## DOKUZ EYLUL UNIVERSITY GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

# MODELLING OF TRAFFIC FLOW INTERACTIONS AT UNINTERRUPTED FLOW FACILITIES

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> July, 2011 İZMİR

## MODELLING OF TRAFFIC FLOW INTERACTIONS AT UNINTERRUPTED FLOW FACILITIES

A Thesis Submitted to the Graduate School of Natural and Applied Sciences of Dokuz Eylül University In Partial Fulfillment of the Requirements for the Degree of Master of Science in Civil Engineering, Transportation Engineering Program

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> > July, 2011 İZMİR

## M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled "MODELLING OF TRAFFIC FLOW INTERACTIONS AT UNINTERRUPTED FLOW FACILITIES" completed by ÖZLEM CEYHAN under supervision of ASSOC. PROF. DR. SERHAN TANYEL and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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## MODELLING OF TRAFFIC FLOW INTERACTIONS AT UNINTERRUPTED FLOW FACILITIES

#### ABSTRACT

The ring roads and freeway passages are proposed and applied as promising solutions to traffic congestion problem in metropolitan areas. However, improper usage of lanes at uninterrupted sections lead to capacity losses at ramp junctions or weaving areas, and long queues and congestion occur especially during the peak hours.

In many traffic applications vehicle time headway is efficiently used to define traffic flow characteristics such as ramp merging, passing, crossing and miscroscopic traffic simulation. Besides, the properties of headways are extensively studied to present a distribution model. Accurate modeling and analysis of vehicle headway distribution helps to minimize vehicle delays and maximize roadway capacity.

Additionally, the characterization of the flow variation between freeway lanes is significant for several reasons. First, lane flow variations and differences of time headway affect the overall capacity of a freeway. Moreover, lane effects are taken into account especially in case of lane selection and lane changing behavior of drivers.

Although the statistical properties of the vehicle time headways at several signalized intersections have been studied before, such an analysis has not been investigated for uninterrupted flows in Turkey.

In the presented study, the statistical characteristics of headway data observed from four different three-lane freeways in İzmir are briefly described. By using Kolmogorov-Smirnov test, the most suitable headway distribution model for a certain traffic condition is proposed. Besides, the analysis of vehicle time headways collected from the same locations is presented in lane by lane principle. Finally, delay parameter and proportion of unbunched vehicles at uninterrupted flows are determined.

**Keywords**: Time headway, three-lane freeways, distribution models, proportion of free vehicles, delay parameter estimation, lane by lane analysis.

## KESİNTİSİZ AKIMLARDA TRAFİK AKIM ETKİLEŞİMLERİNİN MODELLENMESİ

## ÖΖ

Çevre yolları ve otoyollar, büyükşehirlerdeki trafik tıkanıklarına en uygun çözüm olarak önerilir ve uygulanırlar. Ancak, yolların kesintisiz bölümlerinde şeritlerin uygun olmayan kullanımı, yanyol katılımlarında ya da örülme sahalarında kapasite kayıplarına sebep olmaktadır. Bu durum ise, özellikle pik saatlerde uzun kuyruk oluşumlarına ve tıkanıklıklara yol açmaktadır.

Zaman cinsinden aralık değerleri yan yol katılımı, sollama, kavşak geçişi ve mikroskobik trafik simülasyonu gibi birçok trafik uygulamasında etkili bir biçimde kullanılmaktadır. Ayrıca, zaman cinsinden aralıkların özellikleri bir dağılım modeli ortaya koymak için yaygın olarak çalışılmaktadır. Araçların aralık dağılımlarının doğru modellenmesi ve analizi araç gecikmelerini azaltmaya ve yol kapasitesini arttırmaya yardımcı olmaktadır.

Ek olarak, çevre yollarının şeritlerinin arasındaki akım değişiminin karakterizasyonu birçok nedenle önemlidir. Bunlardan ilki, şerit akım değişimleri ve zaman cinsinden aralık farklılıkları tüm çevre yolunun kapasitesini etkilemektedir. Ayrıca şerit etkileri, özellikle sürücüler için şerit seçimi ve şerit değiştirme davranışında dikkate alınmaktadır.

Birçok sinyalize kavşakta araçların zaman cinsinden aralıklarının istatistiksel özellikleri daha önce incelenmiş olmasına rağmen, Türkiye'de kesintisiz akımlar için böyle bir inceleme yapılmamıştır.

Sunulan bu çalışmada, İzmir'de bulunan üç şeritli çevre yollarından farklı dört tanesinden elde edilen zaman cinsinden aralık verilerinin istatistiksel karakteristikleri tanımlanmıştır. Kolmogorov-Smirnov testi kullanılarak belirli trafik koşulları için en uygun dağılım modeli belirlenmiştir. Ayrıca, aynı bölgelerden elde edilen veriler ile

şerit bazında analizler yapılmıştır. Son olarak, kesintisiz akımlarda gecikme parametresi ve serbest araç oranı hesaplanmıştır.

Anahtar sözcükler: Zaman cinsinden aralık, üç şeritli çevre yolları, dağılım modelleri, serbest araç oranı, gecikme parametresi tahmini, şeritler bazında analiz.

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## CHAPTER ONE INTRODUCTION

Vehicle headway is a measure of the temporal space between two vehicles, and is defined as: the elapsed time between the arrival of the leading vehicle and the following vehicle at a specific test point. It is usually measured in seconds. Since the average of vehicle headways is the reciprocal of flow rate, vehicle headways represent microscopic measures of flows passing a point.

In many traffic applications vehicle time headway is efficiently used to define traffic flow characteristics such as ramp merging, passing, crossing and miscroscopic traffic simulation. Besides, the properties of headways are extensively studied to present a distribution model. Accurate modeling and analysis of vehicle headway distribution helps to minimize vehicle delays and maximize roadway capacity.

Many headway models have been developed over the past decades. In general, these models can be classified into two categories: simple statistical distribution models and mixed models of two or more distributions. The mixed models are more flexible to represent headways by decomposing them into following and free-following components, but the calibration process may be too complex for field application. In practice, selection of the most suitable headway distribution for a certain traffic condition remains an open issue (Zhang, Wang, Wei & Chen, 2007).

Furthermore, the characterization of the flow variation between freeway lanes is significant for several reasons. First, lane flow variations and differences of time headway affect the overall capacity of a freeway. Moreover, lane effects are taken into account especially in case of lane selection and lane changing behavior of drivers.

Some of the researchers in Turkey made detailed studies on freeways and their macroscopic traffic properties. Most of the studies were focused on the Bosporus Bridge's connection arterials and some of the studies were focused on link capacity

estimates of freeways in Turkey (Dell'orco et al., 2009; Şahin, 2009; Şahin & Akyıldız, 2005; Şahin & Altun, 2008). Some of the studies also focused on lane utilization characteristics of Turkish drivers (Günay, 2004).

Headway studies are often used to define characteristics of uninterrupted and interrupted traffic flows; to develop capacity and performance functions for unsignalized intersections, roundabouts or on and off ramp junctions etc. In literature, many studies can be found which are focused on headway characteristics of traffic flow (Cowan, 1975; Akçelik, 2003; Al-Ghamdi, 2001; Griffiths and Hunt, 1991; Luttinen, 1996; Sullivan and Troutbeck, 1994; Zwahlen, Öner & Suravaram, 2007). These studies are only some of the studies which form the main principles of headway analysis.

In Turkey, the studies on headway characteristics of traffic flow were mainly dealing with signalized arterials, intersection approaches or roundabouts. Murat and Gedizlioğlu (2007) tried to model headways at signalized intersection approaches. Çalışkanelli and Tanyel (2010) suggested a model for proportion of unbunched vehicles for usage in Cowan's M3 distribution. Tanyel and Yayla (2003) and Çalışkanelli et al. (2009) have studied on the headway distribution modeling at single and multi lane roundabouts. One of the rare studies on headways at different lanes of an uninterrupted section of İzmir ring road was prepared by Aydın (2007). In that study she tried to show the effect of reverse-lane usage on headways and also investigated the effect of opening of new İzmir outer ring road.

This thesis is organized as follows. In Chapter Two, the statistical characteristics of headway data collected from four different freeways in İzmir are briefly described. In Chapter Three, the examination of several commonly used headway distribution models using the same headway data is presented. This is followed by the analysis of vehicle time headways in lane by lane principle. In the following, a model for delay parameter estimation and proportion of unbunched vehicles for application in Cowan's M3 distribution is suggested. The final chapter concludes the research results and proposes further research works.

## CHAPTER TWO DATA CHARACTERISTICS OF UNINTERRUPTED FLOWS

### 2.1 Features of Study Sites

In this study, specific sections of four different freeways entitled Altınyol, Yeşildere, Özkanlar and Karşıyaka Tunnel, were selected for data collection in İzmir, Turkey (Figure 2.1-2.4). All sites have three lanes at each direction and lane widths are approximately 3.6 m. At each section, data was collected at the peak hours of the day, including a morning period (8:00 to 9:00 am), an evening period (6:00 to 7:00 pm) during mid-week days (Tuesday, Wednesday or Thursday). In addition, data collection was repeated in different times of the year, in winter and spring.



Figure 2.1 Observation site I: Altınyol



Figure 2.2 Observation site II: Özkanlar



Figure 2.3 Observation site III: Yeşildere



Figure 2.4 Observation site IV: Karşıyaka Tunnel

These freeways are selected because they represent typical uninterrupted traffic patterns and enable us to determine each headway model under various traffic conditions.

Vehicle headway data was collected using standard video equipment and data was extracted from the recordings by using a counter program. In the following, the fundamental statistical analysis of the data and the distribution models of vehicle headway were examined by a statistical program, Statistica 7.0.

Autocorrelation and randomness were also investigated for the data collected from each freeway. The results of analysis show that, hypothesis of randomness was accepted for all data and no significant autocorrelation was found.

The fundamental statistical characteristics of the headway data for each freeway are shown in Table 2.1 - 2.8.

| Altınyol                           |         |         |         |         |           |         |  |
|------------------------------------|---------|---------|---------|---------|-----------|---------|--|
|                                    | Right   | Lane    | Middle  | e Lane  | Left Lane |         |  |
| Period                             | Morning | Evening | Morning | Evening | Morning   | Evening |  |
| Average Flow<br>Rate (vph)         | 952     | 319     | 1635    | 1018    | 2021      | 1107    |  |
| Mean of<br>Headways (sec)          | 3.803   | 11.347  | 2.253   | 3.582   | 1.828     | 3.291   |  |
| Std deviation of<br>Headways (sec) | 3.224   | 9.805   | 1.369   | 2.445   | 1.097     | 3.162   |  |
| Minimum Value<br>(sec)             | 0.631   | 0.610   | 0.591   | 0.590   | 0.521     | 0.410   |  |
| Maximum Value<br>(sec)             | 20.900  | 72.434  | 15.402  | 17.656  | 10.735    | 21.721  |  |

Table 2.1 Fundamental statistical analysis of the collected headways on Altınyol in winter 2009

Table 2.2 Fundamental statistical analysis of the collected headways on Altınyol in spring 2010

| Altınyol         |         |         |         |         |           |         |  |
|------------------|---------|---------|---------|---------|-----------|---------|--|
|                  | Right   | Lane    | Middle  | e Lane  | Left Lane |         |  |
| Period           | Morning | Evening | Morning | Evening | Morning   | Evening |  |
| Average Flow     | 1536    | 427     | 1666    | 1047    | 1746      | 1101    |  |
| Rate (vph)       | 1550    | 727     | 1000    | 1047    | 1740      | 1101    |  |
| Mean of          | 2.385   | 8.419   | 2.214   | 3.486   | 2.105     | 3.295   |  |
| Headways (sec)   | 2.303   | 017     | 2.211   | 5.100   | 2.105     | 5.275   |  |
| Std deviation of | 1.509   | 6.775   | 1.371   | 2.342   | 1.583     | 2.929   |  |
| Headways (sec)   | 1.007   | 0.775   | 1.571   | 2.3 12  | 1.505     | 2.727   |  |
| Minimum Value    | 0.282   | 0.718   | 0.310   | 0.471   | 0.234     | 0.200   |  |
| (sec)            | 0.202   | 0.710   | 0.010   | 0.171   | 0.201     | 0.200   |  |
| Maximum Value    | 16.844  | 45.510  | 21.125  | 14.831  | 31.860    | 20.536  |  |
| (sec)            | 101011  |         |         | 1.1.001 | 22.000    |         |  |

| Özkanlar                           |         |         |         |         |           |         |  |
|------------------------------------|---------|---------|---------|---------|-----------|---------|--|
|                                    | Right   | Lane    | Middle  | e Lane  | Left Lane |         |  |
| Period                             | Morning | Evening | Morning | Evening | Morning   | Evening |  |
| Average Flow<br>Rate (vph)         | 507     | 338     | 1062    | 1109    | 1221      | 1487    |  |
| Mean of<br>Headways (sec)          | 6.648   | 10.840  | 3.190   | 3.369   | 2.782     | 2.524   |  |
| Std deviation of<br>Headways (sec) | 5.941   | 9.614   | 2.457   | 2.481   | 2.540     | 2.135   |  |
| Minimum Value<br>(sec)             | 0.594   | 0.594   | 0.109   | 0.297   | 0.328     | 0.281   |  |
| Maximum Value<br>(sec)             | 46.940  | 76.562  | 26.750  | 19.266  | 22.469    | 19.203  |  |

Table 2.3 Fundamental statistical analysis of the collected headways on Özkanlar in winter 2009

Table 2.4 Fundamental statistical analysis of the collected headways on Özkanlar in spring 2010

| Özkanlar         |         |         |         |         |           |         |  |
|------------------|---------|---------|---------|---------|-----------|---------|--|
|                  | Right   | Lane    | Middle  | e Lane  | Left Lane |         |  |
| Period           | Morning | Evening | Morning | Evening | Morning   | Evening |  |
| Average Flow     | 309     | 371     | 926     | 897     | 1020      | 1194    |  |
| Rate (vph)       | 309     | 571     | 920     | 097     | 1020      | 1174    |  |
| Mean of          | 7.751   | 9.682   | 3.900   | 4.014   | 3.543     | 3.064   |  |
| Headways (sec)   | 7.751   | 9.002   | 5.700   | 4.014   | 5.515     | 5.004   |  |
| Std deviation of | 5.892   | 8.393   | 2.925   | 2.973   | 3.016     | 2.656   |  |
| Headways (sec)   | 5.072   | 0.375   | 2.725   | 2.715   | 5.010     | 2.050   |  |
| Minimum Value    | 0.721   | 0.561   | 0.611   | 0.391   | 0.400     | 0.270   |  |
| (sec)            | 0.721   | 0.501   | 0.011   | 0.371   | 0.100     | 0.270   |  |
| Maximum Value    | 37.464  | 71.573  | 25.827  | 20.340  | 24.576    | 21.250  |  |
| (sec)            | 57.101  | 11.070  | 23.027  | 20.010  | 21.370    | 21.200  |  |

| Yeşildere                          |         |         |         |         |           |         |  |
|------------------------------------|---------|---------|---------|---------|-----------|---------|--|
|                                    | Right   | Lane    | Middle  | e Lane  | Left Lane |         |  |
| Period                             | Morning | Evening | Morning | Evening | Morning   | Evening |  |
| Average Flow<br>Rate (vph)         | 859     | 448     | 1343    | 1021    | 1669      | 1348    |  |
| Mean of<br>Headways (sec)          | 4.286   | 6.585   | 2.744   | 2.917   | 2.212     | 2.683   |  |
| Std deviation of<br>Headways (sec) | 4.183   | 6.638   | 2.273   | 2.481   | 2.457     | 2.964   |  |
| Minimum Value<br>(sec)             | 0.150   | 0.297   | 0.150   | 0.219   | 0.469     | 0.422   |  |
| Maximum Value<br>(sec)             | 38.844  | 51.930  | 25.540  | 17.157  | 37.812    | 37.390  |  |

Table 2.5 Fundamental statistical analysis of the collected headways on Yeşildere in winter 2009

Table 2.6 Fundamental statistical analysis of the collected headways on Yeşildere in spring 2010

|                  | Yeşildere |         |         |         |           |         |  |  |
|------------------|-----------|---------|---------|---------|-----------|---------|--|--|
|                  | Right     | Lane    | Middle  | e Lane  | Left Lane |         |  |  |
| Period           | Morning   | Evening | Morning | Evening | Morning   | Evening |  |  |
| Average Flow     | 786       | 404     | 1211    | 817     | 1593      | 1085    |  |  |
| Rate (vph)       | 780       | 404     | 1211    | 017     | 1575      | 1005    |  |  |
| Mean of          | 3.769     | 7.868   | 2.480   | 3.608   | 1.913     | 2.727   |  |  |
| Headways (sec)   | 5.707     | 7.000   | 2.400   | 5.000   | 1.715     | 2.727   |  |  |
| Std deviation of | 3.315     | 8.108   | 1.609   | 3.167   | 1.323     | 2.728   |  |  |
| Headways (sec)   | 5.515     | 0.100   | 1.009   | 5.107   | 1.525     | 2.720   |  |  |
| Minimum Value    | 0.302     | 0.672   | 0.302   | 0.235   | 0.344     | 0.406   |  |  |
| (sec)            | 0.202     | 0.072   | 0.202   | 0.200   | 0.011     | 0.100   |  |  |
| Maximum Value    | 23.444    | 350.328 | 14.637  | 31.160  | 24.907    | 28.688  |  |  |
| (sec)            | 23.111    | 2201320 | 1.1007  | 211100  | 2         | 20.000  |  |  |

| Karşıyaka Tunnel                   |         |         |         |         |         |         |  |
|------------------------------------|---------|---------|---------|---------|---------|---------|--|
|                                    | Right   | Lane    | Middle  | e Lane  | Left ]  | Lane    |  |
| Period                             | Morning | Evening | Morning | Evening | Morning | Evening |  |
| Average Flow<br>Rate (vph)         | 59      | 212     | 526     | 602     | 542     | 280     |  |
| Mean of<br>Headways (sec)          | 5.885   | 13.597  | 5.815   | 4.946   | 5.560   | 10.527  |  |
| Std deviation of<br>Headways (sec) | 2.986   | 9.955   | 4.088   | 4.241   | 8.815   | 10.907  |  |
| Minimum Value<br>(sec)             | 2.235   | 0.680   | 1.359   | 0.337   | 0.422   | 0.188   |  |
| Maximum Value<br>(sec)             | 14.608  | 58.740  | 28.546  | 33.281  | 70.687  | 68.599  |  |

Table 2.7 Fundamental statistical analysis of the collected headways on Karşıyaka Tunnel in winter2009

Table 2.8 Fundamental statistical analysis of the collected headways on Karşıyaka Tunnel in spring2010

|                                    | Karşıyaka Tunnel |         |         |         |           |         |  |  |
|------------------------------------|------------------|---------|---------|---------|-----------|---------|--|--|
|                                    | Right            | Lane    | Middle  | e Lane  | Left Lane |         |  |  |
| Period                             | Morning          | Evening | Morning | Evening | Morning   | Evening |  |  |
| Average Flow<br>Rate (vph)         | 240              | 374     | 552     | 1042    | 233       | 714     |  |  |
| Mean of<br>Headways (sec)          | 13.233           | 8.897   | 5.241   | 3.228   | 12.383    | 4.757   |  |  |
| Std deviation of<br>Headways (sec) | 11.667           | 6.930   | 4.155   | 2.473   | 10.695    | 5.070   |  |  |
| Minimum Value<br>(sec)             | 0.867            | 0.594   | 0.703   | 0.447   | 0.985     | 0.515   |  |  |
| Maximum Value<br>(sec)             | 74.790           | 38.750  | 33.328  | 16.406  | 57.391    | 37.313  |  |  |

#### 2.2 Standard Deviation and Mean Association

Al-Ghamdi (2001) stated that the relationship between mean and standard deviation of time headways has important consequences because of estimating the standard deviation directly from the mean of observed flow, which is the reciprocal of the mean headway ( $\bar{t} = 1/q$ ), where q is traffic flow rate (vph).

The relationships between mean and standard deviation of vehicle time headways for investigated sites are shown in Figure 2.5 - Figure 2.8. Griffiths & Hunt (1991) also found a similar linear trend between these statistical characteristics.

The regression equations of each observed freeway as follows:

| Altınyol         | $s = 0.8913\overline{t} - 0.4314$ | , | $R^2 = 0.984$ |
|------------------|-----------------------------------|---|---------------|
| Özkanlar         | $s = 0.8857\bar{t} - 0.2732$      | , | $R^2 = 0.984$ |
| Yeşildere        | $s = 1.0887\bar{t} - 0.5358$      | , | $R^2 = 0.971$ |
| Karşıyaka Tunnel | $s = 0.7533\overline{t} + 1.0811$ | , | $R^2 = 0.796$ |

where s is standard deviation and  $\bar{t}$  is mean time headway.

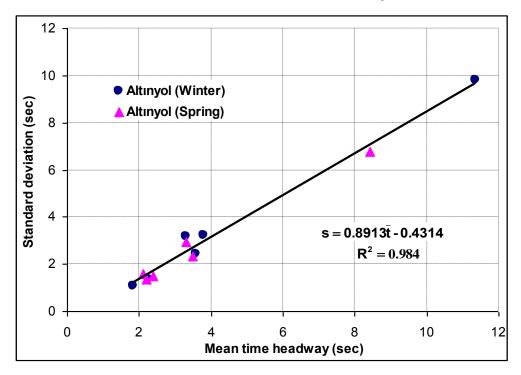


Figure 2.5 Standard deviation versus mean of freeway headways with best fit line for Altinyol

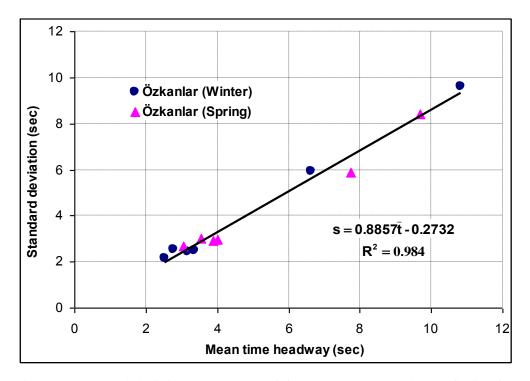


Figure 2.6 Standard deviation versus mean of freeway headways with best fit line for Özkanlar

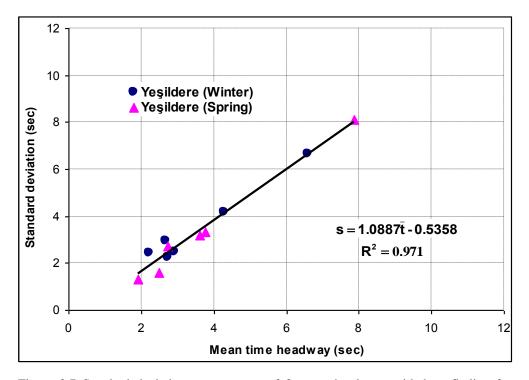


Figure 2.7 Standard deviation versus mean of freeway headways with best fit line for Yeşildere

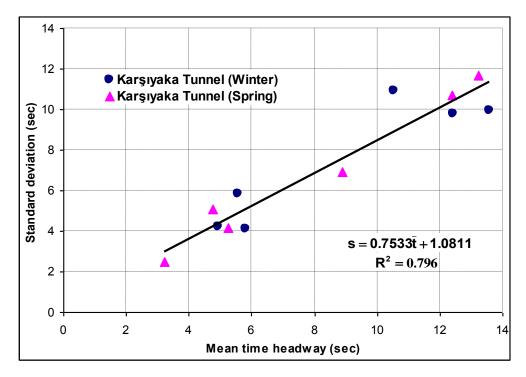


Figure 2.8 Standard deviation versus mean of freeway headways with best fit line for Karşıyaka Tunnel

Figure 2.5 - Figure 2.8 show that Altınyol, Özkanlar and Yeşildere have similar flow characteristics in each other which differs from the flow behavior of Karşıyaka Tunnel. It can be attributed to the low flow rate and the large headways in the traffic flow of Karşıyaka Tunnel.

If the collected data of all sites is examined together, the linear relationship mentioned above is valid as depicted in Figure 2.9. The regression equation for freeway sites as follows:

For investigated freeways; 
$$S = 0.855t - 0.0812$$
 ,  $R^2 = 0.962$ 

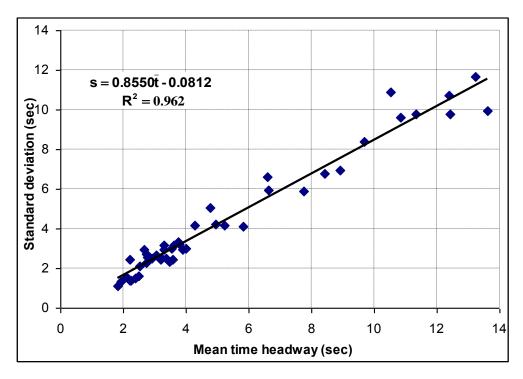


Figure 2.9 Standard deviation of freeway headways versus mean of time headways with best fit line for freeways investigated

#### 2.3 Coefficient of Variation and Flow Rate Association

The sample coefficient of variation (c.v.) is the proportion of sample standard deviation to sample mean. In distribution functions c.v. is the proportion of standard deviation to expectation.

Luttinen (1996) found that polynomial curves fit to the same data for high-speed and low-speed roads. He observed that under heavy traffic, the proportion of freely moving vehicles is small. The variance of headways is accordingly small [c.v. <1 at high flow levels (1.000 to 1.500 vph)].

The c.v. data in the current study have the same interpretation as do Luttinen's data. A third degree polynomial curve fit to the collected data and it is clear from Figure 2.10 - Figure 2.17 that all c.v. values are commonly less than 1.

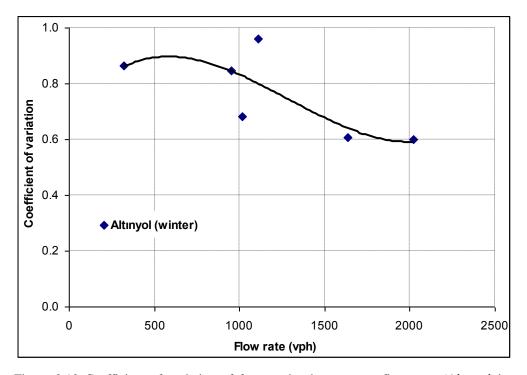


Figure 2.10 Coefficient of variation of freeway headway versus flow rate. (Altınyol in winter)

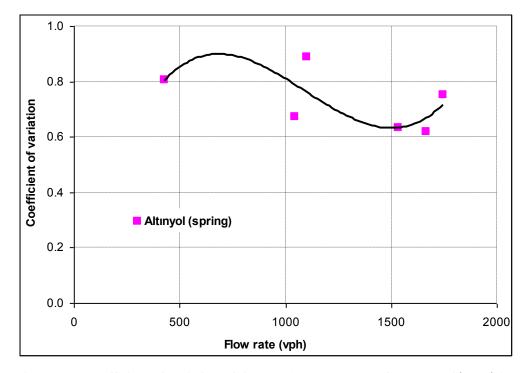


Figure 2.11 Coefficient of variation of freeway headway versus flow rate. (Altınyol in spring)

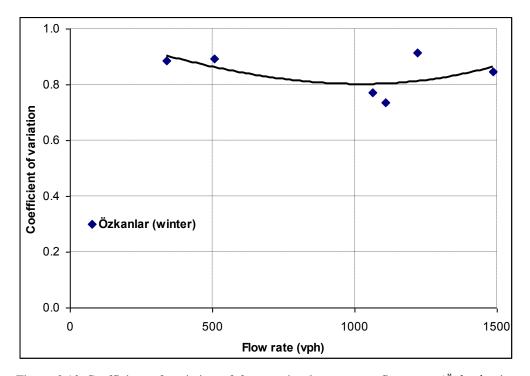


Figure 2.12 Coefficient of variation of freeway headway versus flow rate. (Özkanlar in winter)

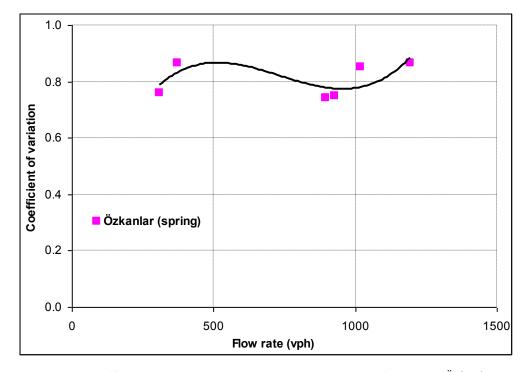


Figure 2.13 Coefficient of variation of freeway headway versus flow rate. (Özkanlar in spring)

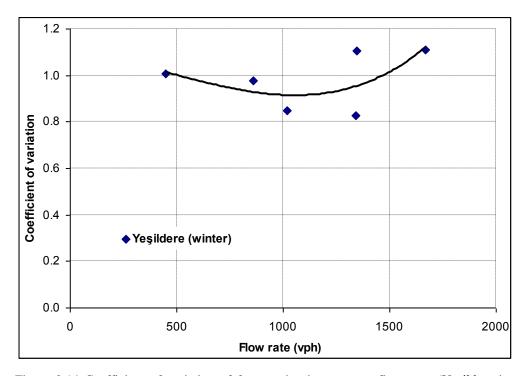


Figure 2.14 Coefficient of variation of freeway headway versus flow rate. (Yeşildere in winter)

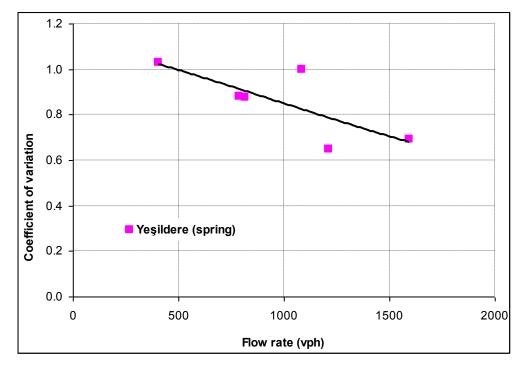


Figure 2.15 Coefficient of variation of freeway headway versus flow rate. (Yeşildere in spring)

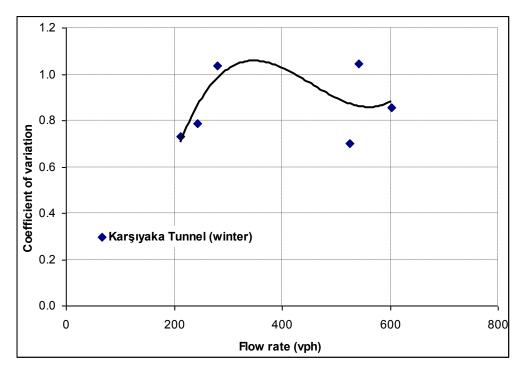


Figure 2.16 Coefficient of variation of freeway headway versus flow rate. (Karşıyaka Tunnel in winter)

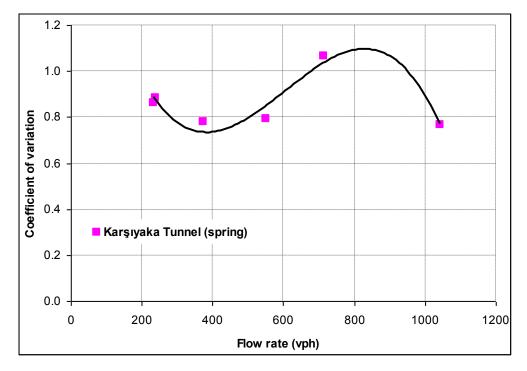


Figure 2.17 Coefficient of variation of freeway headway versus flow rate. (Karşıyaka Tunnel in spring)

The c.v. values of Altinyol and Özkanlar change between 1.0 and 0.6 as depicted in Figure 2.10 - Figure 2.13. For Karşıyaka Tunnel and Yeşildere the c.v. rarely takes values higher than 1.0 (Figure 2.14 - Figure 2.17). In all sites the flow rate takes values between 500 vph and 2000 vph. Unlike the other observation sites, the flow rate values of Karşıyaka Tunnel are in the range of 200 vph and 1000 vph.

As mentioned in previous analysis (Section 2.2), the flow characteristics of Karşıyaka Tunnel are different from the other freeways. Although it can be defined as an uninterrupted flow, the differences in the level of flow rates and vehicle time headways create dissimilarity from the other observation sites.

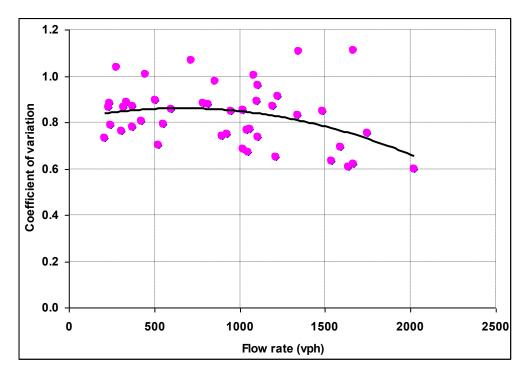


Figure 2.18 Coefficient of variation chart of the time headway data versus flow rate for freeways

Figure 2.18 depicts c.v. values versus flow rate (vph) at uninterrupted flows in İzmir. The study indicates that c.v. is predominantly less than 1.0 and greater than 0.6. Approximately at flow rate 1100 vph, c.v. is regularly distributed between 0.6 and 1.0. As the flow rate increases, it can be seen that the dispersion of c.v. values increases.

## CHAPTER THREE MODEL DESCRIPTION AND ANALYSIS

The evaluation of the headway distribution models is based on three considerations (Luttinen, 1996):

1. Reasonability: It is an advantage if the structure of the model is based on explicit theoretical reasoning about the characteristics of traffic flow. The parameters of such models can give additional information on the properties of traffic flow.

2. Applicability: In mathematical analysis the model should have a simple structure to avoid unsurpassable problems, and the existence and simple form of the Laplace transform is often an advantage. If simulation is considered as an alternative, the generation of pseudorandom variates should be fast and reliable. Parameter estimation should not be too complicated.

3. Validity: The model should give a good approximation of the real world phenomena; i.e., the empirical headway distributions. This is tested first by the identification process and finally by the goodness-of-fit tests.

#### **3.1 Simple Distribution Models**

Several simple distribution models of vehicle headway have been investigated by a number of researchers (Al-Ghamdi, 1999; Cowan, 1975; Mei & Bullen, 1999; Murat & Gedizlioğlu, 2007). Generally used simple distributions are negative exponential, shifted negative exponential, Erlang, Gamma and lognormal distributions.

For constant headway state, the normal distribution is a mathematical distribution that can be used when either the time headways are all constant or when drivers attempt to drive at a constant time headway but driver errors cause the time headways to vary about the intended constant time headway. A unique normal distribution is defined by specifying the mean time headway and the standard deviation of the time headway distribution (May, 1990).

Luttinen (1996) stated that the negative exponential distribution is the interarrival time distribution of the Poisson process. The Markov property makes the distribution analytically simple to use. Consequently, the distribution is widely used in the theory of point processes. In traffic flow theory, the negative exponential distribution has been used since Adams, 1936.

The frequency of unrealistically short headways in the negative exponential distribution is too large. In fact, extremely short headways have the highest probability density. The model gets more distorted as the flow rate increases. On the basis of the Poisson tendency theory, the exponential distribution could be used in the study of low density traffic conditions. Wattleworth (1976) suggests flow rates of 500 vph or less. The coefficient of variation data and the goodness-of-fit tests above suggest even lower volumes (about 100 vph or less). There is, however, not enough low volume data to make accurate recommendations.

The negative exponential distribution has since Adams (1936) played a major role in the theoretical study of traffic flow, and especially in traffic signal control. The studies of Garwood (1940), Darroch et al. (1964), and many others rely on the exponential headway distribution. Also in some modern studies on adaptive traffic signal control, the queue length predictor algorithm is based, at least partially, on Poisson arrivals (Baras, Levine & Lin 1979b, Betro, Schoen & Speranza 1987) (Luttinen, 1996).

The gamma distribution is rather simple in mathematical analysis, especially when dealing with transforms, or if the analysis is restricted to the Erlangian distribution. Efficient subroutines can be found to generate gamma variates. In the tradeoff between applicability and validity, the gamma distribution offers a little more credibility and a little more hard work than the negative exponential distribution. The probability density function of the gamma distribution is appealing, when the shape parameter is greater than unity, but it fails to model both the strong accumulation of headways near the mode and the skewness of headway distributions. If the gamma distribution is used as a model for vehicle headways, the process studied should not be very sensitive to the shape of the headway distribution (Luttinen, 1996).

Luttinen (1996) stated that the lognormal distribution holds if the change in headway during a small time interval is a random proportion of the headway at the start of the interval, and the mean and the variance of the headway remain constant over time.

Daou (1964) reported a good fit to lognormal distribution of spacings within platoons. He considered space headways less than 200 ft (61 m) with speed differences less than 5 ft/sec (5.5 km/h). Two years later Daou (1966) presented a more detailed analysis of the data and also a theoretical justification for using lognormal distribution as a model of constrained headways (Luttinen, 1996).

Greenberg (1966) observed that "there may be some 'universal' law of traffic described (or at least approximated) by log-normal distribution". The lognormal relation holds if the change in the headway of a vehicle in any small interval of time is a random proportion of the headway at the start of the interval, and the mean headway of a vehicle and the variance of its headways remain constant over time. Greenberg also showed a connection between this model and the car-following models (Luttinen, 1996).

According to Baras, Dorsey & Levine (1979), multiplicative, independent, and identically distributed errors by various drivers attempting to follow each other combine to give a lognormal density. Consequently, the lognormal distribution appears to be an attractive model for follower headways.

### **3.2 Mixed Distribution Models**

The main problem of the simple distributions is their inability to describe both the sharp peak and the long tail of the headway distribution. Even at low volume conditions, there is a considerable accumulation of headways near the mode. These properties of the headway distributions suggest two vehicle categories—free flowing and following. Because vehicles in different categories have different headway properties, the division should be included in the model, which accordingly becomes a mixture of two distributions (Luttinen, 1996).

According to Dawson & Chimini (1968) a vehicle is considered free, if:

1. The headway is of "adequate" duration.

2. The free vehicle is able to pass so that it does not have to modify its time-space trajectory, as it approaches the preceding vehicle.

3. A passing vehicle has sustained a positive speed difference after the passing maneuver so that the free vehicle is still able to operate as an independent unit.

The other vehicles are followers. The vehicles could also be classified into four or five categories:

- 1. Free flowing vehicles
- 2. Followers
- 3. Vehicles in a transition stage from free flowing to follower
- 4. Vehicles starting a passing maneuver.

Studies have shown that near the mode of the distribution there is high accumulation between vehicles even at very low volumes while the long tail can be modeled by using negative exponential distribution. This brings us to the solution that vehicles can be categorized as free flowing and bunched vehicles. As a result it is clear that using composite headway distributions may be more informative for the traffic engineer (Çalışkanelli & Tanyel, 2010).

Cowan's M3 distribution is widely used especially in Australia and Europe for capacity and performance analysis, and preferred by traffic engineers for its simple form. Thus the proportion of free vehicles is calculated by using different methods. This is mostly because the driver behavior changes from country to country, even from city to city (Tanyel & Yayla, 2003).

#### 3.3 Evaluating and Selecting Mathematical Distributions

A number of mathematical distribution have been proposed to describe measured time headway distributions in the Section 3.1 and Section 3.2. Some of these distributions may appear to represent measured time headway distributions rather well, whereas, others appear not to be appropriate. However, this evaluation is qualitative, quantitative evaluation techniques are needed.

Two statistical techniques, Chi-Square and Kolmogorov-Smirnov can be applied to evaluate how well a measured distribution can be represented by a mathematical distribution.

The Chi-Square test is a technique that can be used to assess statistically the likelihood that a measured distribution has the attributes of a mathematical distribution. The Chi-Square test can also be used to assess statistically how closely the measured distribution is similar to another measured distribution.

The Kolmogorov–Smirnov test (K–S test) is a form of minimum distance estimation used as a nonparametric test of equality of one-dimensional probability distributions used to compare a sample with a reference probability distribution (onesample K–S test), or to compare two samples (two-sample K–S test). The Kolmogorov–Smirnov statistic quantifies a distance between the empirical distribution function of the sample and the cumulative distribution function of the reference distribution, or between the empirical distribution functions of two samples. It is a more powerful alternative to Chi-Square Goodness-Of-Fit Tests when its assumptions are met. Whereas the Chi-Square test of goodness-of-fit tests whether in general the observed distribution is not significantly different from the hypothesized one, the K-S tests whether this is so even for the most deviant values of the criterion variable. Thus it is a more stringent test.

In this study, the K-S test is used to determine the distance between the distribution functions of the measured data. K-S test results for each lane of all sites are given in Table 3.1 - Table 3.8 and the samples of the resultant graphics obtained from Statistica 7.0 are given in Figure 3.1 - Figure 3.4.

|                            | Altınyol                                  |         |         |           |         |         |         |  |  |  |
|----------------------------|---|---------|---------|-----------|---------|---------|---------|--|--|--|
|                            |   | Right   | Lane    | Left Lane |         |         |         |  |  |  |
| Period                     |   | Morning | Evening | Morning   | Evening | Morning | Evening |  |  |  |
| Average Flow<br>Rate (vph) |   | 952     | 319     | 1635      | 1018    | 2021    | 1107    |  |  |  |
|                            | Normal Distr.<br>( <b>K-S Test</b> )      | 0.161   | 0.124   | 0.145     | 0.096   | 0.156   | 0.151   |  |  |  |
| <b>D-Statistic value</b>   | Exponential Distr.<br>( <b>K-S Test</b> ) | 0.034   | 0.043   | 0.267     | 0.098   | 0.234   | 0.037   |  |  |  |
|                            | Gamma Distr.<br>( <b>K-S Test</b> )       | 0.084   | 0.041   | 0.090     | 0.066   | 0.082   | 0.113   |  |  |  |
|                            | Log-Normal Distr.<br>( <b>K-S Test</b> )  | 0.061   | 0.039   | 0.052     | 0.049   | 0.044   | 0.068   |  |  |  |

Table 3.1 Comparison of goodness-of-fit of simple distributions of Altınyol in winter, 2009

|                            | Altınyol                                  |         |         |             |         |           |         |  |  |
|----------------------------|---|---------|---------|-------------|---------|-----------|---------|--|--|
|                            |   | Right   | Lane    | Middle Lane |         | Left Lane |         |  |  |
| Period                     |   | Morning | Evening | Morning     | Evening | Morning   | Evening |  |  |
| Average Flow<br>Rate (vph) |   | 1536    | 427     | 1666        | 1047    | 1746      | 1101    |  |  |
|                            | Normal Distr.<br>( <b>K-S Test</b> )      | 0.133   | 0.120   | 0.107       | 0.118   | 0.133     | 0.141   |  |  |
| D-Statistic value          | Exponential Distr.<br>( <b>K-S Test</b> ) | 0.281   | 0.078   | 0.105       | 0.205   | 0.102     | 0.033   |  |  |
|                            | Gamma Distr.<br>( <b>K-S Test</b> )       | 0.090   | 0.026   | 0.092       | 0.063   | 0.131     | 0.099   |  |  |
|                            | Log-Normal Distr.<br>( <b>K-S Test</b> )  | 0.057   | 0.014   | 0.049       | 0.045   | 0.056     | 0.060   |  |  |

Table 3.2 Comparison of goodness-of-fit of simple distributions of Altınyol in spring

Table 3.3 Comparison of goodness-of-fit of simple distributions of Özkanlar in winter

|                            | Özkanlar                                  |            |         |             |         |           |         |  |  |
|----------------------------|---|------------|---------|-------------|---------|-----------|---------|--|--|
|                            |   | Right Lane |         | Middle Lane |         | Left Lane |         |  |  |
| Period                     |   | Morning    | Evening | Morning     | Evening | Morning   | Evening |  |  |
| Average Flow<br>Rate (vph) |   | 507        | 338     | 1062        | 1109    | 1221      | 1487    |  |  |
|                            | Normal Distr.<br>( <b>K-S Test</b> )      | 0.132      | 0.135   | 0.115       | 0.107   | 0.139     | 0.169   |  |  |
| <b>D-Statistic value</b>   | Exponential Distr.<br>( <b>K-S Test</b> ) | 0.029      | 0.032   | 0.066       | 0.083   | 0.040     | 0.046   |  |  |
|                            | Gamma Distr.<br>( <b>K-S Test</b> )       | 0.055      | 0.043   | 0.061       | 0.057   | 0.081     | 0.109   |  |  |
|                            | Log-Normal Distr.<br>( <b>K-S Test</b> )  | 0.009      | 0.035   | 0.028       | 0.030   | 0.029     | 0.060   |  |  |

|                            | Özkanlar                                 |            |         |             |         |           |         |  |  |  |
|----------------------------|--|------------|---------|-------------|---------|-----------|---------|--|--|--|
|                            |  | Right Lane |         | Middle Lane |         | Left Lane |         |  |  |  |
| Period                     |  | Morning    | Evening | Morning     | Evening | Morning   | Evening |  |  |  |
| Average Flow<br>Rate (vph) |  | 309        | 371     | 926         | 897     | 1020      | 1194    |  |  |  |
|                            | Normal Distr.<br>( <b>K-S Test</b> )     | 0.094      | 0.124   | 0.131       | 0.122   | 0.145     | 0.145   |  |  |  |
| <b>D-Statistic value</b>   | Exponential Distr.<br>(K-S Test)         | 0.074      | 0.045   | 0.113       | 0.101   | 0.034     | 0.031   |  |  |  |
|                            | Gamma Distr.<br>( <b>K-S Test</b> )      | 0.022      | 0.048   | 0.042       | 0.050   | 0.083     | 0.103   |  |  |  |
|                            | Log-Normal Distr.<br>( <b>K-S Test</b> ) | 0.025      | 0.031   | 0.029       | 0.037   | 0.054     | 0.060   |  |  |  |

Table 3.4 Comparison of goodness-of-fit of simple distributions of Özkanlar in spring

Table 3.5 Comparison of goodness-of-fit of simple distributions of Yeşildere in winter

|                            | Yeşildere                                |                        |         |         |         |           |         |  |  |
|----------------------------|--|------------------------|---------|---------|---------|-----------|---------|--|--|
|                            |  | Right Lane Middle Lane |         |         |         | Left Lane |         |  |  |
| Period                     |  | Morning                | Evening | Morning | Evening | Morning   | Evening |  |  |
| Average Flow<br>Rate (vph) |  | 859                    | 448     | 1343    | 1021    | 1669      | 1348    |  |  |
|                            | Normal Distr.<br>( <b>K-S Test</b> )     | 0.172                  | 0,161   | 0,123   | 0.144   | 0.184     | 0.278   |  |  |
| <b>D-Statistic value</b>   | Exponential Distr.<br>(K-S Test)         | 0.051                  | 0.028   | 0.065   | 0.039   | 0.047     | 0.596   |  |  |
|                            | Gamma Distr.<br>( <b>K-S Test</b> )      | 0.062                  | 0.059   | 0.090   | 0.091   | 0.011     | 0.142   |  |  |
|                            | Log-Normal Distr.<br>( <b>K-S Test</b> ) | 0.013                  | 0.020   | 0.052   | 0.044   | 0.018     | 0.076   |  |  |

| Yeşildere                  |   |            |         |             |         |           |         |
|----------------------------|---|------------|---------|-------------|---------|-----------|---------|
|                            |   | Right Lane |         | Middle Lane |         | Left Lane |         |
| Period                     |   | Morning    | Evening | Morning     | Evening | Morning   | Evening |
| Average Flow<br>Rate (vph) |   | 786        | 404     | 1211        | 817     | 1593      | 1085    |
|                            | Normal Distr.<br>( <b>K-S Test</b> )      | 0.173      | 0.332   | 0.136       | 0.157   | 0.191     | 0.198   |
| D-Statistic value          | Exponential Distr.<br>( <b>K-S Test</b> ) | 0.047      | 0.002   | 0.248       | 0.042   | 0.085     | 0.073   |
|                            | Gamma Distr.<br>( <b>K-S Test</b> )       | 0.076      | 0.002   | 0.092       | 0.076   | 0.117     | 0.153   |
|                            | Log-Normal Distr.<br>( <b>K-S Test</b> )  | 0.056      | 0.002   | 0.057       | 0.052   | 0.075     | 0.103   |

Table 3.6 Comparison of goodness-of-fit of simple distributions of Yeşildere in spring

Table 3.7 Comparison of goodness-of-fit of simple distributions of Karşıyaka Tunnel in winter

| Karşıyaka Tunnel           |   |            |         |             |         |           |         |
|----------------------------|---|------------|---------|-------------|---------|-----------|---------|
|                            |   | Right Lane |         | Middle Lane |         | Left Lane |         |
| Period                     |   | Morning    | Evening | Morning     | Evening | Morning   | Evening |
| Average Flow<br>Rate (vph) |   | 59         | 212     | 526         | 602     | 542       | 280     |
| D-Statistic value          | Normal Distr.<br>( <b>K-S Test</b> )      | 0.116      | 0.086   | 0.130       | 0.138   | 0.264     | 0.167   |
|                            | Exponential Distr.<br>( <b>K-S Test</b> ) | 0.109      | 0.101   | 0.245       | 0.077   | 0.121     | 0.021   |
|                            | Gamma Distr.<br>( <b>K-S Test</b> )       | 0.030      | 0.046   | 0.093       | 0.058   | 0.088     | 0.017   |
|                            | Log-Normal Distr.<br>( <b>K-S Test</b> )  | 0.033      | 0.080   | 0.061       | 0.036   | 0.067     | 0.048   |

| Karşıyaka Tunnel           |   |            |         |             |         |           |         |
|----------------------------|---|------------|---------|-------------|---------|-----------|---------|
|                            |   | Right Lane |         | Middle Lane |         | Left Lane |         |
| Period                     |   | Morning    | Evening | Morning     | Evening | Morning   | Evening |
| Average Flow<br>Rate (vph) |   | 240        | 374     | 552         | 1042    | 233       | 714     |
|                            | Normal Distr.<br>( <b>K-S Test</b> )      | 0.148      | 0.127   | 0.114       | 0.125   | 0.133     | 0.174   |
| D-Statistic value          | Exponential Distr.<br>( <b>K-S Test</b> ) | 0.106      | 0.072   | 0.105       | 0.165   | 0.036     | 0.043   |
|                            | Gamma Distr.<br>( <b>K-S Test</b> )       | 0.044      | 0.039   | 0.044       | 0.065   | 0.060     | 0.056   |
|                            | Log-Normal Distr.<br>( <b>K-S Test</b> )  | 0.034      | 0.014   | 0.033       | 0.032   | 0.040     | 0.022   |

Table 3.8 Comparison of goodness-of-fit of simple distributions of Karşıyaka Tunnel in spring

Simple distribution models have been investigated to test their goodness-of-fit to the sample data. In order to test the goodness-of-fit of the headway models, the K-S test statistics was applied for visual and quantitative comparisons.

As a result, in most cases lognormal distribution is found to be the most appropriate distribution model for headway modelling purposes. Gamma distribution is the second appropriate distribution model in fitting headways for the observed freeways.

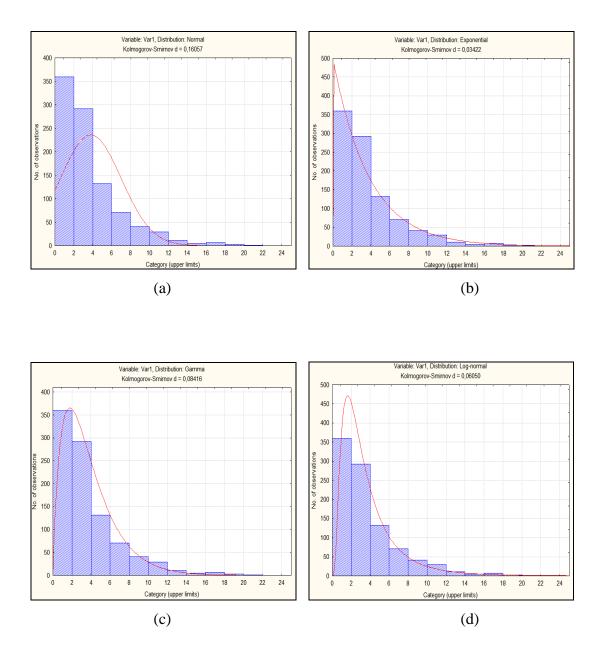


Figure 3.1 Simple distribution models of the vehicle headways for the right lane on Altınyol during the peak hours in the morning (winter data): a) normal distribution, b) negative exponential distribution, c) gamma distribution, d) lognormal distribution

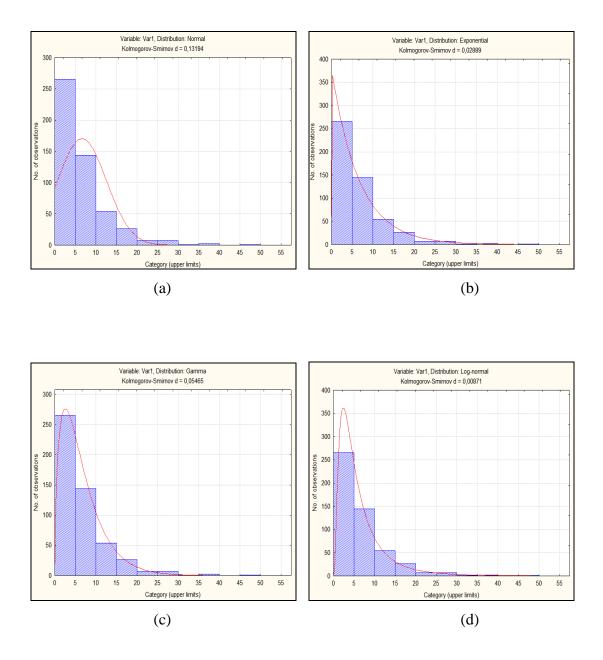


Figure 3.2 Simple distribution models of the vehicle headways for the right lane on Özkanlar during the peak hours in the morning (winter data): a) normal distribution, b) negative exponential distribution, c) gamma distribution, d) lognormal distribution

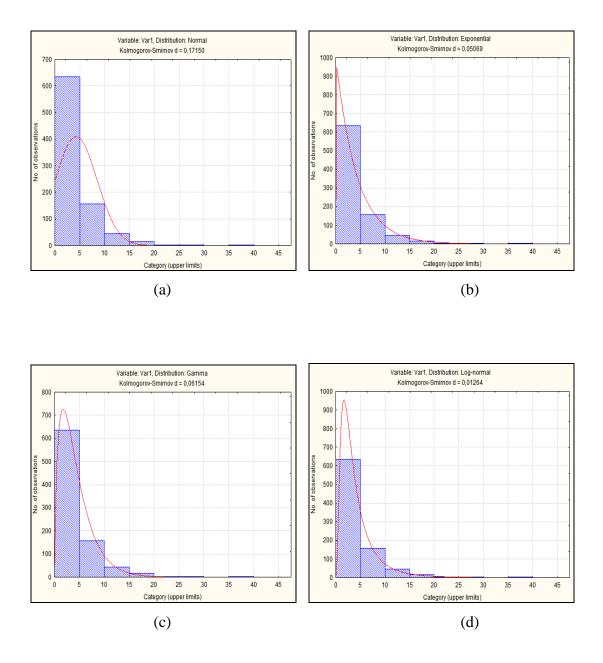


Figure 3.3 Simple distribution models of the vehicle headways for the right lane on Yeşildere during the peak hours in the morning (winter data): a) normal distribution, b) negative exponential distribution, c) gamma distribution, d) lognormal distribution

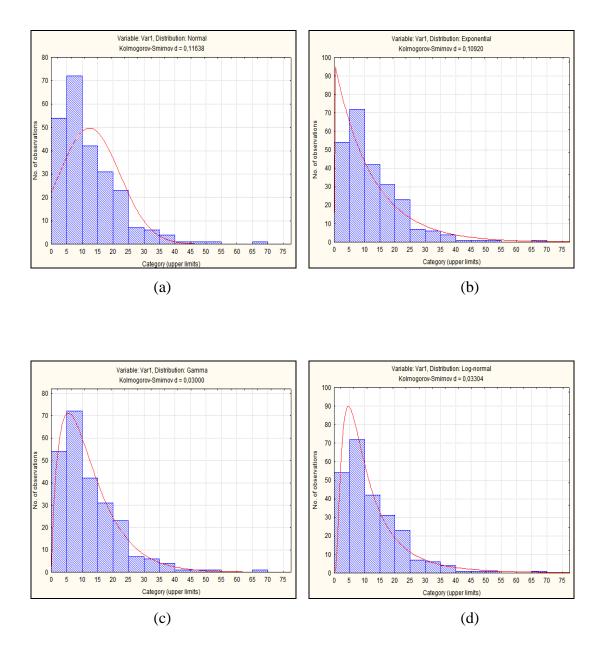


Figure 3.4 Simple distribution models of the vehicle headways for the right lane on Karşıyaka Tunnel during the peak hours in the morning (winter data) : a) normal distribution, b) negative exponential distribution, c) gamma distribution, d) lognormal distribution

# CHAPTER FOUR LANE BY LANE ANALYSIS OF VEHICLE TIME HEADWAYS OF UNINTERRUPTED FLOWS

It is a commonly accepted fact that the traffic flow characteristics of lanes on a freeway can vary considerably. In this chapter, the vehicle time headways at uninterrupted flows in İzmir are presented in lane by lane principle. Each of the following analysis is to demonstrate the differences in the flow conditions in terms of the relationship between statistical characteristics and to determine the proportiom of unbunched vehicles and the delay parameter at all observed flows.

#### 4.1 The Relationship Between Standard Deviation and Mean

As stated in Chapter Two, the relationship between mean and standard deviation of time headways denotes a linear trend. To determine the differences between lane usage behaviors explicitly, an investigation is presented for whether this linear relationship is appropriate for each lane on three-lane freeway, seperately.



Figure 4.1 Standard deviation versus mean of time headways with best fit line for each lane on three-lane freeways

The regression equations of observed headways for each lane are as follows:

| For left lanes;   | $s_{L} = 0.9899\bar{t} - 0.0285$           | , | $R^2 = 0.90$ |
|-------------------|--|---|--------------|
| For middle lanes; | ${\bm s}_{\sf M} = 0.824 \bar{t} - 0.2565$ | , | $R^2 = 0.91$ |
| For right lanes;  | $\mathbf{s}_{R} = 0.7745\bar{t} - 1.2401$  | , | $R^2 = 0.46$ |

Figure 4.1 indicates that mean values of headways for lane are in the range of 2-15 sec. The slope of the trend lines of both right and middle lane are approximately identical. For the values under the mean time headway of 6 sec, the standard deviation of right lane is greater than the standard deviation of other lanes. In addition to the existence of heavy vehicles which make larger gaps, if the proportion of the other vehicles which follow the vehicle ahead with lower time headways increases, the standard deviation increases in right lane. Furthermore, the mean headways in the middle lane are much lower than the other lanes.

#### 4.2 The Relationship Between Mode and Flow Rate

Another analysis, which can be performed for the descriptive statistics of observed data, is to investigate the relation between modes of data and traffic flow rate.

Summala & Vierimaa (1980) describe the mode as an approximation of the headway that most drivers select when they are following the vehicle ahead (Luttinen, 1996).

The modes of time headways versus flow rates for each lane are shown in Figure 4.2, seperately.

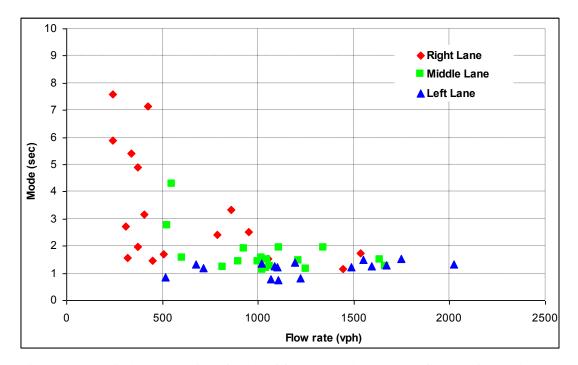


Figure 4.2 Graphical representation of mode of freeway headways versus flow rate in lane by lane principle

The Figure 4.2 indicates that at low flow rate (vph) values the mode varies between 1 and 8 sec in right lanes. The mode varies in the range of 1 and 5 sec in middle lanes whereas it varies around 1 sec in left lanes. The mode of the right lane is greater than the other lanes because of the higher proportion of heavy vehicles.

## 4.3 The Relationship Between Coefficient of Variation and Flow Rate

As stated in Chapter Two, the coefficient of variation (c.v.) is the proportion of the standard deviation to the mean of a random variable (T):

$$C(T) = \frac{\sigma(T)}{\mu(T)}$$

The sample c.v. is the proportion of the sample standard deviation to the sample mean:

$$C = \frac{s}{\bar{t}}$$

Several authors discussed the relationship of c.v. and flow rate in international researches. The studies of Breiman, Lawrence, Goodwin & Bailey (1977), May (1965) and Buckley (1968) are focused on freeway lanes.

The c.v. is less than 1 in the freeway samples of May (1965) and Breiman et al. (1977) (one and two lanes), and near 1 in the samples of Buckley (1968) and Breiman et al. (1977) for three-lane freeway.

The data of Luttinen (1996) is collected from low-speed roads (speed limit 50-70 km/h) and high speed roads (speed limit 80-100 km/h). In his study, c.v. of the samples is near 1 in low-speed roads and is between 1-2 in high-speed roads.

In this study, c.v. values versus flow rates for each lane on three-lane freeway sites are examined. Figure 4.3 depicts that c.v. is less than 1 in all samples. In the range of flow rates 210 vph and 1500 vph in right lane, the c.v. values are between 0.6 and 0.9. In middle lane the c.v. values are between 0.6 and 0.8 in the range of flow rates 500 vph and 1700 vph. c.v. value varies between 0.6 and 1.0 in left lanes in the range of flow rates 250 vph and 2000 vph and it is approximately 1 at flow rate 750 vph. The curves have maxima at flow rates 750 vph for left lanes, 500 vph for right lanes.

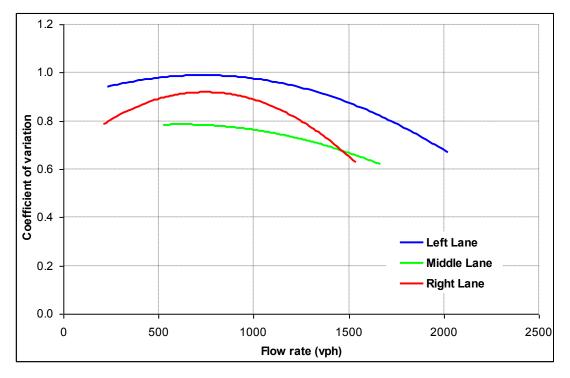


Figure 4.3 Graphical representation of coefficient of variation of freeway headways versus flow rate in lane by lane principle

To evaluate the model C(T) on freeway lanes separately, three polynomial curves fit to the c.v. data. The regression equations of each lane are:

| For left lanes;   | $C_{L} = -2 \times 10^{-7} \lambda^{2} + 0.0003\lambda + 0.8848$              |
|-------------------|---|
| For middle lanes; | $\boldsymbol{C}_{M} = -2 \times 10^{-7}  \lambda^2 + 0.0002 \lambda + 0.7261$ |
| For right lanes;  | $C_{R} = -5 \times 10^{-7} \lambda^{2} + 0.0007 \lambda + 0.6599$             |

where;  $\lambda$  is traffic flow rate (vph) and  $C_L$ ,  $C_M$ ,  $C_R$  are coefficient of variation of vehicle time headways for left, middle and right lanes, respectively.

Due to the fact that c.v. values of middle lane are lower than both left and right lanes, it can be concluded that vehicles in the middle lane are affected by the vehicles which are changing lane from either left or right lane. This is an expected result because of the following reasons: The drivers in the left lane may want to change their location or lane under some circumstances:

- Drivers who are cruising at much higher speeds may force them to move to the middle lane (in Turkey most of the drivers tend to use the left lane for a long period, although it is not permitted to use the left lanes continually).
- They may choose to use the middle lane for a short period to pass a slower vehicle.
- They may exit the freeway from the next off-ramp so they have to drive towards to the right lane.
- > On the other hand, drivers on the right lane may move to the middle lane if:
  - The middle lane is their preference for travel.
  - Drivers may be passing a slower vehicle in the right lane.
  - Drivers prefer to cruise at higher speeds on left lane.

In all these conditions it is clear that, drivers who prefer the middle lane can be defined as the most disturbed drivers in the road section. This causes a much higher interaction between vehicles in the middle lane. As a result, the proportion of bunched vehicles in the middle lane is expected to be higher than other lanes.

## 4.4 Cowan's M3 Distribution

Cowan (1975) proposed four different headway models. One of them is named as Cowan's M3 distribution. Sullivan & Troutbeck (1997) stated that Cowan's M3 distribution is dichotomized, and it is simple enough for capacity model of considerable complexity.

In Cowan's M3 distribution, the cumulative distribution function is given by:

$$F(t) = \begin{cases} 0 & \text{if} \quad t < \Delta \\ 1 - \alpha \exp\left[-\lambda(t - \Delta)\right] & \text{if} \quad t > \Delta \end{cases}$$

where; the headways in platoons are assumed to be constant ( $\Delta$ ) and free-vehicle headways follow shifted-exponential distribution,  $\alpha$  is the proportion of vehicles,  $\Delta$ is the minimum time headway and  $\lambda$  is the shape parameter (Luttinen 2003, 2004).

The shape parameter  $\lambda$  can be expressed as:

$$\lambda = \alpha q / (1 - \Delta q)$$

where; q is traffic flow rate (vph).

This equation was developed from the requirement that the mean headway is equal to 1/q (Troutbeck, 1997).

For estimation of the parameters for Cowan's M3 model, three different methods can be applied. These are method of moments, least squares method and maximum likelihood method.

In this study, least squares method was applied to determine Cowan's M3 model parameters likewise Sullivan & Troutbeck (1994) have used. They used the following equation to predict  $\lambda$ :

$$\lambda = \frac{1}{\sum_{i} t_i / \eta - \varepsilon}$$

where;  $\varepsilon$  is a headway limit value at which vehicles are assumed to be free. It is found  $\varepsilon = 3$  sec by Hagring (1998) and  $\varepsilon = 4$  sec by Troutbeck (1997). It is assumed as  $\varepsilon = 3$  sec in this study.

The average of headways  $\sum_{i} t_i / n$  is greater than  $\varepsilon$ . The location parameter  $\Delta$  was substituted with an exponential threshold that does not introduce any bias to scale parameter estimate, if F(t|  $t > \varepsilon$ ) follows exponential distribution (Luttinen, 1999).

For t> $\varepsilon$ , the cumulative distribution function can be rewritten as:

$$F(t) = 1 - \alpha e^{(-\lambda t - \Delta)}$$
$$= 1 - \gamma e^{(-\lambda t)}$$

where;

$$\gamma = \alpha e^{(\lambda \Delta)}$$

By minimizing the sum of squares of the difference between the measured and expected distributions, an estimate of  $\gamma$  can be found (Hagring, 1996) as:

$$\gamma = \frac{\sum_{i} \{1 - H(t_i)\} e^{-\lambda \Delta_i}}{\sum_{i} e^{-2\lambda \Delta_i}}$$

where;  $H(t_i)$  is the measured cumulative distribution. Satisfying the condition that the mean headway must be equal to the reciprocal of the flow, we find that the proportion of free vehicles by solving the following equality:

$$\alpha e^{-\alpha} = \gamma e^{-\lambda/q}$$

After proportion of free vehicles is calculated,  $\Delta$  can be determined by using following equation:

$$M = \Delta + \frac{\alpha}{\lambda}$$

where; M is the average.

Troutbeck (1997) suggested that especially in prediction of  $\Delta$ , least squares method should be used. Luttinen (1999), like Troutbeck (1997), stated that results would be more precise if  $\Delta$  parameter was estimated by using least squares method. In this study least squares method is used in parameter estimation.

Sullivan & Troutbeck (1997) have defined three issues with Cowan's M3 model that should be explained.

(1)  $\alpha$  and  $\Delta$  are not independent. This inter-relationship between  $\alpha$  and  $\Delta$  can mask important attributes of  $\alpha$  and  $\Delta$ . The modelling of the larger headways greater than  $\varepsilon$  will not be affected if the term  $\alpha \exp(\lambda \Delta)$  and the decay constant  $\Delta$ , remain unchanged. Consequently,  $\Delta$  could be changed to a new value  $\Delta_c$  (less than  $\varepsilon$ ) with  $\alpha_c$  being adjusted accordingly. This action will only have a minor effect on the estimate of the mean, if  $\Delta_c$  is approximately equal to  $\Delta$ . Sullivan & Troutbeck (1994, 1997) found that in most single-lane cases,  $\Delta_c$  could be set to 2 sec and still provide representation of the headway distribution.

(2) The proportion of free vehicles  $\alpha$ , is not able to be measured in the field. It is a calibration term that ensures the distribution of the larger gaps is correct and that the distribution has an appropriate mean. The value of  $\alpha$  will be less than the measured number of headways greater than  $\Delta_c$ .

(3) For the same mean flow but with different  $\alpha$ , the mean headway is the same but the variance (Var) is given by:

$$Var = \alpha (2 - \alpha) / (\lambda^2)$$

Changing  $\alpha$ , is equivalent to changing the variance of the headways. The process is then one where this variance is related to traffic patterns. This process requires a substantial amount of data to calibrate the model.

### 4.5 The Proportion of Unbunched Vehicles and Delay Parameter Estimation

Several researchers investigated the relation between  $\Delta q$  and  $\alpha$  like Tanner (1962), Plank (1982), Akçelik (1998, 2003) and Tanyel & Yayla (2003).

Firstly, Tanner (1962) defined the proportion of free vehicles ( $\alpha$ ) as:

$$\alpha = 1 - \Delta q$$

where;  $\Delta$  is minimum headway and q is traffic flow rate.

Sullivan and Troutbeck (1997) also studied on proportion of unbunched vehicles. They searched the effects of lane width on saturation flow and suggested the following equations:

$$\alpha = \exp(-6.5q)$$
 for L < 3.0 m

For kerb lane

$$\alpha = \exp(-5.25q) \quad \text{for} \quad 3.0 \le L \le 3.5 \text{ m}$$
$$\alpha = \exp(-3.4q) \quad \text{for} \quad L > 3.5 \text{ m}$$

For median lane

$$\alpha = \exp(-7.5q)$$
 for  $3.0 \le L \le 3.5$  m

where; L is the lane width.

Akçelik (1998) suggested a different equation including fraction of following vehicles  $\Delta q$  instead of q:

$$\alpha = \exp(-b\Delta q)$$

where; b is a constant which is 2.5 for roundabouts and 0.5~0.8 for interrupted flows.

In this study, regression analysis was performed for each lane between  $\Delta q$  (independent variable) and  $\alpha$  (dependent variable). The relations between  $\Delta q$  and  $\alpha$  for right lanes, middle lanes and left lanes are shown in Figures 4.4, 4.5 and 4.6, respectively. The linear regression function are plotted on the figures.

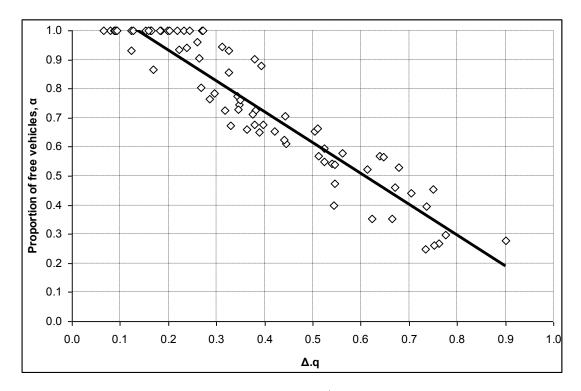


Figure 4.4 Graphical representation of relation between  $\Delta q$  and  $\alpha$  for right lanes

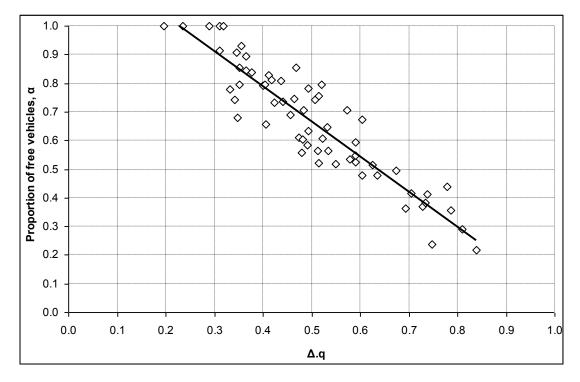


Figure 4.5 Graphical representation of relation between  $\Delta q$  and  $\alpha$  for middle lanes

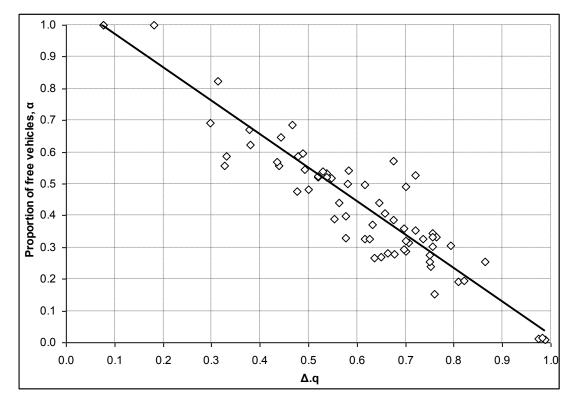


Figure 4.6 Graphical representation of relation between  $\Delta q\,$  and  $\,lpha\,$  for left lanes

The regression equations of each lane are;

Linear function for right lanes:

 $\alpha = 1.15 - 1.06\Delta q$   $0.142 \le \Delta q \le 0.900$   $\mathbb{R}^2 = 0.89$ 

Linear function for middle lanes:

 $\alpha = 1.28 - 1.23 \Delta q$   $0.228 \le \Delta q \le 0.840$   $\mathbb{R}^2 = 0.85$ 

Linear function for left lanes:

 $\alpha = 1.08 - 1.05 \Delta q$   $0.076 \le \Delta q \le 0.988$   $R^2 = 0.78$ 

All three models are shown and compared in Figure 4.7 with respect to traffic flow rate (vph). As the figure indicates, for low and moderate traffic flows, proportion of free vehicles in the middle lane is greater than the other lanes'. This may be the result of the fluctuations in the traffic flow caused by other vehicles passing from or to other lanes.  $\alpha$  values for the right lanes are also greater than the left lanes' and this may be a result of higher proportion of heavy vehicles since they occupy larger areas and create larger gaps.

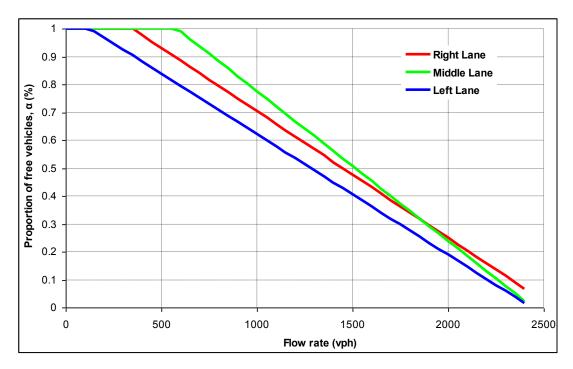


Figure 4.7 Comparison of proportion of free vehicles for different lanes with respect to traffic flow rate (vph)

Akçelik (2003) also proposed a model for the proportion of free vehicles:

$$\alpha = (1 - \Delta q) / [1 - (1 - k_d) \Delta q]$$
 subject to  $\alpha \ge 0.001$ 

where;  $k_d$  is a constant (delay parameter).

Although Akçelik (2003) suggested different values of  $k_d$  for different facilities (0.20 for uninterrupted flows and 2.20 for roundabouts), the parameter  $\alpha$  takes different values depending on  $k_d$  (Çalışkanelli & Tanyel, 2010).

To calculate "k<sub>d</sub>" for each data set, the above equation is reorganized as follows:

$$k_d = (1 - \alpha)(1 - \Delta q)/(\alpha \Delta q)$$

In this study, the applicability of Akçelik's (2003) equation is tested for each lane at uninterrupted flows. By using above equation,  $k_d$  values for right, middle and left lanes are found 0.53, 0.63 and 0.85, respectively.  $\alpha$  values which are computed by using  $k_d$  parameters, are shown in Figure 4.8.

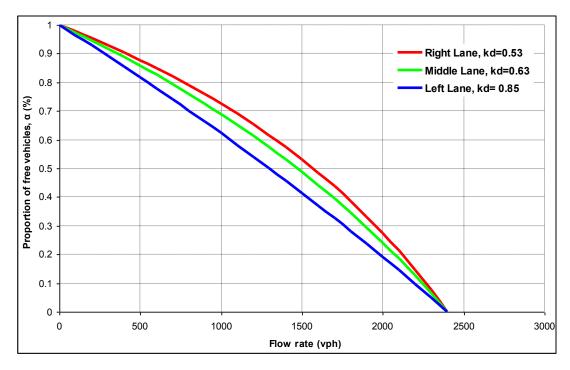


Figure 4.8 Graphical representation of relation between flow rate and  $\alpha$ , proportion of free vehicles

Figure 4.8 denotes that the proportion of free vehicles at right lanes is higher than the proportion of vehicles at both middle and left lanes at certain traffic flow rate values. Because of the higher proportion of heavy vehicles which create larger gaps in right lanes,  $\alpha$  value increases. Furthermore, drivers who are cruising at high speeds in left lanes tend to follow each other with small headways, and this causes a reduction in the proportion of unbunched vehicles.

Although the results given in Figure 4.7 and Figure 4.8 show some differences, especially for middle lanes, they provide useful information for engineers which will design uninterrupted facilities. Both functions for left lanes give very close results to Tanner's equation. This indicates that, the vehicles in the left lanes may be assumed to arrive at the observation points randomly. However, the same result can not be pronounced for the middle and right lanes. It is clear that both lanes are affected by other vehicles which change lanes.

## CHAPTER FIVE CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

This thesis presents the results of the statistical analysis carried out with traffic data collected from four freeway sites in İzmir. Based on the findings of this study the following conclusions may be drawn:

The relationship between standard deviation and mean of the collected headway data is examined for all observation sites. The result indicates a similar linear trend between mean and standard deviation of time headways at uninterrupted flows.

Luttinen (1996) found that polynomial curves fit to the same data for high-speed and low-speed roads. He observed that under heavy traffic, the proportion of freely moving vehicles is small. The variance of headways is accordingly small [c.v. <1 at high flow levels (1.000 to 1.500 vph)]. The c.v. data in the current study have the same interpretation as do Luttinen's data. All c.v. values are commonly less than 1 and a third degree polynomial curve fit to the collected data.

Furthermore, the results show that the c.v. values of Altinyol and Özkanlar change between in 0.6 and 1.0 whereas the c.v. rarely takes values higher than 1.0 for Karşıyaka Tunnel and Yeşildere. When all data is analyzed together, c.v. is regularly distributed between 0.6 and 1.0 at flow rates of approximately 1100 vph. As the flow rate increases, the dispersion of c.v. values increases.

As shown in Section 2.1 and Section 2.2, the flow characteristics of Karşıyaka Tunnel are different from the other freeways. Although it can be defined as an uninterrupted flow, the differences in the level of flow rates and vehicle time headways create dissimilarity from the other observation sites.

The examinations also indicate that, Altinyol and Yeşildere are the roads which drivers prefer left lane mostly, and the flow rate values are greater in the morning peak hours than the evening peak hours. According to the comparison of the seasonal differences, it is clear that the traffic flow rate is higher in winter months than spring months on Altınyol and Yeşildere.

Considering the traffic flow characteristics of Özkanlar, flow rates take nearly the same values during the peak hours both in the morning and the evening. The average flow rate is greater in winter months than spring months. The left lane is mostly preferred by drivers on Özkanlar.

According to the data collected from the entrance of the Karşıyaka Tunnel, the drivers mostly prefer middle lane. Moreover, the traffic flow rate is greater in the evening peak hours than the morning peak hours. In contradistinction to Altınyol, Yeşildere and Özkanlar, the average flow rate of Karşıyaka Tunnel is greater in spring months than winter months.

Knowledge of headway distribution is significant for traffic flow theory and simulation researches. Various simple and mixed distribution models were proposed to adapt to varying traffic situations. Based on the vehicle headway data collected at four different freeway sections, the adaptability and accuracy of several typical headway distribution models were examined. Four simple headway distribution models have been investigated to test their goodness-of-fit to the sample data. In order to test the goodness-of-fit of the headway models, the K-S test statistics was applied for visual and quantitative comparisons. As a result, in most cases lognormal distribution is found to be the most appropriate distribution model for headway modelling purposes. Gamma distribution is the second appropriate distribution model in fitting headways for the observed freeways.

The research findings and recommendations may be appropriate for freeways in an urban setting similar to İzmir. Before the models are exploited in other areas, it is suggested using a small amount of headway data to re-examine their transferability.

However, it is recommended that more vehicle headway distributions should be tried for a thorough investigation with Hyperexponential, Hyperlang and SemiPoisson distributions. Additionally, for further research more data need to be collected and analyzed to identify the associated traffic flow patterns at uninterrupted flows.

In Chapter Four, the relationship between standard deviation and mean of the time headways is investigated for whether the linear trend is appropriate for each lane seperately. The study indicated that each lane has a linear relationship between standard deviation and mean of the collected data.

In the following, the mode of time headways versus flow rate for each lane is examined. It is shown that the mode of the right lane is greater than the other lanes because of the higher proportion of heavy vehicles.

As a next step, the relationship between the coefficient of variation and flow rate is analyzed. The study indicates that c.v. is less than 1.0 in the samples of each lane. The c.v. only approaches to 1.0 at flow rate of 750 vph in left lane. The results also show that the vehicles in the middle lane are affected by the vehicles which are changing lane from either left lane or right lane. In other words, a much higher interaction occurs between vehicles in the middle lane.

In the last part of the Chapter Four, Cowan's M3 distribution was chosen to model time headways of observed sites. Regression analysis was performed for each lane between  $\Delta q$  and  $\alpha$ . Then, the calculated parameters were applied for the k<sub>d</sub> parameter estimation. It was shown that  $\alpha$  and  $\Delta q$  are linearly dependent and k<sub>d</sub> values for right, middle and left lanes were found 0.53, 0.63 and 0.85, respectively.

Besides, the results denote that the proportion of free vehicles at right lanes is higher than the proportion of vehicles at both middle and left lanes at certain traffic flow rate values. Because of the higher proportion of heavy vehicles which create larger gaps in right lanes,  $\alpha$  value increases. Additionally, the  $\alpha$  parameter take lower values in left lanes because of the tendency of drivers for following each other with small time headways. Furthermore, the analysis denotes that, the vehicles in the left lanes may be assumed to arrive at the observation points randomly. However, the same result can not be pronounced for the middle and right lanes. It is clear that both lanes are affected by other vehicles which change lanes.

This thesis gives a framework to evaluate the traffic flow characteristics of uninterrupted flows. In future studies, the number of observation sites may be increased and more accurate results may be obtained with the calibration of the models by using more data. A significant parameter, the proportion of heavy vehicles, is not considered in calculations. It would be useful to further researches to take into account the proportion of heavy vehicles for similar analysis.

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