Physics Education

PAPER • OPEN ACCESS

Teaching large angle pendulum via Arduino based STEM education material

To cite this article: Mustafa Erol and Mehmet Oğur 2023 *Phys. Educ.* **58** 045001

OPEN ACCESS

PAPER

Phys. Educ. **58** (2023) 045001 (12pp) iopscience.org/ped

Teaching large angle pendulum via Arduino based STEM education material

Mustafa Erol[1,*] **and Mehmet Oğur**[2]

[1] Dokuz Eylül University, Education Faculty of Buca, Department of Mathematics and Science Education, Physics Education Division, Buca, İzmir, Turkey
[2] Dokuz Eylül University, Institute of Educational Studies, Physics Education Division, Buca, İzmir, Turkey

E-mail: mustafa.erol@deu.edu.tr

CrossMark

Abstract

This study reports an Arduino based STEM education material that resolves large angle pendulum, alternatively anharmonic oscillator, both experimentally and theoretically, particularly focuses on the time dependence of the instantaneous displacement angles. Instantaneous time dependence of the angles is experimentally measured by using Arduino Nano microprocessor connected to a gyro sensor and a bluetooth transmitter/receiver structured as a STEM education material. Theoretical resolution of the large angle pendulum is also accomplished by considering the approximate solution taking into account the relevant frictional effects. The theoretical and experimental resolutions are managed for the initial angles of $\theta_0 = 1.40$ rad and $\theta_0 = 2.61$ rad, specifically focusing on the angular frequencies, frictional effects and anharmonicity of the motion. The relative errors concerning the angular frequency are found to be % 15.2 and % 6.5 for $\theta_0 = 2.61$ rad and $\theta_0 = 1.40$ rad, respectively. The exponential decrease due to the frictional effects are in perfect agreement with each other and providing a friction coefficient of $\gamma = 0.015\,\mathrm{kg\ ms^{-1}}$ for both experimental and theoretical results. The anharmonicity of the pendulum is

* Author to whom any correspondence should be addressed.

also illustrated by comparing the experimental plots and relevant theoretical plots for the small angle approximations. This work offers a prototype teaching material which can create an education environment where students can feel dynamic and encompass the acquisitions of Science Technology Engineering and Mathematics.

Keywords: physics education, large angle pendulum, STEM education, Arduino, anharmonic oscillator

1. Introduction

Physics education research aims to determine the topics that students have trouble understanding and also aims to eliminate the learning difficulties and to raise students with the required competencies [1]. Hence, it is applicable to focus on the subjects that are recognised to be problematic and teach them based on up-to-date approaches. One of the subjects that students have difficulties in learning within the scope of advanced physics courses is large angle pendulum [2–5]. Harmonic motion stands within the scope of the undergraduate physics courses and clearly observed via the motions of simple pendulum, spring pendulum or physical pendulum within the regime of small angle approximation [6]. However, in the case of initial angles larger than 10° or even much larger ($\theta_0 \geqslant 10°$), the motion becomes nonharmonic and more complicated. There are a number of works in the literature focusing on the large angle pendulum both theoretically and experimentally. In order to mention some, for instance exact solution of a large angle pendulum is achieved newly and the work presents not only the exact formula for the period but also the exact expression of the angular displacement as a function of the time, the amplitude of oscillations and the angular frequency for small oscillations [7]. Similarly, the anharmonicity of a large amplitude pendulum was lately tackled and the effort developed a novel technique to detect the high Fourier components as a function of the amplitude for very large amplitudes [4]. Another current study of large-angle anharmonic oscillations of a physical pendulum using an acceleration sensor is managed by the damping of a bar acting as a physical pendulum subjected to air drag [5]. Likewise, an anharmonic solution to the differential equation describing the oscillations of a simple pendulum at large angles is also discussed recently. Experimentally the resolution of a physical pendulum with an Arduino microprocessor is also managed [8]. There are some works alternatively using smartphones and to mention one, a recent work has been carried out investigating the large angle approach of a physical pendulum [9].

Fundamental problem with teaching the large angle pendulum is that the deviation from the harmonic motion is not normally measurable and accordingly students have certain internalising difficulties. On the other hand, visualisation or experimental justification in teaching is very central in order to realising anticipated educational goals. Hence, it is promising to teach the physical laws if an appropriate application is engaged where related concepts can be observed and measured. One of the recent educational methods used for this purpose is the STEM education method. STEM education offers an interdisciplinary approach to instruction, bypassing the tradition of having separate lines for each field in old-style education. STEM education is based on an interdisciplinary education procedure in which the accomplishments of Science, Technology, Engineering and Mathematics fields take place together [10]. Arduino microcontrollers, on which the STEM based, are electronic devices which can be encoded with the C language and can easily be used for literally any purpose. Arduino microprocessors are employed in technology, engineering at almost all levels and indeed in teaching by basically collecting data from various sensors [11]. Essentially, in order to observe any physical law, a material based on the law and the required electronic connections are made, a data collection process is carried out and the data obtained is analysed with the help of appropriate tools. Thus, Arduino based STEM applications related to all subfields of physics education can easily be achieved. Many studies using Arduino based STEM education are available in the literature [12–14].

The aim of this study is to realise and analyse the motion of a large angle pendulum as an anharmonic oscillator both theoretically and experimentally by focusing on the time dependence of the displacement angle. The present effort especially focuses on the angular frequencies, frictional effects and anharmonicity of the swinging. This work is especially beneficial in terms of using low-cost tools and demanding elementary electronic knowledge in a STEM application.

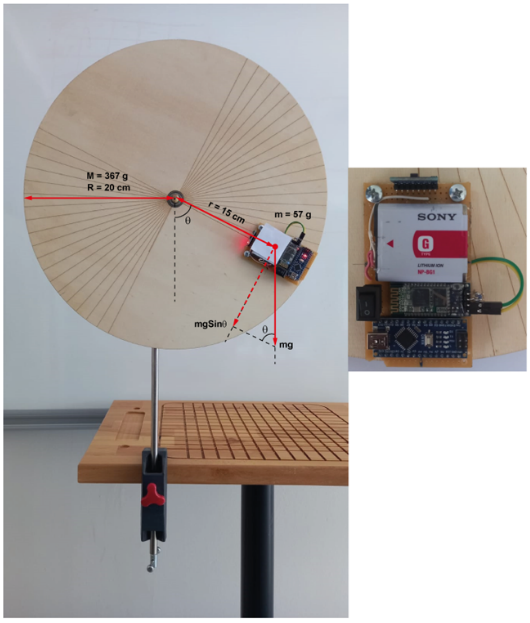

Figure 1. The photograph of the experimental setup which shows the disc, the Arduino on the disc, the radius of the disc and the distance of the Arduino to the centre. The small photo is presented to show the orientation of the gyro sensor.

2. Theory of large angle pendulum

The pendulum under consideration is slightly different compared to simple or physical pendulums in the sense that it is composed of a disc with a mass of M that can rotate around the central axis and a mass of m fixed on the disc as shown in figure 1. Therefore, the equation of the motion, by taking into account of the relevant frictions, can be given by,

$$I\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + r\,mg\sin\theta + r\gamma\frac{\mathrm{d}\theta}{\mathrm{d}t} = 0 \qquad (1)$$

where θ denotes the displacement angle with respect to the vertical, I denotes the inertial moment of the whole system, m is the mass of Arduino system, γ denotes the friction coefficient of the system and r denotes the distance of the mass m to the pivot. In the equation, overall moment of inertia is given by, $I = mr^2 + \frac{1}{2}MR^2$ where M denotes the mass of the disc and R denotes the radius of the disc. Unavoidable frictional effects between the swinging disc and the central axis and also between the disc and the air is defined by the friction force as, $F_f = \gamma\frac{\mathrm{d}\theta}{\mathrm{d}t}$.

In the first step of the solution, it is appropriate to disregard the frictional effects hence assume that the friction coefficient is zero ($\gamma = 0$) for simplicity. The solution of this equation is still not straightforward due to being non-linear by including the term of sin θ. However, it is standard activity to assume small angle approximation, in other words the initial displacement angle is less than $10°$ ($\theta_0 \leqslant 10°$), then the differential equation is linear and the motion is considered to be harmonic so one can use $\sin\theta = \theta$ approximation. To just mention, the exact solution of this equation, by ignoring the friction of the system ($\gamma = 0$), is recently managed by Beléndez *et al* [7] and the final expression was given by,

$$\theta(t) = 2\arcsin\left[\sin\frac{\theta_0}{2}\, sn\left[K\left(\sin^2\frac{\theta_0}{2}\right) - \omega_0 t; \sin^2\frac{\theta_0}{2}\right]\right] \qquad (2)$$

where K denotes complete elliptic integral, $sn\left[K\sin^2\frac{\theta_0}{2} - \omega_0 t; \sin^2\frac{\theta_0}{2}\right]$ denotes the Jacobi elliptic function, θ_0 denotes the initial displacement angle and ω_0 denotes the angular frequency of the pendulum within the small angle regime that is $\omega_0 = \sqrt{\frac{r\,mg}{I}}$. This expression is obviously complicated and beyond the educational aims, so the approximate solution of the system is more convenient for undergraduate teaching purposes.

The approximate solution of the differential equation is straightforward especially if one ignores the frictional effects that means($\gamma = 0$). The approximate solution of the differential equation was managed by Gil *et al* [4] through basically employing the well-known Taylor approximation that is $\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} \ldots$. Accordingly, the approximate solution of the displacement angle is given as a function of time by,

$$\theta(t) \cong \theta_0 \cos\omega t + \frac{\theta_0^3}{192}(\cos\omega t - \cos 3\omega t) + \frac{\theta_0^5}{512} \times \left(\frac{17}{20}\cos\omega t - \frac{1}{6}\cos 3\omega t + \frac{1}{40}\cos 5\omega t\right) + \cdots \tag{3}$$

where ${\omega_0}^2 = \omega^2\left(1 + \frac{\theta_0^2}{8} + \frac{17\theta_0^4}{1536} + \cdots\right)$ and $\omega_0 = \sqrt{\frac{r\,mg}{I}}$ is the small angle angular frequency of the pendulum.

To obtain the full solution one ought to solve the equation of motion without ignoring the frictional effects. Hence, if one solves the equation (1) including the friction force, $F_f = \gamma\frac{\mathrm{d}\theta}{\mathrm{d}t}$, it is reasonably straightforward to see that the friction force influences the general solution by a factor and exponentially reduces the amplitude of the oscillation. Consequently, approximate solution of the equation of motion including the frictions is, in terms of the approximate solution $\theta(t)$, given by,

$$\theta_f(t) \cong \theta(t)\ \mathrm{e}^{-\gamma\frac{r}{2I}t}. \tag{4}$$

In this general solution, $\theta_f(t)$ denotes the displacement angle taking into account of the friction by including the friction coefficient γ and $\theta(t)$ is the approximate value of the displacement angle excluding the friction given by the equation (3). In the derivation which is based on the equation (1), the friction coefficient assumed to be invariant. However, in reality the friction coefficient can obviously be varying due to the physical conditions, environmental effects and more predominantly due to the change in the angular velocity of the pendulum. This final expression approximately describes anharmonic and damped oscillations of the large angle pendulum which can be employed for teaching activities.

Theoretical resolution of the apparatus which consists of a uniform disc and a mass mounted on the disc, the Arduino system, is carried out in the theory part of this work. The equation of the motion is given by the equation (1) and to solve this equation concerning the large angle swinging together with the frictional effects no small angle approximation is considered. Exact solution of the equation is managed by a number of efforts and the work by Beléndez *et al* [7] is mentioned above, however the exact solution, expressed by Jacobi elliptic functions, is considered substantially hard and beyond teaching purposes. Therefore, the present effort considers the approximate solution managed by Gil *et al* [4] and given by the equation (3). Specifically, in the experimental setup employed, the mass of the Arduino Nano system is m = 57.0 g, the distance of the centre of the mass m to the pivot is r = 15.0 cm, the mass of the wooden disc is M = 367.0 g and the radius of the disc is R = 20.0 cm. Using these values, the inertial moment of the whole system is calculated as, $I = mr^2 + \frac{1}{2}MR^2 = 8.62 \times 10^{-3}$ kg m^2. The angular frequency of the pendulum within the small angle regime ($\sin\theta \cong \theta$) is then calculated as, $\omega_0 = \sqrt{\frac{r\,m\,g}{I}} = 3.12\ \frac{\mathrm{rad}}{\mathrm{s}}$. The large angle calculations were carried out for two different large angle initial values, namely $\theta_0 =$ 2.61 rad ($\theta_0 = 150°$) and $\theta_0 =$ 1.40 rad ($\theta_0 = 80°$). The specific angular frequencies for those initial angles are calculated from the equation of ${\omega_0}^2 = \omega^2\left(1 + \frac{\theta_0^2}{8} + \frac{17\theta_0^4}{1536} + \cdots\right)$ and found as, $\omega = 2.04\ \frac{\mathrm{rad}}{\mathrm{s}}$ and $\omega = 2.75\frac{\mathrm{rad}}{\mathrm{s}}$, respectively.

3. Experimental details

In order to perform the experiments, a uniform wooden disc with a radius of R = 20.0 cm and a mass of M = 367.0 g, has been prepared and mounted to a vertical stand such that the disc can rotate or swing around the central axis almost freely (see figure 1). The Arduino Nano system with required equipment, having a total mass of m = 57.0 g and a distance to the centre of r = 15.0 cm, was fixed to the disc such that

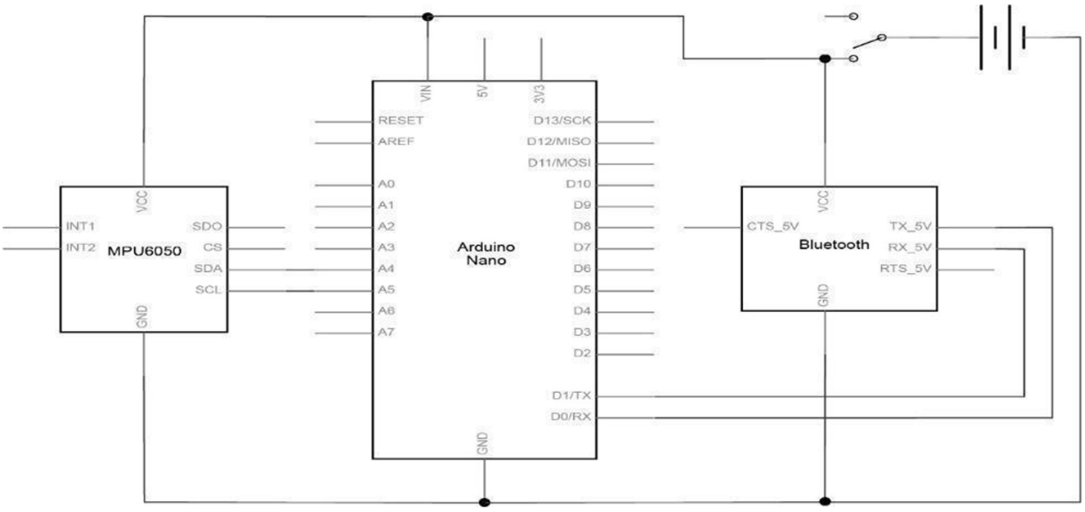

Figure 2. Schematic diagram for the electronic connections relating the whole data collection system, including the Arduino Nano, the gyro sensor, the bluetooth transmitter/receiver and the 4.7 V battery.

the whole system can oscillate by means of the mass of the Arduino. The gyro sensor is positioned and fixed to the Arduino to measure the displacement angle of the disc, θ_f, with respect to the vertical. The data collected by the gyro sensor is then transferred to the computer by means of the bluetooth transmitter/receiver, elegantly preventing any disturbance of the motion of the disc due to any wiring connections. In order to run the experiments, the wooden disc displaced by a large angle of, θ_0, with respect to the vertical and released so that it can freely move and oscillate, allowing the Arduino gyro sensor to collect the angle data ten times in a second. The photograph of the experimental setup is given in figure 1.

The Arduino based measurement system is designed to directly measure the instantaneous displacement angles of a large angle pendulum as a function of time. Hence, the measurement system mainly consists of an Arduino Nano microcontroller, a MPU6050 sensor, a HC05 bluetooth transmitter/receiver, a 4.7 V battery to power the Arduino and finally an ordinary computer. Electronic connections concerning the whole data collection system which comprises the Arduino Nano microprocessor, the gyro sensor, the bluetooth transmitter/receiver and the battery, is given in the figure 2. The measurement system is organised such that the gyro sensor collects 10 data every second, transfers the data to the Arduino Nano, the Arduino Nano then transfers the data to the computer via the bluetooth transmitter/receiver. The data of displacement angle obtained by the computer is then plotted as a function of time by means of the Excel.

The Arduino Nano microprocessor can obviously be used for countless applications by an appropriate code [15]. The code used in the study was written such that the change of the instant displacement angle values can be recorded as a function of time such that ten data every second can be collected. It is essential to express that the bluetooth transmitter/receiver, the Arduino Nano and the computer can communicate wirelessly hence the effects of cable connections on the motion is totally eliminated. It is also crucial to disconnect the data cable between the bluetooth transmitter/receiver and the Arduino Nano during the code loading to prevent any faulty data transfer. The code written and used to run the Arduino Nano microprocessor is given below.

```
#include <Wire.h>
#include <MPU6050.h>
MPU6050 mpu;
void setup()
{
  Serial.begin(9600);
```

```
  while (!mpu.begin(MPU6050_SCALE_
  2000DPS, MPU6050_RANGE_2 G))
  {
    Serial.println('Could not find a valid
    MPU6050 sensor, check wiring!');
    delay(500);
  }
}
void loop()
{
  Vector normAccel = mpu.readNormalize
    Accel();
  float roll = atan2(normAccel.YAxis,
    normAccel.ZAxis);
  Serial.println(roll, 4);
  delay(100);
}
```

The MPU6050 sensor, employed in this work, is a multipurpose instrument that has both accelerometer and gyroscope options. Nevertheless, the accelerometer option is only engaged throughout the experimental measurements. The code prepared for the Arduino Nano is obtained from the 'MPU6050.h' library however a portion of the code which allows to collect data only on accelerometer measurements is obtained. In addition, the code obtained from the library was originally written to collect instantaneous acceleration data for three-dimensional motion, though the code is transformed in order to collect data for two-dimensionl motion. The code is moreover modified to transform the acceleration data into the instantaneous displacement angle data by using the relevant trigonometric equation which can be noticed within the code. Finally, the code is also modified to transform the data in degrees into the data in radians.

4. Results and discussions

4.1. Theoretical resolution

The theoretical graph of the displacement angle as a function of time is, concerning $\theta_0 = 2.61$ rad, plotted in accordance with the equation (3) and presented in the figure 3.

The theoretical graph can also be used to estimate the exponential decrease in the amplitude of the oscillations due to the frictional effects. The standard curve fitting to the exponential decrease straightforwardly gives the equation of $\theta_f = 2.8\ \mathrm{e}^{-0.015\,t}$ leading to a friction coefficient of $\gamma = 0.015\,\mathrm{kg\ ms}^{-1}$. This exponential decrease is due to the existence of frictional effects due both to air and to pivot connection which are theoretically expressed in terms of friction force $F_f = \gamma \frac{\mathrm{d}\theta}{\mathrm{d}t}$. In order to check the angular frequency, it is possible to count the number of periods within a certain time interval and calculate the actual value. In this sense, there are 13 periods within a time interval of 40 s, which gives the mean period as, $T = 3.077$ s. Then the angular frequency can be calculated directly from the graph as, $\omega = 2.04\,\frac{\mathrm{rad}}{\mathrm{s}}$ which is same with the theoretical value of $\omega = 2.04\,\frac{\mathrm{rad}}{\mathrm{s}}$.

Similar procedure can obviously be carried out for other experimentally proposed angle which is $\theta_0 = 1.40$ rad. The theoretical plot of displacement angle, $\theta_f(t)$ for the initial displacement angle of $\theta_0 = 80°$ or $\theta_0 = 1.40$ rad is given in figure 4. The curve fit for the exponential decrease of the amplitude of the oscillations gives

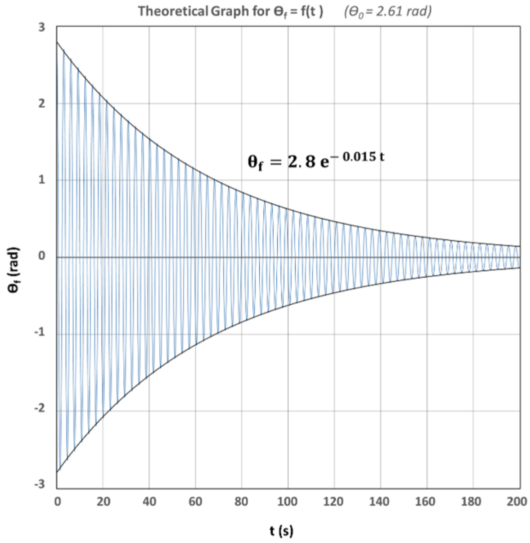

Figure 3. The graph of the oscillations of the large angle pendulum with the initial angle of $\theta_0 = 2.61$ rad and with a corresponding angular frequency of $\omega = 2.04\,\frac{\mathrm{rad}}{\mathrm{s}}$. The mathematical curve fit equation, $\theta_f = 2.80\ \mathrm{e}^{-0.015\,t}$, is produced by GEOGEBRA in terms of θ_f and t.

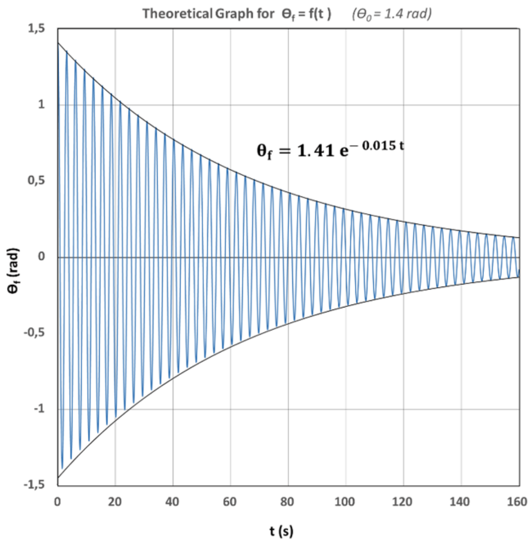

Figure 4. The graph of the displacement angle as a function of time for the large angle pendulum with the initial angle of $\theta_0 =\ 1.40$ rad and with an angular frequency of $\omega = 2.75\,\frac{\text{rad}}{\text{s}}$. The mathematical curve fit equation, $\theta_f = 1.41\ \text{e}^{-\ 0.015\,t}$, is produced by GEOGEBRA.

the equation of $\theta_f = 1.4\ \text{e}^{-0.015\,t}$ leading to a friction coefficient of $\gamma = 0.015\,\text{kg ms}^{-1}$, in full agreement with the previous plot. The theoretical angular frequency for this case is calculated to be $\omega = 2.75\frac{\text{rad}}{\text{s}}$. In this case, there are 35 periods within the time interval of the 80 s, which gives the mean period as, $T = 2.286$ s. Then the angular frequency can be calculated directly from the graph as, $\omega = 2.75\frac{\text{rad}}{\text{s}}$ which is same with the theoretical value of ω.

4.2. Experimental resolution

In order to experimentally resolve the large angle pendulum, the theoretical cases discussed in the previous section are repeated this time by real time measurements. To do so, the pendulum, which is a uniform disc, is displaced by a large angle and released to start swinging. During the motion, the gyro sensor obviously collects the data, specifically ten data in every second, and transfers that data to the bluetooth transmitter/receiver which then transmits the data to the computer to plot the real time graphs. To compare and see the agreement between the theoretical resolution and the experiment, the experiment was run twice for two different initial angles, namely of $\theta_0 =\ 1.40$ rad and of $\theta_0 =\ 2.61$ rad.

Initially, the displacement angle of the disc, θ_f, is plotted as a function of time, t, for the initial angle of $\theta_0 =\ 2.61$ rad and the plot is given in figure 5.

There are a number of points that ought to be underlined concerning teaching activities. Firstly, the experimental plot is smooth, detailed and perfect owing to the Arduino Nano data collection system. It is quite clear that the experimental plot is in a good agreement with the theoretical counterpart in figure 3. The angular frequency for the experimental data can obviously be estimated by counting the number of periods within a certain time interval. Doing so, typically gives 15 periods within 40 s, leading experimentally to an angular frequency of $\omega = 2.35\frac{\text{rad}}{\text{s}}$. For this case, the theoretical angular frequency was $\omega = 2.04\frac{\text{rad}}{\text{s}}$. Hence the relative experimental error can be estimated as % 15.2 which is reasonable.

The motion of the pendulum is, in reality, suppressed by the frictional effects mainly because of the frictional effects at the pivot and air friction and therefore the motion is damped

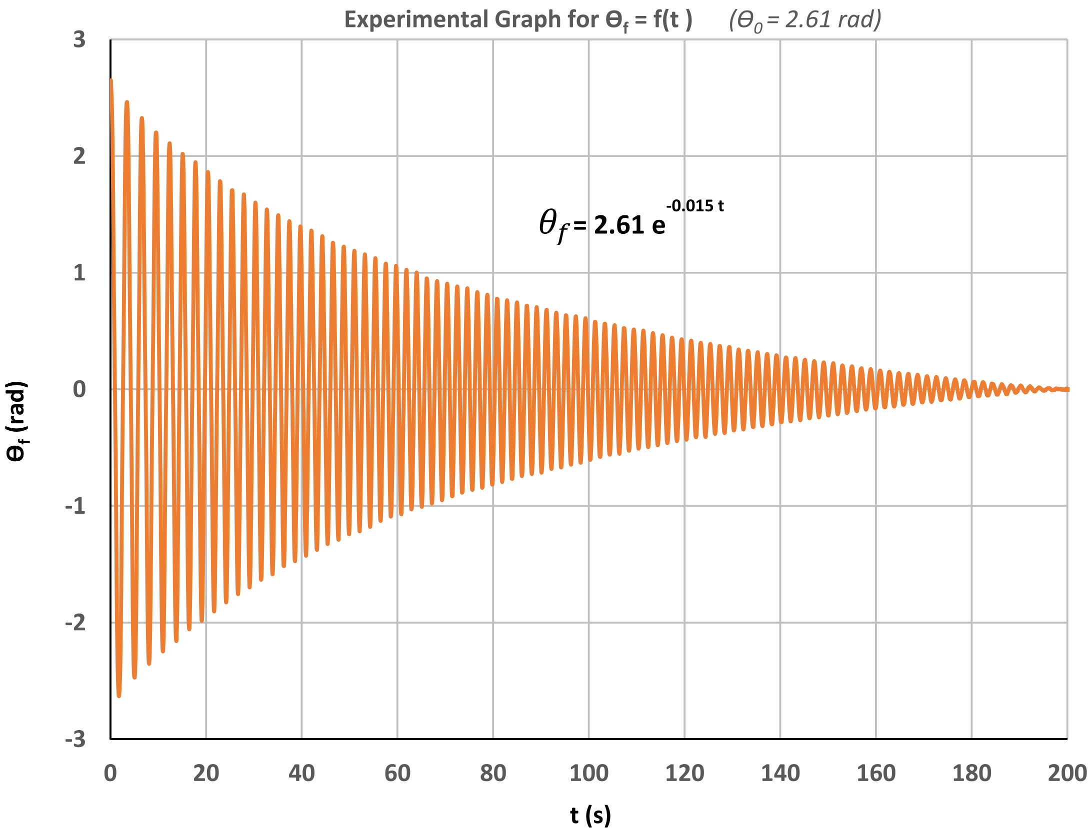

Figure 5. The experimental plot of the displacement angle as a function of time for the large angle pendulum with the initial angle of $\theta_0 =\ 2.61$ rad. Performing curve fit for the exponential decrease of the amplitude leads to the equation of $\theta_f =\ 2.61\ \mathrm{e}^{-0.015\,t}$.

exponentially. In order to quantify the overall frictional effects, the decrease at the amplitude is curve fitted and the mathematical equation is found to be,$\theta_f =\ 2.61\ \mathrm{e}^{-0.015\,t}$ which leads to a friction coefficient of $\gamma = 0.015\,\mathrm{kg\ ms}^{-1}$ and is in perfect agreement with the theoretical estimation. It is obviously notable that the exponential decrease in the amplitude of the oscillation changes character and speeds up at about $t = 140$ s and $\theta\ =\ 0.4$ rad (22.9°) hence the damping increases. This increase is attributed to a somehow change in the frictional effects most likely at the pivot.

The same measurement is repeated this time for $\theta_0 =\ 1.40$ rad which in degrees corresponds to 80°. Figure 6 shows the experimental graph of the displacement angle as a function if time for the initial angle of $\theta_0 = 1.40$ rad.

In this case, the measurement of periods within a certain time interval gives 28 periods within 60 s, leading experimentally to an angular frequency of $\omega = 2.93\,\frac{\mathrm{rad}}{\mathrm{s}}$. The theoretical angular frequency was $\omega = 2.75\,\frac{\mathrm{rad}}{\mathrm{s}}$, hence the relative experimental error can be calculated as % 6.5 which is pretty small. The decrease at the amplitude is similarly curve fitted to estimate the friction coefficient and found same with the theoretical one, that is $\theta_f =\ 1.41\ \mathrm{e}^{-0.015\,t}$. Using this equation the friction coefficient is again estimated to be $\gamma = 0.015\,\mathrm{kg\ ms}^{-1}$ which is in perfect agreement with the theoretical estimation. This in fact shows the consistency in measurements and the frictional influences on the motion of the disc.

The character change in the exponential decrease is similarly detected for this

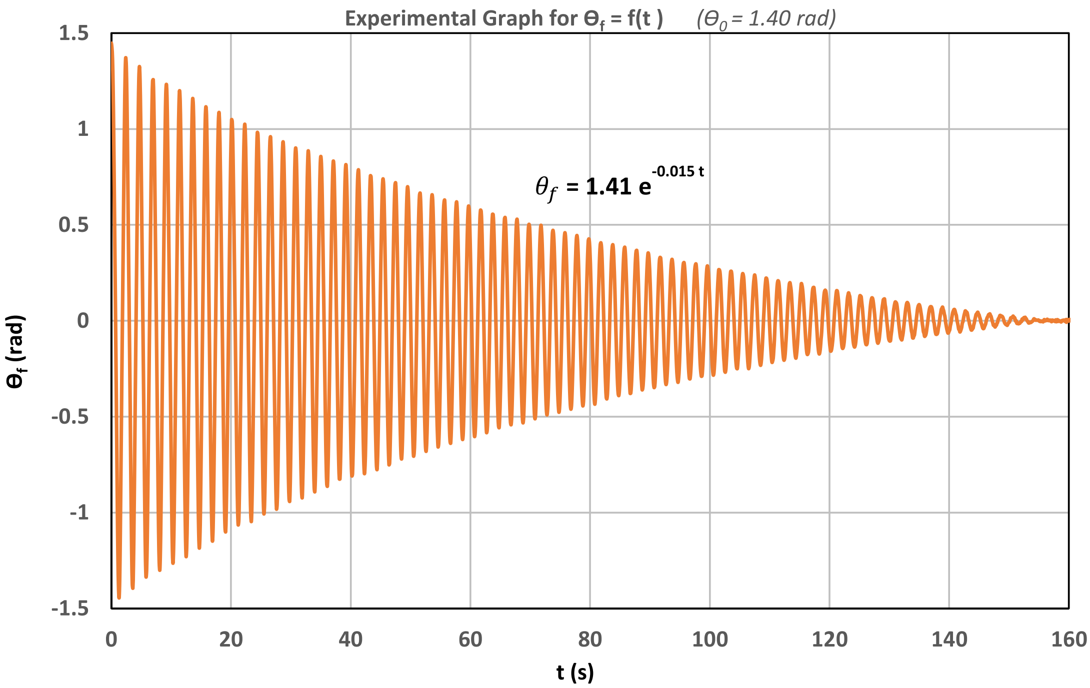

Figure 6. The experimental plot of the displacement angle as a function of time for the large angle pendulum with the initial angle of $\theta_0 =$ 1.40 rad. Performing curve fit for the exponential decrease of the amplitude leads to the equation of $\theta_f =$ 1.41 $\mathrm{e}^{-0.015\,t}$.

measurement, however this time at about $t = 80$ s. Interestingly, the corresponding angle is about the same with the previous situation and is around $\theta = 0.4$ rad (22.9°). Consequently, it is legitimate to think that at about $\theta = 0.4$ rad and below, at small angles and small angular velocities, the friction coefficient changes character and increases.

In order to see the apparent change concerning the frictional effects and hence friction coefficient of the pendulum, semi logarithmic graph between the displacement angle ($\ln \theta_f$) and time (t) is plotted for both theoretical and experimental resolutions. The graphs are shown in figures 7 and 8 for the initial angles of $\theta_0 =$ 1.40 rad and $\theta_0 =$ 2.61 rad, respectively.

The semi logarithmic graphs clearly demonstrate the character change of the friction coefficient as the pendulum moves into smaller displacement angles by the time. The point at which the change of the friction coefficient initiates can approximately be estimated from the plots as $t = 90$ s for $\theta_0 =$ 1.40 rad and $t = 135$ s for $\theta_0 =$ 2.61 rad. The corresponding displacement angles can also be determined from the experimental graphs of the figures 5 and 6 and doing so leads to displacement angles of $\theta_f = 0.3534$ rad for $\theta_0 =$ 1.40 rad plot and $\theta_f =$ 0.3038 rad for $\theta_0 =$ 2.61 rad plot.

4.3. *Anharmonicity of the pendulum*

The harmonicity or anharmonicity of a pendulum is an important issue and ought to be clarified and instructed properly concerning the undergraduate physics courses. However, almost all attempts simplifies the equation of motion that is (1) by only considering the small angle approximations ($\theta_0 \leqslant 10°$). The small angle approximation assumes $\sin\theta \cong \theta$ which leads to a linear differential equation and accordingly to only the harmonic motion. Anharmonic motion in this sense is generally left out of the scope of teaching efforts and therefore the vast majority of students are unfortunately unaware of the anharmonicity. Therefore,

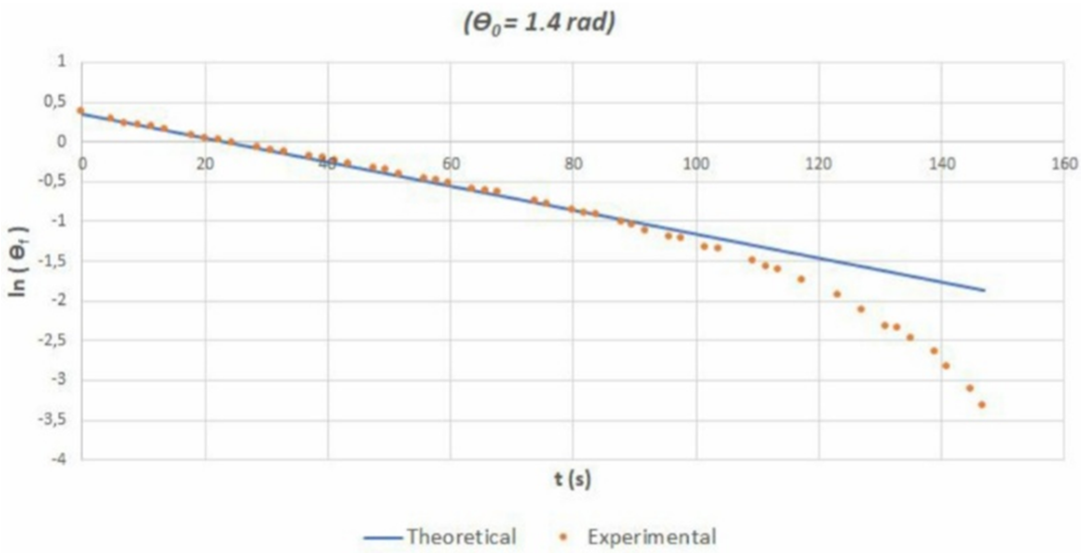

Figure 7. Semi logarithmic graph between the displacement angle ($\ln\theta_f$) and time (t) is plotted for both theoretical and experimental resolutions, concerning the initial angle of $\theta_0 = 1.40$ rad.

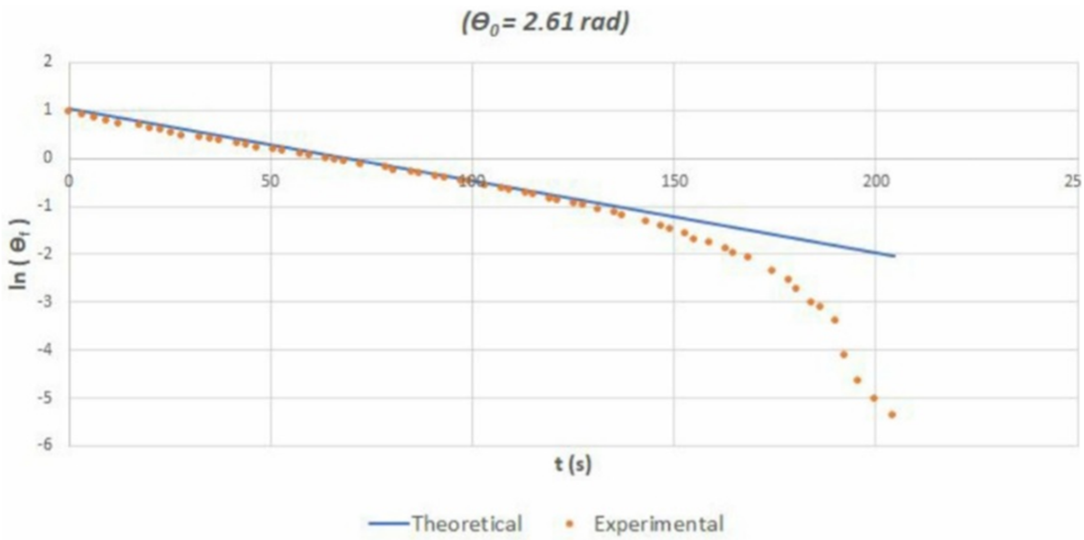

Figure 8. Semi logarithmic graph between the displacement angle ($\ln\theta_f$) and time (t) is plotted for both theoretical and experimental resolutions, concerning the initial angle of $\theta_0 = 2.61$ rad.

this effort specifically focuses on the anharmonicity and tries to clarify the difference. In order to underline the anharmonicity, both harmonic and anharmonic motions are experimentally plotted together in a single graph given in figure 9.

The dotted line shows the cosine function to represent the pure simple harmonic motion describing the small angle approximation. The solid line represents the real-time experimental plot for the large angles specifically for $\theta_0 = 2.80$ rad showing the effect of anharmonicity for one period. The deflection from the pure and original cosine function is clearly detected by means of the Arduino Nano measurement system without giving any complicated details of the theoretical resolution.

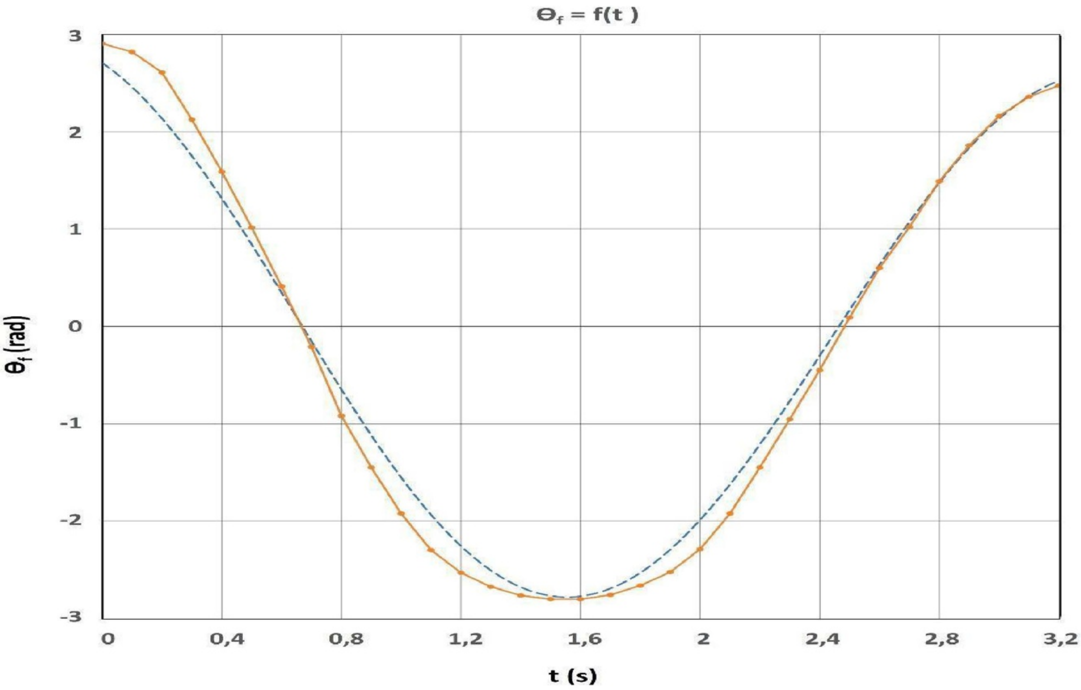

Figure 9. The displacement angle is plotted as a function of time experimentally for the large angle approximation and for small angle approximation.

5. Conclusions

This work has mainly aimed to experimentally and theoretically analyse the motion of a large angle pendulum for teaching purposes by focusing on the angular frequency, the frictional effects and the anharmonicity. The analysis of the pendulum has been achieved by taking the data of displacement angles as a function of time. In the data analysis process, experimentally measured θ_f values are compared with the theoretical expression of $\theta_f(t) \cong \theta(t)\ \mathrm{e}^{-\gamma \frac{r}{2I} t}$. The theoretical and experimental resolutions were carried out for the initial angles of $\theta_0 = 1.40$ rad and $\theta_0 = 2.61$ rad. The relative errors concerning the theoretical results and experimental measurements are % 15.2 and % 6.5 for $\theta_0 = 2.61$ rad and $\theta_0 = 1.40$ rad, respectively. The exponential decrease due to the frictional effects are experimentally found to be $\theta_f = 2.61\ \mathrm{e}^{-0.015\,t}$ and $\theta_f = 1.41\ \mathrm{e}^{-0.015\,t}$ for $\theta_0 = 2.61$ rad and $\theta_0 = 1.40$ rad, respectively. These results are in perfect agreement with each other and providing a friction coefficient of $\gamma = 0.015\,\mathrm{kg\ ms^{-1}}$ for both experimental and theoretical results. The anharmonicity of the pendulum is also clearly revealed by comparing the experimental plot and relevant theoretical plot for the small angle approximation.

The materials used in the study are low-cost and accessible materials. For this reason, teaching activities can easily be included in the physics education which can develop technological literacy skills for programming in schools with limited opportunities. The approach developed and reported is an alternative scheme that has not been trialled on students or educators yet, however it is highly promising and can easily be carried out. The procedure has been prepared in such a way, it can be applied to the students from the material development step to the data analysis stage. The material reported here could be suitable for the educational systems that aim 21st century skills via the STEM education approach [16].

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

ORCID iDs

Mustafa Erol https://orcid.org/0000-0003-1022-4975
Mehmet Oğur https://orcid.org/0000-0002-7751-2245

Received 21 January 2023, in final form 25 March 2023
Accepted for publication 20 April 2023
https://doi.org/10.1088/1361-6552/accef4

References

[1] Soh T M T, Arsad N M and Osman K 2010 The relationship of 21st century skills on students' attitude and perception towards physics *Proc. Soc. Behav. Sci.* **7** 546–54
[2] Beléndez A, Rodes J J, Beléndez T and Hernández A 2009 Approximation for a large-angle simple pendulum period *Eur. J. Phys.* **30** L25
[3] Johannessen K 2010 An approximate solution to the equation of motion for large-angle oscillations of the simple pendulum with initial velocity *Eur. J. Phys.* **31** 511
[4] Gil S, Legarreta A E and Di Gregorio D E 2008 Measuring anharmonicity in a large amplitude pendulum *Am. J. Phys.* **76** 843–7
[5] Fernandes J C, Sebastião P J, Gonçalves L N and Ferraz A 2017 Study of large-angle anharmonic oscillations of a physical pendulum using an acceleration sensor *Eur. J. Phys.* **38** 045004
[6] Sears F W, Zemansky M W and Young H D 1987 *University Physics* 7th edn (Reading, MA: Addison-Wesley Company)
[7] Beléndez A, Pascual C, Méndez D I, Beléndez T and Neipp C 2007 Exact solution for the nonlinear pendulum *Rev. Bras. Ensino Fis.* **29** 645–8
[8] Lukovic M, Lukovic V, Bozic M and Vujicic V 2021 Inexpensive physical pendulum with Arduino *Phys. Teach.* **59** 432–5
[9] Chatchawaltheerat T, Khemmani S and Puttharugsa C 2021 Investigating the large angle of a physical pendulum using a smartphone's sensors *Phys. Educ.* **56** 045023
[10] Stohlmann M, Moore T J and Roehrig G H 2012 Considerations for teaching integrated STEM education *J. Pre-Coll. Eng. Educ. Res.* **2** 4
[11] Jamieson P 2011 Arduino for teaching embedded systems. Are computer scientists and engineering educators missing the boat? *Proc. Int. Conf. on Frontiers in Education: Computer Science and Computer Engineering (FECS)* (The Steering Committee of the World Congress in Computer Science, Computer Engineering and Applied Computing (WorldComp)) p 1
[12] Çoban A and Erol M 2021 Arduino-based STEM education material: work-energy theorem *Phys. Educ.* **56** 023008
[13] Çoban A and Erol M 2020 Validation of Newton's second law using Arduino: STEM teaching material *Phys. Educ.* **56** 013004
[14] Çoban A and Erol M 2021 Teaching impulse-momentum law by Arduino based STEM education material *Phys. Educ.* **3** 2150006
[15] Petry C A, Pacheco F S, Lohmann D, Correa G A and Moura P 2016 Project teaching beyond Physics: integrating Arduino to the laboratory *2016 Technologies Applied to Electronics Teaching (TAEE)* pp 1–6
[16] Grayson D J 2020 Physics education for 21st century graduates *J. Phys.: Conf. Ser.* **1512** 012043

Prof. Dr Mustafa EROL-CV Mustafa Erol graduated from the Department of Physics of Ege University, in 1986. After serving about a year as a research assistant at the department, he was awarded a scholarship to study MSc and PhD degrees. He completed his PhD at Lancaster University on Condensed Matter Physics, specifically on the Electron-Phonon Interactions in Low Dimensional Structures. He started to work in the Education Faculty of Buca, Dokuz Eylül University, he became assistant professor in 1993, associate Professor in 1995 and full time Professor in 2001. Through the years he published some 102 scientific papers, 1 book, 3 book chapters and he achieved about 102 oral or poster presentations. In addition, he was invited a number of times as invited speakers in a variety of conferences. Through the years, his research interests range from Physics Education to Foundations of Quantum Physics and Quantum Consciousness. Mustafa Erol held the presidency of the İzmir branch of the Turkish Physical Society for 14 years between 2002 and 2016.