DOKUZ EYLÜL UNIVERSITY

GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

COMPARING COINTEGRATION TEST IN PRESENCE OF STRUCTURAL BREAKS

by

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İZMİR

COMPARING COINTEGRATION TEST IN PRESENCE OF STRUCTURAL BREAKS

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M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"COMPARING COINTEGRATION TEST IN PRESENCE OF STRUCTURAL BREAKS"** completed by **BERHAN ÇOBAN** under supervision of **ASSOCIATE PROF. DR. ESİN FİRUZAN** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

Cointegration analysis is a method developed for revealing whether there is a long term linear relation between more than one time series. Structural breaks may occur in the data generating processes of the time series due to reasons such as policy change, financial crisis and natural disasters.

Not including the structural breaks into the analysis, in time series analysis, may cause the unit root and cointegration tests to give incorrect results. These results decrease the power of the test used. The widely used Dickey-Fuller unit root test and Engle-Granger and Johansen Cointegration tests may have erroneous results since they investigate the unit root and long term relation without considering structural breaks.

The study gives brief information on the Zivot and Andrews and Perron (1989) unit root tests and Gregory-Hansen (G-H) cointegration test, which have been developed to avoid the incorrect results. A comparison of Engle-Granger (E-G) test, which investigates long term relations without taking structural breaks into consideration, and Gregory-Hansen test, which does the same taking the breaks into consideration, is conducted.

For this comparison the data generating process was conducted by Monte-Carlo simulation using the MATLAB (R2009a) software. Each data pair, produced for the cointegration tests, were repeated 10000 times and results for both tests were obtained and presented in tables.

Keywords : Cointegration, Unit Root, Structural Break, Engle- Granger Test, Gregory-Hansen Test

YAPISAL KIRILMANIN VARLIĞI DURUMUNDA EŞBÜTÜNLEŞME TESTLERİNİN KARŞILAŞTIRILMASI

ÖZ

Eşbütünleşme analizi, birden fazla seri arasında uzun dönemli doğrusal bir ilişki olup olmadığını ortaya çıkarmak için geliştirilmiş bir yöntemdir. Zaman serilerinin veri üretim süreçlerinde, politika değişikliği, finansal krizler, doğal afetler gibi birçok nedenden dolayı yapısal değişimler meydana gelebilmektedir

Zaman serisi analizlerinde yapısal kırılmaların analize dahil edilmemesi birim kök ve eşbütünleşme testlerinin sonuçlarının hatalı çıkmasına neden olabilmektedir. Bu sonuçlar ise kullanılan testin gücünü azaltmaktadır. Yaygın kullanılan Dickey-Fuller birim kök testi, Engle- Granger ve Johansen Eşbütünleşme testleri kırılmaları dikkate almadan birim kökü ve uzun dönemli ilişkiyi araştırdıkları için sonuçları hatalı olabilmektedir.

Çalışmada bu sorunun giderilebilmesi için geliştirilmiş Zivot and Andrews, Perron (1989) birim kök testleri ile Gregory- Hansen (G-H) eşbütünleşme testi hakkında bilgi verilmiştir. Yapısal kırılmaları dikkate almayan Engle- Granger (E-G) testi ile yapısal kırılmaları dikkate alarak uzun dönemli ilişkiyi araştıran Gregory-Hansen testlerinin karşılaştırılması yapılmıştır.

Bu karşılaştırma için Monte-Carlo simulasyonu ile MATLAB (R2009a) programı kullanılarak veri üretimi yapılmıştır. Eşbütünleşme testleri için üretilen her bir veri çifti 10000 kez tekrarlanarak her iki test için de sonuçlar elde edilmiş ve tablolarla gösterilmiştir.

Anahtar Sözcükler: Eşbütünleşme, Birim Kök, Yapısal Kırılma, Engle- Granger Testi, Gregory-Hansen Testi

CONTENTS

CHAPTER ONE

INTRODUCTION

Time series analysis is useful technique for identifying the nature of the phenomenon representing by the sequences of observation. The aim of the time series analysis is extrapolate the identified pattern to predict future events.

While time series analysis may depend on single variable analysis, modeling and analysis can also be performed on more than one series together. This analyis is called the multivariate time series in the literature. One of the multivariate time series analysis is the cointegration analysis. Cointegration analysis is a method developed to reveal whether there is a long term linear correlation between time series. In this method, first a linear model between two or more nonstationary series is constructed. Then, referring to the stationarty feature of error terms produced by this model, it is decided whether the series are cointegrated or not.

In order to determine the cointegrated correlation between the series, various test according to the features of the series have been developed.

The first chapter gives information on the Engle-Granger and Johansen tests, two of widely used cointegration test. Engle-Granger test tries to reveal the cointegrated structure of the series with respect to the stationarity feature of the error terms of a linear combination between two nonstationary time series. If the error terms obtained from the linear combination are stationary then the series are cointegrated. Although, there are various methods for the stationarity test of the error terms, generally the Dickey-Fuller unit root test is used.

The other cointegration test mentioned in the study is the Johansen cointegration test. In this method, the cointegration correlation between the series is determined by the Maximum Likelihood Estimation (MLE) approach.
Instead of the cause-effect relation built between variables in Engle – Granger method, a vector-autoregressive model (VAR) is formed in this method.

With this feature it is possible to test whether more than two series are cointegrated or not, at the same time.

There are two test statistics to determine the number of the cointegration vectors between the series for the Johansen method which can test the cointegrated structure between more than two series. These are trace and maximum eigenvalue tests.

In the second chapter, the characteristic features of the structural breaks, the factors causing the breaks, their effects on the unit root and cointegration tests are examined. Structural changes may occur in the data generating processes of the time series due to reasons such as policy change, financial crisis and natural disasters. These changes in the series, without any exact definition, are generally called as the structural change in the model parameters. Structural breaks may occur in the intercepts or/and the trends of the series. The existence of the outlier observations may cause various problems such as biases and inconsistent estimation results, biased parameter estimation, poor predictions and modelling of a linear model as a non-linear model. Therefore, the effects of outlier observations should be included in the model while analyzing the series.

The widely used ADF and Philips – Perron (PP) unit root tests, which are used for checking the stationarity hypothesis, and the Engle – Granger and Johansen Cointegration approaches, which investigate the long term equilibrium relation, are methods that do not take the possible structural breaks in the series into consideration. Therefore, using these tests on series with structural breaks may yield the aforementioned problems. In order to avoid these problems, unit root and cointegration tests take the structural breaks into consideration.

In the second chapter of the study, Perron (1989), Unit Root Test, Zivot and Andrews Unit Root Test and Gregory-Hansen (1996) Cointegration Test among these test are mentioned.

Perron (1989) test, one of the unit root tests that considers the structural break is a test method in which the break point in the series is known as an external information and it is based on the hypothesis that there is only one structural break in the series.

The knowledge of the break point enables the addition of these shocks into the model as dummy variables. Perron (1989) test investigates the existence of the break in three different models. Another test applied on the time series with structural breaks is the Zivot and Andrews test. Zivot and Andrews (1992), differently from the Perron (1989) test, developed a test which considers the break period internally. The information, models and hypotheses of these two tests are given in the second chapter.

One of the cointegration tests which are used in the presence of a structural break is the Gregory-Hansen (1996) test. Gregory – Hansen (1996) test investigates the determination of structural breaks in long term relation under three different models

These models are the level shift (C) which expresses the break in the intercept of the series, the level shift with trend (C/T) which expresses the break in the intercept with a trend and the Regime Shift (C/S) model which expresses the break both in the intercept and the slope of the series. In Gregory- Hansen tests, the Dickey-Fuller are Philips-Perron test statistics used for the analysis of the break.

In chapter four, the power comparison of Engle-Granger and Gregory-Hansen tests using a Monte-Carlo Simulation is done.

For this comparison the data production is conducted using the MATLAB (R2009a) software. The series are generated for the three different models according to the Gregory-Hansen test procedure as break in the intercept, break in the intercept with trend, and break in both the slope and the intercept. The data are generated from the autoregressive AR(1) process with a sample size of 50, 100, 200 and with the $\phi = 0.1$, $\phi = 0.5$ and $\phi = 0.9$ parameters.

Since it is thought that the magnitude of the break in the series would have effect on the power of the test, the performances of the test with break magnitudes of 1, 5 and 10. Similarly, the breaks' occurring in different regions of the series are thought to affect on the power of the tests, the breaks are applied in the first quarter *(0.25T)*, second quarter *(0.50T)* and the third quarter *(0.75T)* and the power comparison between the Engle- Granger and Gregory – Hansen (1996) is performed.

Chapter Five, the last chapter of the study presents a general comparison of the Engle-Granger and Gregory-Hansen tests on the series obtained after the data generation. In this chapter, the effects of variables such as break magnitude, break point and the values of AR(1) variable, on the power values of the tests are presented.

CHAPTER TWO

UNIT ROOT AND COINTEGRATION TESTS

A time series is simply defined as sequences of measurements that follow non-random orders. A time series is a set of observation X_t, each successive value represents consecutive measurement takes at equally spaced time intervals. The basic nature of a time series is that its observation is dependent or correlated, hence statistical methods are not applicable because of independent assumption. Time series analysis is useful technique for identifying the nature of the phenomenon representing by the sequences of observation. The aim of the time series analysis is extrapolate the identified pattern to predict future events.

Time series analysis may depend on univariate analysis or an analysis and a modeling can be conducted by considering more than one time series together. This method is called, in the literature, as vector or multivariate time series analysis. Multivariate time series analysis is used not only to analyze only one series, but also to analyze the cross-relations between series.

One of the time series analysis is the cointegration analysis. The cointegration analysis is a method developed to reveal whether there is a long term linear correlation between series. In this model, first a linear model is built between two or more non-stationary series. The series are determined as cointegrated or not depending on whether or not the error terms produced by the model have the property of stationarity. The error terms' being stationary – or not including unit root – indicates that the series are cointegrated, otherwise the series are not cointegrated.

Cointegration analysis enables the inclusion of the original values of the series which are not stationary, but which become stationary when their differences of the same degree are calculated. Thus, the possible errors of obtaining difference operations during the analysis are prevented and the statistically significant relations between the series are revealed.

Various tests have been developed in order to determine the cointegrated correlations between the series. The most widely used ones, among these tests, are the Engle – Granger (1987) and the Johansen (1988) cointegration tests.

1.1 Engle – Granger Cointegration Test

One of the most widely used tests for determining the long term correlations between time series is the Engle – Granger cointegration test. The basic approach in Engle – Granger method is the error terms of a linear combination between two non-stationary time series having the property of stationarity.

$$Y_t = \beta X_t + u_t \tag{1}$$

A general model that can be built between two series can be presented as in equation (1). In this model the dependent variable Y_t, the independent variable X_t, and the error term u_t, which is random, is presented. In order to variables in the model to be cointegrated, it is both assumed that the difference of both variables are obtained (I(1) distributed) and at the same time the error term is non-differenced (I(0) distributed). In other words, the error term is $u_t \sim IN(0, \sigma^2)$.

In order to determine the existence of the linear correlation between the series Engle – Granger proposed a procedure comprising of two steps. According to this procedure, first a linear equation (ordinary least squares, OLS) is built and the parameter estimations are obtained by using the least square method. As the second step the unit root test is applied on the error terms obtained from the model. In order to determine whether the error terms are stationary or not, the Dickey-Fuller test is widely used.

1.1.1 Dickey-Fuller Test

The Dickey – Fuller test which analyzes whether any series included has unit root or not, gives information about whether the series are cointegrated or not, since a similar operation is applied on the error term in cointegration analysis. As the error terms obtained from the linear correlation between the series, under cointegration investigation, can be modeled with their lagged values, Dickey – Fuller test can be applied on this data.

Before conducting the Dickey – Fuller analysis for determining whether the error terms obtained from the linear model of the two series under cointegration investigation, the procedure of Dickey – Fuller test will be briefly explained.
In Engle - Granger Cointegration test, the Dickey-Fuller test unit root test of the Y_t and X_t series with the assumption I(1) can be performed as below:

Consider the simplest imaginable AR(1) model,

$$X_t = \phi \ X_{t-1} + e_t \tag{2}$$

where e_t *is* white noise with variance 1. When ϕ_1 = 1, this model has a unit root and becomes a random walk process. If X_{t-1} is subtracted from each variable in equation (2), equation (3) will be as follows:

$$\Delta X = (\phi \ -1)X_{t-1} + e_t \tag{3}$$

Thus, in order to test the null hypothesis of a unit root, we can simply test the hypothesis that the coefficient of X_{t-1} in equation (3) is simply equal to 0. The hypotheses which are relevant to Dickey - Fuller are as follows:

$$H_0 : \rho = 0 \qquad \rho = (\phi - 1)$$

$$H_a : \rho < 0$$

Test statistic is

$$t_{\phi=0} = \frac{\hat{\phi}}{SE(\hat{\phi})}$$

where $\hat{\phi}$ is the least squares estimate and SE($\hat{\phi}$) is the usual standard error estimate. The test is a one-sided and lower tailed test.

The obvious way to test the unit root hypothesis is to use the *t* statistic for the hypothesis $(\phi_1 - 1) = 0$ in equation (3). In fact, this statistic is called as τ statistic, not as *t* statistic, because, its distribution is not the same as that of an ordinary *t* statistic, even asymptotically.

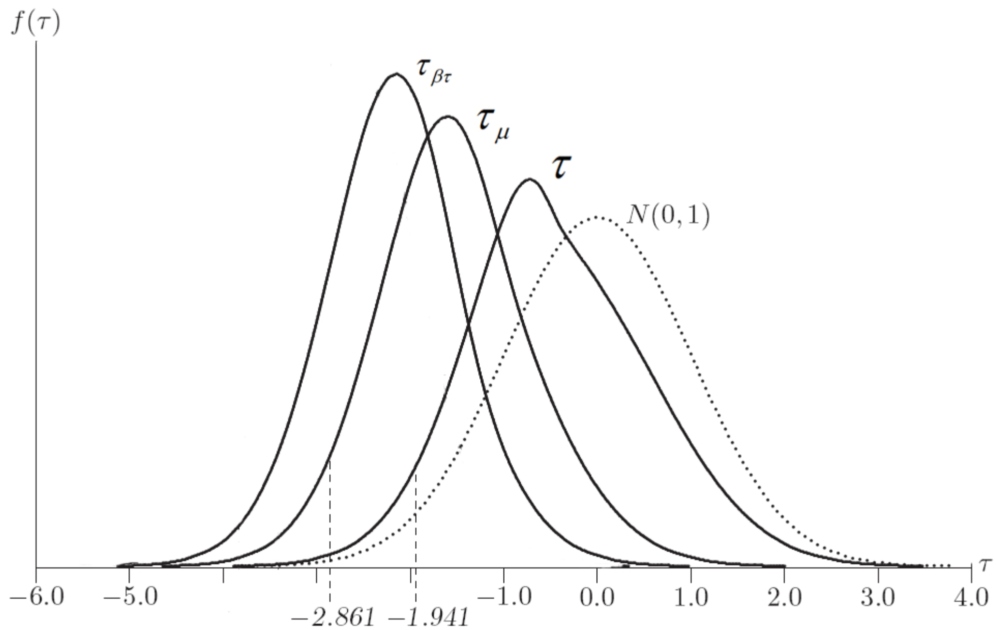

Figure 1 Asymptotic densities of Dickey-Fuller τ tests

The asymptotic densities of the τ , τ_μ, and $\tau_{\beta\tau}$, statistics are shown in Figure 1. For comparison, the standard normal density is also shown. The differences between it and the three Dickey-Fuller τ distributions are skewed and peaked.

The critical values for one-tail tests at the .05 level based on the Dickey-Fuller distributions are also marked on the figure. These critical values become greater than normal distribution.

Dickey and Fuller (1981) consider three different regression equations that can be used to test for the presence of a unit root:

$$\Delta X_t = \phi X_{t-1} + e_t \tag{4}$$

$$\Delta X_t = \alpha + \phi X_{t-1} + e_t \tag{5}$$

$$\Delta X_t = \alpha + \phi X_{t-1} + \beta t + e_t \tag{6}$$

The difference between the three regression equations concerns the presence of the deterministic elements α and βt . The first one is a pure random walk model, the second one involves an intercept or drift term, and the third one includes both a drift and linear time trend.

The unit root can be tested by $\phi = 0$ parameter in all the regression equation. The test involves estimating one of the equations above using OLS in order to obtain the estimated value of ϕ and associated standard error. Comparing the results of *t*-statistic with the appropriate value which is reported in the Dickey-Fuller tables and, it can be determined whether to reject the null hypothesis ϕ=0.

The critical values of the *t*-statistics depend on whether an intercept and time trend is included in the regression equation. In Monte Carlo study, Dickey and Fuller detained that the critical values for ϕ=0 depend on the form of the regression and sample size.

Dickey and Fuller (1981), Said and Dickey (1984), Phillips and Perron (1988) and others improved the Dickey Fuller test when e_i was not white noise. This test is called the “Augmented” Dickey Fuller test. Hence regression equations are:

$$\Delta X_t = \phi X_{t-1} + \sum_{i=1}^{k} \delta_i \Delta X_{t-i+1} + e_t \tag{7}$$

$$\Delta X_t = \alpha + \phi X_{t-1} + \sum_{i=1}^{k} \delta_i \Delta X_{t-i+1} + e_t \tag{8}$$

$$\Delta X_t = \alpha + \phi X_{t-1} + \beta t + \sum_{i=1}^{k} \delta_i \Delta X_{t-i+1} + e_t \tag{9}$$

The statistics are called as τ, τ_μ and τ_τ used for equations (4),(5),(6) respectively. Summary of Dickey - Fuller test process is shown in Table 1.

Table 1: Summary of Dickey – Fuller Tests for n=100

Model	**Hypothesis**	**Test Statistic**	**Critical values for 95% and 99% Confidence Intervals**
$\Delta X_t = \phi X_{t-1} + \varepsilon_t$	$\phi = 0$	τ	-1.95 and -2.60
$\Delta X_t = \alpha + \phi X_{t-1} + \varepsilon_t$	$\phi = 0$	τ_μ	-2.89 and -3.51
	$\alpha = 0$ given $\phi = 0$	$\tau_{\alpha\mu}$	2.54 and 3.22
	$\alpha = \phi = 0$	F_1^*	4.71 and 6.70
$\Delta X_t = \alpha + \phi X_{t-1} + \beta t + \varepsilon_t$	$\phi = 0$	τ_τ	-3.45 and -4.04
	$\alpha = 0$ given $\phi = 0$	$\tau_{\alpha\tau}$	3.11 and 3.78
	$\beta = 0$ given $\phi = 0$	$\tau_{\beta\tau}$	2.79 and 3.53
	$\phi = \beta = 0$	F_3^*	6.49 and 8.73
	$\alpha = \phi = \beta = 0$	F_2^*	4.88 and 6.50

The all τ , τ_μ and τ_τ statistics are used to test the hypotheses ϕ=0. Dickey and Fuller (1981) provide three additional F-statistics (F_1^*, F_2^*, F_3^*) to test joint hypotheses on the coefficient. With (5) or (8), the null hypothesis $\phi = \alpha$ =0 is tested using the F_1^* statistics. Including a time trend in the regression- so that (6) or (9) is estimated- the joint hypotheses $\alpha = \phi = \beta = 0$ is tested using the F_2^* statistics and the joint hypotheses $\phi = \beta$=0 is tested using the F_3^* statistics.

The F_1^*, F_2^*, F_3^* statistics are constructed in exactly the same way as ordinary F-tests are:

$$F_i^* = \frac{[RSS(restricted) - RSS(unrestricted)]/r}{RSS(unrestricted)/(T-k)}$$

where RSS (sums of the squares residuals for restricted models) and RSS (the unrestricted sums of the squares residuals) models.

r = number of restrictions

T = total observations

k = number of parameters in the unrestricted model

T-k = degrees of freedom in the unrestricted model

The Dickey-Fuller test procedure can also be applied for the error term of the model. If the error term u_t is expressed with delay as below, the existence of unit root is performed depending on the statistical significance of ρ .

Dickey-Fuller test applied to a series of any of the above process can also be applied to the model error term. If the error term u_t expressed below, presence of structural breaks analyses depend on significance ρ .

If the error term u_t is leave alone in equation (1), the equation converts into $u_t = Y_t - \beta X_t$. The error term is modeled with lagged values; the equation can be expressed as below:

$$\Delta u_t = \rho u_{t-1} + e_t$$

The hypotheses for these test are;

$H_o : \rho = 0$ means that u_t has unit root. In other words, X_t and Y_t are not cointegrated.

$H_1 : \rho \neq 0$ means that u_t has not unit root. In other words, X_t and Y_t are cointegrated.

$\tau = \frac{\rho}{S_\rho}$ is in the form of test statistics for these hypotheses. The critical values for this test statistics are compared to the values produced by Dickey-Fuller instead of the standard *t* table. In a similar way, a modeling can be performed with the Augmented Dickey – Fuller test which is obtained by adding the *k* delayed values of the error terms to the model.

$$\Delta u_t = \rho u_{t-1} + \sum_{i=1}^{k} \beta_i \Delta u_{t-i} + e_t$$

The unit root hypotheses and the critical values of the Augmented Dickey – Fuller (ADF) test are the same with the general model.

1.2 The Estimation of Engle-Granger Cointegration Vector in Two Dimensional Vector Autoregressive Processes VAR (2)

It is possible to separate any non-stationary series into its stationary and non-stationary parts via the equalities that can be formed using the cointegration vector components. If it is possible to handle a non-stationary vector autoregressive time series of the first degree with two dimensions to estimate the cointegration vector. Let U_t represents a unit rooted series, and S_t represents a stationary series; it can be expressed the equation as below:

$$X_t = a_{11}U_t + a_{12}S_t$$

$$Y_t = a_{21}U_t + a_{22}S_t$$

The equation can be expressed as equation (10) when the required transformations are performed on the series.

$$Y_t - \frac{a_{21}}{a_{11}} X_t = \left(a_{22} - \frac{a_{21}}{a_{11}} a_{12} \right) S_t \tag{10}$$

Beginning from equation (10), as equation (1) can be expressed as a function of S_t series, the system comes to a stationary state. In this equation, knowing the $\beta = \frac{a_{21}}{a_{11}}$ proportion is sufficient for obtaining the cointegration equation (Akdi, 2003).

Let

$$\hat{\beta}_n = \frac{\sum_{t=1}^{n} X_t Y_t}{\sum_{t=1}^{n} X_t^2}$$

$$\frac{1}{n^2}\sum_{t=1}^{n} X_t Y_t = a_{21} a_{11} \sum_{t=1}^{n} U_t^2 + O_p\left(1 \big/ \sqrt{n} \right)$$

and

$$\frac{1}{n^2}\sum_{t=1}^{n} X_t^2 = a_{11}^2 \sum_{t=1}^{n} U_t^2 + O_p\left(1 \big/ \sqrt{n} \right)$$

result as

$$\hat{\beta}_n = \frac{\sum_{t=1}^{n} X_t Y_t}{\sum_{t=1}^{n} X_t^2} = \frac{a_{21}}{a_{11}} + O_p\left(1 \big/ \sqrt{n} \right)$$

If $O_p\left(1 \big/ \sqrt{n} \right)$ term is neglected, the stationary series

$$Z_T = Y_t - \hat{\beta}_n X_t = (a_{21} U_t + a_{22} S_t) - \frac{a_{21}}{a_{11}} (a_{11} U_t + a_{12} S_t) = C S_t$$

is obtained where *C* represents a constant (Akdi 2003).

The regression equation, according to these results, indicates the $(-\hat{\beta}_n, 1)'$ cointegration vector.

1.3 Engle – Granger Cointegration Test in VAR (p) (p>2)

Although Engle– Granger Cointegration test is widely used, its area of use is limited due to some constraints. As this test has the property of “unique solution”, it can analyze the cointegration of only two series. For exemplifying this situation;

Let X_t, Y_t, W_t, Z_t series be I(1); when the V linear transformation of

$$V = W_t - \alpha_1 X_t - \alpha_2 Y_t - \alpha_3 Z_t$$

is considered as having only one linear cointegrated structure; the components' having separate cointegration relations disrupts the cointegrated structure of V. Let V_1 be defined as below having a cointegration relation between W_t and X_t :

$$V_1 = W_t - \beta_1 X_t$$

It is obvious that the error terms obtained from this regression are stationary.

Similarly let V_2 cointegration between Y_t and Z_t and Y defined as below:

$$V_2 = Y_t - \beta_2 Z_t$$

In this equation, it can be said that the error terms are stationary. When the V series comprising of V_1 and V_2 series are considered again, it is seen that both V_1 and V_2 series are I(0); and therefore, it poses a great problem in defining the V Series (Kadılar,2000).

Due to such constraints of Engle– Granger analysis, Johansen method has been developed to perform the cointegration analyses of more than two series.

1.4 Johansen Cointegration Test

Another common method used in revealing the cointegrated structure between time series is the Johansen cointegration test. In this method, the cointegration correlation between the series is determined by the Maximum Likelihood Estimation (MLE) approach. Instead of the cause-effect relation built between variables in Engle– Granger method, a vector-autoregressive model (VAR) is formed in this method. With this feature it is possible to test whether more than two series are cointegrated or not, at the same time.

The aim of the Johansen approach is to determine the cointegrated vector number and to find the MLE estimation of the with respect to parameters of the cointegrated vector.

Johansen method makes use of the eigen-value of the parameters matrix, in order to determine whether the series are cointegrated.

Let a first degree VAR(1) be given in equation (11).

$$X_t = AX_{t-1} + e_t \qquad t = 1,2,3,...,n \tag{11}$$

In the VAR(1) model above while e_t terms represent the error terms which are the variance covariance matrix Σ, the matrix *A* shows the parameter matrix of *kx1* dimensions.

e_t error term has the following features:

$$E(e_t) = 0, \quad E(e_t e_t') = \Sigma \text{ and } E(e_t e_{t+h}') = 0$$

Considering that VAR (1) model is a first degree stationary series, the stationary system will be as below when X_{t-1} is subtracted from both sides of the equation for enabling the stationarity of the system,

$$\Delta X_t = (A - I)X_{t-1} + e_t \,.$$

If expression $(A - I)$ is taken as π, *VAR(1)* model turns into equation (12)

$$\Delta X_t = \pi X_{t-1} + e_t \tag{12}$$

Johansen approach tries to determine the cointegration correlation between the rank of π . If $\pi = \alpha\beta'$, and *B* is a non-single matrix, an infinite number of α and β vectors can be obtained, since it is possible to write $\pi = \alpha BB^{-1}\beta'$. Therefore, Johansen approach builds tests on the rank of π matrix instead of the estimation of β vector (Akdi,2003).

r , the rank of the π matrix*;* assuming the number of variables as *k*

if *r=k* then the series is stationary.

if *r=0* then the series is not stationary. There is not any cointegration.

if *0 <r <k* then the series is cointegrated.

Then, $\pi = \alpha\beta'$ equation can be expressed. Here, β indicates the cointegration vector while α is called the adjustment coefficient. Here, the α matrix shows the adjustment rate of the deviation of variables from long term equilibrium. Therefore, while X_t series is not stationary, and provided that ΔX_t is stationary, the linear combination indicated with $\beta' X_t$ are stationary, considering $\pi = \alpha\beta'$. $\beta' X_t$ which has a stationary structure is a cointegrated process.

Under the light of this information, the aim of Johansen method is to reveal the cointegration structure as a result of estimating *A* and π parameter matrices. The estimation of *A* matrix with OLS method can be shown as below:

$$X_t = AX_{t-1} + e_t \quad t = 1,2,3,...,n$$

$$A = \left[\sum_{t=1}^{n} X_t X'_{t-1} \right] \left[\sum_{t=1}^{n} X_{t-1} X'_{t-1} \right]^{-1}$$

In order to find the cointegration structure, it is not necessary to know *A* matrix. The series can be separated into its stationary and non-stationary components by solving π matrix with the expression $\pi = (A - I)$ (Akdi, 2003).

The estimation of π matrix can be performed by Maximum Likelihood Estimation method. Under the assumptions of $X_0 = 0$ and the normal distribution of the error terms, when $|\Sigma|$ shows the determinant of Σ matrix, the likelihood function can be expressed as:

$$\ell = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left[-\frac{1}{2}\sum_{t=1}^{n}(\Delta X_t - \Pi X_{t-1})'\Sigma^{-1}(\Delta X_t - \prod X_{t-1}) \right]$$

Here, the maximum likelihood estimator of π can be expressed as;

$$\hat{\pi} = \left[\sum_{t=1}^{n} \Delta X_t X'_{t-1}\right]\left[\sum_{t=1}^{n} \Delta X_{t-1} X'_{t-1}\right]^{-1} = S_{01}S_{11}^{-1}$$

And the MLE estimator of Σ matrix can be shown as,

$$\hat{\Sigma}_n = \frac{1}{n}\sum_{t=1}^{n}(\Delta X_t - \hat{\pi}X_{t-1})(\Delta X_t - \hat{\pi}X_{t-1})' = S_{00} - \hat{\pi}S_{11}\hat{\pi}'$$

The hypothesis to be tested is $H_0 : \pi = \alpha\beta'$. Here π matrix is a matrix of kxk dimensions and r rank, α and β matrices are of kxr dimensions. In the context of H_0 null hypothesis, the likelihood function is:

$$\ell(\alpha,\beta) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left\{ -\frac{1}{2}\sum_{t=1}^{n}(\Delta X_t - \alpha\beta' X'_{t-1})'\Sigma^{-1}(\Delta X_t - \alpha\beta' X_{t-1}) \right\}$$

A maximization process should be conducted in the likelihood function above. This process is performed in two steps. First, β' is kept as a constant and the maximum likelihood estimator of α is obtained. For this the equation $\Delta X_t = \alpha\beta' X_{t-1} + e_t$ can be used; and the result

$$\hat{\alpha} = \left[\sum_{t=1}^{n} \beta' X_{t-1} X'_{t-1} \beta\right]^{-1} \left[\sum_{t=1}^{n} \beta'(X_{t-1}\Delta X'_t)\right]$$

$$= \left[\beta' S_{11} \beta\right]^{-1} \left[\beta' S_{10}\right]$$

can be obtained.

As it can be seen, the likelihood function β is a function of Σ. The maximum likelihood estimator of β can be obtained by placing this value in the likelihood function. In order to do this, let $Y = \Delta X_t - \alpha\beta' X_{t-1}$ and $\hat{\Sigma}(\beta)$ indicates the variance – covariance matrix of β,

$$\exp\left[-\frac{n}{2} trace(Y(Y'Y)^{-1}Y')\right] = \exp\left[-\frac{n}{2} trace((Y'Y)^{-1}Y'Y)\right]$$
$$= \exp\left[-\frac{nk}{2}\right]$$

In the context of these information, the likelihood function can be expressed as:

$$\ell = \frac{1}{(2\pi)^{nk/2} \left|\hat{\Sigma}(\beta)\right|^{n/2}} \exp\left[-\frac{nk}{2}\right]$$

In other words, the maximization of the likelihood function depends on the minimization of $\left|\hat{\Sigma}(\beta)\right|^{n/2}$.

Thus, the problem turns into

$$\min_{\beta}\left|\hat{\Sigma}(\beta)\right| = \min_{\beta}\left|S_{00} - S_{01}\beta(\beta' S_{11}\beta)^{-1}\beta' S_{10}\right|$$

In order to determine the number of cointegration vectors in Johansen method, the two different test statistics are used. These are Trace test and Maximum Eigen-Value Tests. Brief information on these tests are provided below.

1.4.1 Trace Test

Trace test hypotheses are constructed as below assuming r_0 shows the maximum number of cointegration vector:

$H_0 : r \leq r_0$

$H_1 : r > r_0$

The rank of π matrix being *r* means that there are *r* numbers of linearly independent cointegration correlation. Therefore, $H_0 : r \leq r_0$ hypothesis means the test of the null hypothesis of "there are at most r_0 linearly independent cointegration correlation" versus the $H_1 : r > r_0$ alternative hypothesis. In order to do this, let the test statistics of the likelihood proportion λ_i indicate the eigen-value of π matrix;

$$LR = \frac{\max\limits_{H_0} \ell(\alpha,\beta)}{\max\limits_{H_0 \cup H_1} \ell(\alpha,\beta)} = \frac{\left|\hat{\Sigma}_1\right|^{n/2}}{\left|\hat{\Sigma}_0\right|^{n/2}}$$

$$= \left[\frac{\prod\limits_{i=0}^{k}(1-\hat{\lambda}_i)}{\prod\limits_{i=0}^{r_0}(1-\hat{\lambda}_i)}\right]^{n/2} = \left[\prod_{i=r_0+1}^{k}(1-\hat{\lambda}_i)\right]^{n/2}$$

Here, the values of the test statistics

$$\lambda_{trace} = -n\sum_{i=r_o+1}^{k}\ln(1-\hat{\lambda}_i) \tag{13}$$

are compared to the critical value in Johansen (1988). If these values are greater than the critical value, $H_0 : r \le r_0$ or $H_0 : r = r_0$ null hypotheses are rejected. Under these circumstances, *r* cointegrated vectors can be defined. The process is continued until H_0 is not rejected and the number of cointegrated vectors is obtained. Here, $k - r_0$ canonical correlations, assuming $\lambda_{r_0+1} > \lambda_{r_0+2} > \ldots > \lambda_p$, are used (Akdi 2003).

As it can be seen in the equation (13), if $\hat{\lambda}$ equals to zero, the value of the test is higher. Therefore, it is easy to reject H_0.

1.4.2 Maximum Eigen-Value Test

Maximum Eigen-Value test, on the other hand, determines the number of the cointegration vectors by testing the r_0 empty hypothesis against r_0+1 alternative hypothesis. The hypotheses are;

$H_0 : r = r_0$

$H_1 : r = r_0 + 1$

and the test statistic is

$$\lambda_{\max} = -n \ln(1 - \lambda_{r_0+1})$$

where *T* is the sample size and $\hat{\lambda}_i$ is the *i*th largest canonical correlation.

The values in Johansen (1990) are used for critical values, since the limit distributions of these test statistics are different from standard distributions.

CHAPTER THREE

UNIT ROOT AND COINTEGRATION TESTS IN PRESENCE OF STRUCTURAL BREAKS

In a time series, outlier observations, which are placed away from other observations and/or which cause changes in the realization of the series, affect significantly the analysis of the series. The existence of the outlier observations may cause various problems such as biases and inconsistent estimation results, biased parameter estimation, poor predictions and modeling of a linear model as a non-linear model. Therefore, the effects of outlier observations should be included in the model while analyzing the series.

The structural breaks which cause the interruption of the series and/or long termed changes in their trends are expressed as outlier observations. Structural changes may occur in the data generating processes of time series due to policy changes, financial crises and natural disasters. These changes in the series, without any exact definition, are generally called as the change in the model parameters.

The widely used ADF and Philips – Perron (PP) unit root tests, which are used for testing the stationarity hypothesis, and the Engle – Granger and Johansen Cointegration approaches, which investigate the long term equilibrium relation, are methods that do not take the possible structural breaks in the series into consideration. Therefore, using these tests on series with structural breaks may yield the aforementioned problems.

3.1 Unit Root Tests Developed in Presence of Structural Breaks

If there is a structural break in the time series used in stationarity analysis; and the unit root analysis is conducted without considering the break, the unit root result of the series can be unreliable. These results decreases the power of the test used.

Thus, in order to attain correct results in unit root analysis, Perron (1989), Christiano (1992), Banarjee, Lumsdaine and Stock (1992), Zivot and Andrews (1992), Perron and Vogelsang (1992), Lee and Strazicich, and Bai – Perron and Perron (1997) tests which take structural breaks in time series into consideration, are used.

In this study, Perron (1989) test and Zivot and Andrews (1992) test are explained.

3.1.1 Perron (1989) Unit Root Test

Perron (1989) developed a new test method in which the break point in the series is known as external information and which are based on the hypotheses that there is only one structural break. Knowledge of the break point enables the inclusion of these shocks into to model as dummy variables. Such inclusion of the break into the model as a dummy variable does not express the models which are built for the variables representing the series, but it is used to remove the effects of the shocks in the series, only.

Perron (1989) examined the unit root analysis on three different models. Of these models, Model A is constructed by taking a structural change in the level (intercept) of the series into consideration; Model B, a structural change in the slope of the series; and Model C, taking into consideration the structural changes both in the level and the slope of the series. The hypotheses for Perron (1989) test can be expressed as below:

H_o: There is a stochastic trend in the series. Series is not stationary.

H_1: There is a deterministic trend in the series. The series is stationary with a break in the trend.

The H_o null hypotheses which vary with respect to the structural break being in different parameters of the series can be presented as below:

MODEL A $Y_t = \mu + dD(TB)_t + Y_{t-1} + e_t$ (14)

MODEL B $Y_t = \mu_1 + Y_{t-1} + (\mu_2 - \mu_1)DU_t + e_t$ (15)

MODEL C $Y_t = \mu_1 + Y_{t-1} + dD(TB)_t + (\mu_2 - \mu_1)DU_t + e_t$ (16)

In the models above let T_B be $1 < T_B < T$ and indicate the time of break, the variables are defines as below:

$$D(TB)_t = \begin{cases} 1, & t = T_B + 1 \\ 0, & otherwise \end{cases} \qquad DU_t = \begin{cases} 1, & t > T_B \\ 0, & otherwise \end{cases}$$

The alternative hypotheses of the models are as below:

MODEL A $Y_t = \mu_1 + \beta t + (\mu_2 - \mu_1)DU_t + e_t$ (17)

MODEL B $Y_t = \mu + \beta_1 t + (\beta_2 - \beta_1)DT_t^* + e_t$ (18)

MODEL C $Y_t = \mu_1 + \beta_1 t + (\mu_2 - \mu_1)DU_t + (\beta_2 - \beta_1)DT_t + e_t$ (19)

In the alternative hypotheses of models above, let T_B be $1 < T_B < T$ and indicate the time of break dummy variables are defined as following.

$$DT_t^* = \begin{cases} t - T_B, & t > T_B \\ 0, & otherwise \end{cases} \qquad DT_t = \begin{cases} t, & t > T_B \\ 0, & otherwise \end{cases}$$

$$DU_t = \begin{cases} 1, & t > T_B \\ 0, & otherwise \end{cases}$$

The null hypothesis of Model A shows that the structural break caused a change in the intercept of the trend line via an external shock. The $dD(TB)$ expression in the equation takes the value 1 for the first period after the break time, and takes the value 0 for other period. When the alternative hypothesis is examined, DU_t in the model is a dummy variable which takes the value 0 until the time of break, and which takes the value 1 for the periods after it; and $(\mu_2 - \mu_1)$ expression is the difference the structural change caused in the trend function.

The null hypothesis of Model B shows that the structural break caused a change in the slope of the trend line via an external shock. In the alternative hypothesis, the dummy variable of slope coefficient DT^* takes the values 1,2,3,…T if there is an increase in the slope of the trend after the time of break, takes the value 0 in other otherwise. $(\beta_2 - \beta_1)$ expression in the hypothesis indicates the difference in the slope of trend function caused by the structural change.

The hypotheses defined for Model C are in the form of a combination of Model A and Model B. When Model C is examined, it is assumed that the structural break caused a change in both the intercept and the slope of the trend line.

The ADF test method can be used for the Perron (1989) procedure test statistics. In this respect, in order to test stationarity about the trend function of any Y_t series, equation (20) below is used.

$$Y_t = \tilde{\mu} + \tilde{\beta}t + \tilde{\alpha}Y_{t-1} + \sum_{i=1}^{k}\tilde{c}_i\Delta Y_{t-i} + \tilde{e}_t \tag{20}$$

When the Perron (1989) test models are taken in this context, the models turn into:

$$Y_t = \hat{\mu}^A + \hat{\theta}^A DU_t + \hat{\beta}^A t + \hat{d}^A D(TB)_t + \hat{\alpha}^A Y_{t-1} + \sum_{i=1}^{k}\hat{c}_i\Delta Y_{t-i} + \hat{e}_t \tag{21}$$

$$Y_t = \hat{\mu}^B + \hat{\theta}^B DU_t + \hat{\beta}^B t + \hat{\gamma}^B DT_t^* + \hat{\alpha}^B Y_{t-1} + \sum_{i=1}^{k}\hat{c}_i\Delta Y_{t-i} + \hat{e}_t \tag{22}$$

$$Y_t = \hat{\mu}^C + \hat{\theta}^C DU_t + \hat{\beta}^C t + \hat{\gamma}^C DT_t + \hat{d}^C D(TB)_t + \hat{\alpha}^C Y_{t-1} + \sum_{i=1}^{k}\hat{c}_i\Delta Y_{t-i} + \hat{e}_t \tag{23}$$

The parameter constraints for the models are as below;

$$MODEL \quad A: \quad \alpha^A = 1, \quad \beta^A = 0, \quad \theta^A = 0$$

$$MODEL \quad B: \quad \alpha^B = 1, \quad \beta^B = 0, \quad \theta^B = 0$$

$$MODEL \quad C: \quad \alpha^C = 1, \quad \beta^C = 0, \quad \theta^C = 0$$

Under the light of this information, the Perron (1989) test procedure is conducted through the following steps.

Step 1

Detrended series is obtained. The error terms of these models are shown as u_t.

Step 2

The modeling of the error terms with their past values can be expressed as below:

$$\Delta u_t = \rho u_{t-1} + e_t$$

The unit root test is applied, under the assumption of $e_t \sim N(0,\sigma^2)$ .Here the distribution of ρ will depend on the ratio of the time of break. This ratio shows the ratio of the number of observations prior to the break to the total number of breaks and expressed as λ. $\lambda = T_B / T$ ratio is also used to find the critical values in the table.

Hypotheses are expressed as follows;

$$H_o : \rho = 0$$

$$H_1 : \rho \neq 0$$

and the test statistics is calculated by the following equation.

$$\tau = \frac{\rho}{S_\rho}$$

S_ρ indicates the standard error of the parameters. H_o hypothesis means that the detrend operation did not convert the series to stationary, therefore the series has unit root; the alternative hypothesis, on the other hand, means that the detrend operation did make the series stationary. The series, analyzed based on these results, is stationary with the structural break around the trend,

Step 3

The diagnostic control of the model, obtained in step two, is performed. If there is an autocorrelation in the error terms of the model, the equation below is obtained by adding the lagged terms.

$$\Delta u_t = \rho u_{t-1} + \sum_{i=1}^{k} \beta_i \Delta u_{t-i} + e_t$$

Step 4

In the last step, the test statistics on the significance of ρ is calculated. This test statistics is compared to the Perron (1989) test's critical values, and it is decided whether or not the null hypothesis could be rejected. If the test statistics is absolute greater than the Perron (1989) critical value, H_o hypothesis is rejected.

One of the important assumptions of Perron (1989) test is the prior information about the break time period. However, in practice, generally the time of the structural change is not known. In addition, determining the break time as a false prior knowledge may cause the results of the test become incorrect. Numerous test have been developed to remove these errors and to determine the time of break internally. Some of these studies are Christiona(1992), Banarjee Lumsdaine and Stock (1992),Perron and Vogelsang (1992), Perron (1997), Zivot and Andrews (1992). This study gives brief information on Zivot and Andrews test.

3.1.2 Zivot and Andrews Unit Root Test

One of the most widely used unit root test in presence of a structural break is the Zivot and Andrews test. As mentioned above, the Perron (1989) test includes the time of break into the model externally. Zivot and Andrews (1992), on the other hand, developed a test that includes the time of the break internally.

Zivot and Andrews (1992) test which takes the possible structural break into consideration allows, as in Perron (1989) test, only a single structural break in the trend function. Zivot and Andrews (1992) performed the unit root test on three different models.

Model A, of these models, allows a change in the level (intercept) of the series; Model B allows a change in the slope of the series; Model C allows change in both the level and the slope of the series. The hypothesis for Zivot and Andrews (1992) test can be expressed as below;

H_o: There is unit root in the series.

H_1: The series is stationary with a structural break in the trend.

Zivot and Andrews (1992) expressed the H_o null hypothesis for the three model of Perron (1989) test in equations (17) – (19) as below

$$H_o : Y_t = \mu + Y_{t-1} + e_t \tag{24}$$

Alternative hypotheses with comparison to the null hypothesis above are formed as below:

$$Y_t = \hat{\mu}^A + \hat{\theta}^A DU_t(\hat{\lambda}) + \hat{\beta}^A t + \hat{\alpha}^A Y_{t-1} + e_t$$

$$Y_t = \hat{\mu}^B + \hat{\beta}^B t + \hat{\gamma}^B DT_t^*(\hat{\lambda}) + \hat{\alpha}^B Y_{t-1} + e_t$$

$$Y_t = \hat{\mu}^C + \hat{\theta}^C DU_t(\hat{\lambda}) + \hat{\beta}^C t + \hat{\gamma} DT_t^*(\hat{\lambda}) + \hat{\alpha}^C Y_{t-1} + e_t$$

While $DU_t(\hat{\lambda})$ and $DT^*(\hat{\lambda})$ represent the breaks in the constant and slope, respectively, of the trend line, e_t indicates the error term.

If the models built for the Zivot and Andrews (1992) hypothesis test are adapted to the ADF test procedure using equation (20), as in Perron (1989) test, the models turn into forms as below;

$$Y_t = \hat{\mu}^A + \hat{\theta}^A DU_t(\hat{\lambda}) + \hat{\beta}^A t + \hat{\alpha}^A Y_{t-1} + \sum_{j=1}^{k} \hat{c}_J^A \Delta Y_{t-J} + \hat{e}_t$$

$$Y_t = \hat{\mu}^B + \hat{\beta}^B t + \hat{\gamma}^B DT_t^*(\hat{\lambda}) + \hat{\alpha}^B Y_{t-1} + \sum_{j=1}^{k} \hat{c}_J^B \Delta Y_{t-J} + \hat{e}_t$$

$$Y_t = \hat{\mu}^C + \hat{\theta}^C DU_t(\hat{\lambda}) + \hat{\beta}^C t + \hat{\gamma} DT_t^*(\hat{\lambda}) + \hat{\alpha}^C Y_{t-1} + \sum_{j=1}^{k} \hat{c}_J^C \Delta Y_{t-J} + \hat{e}_t$$

While $DU_t(\hat{\lambda})$ and $DT_t^*(\hat{\lambda})$ represent the breaks in the constant and slope, respectively, of the trend line, $\hat{e}_t$ indicates the error term.

The dummy variables are defined as below:

$$DU_t(\hat{\lambda}) = \begin{cases} 1, & t > T_B \\ 0, & otherwise \end{cases} \qquad DT_t^*(\hat{\lambda}) = \begin{cases} t - T_B, & t > T_B \\ 0, & otherwise \end{cases}$$

As seen in equation (24), while the null hypothesis includes a unit root, the alternative hypotheses indicate that the series has a trend-stationary with a break in an unknown point. Model A of the alternative hypotheses shows that the change is in the intercept of the model with an unknown break point, model B shows that the change is in the slope of the model with an unknown break point; and the combination of these two models, Model C shows that shows that the change is in both the intercept and the slope of the model with an unknown break point.

Differently from Perron (1989) test, the break point T_B which is not included in the models is determined internally in Zivot and Andrews test. In this step, for the estimation of the break time, each time period is taken as the possible break time and *(T-2)* regressions is obtained with the OLS method until *t=2,…,(T-2)*. After this process is applied for all observation values, the value which is the minimum for the *t* statistics of $\hat{\alpha}$, which is the coefficient of Y_{t-1} variable, is selected as the break point.

The *t* statistics obtained is compared to the critical values prepared by Zivot and Andrews (1992). If the test statistics is greater than the critical values as an absolute value, H_o hypothesis is rejected. A series, analyzed according to this result, is stationary with a break on the trend line.

Zivot and Andrews (1992) imposed a constraint such as λ being between *0.15T* and *0.85T* due to the approaching the asymptotic distribution of test statistics to the infinity when the extreme of the series are included.

Zivot and Andrews acknowledged this constraint acceptable with respect to the size of the sample and comparison of test's power. Under the light of these information, the possible location of break (the beginning or the end of the series) affects the power of the test.

3.2 Cointegration Test in Presence of Structural Break

The presence of structural breaks in time series revealed that the time series showed a great tendency to be non-stationary in terms of unit root test results in the unit root analysis. In order to avoid these problems, many tests have been developed in the literature. In series with a break in their unit roots, the analysis should be performed by taking the possible breaks into consideration. The exclusion of the structural breaks from the analysis can yield incorrect results, since it causes the cointegration parameters to get different values between periods. The widely used Engle – Granger and Johansen cointegration tests may give incorrect results since they investigate long term relations without considering breaks.

In this context, using tests which investigate the cointegration relations by considering structural breaks will give efficient results in detection the cointegration relation. The widely used cointegration test involving structural breaks are Hansen (1992), Quintos and Phillips (1993), Gregory – Hansen (1996) and Hatemi – J tests.

3.2.1 Gregory – Hansen (1996) Cointegration Test

One of the most widely used cointegration tests in presence of a structural break is Gregory – Hansen (1996). This test is similar to the Engle – Granger cointegration method. The breaks are tried to be determined by adding dummy variables to the Engle - Granger method. Gregory – Hansen (1996) test can be thought as a sequel to Zivot and Andrews (1992) test. In this context, Gregory - Hansen test, as Zivot and Andrews test, investigates only one single break. However, while Zivot and Andrews test tries to determine the break in the series, Gregory – Hansen test tries to determine the break in the error terms of the cointegrated relation.

Gregory – Hansen (1996) test investigates the determination of structural breaks in long term relation under three different models.

The hypotheses tested for all models are below:

H_o: There is not any cointegration between the series.

H_1: There is cointegration between the series.

MODEL A

$$Y_t = \mu_1 + \mu_2 \varphi_{1\tau} + \alpha^T X_t + e_t \quad t = 1,2,...,n$$

This model has been developed to determine the break in the constant term which is also expressed as *Level Shift (C),* in the literature. In Model A, μ_1 indicates the constant term before the break, μ_2 the change in the constant term in during the structural break, α^T the coefficient of the independent variable, and $\varphi_{1\tau}$ indicates the dummy variable which reflects the break effect on the model.

In the equation above, the analyses are conducted with the assumption that the value of α^T is constant.

Model B

$$Y_t = \mu_1 + \mu_2\varphi_{1\tau} + \beta t + \alpha^T X_t + e_t \quad t = 1,2,\ldots,n$$

This model has been developed to determine the break in the constant term of series with trend, which is also named as *Level Shift with trend (C/T)*. In Model B, differently from the previous model a trend variable such as βt is included to the model.

MODEL C

$$Y_t = \mu_1 + \mu_2\varphi_{1\tau} + \alpha_1^T X_t + \alpha_2^T X_t\varphi_{1\tau} + e_t \quad t = 1,2,\ldots,n$$

In Model C, which is expressed as *Regime Shift (C/S)* in the literature, μ_1 and μ_2 is the same with the break in constant model. While α_1^T shows the slope coefficient before the break, α_2^T shows the change in the slope coefficient after the break. Model C is different from Model B, because this model does not any trend variable.

The dummy variable $\varphi_{1\tau}$ which is included in the model for the determination of the structural break can be defined as below

$$\varphi_{1\tau} = \begin{cases} 1, & t > [n\tau] \\ 0, & t < [n\tau] \end{cases}$$

Here, *n* represents the number of observations, while τ is a coefficient which shows the break period between (0.15T, 0.85T) and takes the value of 0 or 1.

As the break time is not known previously, all data is analyzed as possible break time and the smallest one is determined as the break time.

The error terms ($\hat{e}_{1\tau}$) obtained by alternative ways are estimated by OLS method. These error terms depend on the selection of break time τ .The first-order autocorrelation coefficients of these error terms can be expressed as below:

$$\hat{\rho}_{\tau} = \frac{\sum_{t=1}^{n-1} \hat{e}_{t\tau} \hat{e}_{t+1\tau}}{\sum_{t=1}^{n-1} \hat{e}^{2}{}_{t\tau}}$$

The bias-corrected first-order serial correlation coefficient estimate is given by

$$\hat{\rho}_{t}^{*} = \frac{\sum_{t=1}^{n-1} (\hat{e}_{t\tau} \hat{e}_{t+1\tau} - \hat{\lambda}_{\tau})}{\sum_{t=1}^{n-1} \hat{e}^{2}{}_{t\tau}}$$

Where $\hat{\lambda}$ is estimate of a weighted sum of autocovariances.

Using this equation the Phillips test statistics can be written as below:

$$Z_{\alpha}(\tau) = n(\hat{\rho}_{\tau}^{*} - 1)$$

$$Z_{t}\,(\tau) = (\hat{\rho}_{\tau}^{*} - 1) / \hat{s}_{\tau} \qquad \hat{s}_{\tau} = \hat{\sigma}_{\tau}^{2} \Big/ \sum_{t=1}^{n-1} \hat{e}_{t\tau}^{2}$$

Gregory –Hansen (1996) test makes use of the Augmented Dickey Fuller (ADF) statistics in order to determine the structural break.

ADF test statistics $ADF = tstat(\hat{e}_{t-1\tau})$

The ADF and Phillips statistics are standard statistics that are used without any regime changes. In Gregory –Hansen (1996) test, on the other hand, in order to reject the hypothesis that there is not any cointegration the smallest of the $\tau \in T$ values is used.

Thus ADF and Phillips (ADF^*, Z_α^*, Z_t^*) test statistics are;

$$Z_\alpha^* = \inf_{\tau \in T} Z_\alpha(\tau)$$

$$Z_t^* = \inf_{\tau \in T} Z_t(\tau)$$

$$ADF^* = \inf_{\tau \in T} ADF(\tau)$$

If these test statistics calculated from the residuals, are greater than the critical values calculated by Gregory – Hansen (1996), then the H_o hypothesis is rejected. In this case, the series can be expressed as cointegrated with the structural breaks.

Gregory – Hansen (1996) test, which analyzes the long term relations between series considering the structural breaks, conducts the analysis by taking only one break into consideration as in Zivot and Andrews test. This test method becomes invalid for series involving more than one break. In such situations, tests which consider more than one break are used.

CHAPTER FOUR

SIMULATION

Cointegration analysis is a method developed to reveal whether there is a long term linear relation between more than one time series. In this method, first a linear model is constructed between two or more nonstationary series. Then, it is decided whether these series are cointegrated depending on the error terms produced by this model being stationary or not.

Various structural changes may occur in the data generation processes of the time series due to various reasons such policy changes, financial crises and natural disasters. These changes which occur in the series, and which do not have a certain definition are generally expressed as the changes in model parameters. These changes cause structural breaks in the series.

The presence of structural breaks in time series revealed that the unit root analyses of the series tend to result in being nonstationary. The cointegration analyses should be conducted considering the possible breaks in series with breaks in unit root analyses. The exclusion of structural breaks from the analyses could yield erroneous results since they would enable the cointegration parameters to take different values between periods. The widely used E-G and Johansen cointegration tests may have erroneous results since they investigate the long term relation without taking the breaks into consideration. Under such circumstances cointegration methods which consider the structural break. One of the cointegration tests which take the structural break is Gregory – Hansen (1996) test.

4.1 Power Comparison of E-G and G-H Tests for Models

This chapter compares the E-G and G-H tests using Monte-Carlo simulation. The basic approach of the Engle – Granger test, which is one of the most widely used cointegration analysis tests to investigate the long term relations between time series, is the stationary property of the error terms of a linear transformation that can be formed between two nonstationary time series.

The general model for two time series can be expressed as below:

$$Y_t = \beta X_t + u_t$$

u_t, which shows the error term in this model. If u_t is stationary, it indicates that X_t ,and Y_t series are cointegrated. The Dickey-Fuller unit root test is performed to check whether the error terms are stationary or not.

Gregory – Hansen (1996) test, which is one of the cointegration test used in the presence of structural breaks, tries to determine the structural break by adding some dummy variables to the model. Gregory – Hansen (1996) test investigates the determination of the structural breaks in the cointegration analysis in three models such as break in intercept, break in intercept with trend and break in both slope and intercept. The information about these models is given in chapter three.

The null and alternative hypotheses for both tests can be expressed as below:

H_o: The series are not cointegrated. The error terms are nonstationary.

H_1: The series are cointegrated. The error terms are stationary.

For this comparison the data production was performed using MATLAB (R2009a) software. The series are generated for three different model as break in intercept, break in intercept with trend and break in both slope and intercept according to the G – H test process.

Each data pair, generated for the cointegration tests, was repeated 10000 times and results for both tests were obtained. The data were produced from the autoregressive AR(1) process with 50, 100, 200 sample size and with $\phi = 0.1$, $\phi = 0.5, \phi = 0.9$ parameters.

Since it was thought that the magnitude of the breaks in the series could affect the power of the tests, the performances of the tests were investigated with 1, 5 and 10 breaks' magnitudes. Similarly, since it was also thought that the breaks' occurring in different positions of the series could affect the power of the tests, the breaks were applied in the first quarter *(0.25T)*, the second quarter *(0.50T)* and in the third quarter *(0.75T)* and the power of the E-G test and Gregory – Hansen (1996) test is compared.

4.1.1 Level Shift

The performances of Engle – Granger (E-G) and Gregory – Hansen (G-H) tests in the presence structural breaks which cause changes in the intercepts of the series are presented in the below.

Table 4.1 Level shift Model

LEVEL SHIFT RESULT											
Tests	Break Point	Break	n=50			n=100			n=200		
			0,1	0,5	0,9	0,1	0,5	0,9	0,1	0,5	0,9
E-G	0.25T	1	1	0,9606	0,2256	1	1	0,4322	1	1	0,8931
G-H	0.25T	1	0,9971	0,6402	0,1	1	0,9975	0,1106	1	1	0,2748
E-G	0.50T	1	0,9999	0,9309	0,2153	1	1	0,4309	1	1	0,8779
G-H	0.50T	1	0,9974	0,665	0,0933	1	0,9977	0,107	1	1	0,2857
E-G	0.75T	1	0,9999	0,9478	0,2149	1	1	0,4259	1	1	0,8862
G-H	0.75T	1	0,9981	0,6576	0,0939	1	0,9977	0,1116	1	1	0,2714
E-G	0.25T	5	0,1713	0,1057	0,1254	0,9395	0,5446	0,206	1	0,9998	0,5007
G-H	0.25T	5	0,9998	0,8763	0,1744	1	1	0,1813	1	1	0,3631
E-G	0.50T	5	0,0153	0,0223	0,0965	0,3281	0,0982	0,1382	1	0,9148	0,293
G-H	0.50T	5	1	0,8934	0,198	1	1	0,1919	1	1	0,3767
E-G	0.75T	5	0,0326	0,0309	0,0964	0,5769	0,1747	0,1433	1	0,9804	0,3455
G-H	0.75T	5	0,9997	0,8857	0,1965	1	1	0,1959	1	1	0,3601
E-G	0.25T	10	0,007	0,0227	0,0564	0,0199	0,0293	0,0733	0,226	0,0886	0,1095
G-H	0.25T	10	1	0,9986	0,6325	1	1	0,6178	1	1	0,7796
E-G	0.50T	10	0,0017	0,0072	0,047	0,0019	0,0051	0,0449	0,011	0,0148	0,0397
G-H	0.50T	10	1	0,9995	0,7159	1	1	0,3364	1	1	0,7981
E-G	0.75T	10	0,0011	0,0067	0,0366	0,0017	0,0062	0,0346	0,017	0,0189	0,0405
G-H	0.75T	10	1	0,9984	0,6839	1	1	0,6649	1	1	0,7927

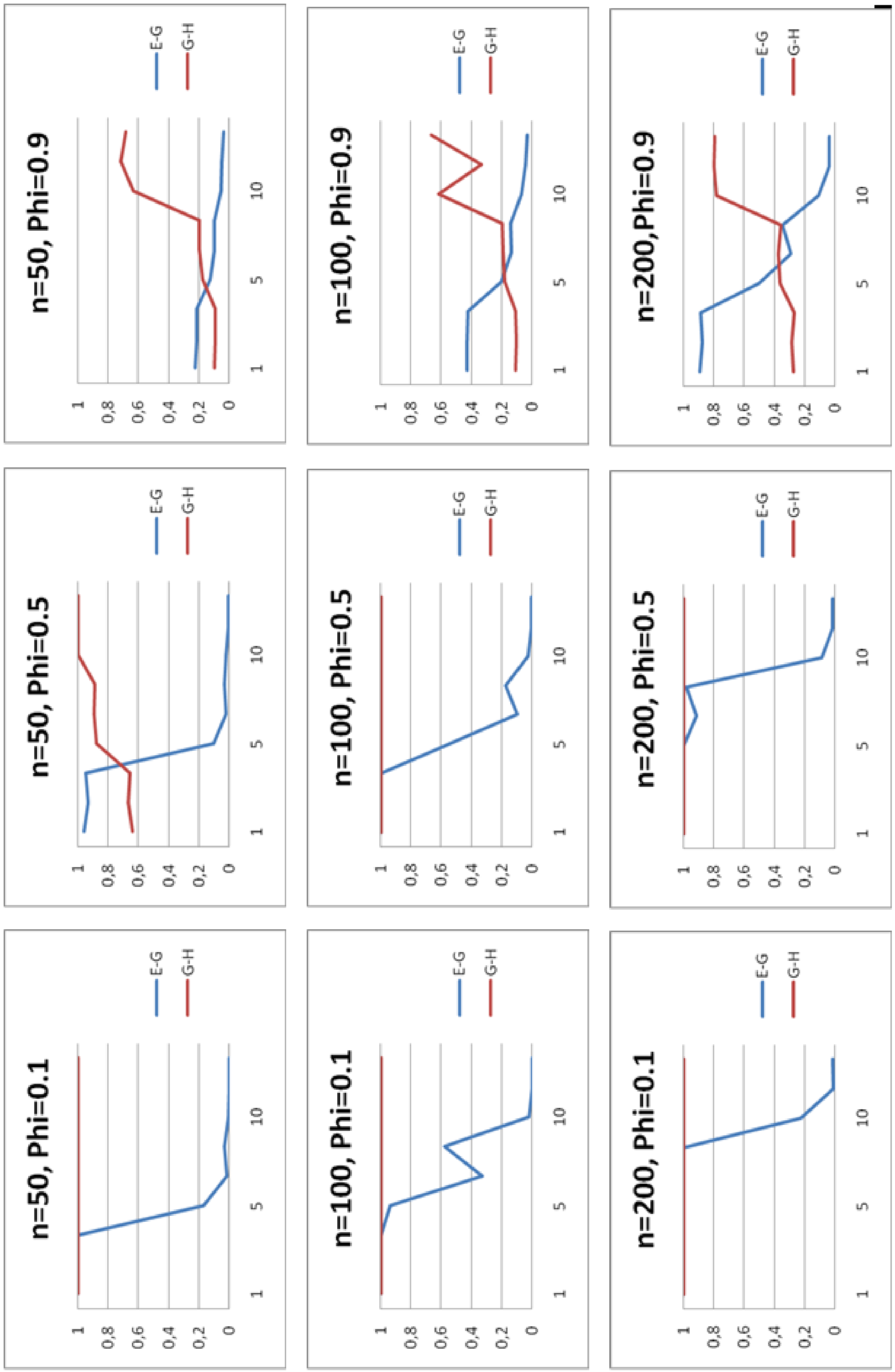

Figure 4.1 Level Shift Model

When one unit break is applied to the intercept of AR(1) model which is generated with a sample size of 50, and $\phi = 0.1$, the result of E-G cointegration test shows that all series are stationary and also cointegrated, whereas the result of G-H test shows that 99% percent out of all series are stationary and cointegrated. Since the autocorrelation degree is low, both tests' powers are found almost independent from the break point, but when the size of the break is 5 or 10, the power of E-G test decreases dramatically. In contrast with this change, as the G-H test is sensitive to breaks, there is not any loss in its power.

In other words, as the size of break increases the power of the E-G test to reveal the cointegrated structure decreases. But G-H test power does not decrease in presence of this change. The difference between two tests increases gradually. Increasing the ϕ parameter in the same sample size generally affects the power of both tests negatively.

When the sample size is 100, it is seen that the power of the E-G and G-H tests increases in all values of the phi parameter of AR(1). On the other hand, it can be argued that the E-G test is more powerful when the break magnitude is 5 compared to time when n=50.

When the sample size is 200, it is seen that the power of the E-G and G-H tests increases in all values of the phi parameter of AR(1). It can be said that the E-G test is more powerful when the break magnitude is 5 compared and when the sample size is 50 and 100.

In general, it was observed that the power E-G and G-H tests, in break in intercept model, increased as the sample size increased, but decreased as the phi parameter gets greater. While the power of E-G test decreases with the increasing break magnitude, the power of G-H test increases.

4.1.2 Level Shift with Trend

The performances of Engle – Granger (E-G) and Gregory – Hansen (G-H) tests in the presence structural breaks which cause changes in the intercepts of the series with trend are presented in the below.

Table 4.2 Level Shift with Trend (trend=0.1)

LEVEL SHIFT WITH TREND (Trend=0.1)											
Tests	Break Point	Break	n=50			n=100			n=200		
			0,1	0,5	0,9	0,1	0,5	0,9	0,1	0,5	0,9
E-G	0.25T	1	1	0,9689	0,2586	1	1	0,4733	1	1	0,9058
G-H	0.25T	1	0,9897	0,6419	0,1408	1	0,9977	0,1737	1	1	0,3912
E-G	0.50T	1	0,9999	0,9662	0,2603	1	1	0,4844	1	1	0,9151
G-H	0.50T	1	0,9918	0,6716	0,149	1	0,997	0,1799	1	1	0,4007
E-G	0.75T	1	0,9999	0,9613	0,254	1	1	0,4766	1	1	0,911
G-H	0.75T	1	0,9918	0,6541	0,1418	1	0,9976	0,1766	1	1	0,3908
E-G	0.25T	5	0,9894	0,7808	0,1924	1	0,9995	0,3659	1	1	0,802
G-H	0.25T	5	0,9971	0,8002	0,1884	1	0,9984	0,2011	1	1	0,4133
E-G	0.50T	5	0,9962	0,8592	0,2265	1	0,9999	0,4122	1	1	0,8412
G-H	0.50T	5	0,9983	0,8317	0,2013	1	0,999	0,2051	1	1	0,4006
E-G	0.75T	5	0,9715	0,733	0,2234	1	0,9997	0,392	1	1	0,804
G-H	0.75T	5	0,9982	0,8139	0,197	1	0,9996	0,2037	1	1	0,4025
E-G	0.25T	10	0,9088	0,444	0,0898	1	0,925	0,1776	1	1	0,4862
G-H	0.25T	10	0,9999	0,9853	0,4896	1	1	0,4235	1	1	0,5817
E-G	0.50T	10	0,9796	0,6739	0,1579	1	0,9918	0,2767	1	1	0,6451
G-H	0.50T	10	0,9999	0,9892	0,4891	1	1	0,4341	1	1	0,5886
E-G	0.75T	10	0,8154	0,3977	0,1558	1	0,925	0,2313	1	1	0,5207
G-H	0.75T	10	1	0,9887	0,4875	1	1	0,4327	1	1	0,5897

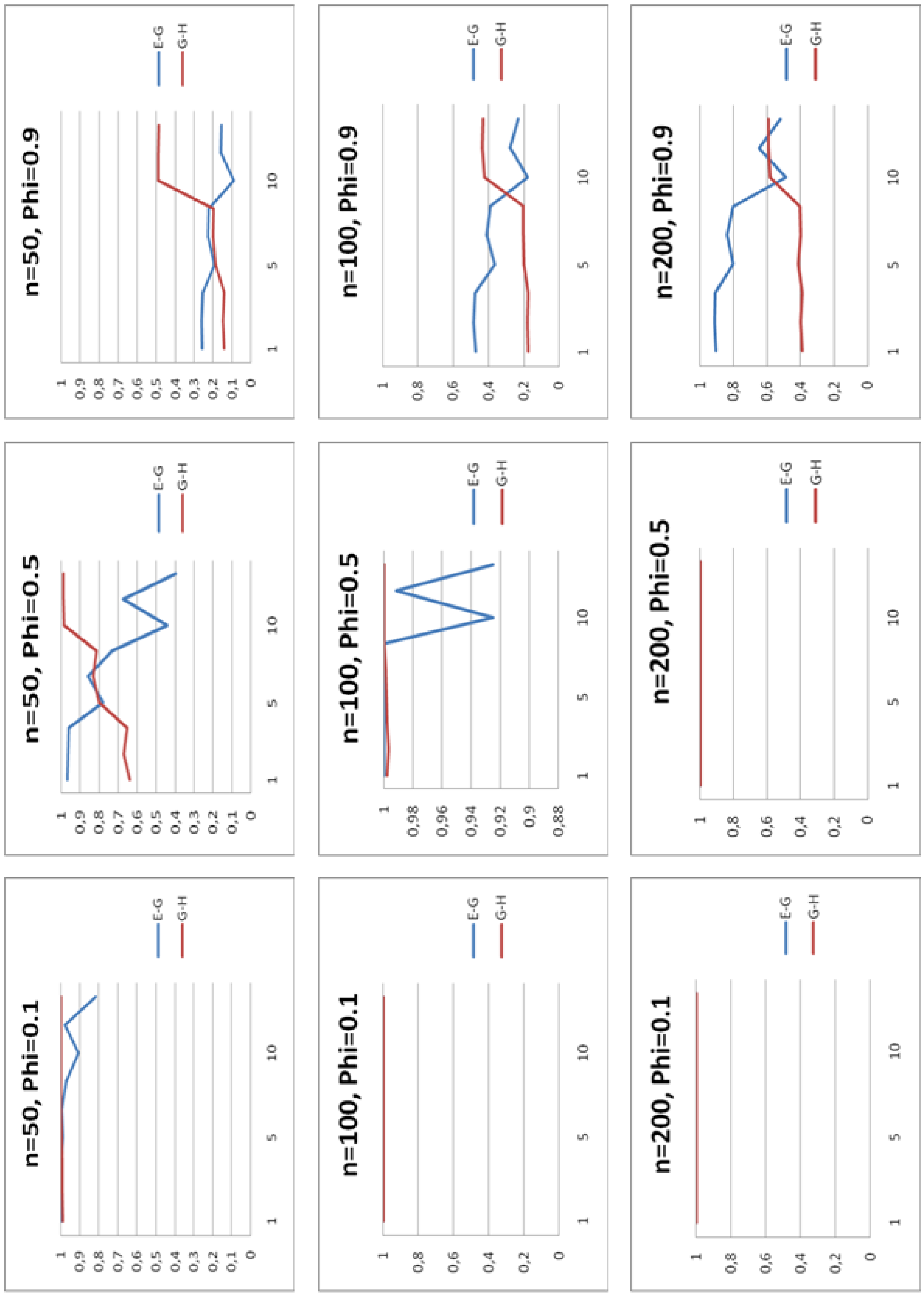

Figure 4.2 Level Shift with Trend (trend=0.1)

When the table presenting the breaks in the intercept of series with trend is examined, it is seen that G-H and E-G tests reveal the series as cointegrated significantly in all sample size as the phi parameter is 0.1. In cases when the break magnitude is 1 and 5, and the phi parameter is defined 0.5 and 0.9, it can be said that E-G test is more powerful than G-H test. On the other hand, when the break magnitude is 10 the power of G-H test increases as the power of E-G test decreases.

Generally, when the trend coefficient is 0.1 (trend slope is smaller) and when a break occurs in the intercept, there is not any statistically significant difference between tests with 0.1 phi coefficient in reveal the cointegrated structure. The trend in the series reduces the specificity of the breaks. Therefore, some deformation occur in the cointegrated structure.

On the other hand, when the break magnitude is 10, the number of the E-G cointegrated series decreases, while the G-H test can reveal this break in significant..

Table 4.3 Level Shift with Trend (trend=0.9)

LEVEL SHIFT WITH TREND(Trend=0.9)											
Tests	**Break Point**	**Break**	**n=50**			**n=100**			**n=200**		
			0,1	**0,5**	**0,9**	**0,1**	**0,5**	**0,9**	**0,1**	**0,5**	**0,9**
E-G	**0.25T**	**1**	1	0,9825	0,2673	1	1	0,4878	1	1	0,9105
G-H	**0.25T**	**1**	0,9913	0,7712	0,1502	1	0,9981	0,1839	1	1	0,3923
E-G	**0.50T**	**1**	1	0,9842	0,2658	1	1	0,4815	1	1	0,9176
G-H	**0.50T**	**1**	0,9995	0,7705	0,1517	1	0,9974	0,1793	1	1	0,4099
E-G	**0.75T**	**1**	1	0,9811	0,2594	1	1	0,4813	1	1	0,9127
G-H	**0.75T**	**1**	0,9993	0,2304	0,1479	1	0,9978	0,1859	1	1	0,3992
E-G	**0.25T**	**5**	0,9831	0,7068	0,1883	1	0,9981	0,3556	1	1	0,7922
G-H	**0.25T**	**5**	0,9994	0,826	0,1823	1	0,9996	0,1946	1	1	0,3949
E-G	**0.50T**	**5**	0,9978	0,8401	0,2236	1	0,9999	0,407	1	1	0,8402
G-H	**0.50T**	**5**	0,9991	0,8187	0,1762	1	0,9998	0,1906	1	1	0,3938
E-G	**0.75T**	**5**	0,9893	0,7366	0,2217	1	0,9994	0,3905	1	1	0,8092
G-H	**0.75T**	**5**	0,9999	0,8253	0,185	1	0,9988	0,2013	1	1	0,4068
E-G	**0.25T**	**10**	0,4101	0,0983	0,0639	0,9888	0,6427	0,1603	1	0,9998	0,4526
G-H	**0.25T**	**10**	1	0,9804	0,4558	1	1	0,406	1	1	0,5715
E-G	**0.50T**	**10**	0,8171	0,345	0,1314	0,9999	0,9303	0,258	1	1	0,6188
G-H	**0.50T**	**10**	0,9999	0,9745	0,42	1	1	0,4114	1	1	0,5879
E-G	**0.75T**	**10**	0,515	0,1522	0,1341	0,9966	0,7621	0,2185	1	0,9998	0,5037
G-H	**0.75T**	**10**	1	0,9811	0,4416	1	1	0,3999	1	1	0,5808

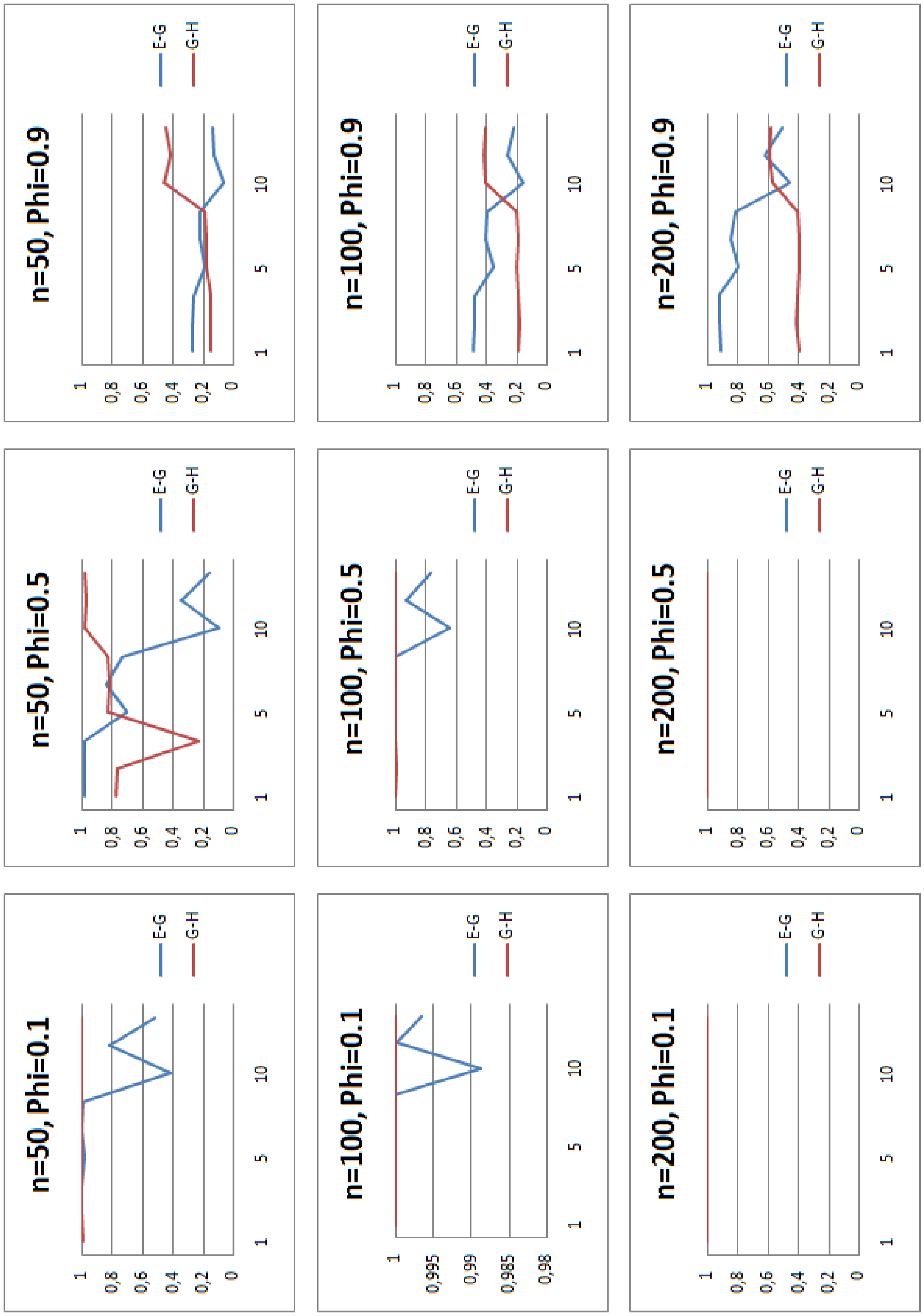

Figure 4.3 Level Shift with Trend (trend=0.9)

When the table presenting the breaks in the intercept of series with trend is examined, it is seen that G-H and E-G tests reveal the series as cointegrated significantly in all sample size and all break magnitudes as the phi parameter is 0.1.

When the break magnitude is 1 and 5, the phi parameter is 0.5 and 0.9, and the sample size 100 and 200, it can be said that E-G test is more powerful than G-H test. On the other hand, when the sample size is 50,100 and the break magnitude is 10 the power of G-H was found more powerful.

Generally, when the trend coefficient is defined as 0.9 (trend slope is high) and when a break occurs in the intercept, there is not any statistically significance difference between tests with 0.1 phi coefficient. The trend in the series reduces the specificity of the breaks. Therefore, some deformation occur in the cointegrated structure.

On the other hand, when the break magnitude is 10, the power of the E-G test decreases, while the G-H test can reveal this break. When compared to the table in which trend coefficient is 0.1, and when all phi coefficient and sample size is 50 it can be said that E-G test is more powerful than the trend is 0.1.

4.1.3. Regime Shift Model

The performances of the cointegration tests Engle-Granger (E-G) and Gregory-Hansen (G-H) in the presence of a structural break in both the slope and the intercept of the series are given in the tables below.

The information on the cointegration results of the series in which the slope value increased to 0.9 from 0.1 after the break is given in table 4.4.

Table 4.4 Regime shift (slope1=0.1, slope2=0.9)

REGIME SHIFT (trend1=0.1, trend2=0.9)											
Tests	**Break Point**	**Break**	**n=50**			**n=100**			**n=200**		
			0,1	**0,5**	**0,9**	**0,1**	**0,5**	**0,9**	**0,1**	**0,5**	**0,9**
E-G	**0.25T**	**1**	1	0,9833	0,2566	1	1	0,4836	1	1	0,9082
G-H	**0.25T**	**1**	0,9994	0,7724	0,1541	1	1	0,1869	1	1	0,4127
E-G	**0.50T**	**1**	1	0,9856	0,2773	1	1	0,4879	1	1	0,9157
G-H	**0.50T**	**1**	0,9998	0,7752	0,153	1	1	0,189	1	1	0,4124
E-G	**0.75T**	**1**	1	0,9882	0,2701	1	1	0,484	1	1	0,9124
G-H	**0.75T**	**1**	0,9998	0,7494	0,1382	1	1	0,1861	1	1	0,3929
E-G	**0.25T**	**5**	0,9917	0,7683	0,1969	1	1	0,3652	1	1	0,806
G-H	**0.25T**	**5**	0,9997	0,8392	0,1584	1	1	0,2097	1	1	0,4108
E-G	**0.50T**	**5**	0,9997	0,928	0,2417	1	1	0,4403	1	1	0,8712
G-H	**0.50T**	**5**	0,999	0,8164	0,4515	1	1	0,2378	1	1	0,4388
E-G	**0.75T**	**5**	1	0,982	0,2669	1	1	0,4608	1	1	0,889
G-H	**0.75T**	**5**	0,9998	0,7563	0,37	1	1	0,2306	1	1	0,4283
E-G	**0.25T**	**10**	0,5477	0,1695	0,0881	0,9977	0,7856	0,1722	1	1	0,5018
G-H	**0.25T**	**10**	1	0,9721	0,2276	1	1	0,4223	1	1	0,585
E-G	**0.50T**	**10**	0,9754	0,66	0,1709	1	0,9967	0,3195	1	1	0,7235
G-H	**0.50T**	**10**	0,9999	0,9044	1	1	1	0,4137	1	1	0,5962
E-G	**0.75T**	**10**	0,9998	0,9691	0,2336	1	1	0,3994	1	1	0,8215
G-H	**0.75T**	10	0,9979	0,7324	1	1	1	0,3743	1	1	0,5792

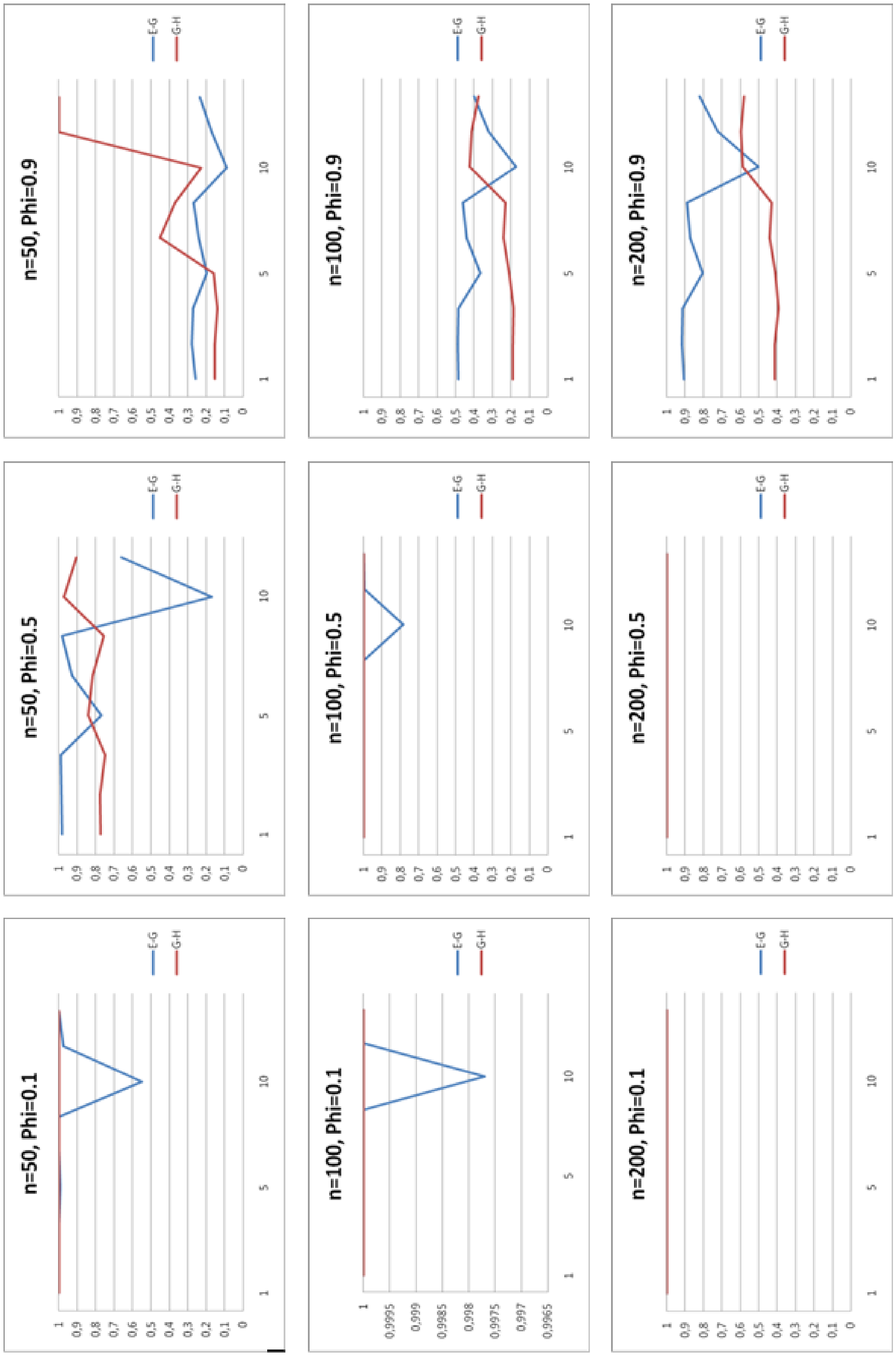

Figure 4.4 Regime shift (slope1=0.1, slope2=0.9)

Where sample size is 50, phi coefficient is 0.1 and 0.5, and the magnitude of the break is 1 and 5, it is seen that E-G test could reveal more cointegrated structures than G-H test. On the other hand, when the magnitude of the break is 10, the power of G-H test is more than E-G test. With the same sample size, when phi coefficient becomes 0.9 and break location is 0.25T the power of both test decreases.

When the sample size is 100, phi coefficient is 0.1 and 0.5, and the break point is 1 and 5, E-G and G-H tests can be argued to have a high power. On the other hand determining the phi coefficient as 0.9 decreases the power of both tests.

Where the sample size is 200, phi coefficient is 0.1 and 0.5, and break point is 1 and 5, it can be argued that both E-G and G-H tests have high power. On the other hand, determining the phi coefficient as 0.9 decreases the power of both tests. When these results are compared with the one, in which the sample size is 100, it can be said that there is a relative increase in the power of both tests.

Whatever the sample size is, if the break magnitude is taken the minimum (1), and the phi parameter is 0.1 and break is positioned at the beginning of the series, both tests have the same performance in catching up the cointegrated series.

When sample size is 50, the phi parameter is 0.1 both tests give similar results. However, as the phi parameter and the sample size increase, both of tests becomes a more powerful. In cases where the break magnitude is 10, the G-H test becomes more powerful. The main point here is that the slope is at a low level before the break; and although the slope after the break increased significantly as the break magnitude increased, the E-G test is not affected from this situation as much as the G-H test.

The information on the cointegration results of the series with a slope value of 0.5 prior to the break and with a slope value 0.9 after it is given in table 4.5

Table 4.5 Regime shift (slope1=0.5, slope2=0.9)

REGIME SHIFT (trend1=0.5, trend2=0.9)											
Tests	Break Point	Break	n=50			n=100			n=200		
			0,1	0,5	0,9	0,1	0,5	0,9	0,1	0,5	0,9
E-G	**0.25T**	**1**	1	0,9853	0,2538	1	1	0,4833	1	1	0,9104
G-H	**0.25T**	**1**	0,999	0,7763	0,15	1	1	0,1828	1	1	0,3964
E-G	**0.50T**	**1**	1	0,9861	0,2641	1	1	0,4838	1	1	0,9184
G-H	**0.50T**	**1**	0,9996	0,7843	0,1483	1	1	0,1831	1	1	0,4062
E-G	**0.75T**	**1**	1	0,9882	0,2625	1	1	0,4862	1	1	0,924
G-H	**0.75T**	**1**	0,9992	0,7669	0,1467	1	1	0,1836	1	1	0,3985
E-G	**0.25T**	**5**	0,9922	0,7388	0,1888	1	1	0,3566	1	1	0,7973
G-H	**0.25T**	**5**	0,9996	0,8235	0,1819	1	1	0,1972	1	1	0,4074
E-G	**0.50T**	**5**	0,9991	0,905	0,2305	1	1	0,4227	1	1	0,8526
G-H	**0.50T**	**5**	0,9997	0,8156	0,1825	1	1	0,2056	1	1	0,4178
E-G	**0.75T**	**5**	1	0,9594	0,2418	1	1	0,4174	1	1	0,8403
G-H	**0.75T**	**5**	0,9987	0,8046	0,1834	1	1	0,2158	1	1	0,4274
E-G	**0.25T**	**10**	0,4497	0,1174	0,0721	0,9946	0,7099	0,1576	1	1	0,4632
G-H	**0.25T**	**10**	1	0,9768	0,4515	1	1	0,4075	1	1	0,5719
E-G	**0.50T**	**10**	0,9487	0,5277	0,1501	1	0,9866	0,2806	1	1	0,6622
G-H	**0.50T**	**10**	0,9997	0,9317	0,4033	1	1	0,393	1	1	0,5722
E-G	**0.75T**	**10**	0,9998	0,8136	0,1718	1	0,9998	0,2825	1	1	0,6348
G-H	**0.75T**	10	0,9986	0,8799	0,4116	1	1	0,4283	1	1	0,5823

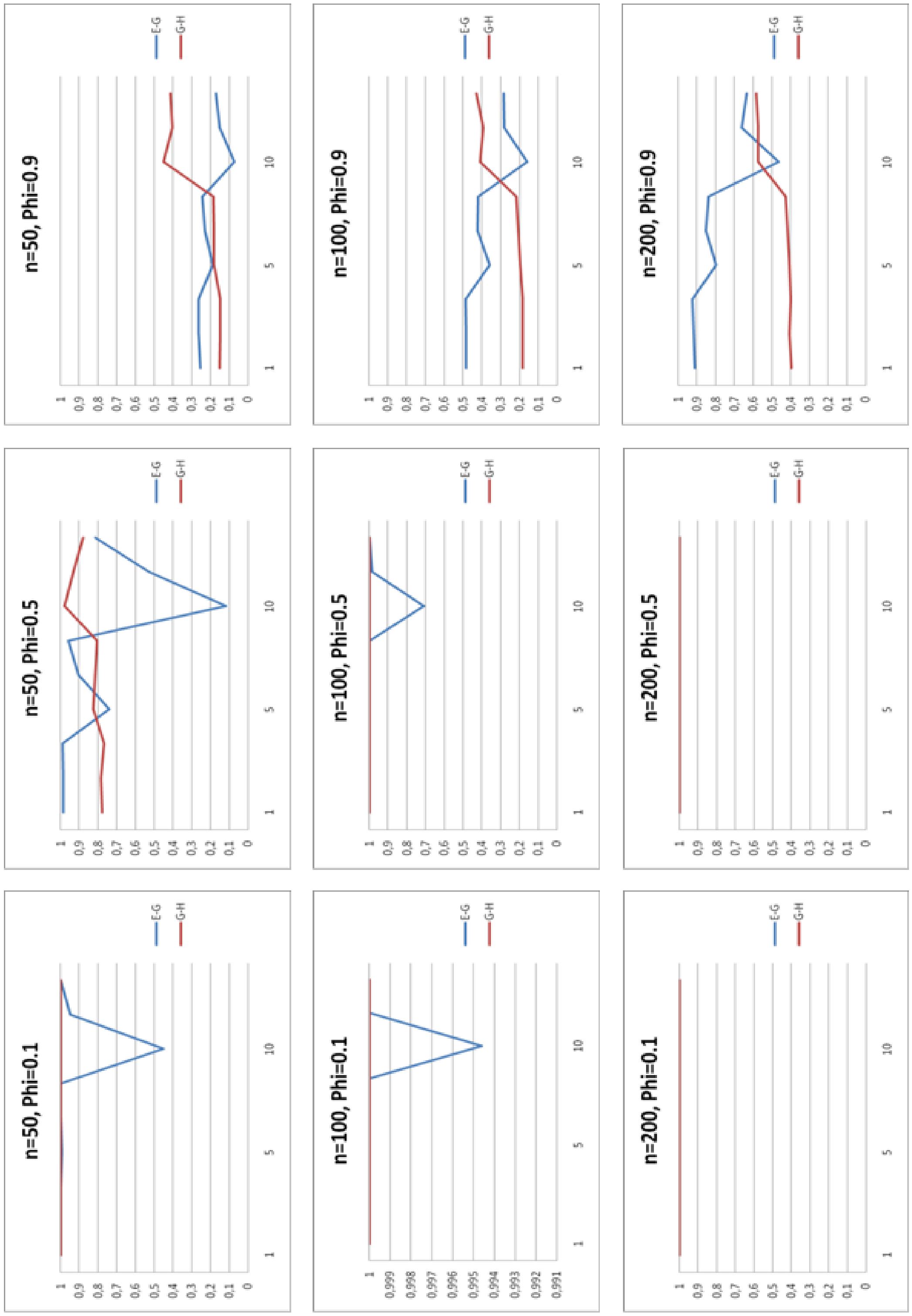

Figure 4.5 Regime shift (slope1=0.5, slope2=0.9)

Where the sample size is 50, phi coefficient is 0.1 and 0.5, and the break magnitude is 1 and 5, it can be said that E-G test has greater success in catching up more cointegrated structures than G-H test does. On the other hand, when the break magnitude is 10, the power of G-H test is more than the E-G test. With the same sample size, when the phi coefficient is 0.9, power of both tests decreases. In addition to this result, in cases where phi coefficient is 0.9 and break magnitude is 1 and 5 E-G gives more powerful results compared to G-H test, but break magnitude being 10 increases the power of G-H test.

When sample size is 100 and 200, phi coefficient is 0.1 and 0.5, and break magnitude is 1 and 5, it can be said that both E-G and G-H test have high power. On the other hand, when the phi coefficient is 0.9, the power of both tests decreases.

When the results of sample size 100 and 200 are compared, it can be argued that expansion of the sample size causes an increase in the power values of both tests. Here the main point to be emphasized is that the power of the tests decreases as the break magnitude increases. In addition, the G-H test is affected from the break point. In cases when there is a middle slope before the break, and there is a strong break and thus the trend slope becomes higher, it can be argued that G-H test has less power than E-G test.

The information on the cointegration results of series with a slope value 0.9 before the break and a slope value 0.1 after it is given in table 4.6.

Table 4.6 Regime shift (slope1=0.9, slope2=0.1)

REGIME SHIFT (trend1=0.9, trend2=0.1)											
Tests	**Break Point**	**Break**	**n=50**			**n=100**			**n=200**		
			0,1	**0,5**	**0,9**	**0,1**	**0,5**	**0,9**	**0,1**	**0,5**	**0,9**
E-G	**0.25T**	**1**	1	0,9812	0,2667	1	1	0,4729	1	1	0,9136
G-H	**0.25T**	**1**	0,9994	0,7756	0,1575	1	0,9991	0,1817	1	1	0,3971
E-G	**0.50T**	**1**	1	0,985	0,2649	1	1	0,4864	1	1	0,9141
G-H	**0.50T**	**1**	0,999	0,7775	0,1453	1	1	0,1779	1	1	0,3988
E-G	**0.75T**	**1**	1	0,9826	0,2575	1	1	0,4829	1	1	0,9096
G-H	**0.75T**	**1**	0,9994	0,7683	0,1475	1	0,9993	0,1839	1	1	0,3984
E-G	**0.25T**	**5**	0,9877	0,7082	0,1925	1	0,9987	0,3582	1	1	0,7879
G-H	**0.25T**	**5**	0,9995	0,8342	0,1792	1	0,9991	0,1965	1	1	0,399
E-G	**0.50T**	**5**	0,9976	0,8466	0,2177	1	0,9996	0,4077	1	1	0,8445
G-H	**0.50T**	**5**	0,9994	0,8162	0,1756	1	0,9988	0,1968	1	1	0,4042
E-G	**0.75T**	**5**	0,9962	0,7794	0,2226	1	0,9997	0,3907	1	1	0,8114
G-H	**0.75T**	**5**	0,9994	0,8291	0,1825	1	0,9995	0,2037	1	1	0,4117
E-G	**0.25T**	**10**	0,4041	0,1011	0,0701	0,9878	0,6509	0,158	1	1	0,455
G-H	**0.25T**	**10**	1	0,9761	0,4492	1	1	0,4073	1	1	0,5769
E-G	**0.50T**	**10**	0,8504	0,3677	0,1392	1	0,9429	0,2599	1	1	0,6358
G-H	**0.50T**	**10**	0,9999	0,9727	0,4267	1	1	0,3892	1	1	0,5681
E-G	**0.75T**	**10**	0,7181	0,2111	0,1264	0,9998	0,8476	0,2141	1	1	0,5168
G-H	**0.75T**	**10**	1	0,9749	0,4419	1	1	0,4119	1	1	0,5735

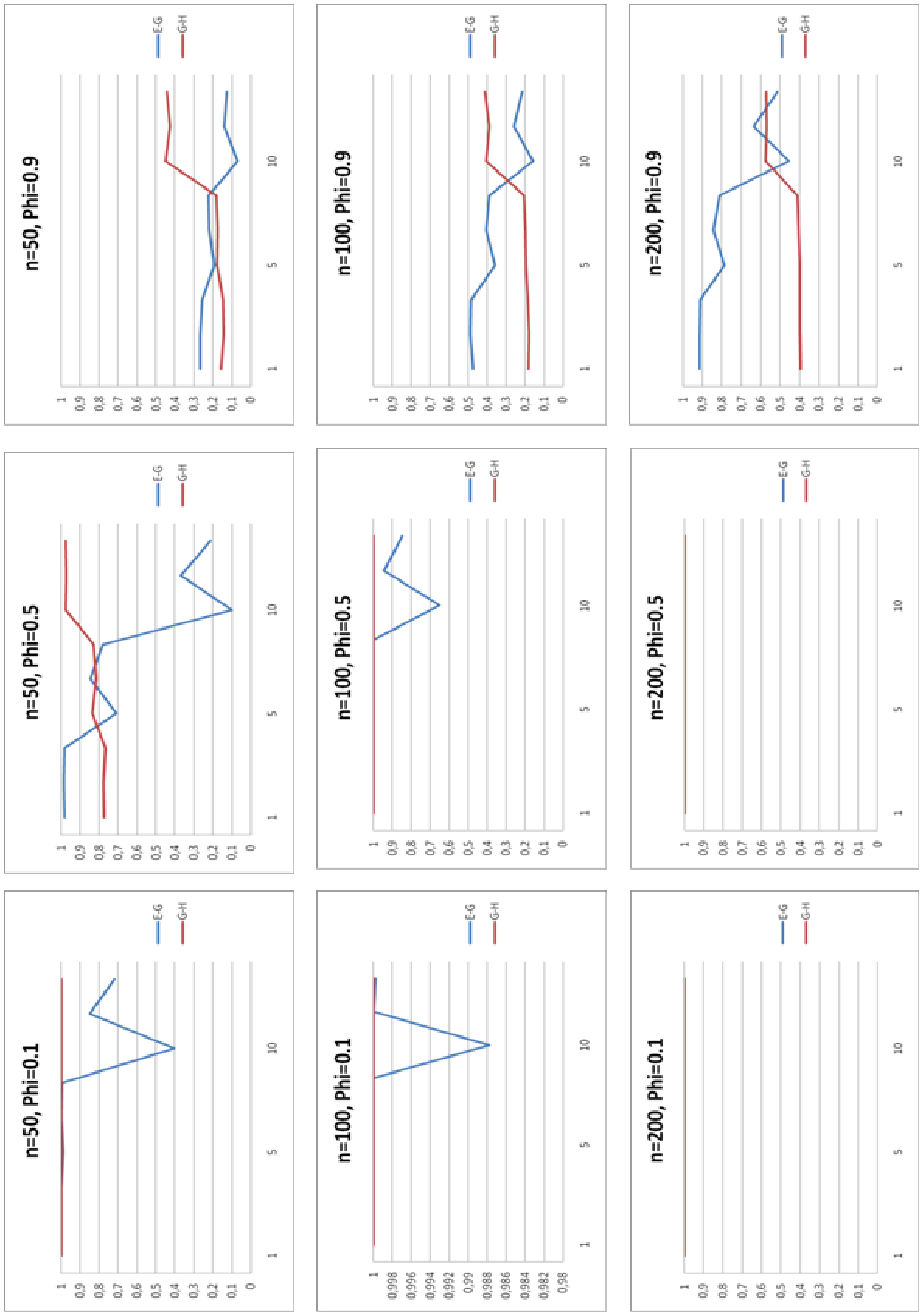

Figure 4.6 Regime shift (slope1=0.9, slope2=0.1)

Where the sample size is 50, phi coefficient is 0.1 and 0.5, and the break magnitude is 1 and 5, it can be said that both E-G and G-H test have the same and high power. On the other hand, when the break magnitude is 10, the power of G-H test is more than the E-G test. With the same sample size, when the phi coefficient is 0.5 and 0.9, and break magnitude is 1 and 5, it is seen that E-G is more powerful than G-H test. In addition, in cases where phi coefficient is 0.9 and break magnitude is 10 the power G-H test is more than E-G test.

In cases when the sample size is 100 and 200, phi coefficient is 0.1, 0.5, and the break magnitude is 1 and 5, it can be said that both of test are powerful. On the other hand, when the break magnitude is 10, and phi coefficient is 0.9, G-H is more powerful than E-G test.

When the results of sample size 100 and sample size 200 are compared, it can be argued that expansion of the sample size causes an increase in the power values of both tests.

The information on the cointegration results of series with a slope value 0.9 before the break and a slope value -0.1 after break, it is given in table 4.7.

Table 4.7 Regime shift (slope1=0.9, slope2= -0.1)

REGIME SHIFT (trend1=0.9, trend2=-0.1)											
Tests	Break Point	Break	n=50			n=100			n=200		
			0,1	0,5	0,9	0,1	0,5	0,9	0,1	0,5	0,9
E-G	0.25T	1	1	0,9824	0,2673	1	1	0,4715	1	1	0,9111
G-H	0.25T	1	0,9993	0,7677	0,1498	1	0,9994	0,1842	1	1	0,3969
E-G	0.50T	1	0,9999	0,9839	0,2653	1	1	0,4763	1	1	0,9093
G-H	0.50T	1	0,9991	0,7668	0,1519	1	0,9991	0,1769	1	1	0,403
E-G	0.75T	1	0,9999	0,9808	0,259	1	1	0,4792	1	1	0,9043
G-H	0.75T	1	0,9991	0,7667	0,1476	1	0,9995	0,1831	1	1	0,4054
E-G	0.25T	5	0,982	0,704	0,1883	1	0,9987	0,3608	1	1	0,7913
G-H	0.25T	5	0,9995	0,8256	0,1822	1	0,9994	0,1929	1	1	0,4064
E-G	0.50T	5	0,9957	0,827	0,2222	1	0,9985	0,4071	1	1	0,8387
G-H	0.50T	5	0,999	0,8181	0,1752	1	0,9998	0,1944	1	1	0,3859
E-G	0.75T	5	0,97	0,6831	0,2195	1	0,9989	0,3807	1	1	0,8052
G-H	0.75T	5	0,9997	0,8221	0,1838	1	0,9994	0,1967	1	1	0,401
E-G	0.25T	10	0,4139	0,098	0,0637	0,9876	0,6378	0,1622	1	1	0,4506
G-H	0.25T	10	1	0,9811	0,4564	1	1	0,4021	1	1	0,5737
E-G	0.50T	10	0,7573	0,3158	0,1302	0,9994	0,9147	0,2561	1	1	0,6192
G-H	0.50T	10	0,9999	0,9766	0,4218	1	1	0,3981	1	1	0,559
E-G	0.75T	10	0,278	0,0973	0,1301	0,9667	0,6325	0,201	1	1	0,492
G-H	0.75T	10	1	0,9824	0,4437	1	1	0,4154	1	1	0,5749

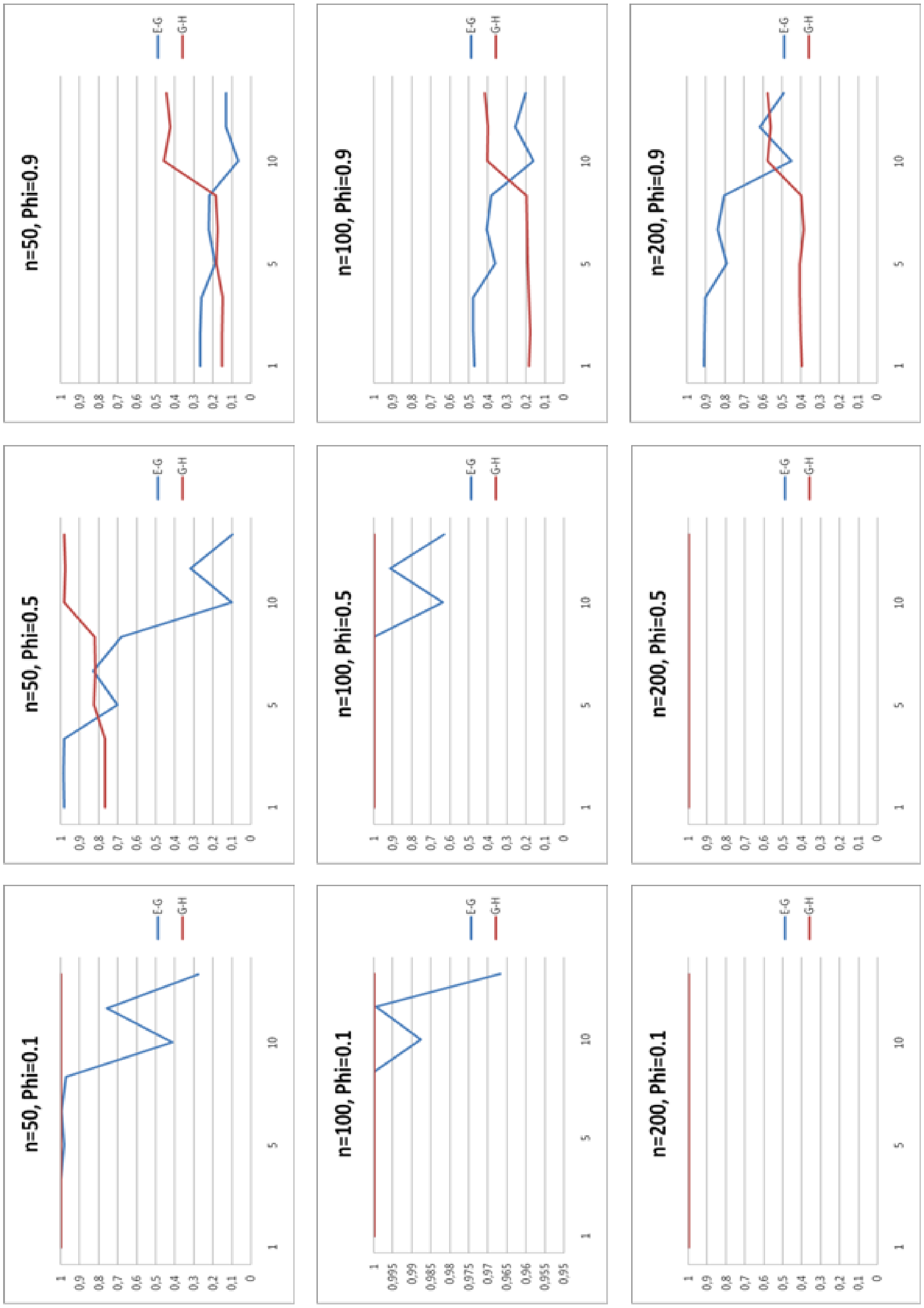

Figure 4.7 Regime shift (slope1=0.9, slope2= -0.1)

In cases where the sample size is 50, phi coefficient is 0.1 and 0.5, and the break magnitude is 1 and 5, it can be said that both E-G and G-H test have the same and high power. On the other hand, when the break magnitude is 10, the power of G-H test is more than the E-G test. With the same sample size, when the phi coefficient is 0.5 and 0.9, and break magnitude is 1 and 5, it is seen that E-G is more powerful than G-H test. In addition, in cases where phi coefficient is 0.9, 0.5 and break magnitude is 10 the power G-H test is more than E-G test.

In cases when the sample size is 100 and 200, phi coefficient is 0.1, and 0.5, in all break magnitudes the power of both tests is generally high. The only process not conforming to this result is when the sample size is 100 and the break magnitude is 10. In this kind of series, the power of E-G test is approximately 86%. In cases where phi coefficient is 0.9, a dramatic decrease in the power of both tests was observed. In addition to this decrease, the power values of the E-G test are greater than the G-H test. On the other hand, an increase in the sample size causes an increase in the power of both tests.

The main point to be emphasized here is the E-G test which is more powerful than G-H test in cases where the slope is strong before the break and becomes weaker or gets even negative values and when the break magnitude is weak; however as the break magnitude increases E-G test have less power than G-H test.

The information on the cointegration results of series with a slope value 0.1 before the break and a slope value -0.5 after the break, it is given in table 4.8.

Table 4.8 Regime shift (slope1=0.1, slope2=-0.5)

REGIME SHIFT (trend1=0.1, trend2=-0.5)											
Tests	**Break Point**	**Break**	**n=50**			**n=100**			**n=200**		
			0,1	**0,5**	**0,9**	**0,1**	**0,5**	**0,9**	**0,1**	**0,5**	**0,9**
E-G	**0.25T**	**1**	1	0,9783	0,2373	1	1	0,4764	1	1	0,9094
G-H	**0.25T**	**1**	0,9992	0,7339	0,1199	1	0,9988	0,1538	1	1	0,3848
E-G	**0.50T**	**1**	1	0,9833	0,1909	1	1	0,4232	1	1	0,8979
G-H	**0.50T**	**1**	0,9986	0,7475	0,1116	1	0,999	0,1526	1	1	0,3566
E-G	**0.75T**	**1**	1	0,9857	0,2226	1	1	0,4421	1	1	0,8968
G-H	**0.75T**	**1**	0,999	0,7063	0,1211	1	0,9992	0,1506	1	1	0,3559
E-G	**0.25T**	**5**	0,9718	0,6106	0,1562	1	0,9986	0,3445	1	1	0,7646
G-H	**0.25T**	**5**	0,9977	0,6421	0,1379	1	0,9979	0,1573	1	1	0,3575
E-G	**0.50T**	**5**	0,9987	0,8795	0,0888	1	1	0,2287	1	1	0,6209
G-H	**0.50T**	**5**	0,9973	0,6579	0,1265	1	0,9973	0,1532	1	1	0,3482
E-G	**0.75T**	**5**	1	0,9444	0,0871	1	1	0,1943	1	1	0,5369
G-H	**0.75T**	**5**	0,9977	0,698	0,1705	1	0,9974	0,1922	1	1	0,3859
E-G	**0.25T**	**10**	0,0457	0,0104	0,0675	0,952	0,4698	0,1622	1	1	0,4333
G-H	**0.25T**	**10**	0,9997	0,82	0,4397	1	0,9999	0,4005	1	1	0,5517
E-G	**0.50T**	**10**	0,7348	0,2411	0,0238	1	0,9827	0,0597	1	1	0,1619
G-H	**0.50T**	**10**	0,99	0,436	0,3776	1	0,9968	0,3643	1	1	0,5392
E-G	**0.75T**	**10**	0,9989	0,6075	0,0091	1	0,9998	0,0157	1	1	0,0529
G-H	**0.75T**	**10**	0,9924	0,6407	0,4221	1	0,9969	0,4008	1	1	0,5631

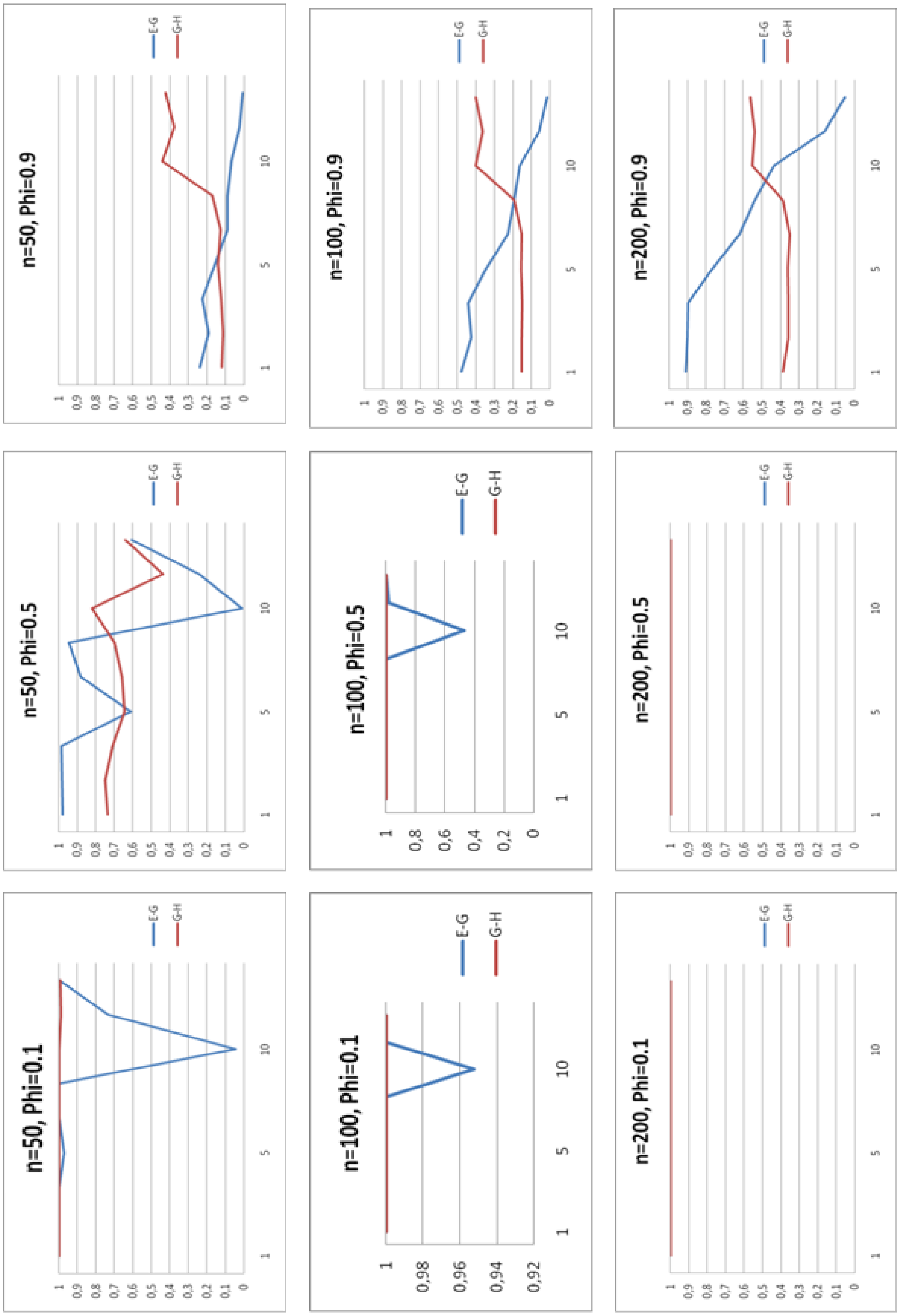

Figure 4.8 Regime shift (slope1=0.1, slope2=-0.5)

In cases where the sample size is 50, phi coefficient is 0.1 and 0.5, and the break magnitude is 1 and 5, it can be said that both E-G and G-H test have the same and high power. On the other hand, when the break magnitude is 10, the power of G-H test is more than the E-G test. With the same sample size, when the phi coefficient is 0.5 and 0.9, and break magnitude is 1 and 5, it is seen that E-G is more powerful than G-H test. In addition, in cases where phi coefficient is 0.9 and break magnitude is 10 the power G-H test is more than E-G test.

In cases when the sample size is 100 and 200, phi coefficient is 0.1, and 0.5, in all break magnitudes the power of both tests is generally high. In cases where phi coefficient is 0.9, a dramatic decrease in the power of both tests was observed. In addition to this decrease, the power values of the E-G test are greater than the G-H test. In addition, break magnitude is 10 the power G-H test is more than E-G test. On the other hand, an increase in the sample size causes an increase in the power of both tests.

Here the essential point is, when the slope before the break is weakly positive, and it becomes a middle negative with the magnitude of the break, and when the break magnitude is weak, E-G test is more powerful than G-H test independent from the sample size. The power of G-H test gets greater than E-G test as the break magnitude increases. When the results are examined with respect to break point, while an inference for E-G test in strong breaks could not be done; for G-H test the break occurring in the middle region of the series increases the power of the test.

The information on the cointegration results of series with a slope value 0.5 before the break and a slope value -0.9 after the break, it is given in table 4.9.

Table 4.9 Regime shift (slope1=0.5, slope2=-0.9)

REGIME SHIFT (trend1=0.5, trend2=-0.9)											
Tests	Break Point	Break	n=50			n=100			n=200		
			0,1	0,5	0,9	0,1	0,5	0,9	0,1	0,5	0,9
E-G	**0.25T**	**1**	1	0,9758	0,2668	1	1	0,4766	1	1	0,91
G-H	**0.25T**	**1**	0,998	0,6079	0,1491	1	0,9978	0,1751	1	1	0,3899
E-G	**0.50T**	**1**	1	0,9751	0,259	1	1	0,4798	1	1	0,9159
G-H	**0.50T**	**1**	0,9984	0,6707	0,1443	1	0,9973	0,1784	1	1	0,4012
E-G	**0.75T**	**1**	1	0,9737	0,2525	1	1	0,4775	1	1	0,9103
G-H	**0.75T**	**1**	0,9983	0,3097	0,1418	1	0,9977	0,1754	1	1	0,3956
E-G	**0.25T**	**5**	0,9882	0,6107	0,1803	1	0,9931	0,3543	1	1	7,8889
G-H	**0.25T**	**5**	0,9971	0,5574	0,1675	1	0,9947	0,1917	1	1	0,3877
E-G	**0.50T**	**5**	0,9999	0,2974	0,2053	1	0,9663	0,3818	1	1	0,815
G-H	**0.50T**	**5**	0,9968	0,7609	0,167	1	0,9994	0,1763	1	1	0,3684
E-G	**0.75T**	**5**	0,9989	0,2092	0,1866	1	0,9353	0,3336	1	1	0,7441
G-H	**0.75T**	**5**	0,999	0,821	0,1651	1	0,9993	0,1798	1	1	0,377
E-G	**0.25T**	**10**	0,147	0,3815	0,0681	0,9919	0,574	0,1527	1	0,999	0,4419
G-H	**0.25T**	**10**	0,9981	0,7526	0,0469	1	0,9998	0,4128	1	1	0,5683
E-G	**0.50T**	**10**	0,7754	0,0006	0,1154	1	0,0093	0,2167	1	0,867	0,5396
G-H	**0.50T**	**10**	0,9954	0,8968	0,4475	1	1	0,3975	1	1	0,5595
E-G	**0.75T**	**10**	0,6191	0	0,075	1	0,0009	0,1248	1	0,615	0,3171
G-H	**0.75T**	**10**	0,9987	0,9453	0,4686	1	1	0,4093	1	1	0,5653

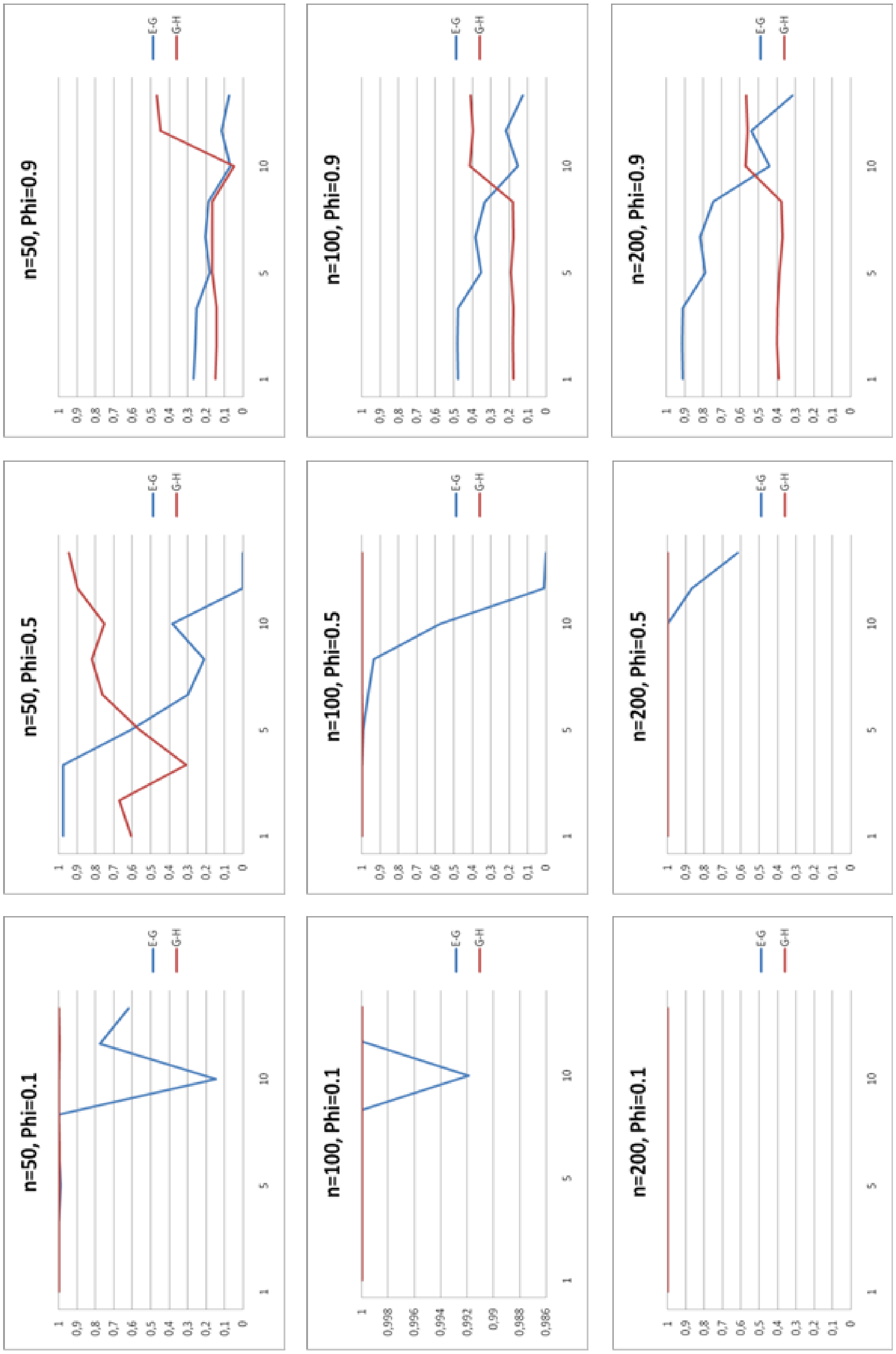

Figure 4.9 Regime shift (slope1=0.5, slope2=-0.9)

In cases where the sample size is 50, phi coefficient is 0.1 and 0.5, and the break magnitude is 1 and 5, it can be said that both E-G and G-H test have the same and high power. On the other hand, when the break magnitude is 10, the power of G-H test is more than the E-G test. With the same sample size, the phi coefficient is 0.5 and 0.9, and break magnitude is 1 and 5, it is seen that E-G is more powerful than G-H test. In addition, in cases where phi coefficient is 0.9 and break magnitude is 10 the power G-H test is more than E-G test.

In cases when the sample size is 100 and 200, phi coefficient is 0.1, and 0.5, in all break magnitudes the power of both tests is high. The only process not conforming to this result is when the break magnitude is 10. In this kind of series, the power of E-G test dramatically decreases. In cases where phi coefficient is 0.9, a dramatic decrease in the power of both tests was observed. In addition to this decrease, the power values of the E-G test are greater than the G-H test when the break magnitude is 1 and 5. When the break magnitude is 10, the power of G-H test is powerful than E-G test. On the other hand, an increase in the sample size causes an increase in the power of both tests.

Here the main point is, when the slope before the break is middle positive, and it becomes a strong negative with the magnitude of the break, and when the break magnitude is weak, E-G test is more powerful than G-H test independent from the sample size. The power of G-H test gets greater than E-G test as the break magnitude increases.

CHAPTER FIVE

CONCLUSION

This study presented a power comparison of the widely used Engle-Granger and Gregory-Hansen (1996) tests using the Monte-Carlo Simulation. For this comparison, data generated is performed using MATLAB (R2009a) software. The series are produced for three different models according to the Gregory-Hansen test procedure as break in intercept, break in intercept with trend, and break in both the slope and the intercept. The data are generated from the AR(1) procedure with a sample size of 50, 100, 200 and with $\phi = 0.1$, $\phi = 0.5$ and $\phi = 0.9$ parameters.

Since it is thought that break magnitude and the point in the series would have effect on the power of the tests, break magnitudes 1, 5 and 10 and breaks points first quarter *(0.25T)*, second quarter *(0.50T)* and the third quarter *(0.75T)* are applied on the series and the power comparison between the Engle- Granger and Gregory – Hansen tests is conducted.

According to the results obtained from the models constructed using different break types; for the model with break in intercept, it was found that both E-G and G-H tests had high power with small sample sizes and low break magnitude. With the increase in the break magnitude the power of E-G test decreases while the power of G-H test does not change since this test is sensitive to breaks. Increasing the value of the phi parameter with the same sample size generally affects the power of both tests negatively.

Generally, it was observed that the power of E-G and G-H tests increased as the sample size increased, and decreased as the value of the phi parameter increased. While the power of E-G test decreases with the increase in the break magnitude, the power of G-H tests increases.

When the results of break in intercept with trend model are investigated; in all sample sizes and all break magnitude, when the phi parameter is defined as 0.1, E-G and G-H tests catch up the cointegrated structure. It can be argued that the E-G test is more powerful than G-H test when the break magnitude is 1 and 5 and the phi parameter is 0.5 and 0.9. On the other hand, when the break magnitude is 10 the power values of the G-H test increases while the values of E-G test decreases. In addition to all these results, the trend in the series reduces the specificity of the breaks. Therefore, some deformation occur in the cointegrated structure. Generally when the trend increases the power of E-G test smaller than the G-H test powers. Here, the main point is that while E-G test is affected from the break point, this does not change the power of G-H test.

For the regime shift model, which expresses breaks in both the slope and the intercept of the series, series were generated with different slope options and the power of the tests were compared. Considering all these options, it can be argued that the power of the tests increases as the sample size increases, and the tests have high power when the phi parameter is 0.1, regardless of sample size. In addition, it is observed that the power of E-G is higher than G-H test when the break magnitude is 1 and 5, and the G-H test become more powerful than E-G test with the increase in break magnitude.

Considering all models, defining the sample size as 50, it is observed that the power of E-G test dramatically decreases with the increase of break magnitude; on the other hand, the power parameters of G-H tests increases. Similarly, it was observed that the power values of the tests are high when the AR(1) parameter phi coefficient is low (0.1, 0.5), but the values decreases with the increase in the phi coefficient. Generally, it is found that there are decreases in the power values of E-G test, and increases in the power values of G-H test with the increase in the break magnitude.

The aim of the study is not only to investigate the power of E-G and G-h tests, two most widely used cointegration tests in the literature, with regard to the sample size, but also to show that the power of the tests depend on the structural break point, break magnitude, the condition that the AR(1) parameter phi having low, middle and high autocorrelation. The most significant finding obtained at the end of the study is revealing that G-H test overtops the E-G test by investigating breaks in different models with various parameters such as break magnitude, break point, presence of a trend in the series with break, and the slope degree of the trend.

REFERENCES

Akdi, Y. (2003). *Zaman serileri analizi (birim kökler ve kointegrasyon.* Ankara: Bıçaklar Kitabevi

Aktan, H. (2007). *Yapısal kırılma ortak bütünleme ve nedensellik analizi dört ülke uygulaması: Türkiye, Yunanistan, Kuzey Kıbrıs Türk Cumhuriyeti ve Güney Kıbrıs Rum Kesimi,* İstanbul: Yüksek Lisans Tezi, Marmara Üniversitesi Sosyal Bilimler Enstitüsü Ekonometri Anabilim Dalı

Elma, Ç. A. (2008).*Yapısal kırılmalar altında birim kök testleri ve eşbütünleşme analizi: Para talebi istikrarı,* Ankara: Yüksek Lisans Tezi, Gazi Üniversitesi Sosyal Bilimler Enstitüsü Ekonometri Anabilim Dalı

Enders, W. (1995). Applied econometric time series (2nd ed.). Iowa State University: John Wiley & Sons.Inc.

Gregory, A. W., & Hansen B. E. (1996) Residual-based tests for cointegration in models with regime shifts. *Journal of Econometrics*, 70, 99-126

Hamilton, J. D. (1994). *Time series analysis.* New Jersey: Princeton University Press.

Kadılar, C. (2000). *Uygulamalı Çok Değişkenli Zaman Serileri Analizi*. Ankara: Bizim Büro Basımevi.

Maddala, G. S. (1992). Introduction to econometrics (2nd ed.). New York: Macmillan Publishing Company

Maddala G. S. & Kim I.M. (2003). *Unit roots, cointegration and structural change* (5th ed.). Cambridge: Cambridge University Press.

Perron, P.(1989). The great crash, the oil-price shock, and the unit-root hypothesis. Econometrica, 57 (6), 1361-1401.

Yurdakul, F. (2000). Yapısal kırılmaların varlığı durumunda geliştirilen birim-kök testleri. *Gazi Üniversitesi İ.İ.B.F. Dergisi*, 2, 21-34

Zivot, E., & Andrews D.W.K. (1992). Further evidence on the great crash, the oil-price shock, and the unit-root hypothesis. Journal of Business & Economic Statistics 10 (3), 251-270