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THERMAL ELASTIC-PLASTIC STRESS ANALYSIS OF COMPOSITE LAMINATED PLATES

by

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THERMAL ELASTIC-PLASTIC STRESS ANALYSIS OF COMPOSITE LAMINATED PLATES

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We certify that we have read the thesis, entitled **"THERMAL ELASTIC-PLASTIC STRESS ANALYSIS OF COMPOSITE LAMINATED PLATES"** completed by **Mehmet ŞENEL** under supervision of **Prof. Dr. Onur SAYMAN** and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy.

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ABSTRACT

Composite materials are those formed by combining more than one bonded material, each with different structural properties. Since composite materials have the ability to tailor many properties, such as a high ratio of stiffness to weight, strength, thermal properties, corrosion resistance, wear resistance, fatigue life, they have been extensively used in many structural applications. Therefore, many manufacturing techniques have been developed recently. The aim is usually to obtain stronger, stiffer materials with lower density.

Metal fiber-metal matrix plates consist of a low ductile, usually low strong metal matrix and elastic or ductile and strong metal fibres. The strength and the stiffness of the fiber and the ductility of the matrix provide a new material with superior properties.

In this study, thermal elastic-plastic stress analysis is carried out on simply supported antisymmetric cross-ply $[0^0/90^0]_2$ and angle-ply $[30^0/-30^0]_2$, $[45^0/-45^0]_2$, $[60^0/-60^0]_2$, $[15^0/-15^0]_2$, $[15^0/-30^0]_2$, $[15^0/-45^0]_2$, $[15^0/-60^0]_2$ aluminium metal-matrix laminated plates under the constant temperature change through the thickness of plates. The plates are composed of four orthotropic layers bonded antisymmetrically. In the solutions, a special computer program has been employed. Plastic and residual stresses are determined in the antisymmetric cross-ply and angle-ply laminated plates for small deformations. In the solution of the elastic part, classical laminated theory has been used. For the elastic-plastic solution, the Tsai-Hill Theory is used a yield criterion. Cross-ply and angle-ply laminated plates are evaluated for elastic-perfectly plastic and lineer strain hardening material properties. For those laminated plates, residual stresses and plastic flow are obtained by using incremental stress method. Additionally, it is assumed that the lamina properties do not change over the temperature range. Finally, results obtained are presented in tables.

ÖZET

Kompozit malzemeler, her biri farklı özelliklere sahip birden fazla malzemenin bir araya getirilerek şekillenmesiyle oluşan malzemelerdir. Kompozit malzemeler ağırlık, mukavemet, yüksek sertlik oranı, ısıl özellikler, korozyon direnci, sürtünme direnci, yorulma ömrü gibi üstün özelliklere sahip olduklarından birçok yapısal uygulamalarda yaygın olarak kullanılırlar. Bundan dolayı şimdiye kadar birçok üretim teknikleri geliştirilmiştir. Burada amaç genellikle düşük yoğunluk ile mukavim malzeme elde etmektir.

Metal fiber-metal matris plaklar yumuşak matris ve sert, mukavemetli fiberlerden oluşurlar. Fiberin mukavemet ve sertliği ile matrisin hafifliği yeni bir süper özellikli malzemenin oluşmasını sağlamaktadır.

Bu çalışmada, plak kalınlığı boyunca sabit sıcaklık değişimi altındaki basit destekli antisimetrik çapraz takviyeli $[0^0/90^0]_2$ ve açılı takviyeli $[30^0/-30^0]_2$, $[45^0/-45^0]_2$, $[60^0/-60^0]_2$, $[15^0/-15^0]_2$, $[15^0/-30^0]_2$, $[15^0/-45^0]_2$, $[15^0/-60^0]_2$ alüminyum metal-matrisli tabakalı kompozit plakların ısıl elastik-plastik gerilme analizi üzerine bir çalışma yapılmıştır. Plaklar antisimetrik şekilde dört tabakalı olarak birleştirilmişlerdir. Çözüm için özel bir bilgisayar programı geliştirilmiştir. Plastik ve artık gerilmeler çapraz takviyeli ve açılı takviyeli antisimetrik plaklarda küçük deformasyonlar için belirlenmiştir. Elastik kısmın çözümünde klasik tabaka teorisi kullanılmıştır. Elastik-plastik çözüm içinde Tsai-Hill akma kriteri kullanılmıştır. Çapraz takviyeli ve açılı takviyeli tabakalı plakların ideal plastik ve lineer sertleşebilen malzeme özelliklerine sahip oldukları kabul edilmiştir. Plastik akıma ve artık gerilmeler gerilme artırma metodu kullanılarak bulunmuştur. Ayrıca tabaka özelliklerinin sıcaklık artışı ile değişmediği kabul edilmiştir. En sonunda bulunan sonuçlar tablolarda gösterilmiştir.

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NOMENCLATURE

u_0, v_0, w : the displacements in the x, yand z directions.

$\varepsilon_1, \varepsilon_2, \varepsilon_3$: the principle strains in the principal 1, 2 and 3 directions.

$\varepsilon_x, \varepsilon_y, \varepsilon_z$: the strains in the x, y and z directions.

$\varepsilon_1^T, \varepsilon_2^T, \varepsilon_3^T$: the thermal principle strains in the principal 1, 2 and 3 directions.

$\varepsilon_x^T, \varepsilon_y^T, \varepsilon_z^T$: the thermal strains in the x, y and z directions.

$\varepsilon_x^M, \varepsilon_y^M, \varepsilon_z^M$: the mechanical strains in the x, y and z directions.

$\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0$: the strain components on the reference plane.

$d\varepsilon_1, d\varepsilon_2, d\varepsilon_3$: the total strain increments in the principal directions.

$d\varepsilon_x, d\varepsilon_y, d\varepsilon_z$: the total strain increments in the x, y and z directions.

$\Delta\varepsilon_x, \Delta\varepsilon_y, \Delta\varepsilon_z$: differential form of strain increments in the x, y and z directions.

ε_p : the equivalent plastic strain.

K : the plastic constant.

$\sigma_1, \sigma_2, \sigma_3$: the principle stresses in the principal 1, 2 and 3 directions.

$\sigma_x, \sigma_y, \sigma_z$: the transformed stresses in the x, y and z directions.

$\bar{\sigma}, \sigma_{eq}$: the equivalent stresses.

σ_Y, σ_0 : the yield stresses.

$\tau_{12}, \tau_{13}, \tau_{23}$: the principle shear stresses in the principal 1, 2 and 3 directions.

$\tau_{xy}, \tau_{xz}, \tau_{yz}$: the transformed shear stresses in the x, y and z directions.

$\kappa_x, \kappa_y, \kappa_z$: the curvatures in the x, y and z directions.

N_x^T, N_y^T : the thermal resultant forces for a laminate.

N_{xy}^T : the thermal resultant shear force for a laminate.

M_x^T, M_y^T	: the thermal resultant bending moments for a laminate.
M_{xy}^T	: the thermal resultant twisting moment for a laminate.
X	: the longitudinal tensile or compressive strength.
Y	: the transverse tensile or compressive strength.
S	: the shear strength.
ΔT	: uniform change temperature.
$[T^*],[T^{**}]$	: the thermal force and moment resultant matrix.
$[Q_{ij}]$	: the lamina stiffnesses matrix for principal directions.
$[\overline{Q}_{ij}]$	: the lamina transformed stiffnesses matrix.
A_{ij}	: the laminate extensional stiffnesses.
B_{ij}	: the laminate coupling stiffnesses.
D_{ij}	: the laminate bending or flexural stiffnesses.
S_{ij}	: the principal lamina compliances.
α_1, α_2	: the principal coefficients of thermal expansion.
$\alpha_x, \alpha_y, \alpha_{xy}$	: the transformed coefficients of thermal expansion.
α_f, α_m	: the fiber and matrix coefficients of thermal expansion.
E_1, E_2	: the composite modulus first and second material directions.
E_f, E_m	: the fiber and matrix moduli.
υ_1, υ_2	: Poisson's ratios.
υ_f, υ_m	: Poisson's ratios of fiber and matrix.
$\sigma_1^{(i)}, \sigma_2^{(i)}, \tau_{12}^{(i)}$	: the total principal stresses at *i* th step.
$\sigma_x^p, \sigma_y^p, \tau_{xy}^p$	: the plastic stress components.
$\sigma_x^e, \sigma_y^e, \tau_{xy}^e$	: the stress components during the releasing of external forces.
$\sigma_x^r, \sigma_y^r, \tau_{xy}^r$	: the residual stress components.

CHAPTER 1
INTRODUCTION

1. 1. Introduction

Composite materials are those formed by combining two or more materials on a macroscopic scale such that have better engineering properties than the conventional materials, for, example, metals. Some of the properties that can be improved by forming a composite material are stiffness, strength, weight reduction, corrosion resistance, thermal properties, fatigue life and wear resistance. Most man-made composite materials are make from two materials: a reinforcement material called fiber and a base material called matrix material.

The behavior of composite materials is often sensitive to changes in temperature. This arises for two main reasons. Firstly the response of matrix to an applied load is temperature-dependent and, secondly, changes in temperature can cause internal stresses to be set up as a result of differential thermal contraction and expansion of the constituents. These stresses affect the thermal expansivity (expansion coefficient) of the composite. Furthermore, significant stresses are normally present in the material at ambient temperatures, since it has in most cases been cooled at the end of the fabrication process. Changes in internal stress state on altering the temperature can be substantial and may strongly influence the response of the material to applied load. Thermal conductivity of composite materials is of interest, since many applications and processing procedures involve heat flow of some type. This property can be predicted from the conductivities the constituents, although the situation may be complicated by poor thermal contact across the interfaces.

Data for the thermal expansion coefficients (α) of matrices and reinforcements, as a function of temperature. Polymer and metals generally expand more than ceramic.

It can be seen that the differences in expansivity between fibre and matrix are large in many cases. Since fabrication almost inevitably involves consolidation relatively high temperatures composites usually contain significant differential thermal contraction stresses at ambient temperatures. In additional, for resin-based composites, the matrix undergoes shrinkage during curing, which represent a further contribution to differential contraction.

Analysis of stresses due to change in temperature allows the coefficient of thermal expansion (CTE) to be predicted. These stresses will have associated strains and the net effect of these on the length of the composite, in any given direction, can be calculated or estimated. This net length change arising from the internal stresses is simply added to the natural thermal expansion of the matrix to give the overall length change and hence the composite expensivity. This simple view of thermal expansion allows certain points to be identified immediately. For example: a porous material, regarded as a composite of voids in a matrix, does not develop any internal stress on heating, because the inclusions have zero stiffness. Hence the presence of pores (of whatever shape, size and volume fraction) does not affect the CTE, although they give rise to sharp reductions in stiffness.

Problem associated with thermal stresses can be severe with laminates. Thermal misfit strains now arise, not only between fibre and matrix, but between the individual plies of the laminate. For example, since a lamina usually has a much large expansion coefficient in the transverse direction than in the axial direction, heating of a cross-ply laminate will lead to the transverse expansion of each ply being strongly inhibited by the presence of the other ply. This is useful in the sense that it will make the dimensional changes both less pronounced and more isotropic, but it does lead to internal stress in the laminate. These can cause the laminate to distort, so that it is no longer flat. However, even if such distortions do not occur, the stresses which arise on changing the temperature can cause micro structural damage and impairment of properties.

In this study, thermal stress with in a laminate can be calculated using the analytical solution method. For the laminated plates, residual stresses and plastic flow are obtained by using incremental stress method. This method has not been met in the literature survey, but it has only used by Numan Bektaş who is my adviser's student.

1. 2. Literature Survey

Shabana and Noda (2001) analyzed elasto-plastic thermal stresses in a rectangular plates. In the analysis, the thermal stresses constitutive equation of a particle-reinforced composite taking temperature change and damage process into consideration is used. The effects of the particle volume fraction and the three kinds of temperature conditions on the stress in the matrix, stresses in the particle and macroscopic stress are discussed.

Teixeira-Dias and Menezes (2001) purposed to present a fully three-dimensional finite element algorithm dedicated to the numerical calculation of thermaly induced residual stresses in metal matrix composite materials. They said that the mechanical model considers the reinforcement component to behave thermo-elastically and the matrix material to have a thermo-elastoviscoplastic behaviour. They carried out the developed code which is tested with some numerical examples concerning a SiC reinforcement in an aluminium matrix in order to validate and optimize the numerical algorithms that were implemented.

Yu and Lee (2000) studied on design and fabrication of an alumina reinforced aluminum composite material. The purpose of their study was to investigate the effects of geometrical parameters on the mechanical properties of the composite material. They used finite element analysis for a representative unit cell of the composite using the elastic-plastic theory. The finite element analysis demonstrated that detailed geometry parameters of the reinforcement as well as its volume fraction are significant while designing the composite to achieve better mechanical properties.

Jeronimidis and Parkyn (1998) investigated the residual stress in carbon fiber-thermoplastic matrix laminates. In their work, residual stresses in APC-2 cross-ply laminates were obtained by using lamination theory. They compared their results to measured levels of residual stress obtained from a number of experimental technique. The analysis of the results shows that accurate predictions can be made to provided that the changes in thermoelastic properties of the materials with temperature while taking into account.

Li, et al. (2000) purposed to develop such materials that can potentially reduce residual stresses in laser deposited parts significantly while maintaining or even improving all other properties (strength, toughness, corrosion and wear resistance). They show that the new materials yield exceptionally low CTE, reasonable ductility, high hardness, and significantly improved yield strength in the experimental results.

Sayman (2002) studied an elastic-plastic thermal stress analysis in aluminum metal-matrix composite beam. In his solutions, he used an analytic method. Temperature was chosen to vary linearly along the section of the beam. The intensity of the residual stress component of σ_x was maximized at the upper and lower surfaces of the beam and for an orientation of zero. He found that the equivalent plastic strain to be greatest at the same orientation angle. Plastic yielding beings on the beam at higher orientation angles at the lower temperature.

Agbossou and Pastor (1997) developed a thermal self-consistent (TCS) model for n-layered fiber-reinforced composites. They show that the thermal problem under conditions of no external loading can be treated without having to determine the elastic characteristics of the equivalent medium. They found that fiber anisotropy and interphase imperfections significantly change the thermal expansion coefficients and the thermal microstresses. The thermal stresses in the fiber decreased when adhesion between fiber and matrix was poor.

Akbulut and Şenel (2001) studied on residual stresses in a stainless steel metal-fiber reinforced aluminum metal-matrix flat plate with a central square hole. They show that a central hole or cut-out in a plate causes the residual stresses in a considerable quantity particularly near itself, which increases strength of the plate. It was also seen that the hole geometry and the orientation angle have affected residual stresses.

Çallıoğlu, et al. (2001) worked on a steel fibre high density polyethylene matrix composite beam. They found the distribution of residual stress and deformations analytically. They show that thermal stresses are important in the design because they lead to plastic yielding or failure of the material.

Sayman and Zor (2000) investigated elastic-plastic stress analysis in a woven steel reinforced thermoplastic composite cantilever beam loaded uniformly at the upper surface. They proposed a closed form solution to satisfy both the governing differential equation and boundary conditions. They determined the expansion of the plastic region and the distribution of the residual stresses along the sections of the beam. The solution is carried out for small plastic deformations. Residual stresses are determined by subtracting elastic stresses from elastic-plastic stresses. They found the intensity of the residual stress components as maximum at the upper and/or lower surfaces of the beam. They also found the intensity of the residual stress component of the shear stress as maximum on around the neutral axis.

Sayman, et al. (2000) studied on elasto-plastic stress analysis of aluminum metal-matrix composite laminated plates under in-plane loading. They found that load carrying capacity of the laminated plate can be increased by means of residual stresses. The yield point of symmetric laminated plates is higher than that of the yield point of anti-symmetric laminated plates, and the orientation angle effects the yield points of laminated plates.

Todd, et al. (1999) measured the thermal residual stresses with fluorescence spectroscopy. Tensile residual stresses were found in platelets, implying the presence of compressive residual stresses in the glass matrix. The measured data were used to estimate the contribution of the residual stresses to the toughening effect of the platelets. They found that the measured stresses lay between the prediction of models for spherical particles and thin platelets, but were significantly closer to the former.

Kojic et al. (1995) analyzed elastic-plastic deformation of an orthotropic multilayered beam. They used Tsai-Hill criterion as a yield criterion. First author is developed incremental relations for stress calculation which are based on the governing parameter method. They presented that the consistent differentiation of the governing relations has lead to derivation of the tangent moduli, important for high convergence rate in equilibrium iterations on the structural level.

Bektas and Sayman (2001) worked on the elasto-plastic stress analysis in simply supported thermoplastic laminated plates under thermal loads. In their work, symmetric cross-ply and angle-ply thermoplastic laminated plates are analyzed. They used Tsai-Hill criterion and assumed perfectly plastic material properties. They found plastic stresses and residual stresses.

Sayman (2000) studied thermal elastic-plastic stress analysis on simply supported symmetric cross-ply and angle-ply aluminum metal-matrix composite laminated plates. In his work, Tsai-Hill theory is used as a yield criterion. He found the intensity of the residual stress components in the cross-ply laminated plate higher than those in the angle-ply laminated plates. He also found that the residual stress component of the shear stress alone in the angle-ply laminated plates is different from zero.

Özcan (2000) carried out an elastic-plastic stress analysis in thermoplastic composite laminated plate for in-plane loading. He obtained yield points and stresses for symmetric and antisymmetric laminated plates with or without hole. He compared yield points and the expansion of plastic zones with different plates.

Hu and Weng (1998) analyzed the influence of thermal residual stress on the deformation behavior of a composite with a new micromechanical method. Their method was based on secant moduli approximation and a new homogenized effective stress to characterize the plastic state of the matrix. They show that the induced plastic strain (after a temperature change) and their influence on the mechanical behavior depends significantly on the inclusion shape.

Sayman and Çallıoğlu (2000) studied an elastic-plastic stress analysis on a steel fibre reinforced high-density thermoplastic composite cantilever beam loaded by a bending moment at its free end. During the solution of the problem, beam was assumed to be strain-hardening. They used Bernoulli-Navier assumptions in the investigation. They found the maximum residual stress at the upper and lower surfaces or at the boundary of the elastic and plastic regions.

Karakuzu, Özel and Sayman (1997) worked on elastic-plastic finite element analysis in steel aluminum composite plates with semicircular and "U" shaped edge notches under various uniform distributed loads. They used an initial stress method in finite element solution. The spread of plastic zones around notches has been shown for different orientation angles and various loads.

Karakuzu R. and Özcan R., (1996) carried out an analytical elasto-plastic stress analysis for a fiber-reinforced metal-matrix composite beam to a single transverse force applied to the free end of the beam and a uniformly distributed load. They calculated elastic, elasto-plastic and residual normal and shear stresses in a composite beam. The location of elasto-plastic boundary of the beam was obtained according to the x coordinate of the beam. They presented that when the value of the orientation angle decreases, length of the beam increases.

Kang and Kang (1994) reviewed the work on short fiber-reinforced aluminium composite material fabricated by the stirring method and extruded at high temperatures at various extrusion ratios. In that study, the tensile strength of the hot extrusion billet was experimentally determined for different extrusion ratios, and compared with the theoretically calculated strength.

Shen and Lin (1995) investigated thermal post-buckling analysis of imperfect laminated plates. In their analysis, they used a mixed Galerkin-perturbation technique to determine the thermal buckling loads and post-buckling equbilirium paths. They show that the characteristic of thermal post-buckling are significantly influenced by fibre orientation, plate aspect ratio, the thermal load ratio and initial geometrical imperfection.

Owen and Figueiras (1983) used a finite element displacement method on anisotropic plates and shells. They adopted initial stiffness approach for elasto-plastic numerical solution and flow rule. They applied the nine-node Lagrangian element and Heterosis element to thick and thin plate and shell. For shell applications, an appropriate shear correction factor was not considered, due to the lack of available composite shell solution. And they obtained very good agreement with other published plate solution.

Barnes (1993) investigated the thermal-expansion behaviors of a variety of thermoplastic composites based on the ICI Victrex polymers, Polyarylether etherketone (PEEK), Intermediate Temperature Semi-crystalline Polymer "Victrex", ITX (an aromatic copolymer), Victrex Intermediate Temperature Amorphous Polymer ITA (a polyether sulphone) and Victrex High Temperature Amorphous Polymer HTA (a bisphenyl-modified polyether sulphone).

Rolfes, et al. (1998) presented a post processing procedure for the evaluation of the transverse thermal stresses in composite laminated plates based on the first order shear deformation theory.

Szyszkowski and King (1995) analyzed the thermal stress concentration at the surface of the composite using the ADINA program. They show that the residual stress at the free surface is a dominating stress component should be primarly responsible for the cracks observed experimentally at the fibre-matrix interface. And this cracks was initiated during the cooling phase of the temperature fluctuations, when the large radial stress was tensile.

Chung (2000) reviewed the work on the structural composite material to illustrate the application of thermal analysis in the form of the measurement of electrical resistance as a function of temperature. In this review, the experimental methods and data interpretation was covered. The used technique was enabled by the electrical conductivity of carbon fibers.

Ghorbel (1997) performed to identify the main mechanisms leading to the failure of metal matrix composite components subjected to in-service temperature fluctuations. He investigated the thermo-mechanical behavior of composites with micro-structural analysis and mechanical tests.

Ifju, et al. (1999) developed a novel method or a non-destructive method called the Cure Referancing Method (CRM) to measure the residual stresses in laminated composites. They used this method to determine the residual stresses in three different multidirectional laminate lay-ups of AS4/3501-6 graphite/epoxy composite. They found that the repeatability of the method was reasonable.

Meijer, et al. (2000) investigated the influence of inclusion geometry and thermal residual stresses and strains on the mechanical behavior of a Al_2O_3 particulate reinforced 6061-TO Al alloy metal matrix composite by finite element method, the introduction of the residual thermal stress/strains prior to external loading leads to a decrease in the proportional limit, 0.2% offset yield stress and the apparent stiffness.

Atas and Sayman (2000) used finite element method for elastic-plastic stress analysis and expansion of plastic zone in clamped and simply supported aluminum metal-matrix laminated plates. They used first-order shear deformation theory for small deformations. In their work, plate is simply supported or clamped boundary conditions. Laminated plates of constant thickness are formed by stacking four layers bonded symmetrically or anti-symmetrically. It is assumed that the laminated plates are subjected to transverse uniform loads. They used the Tsai-Hill theory as a yield criterion, and used a nine-node iso-parametric quadrilateral element. Their results for both clamped and simply supported plates: the yield points in symmetric cross-ply and angle-ply laminated plates are higher than those in anti-symmetric cross ply and angle ply laminates. The yield point for cross-ply laminated plates is always smaller than that for angle ply-laminated plates. The yield loads in clamped laminated plates are higher than those of simply supported plates..

Atas, et al. (2001) offered a residual stress analysis and plastic zone behavior in antisymmetric aluminium metal matrix laminated plates. They considered first-order shear deformation theory and finite element method in the solution for small deformations. They are assumed that the laminated plates are subjected to transverse uniform loads.

Hsueh, et al. (2001) proposed to examine residual thermal stresses and effective CTEs of intergranular two phases composites in a two dimensional sense. In their solutions, they used analytical modeling and numerical simulations. The results show that in predicting CTEs, the simply analytical model represents the two- dimensional composite well except that brick wall-array grains, which included significant anisotropic CTEs in the composites.

Sayman, et al. (2000) manufactured aluminum metal-matrix composite laminated plates reinforced by steel fibers and analyzed with finite element technique. They used the first-order shear deformation theory and nine-node Lagrangian finite element in analysis. In their work, the expansion of the plastic zone and residual stresses are determined in the symmetric and/or anti-symmetric cross-ply and angle-ply laminated plates. They presented that load carrying capacity of the laminated plate can be increased by means of residual stresses.

Trende et al. (2000) investigated of residual stresses and dimensional changes in the compression moulded glass-mat reinforced thermoplastic (GMT) parts. They used a heat transfer and crystallization model with temperature dependent matrix properties to obtain input to the subsequent thermal stress analysis. In their work, finite element method is used for investigation of both an isotropic viscoelastic and transversely isotropic elastic material model. Their results show that the elastic material model is sufficient for prediction of geometrical changes if the temperature history is symmetric through the thickness of the specimen, but insufficient for prediction of residual stresses and dimensional changes if the temperature field is not symmetric.

Kim, et al. (1997) proposed a method to actively control interlaminar stresses near the free edges of laminated composites by through thickness thermal gradients.

Unger, et al. (1998) presented a method accounting for the effect of process-induced thermal residual stresses on the free-edge delamination behavior of fibre-reinforced laminates. They developed a hybrid crack closure-finite element mode which includes the effect of thermal residual stresses in the calculation of strain energy release rate for interlaminar cracking. Their works also illustrate the significant reduction in material strength which these residual stress can produce.

Sayman (1998) used finite element technique for an elasto-plastic stress analysis in stainless steel fiber reinforced aluminum metal matrix laminated plates loaded transversely. In his work, the expansion of the plastic zone and residual stresses are determined in the symmetric and/or anti-symmetric cross-ply and angle-ply laminated plates for small deformations. He found the yield points in symmetric laminates higher than those in anti-symmetric laminates. He noticed that the strength of the laminated plates can be increased by using residual stresses.

Wang, et al. (2000) developed an analytical method that based on inclusion analogy for determining the thermal residual stresses in an isotropic plate reinforced with a circular orthotropic patch. They obtained approximate solutions to quantify the size effect associated with repairing finite size structures. These solutions provided a convenient means for the design and analysis of bonded repairs in relation to the thermal stresses.

Jiang, et al. (1998) developed an analytical method to study the influence of thermal stresses on elastic and yield behaviors of aligned short fiber–reinforced metal-matrix composites. They calculated the thermal stresses and distribution of the stresses under tensile and compressive loading as a function of the material parameters. Finally, they found that the residual stresses affect the elastic modulus and yield strength primarily through influencing the stress transfer.

Yoon and Kim (2000) proposed a computational method based on the classical lamination theory to predict the change of coefficients of thermal expansion (CTE) of Carbon/Epoxy laminates for temperature variation. They measured and compared the coefficients of thermal expansion of various angle-ply laminates with those predicted. In their work, experimental results show that the changes of CTEs of general laminate for temperature variation can be predicted well by using the proposed method.

Chou, et al. (1985) investigated the fibre-reinforced metal-matrix composites involving fabrication methods, mechanical properties, secondary working techniques and interfaces.

Yeh and Krempl (1993) examined the influence of cool-down temperature histories on the residual stresses in fibrous metal-matrix composites. They found that the residual stresses at room temperature change with temperature history. The matrix residual stress upon reaching room temperature, is the highest for the fastest cooling rate, but after third days rest the infuence of cooling rate is hardly noticeable since relaxtion takes place.

CHAPTER 2

ELASTIC BEHAVIOR OF LAMINATED PLATES

2. 1. Laminated Plates

A laminate is constructed by stacking a number of laminas in thickness (z) direction. Examples of a few special types of laminates and the standard lamination code are given below:

- Unidirectional Laminates: In unidirectional laminates, fiber orientation angle is the same in all laminas (Fig.1a). In unidirectional 0^0 laminates, for example, $\theta=0^0$ in all laminas.
- Angle-ply laminates: In angle-ply laminates, fiber orientation angles in alternate layers are [...../θ/- θ/θ/-θ/....]when $\theta \neq 0$ or 90^0 (Fig.1b).
- Cross-ply laminates: In cross-ply laminates, the angles in alternate layers are [.../0^0/90^0/0^0/90^0/....] (Fig.1c).
- Symmetric laminates: In symmetric laminates, the ply orientation is symmetrical about the centerline of the laminate; i.e., for each ply above the midplane, there is an identical ply (including in material, thickness, and fiber orientation angle) at an equal distance below the midplane. For example:[0/+45/90/90/+45/0] ⇔ $[0/+45/90]_s$ or [0/90/0/90/0/90/0/90/0] ⇔ $[(0/90)_2/0]_s$(symmetric cross-ply) (Fig. 1c).
- Antisymmetric laminates: In antisymmetric laminates, the ply orientation is antisymmetrical about the centerline of the laminate; i.e., for each ply above the midplane. For example: [0/90/0/90...](antisymmetric cross-ply) or [-45/40/-15/15/-40/45] (antisymmetric angle-ply).
- Quasi-isotropic laminates: These are made of three or more laminas of identical thickness and materials with equal angles between each adjacent lamina. Thus, if the total number of laminas is n, the orientation angles of the

laminas are at increments of π/n. The resulting laminate exhibits an isotropic elastic behavior in the xy plane. Examples are [+60/0/-60] and [+45/0/-45/90]. Other combinations of these stacking sequences, such as [0/+60/-60] and [0/+45/-45/90], also exhibit in plane isotropic elastic behavior. A very common and widely used quasi-isotropic symmetrical stacking sequence is $[0/\pm 45/90]_s$.

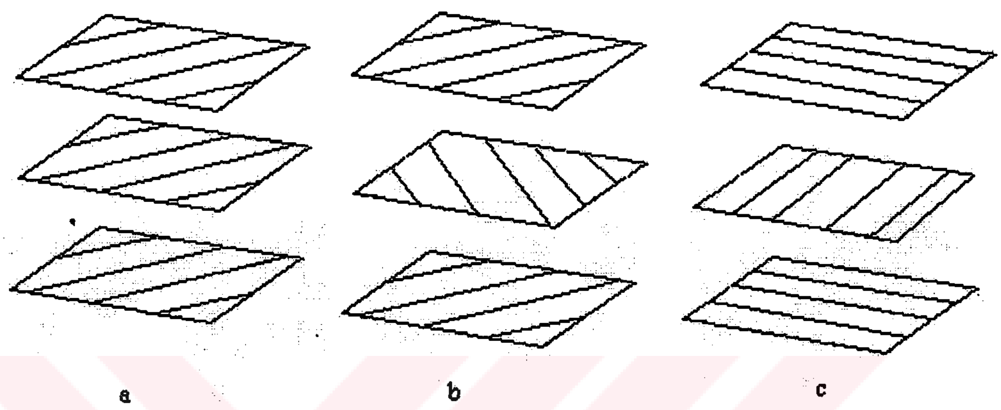

Figure 2. 1 a) Unidirectional Laminate, b) Angle-ply Laminate, c) Cross-ply Laminate.

2. 2. Classic Laminated Plate Theory

2. 2. 1. Assumptions

The classical laminated plate theory is an extension of the classical plate theory to composite laminates. In the classical laminated plate theory (CLPT) it is assumed (an assumption is that which is necesarry for the development of the mathematical model, whereas a restriction is not condition for the development of the theory) that the Kirchhoff hypothesis holds:

- Straight lines perpendicular to the midsurface (i.e., transverse normals) before deformation remain straight after deformation.
- The transverse normals do not experience elongation (i.e., they are inextensible).

- The transverse normals rotate such that they remain perpendicular to the midsurface after deformation.

The first two assumptions imply that the transverse displacement is independent of the transverse (or thickness) coordinate and the transverse shear strains, $\varepsilon_{xz} = \varepsilon_{yz} = 0$.

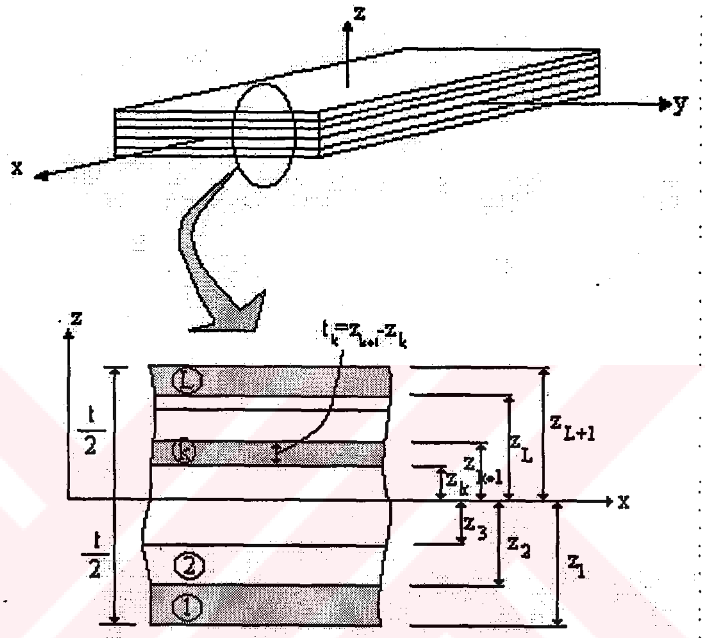

Figure 2. 2 Coordinate system and layer numbering used for a typical laminated plate.

2. 2. 2. Strain-Displacement Relations

A section of the laminate remains normal to the y-axis before and after deformation. The x-y plane is equidistant from the top and bottom surfaces of the laminate and is called the reference plane. The reference plane displacements u_0 and v_0 in the x and y- directions and the out-of-plane displacement w in the z-direction are functions of x and y only:

$$\begin{aligned} u_0 &= u_0(x, y) \\ v_0 &= v_0(x, y) \\ w &= f(x, y) \end{aligned} \tag{2.1}$$

The rotations of the x-and y-axes are

$$\alpha_x = \frac{\partial w}{\partial x}$$
$$\alpha_y = \frac{\partial w}{\partial y} \qquad (2.2)$$

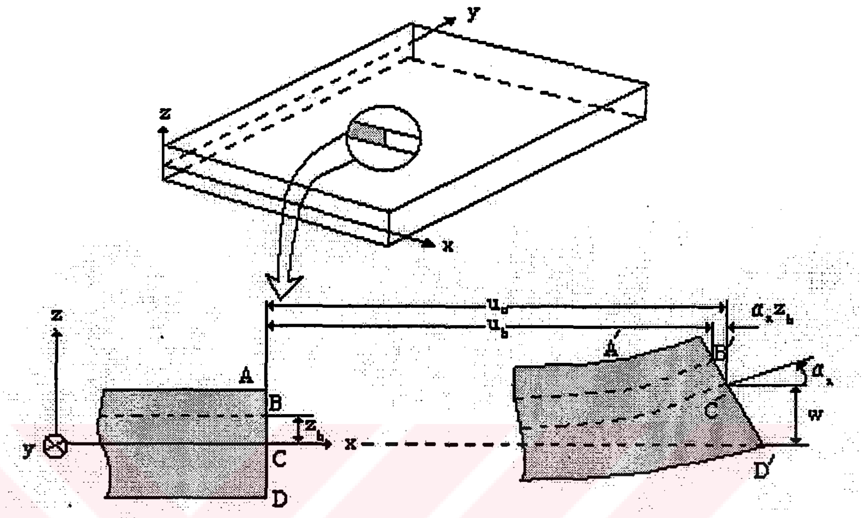

Figure 2. 3 Laminate section before (ABCD) and after (A'B'C'D') deformation.

The in-plane displacement components of a point B of coordinate z_b are

$$u_b = u_0 - \alpha_x . z_b$$
$$v_b = v_0 - \alpha_y . z_b \qquad (2.3)$$

and in general

$$u = u_0 - z\frac{\partial w}{\partial x}$$
$$v = v_0 - z\frac{\partial w}{\partial y} \qquad (2.4)$$

where z is the coordinate variable of a general point of the cross section.

For small displacements, the classical strain-displacement relations of elasticity yield

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w}{\partial y^2} \qquad \Rightarrow \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \qquad (2.5)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y}$$

$$\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0$$

Noting that strain components on the reference plane are expressed as

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x}$$

$$\varepsilon_y^0 = \frac{\partial v_0}{\partial y} \qquad \Rightarrow \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{bmatrix} \qquad (2.6)$$

$$\gamma_{xy}^0 = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}$$

and the curvatures of the laminate as

$$\kappa_x = -\frac{\partial^2 w}{\partial x^2}$$

$$\kappa_y = -\frac{\partial^2 w}{\partial y^2} \qquad \Rightarrow \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = -\begin{bmatrix} \dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial^2 w}{\partial y^2} \\ 2\dfrac{\partial^2 w}{\partial x \partial y} \end{bmatrix} \qquad (2.7)$$

$$\kappa_{xy} = -2\frac{\partial^2 w}{\partial x \partial y}$$

The strains at any point in the laminate can be written with the reference plane strains and the laminate curvatures as follows:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_j = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + z_j \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix} \tag{2.8}$$

where z_j : distance from the laminate midplane to the midplane of the jth lamina

2. 2. 3. Stress-Strain Relations of One Layer Within a Laminate

This theory is useful in calculating stresses and strains in each lamina of a laminated structure. Beginning with the stiffness matrix of each lamina, step-by-step procedure in lamination theory includes;

1. Calculation of stiffness matrices for the laminate
2. Calculation of midplane strains and curvatures for the laminate due to a given set applied forces and moments
3. Calculation of in-plane strains $\varepsilon_x, \varepsilon_y$ and γ_{xy} for each lamina
4. Calculation of in-plane stresses σ_x, σ_y and τ_{xy} in each lamina.

Consider an individual layer k in a laminate whose midplane is at a distance z_k from the laminate reference plane (Fig.2.3). The stress-strain relations for this layer referred to its material axes are

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}_k \tag{2.9}$$

where [Q] represents the stiffness matrix for a specially orthotropic lamina. Various elements in the [Q] matrix are;

$$Q_{11} = \frac{E_{11}}{1-\upsilon_{12}\upsilon_{21}}$$
$$Q_{22} = \frac{E_{22}}{1-\upsilon_{12}\upsilon_{21}}$$
$$Q_{12} = Q_{21} = \frac{\upsilon_{12} E_{22}}{1-\upsilon_{12}\upsilon_{21}} = \frac{\upsilon_{21} E_{11}}{1-\upsilon_{12}\upsilon_{21}} \tag{2.10}$$
$$Q_{66} = G_{12}$$

and after transformation to the laminate coordinate system;

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{2.11}$$

where $[\overline{Q}]$ represents the stiffness matrix for the layer. Various elements in the $[\overline{Q}]$ matrix are expressed in terms of the elements in [Q] matrix as;

$$\begin{aligned}
\overline{Q}_{11} &= Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta \\
\overline{Q}_{22} &= Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta \\
\overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\sin^4\theta + \cos^4\theta) \\
\overline{Q}_{16} &= (Q_{11} - Q_{22} - 2Q_{66})\sin\theta in\theta^3\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin^3\theta\cos\theta \\
\overline{Q}_{26} &= (Q_{11} - Q_{22} - 2Q_{66})\sin^3\theta\cos\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin\theta in\theta^3\theta \\
\overline{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta + \cos^4\theta)
\end{aligned} \tag{2.12}$$

Stresses in the k^{th} lamina can be calculated using its stiffness matrix. Thus;

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left\{ \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \right\} \tag{2.13}$$

or

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = [\overline{Q}]_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [\overline{Q}]_k \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + [\overline{Q}]_k z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{2.14}$$

2. 2. 4. Resultant Laminate Forces and Moments

The resultant forces and moments acting on a laminate are obtained by integration of the stresses in each layer k of laminate through the laminate thickness as given below:

$$N_x^k = \int_{-t/2}^{t/2} \sigma_x dz \quad \textit{: normal forces per unit length}$$
$$N_y^k = \int_{-t/2}^{t/2} \sigma_y dz \quad \textit{: normal forces per unit length} \tag{2.15}$$
$$N_{xy}^k = \int_{-t/2}^{t/2} \tau_{xy} dz \quad \textit{: shear forces per unit length}$$

and

$$M_x^k = \int_{-t/2}^{t/2} \sigma_x z dz \quad \textit{: bending moments per unit length}$$
$$M_y^k = \int_{-t/2}^{t/2} \sigma_y z dz \quad \textit{: bending moments per unit length} \tag{2.16}$$
$$M_{xy}^k = \int_{-t/2}^{t/2} \tau_{xy} z dz \quad \textit{: twisting moments per unit length}$$

where t is layer thickness and z is the coordinate variable of a point in the cross section.

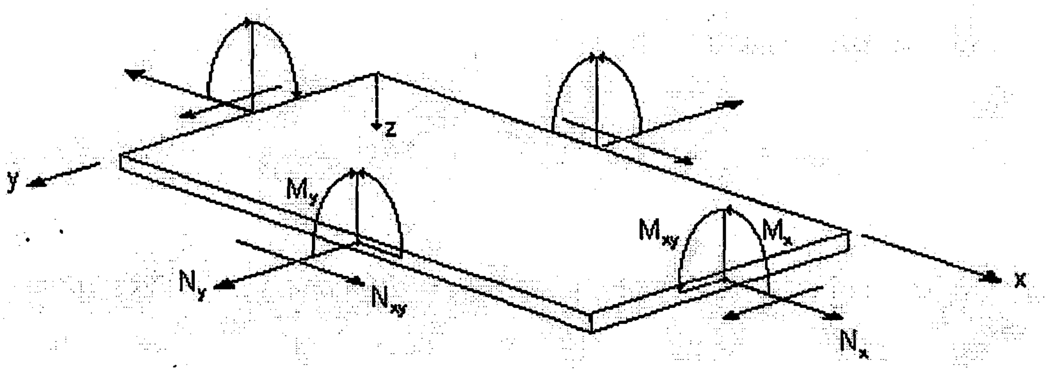

Figure 2. 4 In-plane forces and moments on a flat laminate.

For the n-ply laminate in Fig.2.4, the force and moment resultants are obtained as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k dz$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k zdz \qquad (2.17)$$

where z_k and z_{k-1} are the z-coordinates of the upper and lower surface of layer k.

Substituting Eq.(2.13) for the layer stresses in Eqs.(2.17) above, we obtain,

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix}_k = \sum_{k=1}^{n} \left\{ \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \int_{z_{k-1}}^{z_k} dz + \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \int_{z_{k-1}}^{z_k} zdz \right\} \qquad (2.18)$$

and

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix}_k = \sum_{k=1}^{n} \left\{ \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \int_{z_{k-1}}^{z_k} zdz + \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \int_{z_{k-1}}^{z_k} z^2dz \right\} \qquad (2.19)$$

In the expression above, the stiffnesses $[\overline{Q}]_{1,2}^k$, reference plane strains $[\varepsilon^0]_{x,y}$, and curvatures $[\kappa]_{x,y}$ are taken outside the integration operation since they are not functions of z. Of these quantities only the stiffnesses are unique for each layer k, whereas the reference plane strains and curvatures refer to the entire laminate and are the same for all plies.

Thus, in full form the force-deformation relations are

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x^0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \end{Bmatrix} \tag{2.20}$$

and moment-deformation relations are

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x^0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \end{Bmatrix} \tag{2.21}$$

where

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{n} \overline{Q}_{ij}^{k} \left(z_k - z_{k-1}\right) \\ B_{ij} &= \frac{1}{2}\sum_{k=1}^{n} \overline{Q}_{ij}^{k} \left(z_k^2 - z_{k-1}^2\right) \\ D_{ij} &= \frac{1}{3}\sum_{k=1}^{n} \overline{Q}_{ij}^{k} \left(z_k^3 - z_{k-1}^3\right) \end{aligned} \tag{2.22}$$

where A_{ij} are extensional stiffnesses, or in-plane laminate moduli, relating in-plane loads to in-plane strains, B_{ij} are coupling stiffnesses, or in-plane/flexure coupling laminate moduli, relating in-plane loads to curvatures and moments to in-plane strains. Thus, if $B_{ij} \neq 0$, in-plane forces produce flaxural and twisting deformations; moments produce extension of the middle surface in addition to flexure and twisting and D_{ij} are bending or flexural laminate stiffnesses relating moments to curvatures.

The complete set of equations can be expressed in matrix form as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{2.23}$$

or briefly

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} \tag{2.24}$$

If we want to write inversion of above load-deformation relation we can write as

$$\begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{Bmatrix} N \\ M \end{Bmatrix} \tag{2.25}$$

where

$$\begin{aligned} &[A'] = [A^*] - [B^*][D^*]^{-1}.[C^*] &, \quad &[A^*] = [A]^{-1} \\ &[B'] = [B^*][D^*]^{-1} &, \quad &[B^*] = -[A]^{-1}.[B] \\ &[C'] = -[D^*]^{-1}.[C^*] = [B']^T = [B'] &, \quad &[C^*] = [B][A]^{-1} \\ &[D'] = [D^*]^{-1} &, \quad &[D^*] = [D] - [B][A]^{-1}.[B] \end{aligned} \tag{2.26}$$

2. 3. Antisymmetric Laminated Plates

The general class of antisymmetric laminates must have an even number of layers if adjacent laminae have alternating signs of the principal material property directions with respect to the laminate axes. In addition, each pair of laminae must have the same thickness. The extensional coupling stiffness of an anti-symmetric laminate is written as

$$A_{16} = \sum_{k=1}^{n} \overline{Q}_{16}^{k}\left(z_k - z_{k-1}\right) = 0 \tag{2.27}$$

and layers symmetric about the middle surface have equal thickness and hence the same value of the geometric term multiplying $\overline{Q}_{16}^{k}$. Similarly, A_{26} is zero as is the bending twist coupling stiffnesses D_{16},

$$D_{16} = \frac{1}{3}\sum_{k=1}^{n} \overline{Q}_{16}^{k}\left(z_k^3 - z_{k-1}^3\right) = 0 \tag{2.28}$$

since $\left(z_k^3 - z_{k-1}^3\right) = \left(z_{k'}^3 - z_{k-1'}^3\right)$ and $\overline{Q}_{16}^{k} = -\overline{Q}_{16}^{k'}$ for the symmetrically situated balanced pair of k and k' (or +θ and -θ) layers.

The coupling stiffness B_{ij} for anti-symmetric laminates are in general nonzero, and they vary according to the specific lay-up. The overall-deformation relations for this class of laminates are

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & B_{26} \\ 0 & 0 & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & 0 \\ B_{16} & B_{26} & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_x^0 \\ \varepsilon_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{2.29}$$

2. 3. 1. Antisymmetric Cross-Ply Laminated Plates

Antisymmetric cross-ply laminates consist of 0^0 and 90^0 plies arranged in such a way that for every 0^0 ply at a distance z from the midplane there is a 90^0 ply of the same material and thickness at a distance –z from the midplane. By definition then, this laminate has an even number of plies.

For every pair k and k` of 0^0 and 90^0 plies we have

$$\begin{aligned} \overline{Q}_{11}^{k} &= \overline{Q}_{22}^{k^1} \\ \overline{Q}_{22}^{k} &= \overline{Q}_{11}^{k^1} \\ \overline{Q}_{12}^{k} &= \overline{Q}_{12}^{k^1} \\ \overline{Q}_{16}^{k} &= \overline{Q}_{26}^{k} = \overline{Q}_{16}^{k^1} = \overline{Q}_{26}^{k^1} = 0 \end{aligned} \tag{2.30}$$

that, it follows from the definitions of the laminate stiffinesses that

$$\begin{aligned} A_{11} &= A_{22} \\ A_{16} &= A_{26} = 0 \\ B_{12} &= B_{16} = B_{26} = B_{66} = 0 \\ D_{11} &= D_{22} \\ D_{16} &= D_{26} = 0 \end{aligned} \tag{2.31}$$

the overall laod-deformation relation are

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & 0 & 0 \\ A_{12} & A_{22} & 0 & 0 & -B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & 0 \\ B_{11} & 0 & 0 & D_{11} & D_{12} & 0 \\ 0 & -B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_x^0 \\ \varepsilon_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{2.32}$$

For the cross-ply laminates with alternating 0^0 and 90^0 plies, the coupling stiffness B_{11} approaches zero as the number of plies increases for a constant laminate thickness.

2. 3. 2. Antisymmetric Angle-Ply Laminated Plates

Antisymmetric angle-ply laminates consist of pairs of $+\theta_i$ and $-\theta_i$ orientations ($0<\theta_i<90$), symmetrically situated about the middle plane and having the same thickness and elastic properties. Because of anti-symmetry
$A_{i6} = D_{i6} = 0$ with i=1,2.

For every balanced pair of k and k` plies with orientations θ and -θ we have

$$\begin{aligned}
\overline{Q}_{11}^{k} &= \overline{Q}_{11}^{k'} \\
\overline{Q}_{22}^{k} &= \overline{Q}_{11}^{k'} \\
\overline{Q}_{12}^{k} &= \overline{Q}_{12}^{k'} \\
\overline{Q}_{16}^{k} &= \overline{Q}_{16}^{k'} \\
\overline{Q}_{26}^{k} &= -\overline{Q}_{26}^{k'} \\
\overline{Q}_{66}^{k} &= \overline{Q}_{66}^{k'}
\end{aligned} \tag{2.33}$$

Then, from the definition of B_{ij} it follows that $B_{11}=B_{22}=B_{12}=B_{66}=0$ and the overall load-deformation relations take the form

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & B_{16} \\ A_{12} & A_{22} & 0 & 0 & 0 & B_{26} \\ 0 & 0 & A_{66} & B_{16} & B_{26} & 0 \\ 0 & 0 & B_{16} & D_{11} & D_{12} & 0 \\ 0 & 0 & B_{26} & D_{12} & D_{22} & 0 \\ B_{16} & B_{26} & 0 & 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_x^0 \\ \varepsilon_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{2.34}$$

A more special case of this class of laminates is the anti-symmetric regular angle-ply laminates, consist of an even number of plies alternating between θ and -θ in orientation, i.e., [θ/-θ/.../θ/-θ].

The nonzero coupling stiffness B_{16} and B_{26} decrease and approach zero as the number of plies increases for the same overall laminate thickness.

CHAPTER 3
THERMOELASTIC BEHAVIOR OF LAMINATED PLATES

3. 1. Thermal Strains

If a temperature variation ΔT is involved, lamina strains will be

$$\begin{aligned} \varepsilon_x &= \varepsilon_x^M + \varepsilon_x^T = \varepsilon_x^0 + z\kappa_x \\ \varepsilon_y &= \varepsilon_y^M + \varepsilon_y^T = \varepsilon_y^0 + z\kappa_y \\ \gamma_{xy} &= \gamma_{xy}^M + \gamma_{xy}^T = \gamma_{xy}^0 + z\kappa_{xy} \end{aligned} \tag{3.1}$$

where the superscripts M and T denote the mechanical and thermal strains, respectively.

It should be pointed out that the thermal strains are due to free expansions (or contractions) caused by temperature variation, but mechanical strains are due to both applied loads and thermal loads. Thermal loads appear due to restrictions imposed by various layers against their free thermal expansion. In many applications involving polymeric matrix composites, moisture can also influence the laminate strains owing to volumetric expansion (swelling) or contraction of the matrix caused by moisture absorption or desorption. In such case, a third term representing hygroscopic strains must be added in the middle column of Eq.(3.1). (Mallick, 1988)

3. 2. Thermal Forces and Moments

Laminate Forces and Moments for thermal effects, we can write

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = [A].\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + [B].\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} - [T^*].\Delta T \tag{3.2}$$

and

$$\begin{bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} = [B].\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + [D].\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} - [T^{**}].\Delta T \tag{3.3}$$

where

$$[T^*] = \begin{bmatrix} \sum_{k=1}^{N} \left[(\overline{Q}_{11})_k (\alpha_x)_k + (\overline{Q}_{12})_k (\alpha_y)_k + (\overline{Q}_{16})_k (\alpha_{xy})_k \right] (z_k - z_{k-1}) \\ \sum_{k=1}^{N} \left[(\overline{Q}_{12})_k (\alpha_x)_k + (\overline{Q}_{22})_k (\alpha_y)_k + (\overline{Q}_{26})_k (\alpha_{xy})_k \right] (z_k - z_{k-1}) \\ \sum_{k=1}^{N} \left[(\overline{Q}_{16})_k (\alpha_x)_k + (\overline{Q}_{26})_k (\alpha_y)_k + (\overline{Q}_{66})_k (\alpha_{xy})_k \right] (z_k - z_{k-1}) \end{bmatrix} \tag{3.4}$$

$$[T^{**}] = \frac{1}{2} \begin{bmatrix} \sum_{k=1}^{N} \left[(\overline{Q}_{11})_k (\alpha_x)_k + (\overline{Q}_{12})_k (\alpha_y)_k + (\overline{Q}_{16})_k (\alpha_{xy})_k \right] (z_k^2 - z_{k-1}^2) \\ \sum_{k=1}^{N} \left[(\overline{Q}_{12})_k (\alpha_x)_k + (\overline{Q}_{22})_k (\alpha_y)_k + (\overline{Q}_{26})_k (\alpha_{xy})_k \right] (z_k^2 - z_{k-1}^2) \\ \sum_{k=1}^{N} \left[(\overline{Q}_{16})_k (\alpha_x)_k + (\overline{Q}_{26})_k (\alpha_y)_k + (\overline{Q}_{66})_k (\alpha_{xy})_k \right] (z_k^2 - z_{k-1}^2) \end{bmatrix} \tag{3.5}$$

For a balanced symmetric laminate elements in $[T^{**}]$ matrix are zero. There will be no curvatures.

3. 3. Element Stiffness Matrices

[A] : extensional stiffness matrix for the laminate (N/m).

$$[A]=\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \quad (3.6)$$

[B] : coupling stiffness matrix for the laminate (N).

$$[B]=\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \quad (3.7)$$

[D] : bending stiffness matrix for the laminate (N-m).

$$[D]=\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \quad (3.8)$$

3. 4. Thermal Expansion Coefficients

The longitudinal coefficient for continuous fiber composite is given by the ralation (Isaac, 1994)

$$\alpha_1 = \frac{E_f \alpha_f V_f + E_m \alpha_m V_m}{E_f V_f + E_m V_m} = \frac{(E\alpha)_1}{E_1} \quad (3.9)$$

where

E_f, E_m : Fiber and matrix moduli.

α_f, α_m: Fiber and matrix coefficients of thermal expansion.

V_f, V_m : Fiber and matrix volume ratios.

E_1 : Longitudinal composite modulus.

The relation for the transverse coefficient of thermal expansion based on energy principles is

$$\alpha_2 = \alpha_f V_f (1+\upsilon_f) + \alpha_m V_m (1+\upsilon_m) - \upsilon_{12}\alpha_1 \tag{3.10}$$

where

υ_f, υ_m : Poisson's ratios of fiber and matrix.

$\upsilon_{12} = \upsilon_f V_f + \upsilon_m V_m$: Major Poisson's ratio of composite lamina.

α_1 : Longitudinal coefficient of thermal expansion of lamina.

The principle coefficients α_1 and α_2 are known, the coefficients reffered to any system of coordinates *x*, *y* can be obtained by the following transformation relations, which are the same as those for strain transformation

$$\begin{aligned} \alpha_x &= \alpha_1 \cos^2\theta + \alpha_2 \sin^2\theta \\ \alpha_x &= \alpha_1 \sin^2\theta + \alpha_2 \cos^2\theta \\ \alpha_{xy} &= 2\cos\theta\cos\theta(\alpha_1 - \alpha_2) \end{aligned} \tag{3.11}$$

3. 5. Thermal Stress and Strain Relations

For plane stress on an orthotropic lamina in principle material coordinates,

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 - \alpha_1 \Delta T \\ \varepsilon_2 - \alpha_2 \Delta T \\ \gamma_{12} - \alpha_{12} \Delta T \end{Bmatrix} \tag{3.12}$$

The stress is the laminate coordinate for the k^{th} layer are obtained by transformation of coordinates as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x - \alpha_x \Delta T \\ \varepsilon_y - \alpha_y \Delta T \\ \gamma_{xy} - \alpha_{xy} \Delta T \end{Bmatrix}_k \tag{3.13}$$

or

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left\{ \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} - \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} \Delta T \right\}_k \tag{3.14}$$

Note that even if no external loads are applied, i.e., if [N]=[M]=[0], there may be midplane strains and curvatures due to thermal effects, which in turn will create thermal stresses in various laminas. These stresses can be calculated using midplane strains and curvatures due to thermal effects in Eq.(2.14).

When a composite laminate is cooled from the curing temperature to the room temperature, significant curing (residual) stresses may developed owing to the thermal mismatch of various laminas. In some cases, these curing stresses may be sufficiently high to cause interlaminar cracks. Therefore, it may be prudent to consider them in the analysis of composite laminates (Hahn, 1976).

Equations (3.3), (3.4) and (3.5) are also useful in calculating the coefficients of thermal expansion and cured shapes of a laminate.

CHAPTER 4
THERMAL ELASTIC-PLASTIC ANALYSIS OF LAMINATED PLATES FOR ELASTIC-PERFECTLY PLASTIC MATERIAL

4. 1. Introduction

Thermal stresses are of great importance in composite structures. In this investigation, a thermal elastic-plastic stress analysis is carried out in $[0^0/90^0]_2$, $[30^0/-30^0]_2$, $[45^0/-45^0]_2$, $[60^0/-60^0]_2$, $[15^0/-15^0]_2$, $[15^0/-30^0]_2$, $[15^0/-45^0]_2$, $[15^0/-60^0]_2$ antisymmetric aluminum metal-matrix laminated plates. Temperature is chosen to be constant along the cross sections of the plates. The plates are composed of four orthotropic layers bonded antisymmetrically. In the solution, a special computer program has been employed. The residual stress components of σ_x, σ_y and τ_{xy} are illustrated in the layers of the laminated plates for different thermal loading. The composite materials are assumed to be elastic-perfectly plastic. Thsai-Hill Theory is used as a yield criterion. Plastic and residual stresses are determined in the antisymmetric cross-ply and angle-ply laminated plates for small deformations.

Elastic-plastic and residual stress components are given in Tables. Laminate mechanical properties are given in Table 1. Results for the stress components at the beginning of the plastic yielding in the first layer are shown in Table 2 for all material properties. Elastic-plastic, residual stress components, equivalent plastic strain for $[0^o/90^o]_2$ antisymmetric cross-ply and for $[30^o/-30^o]_2$, $[45^o/-45^o]_2$, $[60^o/-60^o]_2$, $[15^0/-15^0]_2$, $[15^0/-30^0]_2$, $[15^0/-45^0]_2$, and $[15^0/-60^0]_2$ antisymmetric angle-ply laminated plates are shown in Table 3 and Table 4 for elastic-perfectly plastic material properties.

4. 2. Tsai-Hill Yield Criterion

Hill used a yield criterion for orthotropic materials:

$$(G+H)\sigma_1^2+(F+H)\sigma_2^2+(F+G)\sigma_3^2-2H\sigma_1\sigma_2-2G\sigma_1\sigma_3-2F\sigma_2\sigma_3 \\ +2L\tau_{23}^2+2M\tau_{13}^2+2N\tau_{12}^2=1 \quad (4.1)$$

This orthotropic yield criterion is an extension of von Mises' yield criterion. von Mises yield criterion has the form

$$\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2=\sigma_Y^2 \quad (4.2)$$

this form is for a two –dimensional state of stress reffered to the principal stress directions.

The failure strength parameters *F, G, H, L, M* and *N* were related to the usual failure strength *X, Y* ans *S* for a lamina (Jones, 1999).

First, if only τ_{12} acts on the body, then, since its maximum value is S;

$$2N=\frac{1}{S^2} \quad (4.3)$$

Similarly, if only σ_1 acts on the body, then

$$G+H=\frac{1}{X^2} \quad (4.4)$$

and if only σ_2 acts on the body

$$F+H=\frac{1}{Y^2} \quad (4.5)$$

the strength in the 3-direction is denoted by *Z* and σ_3 acts, then

$$F+G=\frac{1}{Z^2} \quad (4.6)$$

then upon combination of Eqs. (4.4), (4.5) and (4.6), the following relations between *F, G, H* and *X, Y, Z* results;

$$2H = \frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}$$
$$2G = \frac{1}{X^2} + \frac{1}{Z^2} - \frac{1}{Y^2} \qquad (4.7)$$
$$2F = \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}$$

For plane stress in the 1-2 plane of a unidirectional lamina with fibers in the 1-direction, $\sigma_3 = \tau_{13} = \tau_{23} = 0$. However, from the cross section of such a lamina in Fig.(4.1) Y=Z from geometrical symmetry of the material construction. Thus, Eq.(4.2) leads to

$$\frac{\sigma_1^2}{X^2} - \frac{\sigma_1\sigma_2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\tau_{12}^2}{S^2} = 1 \qquad (4.8)$$

as the governing failure criterion in terms of the familiar lamina principal strength X, Y and S.

In additional, stress-transformation equations;

$$\sigma_1 = \sigma_x \cos^2\theta$$
$$\sigma_2 = \sigma_y \sin^2\theta \qquad (4.9)$$
$$\tau_{12} = -\sigma_x \sin\theta\cos\theta$$

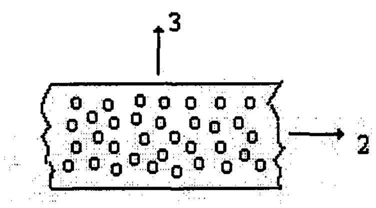

Figure 4. 1 Cross-section of a unidirectional lamina with fibers in the 1-direction.

4. 3. Elastic Solution

The antisymmetric aluminum metal-matrix laminated plate under uniform thermal distribution is shown in Figure 4.2. Its dimensions are chosen as 100mm x 100mm in order to prevent buckling of the plate. It is assumed that the thermal stresses are zero when the temperature is zero. The temperature is chosen to be constant across the thickness of the laminate. The classical lamination theory is used in the solution. The three-dimensional thermo-elastic anisotropic strain-stress relations are written as

$$\varepsilon_i = S_{ij}\sigma_j + \alpha_i T \qquad i,j = 1,2...,6 \tag{4.10}$$

where ε_i, $S_{ij}\sigma_j$ and $\alpha_i T$ are the total strains, mechanical strains and the six thermal strains. S_{ij} is the compliance tensor. The three-dimensional stress-strain relations are obtained by inversion of the above equation:

$$\sigma_i = C_{ij}\left(\varepsilon_j - \alpha_j T\right) \qquad i,j = 1,2...,6 \tag{4.11}$$

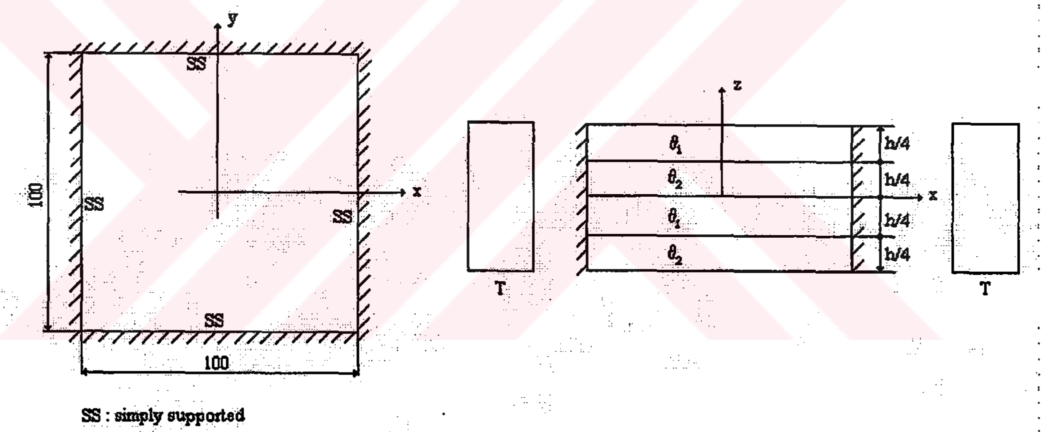

Figure 4. 2 Simply supported antisymmetric composite laminated plate under uniform thermal load.

Elastic solutions are obtained for elastic-perfectly plastic and strain hardening material properties. For each material properties, laminated plates as antisymmetric cross-ply $[0^\circ/90^\circ]_2$ and angle-ply $[30^\circ/-30^\circ]_2$, $[45^\circ/-45^\circ]_2$, $[60^\circ/-60^\circ]_2$, $[15^0/-15^0]_2$, $[15^0/-30^0]_2$, $[15^0/-45^0]_2$, $[15^0/-60^0]_2$ are simply supported at all edges, and subjected to the temperature loads T.

$$\alpha_x = \alpha_1 \cos^2\theta + \alpha_2 \sin^2\theta$$
$$\alpha_y = \alpha_1 \sin^2\theta + \alpha_2 \cos^2\theta \qquad (4.12)$$
$$\alpha_{xy} = 2\cos\theta \sin\theta(\alpha_1 - \alpha_2)$$

α_1 and α_2 are the thermal expansion coefficients in principal material directions.

The Classical Lamination Theory is used during the solution. According to this theory thermal forces and moments can be written as (Reddy, 1997),

$$\{N^T\} = \int [\overline{Q}]_k \{\alpha\}_k T dz = T \sum_{k=1}^{N} [\overline{Q}]_k \{\alpha\}_k (z_k - z_{k-1})$$

and (4.13)

$$\{M^T\} = \int [\overline{Q}]_k \{\alpha\}_k T z dz = \frac{T}{2} \sum_{k=1}^{N} [\overline{Q}]_k \{\alpha\}_k (z_k^2 - z_{k-1}^2)$$

where the subscript k refers to the kth lamina and z is the thickness of the layers.

The general laminate force-deformation equations can be expressed as;

$$\begin{Bmatrix} N^T \\ \hline M^T \end{Bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline B & D \end{array}\right] \begin{Bmatrix} \varepsilon^0 \\ \hline \kappa \end{Bmatrix} \qquad (4.14)$$

the inverted forms of this equation are given by

$$\begin{Bmatrix} \varepsilon^0 \\ \hline \kappa \end{Bmatrix} = \left[\begin{array}{c|c} A' & B' \\ \hline B' & D' \end{array}\right] \begin{Bmatrix} N^T \\ \hline M^T \end{Bmatrix} \qquad (4.15)$$

[A], [B] and [D] are the extensional, coupling and bending stiffnesses matrix respectively. ε^0 and κ are the strain and twist curvature of the middle surface. These midplane strains and curvatures are determinated from thermal forces and moments, then the lamina stresses are obtained from below equations;

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}_k = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}_k \begin{Bmatrix}\varepsilon_x-\alpha_x T\\ \varepsilon_y-\alpha_y T\\ \gamma_{xy}-\alpha_{xy} T\end{Bmatrix} \tag{4.16}$$

The other parameters as Poisson's ratio and stiffness are written as,

$$\upsilon_{21}=\upsilon_{12}\frac{E_2}{E_1},\ Q_{11}^k=\frac{E_1^k}{1-\upsilon_{12}^k\upsilon_{21}^k},\ Q_{12}^k=\frac{\upsilon_{21}^k E_1^k}{1-\upsilon_{12}^k\upsilon_{21}^k}=\frac{\upsilon_{12}^k E_2^k}{1-\upsilon_{12}^k\upsilon_{21}^k},$$

$$Q_{22}^k=\frac{E_2^k}{1-\upsilon_{12}^k\upsilon_{21}^k},\ Q_{66}^k=G_{12}^k,\ Q_{16}^k=0 \text{ and } Q_{26}^k=0.$$

The temperature causing to yield layers can be found by using the stress components. The stress component in the principal material directions are written as,

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix}=\begin{bmatrix}c^2 & s^2 & 2sc\\ s^2 & c^2 & -2sc\\ -sc & sc & c^2-s^2\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} \tag{4.17}$$

where, c=cosθ and s=sinθ.

These principal stresses are substituted in the Tsai-Hill Criterion in order to find the temperature which is causing to yield laminas, (Hyer, 1981).

4. 4. Finding of The Yield Temperature

The stress-strain relation for the laminated plates can be written as;

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & Q_{16} & \bar{Q}_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} \tag{4.18}$$

or strain-stress relation as;

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \quad (4.19)$$

for the thermal case;

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} & \bar{a}_{16} \\ \bar{a}_{12} & \bar{a}_{22} & \bar{a}_{26} \\ \bar{a}_{16} & \bar{a}_{26} & \bar{a}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} + \begin{Bmatrix} \alpha_x T \\ \alpha_y T \\ \alpha_{xy} T \end{Bmatrix} \quad (4.20)$$

where;

$$\bar{a}_{11} = S_{11}Cos^4\theta + (2S_{12} + S_{66})Sin^2\theta Cos^2\theta + S_{22}Sin^4\theta$$
$$\bar{a}_{12} = S_{12}\left(Sin^4\theta + Cos^4\theta\right) + (S_{11} + S_{22} - S_{66})Sin^2\theta Cos^2\theta$$
$$\bar{a}_{22} = S_{11}Sin^4\theta + (2S_{12} + S_{66})Sin^2\theta Cos^2\theta + S_{22}Cos^4\theta$$
$$\bar{a}_{16} = (2S_{11} - 2S_{12} - S_{66})Sin\theta Cos^3\theta - (2S_{22} - 2S_{12} - S_{66})Sin^3\theta Cos\theta$$
$$\bar{a}_{26} = (2S_{11} - 2S_{12} - S_{66})Sin^3\theta Cos\theta - (2S_{22} - 2S_{12} - S_{66})Sin\theta Cos^3\theta$$
$$\bar{a}_{66} = 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})Sin^2\theta Cos^2\theta + S_{66}\left(Sin^4\theta + Cos^4\theta\right)$$

$$S_{11} = \frac{1}{E_1}, \quad S_{12} = -\frac{\nu_{12}}{E_1}, \quad S_{22} = \frac{1}{E_2}, \quad S_{66} = \frac{1}{G_{12}}$$

from the boundary conditions; $\varepsilon_x = 0$, $\varepsilon_y = 0$, $\gamma_{xy} = 0$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} & \bar{a}_{16} \\ \bar{a}_{12} & \bar{a}_{22} & \bar{a}_{26} \\ \bar{a}_{16} & \bar{a}_{26} & \bar{a}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} + \begin{Bmatrix} \alpha_x T \\ \alpha_y T \\ \alpha_{xy} T \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad (4.21)$$

$$\begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} & \bar{a}_{16} \\ \bar{a}_{12} & \bar{a}_{22} & \bar{a}_{26} \\ \bar{a}_{16} & \bar{a}_{26} & \bar{a}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = - \begin{Bmatrix} \alpha_x T \\ \alpha_y T \\ \alpha_{xy} T \end{Bmatrix} \tag{4.22}$$

$$\begin{aligned} \bar{a}_{11}\sigma_x + \bar{a}_{12}\sigma_y + \bar{a}_{16}\tau_{xy} &= -\alpha_x T \\ \bar{a}_{12}\sigma_x + \bar{a}_{22}\sigma_y + \bar{a}_{26}\tau_{xy} &= -\alpha_y T \\ \bar{a}_{16}\sigma_x + \bar{a}_{26}\sigma_y + \bar{a}_{66}\tau_{xy} &= -\alpha_{xy} T \end{aligned} \tag{4.23}$$

from the Eq.(4.23);

$$\tau_{xy} = -\frac{\alpha_{xy}T + \bar{a}_{16}\sigma_x + \bar{a}_{26}\sigma_y}{\bar{a}_{66}} \tag{4.24}$$

$$\bar{a}_{11}\sigma_x + \bar{a}_{12}\sigma_y + \bar{a}_{16}\left(-\frac{\alpha_{xy}T + \bar{a}_{16}\sigma_x + \bar{a}_{26}\sigma_y}{\bar{a}_{66}}\right) = -\alpha_x T$$

$$\bar{a}_{11}\bar{a}_{66}\sigma_x + \bar{a}_{12}\bar{a}_{66}\sigma_y - \bar{a}_{16}\alpha_{xy}T - \bar{a}_{16}^{\,2}\sigma_x - \bar{a}_{26}\bar{a}_{16}\sigma_y = -\alpha_x\bar{a}_{66}T$$

$$\sigma_x\left(\bar{a}_{11}\bar{a}_{66} - \bar{a}_{16}^{\,2}\right) + \sigma_y\left(\bar{a}_{12}\bar{a}_{66} - \bar{a}_{26}\bar{a}_{16}\right) = -\alpha_x\bar{a}_{66}T + \alpha_{xy}\bar{a}_{16}T$$

$$\sigma_x\underbrace{\left(\bar{a}_{11}\bar{a}_{66} - \bar{a}_{16}^{\,2}\right)}_{R_1} + \sigma_y\underbrace{\left(\bar{a}_{12}\bar{a}_{66} - \bar{a}_{26}\bar{a}_{16}\right)}_{R_2} = \underbrace{\left(\alpha_{xy}\bar{a}_{16} - \alpha_x\bar{a}_{66}\right)}_{U_1}T$$

$$\sigma_x R_1 + \sigma_y R_2 = U_1 T \tag{4.25}$$

in the same way;

$$\bar{a}_{12}\sigma_x + \bar{a}_{22}\sigma_y + \bar{a}_{26}\left(-\frac{\alpha_{xy}T + \bar{a}_{16}\sigma_x + \bar{a}_{26}\sigma_y}{\bar{a}_{66}}\right) = -\alpha_y T$$

$$\bar{a}_{12}\bar{a}_{66}\sigma_x + \bar{a}_{22}\bar{a}_{66}\sigma_y - \bar{a}_{26}\alpha_{xy}T - \bar{a}_{26}^{\,2}\sigma_x - \bar{a}_{26}\bar{a}_{16}\sigma_x = -\alpha_y\bar{a}_{66}T$$

$$\sigma_x\left(\bar{a}_{12}\bar{a}_{66} - \bar{a}_{16}\bar{a}_{26}\right) + \sigma_y\left(\bar{a}_{22}\bar{a}_{66} - \bar{a}_{26}^{\,2}\right) = -\alpha_y\bar{a}_{66}T + \alpha_{xy}\bar{a}_{26}T$$

$$\sigma_x\underbrace{\left(\bar{a}_{12}\bar{a}_{66} - \bar{a}_{16}\bar{a}_{26}\right)}_{R_2} + \sigma_y\underbrace{\left(\bar{a}_{22}\bar{a}_{66} - \bar{a}_{26}^{\,2}\right)}_{R_3} = \underbrace{\left(\alpha_{xy}\bar{a}_{26} - \alpha_x\bar{a}_{66}\right)}_{U_2}T$$

$$\sigma_x R_2 + \sigma_y R_3 = U_2 T \tag{4.26}$$

from the Eq.(4.25),

$$\sigma_y = \frac{U_1 T - \sigma_x R_1}{R_2} \tag{4.27}$$

Putting σ_y in the Eq.(4.26) gives the σ_x,

$$\sigma_x R_2 + \frac{U_1 T - \sigma_x R_1}{R_2} R_3 = U_2 T$$

$$\sigma_x R_2 R_2 + R_3 U_1 T - \sigma_x R_1 R_3 = R_2 U_2 T$$

$$\sigma_x \left(R_2^{\,2} - R_1 R_3\right) = \left(R_2 U_2 - R_3 U_1\right) T$$

$$\sigma_x = \underbrace{\frac{\left(R_2 U_2 - R_3 U_1\right)}{\left(R_2^{\,2} - R_1 R_3\right)}}_{f_1} T \tag{4.28}$$

$$\sigma_x = f_1 T \tag{4.29}$$

and substituting the Eq.(4.28) in the Eq.(4.27) gives the σ_y,

$$\sigma_y = \frac{U_1 T - \frac{\left(R_2 U_2 - R_3 U_1\right)}{\left(R_2^{\,2} - R_1 R_3\right)} T R_1}{R_2} = \frac{U_1 T}{R_2} - \frac{R_1}{R_2} \frac{\left(R_2 U_2 - R_3 U_1\right)}{\left(R_2^{\,2} - R_1 R_3\right)} T$$

$$\sigma_y = \left(\frac{U_1}{R_2} - \frac{R_1 R_2 U_2 - R_1 R_3 U_1}{R_2^{\,3} - R_1 R_2 R_3}\right) T$$

$$\sigma_y = \left(\frac{U_1\left(R_2^{\ 3} - R_1R_2R_3\right) - \left(R_1R_2^2U_2 - R_1R_3U_1\right)R_2}{R_2^4 - R_1R_2^2R_3} \right) T$$

$$\sigma_y = \left(\frac{R_2^2\left(U_1R_2 - R_1U_2\right)}{R_2^2\left(R_2^2 - R_1R_3\right)} \right) T$$

$$\sigma_y = \underbrace{\frac{U_1R_2 - R_1U_2}{R_2^2 - R_1R_3}}_{f_2} T \tag{4.30}$$

$$\sigma_y = f_2 T \tag{4.31}$$

and, putting the Eq.(4.29) and the Eq.(4.31) in the Eq.(4.24) gives τ_{xy};

$$\tau_{xy} = -\frac{\alpha_{xy}T + \bar{a}_{16}\sigma_x + \bar{a}_{26}\sigma_y}{\bar{a}_{66}} = -\frac{\alpha_{xy}T + \bar{a}_{16}f_1T + \bar{a}_{26}f_2T}{\bar{a}_{66}}$$

$$\tau_{xy} = \underbrace{-\frac{\alpha_{xy} + \bar{a}_{16}f_1 + \bar{a}_{26}f_2}{\bar{a}_{66}}}_{f_3} T$$

$$\tau_{xy} = f_3 T \tag{4.32}$$

thus, the stresses as;

$$\begin{aligned} \sigma_x &= f_1 T \\ \sigma_y &= f_2 T \\ \tau_{xy} &= f_3 T \end{aligned} \tag{4.33}$$

then stress components are transformed to the principal material directions as,

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -2mn & 2mn & m^2 - n^2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \tag{4.34}$$

where m=cosθ, n=sinθ.

from here,

$$\sigma_1 = \underbrace{\left(\cos^2\theta f_1 + \sin^2\theta f_2 + 2\cos\theta\sin\theta f_3\right)}_{F_1}T$$
$$\sigma_1 = F_1 T \tag{4.35}$$

$$\sigma_2 = \underbrace{\left(\sin^2\theta f_1 + \cos^2\theta f_2 - 2\cos\theta\sin\theta f_3\right)}_{F_2}T$$
$$\sigma_2 = F_2 T \tag{4.36}$$

$$\tau_{12} = \underbrace{\left(-\cos\theta\sin\theta f_1 + \cos\theta\sin\theta f_2 + \left(\cos^2\theta - \sin^2\theta\right)f_3\right)}_{F_3}T$$
$$\tau_{12} = F_3 T \tag{4.37}$$

from the Tsai-Hill yielding criteria ;

$$\bar{\sigma}^2 = \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\frac{X^2}{Y^2} + \tau_{12}^2\frac{X^2}{S^2} \tag{4.38}$$

at the yielding point;

$$\bar{\sigma} = X \Rightarrow X^2 = \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\frac{X^2}{Y^2} + \tau_{12}^2\frac{X^2}{S^2}$$
$$X^2 = F_1^2T^2 - F_1F_2T^2 + F_2^2\frac{X^2}{Y^2}T^2 + F_3^2\frac{X^2}{S^2}T^2$$
$$X^2 = \left(F_1^2 - F_1F_2 + F_2^2\frac{X^2}{Y^2} + F_3^2\frac{X^2}{S^2}\right)T^2$$

from here,

$$T^2 = \frac{X}{\left(F_1^2 - F_1F_2 + F_2^2\frac{X^2}{Y^2} + F_3^2\frac{X^2}{S^2}\right)}$$

and

$$T = \frac{X}{\sqrt{F_1^2 - F_1F_2 + F_2^2\frac{X^2}{Y^2} + F_3^2\frac{X^2}{S^2}}} \tag{4.39}$$

4. 5. Plastic Solution

4. 5. 1. For Elastic-Perfectly Plastic Materials

For the elastic-plastic solution, the Tsai-Hill theory is used as a yield criterion due to the same yield points in tension and compression for aluminum metal-matrix composite layers. The yield strengths in the first and second principal material directions are X and Y, respectively. It is assumed that the yield strength in the third principal material direction (Z) to be equal to the transverse yield strength (Y). The shear strengths in the 2-3 and 1-3 planes are assumed to be equal to the shear strength (S) in the 1-2 plane.

In this chapter, cross-ply and angle-ply laminated plates are evaluated for elastic-perfectly plastic material properties. For those laminated plates, solutions are obtained by using incremental stress method.

The yield condition for Tsai-Hill criterion can be written as (Jones, 1999);

$$\frac{\sigma_1^2}{X^2} - \frac{\sigma_1\sigma_2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\tau_{12}^2}{S^2} = 1 \tag{4.40}$$

And multiplying it by X, gives the equivalent stress in the first principal material direction

$$\bar{\sigma} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \frac{\sigma_2^2X^2}{Y^2} + \tau_{12}^2\frac{X^2}{S^2}} \tag{4.41}$$

where equivelent stress $\bar{\sigma}$ is equal to X. For an perfectly plastic material, the yield stress is given as;

$$\sigma_Y = \sigma_0 = X \tag{4.42}$$

The plastic strain increments in the principal material directions are obtained by using the potential function $f = \bar{\sigma} - \sigma_Y(\varepsilon_p)$ which is

$$f = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \frac{\sigma_2^2 X^2}{Y^2} + \tau_{12}^2 \frac{X^2}{S^2}} - \bar{\sigma} \tag{4.43}$$

and the strain increments are as:

$$\begin{Bmatrix} d\varepsilon_1^P \\ d\varepsilon_2^P \\ d\varepsilon_{12}^P \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial f}{\partial \sigma_1} d\lambda \\ \dfrac{\partial f}{\partial \sigma_2} d\lambda \\ \dfrac{\partial f}{\partial \tau_{12}} d\lambda \end{Bmatrix} \tag{4.44}$$

Plastic strain increments can be written as; $d\lambda = d\varepsilon_P$, and the solution of these equations may be impossible. The total strain increments in the principal material directions can be written as,

$$\begin{aligned} d\varepsilon_1 &= a_{11} d\sigma_1 + a_{12} d\sigma_2 + \frac{2\sigma_1 - \sigma_2}{2\bar{\sigma}} d\varepsilon_p + \alpha_1 dT \\ d\varepsilon_2 &= a_{12} d\sigma_1 + a_{22} d\sigma_{12} + \frac{-\sigma_1 + 2\sigma_2 \dfrac{X^2}{Y^2}}{2\bar{\sigma}} d\varepsilon_p + \alpha_2 dT \\ d\varepsilon_{12} &= \frac{a_{16}}{2} d\tau_{12} + 2\tau_{12} \frac{\dfrac{X^2}{S^2}}{2\bar{\sigma}} d\varepsilon_p \end{aligned} \tag{4.45}$$

where;

$$a_{11} = \frac{1}{E_{11}},\ a_{12} = -\frac{\nu_{12}}{E_{11}},\ a_{22} = \frac{1}{E_{22}} \text{ and } a_{66} = \frac{1}{G_{12}}.$$

The total strain increments in the x-y plane can be obtained by using the transformation formula,

$$\begin{Bmatrix} d\varepsilon_x \\ d\varepsilon_y \\ d\varepsilon_{xy} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\varepsilon_{12} \end{Bmatrix} \tag{4.46}$$

where, c=cosθ and s=sinθ. For this simply supported composite plate, boundary conditions enable that the strains are assumed zero, $\varepsilon_x = \varepsilon_y = \varepsilon_{xy} = 0$. So that strain increments, $d\varepsilon_x = d\varepsilon_y = d\varepsilon_{xy} = 0$, are zero as well. In these equations, σ_1, σ_2 and τ_{12} are stresses at the beginning of the yielding of the lamina. Eq.(4.45) is written as;

$$\begin{aligned}
d\varepsilon_x &= (a_{11}c^2 + a_{12}s^2)d\sigma_1 + (a_{12}c^2 + a_{22}s^2)d\sigma_2 - a_{66}csd\tau_{12} \\
&\quad + (\frac{2\sigma_1 - \sigma_2}{2\bar{\sigma}}c^2 + \frac{-\sigma_1 + 2\sigma_2 \frac{X^2}{Y^2}}{2\bar{\sigma}}s^2 - \frac{4\tau_{12}\frac{X^2}{Y^2}}{2\bar{\sigma}}cs)d\varepsilon_p + \left(\alpha_1 c^2 + \alpha_2 s^2\right)dT = 0 \\
d\varepsilon_y &= (a_{11}s^2 + a_{12}c^2)d\sigma_1 + (a_{12}s^2 + a_{22}c^2)d\sigma_2 + a_{66}csd\tau_{12} \\
&\quad + (\frac{2\sigma_1 - \sigma_2}{2\bar{\sigma}}s^2 + \frac{-\sigma_1 + 2\sigma_2 \frac{X^2}{Y^2}}{2\bar{\sigma}}c^2 + \frac{4\tau_{12}\frac{X^2}{S^2}}{2\bar{\sigma}}cs)d\varepsilon_p + \left(\alpha_1 s^2 + \alpha_2 c^2\right)dT = 0 \qquad (4.47) \\
d\varepsilon_{xy} &= (a_{11} - a_{12})csd\sigma_1 + (a_{12} - a_{22})csd\sigma_2 + \frac{a_{66}}{2}(c^2 - s^2)d\tau_{12} \\
&\quad + \left[\frac{2\sigma_1 - \sigma_2}{2\bar{\sigma}}cs - \frac{-\sigma_1 + 2\sigma_2 \frac{X^2}{Y^2}}{2\bar{\sigma}}cs + \frac{2\tau_{12}\frac{X^2}{S^2}}{2\bar{\sigma}}(c^2 - s^2)\right]d\varepsilon_p + (\alpha_1 - \alpha_2)cs\,dT = 0
\end{aligned}$$

Form Eq. (4.40) and Eq. (4.42), the Tsai-Hill Criterion can be written as,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 \frac{X^2}{Y^2} + \tau_{12}^2 \frac{X^2}{S^2} - X^2 = 0 \tag{4.48}$$

where $\sigma_0 = X$ since the yielding takes place in the first principal material direction.

Differentiation of the Tsai-Hill criterion gives:

$$\Delta\sigma_1(2\sigma_1 - \sigma_2) + \Delta\sigma_2\left(-\sigma_1 + 2\sigma_2 \frac{X^2}{Y^2}\right) + 2\tau_{12}\frac{X^2}{S^2}\Delta\tau_{12} = 0 \tag{4.49}$$

They can be written in differential form,

$$\begin{aligned}
\Delta\varepsilon_x &= a_1\Delta\sigma_1 + b_1\Delta\sigma_2 + c_1\Delta\tau_{12} + d_1\Delta\varepsilon_p + e_1\Delta T = 0 \\
\Delta\varepsilon_y &= a_2\Delta\sigma_1 + b_2\Delta\sigma_2 + c_2\Delta\tau_{12} + d_2\Delta\varepsilon_p + e_2\Delta T = 0 \\
\Delta\varepsilon_{xy} &= a_3\Delta\sigma_1 + b_3\Delta\sigma_2 + c_3\Delta\tau_{12} + d_3\Delta\varepsilon_p + e_3\Delta T = 0 \\
& a_4\Delta\sigma_1 + b_4\Delta\sigma_2 + c_4\Delta\tau_{12} + d_4\Delta\varepsilon_p = 0.
\end{aligned} \tag{4.50}$$

where $\Delta\sigma_1$, $\Delta\sigma_2$ and $\Delta\tau_{12}$ are incremental stresses at yielding and after yielding. Incremental plastic strain and stresses can be obtained from Eq.(4.50) and the stress components and the equivalent plastic strain are calculated for 1000 load steps in different temperatures,

$$\begin{aligned}
\sigma_1^{(i)} &= \sigma_1^{(i-1)} + \Delta\sigma_1^{(i)} \\
\sigma_2^{(i)} &= \sigma_2^{(i-1)} + \Delta\sigma_2^{(i)} \\
\tau_{12}^{(i)} &= \tau_{12}^{(i-1)} + \Delta\tau_{12}^{(i)} \\
\varepsilon_p^{(i)} &= \varepsilon_p^{(i-1)} + \Delta\varepsilon_p^{(i)}
\end{aligned} \tag{4.51}$$

where i is the number of the load step.

4. 6. Residual Stresses

To find the residual stresses it is necessary to superpose on the stress system due to T, a completely elastic system due to temperature –*T*. The load found in the plastic solution is used in the opposite direction in order to obtain the complete elastic solution. The superposition of the elastic-plastic and elastic stresses gives the residual stresses. The resultant forces and moments per unit length after elastic solution are written as,

$$\begin{Bmatrix} N_x^p \\ N_y^p \\ N_{xy}^p \end{Bmatrix} = \int \begin{Bmatrix} \sigma_x^p \\ \sigma_y^p \\ \tau_{xy}^p \end{Bmatrix}_k dz \tag{4.52}$$

$$\begin{Bmatrix} M_x^p \\ M_y^p \\ M_{xy}^p \end{Bmatrix} = \int \begin{Bmatrix} \sigma_x^p \\ \sigma_y^p \\ \tau_{xy}^p \end{Bmatrix}_k z\,dz \tag{4.53}$$

$$\begin{Bmatrix} N_x^e \\ N_y^e \\ N_{xy}^e \end{Bmatrix} = \int \begin{Bmatrix} \sigma_x^e \\ \sigma_y^e \\ \tau_{xy}^e \end{Bmatrix}_k dz \tag{4.54}$$

$$\begin{Bmatrix} M_x^e \\ M_y^e \\ M_{xy}^e \end{Bmatrix} = \int \begin{Bmatrix} \sigma_x^e \\ \sigma_y^e \\ \tau_{xy}^e \end{Bmatrix}_k z\,dz \tag{4.55}$$

These two kinds of resultant forces and moments are similar or it can be said that the slope of the loading and unloading curves is the same. When the temperature attains to zero degree, the elastic stress components are calculated as,

$$\sigma_x^e = \frac{(\sigma_x)_1^e N_x^p}{N_x^e} + \frac{(\sigma_x)_2^e M_x^p}{M_x^e}$$

$$\sigma_y^e = \frac{(\sigma_y)_1^e N_y^p}{N_y^e} + \frac{(\sigma_y)_2^e M_y^p}{M_y^e} \tag{4.56}$$

$$\tau_{xy}^e = \frac{(\tau_{xy})_1^e N_{xy}^p}{N_{xy}^e} + \frac{(\tau_{xy})_2^e M_{xy}^p}{M_{xy}^e}$$

where

$$\begin{Bmatrix} (\sigma_x)_1^e \\ (\sigma_y)_1^e \\ (\tau_{xy})_1^e \end{Bmatrix}_k = \begin{vmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{vmatrix}_k \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \tag{4.57}$$

$$\begin{Bmatrix} (\sigma_x)_2^e \\ (\sigma_y)_2^e \\ (\tau_{xy})_2^e \end{Bmatrix}_k = \begin{vmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{vmatrix}_k \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} z \tag{4.58}$$

The superposition of the elastic and plastic stresses gives the residual stress components as,

$$\sigma_x^r = \sigma_x^p - \sigma_x^e$$

$$\sigma_y^r = \sigma_y^p - \sigma_y^e \tag{4.59}$$

$$\tau_{xy}^r = \tau_{xy}^p - \tau_{xy}^e$$

4. 7. Production of Laminated Plates

The composite layer consist of stainless steel fiber and aluminum matrix were produced by using moulds having upper and lower parts which were heated by means of electrical resistance. During manufacturing of the composite plates, the production set was insulated to prevent heat loss and corrosion of the componenets of composite material, and to supply a convenient heat distribution [16]. The hydraulic press provided to obtain a pressure of 30 MPa to the upper mould. Manufacturing set was heated up to 600^0C. Under these condutions the steel fiber and aluminum matrix were bounded because of the yield strength of aluminum was exceeded.

Elastic properties and yield points of materials were obtained using an Instron machine and strain-gage. Forces were applied to the layer in principal material directions, and then yield points of the layer were found.

4. 8. Results and Discussion

In this study, an elastic-plastic stress analysis is performed in antisymmetric aluminum metal-matrix laminated plates under thermal load. All the results are obtained by using the analytical solution. Laminate mechanical properties are given in Table 1. The composite material is assumed to be perfectly plastic. The thickness of each layer is 2mm and its dimensions (100x100mm) are chosen small in order to prevent buckling. Four layers are bonded in order to form antisymmetric laminated plates. The plate is subjected to a uniform temperature distribution across the thickness. The plates are assumed that they do not have stresses at the temperature of 0^0C.

The temperature causing to yield the plates is found as 11.26 ^{0}C. For all the laminated plates, stress components at the beginning of the plastic yielding in the first and second layers are shown in Table 2. As seen from this table, σ_x and σ_y are greater than τ_{xy}.

The elastic-plastic stresses, residual stresses and equivalent plastic strains for the antisymmetric plates are given in Tables 3-4-5-6-7-8-9 and 10, for the $[0^o/90^o]_2$, $[30^o/-30^o]_2$, $[45^o/-45^o]_2$, $[60^o/-60^o]_2$, $[15^0/-15^0]_2$, $[15^0/-30^0]_2$, $[15^0/-45^0]_2$, $[15^0/-60^0]_2$ orientations, respectively.

Results for the antisymmetric cross-ply laminated plate, $[0^o/90^o]_2$, are shown in Table 3. As seen from this table, the intensity of the stress component in the first principal material direction is greater than that in the second principal material direction. Therefore, 0^o oriented layers (first and third layers) produce tensile stresses on the 90^0 oriented layers (second and fourth layer). Furtermore, the residual stress componenets on the first and second layer are the same for 0^o and 90^o orientation angles, and shear stress components are zero for the cross-ply laminated plate. If we glance at all tables, we see that the residual stress components are zero except $[0^o/90^o]_2$, $[15^0/-30^0]_2$, $[15^0/-45^0]_2$, $[15^0/-60^0]_2$ orientations.

Results for the antisymmetric angle-ply laminated plate are presented in other tables. The residual stress components are zero for $[30^o/-30^o]_2$, $[45^o/-45^o]_2$, $[60^o/-60^o]_2$, $[15^0/-15^0]_2$ orientations. But for the other angle-ply laminated plates, $[15^0/-30^0]_2$, $[15^0/-45^0]_2$, $[15^0/-60^0]_2$, residual stress components are not zero. At these tables, the intensity of residual stresses increases depending on the temperature increment.

Table 4. 1 Mechanical properties and yield strengths of the composite layer.

E_1 (MPa)	E_2 (MPa)	G_{12} (MPa)	ν_{12}	Axial strength X (MPa)	Transverse strength Y (MPa)	Shear strength S (MPa)	Thickness of each layer (mm)	Thermal expansion coefficient (1/°C)	
								α_1	α_2
85000	74000	30000	0.3	230.0	24.0	48.9	2	18.5E-06	21.0E-06

Table 4. 2 Stress components at the beginning of the plastic yielding in the first and second layer.

		T (°C)	(σ_x) (MPa)		(σ_y) (MPa)		(τ_{xy}) (MPa)	
Layer Number			1	2	1	2	1	2
Orientation angles	$[0^0/90^0]_2$	11.26	-24.90	-23.99	-23.99	-24.90	0.00	0.00
	$[30^0/-30^0]_2$	11.26	-24.68	-24.68	-24.22	-24.22	-0.39	0.39
	$[45^0/-45^0]_2$	11.26	-24.45	-24.45	-24.45	-24.45	-0.46	0.46
	$[60^0/-60^0]_2$	11.26	-24.22	-24.22	-24.68	-24.68	-0.39	0.39
	$[15^0/-15^0]_2$	11.26	-24.84	-24.84	-24.06	-24.06	-0.23	0.23
	$[15^0/-30^0]_2$	11.26	-24.84	-24.68	-24.06	-24.22	-0.23	0.39
	$[15^0/-45^0]_2$	11.26	-24.84	-24.45	-24.06	-24.45	-0.23	0.46
	$[15^0/-60^0]_2$	11.26	-24.84	-24.22	-24.06	-24.67	-0.23	0.39

Table 4. 3 Elastic-plastic, residual stress components and equivalent plastic strain for antisymmetric cross-ply, $[0^o/90^o]_2$, laminated plate.

Oriantation angle(θ)	T (°C)	Layer number	$(\sigma_1)_p$ (MPa)	$(\sigma_2)_p$ (MPa)	$(\tau_{12})_p$ (MPa)	$(\sigma_1)_r$ (MPa)	$(\sigma_2)_r$ (MPa)	$(\tau_{12})_r$ (MPa)	$\varepsilon_p.10^{-4}$
$[0^o/90^o]_2$	20	1	-38.43	-23.88	0.00	-5.58	5.60	0.00	1
		2	-23.88	-38.43	0.00	5.60	-5.58	0.00	
	25	1	-46.10	-23.77	0.00	-9.27	9.29	0.00	
		2	-23.77	-46.10	0.00	9.29	-9.27	0.00	
	30	1	-53.73	-23.64	0.00	-12.94	12.96	0.00	2
		2	-23.64	-53.73	0.00	12.96	-12.94	0.00	
	35	1	-61.30	-23.47	0.00	-16.61	16.63	0.00	
		2	-23.47	-61.30	0.00	16.63	-16.61	0.00	
	40	1	-68.82	-23.29	0.00	-20.26	20.29	0.00	3
		2	-23.29	-68.82	0.00	20.29	-20.26	0.00	
	45	1	-76.28	-23.07	0.00	-23.90	23.93	0.00	
		2	-23.07	-76.28	0.00	23.93	-23.90	0.00	
	50	1	-83.69	-22.82	0.00	-27.54	27.57	0.00	4
		2	-22.82	-83.69	0.00	27.57	-27.54	0.00	

Table 4. 4 Elastic-plastic, residual stress components and equivalent plastic strain for antisymmetric angle-ply, [30°/-30°]₂ , laminated plate.

Oriantation angle(θ)	T (^{0}C)	Layer number	$(\sigma_1)_p$ (MPa)	$(\sigma_2)_p$ (MPa)	$(\tau_{12})_p$ (MPa)	$(\sigma_1)_r$ (MPa)	$(\sigma_2)_r$ (MPa)	$(\tau_{12})_r$ (MPa)	$\varepsilon_p.10^{-4}$
[30°/-30°]$_2$	20	1	-34.80	-27.51	-6.30	0.00	0.00	0.00	1
		2	-34.80	-27.51	6.30	0.00	0.00	0.00	
	25	1	-40.51	-29.35	-9.68	0.00	0.00	0.00	
		2	-40.51	-29.35	9.68	0.00	0.00	0.00	
	30	1	-46.20	-31.15	-13.03	0.00	0.00	0.00	2
		2	-46.20	-31.15	13.03	0.00	0.00	0.00	
	35	1	-51.83	-32.93	-16.38	0.00	0.00	0.00	
		2	-51.83	-32.93	16.38	0.00	0.00	0.00	
	40	1	-57.43	-34.66	-19.72	0.00	0.00	0.00	3
		2	-57.43	-34.66	19.72	0.00	0.00	0.00	
	45	1	-62.98	-36.36	-23.04	0.00	0.00	0.00	
		2	-62.98	-36.36	23.04	0.00	0.00	0.00	
	50	1	-68.47	-38.04	-26.36	0.00	0.00	0.00	4
		2	-68.47	-38.04	26.36	0.00	0.00	0.00	

Table 4. 5 Elastic-plastic, residual stress components and equivalent plastic strain for antisymmetric angle-ply, [45°/-45°]₂ , laminated plate.

Oriantation angle(θ)	T (^{0}C)	Layer number	$(\sigma_1)_p$ (MPa)	$(\sigma_2)_p$ (MPa)	$(\tau_{12})_p$ (MPa)	$(\sigma_1)_r$ (MPa)	$(\sigma_2)_r$ (MPa)	$(\tau_{12})_r$ (MPa)	$\varepsilon_p.10^{-4}$
[45°/-45°]$_2$	20	1	-31.19	-31.11	-7.28	0.00	00.00	0.00	1
		2	-31.19	-31.11	7.28	0.00	00.00	0.00	
	25	1	-34.98	-34.89	-11.17	0.00	0.00	0.00	
		2	-34.98	-34.89	11.17	0.00	0.00	0.00	
	30	1	-38.71	-38.64	-15.05	0.00	0.00	0.00	2
		2	-38.71	-38.64	15.05	0.00	0.00	0.00	
	35	1	-42.43	-42.32	-18.91	0.00	0.00	0.00	
		2	-42.43	-42.32	18.91	0.00	0.00	0.00	
	40	1	-46.11	-45.97	-22.77	0.00	0.00	0.00	3
		2	-46.11	-45.97	22.77	0.00	0.00	0.00	
	45	1	-49.74	-49.59	-26.61	0.00	0.00	0.00	
		2	-49.74	-49.59	26.61	0.00	0.00	0.00	
	50	1	-53.34	-53.12	-30.43	0.00	0.00	0.00	4
		2	-53.34	-53.12	30.43	0.00	0.00	0.00	

Table 4. 6 Elastic-plastic, residual stress components and equivalent plastic strain for antisymmetric angle-ply, [60°/-60°]₂ , laminated plate.

Oriantation angle(θ)	T (^{0}C)	Layer number	$(\sigma_1)_p$ (MPa)	$(\sigma_2)_p$ (MPa)	$(\tau_{12})_p$ (MPa)	$(\sigma_1)_r$ (MPa)	$(\sigma_2)_r$ (MPa)	$(\tau_{12})_r$ (MPa)	$\varepsilon_p .10^{-4}$
[60°/-60°]$_2$	20	1	-27.51	-34.79	-6.30	0.00	0.00	0.00	1
		2	-27.51	-34.79	6.30	0.00	0.00	0.00	
	25	1	-29.34	-40.51	-9.66	0.00	0.00	0.00	
		2	-29.34	-40.51	9.66	0.00	0.00	0.00	
	30	1	-31.17	-46.17	-13.04	0.00	0.00	0.00	2
		2	-31.17	-46.17	13.04	0.00	0.00	0.00	
	35	1	-32.92	-51.83	-16.37	0.00	0.00	0.00	
		2	-32.92	-51.83	16.37	0.00	0.00	0.00	
	40	1	-34.68	-57.39	-19.72	0.00	0.00	0.00	3
		2	-34.68	-57.39	19.72	0.00	0.00	0.00	
	45	1	-36.40	-62.91	-23.05	0.00	0.00	0.00	
		2	-36.40	-62.91	23.05	0.00	0.00	0.00	
	50	1	-38.04	-68.43	-26.35	0.00	0.00	0.00	4
		2	-38.04	-68.43	26.35	0.00	0.00	0.00	

Table 4. 7 Elastic-plastic, residual stress components and equivalent plastic strain for antisymmetric angle-ply, [15°/-15°]₂ , laminated plate.

Oriantation angle(θ)	T (^{0}C)	Layer number	$(\sigma_1)_p$ (MPa)	$(\sigma_2)_p$ (MPa)	$(\tau_{12})_p$ (MPa)	$(\sigma_1)_r$ (MPa)	$(\sigma_2)_r$ (MPa)	$(\tau_{12})_r$ (MPa)	$\varepsilon_p .10^{-4}$
[15°/-15°]$_2$	20	1	-37.42	-24.87	-3.68	0.00	0.00	0.00	1
		2	-37.42	-24.87	3.68	0.00	0.00	0.00	
	25	1	-44.59	-25.26	-5.58	0.00	0.00	0.00	
		2	-44.59	-25.26	5.58	0.00	0.00	0.00	
	30	1	-51.63	-25.61	-7.45	0.00	0.00	0.00	2
		2	-51.63	-25.61	7.45	0.00	0.00	0.00	
	35	1	-58.78	-25.96	-9.37	0.00	0.00	0.00	
		2	-58.78	-25.96	-9.37	0.00	0.00	0.00	
	40	1	-65.74	-26.32	-11.36	0.00	0.00	0.00	3
		2	-65.74	-26.32	11.36	0.00	0.00	0.00	
	45	1	-72.71	-26.59	-13.23	0.00	0.00	0.00	
		2	-72.71	-26.59	13.23	0.00	0.00	0.00	
	50	1	-79.60	-26.86	-15.15	0.00	0.00	0.00	4
		2	-79.60	-26.86	-15.15	0.00	0.00	0.00	

Table 4. 8 Elastic-plastic, residual stress components and equivalent plastic strain for antisymmetric angle-ply, [15°/-30°]$_2$, laminated plate.

Oriantation angle(θ)	T (^{0}C)	Layer number	$(\sigma_1)_p$ (MPa)	$(\sigma_2)_p$ (MPa)	$(\tau_{12})_p$ (MPa)	$(\sigma_1)_r$ (MPa)	$(\sigma_2)_r$ (MPa)	$(\tau_{12})_r$ (MPa)	$\varepsilon_p.10^{-4}$
[15°/-30°]$_2$	20	1	-37.42	-24.87	-3.68	-0.96	1.06	-0.03	1
		2	-34.80	-27.51	6.30	0.97	-1.06	-0.03	
	25	1	-44.59	-25.26	-5.59	-1.62	1.78	0.00	
		2	-40.52	-29.35	9.68	1.63	-1.78	0.00	
	30	1	-51.73	-25.61	-7.45	-2.28	2.49	0.05	2
		2	-46.20	-31.15	13.03	2.30	-2.49	0.05	
	35	1	-58.78	-25.96	-9.37	-2.93	3.19	0.05	
		2	-51.84	-32.93	16.38	2.93	-3.19	0.05	
	40	1	-65.74	-26.32	-11.36	-3.55	3.87	0.01	3
		2	-57.43	-34.67	19.72	3.57	-3.87	0.01	
	45	1	-72.71	-26.59	-13.23	-4.20	4.58	0.05	
		2	-62.98	-36.36	23.04	4.22	-4.58	0.05	
	50	1	-79.59	-26.86	-15.15	-4.83	5.27	0.04	4
		2	-68.47	-38.04	26.36	4.86	-5.27	0.04	

Table 4. 9 Elastic-plastic, residual stress components and equivalent plastic strain for antisymmetric angle-ply, [15°/-45°]$_2$, laminated plate.

Oriantation angle(θ)	T (^{0}C)	Layer number	$(\sigma_1)_p$ (MPa)	$(\sigma_2)_p$ (MPa)	$(\tau_{12})_p$ (MPa)	$(\sigma_1)_r$ (MPa)	$(\sigma_2)_r$ (MPa)	$(\tau_{12})_r$ (MPa)	$\varepsilon_p.10^{-4}$
[15°/-45°]$_2$	20	1	-37.42	-24.87	-3.68	-2.33	2.48	-0.03	1
		2	-31.18	-31.13	-7.28	2.34	-2.48	-0.03	
	25	1	-44.59	-25.26	-5.59	-3.89	4.12	0.00	
		2	-34.97	-34.90	-11.17	3.90	-4.12	0.00	
	30	1	-51.73	-25.61	-7.45	-5.46	5.77	0.05	2
		2	-38.73	-38.63	-15.05	5.47	-5.76	0.05	
	35	1	-58.78	-25.96	-9.37	-7.01	7.39	0.06	
		2	-42.45	-42.32	-18.92	7.01	-7.39	0.06	
	40	1	-65.74	-26.32	-11.36	-8.52	8.99	0.01	3
		2	-46.13	-45.97	-22.77	8.53	-8.99	0.01	
	45	1	-72.71	-26.59	-13.23	-10.07	10.62	0.05	
		2	-49.76	-49.58	-26.61	10.08	-10.62	0.05	
	50	1	-79.60	-26.86	-15.15	-11.59	12.21	0.04	4
		2	-53.36	-53.15	-30.44	11.61	-12.21	0.04	

Table 4. 10 Elastic-plastic, residual stress components and equivalent plastic strain for antisymmetric angle-ply, [15°/-60°]₂ , laminated plate.

Oriantation angle(θ)	T (^{0}C)	Layer number	$(\sigma_1)_p$ (MPa)	$(\sigma_2)_p$ (MPa)	$(\tau_{12})_p$ (MPa)	$(\sigma_1)_r$ (MPa)	$(\sigma_2)_r$ (MPa)	$(\tau_{12})_r$ (MPa)	$\varepsilon_p.10^{-4}$
[15°/-60°]$_2$	20	1	-37.42	-24.87	-3.68	-3.77	3.87	-0.03	1
		2	-27.51	-34.79	-6.30	3.77	-3.87	-0.03	
	25	1	-44.59	-25.26	-5.58	-6.27	6.42	0.00	
		2	-29.34	-40.51	-9.66	6.28	-6.42	0.00	
	30	1	-51.73	-25.61	-7.45	-8.76	8.97	0.05	2
		2	-31.17	-46.17	-13.04	8.77	-8.96	0.05	
	35	1	-58.78	-25.96	-9.37	-11.25	11.51	0.05	
		2	-32.91	-51.83	-16.37	11.27	-11.50	0.05	
	40	1	-65.74	-26.32	-11.36	-13.69	14.00	0.01	3
		2	-34.67	-57.39	-19.72	13.71	-13.99	0.01	
	45	1	-72.71	-26.59	-13.23	-16.16	16.52	0.05	
		2	-36.40	-62.91	-23.05	16.18	-16.51	0.05	
	50	1	-79.60	-26.86	-15.15	-18.63	19.04	0.04	4
		2	-38.04	-68.43	-26.35	18.64	-19.03	0.04	

CHAPTER 5
THERMAL ELASTIC-PLASTIC ANALYSIS OF LAMINATED PLATES FOR LINEAR HARDENING MATERIAL

5. 1. Plastic Solution

The solution is carried out for small plastic deformations. The aluminum metal-matrix composite layers possess the same yield strengths in tension and compression, therefore the Tsai-Hill Criterion is used in the solution. The yield strengths in the first and second principal material directions are X and Y, respectively. It is assumed that the yield strength in the third principal material direction (Z) to be equal to the transverse yield strength (Y). The shear strengths in the 2-3 and 1-3 planes are assumed to be equal to the shear strength (S) in the 1-2 plane. The equivalent stress in the first principal material direction is written as (Jones, 1999),

$$\bar{\sigma}^2 = \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 \frac{X^2}{Y^2} + \tau_{12}^2 \frac{X^2}{S^2} \qquad (5.1)$$

where the equivalent stress at the beginning of plastic yielding is equal to X. The composite layer is assumed to harden linearly. It is written in the Ludwik equation as

$$\bar{\sigma} = \sigma_Y = \sigma_0 + K\varepsilon_p \qquad (5.2)$$

where σ_0 is equal to X and K is the plasticity constant. ε_p denotes the equivalent plastic strain. The plastic strain increments in the principal material directions are determined by using potential function (Owen, 1980), $f = \bar{\sigma} - \sigma_Y(\varepsilon_p)$;

$$\begin{Bmatrix} d\varepsilon_1^P \\ d\varepsilon_2^P \\ d\varepsilon_{12}^P \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial f}{\partial \sigma_1} d\lambda \\ \dfrac{\partial f}{\partial \sigma_2} d\lambda \\ \dfrac{\partial f}{\partial \tau_{12}} d\lambda \end{Bmatrix} \tag{5.3}$$

where $d\lambda$ is equal to the equivalent plastic strain increment $d\varepsilon_p$. The total strain increments in the principal material directions can be written as,

$$d\varepsilon_1 = a_{11} d\sigma_1 + a_{12} d\sigma_2 + \frac{2\sigma_1 - \sigma_2}{2\bar{\sigma}} d\varepsilon_p + \alpha_1 dT$$

$$d\varepsilon_2 = a_{12} d\sigma_1 + a_{22} d\sigma_{12} + \frac{-\sigma_1 + 2\sigma_2 \dfrac{X^2}{Y^2}}{2\bar{\sigma}} d\varepsilon_p + \alpha_2 dT \tag{5.4}$$

$$d\varepsilon_{12} = \frac{a_{16}}{2} d\tau_{12} + 2\tau_{12} \frac{\dfrac{X^2}{S^2}}{2\bar{\sigma}} d\varepsilon_p$$

The total strain increments in the x-y plane can be obtained by using the transformation formula,

$$\begin{Bmatrix} d\varepsilon_x \\ d\varepsilon_y \\ d\varepsilon_{xy} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\varepsilon_{12} \end{Bmatrix} \tag{5.5}$$

where $c = \cos\theta$, $s = \sin\theta$. The strain components ε_x, ε_y and ε_{xy} are zero due to the perfectly restrained boundary conditions of the laminated plate. As a result of this, $d\varepsilon_x$, $d\varepsilon_y$ and $d\varepsilon_{xy}$ are equal to zero. They are written as,

$$d\varepsilon_x = (a_{11}c^2 + a_{12}s^2) d\sigma_1 + (a_{12}c^2 + a_{22}s^2) d\sigma_2 - a_{66} csd\tau_{12}$$
$$+ (\frac{2\sigma_1 - \sigma_2}{2\sigma_Y} c^2 + \frac{-\sigma_1 + 2\sigma_2 \dfrac{X^2}{Y^2}}{2\sigma_Y} s^2 - \frac{4\tau_{12} \dfrac{X^2}{Y^2}}{2\sigma_Y} cs) d\varepsilon_p + (\alpha_1 c^2 + \alpha_2 s^2) dT = 0$$

$$d\varepsilon_y = (a_{11}s^2 + a_{12}c^2)d\sigma_1 + (a_{12}s^2 + a_{22}c^2)d\sigma_2 - a_{66}csd\tau_{12}$$

$$+(\frac{2\sigma_1 - \sigma_2}{2\sigma_Y}s^2 + \frac{-\sigma_1 + 2\sigma_2 \frac{X^2}{Y^2}}{2\sigma_Y}c^2 + \frac{4\tau_{12}\frac{X^2}{S^2}}{2\sigma_Y}cs)d\varepsilon_p + (\alpha_1 s^2 + \alpha_2 c^2)dT = 0 \qquad (5.6)$$

$$d\varepsilon_{xy} = (a_{11} - a_{12})csd\sigma_1 + (a_{12} - a_{22})csd\sigma_2 - \frac{a_{66}}{2}(c^2 - s^2)d\tau_{12}$$

$$+\left[\frac{2\sigma_1 - \sigma_2}{2\sigma_Y}cs - \frac{-\sigma_1 + 2\sigma_2 \frac{X^2}{Y^2}}{2\sigma_Y}cs + \frac{2\tau_{12}\frac{X^2}{S^2}}{2\sigma_Y}(c^2 - s^2)\right]d\varepsilon_p + (\alpha_1 - \alpha_2)cs\,dT = 0$$

The yield criterion can be evaluated in the differential form,

$$(2\sigma_1 - \sigma_2)d\sigma_1 + (-\sigma_1 + 2\sigma_2\frac{X^2}{Y^2})d\sigma_2 + 2\tau_{12}\frac{X^2}{S^2}d\tau_{12} - 2K\bar{\sigma}d\varepsilon_p = 0 \qquad (5.7)$$

The solution of these equations may be undeterminable. Therefore, the solution is carried out by using a numerical method. They are written in the differential and symbolic form as,

$$\Delta\varepsilon_x = a_1\Delta\sigma_1 + b_1\Delta\sigma_2 + c_1\Delta\tau_{12} + d_1\Delta\varepsilon_p + e_1\Delta T = 0$$

$$\Delta\varepsilon_y = a_2\Delta\sigma_1 + b_2\Delta\sigma_2 + c_2\Delta\tau_{12} + d_2\Delta\varepsilon_p + e_2\Delta T = 0$$

$$\Delta\varepsilon_{xy} = a_3\Delta\sigma_1 + b_3\Delta\sigma_2 + c_3\Delta\tau_{12} + d_3\Delta\varepsilon_p + e_3\Delta T = 0 \qquad (5.8)$$

$$a_4\Delta\sigma_1 + b_4\Delta\sigma_2 + c_4\Delta\tau_{12} + d_4\Delta\varepsilon_p = 0.$$

Above equations can be written in the matrix form,

$$\begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{bmatrix} \begin{Bmatrix} \Delta\sigma_1 \\ \Delta\sigma_2 \\ \Delta\tau_{12} \\ \Delta\varepsilon_p \end{Bmatrix} = \begin{Bmatrix} -e_1 \\ -e_2 \\ -e_3 \\ 0 \end{Bmatrix} \Delta T \qquad (5.9)$$

or in symbolic form $|A|\{\Delta\sigma\} = \{C\}$.

ΔT is chosen to be a small value per load step during the solution. Subsequently, $\Delta\sigma_1$, $\Delta\sigma_2$, $\Delta\tau_{12}$ and $\Delta\varepsilon_p$ are obtained from the above equation by using the inversion of the matrix A as,

$$\{\Delta\sigma\} = |A|^{-1}\{C\} \tag{5.10}$$

The stress components and the equivalent plastic strain are calculated for 1000 load steps in different temperatures,

$$\begin{aligned}
\sigma_1^{(i)} &= \sigma_1^{(i-1)} + \Delta\sigma_1^{(i)} \\
\sigma_2^{(i)} &= \sigma_2^{(i-1)} + \Delta\sigma_2^{(i)} \\
\tau_{12}^{(i)} &= \tau_{12}^{(i-1)} + \Delta\tau_{12}^{(i)} \\
\varepsilon_p^{(i)} &= \varepsilon_p^{(i-1)} + \Delta\varepsilon_p^{(i)}
\end{aligned} \tag{5.11}$$

where i is the number of the load step.

5. 2. Residual Stresses

To find the residual stresses it is necessary to superpose on the stress system due to T, a completely elastic system due to temperature $-T$. The load found in the plastic solution is used in the opposite direction in order to obtain the complete elastic solution. The superposition of the elastic-plastic and elastic stresses gives the residual stresses. The resultant forces and moments per unit length after elastic solution are written as,

$$\begin{Bmatrix} N_x^p \\ N_y^p \\ N_{xy}^p \end{Bmatrix} = \int \begin{Bmatrix} \sigma_x^p \\ \sigma_y^p \\ \tau_{xy}^p \end{Bmatrix}_k dz \tag{5.12}$$

$$\begin{Bmatrix} M_x^p \\ M_y^p \\ M_{xy}^p \end{Bmatrix} = \int \begin{Bmatrix} \sigma_x^p \\ \sigma_y^p \\ \tau_{xy}^p \end{Bmatrix}_k z\, dz \tag{5.13}$$

$$\begin{Bmatrix} N_x^e \\ N_y^e \\ N_{xy}^e \end{Bmatrix} = \int \begin{Bmatrix} \sigma_x^e \\ \sigma_y^e \\ \tau_{xy}^e \end{Bmatrix}_k dz \tag{5.14}$$

$$\begin{Bmatrix} M_x^e \\ M_y^e \\ M_{xy}^e \end{Bmatrix} = \int \begin{Bmatrix} \sigma_x^e \\ \sigma_y^e \\ \tau_{xy}^e \end{Bmatrix}_k z\, dz \tag{5.15}$$

These two kinds of resultant forces and moments are similar or it can be said that the slope of the loading and unloading curves is the same. When the temperature attains to zero degree, the elastic stress components are calculated as,

$$\begin{aligned} \sigma_x^e &= \frac{(\sigma_x)_1^e N_x^p}{N_x^e} + \frac{(\sigma_x)_2^e M_x^p}{M_x^e} \\ \sigma_y^e &= \frac{(\sigma_y)_1^e N_y^p}{N_y^e} + \frac{(\sigma_y)_2^e M_y^p}{M_y^e} \\ \tau_{xy}^e &= \frac{(\tau_{xy})_1^e N_{xy}^p}{N_{xy}^e} + \frac{(\tau_{xy})_2^e M_{xy}^p}{M_{xy}^e} \end{aligned} \tag{5.16}$$

where

$$\begin{Bmatrix} (\sigma_x)_1^e \\ (\sigma_y)_1^e \\ (\tau_{xy})_1^e \end{Bmatrix}_k = \begin{vmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{vmatrix}_k \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \tag{5.17}$$

$$\begin{Bmatrix} (\sigma_x)_2^e \\ (\sigma_y)_2^e \\ (\tau_{xy})_2^e \end{Bmatrix}_k = \begin{vmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{vmatrix}_k \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} z \tag{5.18}$$

The superposition of the elastic and plastic stresses gives the residual stress components as,

$$\sigma_x^r = \sigma_x^p - \sigma_x^e$$
$$\sigma_y^r = \sigma_y^p - \sigma_y^e \quad (5.19)$$
$$\tau_{xy}^r = \tau_{xy}^p - \tau_{xy}^e$$

These stresses satisfy the static equilibrium in the laminated plates.

5. 3. Results and Discussions

In this study, antisymmetric aluminum metal-matrix composite laminated plates are used. The mechanical properties and yield strengths of a layer are given in Table 4.1. But, *K*, is carried out into the solution since linear hardening material properties are assumed. The plate is subjected to a uniform temperature distribution across the thickness. It is assumed that the stress components are zero at the zero degree.

When the temperature reaches 11.26 ^{0}C each layer yields. The stress components in first and second layers at this temperature are given in Table 5.1 for the $[0^o/90^o]_2$, $[30^o/-30^o]_2$, $[45^o/-45^o]_2$, $[60^o/-60^o]_2$, $[15^0/-15^0]_2$, $[15^0/ -30^0]_2$, $[15^0/-45^0]_2$, $[15^0/-60^0]_2$ orientations. The magnitude of the stress components of σ_x and σ_y is nearly the same at the beginning of plastic yielding due to the fact that $E\alpha$ is nearly the same in each direction.

The plastic and residual stress components in the first and second layers are given in Tables 5.2, 5.3, 5.4 and 5.5 for the $[0^o/90^o]_2$, $[30^o/-30^o]_2$, $[45^o/-45^o]_2$, $[60^o/-60^o]_2$ orientations. As seen in the table, the magnitude of the plastic stress component of σ_x is higher than that of σ_y, since the yield point in the second principal material direction (Y) is smaller than that in the first principal material direction. The plastic stress component of σ_y is always lower than the yield point in the second principal material direction. The magnitude of the residual stress component of σ_x and σ_y possesses high values in the cross-ply $[0^o/90^o]_2$, laminated plate due to the highest different material properties in each layer. However, the residual stress component of

τ_{xy} is minimized in the cross-ply laminated plate. The residual stress components in $[30^o/-30^o]_2$, $[45^o/-45^o]_2$, and $[60^o/-60^o]_2$, laminated plates possess small values, since the stiffnesses have similar properties in each layer.

The plastic and residual stress components for the antisymmetric, $[15^0/-30^0]_2$, $[15^0/-45^0]_2$, $[15^0/-60^0]_2$ and $[15^0/-15^0]_2$, laminated plates are given in Tables 5.6, 5.7, 5.8 and 5.9. As seen in these tables, the residual stress components for the angle-ply $[15^0/-15^0]_2$laminated plate have small values due to the close stiffnesses in each layer. However, considerable high residual stress component occurs in the antisymmetric angle-ply $[15^0/-30^0]_2$, $[15^0/-45^0]_2$, $[15^0/-60^0]_2$, laminated plates, since stiffnesses in each layer have hugely different values. This difference creates the high residual stresses in the plates. The magnitude of the residual stress components in antisymmetric $[15^0/-60^0]_2$, laminated plates is maximized, due to the hugely different material properties in these orientation angles at T=50 ^{0}C. When the difference between the orientation angle in each layer is increased, the magnitude of the residual stress components becomes higher. For example, the magnitude of the residual stress components in, $[15^0/-45^0]_2$, laminated plate is higher than that in the $[15^0/-30^0]_2$, laminated plates.

Table 5. 1 Mechanical properties and yield strengths of the composite layer.

E_1 (MPa)	E_2 (MPa)	G_{12} (MPa)	ν_{12}	K	Axial strength X (MPa)	Transverse strength Y (MPa)	Shear strength S (MPa)	Thickness of each layer (mm)	Thermal expansion coefficient (1/°C)	
									α_1	α_2
85000	74000	30000	0.3	1250	230.0	24.0	48.9	2	18.5 E-06	21.0E -06

Table 5. 2 Stress components at the beginning of the plastic yielding in the first and second layer.

		T (^{0}C)	(σ_x) (MPa)		(σ_y) (MPa)		(τ_{xy}) (MPa)	
Layer Number			1	2	1	2	1	2
Orientation angles	$[0^0/90^0]_2$	11.26	-24.90	-23.99	-23.99	-24.90	0.00	0.00
	$[30^0/-30^0]_2$	11.26	-24.68	-24.68	-24.22	-24.22	-0.39	0.39
	$[45^0/-45^0]_2$	11.26	-24.45	-24.45	-24.45	-24.45	-0.46	0.46
	$[60^0/-60^0]_2$	11.26	-24.22	-24.22	-24.68	-24.68	-0.39	0.39
	$[15^0/-15^0]_2$	11.26	-24.84	-24.84	-24.06	-24.06	-0.23	0.23
	$[15^0/-30^0]_2$	11.26	-24.84	-24.68	-24.06	-24.22	-0.23	0.39
	$[15^0/-45^0]_2$	11.26	-24.84	-24.45	-24.06	-24.45	-0.23	0.46
	$[15^0/-60^0]_2$	11.26	-24.84	-24.22	-24.06	-24.67	-0.23	0.39

Table 5. 3 Elastic-plastic, residual stress components and equivalent plastic strain at the first and second layers for $[0^o/90^o]_2$ orientation.

Orientation angle (θ)	T (^{0}C)	Layer Number	$(\sigma_x)_p$ (MPa)	$(\sigma_y)_p$ (MPa)	$(\tau_{xy})_p$ (MPa)	$(\sigma_x)_r$ (MPa)	$(\sigma_y)_r$ (MPa)	$(\tau_{xy})_r$ (MPa)	$\varepsilon_p.10^{-4}$
$[0^o/90^o]_2$	20	1	-38.43	-23.88	0.00	-5.58	5.60	0.00	1
	20	2	-23.88	-38.43	0.00	5.60	-5.58	0.00	
	30	1	-53.73	-23.64	0.00	-12.94	12.96	0.00	2
	30	2	-23.64	-53.73	0.00	12.96	-12.94	0.00	
	40	1	-68.82	-23.29	0.00	-20.26	20.29	0.00	3
	40	2	-23.29	-68.82	0.00	20.29	-2.88	0.00	
	45	1	-76.28	-23.07	0.00	-23.90	23.93	0.00	4
	45	2	-23.07	-76.28	0.00	23.93	-23.90	0.00	

Table 5. 4 Elastic-plastic, residual stress components and equivalent plastic strain at the first and second layers for [30°/-30°]$_2$ orientation.

Orientation angle (θ)	T (°C)	Layer Number	$(\sigma_x)_p$ (MPa)	$(\sigma_y)_p$ (MPa)	$(\tau_{xy})_p$ (MPa)	$(\sigma_x)_r$ (MPa)	$(\sigma_y)_r$ (MPa)	$(\tau_{xy})_r$ (MPa)	$\varepsilon_p.10^{-4}$
[30°/-30°]$_2$	20	1	-34.80	-27.50	-6.29	0.00	0.02	0.00	1
	20	2	-34.77	-27.52	6.30	0.02	0.01	0.02	
	30	1	-46.23	-31.11	-12.99	-0.01	0.03	0.00	2
	30	2	-46.18	-31.16	13.02	0.04	-0.02	0.03	
	40	1	-57.45	-34.62	-19.67	0.02	0.01	0.04	3
	40	2	-57.42	-34.64	19.68	0.04	-0.02	-0.02	
	45	1	-62.91	-36.40	-23.03	0.05	-0.03	0.04	4
	45	2	-63.00	-36.31	22.98	-0.03	0.06	-0.09	

Table 5. 5 Elastic-plastic, residual stress components and equivalent plastic strain at the first and second layers for [45°/-45°]$_2$ orientation.

Orientation angle (θ)	T (°C)	Layer Number	$(\sigma_x)_p$ (MPa)	$(\sigma_y)_p$ (MPa)	$(\tau_{xy})_p$ (MPa)	$(\sigma_x)_r$ (MPa)	$(\sigma_y)_r$ (MPa)	$(\tau_{xy})_r$ (MPa)	$\varepsilon_p.10^{-4}$
[45°/-45°]$_2$	20	1	-31.23	-31.23	-7.27	0.00	0.01	0.00	1
	20	2	-31.23	-31.23	7.27	0.00	0.01	0.00	
	30	1	-38.31	-38.31	-15.03	0.67	-0.64	0.00	2
	30	2	-38.31	-38.31	-15.03	0.67	-0.64	0.00	
	40	1	-45.87	-45.87	-22.74	0.49	-0.46	0.00	3
	40	2	-45.87	-45.87	22.74	0.49	-0.46	0.00	
	45	1	-49.66	-49.66	-26.58	-0.02	0.05	0.00	4
	45	2	-49.66	-49.66	26.58	0.02	-0.04	0.00	

Table 5. 6 Elastic-plastic, residual stress components and equivalent plastic strain at the first and second layers for [60°/-60°]$_2$ orientation.

Orientation angle (θ)	T (°C)	Layer Number	$(\sigma_x)_p$ (MPa)	$(\sigma_y)_p$ (MPa)	$(\tau_{xy})_p$ (MPa)	$(\sigma_x)_r$ (MPa)	$(\sigma_y)_r$ (MPa)	$(\tau_{xy})_r$ (MPa)	$\varepsilon_p.10^{-4}$
[60°/-60°]$_2$	20	1	-27.53	-34.76	-6.31	0.00	0.02	0.00	1
	20	2	-27.52	-34.78	6.30	0.01	0.00	-0.01	
	30	1	-31.21	-46.12	-13.05	-0.04	0.05	0.00	2
	30	2	-31.16	-46.18	13.02	0.02	0.00	-0.03	
	40	1	-34.63	-57.43	-19.68	0.07	-0.05	0.02	3
	40	2	-34.73	-57.33	19.73	-0.02	0.05	0.04	
	45	1	-36.38	-62.93	-23.03	-0.02	0.04	0.03	4
	45	2	-36.26	-63.05	22.96	0.10	-0.07	-0.10	

Table 5. 7 Elastic-plastic, residual stress components and equivalent plastic strain at the first and second layers for $[15^0/\text{-}15^0]_2$, orientation.

Orientation angle (θ)	T (^{0}C)	Layer Number	$(\sigma_x)_p$ (MPa)	$(\sigma_y)_p$ (MPa)	$(\tau_{xy})_p$ (MPa)	$(\sigma_x)_r$ (MPa)	$(\sigma_y)_r$ (MPa)	$(\tau_{xy})_r$ (MPa)	$\varepsilon_p.10^{-4}$
$[15^0/\text{-}15^0]_2$	20	1	-37.44	-24.85	-3.65	0.02	-0.02	-0.05	1
	20	2	-37.44	-37.44	-3.64	0.02	-0.01	0.03	
	30	1	-51.69	-25.65	-7.51	0.06	0.08	0.53	2
	30	2	-51.79	-25.55	7.53	-0.18	0.18	-0.70	
	40	1	-65.77	-26.29	-11.29	0.02	-0.01	-0.02	3
	40	2	-65.76	-26.30	11.32	0.03	-0.03	0.04	
	50	1	-79.70	-26.77	-14.96	-0.07	0.08	0.04	4
	50	2	-79.60	-26.88	15.16	0.04	-0.03	0.16	

Table 5. 8 Elastic-plastic, residual stress components and equivalent plastic strain at the first and second layers for $[15^0/ \text{-}30^0]_2$ orientation.

Orientation angle (θ)	T (^{0}C)	Layer Number	$(\sigma_x)_p$ (MPa)	$(\sigma_y)_p$ (MPa)	$(\tau_{xy})_p$ (MPa)	$(\sigma_x)_r$ (MPa)	$(\sigma_y)_r$ (MPa)	$(\tau_{xy})_r$ (MPa)	$\varepsilon_p.10^{-4}$
$[15^0/ \text{-}30^0]_2$	20	1	-37.44	-24.86	-3.65	-0.95	1.04	-0.11	1
	20	2	-34.81	-27.48	-6.28	0.98	-1.07	-0.15	
	30	1	-51.68	-25.65	-7.51	-2.37	2.56	-0.16	2
	30	2	-45.81	-31.53	-13.23	2.57	-2.76	-0.50	
	40	1	-65.77	-26.29	-11.29	-3.60	3.89	0.21	3
	40	2	-57.41	-34.66	-19.69	3.59	-3.87	0.23	
	50	1	-79.70	-26.77	-14.96	-4.87	5.26	0.76	4
	50	2	-68.70	-37.77	-26.17	4.70	-5.09	1.06	

Table 5. 9 Elastic-plastic, residual stress components and equivalent plastic strain at the first and second layers for $[15^0/\text{-}45^0]_2$ orientation.

Orientation angle (θ)	T (^{0}C)	Layer Number	$(\sigma_x)_p$ (MPa)	$(\sigma_y)_p$ (MPa)	$(\tau_{xy})_p$ (MPa)	$(\sigma_x)_r$ (MPa)	$(\sigma_y)_r$ (MPa)	$(\tau_{xy})_r$ (MPa)	$\varepsilon_p.10^{-4}$
$[15^0/\text{-}45^0]_2$	20	1	-37.44	-24.86	-3.65	-2.43	2.56	-0.06	1
	20	2	-30.85	-31.44	-7.27	2.59	-2.72	-0.10	
	30	1	-51.69	-25.65	-7.51	-5.45	5.74	-0.02	2
	30	2	-38.70	-38.64	-15.03	5.48	-5.76	-0.05	
	40	1	-65.77	-26.29	-11.29	-8.05	8.53	0.22	3
	40	2	-48.51	-43.61	-22.72	6.63	-7.14	0.31	
	50	1	-79.70	-26.77	-14.96	-12.02	12.61	0.52	4
	50	2	-51.98	-54.52	-30.39	12.68	-13.27	0.69	

Table 5. 10 Elastic-plastic, residual stress components and equivalent plastic strain at the first and second layers for $[15^0/-60^0]_2$ orientation.

Orientation angle (θ)	T (^{0}C)	Layer Number	$(\sigma_x)_p$ (MPa)	$(\sigma_y)_p$ (MPa)	$(\tau_{xy})_p$ (MPa)	$(\sigma_x)_r$ (MPa)	$(\sigma_y)_r$ (MPa)	$(\tau_{xy})_r$ (MPa)	$\varepsilon_p.10^{-4}$
$[15^0/-60^0]_2$	20	1	-37.44	-24.80	-3.65	-3.82	3.91	-0.25	1
	20	2	-27.51	-34.79	-6.30	3.75	-3.82	-0.41	
	30	1	-51.68	-25.65	-7.51	-10.97	11.45	11.16	2
	30	2	-31.34	-46.00	-13.12	6.52	-6.07	19.22	
	40	1	-65.77	-26.29	-11.29	-13.73	14.03	0.09	3
	40	2	-36.65	-57.41	-19.69	13.73	-14.01	0.03	
	50	1	-79.70	-26.77	-14.96	-18.61	19.03	1.06	4
	50	2	-38.08	-68.39	-26.35	18.71	-19.09	1.40	

CHAPTER 6
CONCLUSIONS AND FURTHER WORKS

6. 1. Summary

In this work, thermal elastic-plastic analysis is carried out on simply supported antisymmetric stainless steel-fiber reinforced aluminium metal-matrix composite laminated plates. The plates are composed of four orthotropic layers bonded antisymmetrically. In the solution, a special computer program has been employed. The composite materials are assumed to be perfectly plastic and linear hardening. In the solution of the elastic part, classical laminated plate theory has been used. For the elastic-plastic solution, the Tsai-Hill Theory is used as a yield criterion. Plastic and residual stresses are determined in the antisymmetric cross-ply and angle-ply laminated plates for small deformations. Plastic yielding occurs in all the laminated plate at the same temperature. The intensity of stress componenets in the antisymmetric cross-ply laminated plate is higher than that in angle-ply laminated plate due to the differences among the stiffnesses of layers. For those laminated plates, residual stresses and plastic flow are obtained by using incremental stress method. Moreover, assuming that the lamina properties do not change over the temperature range.

6. 2. General Conclusions

6. 2. 1. Conclusions For Ideal Plastic Material Properties

6. 2. 1. 1. For Cross-ply Laminated Plate

- The temperature causing plastic yielding in the antisymmetric cross-ply laminated plate $[0^o/90^o]_2$ is 11.26 ^{0}C.

- There is no elastic-plastic shear stresses $(\tau_{xy})_p$ in the antisymmetric cross-ply laminated plate.
- Similarly, there is no residual shear stresses $(\tau_{xy})_r$ in the antisymmetric cross-ply laminated plate.
- The magnitude of residual stress components in the antisymmetric cross-ply, $[0^o/90^o]_2$, laminated plate is maximized.
- The intensity of stress components in the antisymmetric cross-ply laminated plate is higher than that in angle-ply laminated plate due to the differences among the stiffnesses of layers.
- Elastic-plastic and residual stress components $(\sigma_x)_p$, $(\sigma_y)_p$, $(\sigma_x)_r$, $(\sigma_y)_r$ and equivalent plastic strain ε_p increase depending on the temperature increment.

6. 2. 1. 2. For Angle-ply Laminated Plates

- The temperature causing plastic yielding in the antisymmetric angle-ply laminated plates, 11.26 ^{0}C, is the same in all the simply supported case.
- Although some antisymmetric angle-ply laminated plates have magnitude of residual stresses, some antisymmetric angle-ply laminated plates have not.
- The intensity of the equivalent plastic strain is the same in all the stacking sequences for the same temperature.
- The residual stress components for angle-ply $[15^o/-30^o]_2$, $[15^o/-45^o]_2$, $[15^o/-60^o]_2$ laminated plates have magnitudes whereas the residual stress components for angle-ply $[30^o/-30^o]_2$, $[45^o/-45^o]_2$, $[60^o/-60^o]_2$ and $[15^o/-15^o]_2$ laminated plates are zero.
- All the elastic-plastic and residual stress components and equivalent plastic strain ε_p increase depending on the temperature increment.
- The antisymmetric angle-ply laminated plates, $[15^o/-30^o]_2$, $[15^o/-45^o]_2$, $[15^o/-60^o]_2$, have residual stresses.
- The magnitude of the equivalent plastic strain is the same in all the stacking-sequences for the same temperature.

6. 2. 2. Conclusions for Linear Hardening Material Properties

6. 2. 2. 1. For Cross-ply Laminated Plate

- The temperature causing plastic yielding in the antisymmetric cross-ply laminated plate is 11.26 ^{0}C.
- The magnitude of the residual stress components in the antisymmetric cross-ply, $[0^o/90^o]_2$, laminated plate is maximized.
- The elastic-plastic stress components of $(\sigma_x)_p$, and $(\sigma_y)_p$ have some magnitudes but the shear stress component $(\tau_{xy})_p$ is zero, for the antisymmetric cross-ply laminates.
- Similarly, for the antisymmetric cross-ply laminates, the residual stress components of $(\sigma_x)_r$, and $(\sigma_y)_r$ have some magnitudes but the shear stress component $(\tau_{xy})_r$ is zero.
- Elastic-plastic, residual stress components and equivalent plastic strain increase depending on the temperature increment.

6. 2. 2. 2. For Angle-ply Laminated Plates

- The temperature causing plastic yielding in antisymmetric cross-ply and angle-ply laminated plates is the same for all the stacking sequences.
- Antisymmetric laminated plates produce higher stress components as the difference between the absolute values of the angles increases.
- The magnitude of the equivalent plastic strain is the same in all the stacking-sequences for the same temperature.
- All the elastic-plastic and residual stress components and equivalent plastic strain increase depending on the temperature increment.
- For linear hardening materials, angle-ply laminates have magnitudes of the residual stresses but this case is not valid for perfectly platic materials.

6. 3. Further Works

For furter works, a finite element analysis program can be written in order to examine the elastic backling for cross-ply and angle-ply antisymmetric laminated plates. And this case can be investigated for different composite materials and different boundary conditions. Also, the elastic-plastic stress analysis of laminates can be carried out for different thermal loads varying non-linearly through the thickness. In addition, these antisymmetric laminated plates can be analysed as composite plates with a central hole.

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