DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

BLACK-SCHOLES OPTION PRICING MODEL AND TESTING THE OPTION PRICES STABILITY

by
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June, 2011
İZMİR

BLACK-SCHOLES OPTION PRICING MODEL AND TESTING THE OPTION PRICES STABILITY

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M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"BLACK-SCHOLES OPTION PRICING MODEL AND TESTING THE OPTION PRICES STABILITY"** completed by **ERAY AKGÜN** under supervision of **ASSOC. PROF. DR. GÜÇKAN YAPAR** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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BLACK-SCHOLES OPTION PRICING MODEL AND TESTING THE OPTION PRICES STABILITY

ABSTRACT

Derivatives have moved to the center of modern corporate finance, investments, and the management of financial institutions. In our study topics such as derivatives, types of derivatives, their features and differences, derivatives markets, which these derivatives are put into action and their features, were mentioned. Later, options, which is one of the most dependable means of hedging, and their development were investigated in detail. Here, the most important factors for options were mentioned and the basic effects of these factors on determining the price of the option were tried to be analyzed.

Afterwards, option pricing models were defined and how to perform option pricing were sampled. The most widely used model, which revolutionized option pricing, namely Black-Scholes (BS) option model and the hypotheses on which the model is based, were explained as they constituted the basic structure of the study. Then, based on the hypothesis that, in BS model, option prices show a Stable distribution, the relation between Stable distribution and normality was touched on.

In application we tested, using Anderson-Darling Normality Test, whether it is a normal distribution or not, by calculating the option prices of stock issue options in BS model. In this way, we have performed the hypothetical conformity of the BS option pricing model in terms of stock issue options through a statistical approach.

Keywords: Derivatives, Derivative Markets, Option, Option Pricing Models, Black-Scholes Option Pricing Model, Stable Distributions, Anderson-Darling Normality Test.

BLACK-SCHOLES OPSİYON FİYATLAMA MODELİ VE OPSİYON PRİMLERİNİN DURAĞANLIĞININ TEST EDİLMESİ

ÖZ

Türev ürünler, finans, yatırımlar ve finansal kuruluşların yönetiminin modern birlikteliklerinin merkezinde yer almaktadır. Çalışmamızda türev ürünün ne olduğu, hangi türev ürün çeşitlerinin bulunduğu, türev ürünlerinin özellikleri ve birbirlerinden farkları, ürünlerin işleme konulduğu türev piyasaları ve bu piyasaların özelliklerinden bahsedilmiştir. Daha sonra en güvenilir riskten korunma yollarının başında gelen opsiyonlar ve tarihsel gelişimi detaylı bir şekilde incelenmiştir. Burada opsiyonlar için en önemli faktörlere değinilmiş ve bunların opsiyon fiyatının belirlenmesindeki temel etki analiz edilmeye çalışılmıştır.

Daha sonra opsiyon fiyatlama modelleri tanımlanmış, ayrıca nasıl opsiyon fiyatlandırması yapıldığı örneklenmiştir. En çok kullanılan ve opsiyon fiyatlamasında bir devrim niteliği taşıyan Black-Scholes opsiyon modeli ve üzerine kurulduğu varsayımlar, çalışmanın temelini oluşturan temel yapıyı belirleyerek açıklanmıştır. Sonra BS modelinde opsiyon fiyatlarının Durağan (Kararlı) dağılım sergilediği varsayımından yola çıkarak, Durağan dağılımlar ve normallik ilişkisinden bahsedilmiştir.

Uygulamada hisse senedi opsiyonlarının BS model ile opsiyon fiyatını hesap ederek, normal dağılım olup olmadığını Anderson-Darling Normallik Testi ile test ettik. Bu şekilde BS opsiyon fiyatlama modelinin, istatistiksel bir yaklaşım ile hisse senedi opsiyonları açısından varsayımsal tutarlılığını gerçekleştirmiş olmaktayız.

Anahtar Kelimeler: Türev Ürünler, Türev Piyasalar, Opsiyon, Opsiyon Fiyatlama Modelleri, Black-Scholes Opsiyon Fiyatlama Modeli, Durağan (Kararlı) Dağılımlar, Anderson-Darling Normallik Testi.

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CHAPTER ONE
INTRODUCTION

Derivatives have moved to the center of modern corporate finance, investments, and the management of financial institutions. They have also had a profound impact on other management functions such as business strategy, operations management, and marketing. In this study, I begin with an introduction to derivatives (futures, forwards, swaps and options). There have been many developments in derivatives markets over the last 30 years, in chapter two and three; the study has grown to keep up with them. I look in detail at options on stocks, option positions, and option pricing models. The heart of the study is an extensive treatment of the Black-Scholes model.

Options, futures and swaps are examples of derivatives. A derivatives is simply a financial instrument (or even more simply, an agreement between two people) which has a value determined by the price of something else (McDonald, R.L., 2003, 1-2). In figure 1.1 we can see the derivative markets development.

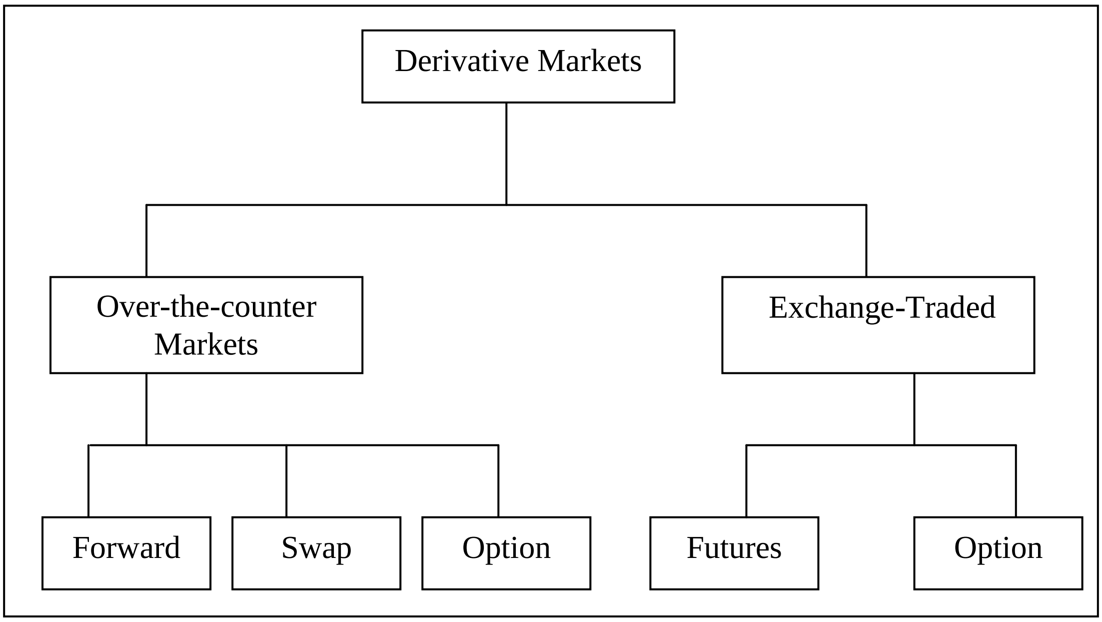

Figure 1.1 Derivative markets and derivative instruments (Aydeniz, Ş., 2008, 39).

Options, futures are examples of what are derivatives of what are termed derivatives. These are instruments whose values depend on the values of other more basic variables (Hull, J., 1995, 13).

For example, a bushel of corn has a value determined by the price corn. However, you could enter into an agreement with a friend that says: If the price of a bushel of corn in one year is grater than $3, you will pay the friend $1. If the price of corn less than $3, the friend will pay you $1. This is a derivative in the sense that you have an agreement with a value depending on the price of something else (corn, in this case) (McDonald, R.L., 2003, 1-2).

1.1 History of Derivatives

Futures markets can be traced back to the middle ages. They were originally developed to meet the needs and merchant consider the position of a farmer in April of a certain year who will harvest grain in June. The farmer is uncertain as to the price he or she will receive for the grain. In years of scarcity, it might be possible to obtain relatively high prices –particularly if the farmer is not in a hurry sell. On the other hand, in years oversupply, the grain might have to be disposed of at fire-sale prices. The farmer and the farmer's family are clearly exposed to great deal of risk.

Consider next a merchant who has an ongoing requirement for grain. The merchant is also exposed to price risk. In some years, an oversupply situation may create favorable prices; in other years, scarcity may cause the prices to be exorbitant. It clearly makes sense for the farmer and the merchant togher in April (even earlier) and agrees on a price for the farmer's anticipated production of grain in June. In other words, it makes sense for them to negotiate a type of futures contract. The contract provides a way for each side to eliminate the risk it faces because of the uncertain price of grain (Hull, J., 1995, 2-3).

Foreign exchange options is a risk management instrument developed to get protected against the foreign exchange rate risk both companies that perform international transactions, banks and financial institutions face since the 1970's (Ersan, İ., 1986).

We explained derivatives history about ancient time but now we should write something about recent history. The restoration in the financial system begins with the Bretton-Woods System after the World War II. According to this system, the party countries to the system accepted that they would fix the foreign exchange rates or at least they would keep the rates 1% below or above the nominal rate. When the Bretton-Woods System was infringed, the world entered a period of rapid changes. In this period the financial world faced with financial risks such as interest rates and especially high exchange rates. As a result, financial risk management started to gain greater importance. In order to avoid or to minimize the financial risks, new financial instruments were developed. As for derivatives, they are the most important ones of these instruments (Chambers, N., 2007, 1-7).

1.2 Derivatives Markets

Financial markets are generally divided into two as spot markets and futures markets (Derivatives markets) according to their terms of swap in purchase and sale transactions. Spot markets are the markets where certain amount of goods or assets and covered money change hands in the day of swap after the transaction. The equity market, bonds and bill market operating under İstanbul Stock Exchange Market can be given as examples to spot markets. As for the futures markets, they are the markets where the purchase and sale transactions are made immediately for a good or a financial instrument, but their delivery or cash settlement are to be made in the future. The Futures Exchange operating under İzmir Derivatives Exchange can be given as example to these markets. The changing economical and political conditions in the world due to the liberalization of internatioal commerce and the increase in its volume, caused fluctuations in foreign exchange rates, interest rates and in prices of particular goods, and thus make difficult the estimation of the price movements in the future. The need for protection against the financial risks arising from this uncertainty of prices in the future has been the greatest factor in the development of derivatives exchanges (Karaahmetoğlu, A., 2006, 3).

Derivatives markets, by acting as an intermediary between the investos who wish to avoid the risks caused by steep price movements in the spot markets and the speculators who are willing to carry these risks, contribute greatly to the development of a country's financial infrastructure, more accurately, to the accomplishment of the capital markets' development in that country (Yılmaz, M.K., 2002, 15).

CHAPTER TWO
TYPE OF DERIVATIVES

2.1 Futures

The transition of Western Markets from fixed exchange rate system to free floating exchange rate system in 1971 became a cornerstone in the development of futures markets. In 1972, with the opening of International Monetary Market under Chicago Mercantile Exchange, first the trade of foreign money futures contracts began. These contracts are the first contracts that can be qualified as financial futures contracts.

Futures contracts are essentially exchange-traded forward contracts. Futures contracts represent a commoment to buy or sell an underlying asset at some future date. Because futures are exchange-traded, they are standardized and have specified delivery dates, locations, and procedures (Ersan, İ., 1998, 7). Futures contracts are traded on an organized exchanged, and the terms of the contract are standardized by the exchange (Hull, J., 1995, 17).

A Futures contract is an agreement comprising of standard period and amount, traded in organized exchange markets and adherent to a daily offset procedure. In daily offset, the losing party is required to make payment to the other party at the end of each transaction day. Futures contracts have two important advantages. These are transaction speed and liquidity. A future contract can be exchanged easily between parties and can be traded in great amounts without affecting the price.

An investor does not have to own the asset as part of the contracts to sell a futures contract. In other words, futures contracts are floated depending on an asset such as foreign exchange, stock issue or bill of exchange. However, an investor can sell futures contracts without owning the aforementioned financial assets. Therefore, the amount of the futures contracts are more than the amount of the financial assets which are subject to trade.

Futures contract is an agreement between two parties to buy or sell an asset at a certain time in the future for a certain price. The largest exchanges on which futures contracts are traded are the Chicago Board of Trade (CBOT) and the Chicago Mercantile Exchange (CME). On these and other exchanges throughout the world, a very wide range of commodities and financial assets forms the underlying assets in the various contracts. The commodities include pork bellies, live cattle, sugar, wool, lumber, copper, aluminum, gold, and tin. The financial assets include stock indices, currencies, and treasury bonds (Chambers, N., 2007, 6-7).

Unlike swap and forward contracts, futures contracts are used intensively, except for its risk management function, for speculation purposes. So much that, 80% to 85% of futures contracts are closed without being due (Ersan, İ., 2003).

2.2 Forwards

A forward contract is an agreement signed between the seller and the buyer with its price determined today and the delivery of an asset is on a determined date in the future (Chance, D. M., 1989, 213). Forward contracts can be organized on all kinds of goods and services. Besides, a forward contract can be organized for financial assets such as foreign exchange, index, stock issues or bill for debt (Chambers, N., 2007, 42).

In another word, forward contracts are similar to futures contracts in that they are agreements to buy or sell an asset at a certain time in the future for a certain price. However, unlike futures contracts, they are not traded on an exchange. They are private agreements between two financial institutions or between a financial institution and one of its corporate clients (Hull, J., 1995, 38).

In addition to a forward contract is a particularly simple derivative. It is an agreement to buy or sell an asset at a certain future time for a certain price. It can be contrasted with a spot contract, which is an agreement to buy or sell an asset today.

A forward is traded in the over the counter market usually between two financial institutions and one of its clients (Hull, J.C., 2003, 2).

2.2.1 Comparison of Futures and Forward Contracts

Together with many similarities, futures and forward contracts have some important differences. The differences between futures contracts and forward contracts are presented below:

- *Contract size*

  Futures: Futures contracts have standard size.

  Forward: The size of the forward contracts is determined in the personal interviews.

- *Organization*

  Futures: Futures contracts traded in well organized and rule-governed official exchanges.

  Forward: Forward contracts are personal and their transactions are performed by banks and financial institutions.

- *Delivery*

  Futures: Future contracts can be delivered at the expiry of term as well as their trade can be done immediately. In futures contracts, delivery is not the purpose.

  Forward: Forward contracts require delivery at the expiry of term. Here delivery is the purpose.

- *Delivery date and procedure*

  Futures: In futures contract specific delivery dates are at issue. Delivery is done in specific places.

  Forward: The delivery of forward contracts is made at the date and in the place determined by the parties.

- *Price volatility*

  Futures: The price is identical for all parties without considering the size of the transaction.

  Forward: The prices can vary due to reasons such as credit risks and trading volume.

- *Price setting*

  Futures: The prices are set by the market forces.

  Forward: Prices are set at the end of the interviews with the bank.

- *Transaction method*

  Futures: Transactions are made in the trading rooms of the stock exchange.

  Forward: Transaction is made between individual buyers and sellers via devices such as phone or fax.

- *Announcement of prices*

  Futures: Prices are open to public.

  Forward: Prices are not open to public.

- *Market place and transaction hours*

  Futures: Transactions are made in hours determined by the stock exchange, in centralized exchange trading rooms with world-wide communication.

  Forward: Transactions are made world-wide over-the-counter 24 hours a day via phone and fax. Forward contracts are non-organized market trading.

- *Deposits and margins*

  Futures: For futures contracts, initial margin, and exchange margin for the daily offset are required.

  Forward: The mortgage amount in exchange for the debts borrowed can be determined with bargain. No margins are required for daily range.

- *Exchange transactions*

  Futures: There is a central exchange room connected with the stock exchange. Here, daily organizations, cash payments and delivery transactions are made. There is the reassurance of the exchange room in case of nonpayment.

  Forward: There is not an exchange room. Therefore there is no assurance against nonpayment.

- *Trading volume*

  Futures: Information on trading volume is published.

  Forward: It is difficult to identify the information related to trading volume.

- *Daily price fluctuations*

  Futures: There are daily price limitations except for FTSE- 100 index.

  Forward: There is not a daily price limitation.

- *Market liquidity and ease of position closing*

  Futures: Due to non-standardized contracts, market liquidity is very high and position closing with other market parties is rather easy.

  Forward: Due to varying contract periods, market liquidity and ease of position closing are limited. Positions are generally closed not with the market participants, but with the real participants of transactions.

- *Credit risks*

  Futures: The exchange room undertakes the credit risk.

  Forward: One party must undertake the credit risk of the other party.

- *Fixing the market (daily cash flow)*

  Futures: One of the most important features of the futures contracts is that there are regulations on daily payments.

Forward: No payment is at issue until the expiriy of term of the contract.

- *Regulations*

Futures: In futures contracts the transactions are subjected to regulations by the stock exchanges.

Forward: Forward markets make their regulations themselves.

As a result of this comparison, it can be said that futures markets are more developed than forward markets together with some exceptions. Undoubtedly, the reason for this is the important advantages the futures markets offer (Chance, D. M., 1989, 213).

2.3 Swaps

Swaps began to show a great development especially from the 1980’s. The most common swap, the foreign exchange swap was first applied in United Kingdom in the 1960’s. The interest rate swap began to take place in the markets since 1981 and found a wide area of application. One of the reasons of this is that all parties of the swap benefit from this transaction. Among the swap parties there are financial institutions, investment and trade banks, agencies of governments and companies (Chambers, N., 2007, 123). It was first tried by Australian Central Bank in 1923 by selling national currency in exchange with British Pound and forward buying it in the spot market (Ersan, İ., 1998, 166). This is a contract that includes particular periodical payment dates for the money exchanged by two parties for a particular period of time (Dönmez, Ç.A., 2002, 3). Money swap can be summarized under three headings:

1. Capitals are exchanged.
2. Interest rates are changed during the term of contract.
3. At the expiry of term capitals are returned (Ersan, İ., 1998, 4, 46).

As mentioned before, swaps have been the leading financial instruments that caught an important growth rate in the financial markets since 1980. Swaps reached to a trading volume of hundreds of million dollars from scratch. It is difficult to calculate the certain trading volume that swaps have reached. One of the reasons for this is that there is not any institution to collect and report the actual data. Beyond question, the trading volume of swaps has reached an enormous amount. It can be argued that, the greatest share in this situation is that swaps are traded in various markets and thus they build a bridge between these markets and the investor. Today, it seems impossible for a head of a company to dominate the future of the company without making a decision on whether or not to enter swap transactions in financial markets.

Swaps are private contracts, on changing the future cash flows caused by a particular financial asset in a predetermined system, between two parties. With this contract, parties try to turn the financial conditions they are in on good account (Chambers, N., 2007, 123).

In addition to, a swap is an agreement two companies to exchange cash flows in the future. The agreement defines the dates when the cash flows are to be paid and the way in which they are to be calculated. Usually the calculation of the cash flows involves the futures values of one or more market variables.

A forward contract can be viewed as a simple example of a swap. Suppose it is March 1, 2002, and a company enters into a forward contract to buy 100 ounces of gold for $300 per ounce in one year. The company can sell the gold in one year as soon as it is received. The forward contract is therefore equivalent to a swap where the company agrees that on March 1, 2003, it will pay $30 000 and receive 100S, where S is the market price of one ounce of gold on that date.

Whereas a forward contract leads to the exchange of cash flows on just one future date, swaps typically lead to cash flow exchanges taking place on several future dates (Hull, J. C., 2003, 125).

The two most common types of swaps are interest rate swap and currency swaps. The most common is a “plain vanilla” interest rate swap. In this, a company agrees to pay cash flows equal to interest at a predetermined fixed rate on a notional principal for the same period of time. And the other popular type of is known as a currency swap. In its simplest form, this involves exchancing principal and interest payments in one currency for principal and interest payments in another currency (Hull, J. C., 2003, 125, 140-146).

2.4 Options

In the International Stock-Exchange Market, the stock issues experience important fluctuations. As a natural result of this, while the stock issue spot market provides opportunity to some investors for great earnings, it exposes some investors with important risks. Futures markets provide important opportunities in order to avoid these kinds of risk faced in spots markets and in order not to leave great earnings opportunities for speculators. One of the products traded in these markets is the option contracts written on stock issues (Yılmaz, M. K., 1998, 1-3).

Put it another way, the option contract is the right to buy or sell a particular amount of assets at a particular price in a particular date in the future or before this date (Ersan, İ., 1998, 94). Options are traded both on exchanges and in the over the counter market (Hull, J. C., 2003, 6).

Option markets are divided into two as organised markets, in which the form provisos of the contracts are standardly determined, and as over the counter markets, in which the form provisos of the contracts are determined freely between parties, taking the mutual needs into consideration.

Over the counter options are progressively growing markets in the world, that are used more and more by the institutional investors. These markets are mostly used by big companies, financial institutions and sometimes by governments, which are well-informed about the credibility of the writer of the option or which guarantee the credit risk being undertaken by several assurances. However, there are always several credit risks that the option buyers using these markets encounter. Over the counter markets are not regulated legally. Their rules are formed as part of honesty and respect depending on commercial common sense. Organized option markets, on the other hand, emerged in order to establish the trading venue, legal infrastructure, standardization of the contracts (usage price, end of term date, etc.) and liquidity, thus accelerated and facilitated the trade of options contracts in the markets like stock isssues, and laid in the emergence of the second hand market, where option contracts can easly change hands (Yılmaz, M.K., 1998, 10). Hereafter we can go through the development of option markets.

2.4.1 History of Option Markets

It is seen that the first use of option contracts date back to ancient Greek and Roman periods. The philosopher Thales, using his knowledge on astronomy, estimated that in the following spring a good crop from olive would be taken and made contracts with the olive press houses in winter months before the harvest. Thales, hitting the mark put in place the contract he had made and made profit by renting the olive presses to other farmers through his contract.

Although the history of options dates back to ancient Greek and Roman era, in its historical development, the options written on tulip bulbs in Netherlends in the 17th century take a rather important place. However, even in this period, many swap problems occurred and options transactions remained off the agenda for some time.

Option markets began to liven up with the contracts written on the stock issues of North Sea Company in 1711 in the United Kingdom. However, again, due to parties not fulfilling their obligations, option markets suffered and options trade was declared unlawful.

The first use of options in America, which collapsed twice in Europe, happens upon the civil war era. The instability in the prices of goods and supplies due to the war carried the farmers to make contracts with merchants and suppliers against uncertainities of prices in the future.

2.4.2 Development of Option Markets

While the use of the concept of option with its general meaning dates back rather early times, the first option contracts were made on goods (mosly agricultural goods) by that times' merchants. As for the options trade on stock issues, it began in the 19th century for the first time.

In the beginning of the 1900's, a group companies which introduced themselves into the market as "Put and Call Brokers and Dealers Association" formed a primitive options market (Chance, D. M., 1989, 22). In this market, if anyone wants to buy an option, the member of the association tries to find a seller who would write the option. On the other hand, if that member cannot find any seller in the market, who is a member of the association, that member itself writes the contract at issue. Put another way, the member company either plays the role of a broker by making the buyer and the seller to meet or bears the role a dealer by becoming a party itself to pozition related to the transaction (Yılmaz, M. K., 1998, 6).

Year 1973 witnessed an important change related to the development of option markets, and an organized exchange with predetermined standards that would only trade options written on stock issues was found by The Chicago Board of Trade (CBOT) which is the oldest and biggest exchange in the world, where commodity futures were traded. The name of this exchange is The Chicago Board Options Exchange (CBOE). In CBOE, buying options written on 16 stock issues were treated in 26 April 1973 for the first time, and the first selling options had began since June 1977 (Whaley, R. E., & Stoll, H. R., 1993, 310). As for foreign exchange options, they were first treated in 1982, in Philadelphia Exchange. In 1983, options on Standard & Poor stock issues were prepared by CBOE (Leblebici, Ü., 1994, 6).

CBOE offered an organised and central market to investors from various sectors, where options would be bought and sold, and increased the liquidity of the market by standardizing the terms and terminology of the contracts. In other words, parties selling or buying an option contract reached the position where they could close their position (by buying or selling) in the market before the expiry of term of the contract. None the less, more important than that, CBOE put a "Corporation clearing house" system, which guarantees the buyer that the seller (writer) of the option would fulfill its obligations, into effect. Thus, counter to over the counter market, the investors buying the option do not need to worry about the credit risks of the writer of the option (Yılmaz, M. K., 1998, 7-8).

From this date forward, many stock exchanges and almost all futures markets in which goods are traded have begun to trade option contracts. The options industry made a rather great progress until the cricis in stock exchange in 1987, many investors affected by this crisis and who previously became parties to option contracts preferred to stay away from the markets in the perido following the aforementioned crisis.

Another important factor that affects the trading volume of the organised markets, where option contracts are treated, is the existence of the over the counter markets emerged as a rival to the organized markets. In the beginning of the 1980's many big companies started to use money interest swaps in order to contain their risks. For these contracts are formed according to the spesific needs of the parties, they became prominent in a very short time. The companies, in the next step, began to trade other contracts such as forward contracts and option contracts, in the over the counter market. However, for the minimum amount of each transaction is very high, individual investord could not find a chance to participate in this new market. The growing of the over the counter markets, which has begun to gain an institutional structure, became an element of oppression on the organized option markets. In the beginning of the 1990's the organized markets, in order to win the battle in institutional trading volume and in order to arouse the investors' interest in options, tried to be more innovative and introduced many sophisticated instrument into the market. If we are to carry out an evaluation as of today, the popularity of the options increases day by day, but the growth becomes dense mostly in over the counter markets. Especially, the emerging markets begin to establish organised futures exchanges and option exchanges one by one, and in especially option trades, the trading volume and the contract diversity in the world still increase with each passing day. When the development of the globalization movement in the world and the increase of access speed between different markets are considered, it is estimated that option markets will grow faster (Yılmaz, M. K., 1998, 9).

2.4.3 Definition of the Option Contracts

Options are contracts which gives the party which sells the right, but no obliging, to buy or sell a certain amount of goods, financial product, capital market instrument or economic indicator that forms a basis for an option for a certain price up to a certain term (or in a certain term) to the party which sells, against a certain premium; on the other hand which obliges the seller of the option to sell (or buy) in a condition of the buyer's request (Türkiye Sermaye Piyasası Aracı Kuruluşlar Birliği Eğitim Notları, Türev Araç Kılavuzu, 208).

Option is an agreement between buyer (holder) and seller (writer). By this agreement, the party selling the option has the right to buy or sell the goods subject to option in a determined price. For this, the buyer pays the seller a premium also called as the rate of option. On the other hand, the seller, by the option contract, undertakes the obligation of delivering the asset in determined price when the buyer makes a request (Chambers, N., 2007, 57).

2.4.4 Basic Option Concepts

2.4.4.1 Kinds of Option

There are two basic kinds of options: Call options and Put options. Besides, there are two parties, as buyer and seller, in each option transaction (Yüksel, A.S., 1997, 423).

Call options: A call option gives the holder the right to buy an asset by a certain date for a certain price (Yılmaz, M.K., 1998, 29). This is a right, not an obligation. In other words, the buyer may prefer not to buy the asset. However, the call option seller has to sell the asset designated in the option contract in case of request. Call options are of vital importance for the investors who think that price of the asset subject to option will increase (Chambers, N., 2007, 58).

Put options: A put option gives the holder the right to sell an asset by a certain date for a certain price (Yılmaz, M.K., 1998, 31). This is not an obligation, but a right. The buyer may prefer not to sell the asset. As fot he put option seller, he has to sell the asset if the buyer demands (Chambers, N., 2007, 59). In table 2.1 we can see the relations of kinds and parts of options.

Table 2.1 Right and obligations in Call and Put options

	Kinds of Option		
Parts of Option		CALL	PUT
	BUYER	The right to buying	The right to selling
	SELLER	The obligation of selling	The obligation of buying

2.4.4.2 Types of Option

There are two basic types of option: American and European. Options can be either both of them (Hull, J., 1995, 173). An European option can be exercised only at maturity (at the end of its life). An American option can be exercised at any time during its life (Hull, J.C., 2003, 5-9).

2.4.4.3 Option Positions

There are two sides for each option contract. On one side is the investor who has taken the long position (i.e., has bought the option). On the other side is the investor who has taken a short position (i.e., has sold or written the option). The writer of an option receives cash up front, but has potential liabilities later. The writer’s profit or loss is the reverse of that for the purchaser of the option. There are four types of option positions:

1. A long position in a call option
2. A long position in a put option
3. A short position in a call option
4. A short position in a put option

We consider the situation of an investor who buys an European call and put options with strike prices in a long position of $100 and $70. Suppose that the option prices are $5 and $7, the expiration date of the option is in four months (we can see the figure 2.2 and 2.3). In the same way we consider an investor who buys an European call and put options with strike prices in a short position of $100 and $70, option prices are $5 and $7 (we can see the figure 2.4 and 2.5).

Now, we should investigate these positions and shows the figures of strategy.

Long Call Example

Profit from buying one European call option: option price = \$5, strike price = \$100, option life = 2 months

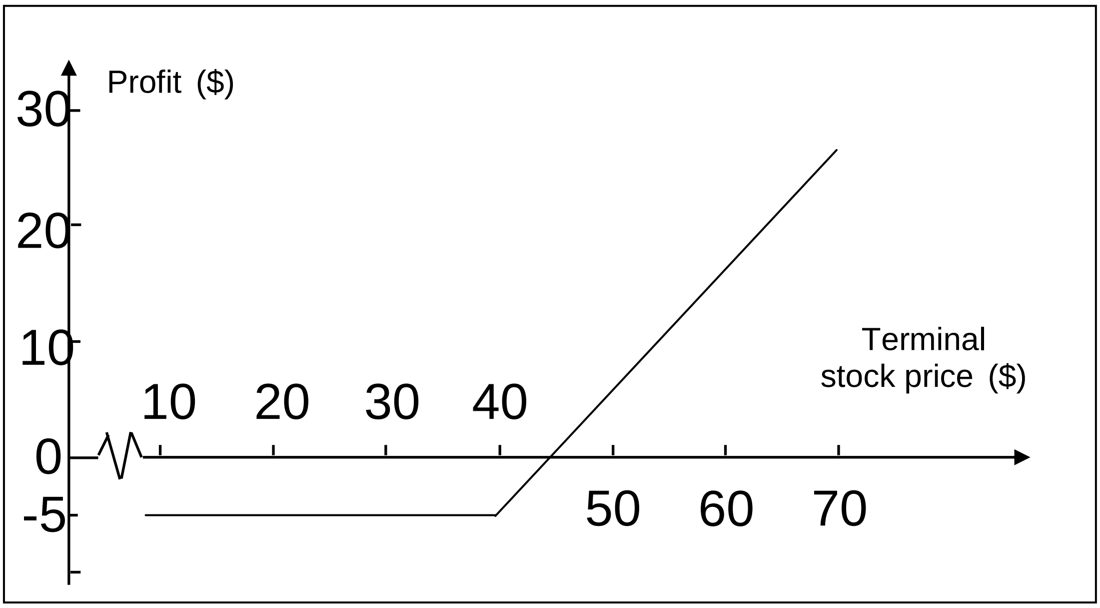

Figure 2.2 Long Call Graph

Long Put Example

Profit from buying a European put option: option price = \$7, strike price = \$70

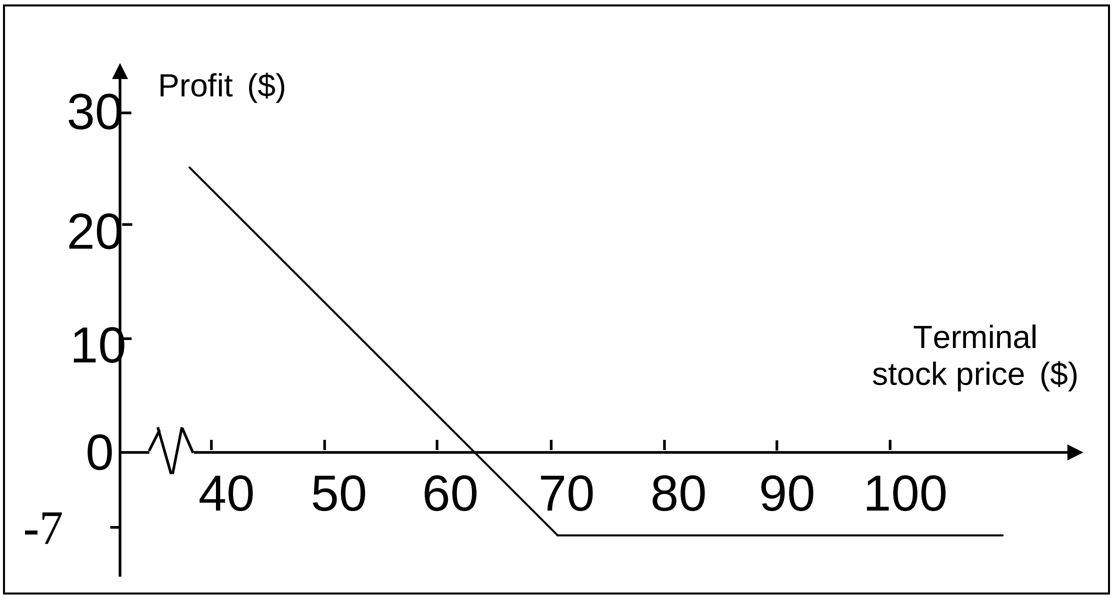

Figure 2.3 Long Put Graph

Short Call Example

Profit from writing one European call option: option price = \$5, strike price = \$100

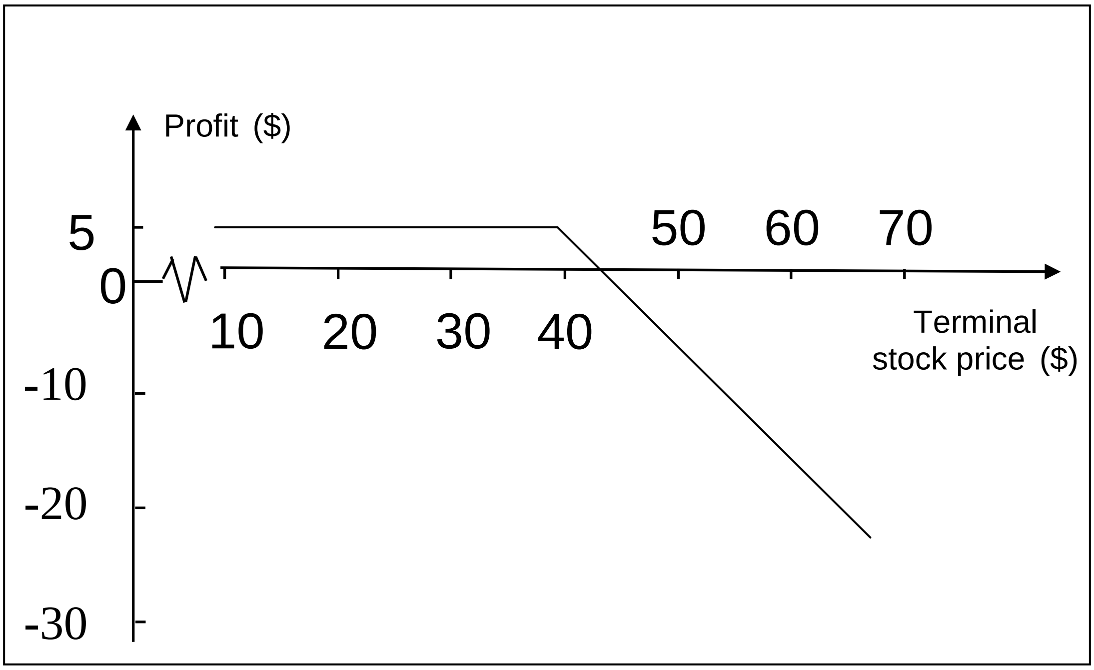

Figure 2.4 Short Call Graph

Short Put Example

Profit from writing a European put option: option price = \$7, strike price = \$70

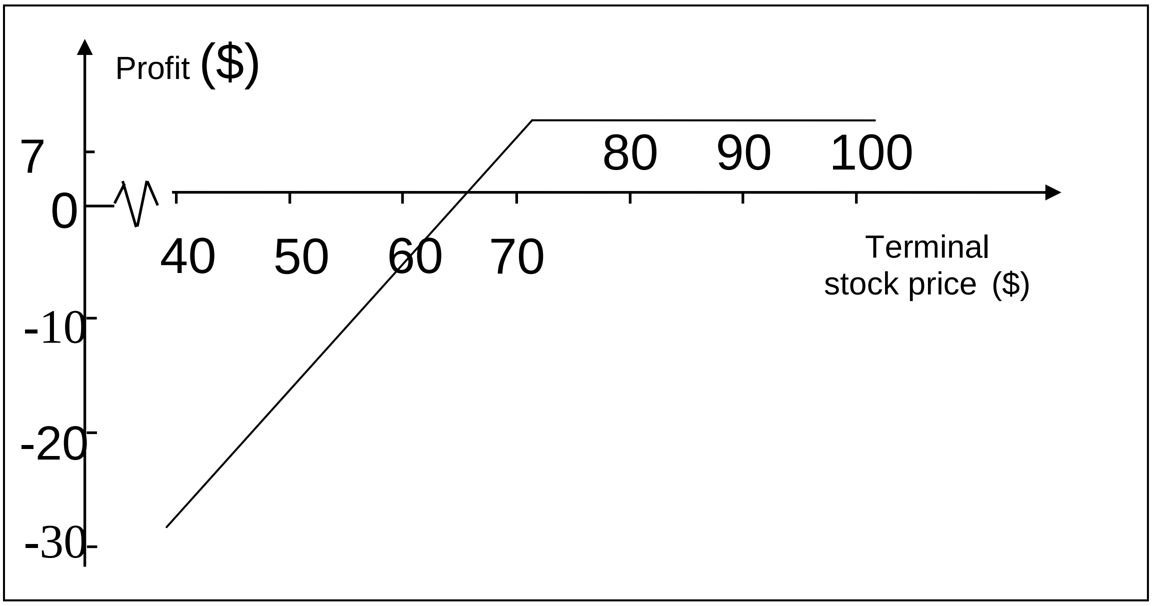

Figure 2.5 Short Put Graph

Payoff and Profit for a Purchased Call Option Example;

The buyer is not obligated to buy the index, and hence will only exercise the option if the payoff is greater than zero. The algebraic expresssion for the payoff to a purchased call is therefore:

Purchased call payoff = max [0, Spot price at expiration – Strike price] (2.1)

For a purchased option, the premium is paid at the time the option is acquired. In computing profit at expiration, suppose we defer the premium payment; then by the time of expiration we accrue 6 months' interest on the premium. The option profit is computed as:

Purchased call profit = max [0, Spot price at expiration- Strike price]
- Future value of option premium (2.2)

Payoff and Profit for a Written Call Option Example;

Now let's look at the option from the point view of the seller. The seller is said to be the option writer, or to have a short position in a call option. The option writer is the counterparty to the buyer. The writer receives the premium for the option and then has an obligation to sell the underlying security in exchange for the strike price if the option buyer exercises the option. The payoff and profit to awritten call are just the opposite of those for a purchased call:

Written call payoff = - max [0, Spot price at expiration- Strike price] (2.3)

Written call profit = -max [0, Spot price at expiration- Stirke price]
+ Future value of option premium (2.4)

Payoff and Profit for a Purchased Put Option Example;

The put option gives the put buyer the right to sell the underlying asset for the strike price. The buyer does this only if the asset is less valuable than the strike price. Thus, the payoff on the put option is:

Put option payoff = max [0, Strike price – Spot price at expiration] (2.5)

The put buyer has a long position in the put.

As with the call, the payoff does not take account of the initial cost of acquiring the position. At the time the option is acquired, the put buyer pays the option premium to the put seller; we need to account for this in computing profit. If we borrow the premium amount, we must pay 6 months' interest. The option profit is computed as:

Purchased put profit = max [0, Strike price- Spot price at expiration]
-Future value of option premium (2.6)

Payoff and Profit for a Written Put Option Example;

The put writer is the counterparty to the buyer. Thus, when the contract is written, the put writer receives the premium. At expiration, if the put buyer elects to sell the underlying asset, the put writer must buy it. The put seller has a short position in the put (McDonald, R.L., 2003, 32-40). The payoff and profit for a written put are the opposite of those for the purchased put:

Written put payoff = - max [0, Strike price- Spot price at expiration] (2.7)

Written put profit = -max [0, Strike price- Spot price at expiration]

+Future value of option premium (2.8)

2.4.4.4 Strike Prices (Exercise Prices)

The exchange chooses the strike prices at which options can be written. For stock options, strike prices are normally spaced \$2.5, \$5, or \$10 apart. (An exception occurs when there has been a stock split or a stock dividend as will be described shortly.) When a new expiration date is introduced, the two strike prices closest to the current stock price are usually selected by the exchange. If one of these is very close to the existing stock price, the third strike price closest to the current stock price may also be selected. If the stock price moves outside the range defined by the highest and lowest strike price, trading is usually introduced in an option with a new strike price (Ersan, İ., 1998, 95).

2.4.4.5 Expiration Dates (Exercise Date, Strike Date)

One of the items used to describe a stock option is the month in which the expiration date occurs. Stock options are on a January, February, or March cycle. If the expiration date for the current month has not yet been reached, options trade with expiration dates in the current month, the following month, and the next two months in its cycle. The last day on which options trade is the third Friday of the expiration month (Campell, T. S., Kracaw, W.A., 1993, 149).

2.4.4.6 In the money, Out of the money, At the money Options

Options are referred to as “in the money”, “out of the money”, or “at the money”. An in-the-money option is one taht would lead to a positive cash flow to the holder if it were exercised immediately. Similarly, an at-the-money option would lead to zero cash flow if it were exercised immediately, and out-of-the-money option would lead to a negative cash flow if it were exercised immediately.

If S is the stock price and K is the strike price, a call option is in-the-money when $S > K$, at-the-money when $S = K$, and out-of-the-money when $S < K$. A put option is in-the-money when $S < K$, at-the-money when $S = K$, and out-of-the-money when

S>K. Clearly, an option will only ever be exercised if it is in-the-money (Ata, B., 2003, 14). In table 2.2 we can see the circumtances of stock and exercice price.

Table 2.2 The relation of option profitableness to Stock Price and Exercise Price

	Call Option	Put Option
Strike Price> Exercise Price	In the money	Out of the money
Strike Price= Exercise Price	At the money	At the money
Strike Price< Exercise Price	Out of the money	In the money

2.4.4.7 Types of Option in Terms of the Asset

By their qualities, there is a real or financial product that each type of derivatives is written on. Options can be written on a great variety of financial or real assets and they are named after the assets they are written on. Among these, goods (stock), gold, stock issue, cotton, futures contract, share index, forward contract and interest options are the basic ones (Gökçe, A.G., 2005, 6).

2.4.4.8 Types of Option Contracts

Stock issue options,
Share index options,
Foreign exchange options,
Interest options,
Options on futures contracts (Sermaye piyasası ve borsa temel bilgi kılavuzu, 2002).

CHAPTER THREE
OPTION PRICING MODELS

3.1 Option Pricing Principles

Each option, by force of the definition of derivative product, is a financial asset dependent on another asset. Therefore, the factors that determine the price, put another way, the value of the option are designated by the features of the asset the option is written on (Hull, J., 1993, 151-153)

3.1.1 Factors Affecting Option Prices

In the beginning, option pricing models take the stock issue options as basis. Both the analytic model Black-Scholes Model and the numerical model Binominal Model (also known as Cox-Ross-Rubinstein Model) are formed by taking stock issue options as basis. This situation has some natural reasons. The first organised formal option market is the CBOE, established by The Chicago Board of Trade in 1973 and the first option contracts traded in this market was the buying options written on stock exchange issues. Thus, the first options, with their pricing becoming a practical requirement, emerged as the options written on stock exchange issues. Besides, stock issues are accepted as having the highest price volatility among the assets, on which the options are written. In this way, it becomes possible to use any pricing model, which grounds on stock issue options, for pricing the options written on other real or financial assets through several adaptations and regulations. There are six factors affecting the price of option (Samuels, J.M., Wilkes, F.M., Brayshaw, R.E., 1995):

i. The current stock price S_0
ii. The strike price K
iii. The time to expiration T
iv. The volatility of the stock price σ
v. The risk-free interest r
vi. The dividends expected during the life of the option

3.1.1.1 Stock price and Strike price

If a call option is exercised at some future time, the payoff will be the amount by which the stock price exceeds the strike price. Call options therefore become more valuable as the stock price increases and less valuable as the strike price increases. For a put option, the payoff on exercise is the amount by which the strike price exceeds the stock price. Put options therefore behave in the opposite way from call options. They become less valuable as the stock price increases and more valuable as the strike price increases (Fabozzi, F.J., Modigliani, F., 1996, 265).

3.1.1.2 Time to Expiration

Both put and call American options become more valuable as the time to expiration increases. Consider two options that differ only as far as the expiration date is concerned. The owner of the long-life option has all the exercise opportunities open to the owner of the short-life option and more. The long-life option must therefore always be worth at least as much as the short-life option.

Although European put and call options usually become more valuable as the time to expiration increases, this is not always the case. Consider two European call optiona on a stock: one with an expiration date in one month, and the other with an expiration date in two months. Suppose that a very large dividend is expected in six weeks. The dividend will cause the stock price to dcline, so that the short-life option could be worth more than the long-life option (Van H., James, C., 1995, 100).

3.1.1.3 Volatility

The volatility of a stock price is a measure of how uncertain we are about future stock price movements. As volatility increases, the chance that the stock will do very well or very poorly increases. For the owner of a stock, these two outcomes tend to offset each other. However, this is not so for the owner of a call or put. The owner of a call benefits from price increases but has limited downside risk in the event of price decreases because the most the owner can lose is the price of the option. Similarly, the owner of a put benefits from price decreases, but has limited downside risk in the event of price increases. The values of both calls and puts therefore increase as volatility increases (Hull, J., 1993, 168).

3.1.1.4 Risk-Free Interest Rate

The risk-free interest rate affects the price of an option in a less clear-cut way. As interest raets in the increase, the expected return required by investors from the stock tends to increase. Also, the present value of any future cash flow received by the holder of the option decreases. The combined impact of these two affects is to decrease the value of put options and increasethe value of call options. It is important to emphasize that we are assuming that interest rates change while all other variables stay the same. In particular, we are assuming that interset rates change while the stock price remains the same. When interest rates rise (fall), the stock price tend to fall (rise). The net effect of an interest rate incerase and the accompanying stock price decrease can be to decrease the value of a call option and increase the value of a put option. Similarly, the net effect of an interest rate decrease and the accompanying stock price incerase can be to increase the value of a call option and decrease the value of a put option (Chance, D. M., 1989, 72-83).

3.1.1.5 Dividends

Dividends have the effect of reducing the stock price on the ex-dividend date. This is bad news for the value of call options and good news for the value of put options. The value of a call option is therefore negatively related to the size of any anticipated dividends, and the value of a put option is positively related to the size of any anticipated dividends (Hull, J., 1993, 168). In table 3.1 we can see the effect of each price on option price.

Table 3.1 Effect of each price on option price

Price Factors	Call Option Value	Put Option Value
*Stock issue price increases	Increases	Decreases
*Strike price increases	Decreases	Increases
*Option term increases	Increases	Increases
*Volatility increases	Increases	Increases
*Interest rate incereases	Increases	Decreases
*Divident paid	Decreases	Increases

3.1.2 The Value of an Option

The price paid for an option conract is called option premium. This premium indicates the value of the option. The value of an option is derived from two sources, real value and time value. Real value is the profit that can be gained from the price movements of the asset. Time value reflects the positive price movement in the period of the term, at the end of the term of the option. The option premium and the relation between the real value and time value can be presented as below (Piesse, J., Peasnell, K., Ward, C., 1995, 195):

Option premium (value) = Real value + Time value (3.1)

Real Value = Stock issue price – Strike Price (3.2)

3.1.2.1 The Intrinsic Value

The intrinsic value of an option is the economic value of the option if it is exercised immediately, except that if there is no positive economic value that results from exercising immediately then the intrinsic value is zero.

For example, if the strike price for a call option is \$100 and the current asset price is \$105, the intrinsic value is \$ 5 gain. That is, an option buyer exercising the option and simultaneously selling the underlying asset would realize \$105 from the sale of the asset, which would be covered by acquiring the asset from the option writer for \$100, thereby netting a \$5 gain.

For a put option, if the strike price of a put option is \$100 and the current asset price is \$92, the intrinsic value is \$8. That is, the buyer of the put option who exercises it and simultaneously sells the underlying asset will net \$8 by exercising.

3.1.2.2 The Time Value

The time value of an option is the amount by which the option price exceeds its intrinsic value. The option buyer hopes that, at some time prior to expiration, changes in the market price of the underlying asset will increase the value of the rights conveyed by the option. For this prospect, the option buyer is willing to pay a premium above the intrinsic value (Cox, J.C., Rubinstein, M., 1985, 298).

3.1.3 Boundary Conditions for Option Pricing

Boundary contions for option pricing have some notations, these are:

S_0 : Current stock price

K : Strike price of option

T : Time to expiration of option

S_T : Stock price at maturity

r : Continuously compounded risk-free rate of interest for an invesment maturing in time T.

C : Value of American call option to buy one share

P : Value of American put option to sell one share

c : Value of European call option to buy one share

p : Value of European put option to sell one share

3.1.3.1 Upper Bounds

An American or European calls option gives gives the holder the right to buy one share of a stock for a certain price. No matter what happens, the option can never be worth more than the stock. Hence, the stock price is an upper bound to the option price:

$$c \leq S_0 \text{ and } C \leq S_0$$

An American or European put option gives the holder the right to sell one share of a stock for K. No matter how low the stock price becomes, the option can never be worth more than K. Hence,

$$p \leq K \text{ and } P \leq K$$

For European options, we know that at option cannot be worth more than K. It follows that it cannot be worth more than the present value of K today (Sermaye piyasası araçlarına dayalı future ve option sözleşmelerinin fiyatlaması, 41):

$$p \leq Ke^{-rT}$$

3.1.3.2 Lower Bounds

3.1.3.2.1 Lower Bound for Calls on Non-Dividend-Paying Stocks

A lower bound for the price of a European call option on a non-dividend-paying stock is

$$S_0 - Ke^{-rT} \tag{3.3}$$

We look at a numerical example:

Suppose that $S_0 = \$20,\ K = \$18,\ r = 10\%$ per annum, $T = 1$ year. In this case,

$$S_0 - Ke^{-rT} = 20 - 18e^{-0.1} = \$3.71$$

For a more formal argument, we consider the following two portfolios:

Portfolio A: one European call option plus an amount of cash equal to Ke^{-rT}

Portfolio B: one share

In portfolio A, the cash, if it is invested at the risk-free interest rate, will grow to K in time T. If $S_T > K,$ the call option exercised at maturity and portfolio A is worth S_T. If $S_T < K,$ the call option expires worthless and the portfolio is worth K. Hence, at time T, portfolio A is worth

$$\max(S_T, K)$$

Portfolio B is worth S_T at time T. Hence, portfolio A is always worth as much as, and can be woth more than, portfolio B at the option's maturity. It follows that in the absence of arbitrage opportunities this much also be true today. Hence,

$$c + Ke^{-rT} \geq S_0 \qquad \text{or} \qquad c \geq S_0 - Ke^{-rT}$$

Because the worst that can happen to a call option is that it expires worthless, its value cannot be negative. This means that $c \geq 0,$ and therefore,

$$c \geq \max(S_0 - Ke^{-rT}, 0) \tag{3.4}$$

3.1.3.2.2 Lower Bound for European Puts on Non-Dividend-Paying Stocks

For a European put option on a non-dividend-paying stock, a lower bound for the price is

$$Ke^{-rT} - S_0 \tag{3.5}$$

We look at a numerical example:

Suppose that $S_0 = \$37, K = \$40, r = 5\%$ per annum, $T = 0.5$ years. In this case,

$$Ke^{-rT} - S_0 = 40e^{-0.5*0.5} - 37 = \$2.01$$

For a more formal argument, we consider the following two portfolios:

Portfolio C: one European put option plus one share

Portfolio D: an amount of cash equal to Ke^{-rT}

If $S_T < K,$ the option in portfolio C is exercised at option maturity, and the portfolio becomes worth K. If $S_T > K,$ the put option expires worthless, and the portfolio is worth S_T at this time. Hence, portfolio C is worth

$$\max(S_T, K)$$

at time T. Assuming the cash is invested at the risk-free interest rate, portfolio D is worth K at time T. Hence, portfolio C is always worth as much as, and can sometimes be worth more than, portfolio D at time T. It follows that in the absence of arbitrage opportunities portfolio C must be worth at least as much as portfolio D today. Hence,

$$p + S_0 \geq Ke^{-rT} \qquad \text{or} \qquad p \geq Ke^{-rT} - S_0$$

Because the worst that can happen to a put option is that it expires worthless, its value cannot be negative. This means that (Hull, J., 2003, 172-174)

$$p \geq \max(Ke^{-rT} - S_0, 0) \qquad (3.6).$$

3.1.4 Put-Call Parity

We derive an important relationship between p and c. Consider the following two portfolios that were used in the previous section:

Portfolio A: one European call option plus an amount of cash equal to Ke^{-rT}

Portfolio C: one European put option plus one share

Both are worth

$$\max(S_T, K)$$

at expiration of the options. Because the options are European, they cannot be exercised prior to the expiration date. The portfolios must therefore have identical values today. This means that

$$c + Ke^{-rT} = p + S_0 \tag{3.7}$$

This relationship is known as put-call parity.

- American Options

Put-call parity holds only for European options. However, it is possible to derive some results for American option prices. It can be shown that (Hull, J., 2003, 174-175)

$$S_0 - K \leq C - P \leq S_0 - Ke^{-rT} \qquad (3.8).$$

3.1.5 Effect of Dividends

The dividends payable during the life of the option can usually be predicted with reasonable accuracy. We will use D to denote the present value of the dividends during the life of the option. In the calculation of D, a dividend is assumed to occur at the time of its ex-dividend date.

3.1.5.1 Lower Bound for Calls and Puts

We can redefine portfolios A and B as follows:

Portfolio A: one European call option plus an amount of cash equal to $D + Ke^{-rT}$

Portfolio B: one share

A similar argument to the one used to derive equation (3.3) shows that

$$c \geq S_0 - D - Ke^{-rT} \tag{3.9}$$

We can also redefine portfolios C and D as follows:
Portfolio C: one European put option plus one share
Portfolio D: an amount of cash equal to $D + Ke^{-rT}$
A similar argument to the one used to derive equation (3.5) shows that

$$p \geq D + Ke^{-rT} - S_0 \qquad (3.10)$$

3.1.5.2 Put-Call Parity

Comparing the value at option maturity of the redefinedportfolşos A and C shows that, with dividends, the put-call parity reesult in equation (3.7) becomes

$$c + D + Ke^{-rT} = p + S_0 \qquad (3.11)$$

Dividends cause equation (3.8) to be modified to (Sermaye piyasası araçlarına dayalı future ve option sözleşmelerinin fiyatlaması, 45)

$$S_0 - D - K \leq C - P \leq S_0 - Ke^{-rT} \qquad (3.12).$$

3.2 The Binomial Option Pricing Model

A useful and very popular technique for pricing a stock option involves constructing a binomial model tree. This is a diagram that represents different possible paths that might be followed by the stock price over the life of the option (Hull, J., 2003, 200). In order to understand the modeling power of the binomial model, we must look at what happens if we increase the number of trading periods between the current date and the expiration date (Cox, J.C., Ross, S.A., Rubinstein, M., 1979, 229).

The following hypotheses on binominal model, used in pricing the options written on stock issues that does not pay dividend are accepted:

1. Markets are perfect and perfectly competitive.
2. Transaction costs and taxes are zero.
3. Short sale is free and investors can use all they reobtained via short sale.
4. Only a single interest rate, r, is available, and investors can borrow and lend at this rate without risks.
5. Periodic interest rate, r, and upticks (u) and downticks (d) of the stock issue prices are known for every period in the future. The stock issue prices move according to this “geometric random walk” only. u, d, and r do not have to be the same for each period, it is accepted that they only are foreknown for each period, namely they have a deterministic nature.
6. Investors prefer high income to low income. Under this hypothesis, all arbitrage possibilities will disappear immediately.
7. Information is a resource without cost, and has a free for all quality.
8. There is no divident pay (Yılmaz, M.K., 1998, 110).

3.2.1 A One-Step Binomial Tree

Suppose that we are interested in valuing a European call option to buy a stock for $21 in three months. A stock price is currently $20. We make a simplifying assumption that at the end of three months the stock price will be either $22 or $18. This means that the option will have one of two values at the end of the three months. If the stock price turns out to be $22, the value of the option will be $1; if the stock price turns out to be $18, the value of the option will be zero. In Figure 3.1 we can see this position.

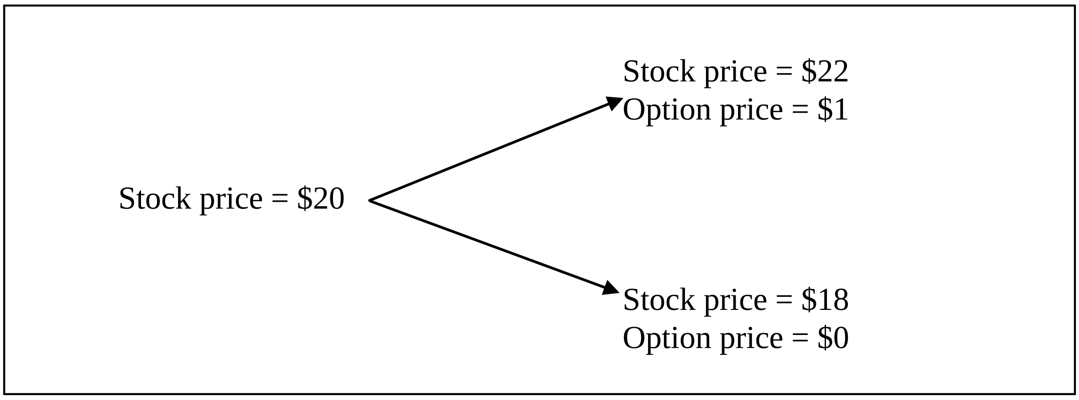

Figure 3.1 Stock price movements in numerical example

In general the argument just presented by considering a stock whose price is S_0 and an option on the stock whose current price is f . We suppose that the option lasts for time T and that during the life of the option the stock price can either move up from S_0 to a new level S_0u or down from S_0 to a new level S_0d , where u>1 and d<1. The proportional increase in the stock price when there is an up movement is u-1; the proportional decrease when there is a down movement is 1-d. If the stock price moves up to S_0u , we suppose that the payoff from the option is f_u ; if the stock price moves down to S_0d , we suppose the payoff from the option is f_d (In Figure 3.2).

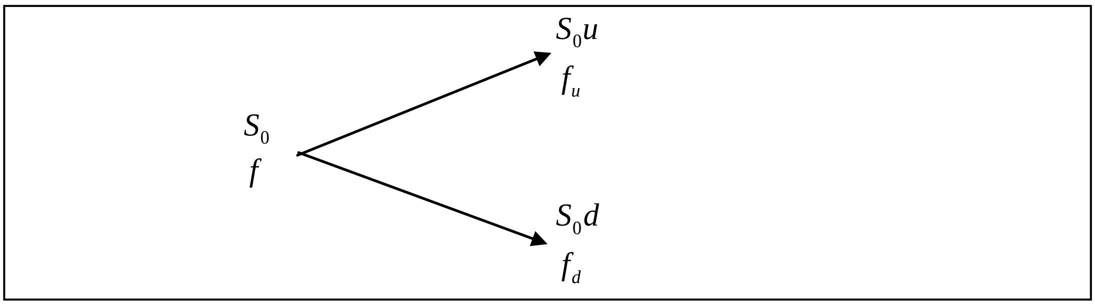

Figure 3.2 Stock and option prices in a general one-step tree

The one-step binomial tree can be expanded as two-step binomial trees.

For the portfolio is risk-free, the current value of the portfolio can be found by discounting the estimated value by the risk-free interest rate (Blake, D., 1990, 204).

$$f = e^{-rT}[pf_u + (1-p)f_d] \tag{3.13}$$

where (Hull, J., 2003, 200)

$$p = \frac{e^{rT} - d}{u - d} \tag{3.14}$$

3.2.2 Two-Step Binomial Trees

We can generalize the case of two time steps by considering the situation given in Figure 3.3 when the stock price is initially S_0. During each time, it either moves up to *u* times its initial value or moves down to d times its initial value. The notation for the value of the option is shown on the tree (For example, after two up movements the value of the option is f_{uu}, we can see this in Figure 3.1).

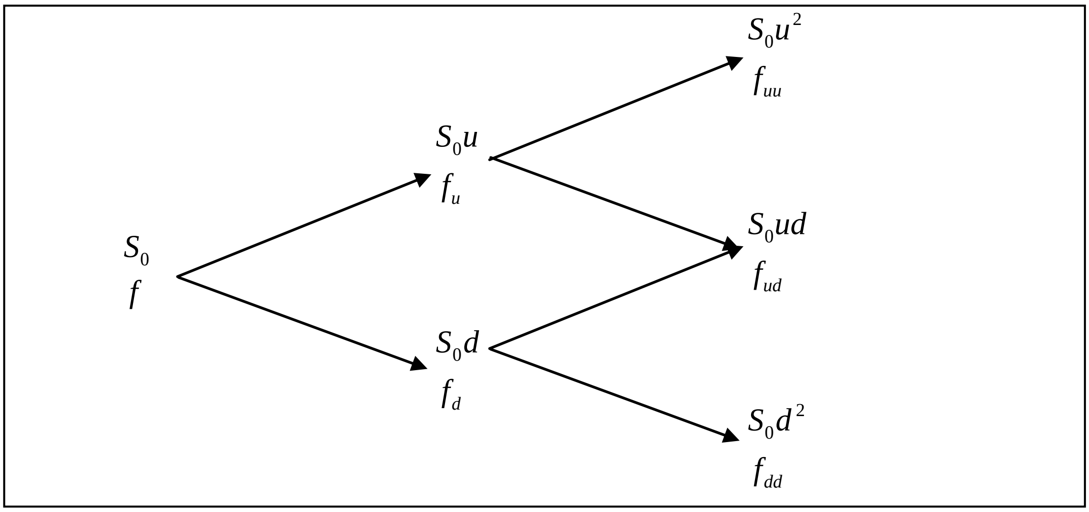

Figure 3.3 Stock and option prices in a general two-step tree

We generalize the use of two-step binomial trees still further by adding more steps to the binomial trees and for the American options (Fabozzi, F.J., Modigliani, F., 1996, 267).

3.3 The Black-Scholes Option Pricing Model

In the early 1970s, Fischer Black, Myron Scholes, and Robert Merton made a major breakthrough in the pricing of stock options (Black, F., Scholes, M., 1973, 637-659, Merton, R.C., 1973, 83-141). This involved the development of what has become known as the Black-Scholes model. The model has had a huge influence on the way that trader's price and hedge options. It has also been pivotal to the growth and success of financial engineering in the 1980s and 1990s. In 1997, the importance of the model was recognized when Robert Merton and Myron Scholes were awarded the Nobel Prize for economics. Sadly, Fischer Black died in 1995; otherwise he also undoubtedly has been one of the recipients of this prize (Bowe, M., 1988, 96).

Although Black-Scholes model was not developed directly from the binominal model, it can be thought that it is the mathematically advanced version of the binominal model. However, Black-Scholes did not derive their models by carrying the binominal models into infinite timeframe. Indeed, when Black-Sholes began studying option pricing models, the binominal had model not been found yet (Yılmaz, M.K., 1998, 140).

Black-Scholes option pricing model is based on several hypotheses (Ersan, İ., 1998, 107, Black, F., Scholes, M., 1973, 640). These are:

1. The price of the derivative instrument follows a geometric Brownian movement (random walk) course with constant μ and σ. Therefore the probabilistic distribution of the prices of the derivative instruments is lognormal distribution.
2. The profits of the assets the option is based on are distributed normally (in a general way of speaking it is a stable distribution).
3. The short term interest ratios are constant.
4. There are no dividends and interest payment during the life of the derivative.
5. Option types are European options.
6. There are no transactions costs or taxes. All securities are perfectly divisible.
7. Security trading is continuous.

8. The short selling of securities with full use of proceeds is permitted.
9. There are no riskless arbitrage opportunities.

3.3.1 Lognormal Property of Stock Prices

Percentage changes in the stock price in a short period of time are normally distributed. Define:

μ : Expected return on stock

σ : Volatility of the stock price

The mean of the percentage change in time δ_t is $\mu\delta_t$ and the standard deviation of this percentage change is $\sigma\sqrt{\delta_t}$, so that

$$\frac{\delta S}{S} \sim N(\mu\delta_t, \sigma\sqrt{\delta_t}) \tag{3.15}$$

where δS is the change in the stock price S in time δ_t, and $N(\mu\delta_t, \sigma\sqrt{\delta_t})$ denotes a normal distribution. The model implies that

$$\ln S_T - \ln S_0 \sim N\left[(\mu - \frac{\sigma^2}{2})T, \sigma\sqrt{T}\right]$$

From this it follows that

$$\ln\frac{S_T}{S_0} \sim N\left[(\mu - \frac{\sigma^2}{2})T, \sigma\sqrt{T}\right] \tag{3.16}$$

and

$$\ln S_T \sim N\left[\ln S_0 + (\mu - \frac{\sigma^2}{2})T, \sigma\sqrt{T}\right] \tag{3.17}$$

Where S_T the stock price is at future time T and S_0 is the stock price at time zero. Equation (3.17) shows that $\ln S_T$ is normally distributed. This means that S_T has a lognormal distribution. From equation (3.17) and the properties of the lognormal distribution, it can be shown that the expected value, $E(S_T)$ of S_T is given by

$$E(S_T) = S_0 e^{\mu T} \qquad (3.18)$$

This fits in with the definition of μ as the expected rate of return. The variance, $\mathrm{var}(S_T)$ of S_T can be shown to be given by (Aitchison, J., Brown, J.A.C., 1966)

$$\mathrm{var}(S_T) = S_0^2 e^{2\mu T}(e^{\sigma^2 T} - 1) \qquad (3.19).$$

3.3.2 The Distribution of the Rate of Return

The lognormal property of stock prices can be used to provide information on the probability distribution of the continuously compounded rate of return earned on a stock between times zero and T. Define the continuously compounded rate of return per annum realized between times zero and T as η. It follows that

$$S_T = S_0 e^{\eta T}$$

so that

$$\eta = \frac{1}{T} \ln \frac{S_T}{S_0} \qquad (3.20)$$

It follows from equation (3.16) that

$$\eta \sim N\left(\mu - \frac{\sigma^2}{2}, \frac{\sigma}{\sqrt{T}}\right) \qquad (3.21)$$

Thus, the continuously compounded rate of return per annum is normally distributed (Hull, J., 2003, 236).

3.3.3 The Expected Return

Equation (3.15) shows that $\mu\delta_t$ is the expected pertentage change in the stock price in a very short period of time δ_t. This means that μ is the expected return in a very short period of time δ_t. It is natural to assume that μ is also the expected continuously compounded return on the stock over a relatively long period of time. However, this is not the case. The continuously compounded return realized over T years is

$$\frac{1}{T}\ln\frac{S_T}{S_0}$$

and equation (3.21) shows that the expected value of this is $\mu - \frac{\sigma^2}{2}$. The reason for the distinction between the μ in equation (3.15) and the $\mu - \frac{\sigma^2}{2}$ in equation (3.21) is subtle but important (Hull, J., 2003, 237-238). So that

$$E[\ln(S_T)] - \ln(S_0) = \mu T, \text{ or } E[\ln(S_T / S_0)] = \mu T$$

3.3.4 Historical Volatility

Historical volatility is a method that investors use to measure volatility. This method is implemented by taking into consideration the price movements the stock issue, on which the option is written, exhibited in a period of time in the past (Blake, D., 1990, 205). To estimate the volatility of a stock price empirically, the stock price is usually observed at fixed intervals of time (e.g., every day, week, or month). Now, in Table 3.2 we can see the computation of historical volatility and price relativity.

Table 3.2 Computation of Historical Volatility

Week	Stock price (S_i)	Price relative (S_i / S_{i-1})	Log price relative [$\ln(S_i / S_{i-1})$]
1	47.38	-	-
2	48.17	1.0167	0.016536
3	49.21	1.0216	0.021360
4	50.79	1.0321	0.031603
5	51.83	1.0205	0.020270
6	52.62	1.0152	0.015127

Let

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right) \qquad \text{for } i = 1, 2, \ldots, n.$$

The usual estimate, s, of the standard deviation of the u_i ’s is given by

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2}$$

or

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}u_i^2 - \frac{1}{n(n-1)}\left(\sum_{i=1}^{n}u_i\right)^2} \tag{3.22}$$

where u is the mean of the u_i ’s.

3.3.5 Implied Volatility

The one parameter in the Black-Scholes pricing formulas that cannot be directly observed volatility of the stock price. In practice, traders usually work with what are known as implied volatilities. These are the volatilities implied by option prices observed in the market (Bowe, M., 1998, 98).

Implied volatilities are used to monitor the market’s opinion about the volatility of a particular stock. Traders like to calculate implied volatilities from actively traded options on a certain asset and interpolate between them to calculate the appropriate volatility for pricing a less actively traded option on the same stock. It is important to note that the prices of deep-in-the-money and deep-out-of-the-money options are relatively insensitive to volatility. Implied volatilities calculated from these options tend, therefore, to be unreliable (Edwards, F. R., & Ma, C. W., 1992, 551).

3.3.6 The Analysis of Black-Scholes Option Pricing Model

The Black-Scholes option pricing models for the prices at time zero of an European call option on a non-dividend-paying stock and an European put option on a non-dividend paying stock are (Fabozzi, F.J., Modigliani, F., 1996, 333)

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2) \tag{3.23}$$

and

$$p = Ke^{-rT} N(-d_2) - S_0 N(-d_1) \tag{3.24}$$

where

$$d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}} \tag{3.25}$$

$$d_2 = \frac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \tag{3.26}$$

In these formulas, the function N(x) is the cumulative probability distribution function for a standardized normal distrbution. In other words, it is the probability that a variable with a standard normal distribution, N(0,1) will be less than x. The remaining variables should be familiar, these are:

S_0 : Current stock price

K : Strike price of option

T : Time to expiration of option

r : Continuously compounded risk-free rate of interest

$c:$ Value of European call option

$p:$ Value of European put option

$N(d_1), N(d_2):$ The cumulative probability distribution functions for a standardized normal distribution.

$\sigma:$ The stock price volatility.

3.3.6.1 Computing a Call Option Price

To example the Black-Scholes option pricing model, assume the following values:

$S_0:$ Current stock price, $S_0 = \$47$

$K:$ Strike price of option, $K = \$45$

$T:$ Time to expiration of option, $T = 0.5$ (183 days / 365 rounded)

$r:$ Continuously compounded risk-free rate of interest, $r = 10\%$

$\sigma:$ The stock price volatility, $\sigma = 25\%$

Substituting these values into the equations (3.25) and (3.26):

$$d_1 = \frac{\ln(47/45) + (0.10 + ([0.25]^2/2))0.5}{0.25\sqrt{0.5}} = 0.6172$$

$$d_2 = 0.6172 - 0.25\sqrt{0.5} = 0.4404$$

From a normal distribution table,

$$N(d_1) = N(0.6172) = 0.7315 \text{ and}$$
$$N(d_2) = N(0.4404) = 0.6702$$

Then

$$c = 47(0.7315) - 45(e^{-(0.10)(0.5)})(0.6702) = \$5.69$$

Here the premium of a European Call Option is calculated as $5.69.

Base Case Call Option:

$$K = \$45, S_0 = \$47, T = (183/365) = 0.5, r = 10\%, \sigma = 25\%$$

Table 3.3 Holding all factors constant except expected return volatility

Expected Price Volatility	Call Option Price
15%	4.69
20%	5.17
25% (base case)	5.69
30%	6.26
35%	6.84
40%	7.42

Table 3.4 Holding all factors constant except the risk-free rate

Risk-Free Interest Rate	Call Option Price
7%	5.27
8%	5.41
9%	5.50
10% (base case)	5.69
11%	5.84
12%	5.99

Table 3.5 Holding all factors constant except time remaining to expiration

Time Remaining to Expiration	Call Option Price
30 days	2.85
60	3.52
91	4.15
183 (base case)	5.69
273	6.99

Table 3.3, 3.4 and 3.5 shows the option value as calculated from the Black-Scholes option pricing model for different assumptions concerning the standard deviation for the stock's return; the risk-free rate; and the time remaining to expiration. Notice that the option price varies directly with three variables: expected return volatility, the risk-free rate, and time remaining to expiration. That is: (Table

3.3) the lower (higher) the expected volatility, the lower (higher) the option price; (Table 3.4) the lower (higher) the risk-free rate, the lower (higher) the option price; and (Table 3.5) the shorter (longer) the time remaining to expiration, the lower (higher) the option price (Geske R., 1979, 375-380).

As it can be understood from these calculations, the option premiums change dependending on volatility, risk-free interest rates and on the time remaining to expiration. However, we examined the hypotheses, on which the Black-Scholes option model was founded, and that affect the option premiums, one by one. In this sense, taking the hypothesis that “the profits of the assets the option is based on are distributed normally (in a general way of speaking it is a stable distribution)” as a starting point, we may investigate the Stable Distributions and their properties, which are considered as exhibited by option premiums.

CHAPTER FOUR
STABLE DISTRIBUTIONS

Stable distributions are a rich class of probability distributions that all skewness and heavy tails and have many intriguing mathematical properties. The class was characterized by Paul Levy in his study of sums of independent identically distributed terms in the 1920's. The lack of closed formulas for densities and distribution functions for all but a few stable distributions (Normal or Gaussian, Cauchy and Levy, see Figure 4.1), has been a drawback to the use of stable distributions by practitioners. There are now reliable computer programs to compute stable densities, distribution functions and quantiles. With these programs, it is possible to use stable models in a variety of practical problems.

The general stable distribution is described by four parameters: an index of stability $\alpha \in (0,2]$, a skewness parameter β, a scale parameter γ and a location parameter δ (Nolan, P. J., 1999, 2).

Stable distributions have been proposed as a model for many types of physical and economic systems. There are several reasons for using a stable distribution to describe a system. The first is where there solid theoretical reasons for expecting a non-Gauussian stable model, e.g. reflection off a rotating mirror yielding a Cauchy distribution, hitting times for a Brownian motion yielding a Levy distribution, the gravitational field of stars yielding the Holtsmark distribution; see Feller (1971) and Uchaikin and Zolotarev (1999) for these. The second reason is the Generalized Central Limit Theorem which states that the only possible non-trivial limit of normalized sums of independent identically distributed terms is stable. It is argued that some observed quantities are the sum of many small terms- the price of a stock, the noisse in a communication system, etc. and hence a stable model should be used to describe such systems. The third argument for modeling with stable distributions is empirical: many large data sets exhibit heavy tails and skewness. The strong empirical evidence for these features combined with the Generalized Central Limit Theorem is used by many to justify the use of stable models (Nolan, P. J., 2009).

4.1 Definitions of Stable Distribution

An importnant property of normal or Gaussian random variables is that the sum of two of them is itself a normal random variable. One consequence of this that if X is normal, then for X_1 and X_2 indepedent copies of X and any positive constants a and b,

$$aX_1 + bX_2 \stackrel{d}{=} cX + d, \tag{4.1}$$

For some positive constant c and some d $\in$ R. (The symbol $\stackrel{d}{=}$ means equality in distribution, i.e. both expressions have the same probability law.) In words, equation (4.1) says that the shape of X is preserved (up to scale and shift) under addition.

Definition 4.1 A random variable X is stable or stable in the broad sense if for X_1 and X_2 independent copies of X and any positive constants a and b, (4.1) holds for some positive c and some d $\in$R. The random variable is strictly or stable in the narrow sense if (4.1) holds with d=0 for all choices of a and b. A random variable is symmetric stable if it is stable and symmetrically distributed around 0, e.g. $X \stackrel{d}{=} -X$.

The addition rule for independent normal random variables says that the mean of the sum is the sum of the means and the variance of the sum is the variances. Suppose $X \sim N(\mu, \sigma^2)$, then the terms on the left hand side above are $N(a\mu, (a\sigma)^2)$ and $N(b\mu, (b\sigma)^2)$ respectively, while the right hand side is $N(c\mu + d, (c\sigma)^2)$. By the addition rule one must have $c^2 = a^2 + b^2$ and $d = (a + b - c)\mu$. Expressions for c and d in the general stable case are given below.

The word stable is used because the shape is stable or unchanged under sums of the type and to distinguish between these distributions and max-stable, multiplication stable and geometric stable distributions. Also, some older literature used slightly different terms: stable aws originally used for what we now call strictly stable, quasi-stable was reserved for what we now call stable.

Two random variables X and Y are said to be of the same type if there exists contants A>0 and B $\in$ R with $X \stackrel{d}{=} AY + B$. The definition of stability can be restated as $aX_1 + bX_2$ has the same type as X.

There are three cases where one can write down closed form expressions for the density and verify directly that they are stable – normal, Cauchy and Levy distributions.

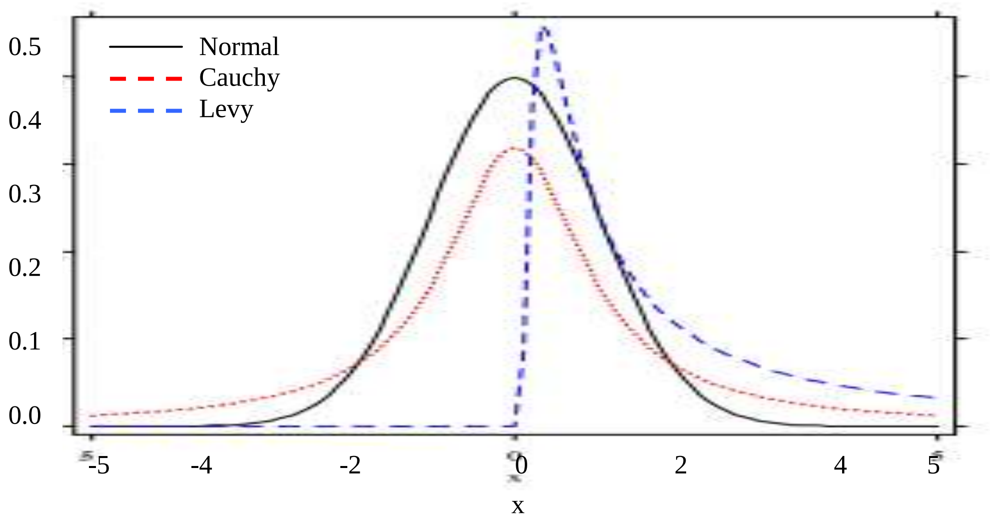

Figure 4.1 Graphs of Standartized Normal N (0, 1), Cauchy (1, 0) and Levy (1, 0) densities.

Table 4.1 Comparison of tail probabilities for Standard Normal, Cauchy and Levy distributions.

c	P(X>c)		
	Normal	Cauchy	Levy
0	0.5000	0.5000	1.0000
1	0.1587	0.2500	0.6827
2	0.0228	0.1476	0.5205
3	0.001347	0.1024	0.4363
4	0.00003167	0.0780	0.3829
5	0.0000002866	0.0628	0.3453

General stable distributions allow for varying degrees of tail heaviness and varying degrees of skewness. Other than the normal distribution, the Cauchy distribution, Levy distribution, and the reflection of the Levy distribution, there are not known closed form expressions for general stable densities and it is unlikely that any other stable distributions have closed forms for their densities. Now we can see the Stable Distributions examples.

Example 4.1 Normal or Gaussian distributions. $X \sim N(\mu, \sigma^2)$ if it has a density

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(x-\mu)^2}{2\sigma^2}), \quad -\infty < x < \infty. \tag{4.2}$$

The cumulative distribution function, for which there is no closed form expression, is $F(x) = P(X \le x) = \Phi((x-\mu)/\sigma)$, where $\Phi(z) =$ probability that a standard normal random variable is less than or equal z.

Example 4.2 Cauchy distributions. $X \sim \mathit{Cauchy}(\gamma, \delta)$ if it has density

$$f(x) = \frac{1}{\pi} \frac{\gamma}{\gamma^2 + (x-\delta)^2}, \quad -\infty < x < \infty. \tag{4.3}$$

These are also called Lorenz distributions in physics.

Example 4.3 Levy distributions. $X \sim \mathit{Levy}(\gamma, \delta)$ if it has density

$$f(x) = \sqrt{\frac{\gamma}{2\pi}} \frac{1}{(x-\delta)^{3/2}} \exp(-\frac{\gamma}{2(x-\delta)}), \quad \delta < x < \infty. \tag{4.4}$$

Note that some authers use the term Levy distribution for all sum stable laws. Consequently,

$N(\mu, \sigma^2)$ Distribution is stable with $(\alpha = 2, \beta = 0, a = \sigma / \sqrt{2}, b = \mu)$,

Cauchy (γ, δ) Distribution is stable with $(\alpha = 1, \beta = 0, a = \gamma, b = \delta)$ and

Levy (γ, δ) Distribution is stable with $(\alpha = 1/2, \beta = 1, a = \gamma, b = \delta)$ (Nolan, P. J., 2009, 5-7).

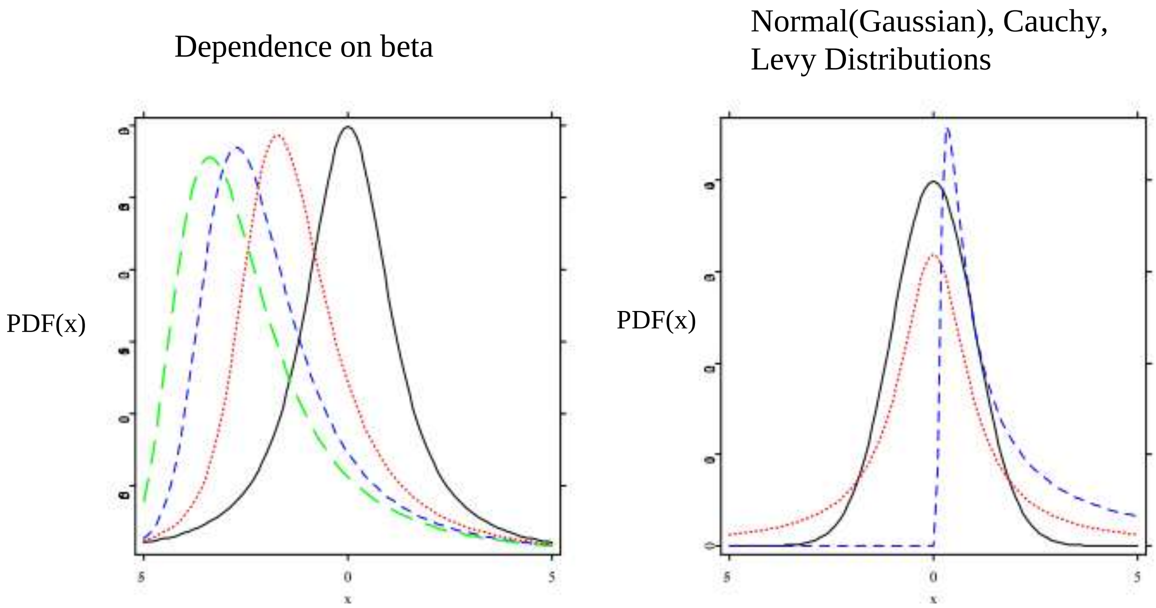

Figure 4.2 *Left panel:* Stable pdfs for $\alpha = 1.2$ and $\beta = 0$ (black solid line), 0.5 (red dotted line), 0.8 (blue dashed line) and 1 (green long-dashed line). *Right panel:* Closed form formulas for densities are known only for three distributions – Gaussian ($\alpha = 2$; black solid line), Cauchy ($\alpha = 1$; red dotted line) and Levy ($\alpha = 0.5, \beta = 1$; blue dashed line). The latter is a totally skewed distribution, i.e. its support is R_{+}. In general, for $\alpha < 1$ and $\beta = 1(-1)$ the distribution is totally skewed to the right (left).

Definition 4.2 For the real numbers $b_1 > 0,\ b_2 > 0$ and c_1, c_2

$$F\left(\frac{x-c_1}{b_1}\right) * F\left(\frac{x-c_2}{b_2}\right) = F\left(\frac{x-c}{b}\right) \tag{4.5}$$

If there is a positive number b and a real number c that derives the equity (4.5), F(x) distribution function is called stable. Here the operation * is the convulsion operation. Generally, if F_1 and F_2, f_1, f_2 are dense constant cumulative distribution functions, their convulsions are expressed as in equation (4.6),

$$F(x) = F_1(x) * F_2(x) \tag{4.6}$$

and defined as below:

$$F(x) = \int_{-\infty}^{\infty} F_1(x-t)\, f_2(t)dt$$

$$= \int_{-\infty}^{\infty} F_2(x-t)\, f_1(t)dt \qquad (4.7)$$

This definition is equivalent to the definition below:

Let X and Y are independent variables which have the cumulative distribution function F. For each $b_1 > 0, b_2 > 0$ pair, if there is a $b_3 > 0$ and c, in $Z \equiv (b_1 X + b_2 Y + c)/b_3$, that have the same cumulative distribution function F, F is stable.

According to both definitions, if the sum of the linear functions of the independent and uniformly distributed random variables belongs to the same distribution family, then this family is stable. For instance, normal distribution provides this property; therefore it is a member of a stable family (Kardiyen, F., 2009, 2-10).

Definition 4.3 Non-degenerate X is stable if and only if for all n>1, there exist constants $c_n > 0$ and $d_n \in$R such that

$$X_1 + ... + X_n \stackrel{d}{=} c_n X + d_n, \qquad (4.8)$$

where $X_1, ..., X_n$ are independent, identical copies of X. X is strictly stable if and only if $d_n = 0$ for all n (Nolan, P. J., 2009, 5-7).

Definition 4.4 If the sum of independent and uniformly distributed random variables has limit distribution, this limit distribution is a member of a stable family (Kardiyen, F., 2009, 4-10).

Definition 4.5 A random variable X is stable if and only if $X \stackrel{d}{=} aZ + b$, where $0 < \alpha \le 2,\ -1 \le \beta \le 1,\ a > 0,\ b \in$ R and Z is a random variable with characteristic function

$$E\exp(iuZ)=\begin{cases}\exp(-|u|^{\alpha}[1-i\beta\tan\frac{\pi\alpha}{2}(\text{sign}u)]) & \alpha\neq 1\\ \exp(-|u|[1+i\beta\frac{2}{\pi}(\text{sign}u)\log|u|]) & \alpha=1\end{cases} \tag{4.9}$$

These distributions are symmetric around when $\beta=0$ and $b=0$, in which case the characteristic function of aZ has the simpler form (Nolan, P. J., 2009, 5-7).

$$\phi(u)=e^{-a^{\alpha}|u|^{\alpha}} \tag{4.10}$$

4.1.1 Parameterizations of Stable Laws

Definition 3.5 shows that a general stable distribution requires four parameters to describe: an index of stability or characteristic exponent $\alpha\in(0,2]$, a skewness parameter $\beta\in[-1,1]$, a scale parameter $\gamma>0$ and a location parameter $\delta\in\mathrm{R}$. We will use γ for the scale parameter and δ for the location parameter to avoid confusion with the symbols α and μ, which will be used exclusively for the standard deviation and mean. The parameters are restricted to the range $\alpha\in(0,2],\beta\in[-1,1],\gamma\geq 0$ and $\delta\in\mathrm{R}$. Generally $\gamma>0$, although $\gamma=0$ will sometimes be used to denote a degenerate distribution concentrated at δ when it simplifies the statement of a result. Since α and β determine the form of the distribution, they may be considered shape parameters (Rozelle, J.P. & Fielitz, B.D., 1980, 1-23).

Various parameter estimation methods have been proposed for stable distributions. Methods using Tail Index Estimation Method, Fama-Roll Estimation Method, Likelihood Method, Percentage Estimation, and Characteristic Function Approach can be given as examples (Kardiyen, F., 2009, 4-10).

Due to the lack of closed form formulas for densities for all but three distributions (the right panel in figure 4.2), the α-stable law can be most conveniently described by its characteristic function $\phi(t)$ - the inverse Fourier transform of the probability density function. However, there are multiple parameterizations for α-stable laws and much confusion has been caused by these different representations in figure 4.3. The variety of formulas is caused by a combination of historical evolution and the numerous problems that have been analyzed using specialized forms of the stable distributions (Borak, S., Hardle, W., Weron, R., 2005, 1-13).

$S_\alpha(\beta,\gamma,\delta)$ Parameterization;

The most popular parameterization by Samorodnitsky and Taqqu (1994) and Weron (2004) for the characteristic function of $X \sim S_\alpha(\beta,\gamma,\delta)$, with the parameters $\alpha,\beta,\gamma,\delta$, and the stable random variable α is as in equation (4.11):

$$\ln\phi(t) = \begin{cases} \left[-\gamma^\alpha |t|^\alpha \left[1 - i\beta\left(\tan\frac{\pi\alpha}{2}\right)(\mathrm{sign}(t))\right] + i\delta t\right] & \alpha \neq 1 \\ \left[-\gamma|t|\left[1 + i\beta\frac{2}{\pi}(\mathrm{sign}(t))\ln|t|\right] + i\delta t\right] & \alpha = 1 \end{cases} \tag{4.11}$$

When the parameter of scale is $\gamma = 1$ and parameter of position is $\delta = 0$, the distribution becomes standardized and the shortened version of the notation $S_\alpha(\beta,1,0)$ is expressed with the symbol $S_\alpha(\beta)$.

On the other hand, when the basic form and the attractive algebraic properties of the characteristic function are considerd, $S_\alpha(\beta,\gamma,\delta)$ parameterization is preferred. $S_\alpha(\beta,\gamma,\delta)$ parameterization with these properties is the most widely used parameterization and is used in argumentations about stable distributions. The basic disadvantage of $S_\alpha(\beta,\gamma,\delta)$ parameterization is that the position of the mode being infinite in any adjacency case of $\alpha = 1$.

$S_\alpha^0(\gamma,\beta,\delta_0)$ Parameterization;

$S_\alpha^0(\gamma,\beta,\delta_0)$ parameterization is a different version, in which the characteristic function, the density function and the distribution function of Zolotarev's M parameterization, are shared continual in all four parameters. This parameterization was proposed for numerical purposes by Nolan (1997).

$$\ln\phi_0(t)=\begin{cases}[-\gamma^\alpha |t|^\alpha [1+i\beta(\tan\frac{\pi\alpha}{2})(\mathrm{sign}(t))(\gamma|t|^{1-\alpha}-1)]+i\delta_0 t] & \alpha\neq 1\\ [-\gamma|t|[1+i\beta\frac{2}{\pi}(\mathrm{sign}(t))\log(\gamma|t|)]+i\delta_0 t] & \alpha=1\end{cases} \tag{4.12}$$

When the parameter of scale is $\gamma=1$ and the parameter of positionis $\delta=0$, the distribution is standardised and the shortened version of notation $S_\alpha^0(\beta,1,0)$ is expressed with the symbol $S_\alpha^0(\beta)$.

When $\alpha=2$, $S_2^0(0,\gamma,\delta_0)$ distribution has a normal distribution with an average of δ and variance of $2\gamma^2$ $(S_2^0(0,\gamma,\delta_0)=N(\delta,2\gamma^2))$.

$S_\alpha^0(\beta,\gamma,\delta_0)$ Parameterization is suggested in numerical studies and statistical inferences on stable distributions. If $X\sim S_\alpha^0(\beta,\gamma,\delta)$, then the distribution $(X-\delta)/\gamma\sim S_\alpha^0(\beta,1,0)$. This situation is not correct for $S_\alpha(\beta,\gamma,\delta)$ parameterization when $\alpha=1$ (Kardiyen, F., 2009, 4-10).

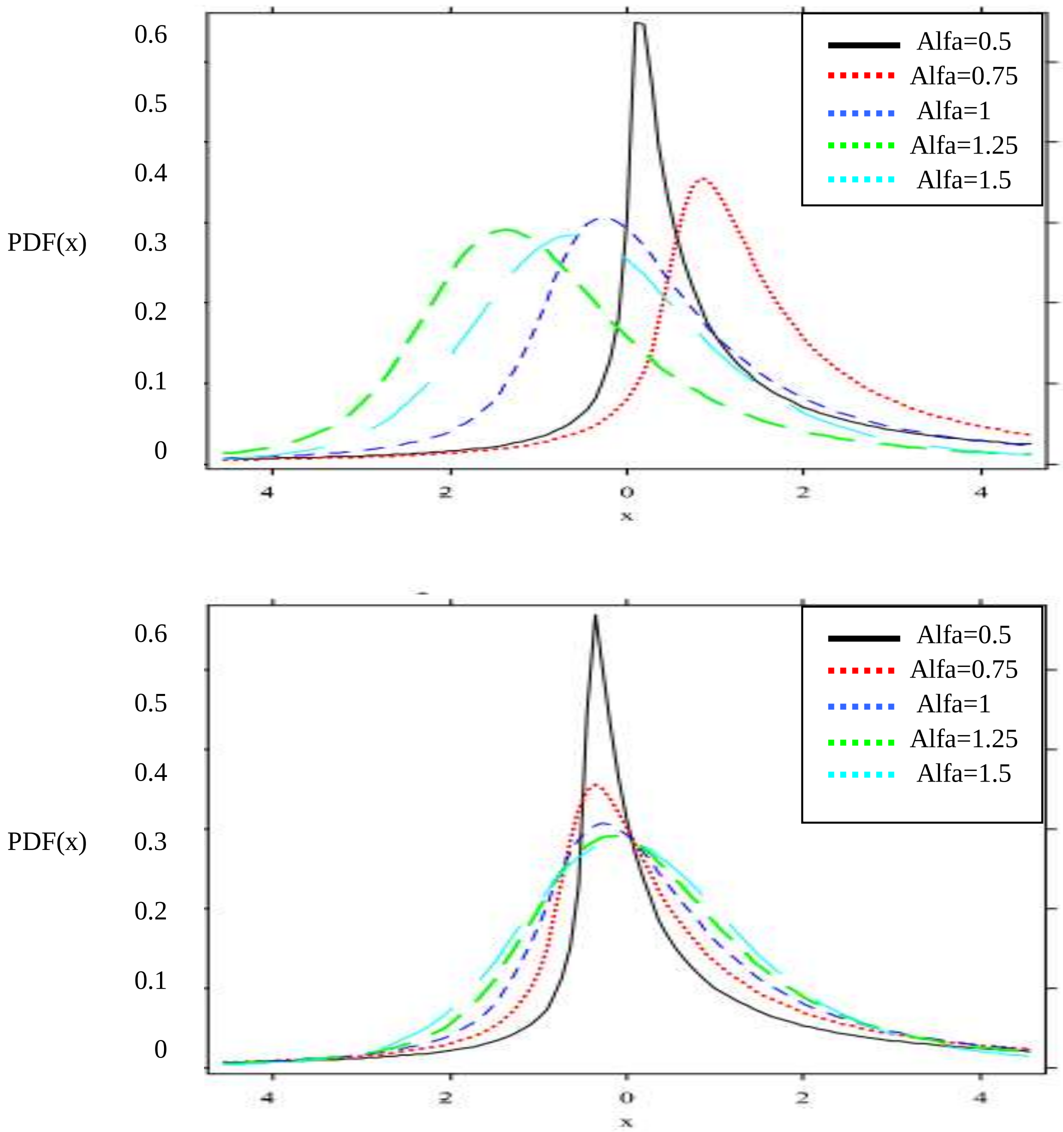

Figure 4.3 Comparison of $S_{\alpha}(\beta,\gamma,\delta)$ and $S_{\alpha}^{0}(\gamma,\beta,\delta_0)$ parameterizations: α -stable pdfs for $\beta = 0.5$ and $\alpha = 0.5,\ 0.75,\ 1,\ 1.25,\ 1.5$.

There is the following correlation between the parameter of position of two parameterizations:

$$\delta = \begin{cases} \delta_0 - \beta\gamma \tan\dfrac{\pi\alpha}{2} & \alpha \neq 1 \\ \delta_0 - \beta\dfrac{2}{\pi}\gamma \ln\gamma & \alpha = 1 \end{cases} \tag{4.13}$$

When $\beta = 0$, the two parameterizations are equivalent and they are expressed as, $S_{\alpha}^{0}(0,\gamma,0) = S_{\alpha}(0,\gamma,0)$ when the parameter of scale is used (Borak, S., Hardle, W., Weron, R., 2005, 1-13).

4.1.2 Densities and Distribution Functions

While there are no explicit formulas for general stable densities, a lot is known about their theoretical properties. The most basic fact is the following.

Theorem 4.1 All (non-degenerate) stable distributions are continuous distributions with an infinitely differentiable density.

To distinguish between the densities and cumulative distribution functions in different parameterizations, $f(x \mid \alpha, \beta, \gamma, \delta; k)$ will denote the density and $F(x \mid \alpha, \beta, \gamma, \delta; k)$ will denote the density function of a $\mathrm{S}(\alpha, \beta, \gamma, \delta; k)$ distribution. When the distribution is standardized, scale $\gamma = 1$, and location $\delta = 0$, $f(x \mid \alpha, \beta; k)$ will be used for the density, and $F(x \mid \alpha, \beta; k)$ will be used for the density function.

Reflection property: For any α and β,Z ~ $\mathrm{S}(\alpha,\beta; k), k = 0,1,2$

$$Z(\alpha, -\beta) \stackrel{d}{=} -Z(\alpha, \beta)$$

Therefore the density and distribution function of a $Z(\alpha, \beta)$ random variable satisfy $f(x \mid \alpha, \beta; k) = f(-x \mid \alpha, -\beta; k)$ and $F(x \mid \alpha, \beta; k) = 1 - F(-x \mid \alpha, -\beta; k)$.

First consider the case when $\beta = 0$. In this case, the reflection property says $f(x \mid \alpha, 0; k) = f(-x \mid \alpha, 0; k)$, so the density and distribution functiona are symmetric around 0. As α decreases, three things occur to the density: the peak gets higher, the region flanking the peak get lower, and the tails get heavier (Nolan, J.P., 2009, 12-13).

Set $k = -\beta \tan\dfrac{\pi\alpha}{2}$. Then the density $f(x \mid \alpha, \beta)$ of a standard α-stable random variable in representation S^0, $X \sim S^0_\alpha(1, \beta, 0)$, can be expressed as (Zolotarev, 1986):

when $\alpha \neq 1$ and $x > k$:

$$f(x \mid \alpha, \beta) = \frac{\alpha (x-k)^{\frac{1}{\alpha-1}}}{\pi \mid \alpha - 1 \mid} \int_{-q}^{\frac{\pi}{2}} V(\theta; \alpha, \beta) \exp\{-(x-k)^{\frac{\alpha}{\alpha-1}} V(\theta; \alpha, \beta)\} d\theta, \quad (4.14)$$

when $\alpha \neq 1$ and $x = k$:

$$f(x \mid \alpha, \beta) = \frac{\Gamma(1+\frac{1}{\alpha})\cos(q)}{\pi (1+k^2)^{\frac{1}{2\alpha}}}, \quad (4.15)$$

when $\alpha \neq 1$ and $x < k$:

$$f(x \mid \alpha, \beta) = f(-x \mid \alpha, -\beta), \quad (4.16)$$

when $\alpha = 1$:

$$f(x \mid 1, \beta) = \begin{cases} \dfrac{1}{2\mid\beta\mid} e^{\frac{-\pi x}{2\beta}} \displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} V(\theta; 1, \beta) \exp\{-e^{\frac{-\pi x}{2\beta}} V(\theta; 1, \beta)\} d\theta, & \beta \neq 0, \\ \dfrac{1}{\pi(1+x^2)}, & \beta = 0 \end{cases} \quad (4.17)$$

where

$$q = \begin{cases} \dfrac{1}{\alpha}\arctan(-k), & \alpha \neq 1, \\ \dfrac{\pi}{2}, & \alpha = 1, \end{cases}$$

and

$$V(\theta; \alpha, \beta) = \begin{cases} (\cos\alpha q)^{\frac{1}{\alpha-1}} \left(\dfrac{\cos\theta}{\sin\alpha(q+\theta)}\right)^{\frac{\alpha}{\alpha-1}} \dfrac{\cos\{\alpha q + (\alpha-1)\theta\}}{\cos\theta}, & \alpha \neq 1, \\ \dfrac{2}{\pi}\left(\dfrac{\frac{\pi}{2}+\beta\theta}{\cos\theta}\right) \exp\{\dfrac{1}{\beta}(\dfrac{\pi}{2}+\beta\theta)\tan\theta\}, & \alpha = 1, \beta \neq 0. \end{cases}$$

The distribution $F(x|\alpha,\beta)$ of a standard α-stable random variable in representation S^0 can be expressed as (Borak, S., Hardle, W., Weron, R., 2005, 7-8):

when $\alpha \neq 1$ and $x > k$:

$$F(x|\alpha,\beta) = c_1(\alpha,\beta) + \frac{sign(1-\alpha)}{\pi} \int_{-q}^{\frac{\pi}{2}} \exp\{-(x-k)^{\frac{\alpha}{\alpha-1}} V(\theta;\alpha,\beta)\} d\theta, \tag{4.18}$$

where

$$c_1(\alpha,\beta) = \begin{cases} \frac{1}{\pi}(\frac{\pi}{2} - q), & \alpha < 1, \\ 1, & \alpha > 1, \end{cases}$$

when $\alpha \neq 1$ and $x = k$:

$$F(x|\alpha,\beta) = \frac{1}{\pi}(\frac{\pi}{2} - q), \tag{4.19}$$

when $\alpha \neq 1$ and $x < k$:

$$F(x|\alpha,\beta) = 1 - F(-x|\alpha,-\beta), \tag{4.20}$$

when $\alpha = 1$:

$$F(x|\alpha,\beta) = \begin{cases} \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \exp\{-e^{\frac{-\pi x}{2\beta}} V(\theta;1,\beta)\} d\theta, & \beta > 0, \\ \frac{1}{2} + \frac{1}{\pi} \arctan x, & \beta = 0, \\ 1 - F(x|1,-\beta), & \beta < 0. \end{cases} \tag{4.21}$$

Now we explain that the normal distributions relation with the option pricing.

4.2 The Normal Distribution

A random variable, x, obeys the normal distribution (or is normally distributed) if the probability that x takes on a particular value is described by the normal density function, which we represent by *N*. The formula for the normal density function is

$$N(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \tag{4.22}$$

In order to calculate a value for N, in addition to x, you need to supply two numbers: a mean, μ, and a standard deviation, σ. For this reason, the normal distribution is said to be a two parameter distribution; it is competely described by the mean and the standard deviation. In figure 4.4 we can see the normal distrbutions,

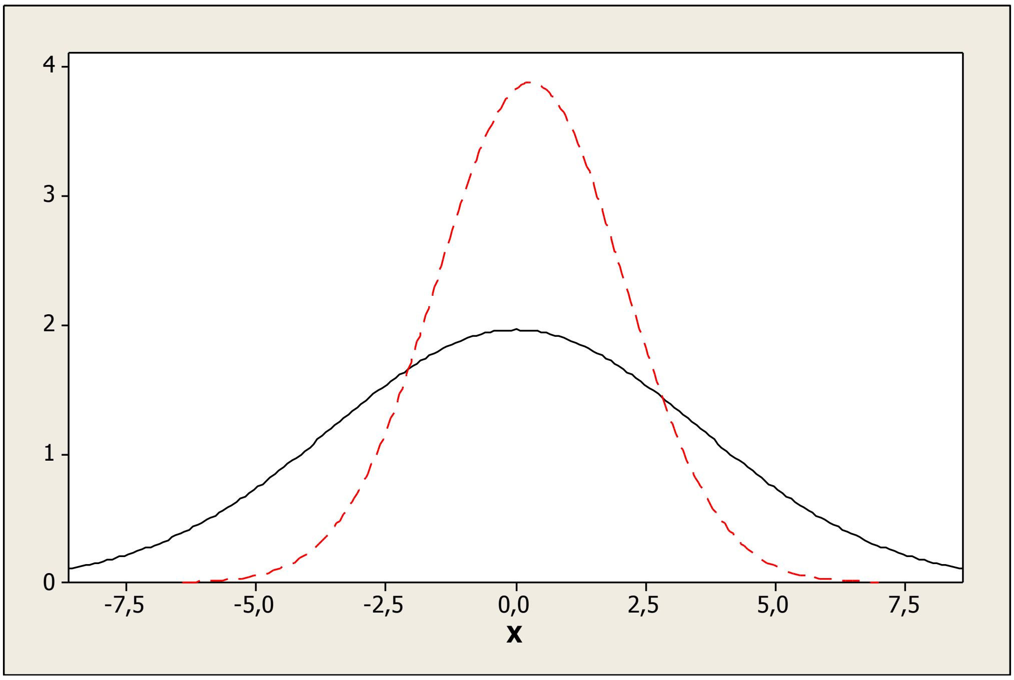

Figure 4.4 Normal Distributions with different standard deviations (N(x; 0, 1) = black solid line, N(x; 0, 1.5) = dotted line).

The normal density with $\mu=0$ and $\sigma=1$ is called the standard normal density. We can compare to the standard normal distribution, the normal distribution in figure (4.4). Increasing the variance spreads out the distribution. The mean locates the center of the distribution, and the standard deviation tells us how spread out it is. The normal density is symmetric aboout the mean, μ , meaning that

$$N(\mu + x; \mu, \sigma) = N(\mu - x; \mu, \sigma)$$

If a random variable x is normally distributedwith mean μ and the standard deviation σ , we write this as

$$x \sim N(\mu, \sigma^2)$$

We will use z to represent a random variable that has the standard normal distribution:

$$z \sim N(0,1)$$

We can use the normal distribution to compute the probability of different events, but we have to be careful about what we mean by an event. Since the distribution is continuous, there are an infinite number of events that can occur when we randomly draw a number from the distribution. The probability of any particular number being drawn from the normal distribution is zero. Thus, we use the normal distribution to describe the probability that a number randomly selected from the normal distribution will be in a particular range (Mood, A.M., Graybill, F.A., Boes, D.C., 1974, 230-239).

4.2.1 Converting a Normal Random Variable to Standard Normal

If we have an arbitrary normal random variable, it is easy to convert it to standard normal. Suppose

$$x \sim N(\mu, \sigma^2)$$

Then we can create a standard normal random variable, z , by subtracting the mean and dividing by the standard deviation:

$$z = \frac{x - \mu}{\sigma} \qquad (4.23)$$

This result will be helpful in interpreting the Black-Scholes option pricing formula. If we have a standard normal random variable z , we can generate a variable $x \sim N(\mu,\sigma^2)$, using the following:

$$x = \mu + \sigma z \tag{4.24}$$

4.2.2 Sums of Normal Random Variables

Suppose we have n random variables $x_i, i = 1,...,n,$ with mean and variance $E(x_i) = \mu_i, Var(x_i) = \sigma_i^2$, and covariance $Cov(x_i, x_j) = \sigma_{ij}$ (The covariance between two random variables measures their tendency to move together. We can also write the covariance in terms of p_{ij}, the correlation between x_i and $x_j : \sigma_{ij} = p_{ij}\sigma_i\sigma_j$). Then the weighted sum of the n random variables has mean

$$E\left(\sum_{i=1}^{n} w_i x_i\right) = \sum_{i=1}^{n} w_i \mu_i \tag{4.25}$$

and variance

$$Var\left(\sum_{i=1}^{n} w_i x_i\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} \tag{4.26}$$

Where w_i and w_j represent arbitrary weights. These formulas for the mean and variance are true for any distribution of the x_i (DeGroot, M.H., 1975, 225-231).

In general, the distribution of the random variables is different from the distribution of the individual random variables. However, the normal distribution is an example of a stable distribution. A distribution is stable if sums of random variables have the same distributions as the original random variables. In this case, the sum of normally distributed random variables is normal. Thus, for normally distributed x_i (McDonald, R.L., 2003, 565-571),

$$\sum_{i=1}^{n} w_i x_i \sim N\left(\sum_{i=1}^{n} w_i \mu_i, \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}\right) \quad (4.27).$$

4.2.3 The Central Limit Theorem

Why does the normal distribution appear in option pricing (and frequently in other contexts)? The normal distribution is important because it arises naturally when random variables are added. The normal distribution was originally discovered by mathematicians studying series of random events, such as gambling outcomes and observational errors (Bernstein, P.L., 1992, 175).

The normal distribution is therefore not just a convenient, aesthetically pleasing distribution, but it arises in nature when outcomes can be characterized as sums of independent random variables with a finite variance. The distribution of such a sum approaches normality. This result is known as the central limit theorem.

In the context of asset returns, the continuously compounded stock return over a year is the sum of the daily continuously compounded returns. If news and other factors are the shocks that cause asset prices to change, and if these changes are independent, then it is natural to think that longer period continuously compounded returns are normally distributed. Since the central limit theorem is a theorem about what happens in the limit, sums of just a few random variables may not appear normal. But the normality of continuously compounded returns seems like a resonable starting point in thinking about stock returns (McDonald, R.L., 2003, 571).

4.3 Normality Tests

Normality tests are methods to determine whether or not the quantitative data obtained from a n-unit group exhibit μ, σ parametered normal distribution. The compliance of the n-unit data to normal distribution is determined using graphical methods or statistical test (normality tests). While graphical methods are explanatory data analysis approaches, normality tests depend on hypothesis and definite parametered normal distribution.

Altohugh the visuality is attractive in graphical methods; they cannot offer objective criteria on determining whether the variables have normal distribution or not. Therefore, in order to determine whether the variables have normal distribution or not, we have to refer to statistical tests.

Statistical tests test the hypotheses of $H_0:$ "The distribution of the variable is normal.", and $H_1:$ "The distribution of the variable is not normal." via n-unit sample data. If, as a result of the test, it is accepted that the variable has normal distribution, (H_0 accepted) parametric tests are carried out. If it is concluded that the distribution of the variable is not normal (H_0 refused) parametric tests cannot be conducted on these data. In this case, the data are testes using an alternative test, which is non-parametric.

Normality tests are categorized in different groups in terms of different statistical approached they use. These are;

- Empirical cumulative density function (ECDF) based tests,
- Skewness or kurtosis statistics based tests,
- Distribution quantiles based tests,
- Correlation based tests,
- Residuals based tests,
- Chi-square based tests.

There are many normality tests developed according to numerous and diverse statistical approaches. Among these tests, Anderson-Darling test will be used in testing whether the option premiums, calculated by Black-Scholes option pricing model, exhibit normal distribution, one of the stable distributions (Özdamar, K., 2004, 289-291).

4.3.1 Anderson-Darling Test

Anderson-Darling (AD) test was developed by Anderson and Darling (1954) based on EDF (empirical distribution function) statistics. It is a very stong normality test. AD test is a modification of Kolmogorov-Simirnov (KS) test. It is a reinforced modification of KS test, by resolving the insensitivity of KS tests against minimum and maximum extreme values. It is assessed that AD test is, in particular conditions, strong as Shapiro-Wilk test.

The data, on which the AD test will be conducted, should be raw data. AD test can be applied to normal, weibull and lognormal distributions and distributions with known functions. With this test, whether or not the differences, between calculated probability functions and the experimental distribution functions of the data obtained from n-units, are in expected comformity.

AD test addresses the compliance of the experimental cumulative probability distribution of the x (i) observations ordered according to greatness. Therefore, the compliance of the probability of stantdard z_i values obtained by standard conversion of each observation to standard normal distribution is investigated. For this purpose, in AD test, whether or not the data is in the form of normal distribution is tested by calculating the A^2 test statistics (Özdamar, K., 2004, 289-291).

$$A^2 = -\frac{1}{n}\sum_{i=1}^{n}(2i-1)[\ln(F_0(Z_i)) + \ln(1-F_0(Z_{n-i+1}))] - n \text{ and } A_\alpha^2 = A^2\left(1+\frac{0.75}{n}+\frac{2.25}{n^2}\right)$$

The momentousness of A^2 value will be realized as, if the A^2 critical value derived for the normal distribution, $A_{0.05}^2 = 0.752, A_{0.025}^2 = 0.873, A_{0.01}^2 = 1.035$ values determined. Anderson-Darling test implemented in this study is conducted using MINITAB software and our hypothesis

$H_0 : A^2 \leq A_\alpha^2$ (The distribution of the variable is normal.),

$H_1 : A^2 > A_\alpha^2$ (The distribution of the variable is not normal.)

CHAPTER FIVE
APPLICATIONS

In this application chapter, our first aim is that, with using Anderson-Darling Normality Test, whether it is a normal distribution or not, by calculating the Disney and IBM option prices of stock issues options in BS model.

5.1 Application of Disney Stock Issues American Call Option Pricing

If we are to examine the calculation of American Call Option of Disney stock issue with expiry of term at the end of January, by 10 November 2009, using Black-Scholes option prices; as of 10 November (the following parameter values were taken CBOE):

S_0 : Current stock price = \$92
K : Strike price of option = \$95
T : Time to expiration of option = 0.136 (50 days / 365 rounded)
r : Continuously compounded risk-free rate of interest = 7.12%
σ : The stock price volatility = 35% are observed as such. From here

$$d_1 = \frac{\ln(92/95) + (0.0712 + ([0.35]^2/2))0.136}{0.35\sqrt{0.136}} = -0.10765$$

$$d_2 = -0.10765 - 0.35\sqrt{0.136} = -0.23719$$

From a normal distribution table,

$$N(d_1) = N(-0.10765) = 0.457138 \text{ and}$$
$$N(d_2) = N(-0.23719) = 0.406256$$

Then

$$c = 92(0.457138) - 95(e^{-(0.0712)(0.136)})(0.406256) = \$3.837$$

Here the valuation of American Call Option of the January dated Disney stock issue on given date was performed according to BS model. In this implementation, the $T = 50, 49, ..., 2,1$ residual dated valuation was calculated in Microsoft Excel and the T residual dated data are presented in Table (5.1) (In the calculations in Table 5.1 Disney stock issues did not pay dividend). Accordingly, it can be observed how the option price indicated with c in the first row of Table 5.1., and the option price valuated in accordance with BS option pricing model are coherent and stable.

According to the information taken from the stock exchange, the price of the January dated Call Option of Disney is $4.00 as of 10 November 2009. The negligible difference is due to Disney stock issues' not paying dividend.

American type call option pricing calculation was used in preparing Table 5.1.

Table 5.1 Option premiums and parameters required for premium calculations

K stock price	S strike price	r (risk-free interest rate)	σ(std. deviation)	ln(S/X)
95	92	7%	35%	-0,03209

time t	T expiration time	(r+(σ²/2))*T	σ*T^(1/2)	d1	d2	N(d1)	N(d2)	(-rT)	(e^(-rT))	K*(e^(-rT))	c (call price
1	0,1370	0,0181	0,1295	-0,1076	-0,2372	0,4571	0,4063	-0,0098	0,9903	94,0779	3,8370
2	0,1342	0,0178	0,1282	-0,1116	-0,2398	0,4556	0,4052	-0,0096	0,9905	94,0963	3,7821
3	0,1315	0,0174	0,1269	-0,1156	-0,2425	0,4540	0,4042	-0,0094	0,9907	94,1146	3,7267
4	0,1288	0,0171	0,1256	-0,1197	-0,2453	0,4524	0,4031	-0,0092	0,9909	94,1330	3,6708
5	0,1260	0,0167	0,1243	-0,1239	-0,2482	0,4507	0,4020	-0,0090	0,9911	94,1514	3,6145
6	0,1233	0,0163	0,1229	-0,1282	-0,2511	0,4490	0,4009	-0,0088	0,9913	94,1697	3,5577
7	0,1205	0,0160	0,1215	-0,1327	-0,2542	0,4472	0,3997	-0,0086	0,9915	94,1881	3,5003
8	0,1178	0,0156	0,1201	-0,1372	-0,2574	0,4454	0,3985	-0,0084	0,9916	94,2065	3,4425
9	0,1151	0,0152	0,1187	-0,1419	-0,2606	0,4436	0,3972	-0,0082	0,9918	94,2249	3,3841
10	0,1123	0,0149	0,1173	-0,1467	-0,2640	0,4417	0,3959	-0,0080	0,9920	94,2432	3,3252
11	0,1096	0,0145	0,1159	-0,1517	-0,2675	0,4397	0,3945	-0,0078	0,9922	94,2616	3,2657
12	0,1068	0,0142	0,1144	-0,1568	-0,2712	0,4377	0,3931	-0,0076	0,9924	94,2800	3,2056
13	0,1041	0,0138	0,1129	-0,1620	-0,2750	0,4356	0,3917	-0,0074	0,9926	94,2984	3,1448
14	0,1014	0,0134	0,1114	-0,1675	-0,2789	0,4335	0,3902	-0,0072	0,9928	94,3168	3,0835
15	0,0986	0,0131	0,1099	-0,1731	-0,2830	0,4313	0,3886	-0,0070	0,9930	94,3352	3,0215
16	0,0959	0,0127	0,1084	-0,1789	-0,2873	0,4290	0,3870	-0,0068	0,9932	94,3536	2,9588
17	0,0932	0,0123	0,1068	-0,1849	-0,2917	0,4267	0,3853	-0,0066	0,9934	94,3720	2,8954
18	0,0904	0,0120	0,1052	-0,1911	-0,2964	0,4242	0,3835	-0,0064	0,9936	94,3904	2,8312
19	0,0877	0,0116	0,1036	-0,1976	-0,3012	0,4217	0,3816	-0,0062	0,9938	94,4088	2,7663
20	0,0849	0,0112	0,1020	-0,2043	-0,3063	0,4191	0,3797	-0,0060	0,9940	94,4273	2,7006
21	0,0822	0,0109	0,1003	-0,2113	-0,3116	0,4163	0,3777	-0,0059	0,9942	94,4457	2,6340
22	0,0795	0,0105	0,0987	-0,2186	-0,3172	0,4135	0,3755	-0,0057	0,9944	94,4641	2,5666
23	0,0767	0,0102	0,0969	-0,2262	-0,3231	0,4105	0,3733	-0,0055	0,9946	94,4825	2,4983
24	0,0740	0,0098	0,0952	-0,2342	-0,3294	0,4074	0,3709	-0,0053	0,9947	94,5010	2,4290
25	0,0712	0,0094	0,0934	-0,2425	-0,3359	0,4042	0,3685	-0,0051	0,9949	94,5194	2,3587
26	0,0685	0,0091	0,0916	-0,2513	-0,3429	0,4008	0,3658	-0,0049	0,9951	94,5378	2,2873
27	0,0658	0,0087	0,0897	-0,2605	-0,3502	0,3972	0,3631	-0,0047	0,9953	94,5563	2,2149
28	0,0630	0,0083	0,0879	-0,2702	-0,3581	0,3935	0,3601	-0,0045	0,9955	94,5747	2,1412
29	0,0603	0,0080	0,0859	-0,2805	-0,3665	0,3895	0,3570	-0,0043	0,9957	94,5932	2,0664
30	0,0575	0,0076	0,0840	-0,2915	-0,3754	0,3854	0,3537	-0,0041	0,9959	94,6116	1,9902
31	0,0548	0,0073	0,0819	-0,3031	-0,3850	0,3809	0,3501	-0,0039	0,9961	94,6301	1,9127
32	0,0521	0,0069	0,0799	-0,3155	-0,3953	0,3762	0,3463	-0,0037	0,9963	94,6486	1,8337
33	0,0493	0,0065	0,0777	-0,3288	-0,4065	0,3711	0,3422	-0,0035	0,9965	94,6670	1,7531
34	0,0466	0,0062	0,0755	-0,3431	-0,4187	0,3657	0,3377	-0,0033	0,9967	94,6855	1,6708
35	0,0438	0,0058	0,0733	-0,3587	-0,4319	0,3599	0,3329	-0,0031	0,9969	94,7040	1,5868
36	0,0411	0,0054	0,0710	-0,3755	-0,4465	0,3536	0,3276	-0,0029	0,9971	94,7224	1,5009
37	0,0384	0,0051	0,0685	-0,3940	-0,4626	0,3468	0,3218	-0,0027	0,9973	94,7409	1,4129
38	0,0356	0,0047	0,0661	-0,4144	-0,4804	0,3393	0,3155	-0,0025	0,9975	94,7594	1,3226
39	0,0329	0,0044	0,0635	-0,4370	-0,5005	0,3310	0,3084	-0,0023	0,9977	94,7779	1,2300
40	0,0301	0,0040	0,0608	-0,4624	-0,5232	0,3219	0,3004	-0,0021	0,9979	94,7964	1,1347

41	0,0274	0,0036	0,0579	-0,4913	-0,5492	0,3116	0,2914	-0,0020	0,9981	94,8149	1,0366
42	0,0247	0,0033	0,0550	-0,5244	-0,5794	0,3000	0,2812	-0,0018	0,9982	94,8334	0,9354
43	0,0219	0,0029	0,0518	-0,5632	-0,6151	0,2866	0,2693	-0,0016	0,9984	94,8519	0,8308
44	0,0192	0,0025	0,0485	-0,6096	-0,6581	0,2711	0,2552	-0,0014	0,9986	94,8704	0,7225
45	0,0164	0,0022	0,0449	-0,6666	-0,7114	0,2525	0,2384	-0,0012	0,9988	94,8889	0,6102
46	0,0137	0,0018	0,0410	-0,7390	-0,7800	0,2299	0,2177	-0,0010	0,9990	94,9074	0,4938
47	0,0110	0,0015	0,0366	-0,8362	-0,8728	0,2015	0,1914	-0,0008	0,9992	94,9259	0,3735
48	0,0082	0,0011	0,0317	-0,9770	-1,0087	0,1643	0,1566	-0,0006	0,9994	94,9444	0,2505
49	0,0055	0,0007	0,0259	-1,2105	-1,2364	0,1130	0,1081	-0,0004	0,9996	94,9629	0,1294
50	0,0027	0,0004	0,0183	-1,7318	-1,7501	0,0417	0,0401	-0,0002	0,9998	94,9815	0,0283

After these calculations, depending on the A^2 values here, when we test for A^2_α, α=0.05; if c call prices A^2 value,

$H_0 : A^2 \leq A^2_\alpha = 0.752$ (The distribution of the variable is normal.),

$H_1 : A^2 > A^2_{0.05} = 0.752$ (The distribution of the variable is not normal.)

Hypothesis with T (expiration time) dated-unit data, we clearly see that the distributions of option prices confirmed the hypothesis $H_0 : A^2 \leq 0.752$ (The distribution of the variable is normal.), for Disney stock issues American call option prices. Because we found $A^2 = 0.613$.

We can see this hypothesis reality in figure 5.1 (AD normality test calculations with 95% confidence interval and $\alpha = 0.05$ in Minitab) and figure 5.2 (probability plot of Disney stock issues c call price).

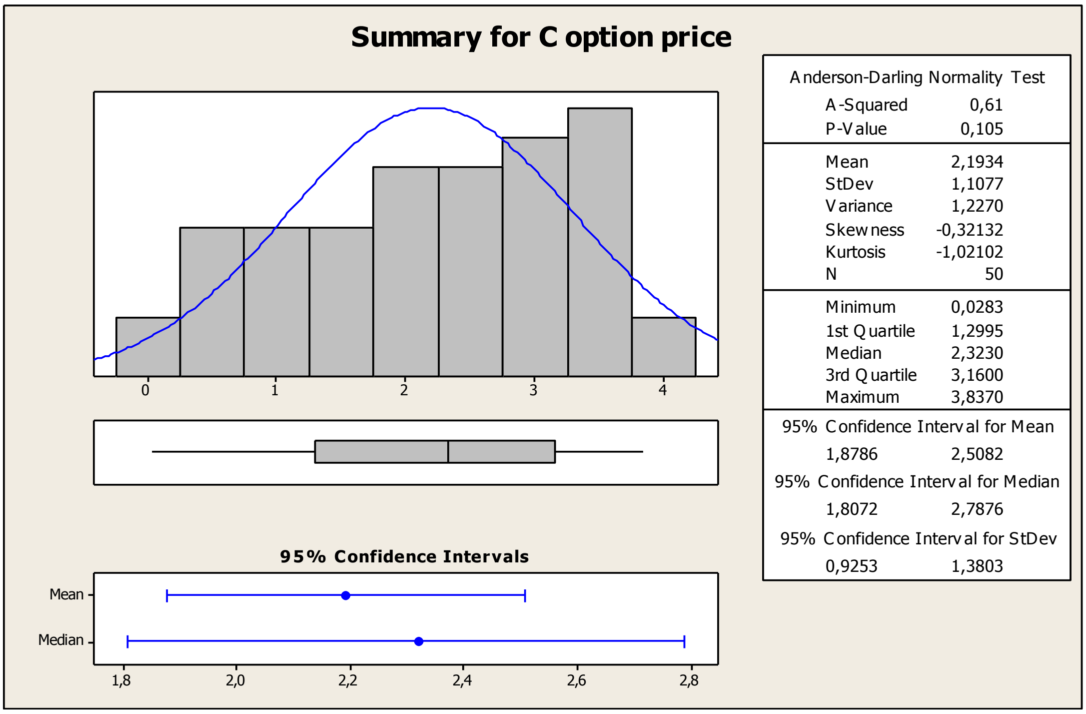

Figure 5.1 Graphical Summary of Anderson-Darling Normality Test for Option Price

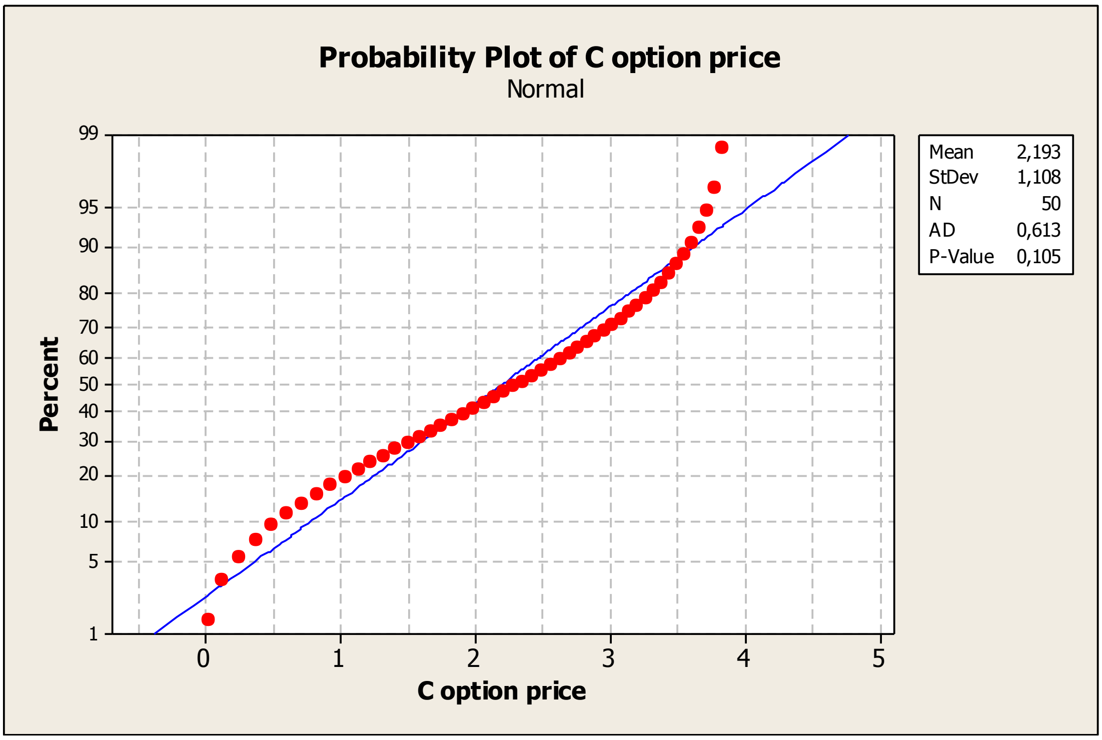

Figure 5.2 Anderson-Darling Normality Test for Option Price

Table 5.2 Disney stock issues c call option price normality variations

(Std. Dev. Is %35 and Constant)

Risk-Free Interest Rate Variation	AD Value Variation (A^2)
2%	0.642
5%	0.624
6%	0.619
7% (base case)	0.613
8%	0.608
9%	0.603
15%	0.579

(Risk Free Interest Rate Is %7 and Constant)

Expected Price Volatility Variation (Std. dev.)	AD Value Variation (A^2)
20%	0.649
30%	0.609
35% (base case)	0.613
40%	0.621
50%	0.636

(Either Std. Dev. or Risk Free Interest Rate Variation. The Expiration Time (T) is 50 Days and Constant)

AD Value for Normality Test		*Risk free interest rate*						
		%2	**%5**	**%6**	**%7***	**%8**	**%9**	**%15**
Volatility (Std.Dev.)	**%20**	0.665	0.654	0.651	0.649	0.647	0.645	0.641
	%30	0.637	0.619	0.614	0.609	0.604	0.599	0.577
	%35*	0.642	0.624	0.619	0.613	0.608	0.603	0.579
	%40	0.649	0.631	0.626	0.621	0.616	0.611	0.586
	%50	0.661	0.646	0.641	0.636	0.632	0.627	0.603

(* base case)

5.2 Application of IBM Stock Issues American Put-Call Option Pricing

If we are to examine the calculation of American Call and Put Options of 6-month dated IBM stock issue using Black-Scholes option prices; as expiry of term (the following parameter values were taken CBOE):

S_0 : Current stock price = $42

K : Strike price of option = $40

T : Time to expiration of option = 0.5 (180 days / 360 rounded)

r : Continuously compounded risk-free rate of interest = 10%

σ : The stock price volatility = 20% are observed as such. From here

$$d_1 = \frac{\ln(42/40) + (0.10 + ([0.20]^2/2))0.5}{0.20\sqrt{0.5}} = 0.769263$$

$$d_2 = 0.769263 - 0.20\sqrt{0.5} = 0.627841$$

From a normal distribution table,

$$N(d_1) = N(0.769263) = 0.779131 \text{ and}$$
$$N(d_2) = N(0.627841) = 0.734946$$

For a put option price,

$$N(-d_1) = N(-0.769263) = 0.220869 \text{ and}$$
$$N(-d_2) = N(-0.627841) = 0.265054$$

Then

$$c = 42(0.779131) - 40(e^{-(0.10)(0.5)})(0.734946) = \$4.759$$

$$p = 40(e^{-(0.10)(0.5)})(0.265054) - 42(0.220869) = \$0.808$$

Here the valuation of American Call and Put Options, of the given date, of 6-month dated IBM stock issues is done in accordance with BS model. In this implementation $T = 180, 179, ..., 2, 1$ residual dated valuation was calculated in Microsoft Excel and T residual dated data are presented in Table (5.3) and (5.5) (In Table 5.3 and 5.5 premium calculation was performed without the IBM stock issues paying any dividends).

Accordingly, it can be observed how the call option price indicated with c in the first row of Table 5.3., and the put option price indicated with p in the first row of Table 5.5 and the option prices valuated in accordance with BS option pricing model are coherent and stable.

American type call and put option pricing calculations were used in preparing Table 5.3 and 5.5.

Table 5.3 Option premiums and parameters required for premium calculations

K stock price	S strike price	ln(s/x)	r (risk-free rate)	σ(std. deviation)
40	42	0,04879	10%	20%

t time	T expiration time	(r+(σ²/2))*T	σ*T^(1/2)	d1	d2	(-rT)	(e^(-rT))	N(d1)	N(d2)	c call price
1	0,5000	0,0600	0,1414	0,7693	0,6278	-0,0500	0,9512	0,7791	0,7349	4,7594
2	0,4972	0,0597	0,1410	0,7690	0,6280	-0,0497	0,9515	0,7791	0,7350	4,7467
3	0,4944	0,0593	0,1406	0,7688	0,6282	-0,0494	0,9518	0,7790	0,7351	4,7341
4	0,4917	0,0590	0,1402	0,7686	0,6284	-0,0492	0,9520	0,7789	0,7351	4,7214
5	0,4889	0,0587	0,1398	0,7684	0,6286	-0,0489	0,9523	0,7789	0,7352	4,7086
6	0,4861	0,0583	0,1394	0,7682	0,6288	-0,0486	0,9526	0,7788	0,7353	4,6959
7	0,4833	0,0580	0,1390	0,7680	0,6290	-0,0483	0,9528	0,7788	0,7353	4,6831
8	0,4806	0,0577	0,1386	0,7678	0,6292	-0,0481	0,9531	0,7787	0,7354	4,6703
9	0,4778	0,0573	0,1382	0,7677	0,6294	-0,0478	0,9533	0,7787	0,7355	4,6575
10	0,4750	0,0570	0,1378	0,7675	0,6296	-0,0475	0,9536	0,7786	0,7355	4,6447
11	0,4722	0,0567	0,1374	0,7673	0,6299	-0,0472	0,9539	0,7786	0,7356	4,6319
12	0,4694	0,0563	0,1370	0,7671	0,6301	-0,0469	0,9541	0,7785	0,7357	4,6190
13	0,4667	0,0560	0,1366	0,7670	0,6304	-0,0467	0,9544	0,7785	0,7358	4,6062
14	0,4639	0,0557	0,1362	0,7668	0,6306	-0,0464	0,9547	0,7784	0,7359	4,5933
15	0,4611	0,0553	0,1358	0,7667	0,6309	-0,0461	0,9549	0,7784	0,7359	4,5804
16	0,4583	0,0550	0,1354	0,7665	0,6311	-0,0458	0,9552	0,7783	0,7360	4,5675
17	0,4556	0,0547	0,1350	0,7664	0,6314	-0,0456	0,9555	0,7783	0,7361	4,5545
18	0,4528	0,0543	0,1346	0,7663	0,6317	-0,0453	0,9557	0,7782	0,7362	4,5415
19	0,4500	0,0540	0,1342	0,7662	0,6320	-0,0450	0,9560	0,7782	0,7363	4,5286
20	0,4472	0,0537	0,1337	0,7660	0,6323	-0,0447	0,9563	0,7782	0,7364	4,5156
21	0,4444	0,0533	0,1333	0,7659	0,6326	-0,0444	0,9565	0,7781	0,7365	4,5025
22	0,4417	0,0530	0,1329	0,7658	0,6329	-0,0442	0,9568	0,7781	0,7366	4,4895
23	0,4389	0,0527	0,1325	0,7657	0,6332	-0,0439	0,9571	0,7781	0,7367	4,4764
24	0,4361	0,0523	0,1321	0,7656	0,6336	-0,0436	0,9573	0,7781	0,7368	4,4633
25	0,4333	0,0520	0,1317	0,7656	0,6339	-0,0433	0,9576	0,7780	0,7369	4,4502
26	0,4306	0,0517	0,1312	0,7655	0,6342	-0,0431	0,9579	0,7780	0,7370	4,4371
27	0,4278	0,0513	0,1308	0,7654	0,6346	-0,0428	0,9581	0,7780	0,7372	4,4240
28	0,4250	0,0510	0,1304	0,7654	0,6350	-0,0425	0,9584	0,7780	0,7373	4,4108
29	0,4222	0,0507	0,1300	0,7653	0,6353	-0,0422	0,9587	0,7780	0,7374	4,3976
30	0,4194	0,0503	0,1295	0,7653	0,6357	-0,0419	0,9589	0,7779	0,7375	4,3844
31	0,4167	0,0500	0,1291	0,7652	0,6361	-0,0417	0,9592	0,7779	0,7377	4,3712
32	0,4139	0,0497	0,1287	0,7652	0,6365	-0,0414	0,9595	0,7779	0,7378	4,3579
33	0,4111	0,0493	0,1282	0,7652	0,6369	-0,0411	0,9597	0,7779	0,7379	4,3446
34	0,4083	0,0490	0,1278	0,7652	0,6374	-0,0408	0,9600	0,7779	0,7381	4,3313
35	0,4056	0,0487	0,1274	0,7652	0,6378	-0,0406	0,9603	0,7779	0,7382	4,3180
36	0,4028	0,0483	0,1269	0,7652	0,6382	-0,0403	0,9605	0,7779	0,7383	4,3047
37	0,4000	0,0480	0,1265	0,7652	0,6387	-0,0400	0,9608	0,7779	0,7385	4,2913
38	0,3972	0,0477	0,1261	0,7652	0,6392	-0,0397	0,9611	0,7779	0,7386	4,2779
39	0,3944	0,0473	0,1256	0,7653	0,6396	-0,0394	0,9613	0,7779	0,7388	4,2645
40	0,3917	0,0470	0,1252	0,7653	0,6401	-0,0392	0,9616	0,7780	0,7390	4,2511
41	0,3889	0,0467	0,1247	0,7654	0,6406	-0,0389	0,9619	0,7780	0,7391	4,2376
42	0,3861	0,0463	0,1243	0,7654	0,6411	-0,0386	0,9621	0,7780	0,7393	4,2242
43	0,3833	0,0460	0,1238	0,7655	0,6417	-0,0383	0,9624	0,7780	0,7395	4,2106
44	0,3806	0,0457	0,1234	0,7656	0,6422	-0,0381	0,9627	0,7780	0,7396	4,1971
45	0,3778	0,0453	0,1229	0,7657	0,6428	-0,0378	0,9629	0,7781	0,7398	4,1836

46	0,3750	0,0450	0,1225	0,7658	0,6433	-0,0375	0,9632	0,7781	0,7400	4,1700
47	0,3722	0,0447	0,1220	0,7659	0,6439	-0,0372	0,9635	0,7781	0,7402	4,1564
48	0,3694	0,0443	0,1216	0,7660	0,6445	-0,0369	0,9637	0,7782	0,7404	4,1428
49	0,3667	0,0440	0,1211	0,7662	0,6451	-0,0367	0,9640	0,7782	0,7406	4,1291
50	0,3639	0,0437	0,1206	0,7663	0,6457	-0,0364	0,9643	0,7783	0,7408	4,1154
51	0,3611	0,0433	0,1202	0,7665	0,6463	-0,0361	0,9645	0,7783	0,7410	4,1017
52	0,3583	0,0430	0,1197	0,7667	0,6470	-0,0358	0,9648	0,7784	0,7412	4,0880
53	0,3556	0,0427	0,1193	0,7669	0,6476	-0,0356	0,9651	0,7784	0,7414	4,0743
54	0,3528	0,0423	0,1188	0,7671	0,6483	-0,0353	0,9653	0,7785	0,7416	4,0605
55	0,3500	0,0420	0,1183	0,7673	0,6490	-0,0350	0,9656	0,7786	0,7418	4,0467
56	0,3472	0,0417	0,1179	0,7676	0,6497	-0,0347	0,9659	0,7786	0,7421	4,0328
57	0,3444	0,0413	0,1174	0,7678	0,6504	-0,0344	0,9661	0,7787	0,7423	4,0190
58	0,3417	0,0410	0,1169	0,7681	0,6512	-0,0342	0,9664	0,7788	0,7425	4,0051
59	0,3389	0,0407	0,1164	0,7683	0,6519	-0,0339	0,9667	0,7789	0,7428	3,9912
60	0,3361	0,0403	0,1160	0,7686	0,6527	-0,0336	0,9669	0,7789	0,7430	3,9772
61	0,3333	0,0400	0,1155	0,7689	0,6535	-0,0333	0,9672	0,7790	0,7433	3,9633
62	0,3306	0,0397	0,1150	0,7693	0,6543	-0,0331	0,9675	0,7791	0,7435	3,9493
63	0,3278	0,0393	0,1145	0,7696	0,6551	-0,0328	0,9678	0,7792	0,7438	3,9352
64	0,3250	0,0390	0,1140	0,7700	0,6560	-0,0325	0,9680	0,7793	0,7441	3,9212
65	0,3222	0,0387	0,1135	0,7703	0,6568	-0,0322	0,9683	0,7795	0,7444	3,9071
66	0,3194	0,0383	0,1130	0,7707	0,6577	-0,0319	0,9686	0,7796	0,7446	3,8930
67	0,3167	0,0380	0,1125	0,7712	0,6586	-0,0317	0,9688	0,7797	0,7449	3,8788
68	0,3139	0,0377	0,1121	0,7716	0,6595	-0,0314	0,9691	0,7798	0,7452	3,8646
69	0,3111	0,0373	0,1116	0,7720	0,6605	-0,0311	0,9694	0,7800	0,7455	3,8504
70	0,3083	0,0370	0,1111	0,7725	0,6614	-0,0308	0,9696	0,7801	0,7458	3,8362
71	0,3056	0,0367	0,1106	0,7730	0,6624	-0,0306	0,9699	0,7802	0,7462	3,8219
72	0,3028	0,0363	0,1101	0,7735	0,6634	-0,0303	0,9702	0,7804	0,7465	3,8076
73	0,3000	0,0360	0,1095	0,7740	0,6645	-0,0300	0,9704	0,7805	0,7468	3,7933
74	0,2972	0,0357	0,1090	0,7746	0,6655	-0,0297	0,9707	0,7807	0,7471	3,7789
75	0,2944	0,0353	0,1085	0,7751	0,6666	-0,0294	0,9710	0,7809	0,7475	3,7645
76	0,2917	0,0350	0,1080	0,7757	0,6677	-0,0292	0,9713	0,7811	0,7478	3,7501
77	0,2889	0,0347	0,1075	0,7764	0,6689	-0,0289	0,9715	0,7812	0,7482	3,7356
78	0,2861	0,0343	0,1070	0,7770	0,6700	-0,0286	0,9718	0,7814	0,7486	3,7211
79	0,2833	0,0340	0,1065	0,7777	0,6712	-0,0283	0,9721	0,7816	0,7490	3,7066
80	0,2806	0,0337	0,1059	0,7784	0,6724	-0,0281	0,9723	0,7818	0,7493	3,6920
81	0,2778	0,0333	0,1054	0,7791	0,6737	-0,0278	0,9726	0,7820	0,7497	3,6774
82	0,2750	0,0330	0,1049	0,7798	0,6750	-0,0275	0,9729	0,7823	0,7501	3,6628
83	0,2722	0,0327	0,1043	0,7806	0,6763	-0,0272	0,9731	0,7825	0,7506	3,6481
84	0,2694	0,0323	0,1038	0,7814	0,6776	-0,0269	0,9734	0,7827	0,7510	3,6334
85	0,2667	0,0320	0,1033	0,7822	0,6790	-0,0267	0,9737	0,7830	0,7514	3,6186
86	0,2639	0,0317	0,1027	0,7831	0,6804	-0,0264	0,9740	0,7832	0,7519	3,6038
87	0,2611	0,0313	0,1022	0,7840	0,6818	-0,0261	0,9742	0,7835	0,7523	3,5890
88	0,2583	0,0310	0,1017	0,7849	0,6833	-0,0258	0,9745	0,7838	0,7528	3,5742
89	0,2556	0,0307	0,1011	0,7859	0,6848	-0,0256	0,9748	0,7840	0,7533	3,5593
90	0,2528	0,0303	0,1006	0,7869	0,6863	-0,0253	0,9750	0,7843	0,7537	3,5443
91	0,2500	0,0300	0,1000	0,7879	0,6879	-0,0250	0,9753	0,7846	0,7542	3,5293
92	0,2472	0,0297	0,0994	0,7890	0,6895	-0,0247	0,9756	0,7849	0,7548	3,5143
93	0,2444	0,0293	0,0989	0,7901	0,6912	-0,0244	0,9759	0,7853	0,7553	3,4993
94	0,2417	0,0290	0,0983	0,7912	0,6929	-0,0242	0,9761	0,7856	0,7558	3,4842
95	0,2389	0,0287	0,0978	0,7924	0,6946	-0,0239	0,9764	0,7859	0,7564	3,4690
96	0,2361	0,0283	0,0972	0,7936	0,6964	-0,0236	0,9767	0,7863	0,7569	3,4538
97	0,2333	0,0280	0,0966	0,7949	0,6982	-0,0233	0,9769	0,7867	0,7575	3,4386
98	0,2306	0,0277	0,0960	0,7962	0,7001	-0,0231	0,9772	0,7870	0,7581	3,4233
99	0,2278	0,0273	0,0955	0,7975	0,7021	-0,0228	0,9775	0,7874	0,7587	3,4080
100	0,2250	0,0270	0,0949	0,7989	0,7040	-0,0225	0,9778	0,7878	0,7593	3,3927

101	0,2222	0,0267	0,0943	0,8003	0,7061	-0,0222	0,9780	0,7882	0,7599	3,3773
102	0,2194	0,0263	0,0937	0,8018	0,7081	-0,0219	0,9783	0,7887	0,7606	3,3618
103	0,2167	0,0260	0,0931	0,8034	0,7103	-0,0217	0,9786	0,7891	0,7612	3,3463
104	0,2139	0,0257	0,0925	0,8050	0,7125	-0,0214	0,9788	0,7896	0,7619	3,3308
105	0,2111	0,0253	0,0919	0,8066	0,7147	-0,0211	0,9791	0,7901	0,7626	3,3152
106	0,2083	0,0250	0,0913	0,8083	0,7170	-0,0208	0,9794	0,7905	0,7633	3,2996
107	0,2056	0,0247	0,0907	0,8101	0,7194	-0,0206	0,9797	0,7911	0,7641	3,2839
108	0,2028	0,0243	0,0901	0,8119	0,7219	-0,0203	0,9799	0,7916	0,7648	3,2681
109	0,2000	0,0240	0,0894	0,8138	0,7244	-0,0200	0,9802	0,7921	0,7656	3,2524
110	0,1972	0,0237	0,0888	0,8158	0,7270	-0,0197	0,9805	0,7927	0,7664	3,2365
111	0,1944	0,0233	0,0882	0,8178	0,7296	-0,0194	0,9807	0,7933	0,7672	3,2206
112	0,1917	0,0230	0,0876	0,8199	0,7323	-0,0192	0,9810	0,7939	0,7680	3,2047
113	0,1889	0,0227	0,0869	0,8221	0,7352	-0,0189	0,9813	0,7945	0,7689	3,1887
114	0,1861	0,0223	0,0863	0,8243	0,7380	-0,0186	0,9816	0,7951	0,7698	3,1726
115	0,1833	0,0220	0,0856	0,8267	0,7410	-0,0183	0,9818	0,7958	0,7707	3,1565
116	0,1806	0,0217	0,0850	0,8291	0,7441	-0,0181	0,9821	0,7965	0,7716	3,1404
117	0,1778	0,0213	0,0843	0,8316	0,7472	-0,0178	0,9824	0,7972	0,7725	3,1242
118	0,1750	0,0210	0,0837	0,8342	0,7505	-0,0175	0,9827	0,7979	0,7735	3,1079
119	0,1722	0,0207	0,0830	0,8368	0,7538	-0,0172	0,9829	0,7987	0,7745	3,0916
120	0,1694	0,0203	0,0823	0,8396	0,7573	-0,0169	0,9832	0,7994	0,7756	3,0752
121	0,1667	0,0200	0,0816	0,8425	0,7609	-0,0167	0,9835	0,8002	0,7766	3,0587
122	0,1639	0,0197	0,0810	0,8455	0,7645	-0,0164	0,9837	0,8011	0,7777	3,0422
123	0,1611	0,0193	0,0803	0,8486	0,7683	-0,0161	0,9840	0,8019	0,7789	3,0256
124	0,1583	0,0190	0,0796	0,8518	0,7722	-0,0158	0,9843	0,8028	0,7800	3,0090
125	0,1556	0,0187	0,0789	0,8552	0,7763	-0,0156	0,9846	0,8038	0,7812	2,9923
126	0,1528	0,0183	0,0782	0,8586	0,7805	-0,0153	0,9848	0,8047	0,7824	2,9755
127	0,1500	0,0180	0,0775	0,8623	0,7848	-0,0150	0,9851	0,8057	0,7837	2,9587
128	0,1472	0,0177	0,0767	0,8660	0,7893	-0,0147	0,9854	0,8068	0,7850	2,9418
129	0,1444	0,0173	0,0760	0,8699	0,7939	-0,0144	0,9857	0,8078	0,7864	2,9248
130	0,1417	0,0170	0,0753	0,8740	0,7987	-0,0142	0,9859	0,8089	0,7878	2,9078
131	0,1389	0,0167	0,0745	0,8782	0,8037	-0,0139	0,9862	0,8101	0,7892	2,8907
132	0,1361	0,0163	0,0738	0,8826	0,8088	-0,0136	0,9865	0,8113	0,7907	2,8735
133	0,1333	0,0160	0,0730	0,8872	0,8141	-0,0133	0,9868	0,8125	0,7922	2,8563
134	0,1306	0,0157	0,0723	0,8920	0,8197	-0,0131	0,9870	0,8138	0,7938	2,8389
135	0,1278	0,0153	0,0715	0,8969	0,8254	-0,0128	0,9873	0,8151	0,7954	2,8215
136	0,1250	0,0150	0,0707	0,9021	0,8314	-0,0125	0,9876	0,8165	0,7971	2,8041
137	0,1222	0,0147	0,0699	0,9076	0,8376	-0,0122	0,9879	0,8179	0,7989	2,7865
138	0,1194	0,0143	0,0691	0,9132	0,8441	-0,0119	0,9881	0,8194	0,8007	2,7689
139	0,1167	0,0140	0,0683	0,9192	0,8508	-0,0117	0,9884	0,8210	0,8026	2,7512
140	0,1139	0,0137	0,0675	0,9254	0,8579	-0,0114	0,9887	0,8226	0,8045	2,7334
141	0,1111	0,0133	0,0667	0,9319	0,8652	-0,0111	0,9890	0,8243	0,8065	2,7155
142	0,1083	0,0130	0,0658	0,9387	0,8728	-0,0108	0,9892	0,8260	0,8086	2,6976
143	0,1056	0,0127	0,0650	0,9458	0,8808	-0,0106	0,9895	0,8279	0,8108	2,6796
144	0,1028	0,0123	0,0641	0,9533	0,8892	-0,0103	0,9898	0,8298	0,8130	2,6614
145	0,1000	0,0120	0,0632	0,9612	0,8979	-0,0100	0,9900	0,8318	0,8154	2,6432

146	0,0972	0,0117	0,0624	0,9695	0,9071	-0,0097	0,9903	0,8338	0,8178	2,6250
147	0,0944	0,0113	0,0615	0,9782	0,9167	-0,0094	0,9906	0,8360	0,8204	2,6066
148	0,0917	0,0110	0,0606	0,9874	0,9268	-0,0092	0,9909	0,8383	0,8230	2,5881
149	0,0889	0,0107	0,0596	0,9971	0,9375	-0,0089	0,9912	0,8406	0,8257	2,5696
150	0,0861	0,0103	0,0587	1,0074	0,9487	-0,0086	0,9914	0,8431	0,8286	2,5509
151	0,0833	0,0100	0,0577	1,0183	0,9605	-0,0083	0,9917	0,8457	0,8316	2,5322
152	0,0806	0,0097	0,0568	1,0298	0,9730	-0,0081	0,9920	0,8485	0,8347	2,5134
153	0,0778	0,0093	0,0558	1,0421	0,9863	-0,0078	0,9923	0,8513	0,8380	2,4945
154	0,0750	0,0090	0,0548	1,0551	1,0003	-0,0075	0,9925	0,8543	0,8414	2,4756
155	0,0722	0,0087	0,0537	1,0690	1,0152	-0,0072	0,9928	0,8575	0,8450	2,4565
156	0,0694	0,0083	0,0527	1,0838	1,0311	-0,0069	0,9931	0,8608	0,8488	2,4373
157	0,0667	0,0080	0,0516	1,0997	1,0481	-0,0067	0,9934	0,8643	0,8527	2,4181
158	0,0639	0,0077	0,0506	1,1168	1,0662	-0,0064	0,9936	0,8680	0,8568	2,3988
159	0,0611	0,0073	0,0494	1,1352	1,0857	-0,0061	0,9939	0,8718	0,8612	2,3795
160	0,0583	0,0070	0,0483	1,1550	1,1067	-0,0058	0,9942	0,8759	0,8658	2,3600
161	0,0556	0,0067	0,0471	1,1764	1,1293	-0,0056	0,9945	0,8803	0,8706	2,3406
162	0,0528	0,0063	0,0459	1,1997	1,1538	-0,0053	0,9947	0,8849	0,8757	2,3210
163	0,0500	0,0060	0,0447	1,2251	1,1804	-0,0050	0,9950	0,8897	0,8811	2,3015
164	0,0472	0,0057	0,0435	1,2530	1,2095	-0,0047	0,9953	0,8949	0,8868	2,2819
165	0,0444	0,0053	0,0422	1,2837	1,2415	-0,0044	0,9956	0,9004	0,8928	2,2623
166	0,0417	0,0050	0,0408	1,3176	1,2768	-0,0042	0,9958	0,9062	0,8992	2,2428
167	0,0389	0,0047	0,0394	1,3554	1,3159	-0,0039	0,9961	0,9124	0,9059	2,2233
168	0,0361	0,0043	0,0380	1,3978	1,3598	-0,0036	0,9964	0,9189	0,9130	2,2039
169	0,0333	0,0040	0,0365	1,4457	1,4092	-0,0033	0,9967	0,9259	0,9206	2,1847
170	0,0306	0,0037	0,0350	1,5005	1,4655	-0,0031	0,9969	0,9333	0,9286	2,1656
171	0,0278	0,0033	0,0333	1,5637	1,5304	-0,0028	0,9972	0,9411	0,9370	2,1468
172	0,0250	0,0030	0,0316	1,6377	1,6061	-0,0025	0,9975	0,9493	0,9459	2,1284
173	0,0222	0,0027	0,0298	1,7259	1,6961	-0,0022	0,9978	0,9578	0,9551	2,1105
174	0,0194	0,0023	0,0279	1,8331	1,8052	-0,0019	0,9981	0,9666	0,9645	2,0932
175	0,0167	0,0020	0,0258	1,9671	1,9413	-0,0017	0,9983	0,9754	0,9739	2,0768
176	0,0139	0,0017	0,0236	2,1407	2,1171	-0,0014	0,9986	0,9839	0,9829	2,0613
177	0,0111	0,0013	0,0211	2,3776	2,3565	-0,0011	0,9989	0,9913	0,9908	2,0470
178	0,0083	0,0010	0,0183	2,7271	2,7089	-0,0008	0,9992	0,9968	0,9966	2,0341
179	0,0056	0,0007	0,0149	3,3177	3,3028	-0,0006	0,9994	0,9995	0,9995	2,0223
180	0,0028	0,0003	0,0105	4,6603	4,6497	-0,0003	0,9997	1,0000	1,0000	2,0111

After the second application, on the contrary of first Disney stock issues call price applications, depending on the A^2 values here, when we test for A^2_α, α=0.05; if c call prices A^2 value,

$H_0 : A^2 \le A^2_{0.05} = 0.752$ (The distribution of the variable is normal.),

$H_1 : A^2 > A^2_{0.05} = 0.752$ (The distribution of the variable is not normal.)

Hypotheses with T (expiration time) dated-unit data, we clearly see that the distributions of option prices confirmed the hypothesis $H_1 : A^2 > A^2_{0.05} = 0.752$ (The distribution of the variable is not normal.), for the IBM stock issues American call option prices. Because we found $A^2 = 2.061$.

We can see this hypothesis reality in figure 5.3 (AD normality test calculations with 95% confidence interval and $\alpha = 0.05$ in Minitab) and figure 5.4 (probability plot of IBM stock issues c call price).

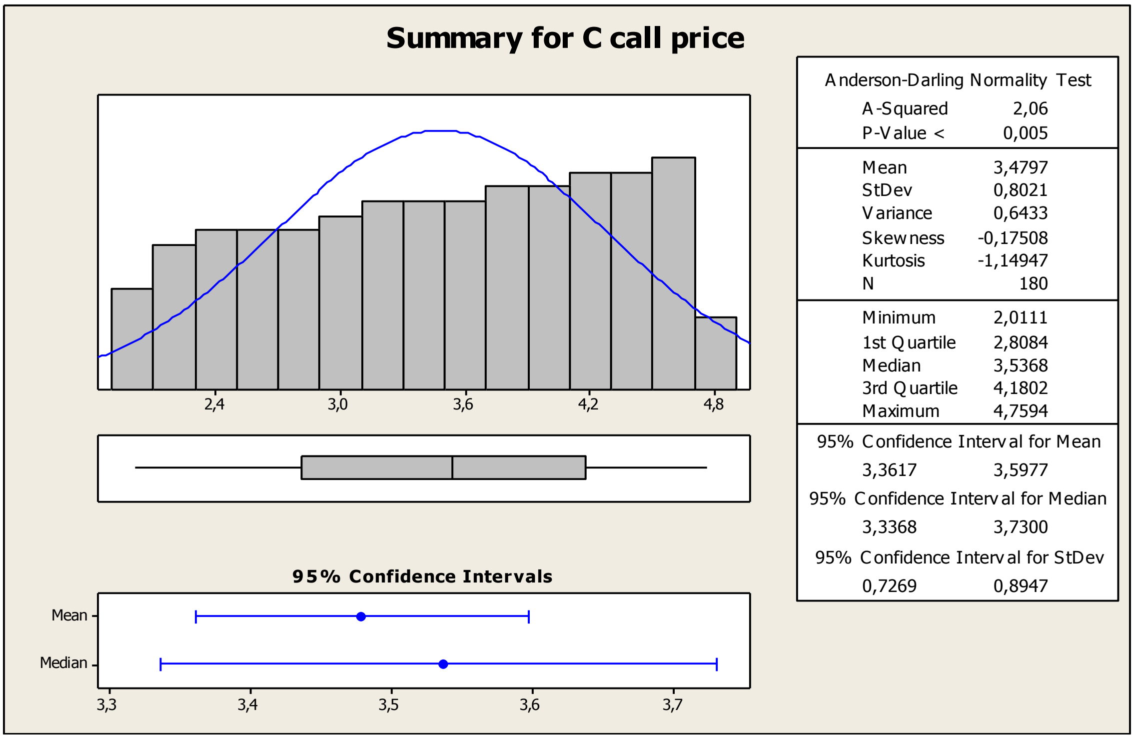

Figure 5.3 Graphical Summary of Anderson-Darling Normality Test for Call Option Price

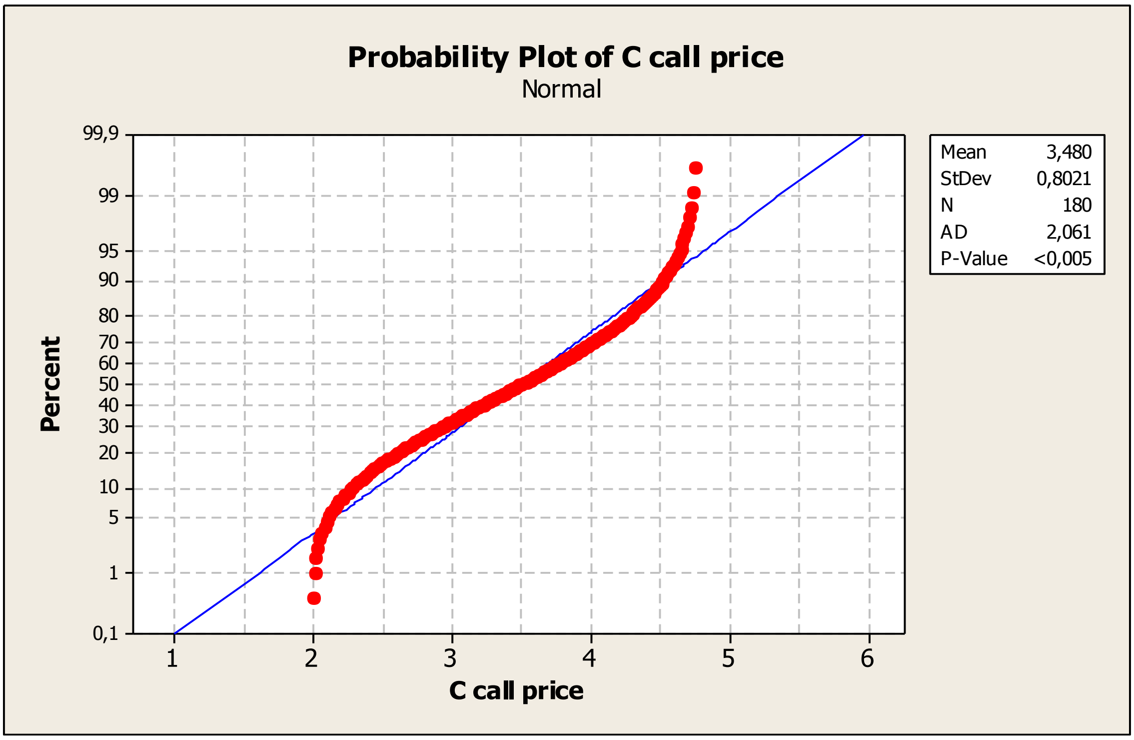

Figure 5.4 Anderson-Darling Normality Test for Call Option Price

Table 5.4 IBM stock issues p put option price normality variations

(Std. Dev. Is %20 And Constant)

Risk-Free Interest Rate Variation	AD Value Variation (A^2)
2%	2.607
8%	3.517
9%	3.750
10% (base case)	4.014
11%	4.312
12%	4.648
20%	9.448

(Risk Free Interest Rate Is %10 And Constant)

Expected Price Volatility Variation (Std.dev.)	AD Value Variation (A^2)
10%	4.765
15%	4.265
20% (base case)	4.014
25%	3.840
30%	3.709

(Either Std. Dev. or Risk Free Interest Rate Variation. The Expiration Time (T) is 180 Days and Constant)

AD Value for Normality Test		*Risk free interest rate*						
		%2	**%8**	**%9**	**%10***	**%11**	**%12**	**%20**
Volatility (Std.Dev.)	**%10**	3.411	4.045	4.358	4.765	5.289	5.965	16.987
	%15	2.717	3.652	3.934	4.265	4.656	5.113	12.862
	%20*	2.607	3.517	3.750	4.014	4.312	4.648	9.448
	%25	2.601	3.429	3.624	3.840	4.079	4.342	7.713
	%30	2.615	3.361	3.528	3.709	3.907	4.123	6.686

(* base case)

Table 5.5 Option premiums and parameters required for premium calculations

K stock price	S strike price	ln(s/x)	r (risk-free rate)	σ(std. deviation)
40	42	0,04879	10%	20%

t time	T expiration time	(r+(σ²/2))*T	σ*T^(1/2)	d1	d2	(-rT)	(e^(-rT))	N(-d1)	N(-d2)	p put price
1	0,5000	0,0600	0,1414	0,7693	0,6278	-0,0500	0,9512	0,2209	0,2651	0,8086
2	0,4972	0,0597	0,1410	0,7690	0,6280	-0,0497	0,9515	0,2209	0,2650	0,8065
3	0,4944	0,0593	0,1406	0,7688	0,6282	-0,0494	0,9518	0,2210	0,2649	0,8044
4	0,4917	0,0590	0,1402	0,7686	0,6284	-0,0492	0,9520	0,2211	0,2649	0,8022
5	0,4889	0,0587	0,1398	0,7684	0,6286	-0,0489	0,9523	0,2211	0,2648	0,8001
6	0,4861	0,0583	0,1394	0,7682	0,6288	-0,0486	0,9526	0,2212	0,2647	0,7979
7	0,4833	0,0580	0,1390	0,7680	0,6290	-0,0483	0,9528	0,2212	0,2647	0,7958
8	0,4806	0,0577	0,1386	0,7678	0,6292	-0,0481	0,9531	0,2213	0,2646	0,7936
9	0,4778	0,0573	0,1382	0,7677	0,6294	-0,0478	0,9533	0,2213	0,2645	0,7914
10	0,4750	0,0570	0,1378	0,7675	0,6296	-0,0475	0,9536	0,2214	0,2645	0,7892
11	0,4722	0,0567	0,1374	0,7673	0,6299	-0,0472	0,9539	0,2214	0,2644	0,7869
12	0,4694	0,0563	0,1370	0,7671	0,6301	-0,0469	0,9541	0,2215	0,2643	0,7847
13	0,4667	0,0560	0,1366	0,7670	0,6304	-0,0467	0,9544	0,2215	0,2642	0,7824
14	0,4639	0,0557	0,1362	0,7668	0,6306	-0,0464	0,9547	0,2216	0,2641	0,7801
15	0,4611	0,0553	0,1358	0,7667	0,6309	-0,0461	0,9549	0,2216	0,2641	0,7778
16	0,4583	0,0550	0,1354	0,7665	0,6311	-0,0458	0,9552	0,2217	0,2640	0,7755
17	0,4556	0,0547	0,1350	0,7664	0,6314	-0,0456	0,9555	0,2217	0,2639	0,7732
18	0,4528	0,0543	0,1346	0,7663	0,6317	-0,0453	0,9557	0,2218	0,2638	0,7708
19	0,4500	0,0540	0,1342	0,7662	0,6320	-0,0450	0,9560	0,2218	0,2637	0,7685
20	0,4472	0,0537	0,1337	0,7660	0,6323	-0,0447	0,9563	0,2218	0,2636	0,7661
21	0,4444	0,0533	0,1333	0,7659	0,6326	-0,0444	0,9565	0,2219	0,2635	0,7637
22	0,4417	0,0530	0,1329	0,7658	0,6329	-0,0442	0,9568	0,2219	0,2634	0,7613
23	0,4389	0,0527	0,1325	0,7657	0,6332	-0,0439	0,9571	0,2219	0,2633	0,7588
24	0,4361	0,0523	0,1321	0,7656	0,6336	-0,0436	0,9573	0,2219	0,2632	0,7564
25	0,4333	0,0520	0,1317	0,7656	0,6339	-0,0433	0,9576	0,2220	0,2631	0,7539
26	0,4306	0,0517	0,1312	0,7655	0,6342	-0,0431	0,9579	0,2220	0,2630	0,7514
27	0,4278	0,0513	0,1308	0,7654	0,6346	-0,0428	0,9581	0,2220	0,2628	0,7489
28	0,4250	0,0510	0,1304	0,7654	0,6350	-0,0425	0,9584	0,2220	0,2627	0,7464
29	0,4222	0,0507	0,1300	0,7653	0,6353	-0,0422	0,9587	0,2220	0,2626	0,7439
30	0,4194	0,0503	0,1295	0,7653	0,6357	-0,0419	0,9589	0,2221	0,2625	0,7413
31	0,4167	0,0500	0,1291	0,7652	0,6361	-0,0417	0,9592	0,2221	0,2623	0,7387
32	0,4139	0,0497	0,1287	0,7652	0,6365	-0,0414	0,9595	0,2221	0,2622	0,7361
33	0,4111	0,0493	0,1282	0,7652	0,6369	-0,0411	0,9597	0,2221	0,2621	0,7335
34	0,4083	0,0490	0,1278	0,7652	0,6374	-0,0408	0,9600	0,2221	0,2619	0,7309
35	0,4056	0,0487	0,1274	0,7652	0,6378	-0,0406	0,9603	0,2221	0,2618	0,7283
36	0,4028	0,0483	0,1269	0,7652	0,6382	-0,0403	0,9605	0,2221	0,2617	0,7256
37	0,4000	0,0480	0,1265	0,7652	0,6387	-0,0400	0,9608	0,2221	0,2615	0,7229
38	0,3972	0,0477	0,1261	0,7652	0,6392	-0,0397	0,9611	0,2221	0,2614	0,7202
39	0,3944	0,0473	0,1256	0,7653	0,6396	-0,0394	0,9613	0,2221	0,2612	0,7175
40	0,3917	0,0470	0,1252	0,7653	0,6401	-0,0392	0,9616	0,2220	0,2610	0,7147
41	0,3889	0,0467	0,1247	0,7654	0,6406	-0,0389	0,9619	0,2220	0,2609	0,7119
42	0,3861	0,0463	0,1243	0,7654	0,6411	-0,0386	0,9621	0,2220	0,2607	0,7091
43	0,3833	0,0460	0,1238	0,7655	0,6417	-0,0383	0,9624	0,2220	0,2605	0,7063
44	0,3806	0,0457	0,1234	0,7656	0,6422	-0,0381	0,9627	0,2220	0,2604	0,7035
45	0,3778	0,0453	0,1229	0,7657	0,6428	-0,0378	0,9629	0,2219	0,2602	0,7006

46	0,3750	0,0450	0,1225	0,7658	0,6433	-0,0375	0,9632	0,2219	0,2600	0,6978
47	0,3722	0,0447	0,1220	0,7659	0,6439	-0,0372	0,9635	0,2219	0,2598	0,6949
48	0,3694	0,0443	0,1216	0,7660	0,6445	-0,0369	0,9637	0,2218	0,2596	0,6920
49	0,3667	0,0440	0,1211	0,7662	0,6451	-0,0367	0,9640	0,2218	0,2594	0,6890
50	0,3639	0,0437	0,1206	0,7663	0,6457	-0,0364	0,9643	0,2217	0,2592	0,6860
51	0,3611	0,0433	0,1202	0,7665	0,6463	-0,0361	0,9645	0,2217	0,2590	0,6831
52	0,3583	0,0430	0,1197	0,7667	0,6470	-0,0358	0,9648	0,2216	0,2588	0,6801
53	0,3556	0,0427	0,1193	0,7669	0,6476	-0,0356	0,9651	0,2216	0,2586	0,6770
54	0,3528	0,0423	0,1188	0,7671	0,6483	-0,0353	0,9653	0,2215	0,2584	0,6740
55	0,3500	0,0420	0,1183	0,7673	0,6490	-0,0350	0,9656	0,2214	0,2582	0,6709
56	0,3472	0,0417	0,1179	0,7676	0,6497	-0,0347	0,9659	0,2214	0,2579	0,6678
57	0,3444	0,0413	0,1174	0,7678	0,6504	-0,0344	0,9661	0,2213	0,2577	0,6647
58	0,3417	0,0410	0,1169	0,7681	0,6512	-0,0342	0,9664	0,2212	0,2575	0,6615
59	0,3389	0,0407	0,1164	0,7683	0,6519	-0,0339	0,9667	0,2211	0,2572	0,6583
60	0,3361	0,0403	0,1160	0,7686	0,6527	-0,0336	0,9669	0,2211	0,2570	0,6551
61	0,3333	0,0400	0,1155	0,7689	0,6535	-0,0333	0,9672	0,2210	0,2567	0,6519
62	0,3306	0,0397	0,1150	0,7693	0,6543	-0,0331	0,9675	0,2209	0,2565	0,6486
63	0,3278	0,0393	0,1145	0,7696	0,6551	-0,0328	0,9678	0,2208	0,2562	0,6454
64	0,3250	0,0390	0,1140	0,7700	0,6560	-0,0325	0,9680	0,2207	0,2559	0,6421
65	0,3222	0,0387	0,1135	0,7703	0,6568	-0,0322	0,9683	0,2205	0,2556	0,6387
66	0,3194	0,0383	0,1130	0,7707	0,6577	-0,0319	0,9686	0,2204	0,2554	0,6354
67	0,3167	0,0380	0,1125	0,7712	0,6586	-0,0317	0,9688	0,2203	0,2551	0,6320
68	0,3139	0,0377	0,1121	0,7716	0,6595	-0,0314	0,9691	0,2202	0,2548	0,6286
69	0,3111	0,0373	0,1116	0,7720	0,6605	-0,0311	0,9694	0,2200	0,2545	0,6251
70	0,3083	0,0370	0,1111	0,7725	0,6614	-0,0308	0,9696	0,2199	0,2542	0,6217
71	0,3056	0,0367	0,1106	0,7730	0,6624	-0,0306	0,9699	0,2198	0,2538	0,6182
72	0,3028	0,0363	0,1101	0,7735	0,6634	-0,0303	0,9702	0,2196	0,2535	0,6147
73	0,3000	0,0360	0,1095	0,7740	0,6645	-0,0300	0,9704	0,2195	0,2532	0,6111
74	0,2972	0,0357	0,1090	0,7746	0,6655	-0,0297	0,9707	0,2193	0,2529	0,6075
75	0,2944	0,0353	0,1085	0,7751	0,6666	-0,0294	0,9710	0,2191	0,2525	0,6039
76	0,2917	0,0350	0,1080	0,7757	0,6677	-0,0292	0,9713	0,2189	0,2522	0,6003
77	0,2889	0,0347	0,1075	0,7764	0,6689	-0,0289	0,9715	0,2188	0,2518	0,5966
78	0,2861	0,0343	0,1070	0,7770	0,6700	-0,0286	0,9718	0,2186	0,2514	0,5929
79	0,2833	0,0340	0,1065	0,7777	0,6712	-0,0283	0,9721	0,2184	0,2510	0,5892
80	0,2806	0,0337	0,1059	0,7784	0,6724	-0,0281	0,9723	0,2182	0,2507	0,5854
81	0,2778	0,0333	0,1054	0,7791	0,6737	-0,0278	0,9726	0,2180	0,2503	0,5816
82	0,2750	0,0330	0,1049	0,7798	0,6750	-0,0275	0,9729	0,2177	0,2499	0,5778
83	0,2722	0,0327	0,1043	0,7806	0,6763	-0,0272	0,9731	0,2175	0,2494	0,5739
84	0,2694	0,0323	0,1038	0,7814	0,6776	-0,0269	0,9734	0,2173	0,2490	0,5700
85	0,2667	0,0320	0,1033	0,7822	0,6790	-0,0267	0,9737	0,2170	0,2486	0,5661
86	0,2639	0,0317	0,1027	0,7831	0,6804	-0,0264	0,9740	0,2168	0,2481	0,5621
87	0,2611	0,0313	0,1022	0,7840	0,6818	-0,0261	0,9742	0,2165	0,2477	0,5581
88	0,2583	0,0310	0,1017	0,7849	0,6833	-0,0258	0,9745	0,2162	0,2472	0,5541
89	0,2556	0,0307	0,1011	0,7859	0,6848	-0,0256	0,9748	0,2160	0,2467	0,5500
90	0,2528	0,0303	0,1006	0,7869	0,6863	-0,0253	0,9750	0,2157	0,2463	0,5459
91	0,2500	0,0300	0,1000	0,7879	0,6879	-0,0250	0,9753	0,2154	0,2458	0,5417
92	0,2472	0,0297	0,0994	0,7890	0,6895	-0,0247	0,9756	0,2151	0,2452	0,5376
93	0,2444	0,0293	0,0989	0,7901	0,6912	-0,0244	0,9759	0,2147	0,2447	0,5333
94	0,2417	0,0290	0,0983	0,7912	0,6929	-0,0242	0,9761	0,2144	0,2442	0,5291
95	0,2389	0,0287	0,0978	0,7924	0,6946	-0,0239	0,9764	0,2141	0,2436	0,5248
96	0,2361	0,0283	0,0972	0,7936	0,6964	-0,0236	0,9767	0,2137	0,2431	0,5205
97	0,2333	0,0280	0,0966	0,7949	0,6982	-0,0233	0,9769	0,2133	0,2425	0,5161
98	0,2306	0,0277	0,0960	0,7962	0,7001	-0,0231	0,9772	0,2130	0,2419	0,5117
99	0,2278	0,0273	0,0955	0,7975	0,7021	-0,0228	0,9775	0,2126	0,2413	0,5072
100	0,2250	0,0270	0,0949	0,7989	0,7040	-0,0225	0,9778	0,2122	0,2407	0,5027

101	0,2222	0,0267	0,0943	0,8003	0,7061	-0,0222	0,9780	0,2118	0,2401	0,4982
102	0,2194	0,0263	0,0937	0,8018	0,7081	-0,0219	0,9783	0,2113	0,2394	0,4936
103	0,2167	0,0260	0,0931	0,8034	0,7103	-0,0217	0,9786	0,2109	0,2388	0,4890
104	0,2139	0,0257	0,0925	0,8050	0,7125	-0,0214	0,9788	0,2104	0,2381	0,4843
105	0,2111	0,0253	0,0919	0,8066	0,7147	-0,0211	0,9791	0,2099	0,2374	0,4796
106	0,2083	0,0250	0,0913	0,8083	0,7170	-0,0208	0,9794	0,2095	0,2367	0,4749
107	0,2056	0,0247	0,0907	0,8101	0,7194	-0,0206	0,9797	0,2089	0,2359	0,4700
108	0,2028	0,0243	0,0901	0,8119	0,7219	-0,0203	0,9799	0,2084	0,2352	0,4652
109	0,2000	0,0240	0,0894	0,8138	0,7244	-0,0200	0,9802	0,2079	0,2344	0,4603
110	0,1972	0,0237	0,0888	0,8158	0,7270	-0,0197	0,9805	0,2073	0,2336	0,4554
111	0,1944	0,0233	0,0882	0,8178	0,7296	-0,0194	0,9807	0,2067	0,2328	0,4504
112	0,1917	0,0230	0,0876	0,8199	0,7323	-0,0192	0,9810	0,2061	0,2320	0,4453
113	0,1889	0,0227	0,0869	0,8221	0,7352	-0,0189	0,9813	0,2055	0,2311	0,4402
114	0,1861	0,0223	0,0863	0,8243	0,7380	-0,0186	0,9816	0,2049	0,2302	0,4351
115	0,1833	0,0220	0,0856	0,8267	0,7410	-0,0183	0,9818	0,2042	0,2293	0,4299
116	0,1806	0,0217	0,0850	0,8291	0,7441	-0,0181	0,9821	0,2035	0,2284	0,4246
117	0,1778	0,0213	0,0843	0,8316	0,7472	-0,0178	0,9824	0,2028	0,2275	0,4193
118	0,1750	0,0210	0,0837	0,8342	0,7505	-0,0175	0,9827	0,2021	0,2265	0,4140
119	0,1722	0,0207	0,0830	0,8368	0,7538	-0,0172	0,9829	0,2013	0,2255	0,4086
120	0,1694	0,0203	0,0823	0,8396	0,7573	-0,0169	0,9832	0,2006	0,2244	0,4031
121	0,1667	0,0200	0,0816	0,8425	0,7609	-0,0167	0,9835	0,1998	0,2234	0,3976
122	0,1639	0,0197	0,0810	0,8455	0,7645	-0,0164	0,9837	0,1989	0,2223	0,3920
123	0,1611	0,0193	0,0803	0,8486	0,7683	-0,0161	0,9840	0,1981	0,2211	0,3864
124	0,1583	0,0190	0,0796	0,8518	0,7722	-0,0158	0,9843	0,1972	0,2200	0,3807
125	0,1556	0,0187	0,0789	0,8552	0,7763	-0,0156	0,9846	0,1962	0,2188	0,3749
126	0,1528	0,0183	0,0782	0,8586	0,7805	-0,0153	0,9848	0,1953	0,2176	0,3691
127	0,1500	0,0180	0,0775	0,8623	0,7848	-0,0150	0,9851	0,1943	0,2163	0,3632
128	0,1472	0,0177	0,0767	0,8660	0,7893	-0,0147	0,9854	0,1932	0,2150	0,3572
129	0,1444	0,0173	0,0760	0,8699	0,7939	-0,0144	0,9857	0,1922	0,2136	0,3512
130	0,1417	0,0170	0,0753	0,8740	0,7987	-0,0142	0,9859	0,1911	0,2122	0,3451
131	0,1389	0,0167	0,0745	0,8782	0,8037	-0,0139	0,9862	0,1899	0,2108	0,3390
132	0,1361	0,0163	0,0738	0,8826	0,8088	-0,0136	0,9865	0,1887	0,2093	0,3328
133	0,1333	0,0160	0,0730	0,8872	0,8141	-0,0133	0,9868	0,1875	0,2078	0,3265
134	0,1306	0,0157	0,0723	0,8920	0,8197	-0,0131	0,9870	0,1862	0,2062	0,3201
135	0,1278	0,0153	0,0715	0,8969	0,8254	-0,0128	0,9873	0,1849	0,2046	0,3137
136	0,1250	0,0150	0,0707	0,9021	0,8314	-0,0125	0,9876	0,1835	0,2029	0,3072
137	0,1222	0,0147	0,0699	0,9076	0,8376	-0,0122	0,9879	0,1821	0,2011	0,3006
138	0,1194	0,0143	0,0691	0,9132	0,8441	-0,0119	0,9881	0,1806	0,1993	0,2940
139	0,1167	0,0140	0,0683	0,9192	0,8508	-0,0117	0,9884	0,1790	0,1974	0,2872
140	0,1139	0,0137	0,0675	0,9254	0,8579	-0,0114	0,9887	0,1774	0,1955	0,2804
141	0,1111	0,0133	0,0667	0,9319	0,8652	-0,0111	0,9890	0,1757	0,1935	0,2736
142	0,1083	0,0130	0,0658	0,9387	0,8728	-0,0108	0,9892	0,1740	0,1914	0,2666
143	0,1056	0,0127	0,0650	0,9458	0,8808	-0,0106	0,9895	0,1721	0,1892	0,2596
144	0,1028	0,0123	0,0641	0,9533	0,8892	-0,0103	0,9898	0,1702	0,1870	0,2524
145	0,1000	0,0120	0,0632	0,9612	0,8979	-0,0100	0,9900	0,1682	0,1846	0,2452

146	0,0972	0,0117	0,0624	0,9695	0,9071	-0,0097	0,9903	0,1662	0,1822	0,2380
147	0,0944	0,0113	0,0615	0,9782	0,9167	-0,0094	0,9906	0,1640	0,1796	0,2306
148	0,0917	0,0110	0,0606	0,9874	0,9268	-0,0092	0,9909	0,1617	0,1770	0,2231
149	0,0889	0,0107	0,0596	0,9971	0,9375	-0,0089	0,9912	0,1594	0,1743	0,2156
150	0,0861	0,0103	0,0587	1,0074	0,9487	-0,0086	0,9914	0,1569	0,1714	0,2080
151	0,0833	0,0100	0,0577	1,0183	0,9605	-0,0083	0,9917	0,1543	0,1684	0,2003
152	0,0806	0,0097	0,0568	1,0298	0,9730	-0,0081	0,9920	0,1515	0,1653	0,1925
153	0,0778	0,0093	0,0558	1,0421	0,9863	-0,0078	0,9923	0,1487	0,1620	0,1846
154	0,0750	0,0090	0,0548	1,0551	1,0003	-0,0075	0,9925	0,1457	0,1586	0,1767
155	0,0722	0,0087	0,0537	1,0690	1,0152	-0,0072	0,9928	0,1425	0,1550	0,1686
156	0,0694	0,0083	0,0527	1,0838	1,0311	-0,0069	0,9931	0,1392	0,1512	0,1605
157	0,0667	0,0080	0,0516	1,0997	1,0481	-0,0067	0,9934	0,1357	0,1473	0,1523
158	0,0639	0,0077	0,0506	1,1168	1,0662	-0,0064	0,9936	0,1320	0,1432	0,1441
159	0,0611	0,0073	0,0494	1,1352	1,0857	-0,0061	0,9939	0,1282	0,1388	0,1358
160	0,0583	0,0070	0,0483	1,1550	1,1067	-0,0058	0,9942	0,1241	0,1342	0,1274
161	0,0556	0,0067	0,0471	1,1764	1,1293	-0,0056	0,9945	0,1197	0,1294	0,1190
162	0,0528	0,0063	0,0459	1,1997	1,1538	-0,0053	0,9947	0,1151	0,1243	0,1105
163	0,0500	0,0060	0,0447	1,2251	1,1804	-0,0050	0,9950	0,1103	0,1189	0,1020
164	0,0472	0,0057	0,0435	1,2530	1,2095	-0,0047	0,9953	0,1051	0,1132	0,0935
165	0,0444	0,0053	0,0422	1,2837	1,2415	-0,0044	0,9956	0,0996	0,1072	0,0850
166	0,0417	0,0050	0,0408	1,3176	1,2768	-0,0042	0,9958	0,0938	0,1008	0,0765
167	0,0389	0,0047	0,0394	1,3554	1,3159	-0,0039	0,9961	0,0876	0,0941	0,0681
168	0,0361	0,0043	0,0380	1,3978	1,3598	-0,0036	0,9964	0,0811	0,0870	0,0597
169	0,0333	0,0040	0,0365	1,4457	1,4092	-0,0033	0,9967	0,0741	0,0794	0,0516
170	0,0306	0,0037	0,0350	1,5005	1,4655	-0,0031	0,9969	0,0667	0,0714	0,0436
171	0,0278	0,0033	0,0333	1,5637	1,5304	-0,0028	0,9972	0,0589	0,0630	0,0359
172	0,0250	0,0030	0,0316	1,6377	1,6061	-0,0025	0,9975	0,0507	0,0541	0,0286
173	0,0222	0,0027	0,0298	1,7259	1,6961	-0,0022	0,9978	0,0422	0,0449	0,0217
174	0,0194	0,0023	0,0279	1,8331	1,8052	-0,0019	0,9981	0,0334	0,0355	0,0155
175	0,0167	0,0020	0,0258	1,9671	1,9413	-0,0017	0,9983	0,0246	0,0261	0,0101
176	0,0139	0,0017	0,0236	2,1407	2,1171	-0,0014	0,9986	0,0161	0,0171	0,0058
177	0,0111	0,0013	0,0211	2,3776	2,3565	-0,0011	0,9989	0,0087	0,0092	0,0026
178	0,0083	0,0010	0,0183	2,7271	2,7089	-0,0008	0,9992	0,0032	0,0034	0,0007
179	0,0056	0,0007	0,0149	3,3177	3,3028	-0,0006	0,9994	0,0005	0,0005	0,0001
180	0,0028	0,0003	0,0105	4,6603	4,6497	-0,0003	0,9997	0,0000	0,0000	0,0000

After the second application, on the contrary of first Disney stock issues call price applications, depending on the A^2 values here, when we test for A^2_α, $\alpha=0.05$; if p put prices A^2 value,

$H_0 : A^2 \leq A^2_{0.05} = 0.752$ (The distribution of the variable is normal.),

$H_1 : A^2 > A^2_{0.05} = 0.752$ (The distribution of the variable is not normal.)

Hypotheses with T (expiration time) dated-unit data, we clearly see that the distributions of option prices confirmed the hypothesis $H_1: A^2 > A^2_{0.05} = 0.752$ (The distribution of the variable is not normal.), for the IBM stock issues American put option prices. Because we found $A^2 = 4.014$.

We can see this hypothesis reality in figure 5.5 (AD normality test calculations with 95% confidence interval and $\alpha = 0.05$ in Minitab) and figure 5.6 (probability plot of IBM stock issues p put price).

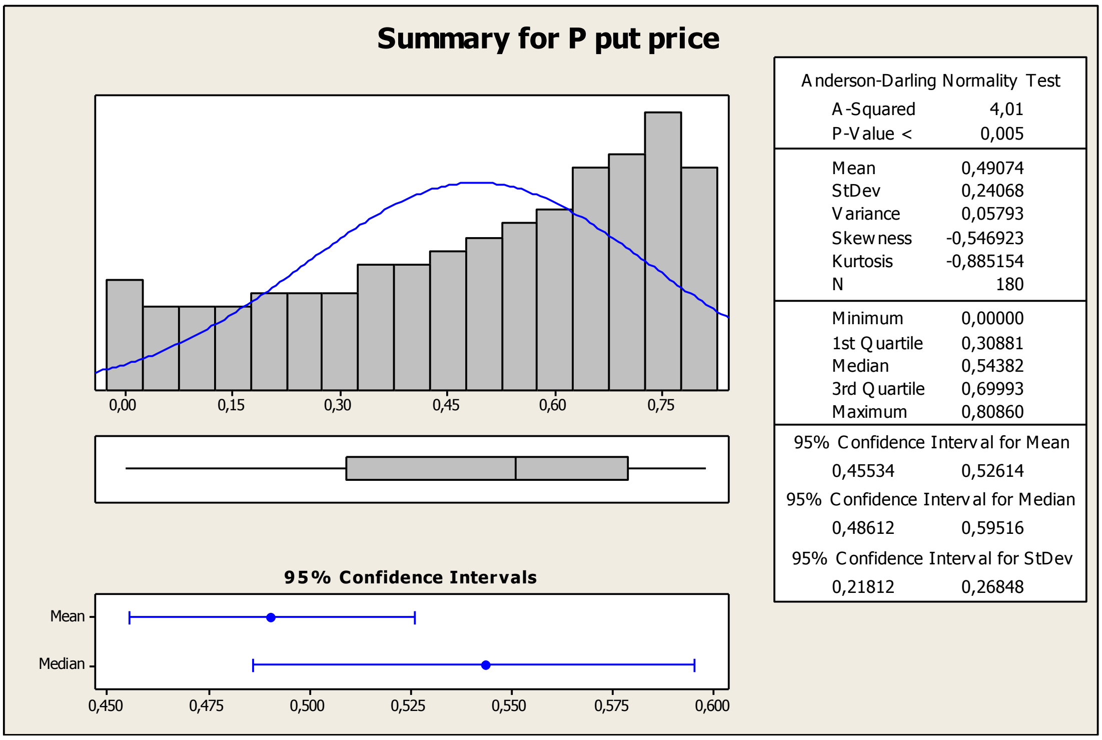

Figure 5.5 Graphical Summary of Anderson-Darling Normality Test for Put Option Price

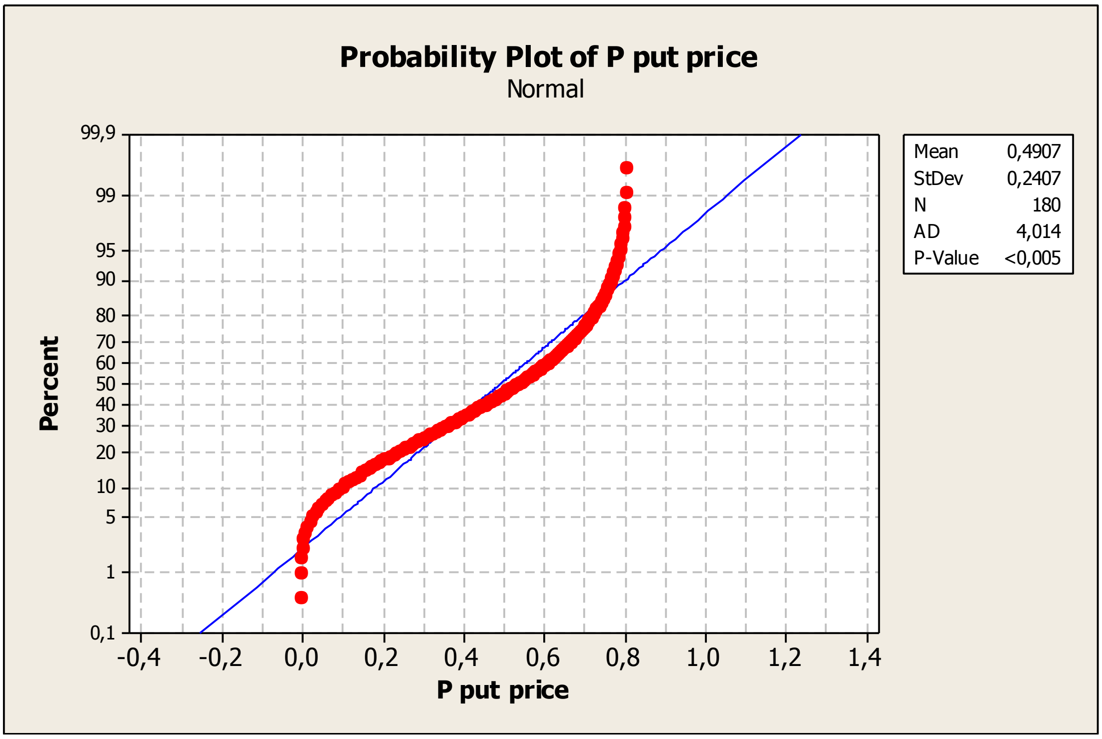

Figure 5.6 Anderson-Darling Normality Test for Put Option Price

Table 5.6 IBM stock issues c call option price normality variations

(Std. Dev. Is %20 And Constant)

Risk-Free Interest Rate Variation	AD Value Variation (A^2)
2%	2.321
8%	2.105
9%	2.081
10% (base case)	2.061
11%	2.043
12%	2.026
20%	1.950

(Risk Free Interest Rate Is %10 And Constant)

Expected Price Volatility Variation (Std.dev.)	AD Value Variation (A^2)
10% 15% 20% (base case) 25% 30%	2.058 2.076 2.061 2.068 2.096

(Either Std. Dev. or Risk Free Interest Rate Variation. The Expiration Time (T) is 180 Days and Constant)

AD Value for Normality Test		*Risk free interest rate*						
		%2	**%8**	**%9**	**%10***	**%11**	**%12**	**%20**
Volatility (Std.Dev.)	**%10**	2.594	2.105	2.079	2.058	2.042	2.028	1.984
	%15	2.400	2.123	2.098	2.076	2.057	2.041	1.973
	%20*	2.321	2.105	2.081	2.061	2.043	2.026	1.950
	%25	2.321	2.114	2.090	2.068	2.049	2.031	1.941
	%30	2.347	2.144	2.119	2.096	2.076	2.056	1.953

(* base case)

The Expiration Time (T)Variations;

When we choose the expiration time (T) different days for the IBM stock issues (the other values are constant and r=%10 and σ =%20), *p put option normality test*

for T=30days (30/360), AD value=0.648,

for T=60days (60/360), AD value=1.000,

for T=120days (120/360), AD value=2.191,

for T=180days (180/360), AD value=4.014,

for T=240days (240/360), AD value=6.523.

Our hypothesis, when we test for A_{α}^{2}, α=0.05; if p put prices A^2 value,

$H_0 : A^2 < A_{0.05}^2 = 0.752$ (The distribution of the variable is normal.),

$H_1 : A^2 > A_{0.05}^2 = 0.752$ (The distribution of the variable is not normal.)

Then hypothesis with T (expiration time=30 days) dated-unit data, we clearly see that the distributions of option prices confirmed the hypothesis $H_0 : A^2 < A_{0.05}^2 = 0.752$ (The distribution of the variable is not normal.), for the IBM stock issues American put option prices. Because $A^2 = 0.648$.

c call option normality test

for T=30days (30/360), AD value=0.425,
for T=60days (60/360), AD value=0.769,
for T=120days (120/360), AD value=1.424,
for T=180days (180/360), AD value=2.061,
for T=240days (240/360), AD value=2.686.

When we test for A_{α}^{2}, α=0.05; if c call price A^2 value,

$H_0 : A^2 < A_{0.05}^2 = 0.752$ (The distribution of the variable is normal.),

$H_1 : A^2 > A_{0.05}^2 = 0.752$ (The distribution of the variable is not normal.)

Hypothesis with T (expiration time=30days) dated-unit data, we clearly see that the distributions of option prices confirmed the hypothesis $H_0 : A^2 < A_{0.05}^2 = 0.752$ (The distribution of the variable is not normal.), for the IBM stock issues American call option prices. Because $A^2 = 0.425$.

5.3 Findings

In the implementation in 5.1 the American call option price of the Disney stock issues is found c = \$3.837 as of T = 50 term date. In addition to this, in Table 5.1, the option prices calculations for other term dates were done, and the AD value, namely the A^2 values of these observed option premiums by Anderson-Darling Normality Test; the result is $A^2 = 0.613$.

In the implementation in 5.2, American Call and Put option prices of IBM stock issues are found c = $4.759 and p = $0.808 as of T=180 term date. Again, option prices calculations were done in Table 5.2; and 5.3, the A^2 value for the observed option premiums were calculated using AD Normality Test; the results are $A^2 = 2.061$ for call option price and $A^2 = 4.014$ for put option price.

CHAPTER SIX
CONCLUSIONS

In our study we performed a calculation of Disney and IBM stock issues of a given date, which are treated in CBOE, in Microsoft Excel, using the Black-Scholes Option Pricing Model, which is known as the most facile option pricing model both theoretically and practically for the premiums of options, which are accepted as the most secure way of hedging.

After the option calculations were conducted, depending on the basic hypotheses of BS option pricing model, especially depending on the hypothesis that “The profits of the assets the option is based on are distributed normally (in a general way of speaking it is a stable distribution)” the required distribution features were determined for the option prices obtained.

By definition of the hypothesis, the option prices should present Stable distribution. Therefore, in order to test whether option prices distributes normal or not normal distributions of option prices, Anderson-Darling Normality test, one of the strongest normality tests developed for normal distributions as part of Stable distributions, was conducted using MINITAB software package. The AD test values or the A^2 values were addressed in the “findings” section. From above mentioned applications we realized the BS model closely result in common stable or to be considerable stable for option prices.

As a result, we tested the compliance of the option premiums of the stock issues, calculated using the Black-Scholes option pricing model, as they confirmed the hypotheses we formed using Anderson-Darling Normality Test to underpin the BS model hypotheses. However, one of the handicaps of the BS model appears right at this point. We can ask the question “Is there any situation that option premiums do not show a stable distribution?” (We tested it existence for IBM stock issues of American call and put option prices.) If there is such a situation, what kind of distribution it exhibits and in case of any distribution other than stable distribution, what kinds of new developments does the BS model cause? (Therefore, BS option pricing model can be used instead of derivative GARCH option pricing model. In the case of high or low initial conditioned variance, BS model makes lower or higher pricing, respectively, with respect to GARCH model.) All of these questions present us new and meaningful areas of study based on this study.

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ABBREVIATIONS LIST

AD: Anderson-Darling

BS: Black-Scholes

CBOE: Chicago Board Options Exchange

CBOT: Chicago Board of Trade

CME: Chicago Mercantile Exchange

D: Present value of the dividends

ECDF: Empirical Cumulative Density Function

EDF: Empirical Distribution Function

İMKB: İstanbul Mercantile Exchange

IMM: International Money Market

K: Strike Price

KS: Kolmogorov-Simirnov

NYSE: New York Stock Exchange

r: Risk-free interest rate

S: Stock Price

T: Time to Expiration