:0.2
hsweep = 0.01:0.01:0.99;
CA = zeros(length(hsweep),1);
i=0;%deefine loop counter
for h = hsweep,
%dbg_print
%imshow(S_function_B);title('s func of B');
%iterate counter
i = i+1

noisy_pixels = (0.5 - h) < abs(S_function_B - 0.5);
neighborhood_of_noisy_pixels = imdilate(noisy_pixels,[
    1 1 1;
    1 0 1;
    1 1 1;
    ]);

masked_neigborhoods = S_function_B.*neighborhood_of_noisy_pixels;
noisy_pixels_recalculated = imfilter(masked_neigborhoods,[
    1/8 1/8 1/8;
    1/8 0 1/8;
    1/8 1/8 1/8;
]);

% build C image (denoised)
C = zeros(size(A));
```

```
C(~noisy_pixels) = S_function_B(~noisy_pixels);%bozulmamış
pikselleri al;
C(noisy_pixels) = noisy_pixels_recalculated(noisy_pixels);


%%%%% calculate mu_c or S_function_C
max_pixel=max(max(C));
min_pixel=min(min(C));
mid_pixel=(max_pixel+min_pixel)/2;

%Resmin boyutları.
[row_number,column_number]=size(C);
S_function_C = zeros(row_number,column_number);

%C nin S fonksiyonu.
for row=1:row_number
    for column=1:column_number

        if C(row,column)<=min_pixel
            S_function_C(row,column)=0;

        elseif min_pixel<C(row,column) && C(row,column)<=mid_pixel
            S_function_C(row,column)=(((double(C(row,column))-
double(min_pixel)))^2)/((double(mid_pixel-
min_pixel))*(double(max_pixel-min_pixel)));

        elseif mid_pixel<C(row,column) && C(row,column)<max_pixel
            S_function_C(row,column)=1-((((double(C(row,column))-
double(max_pixel)))^2)/((double(max_pixel-
mid_pixel))*(double(max_pixel-min_pixel))));

        elseif C(row,column)>=max_pixel
            S_function_C(row,column)=1;

        end
    end
end

%dbg_print
%figure,imshow(S_function_C,[]);

%%% calculate cost , warning, S_function_C is changed!
diff_A2C_sqr = (S_function_C-S_function_A).^2;
mask_C = S_function_C > 0.5;
S_function_C(mask_C) = 1-S_function_C(mask_C);
sqr_c = S_function_C.^2;

CA(i) = sqrt(sum(diff_A2C_sqr(:))/(row_number*column_number)) +
sqrt( sum(sqr_c(:))/(row_number*column_number));
end
figure,plot(hsweep,CA)
xlabel('t threshold value')
ylabel('Entropy value')
%figure, imshow(C);
%
```