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GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

TABU SEARCH BASED SOLUTION APPROACHES FOR LOT STREAMING PROBLEMS IN FLOW SHOPS

by
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İZMİR

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Ph.D. THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"TABU SEARCH BASED SOLUTION APPROACHES FOR LOT STREAMING PROBLEMS IN FLOW SHOPS"** completed by **RAHİME SANCAR EDİS** under supervision of **ASSOC. PROF. DR. ARSLAN ÖRNEK** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy.

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TABU SEARCH BASED SOLUTION APPROACHES FOR LOT STREAMING PROBLEMS IN FLOW SHOPS

ABSTRACT

Lot streaming (LS) splits the production lot into sublots, and schedules these sublots in an overlapping way on the machines in order to accelerate the process of orders and improve the overall system performance. In this thesis, a comprehensive review on LS is presented and a number of LS problems all of which aim to minimize makespan in multi machine flow shops are investigated. The first problem considers a single product case in stochastic flow shops. For this problem, a solution approach that integrates tabu search (TS) and simulation is proposed. The sublot size configurations are searched via TS and the stochastic behavior of the system is handled by simulation. The remaining three problems deal with multi product cases in deterministic flow shops. These problems differ from each other by sublot types and divisibility of sublot sizes. In the solution approaches, the entire problem is partitioned into sequencing and sublot allocation sub-problems. For the sequencing sub-problem, a number of simple and efficient sequencing heuristics developed for general flow shops are modified according to LS requirements. For the sublot allocation sub-problem, mixed integer programming (MIP) based solution approaches are proposed. For the entire problem, a hybrid solution approach which uses the best sequencing heuristic (i.e., NEH(D,TPLS)) in sequencing sub-problem and applies MIP based approaches for the sublot allocation sub-problem, is proposed. The proposed approach not only gives efficient results for small/medium sized problems in short computation times but also solves large sized problems in reasonable times. Finally, to improve the solution quality in small and medium sized problems, the same approach is also integrated to a solution procedure where the initial sequence is taken as NEH(D, TPLS) and the alternative sequences are evaluated via TS. This heuristic performs better than the MIP model of entire problem under a given run time limit.

Keywords : Lot streaming, Flow shops, Tabu search, Sequencing Rules

AKIŞ TİPİ SİSTEMLERDE, KAFİLE BÖLME VE KAYDIRMA PROBLEMLERİ İÇİN TABU ARAMA TABANLI ÇÖZÜM YAKLAŞIMLARI

ÖZ

Kafile bölme ve kaydırma (KBK), üretimi hızlandırmak ve sistem performansını iyileştirmek için, üretim kafilesini daha küçük alt kafilelere bölme ve bu alt kafileleri makineler boyunca çizelgeleme yöntemidir. Bu tezde, KBK çalışmalarının kapsamlı bir yazın taraması yapılmış ve çok makineli akış tipi sistemlerde toplam üretim süresini en küçüklemeyi amaçlayan bir dizi KBK problemi çalışılmıştır. İlk problemde, tek ürünlü stokastik bir akış tipi sistem incelenmiştir. Bu problem için tabu arama ve benzetim yöntemlerini bütünleştiren bir çözüm yaklaşımı önerilmiştir. Bu yaklaşımda, alt kafile büyüklük seçeneklerini değerlendirmek için tabu arama yöntemi ve sistemin stokastik yapısını yansıtabilmek üzere benzetim yöntemi kullanılmıştır. Çalışılan diğer üç problemde, çok ürünlü deterministik akış tipi sistemler ele alınmıştır. Bu problemler, birbirlerinden alt kafile tipi ve alt kafilenin bölünebilirliği karakteristikleri açısından farklılaşmaktadır. Önerilen çözüm yöntemlerinde, çok ürünlü KBK problemi; sıralama ve alt kafile bölme/kaydırma alt problemlerine ayrıştırılmıştır. Sıralama alt problemi için, genel akış tipi sistemlerde geliştirilmiş olan basit ve etkin sıralama algoritmaları, KBK probleminin gereksinimleri doğrultusunda revize edilmiştir. Alt kafile bölme/kaydırma problemi için ise, karışık tam sayılı programlama tabanlı çözüm yaklaşımları geliştirilmiştir. Çok ürünlü KBK problemini çözmek için; sıralama alt problemini, revize edilmiş sezgisel yöntemlerden en iyi sonucu veren (NEH,TPLS) yöntem ile ele alan ve alt kafile bölme/kaydırma problemini önerilen karışık tam sayılı programlama yaklaşımı ile çözen melez bir çözüm prosedürü geliştirilmiştir. Önerilen melez yöntem, sadece küçük ve orta ölçekli problemlere kısa zamanda etkin sonuçlar vermekle kalmayıp aynı zamanda büyük ölçekli problemler için de çözüm sunabilmektedir. Son olarak, küçük ve orta ölçekli problemlerde çözüm etkinliğini arttırmak için, yukarıda tanımlanan melez yaklaşımı içeren ve alternatif ürün sıralarını tabu arama yöntemi ile değerlendirerek geliştiren bir diğer çözüm yöntemi önerilmiştir. Bu çözüm yönteminin sonuçları, bütün problemin çözümü için geliştirilen karışık tam sayılı programlama modelininden elde edilen sonuçlarla karşılaştırılmış ve aynı çözüm süresi verildiğinde önerilen çözüm yönteminin daha iyi bir performans sergilediği gösterilmiştir.

Anahtar sözcükler: Kafile Bölme ve Kaydırma Problemleri, Akış Tipi Sistemler, Tabu Arama, Sıralama Kuralları

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CHAPTER ONE
INTRODUCTION

1.1 Background and Motivation

Material Requirements Planning (MRP) was introduced in 1970s to build time based plans for the delivery of raw materials, the processing stages on the basis of bill of materials and the lead time of end products. However, MRP has some drawbacks. First, it assumes that production parameters such as lot sizes and lead times are priori known and kept fixed. This causes high work-in-process (WIP) inventory and long lead times. Secondly, it may generate infeasible schedules due to infinite capacity constraints. Finally, in an MRP system, a production lot is treated as a single entity which means the items in a lot has to finish their operations in the current machine before transferring to the next one. The MRP II method then introduced to overcome the second drawback by considering the limited capacity of relevant resources. (Sarin & Jaiprakash, 2007, p.20)

In 1980s, the concept of minimizing waste is handled by the just-in-time (JIT) manufacturing technology. The waste is defined as anything that does not add value to the manufacturing process (Ohno, 1988, p.58). To eliminate the wasted inventory, JIT limits the WIP inventory between machines by kanbans. Kanban is a sign card attached to components. The aim in kanban systems is to set the number of kanbans to control the flow of items overall and keep stock in minimum, and to provide visual control to perform these functions accurately (Shingo, 1989, p.188). Similarly, JIT tries to minimize the wasted times and lead times by allowing overlapping operations where one item (i.e., unit size) is transferred at a time between machines. However, this type of transfers (unit sized) between machines might result in reverse direction of the purpose where significant amounts of transfer times and setup times are incurred. At the same time, another technique named optimized production technology (OPT) is appeared. It also aims to eliminate the waste in manufacturing but taking the critical resources such as bottlenecks into account. OPT, first, determines the bottleneck and non-bottleneck machines, and then, builds production plans such that the bottleneck is fully utilized. Using large process batches reduces number of setups and consequently setup costs, and small transfer batches decrease inventory carrying costs. This provides a significant reduction in overall cost and lead times. However, OPT lies on a number of assumptions. First, it assumes that the sublot sizes and sequence of lots are priori known. Second, there exists a single bottleneck in the system. Third, the transfer batches are used only in bottleneck machines. (Sarin & Jaiprakash, 2007, p.21)

In traditional production systems, lots are transferred to the next machine if and only if all items in the lot finish their operations on the current machine. This causes the produced items to spend most of their time waiting for other items that are not produced yet. Inessential waiting times result in long completion times and high WIP inventory. In order to reduce the non-value-added waiting times, the whole lot can be divided into sublots that contain a portion of the lot. Then, the operations of these sublots on successive machines can be performed simultaneously. By this arrangement on the sublots, they can move along the machines immediately and the completion time of the whole lot decreases. This technique is originally introduced by Reiter in 1966 and called “lot streaming”. Formally, lot streaming (LS) is a technique in which a production lot is split into several sublots and overlapping operations in different manufacturing workstations (i.e., stages) are performed. In this way, production can be accelerated.

By the introduction of JIT and OPT concepts in 1980s, LS has been inherently used to overcome the restrictions of these two concepts. Since then, LS has been extensively studied in academic as well as industrial fields and has been shown to be an effective technique for compressing manufacturing lead time.

The advantages of LS are not only limited by reduction in waiting times and manufacturing lead times. Truscott (1986) lists the advantages of LS as (see Chapter 2 for details):

- reduction in completion times which generates better lead times,
- reduction in average WIP inventory level which decreases inventory costs,

- reduction in space and storage requirements within the production area,
- reduction in material handling system capacity requirements.

The LS technique is widely applied to flow shop scheduling problems in the literature. These studies can inherently be categorized into two cases: single and multi product problems.

The aim in single product LS problems is to determine the number of sublots that the production lot is going to be divided into and their sizes (i.e., the number of items). Although a restricted number of single product LS problems can be solved by polynomial time algorithms due to their simple LS properties, most of them are NP-hard in that sense (see Trietsch & Baker, 1993).

On the other hand, multi product LS problems require sequencing the products through the machines as well as sublot allocations of products. The first sub-problem, sequencing the products, is NP-complete for more than three machines (Garey, Johnson & Sethi, 1976). Therefore, multi product LS problems are strongly NP-hard especially for multi machine (m>3) cases.

In this thesis, single and multi product LS problems, which have not received much attention in literature, are studied. All investigated research problems aim to minimize makespan in permutation flow shops. Figure 1.1 indicates the position of these research problems in terms of LS problem characteristics (see Section 2.1 Classification and Section 2.2 Terminology for details). For these research problems, tabu search based solution approaches are proposed.

1.2 Research Objective and Methodology

The main purpose of this thesis is to develop efficient solution algorithms for a class of LS problems in both stochastic and deterministic production environments. In order to fulfill this purpose, this study covers the following issues in details:

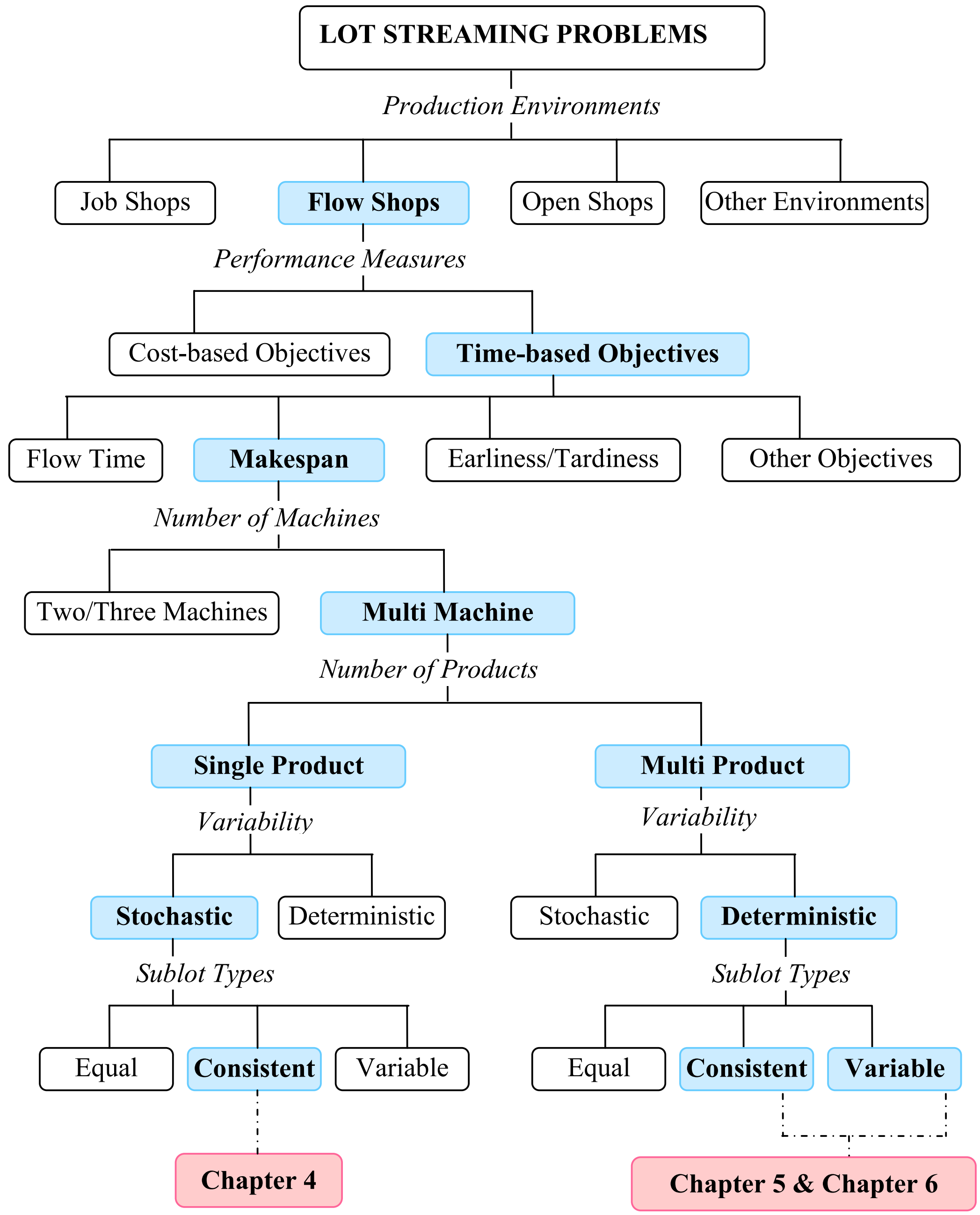

Figure 1.1 Characteristics of the research problems

1. Investigate a range of LS problems with various problem characteristics and properties, which have not received much attention in LS literature.
2. Explore the exact and approximate solution algorithms that have previously been applied to the investigated LS problems.
3. Develop efficient solution approaches to the investigated LS problems.

4. Evaluate the efficiency of the proposed solution approaches against the existing ones.
5. Apply the proposed solution approaches to a wide range of medium and large sized test instances to describe the applicable problem size.
6. Clarify the research fields in LS literature that are still open for further researches.

1.3 Contributions

The research proposed in this thesis provides several contributions. These contributions may be presented in two aspects:

- contributions related with the problem characteristics
- contributions related with the solution approaches

In this thesis, several research problems are handled. The first research problem deals with a single product LS problem in stochastic environments. The other three studies focus on multi-product multi-machine LS problems with non-intermingling schedules in deterministic permutation flow shops. These problems have not been studied widely in the literature due to their complex structures.

The contributions in terms of problem characteristics are given in the following.

- Only a limited number of studies address the stochastic nature of LS problems, although it is widely encountered in real life applications. One of our research problems explores a single product LS problem in stochastic flow shops.

- The multi product LS problem with variable sublots is one of the hardest cases in the LS literature. To the best of author's knowledge, there exists only one study (i.e., Liu, Chen & Liu, 2006) for this class of problems. A research problem of this thesis deals with multi product LS problems with variable sublot types.

- The related studies in the literature generally deal with small to medium size LS problems especially in multi product cases. However, most of real life applications require quite large problems to be solved. In this thesis, medium to large sized test instances of investigated problems are tried to be solved.

The solution approaches developed for LS problems in the literature are directly affected by the problem characteristics. Exact approaches are available for simpler LS problems whereas heuristic and meta-heuristic approaches are widely used for problems that are more complex. The contributions in terms of solution approaches are given in the following.

- The aim of single product LS studies is to find the number of sublots and their sizes with respect to some performance criterion. This aim may not be easily achieved in stochastic systems, since the existing approaches (e.g. LP, dominance relations) developed for deterministic systems may not be appropriate to solve LS problems in stochastic environments. Therefore, the stochastic LS studies in the literature only analyze the performance of pre-determined experimental sublot sizes instead of optimizing them. As far as we know, no study, so far, has proposed a heuristic search algorithm that finds discrete sublot sizes in stochastic flow shops. In this thesis, a tabu search based heuristic approach is proposed to search sublot size configurations of a single product LS problem in a stochastic environment. Due to the stochastic structure of the problem, the proposed solution approach is a hybrid one that integrates tabu-search and simulation.

- Multi product LS problems require sequencing the products through the machines as well as sublot allocations of products. The sequencing part of the problem has received much attention in the literature. However, these studies generally focus on small or medium sized multi product LS problems. To solve large sized problems in reasonable times and to get efficient results for small and medium sized problems in small computation times, a number of simple and efficient sequencing heuristics developed for pure flow shops are modified according to the requirements of LS. The best one of these sequencing heuristics is suggested to be used in multi product LS problems.

- If the sequence is given, there only remains the sublot allocation sub-problem. However, even with the given sequence, it may still be difficult to find optimal number of sublots with optimal sizes in multi product LS problems. Therefore, the studies in the literature generally assume unit or equal sized sublots to eliminate the sublot allocation sub-problem. On the contrary, this thesis also incorporates solution approaches that handle this sub-problem as well as sequencing sub-problem. Particularly, the solution approach proposed for solving sublot allocation sub-problem of continuous sized variable sublots is novel in the LS literature.

- Most of the studies in the multi product LS literature develop heuristic or meta-heuristic approaches. The studies that present mixed integer programming (MIP) models of more complex LS problems are rather new (Biskup and Feldmann, 2006; Feldmann & Biskup, 2008). Hybrid methods that utilize the complementary strengths of heuristic/meta-heuristic algorithms and MIP models may produce more efficient results. Therefore, our solution approaches utilize the benefit of heuristic/meta-heuristic approaches in sequencing and of MIP models in sublot sizing. In addition, for variable sublot types, an alternative MIP model formulation is proposed based on the MIP models of Biskup & Feldmann (2006) and Feldmann & Biskup (2008).

1.4 Organization of the Thesis

The remainder of the thesis is organized as follows.

Chapter 2 describes the relevant terminology of the LS with a classification scheme. Then, brief information on the components of the LS problems is given. Lastly, the dominance relations of the LS components are discussed.

Chapter 3 presents a comprehensive and categorized literature review with respect to past research work on LS problems related with time based objectives. The LS literature in flow shops is divided into four categories based on the number of products and machines. The problem characteristics and solution approaches of the studies falling into these four categories are investigated in detail.

In Chapter 4, a single product multi machine LS problem with discrete sized consistent sublots is investigated in stochastic flow hops. A tabu search based solution approach integrated with simulation is proposed for this problem and its results are compared for both deterministic and stochastic flow shops.

Multi product, multi machine LS problems are studied in Chapter 5 and 6. With respect to sublot type and sublot size characteristics, three different versions of this problem type are investigated: continuous sized consistent sublots, discrete sized consistent sublots and continuous sized variable sublots.

In Chapter 5, a number of simple and efficient sequencing heuristics developed for pure flow shops are modified according to the requirements of LS. To analyze the relative performances of these sequencing heuristics, computational experiments are carried out and the best sequencing heuristic is proposed to be used in multi product LS problems. In addition, for each investigated problem, solution approaches are proposed to find the sublot sizes under a given sequence.

Chapter 6 proposes tabu search based solution approaches for three investigated multi product multi machine LS problems by utilizing the best sequencing heuristics presented in Chapter 5. The results of the proposed algorithms are compared with the ones of MIP models.

Finally, Chapter 7 summarizes the proposed research of this thesis, gives the main contributions and presents future research directions.

CHAPTER TWO
LOT STREAMING PROBLEM

Consider a scenario where lots consisting of several identical items are to be processed on several machines. Instead of transferring the entire lot after all of its items have been processed on a machine (like in traditional production systems), transferring the items of the lot can be made by small batches which are called sublots (Sarin & Jaiprakash, 2007, p.1). Then, the operations of these sublots on successive machines can be performed simultaneously. By this arrangement on the sublots, they can move along the machines immediately and the completion time of the whole lot decreases. This technique of splitting a lot into sublots and processing their movement over the machines is called “lot streaming” in the literature. In a more compact form, it can be defined as the process of splitting a production lot into sublots, and then scheduling the sublots in an overlapping fashion on the machines, in order to accelerate the progress of orders in production, and to improve the overall performance of the production system (Kalir & Sarin, 2000).

To clarify the benefits of LS, consider that a product with 64 items is going to be produced in a four machine flow shop system, where each machine processes an item in 2, 7, 6 and 3 minutes, respectively. The Gantt chart of the schedule without LS is given in Figure 2.1. The corresponding total completion time is 1152 minutes.

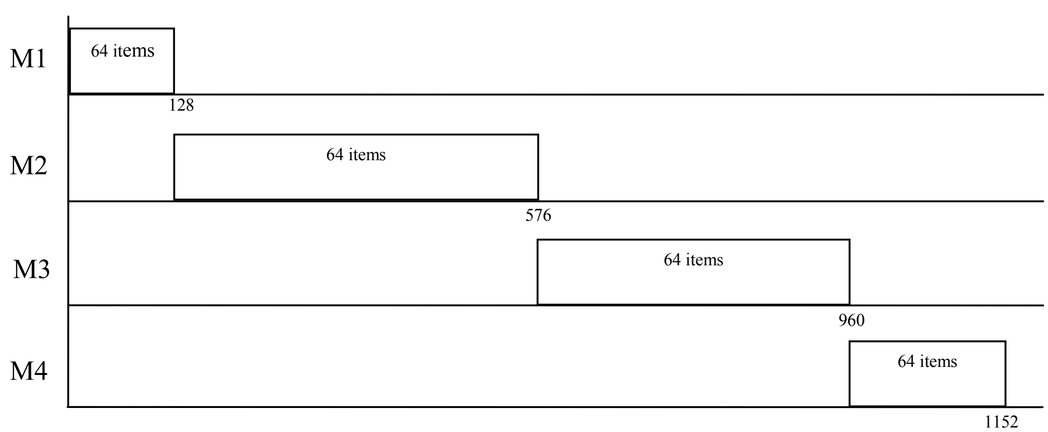

Figure 2.1 Gantt chart of the example without lot streaming

If we apply the LS technique and divide the whole production lot into four sublots each having the same number of items (i.e., 16 items), the total completion time decreases to 624 minutes providing a 45.8% improvement in comparison to the case without LS. It can be seen in Figure 2.2.

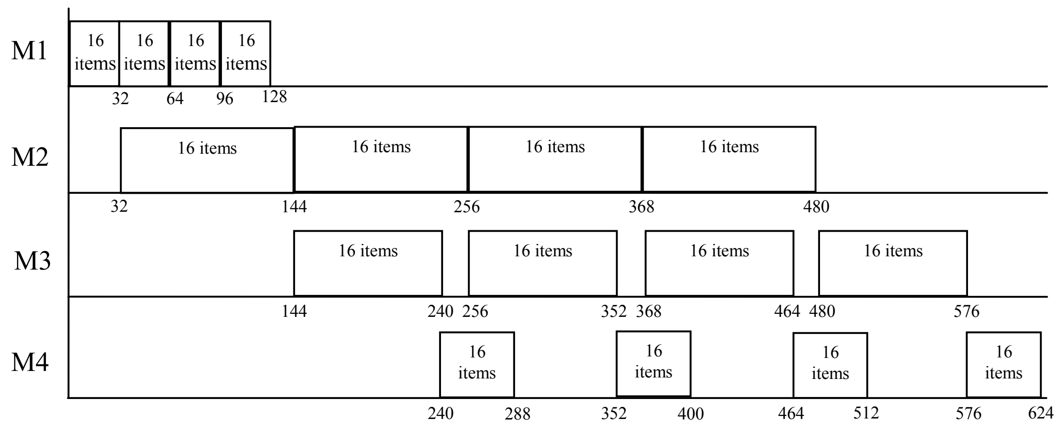

Figure 2.2 Equal sublots

LS has a number of advantages. Its main advantage appears in the reduction of total completion time. This reduction provides better due date performance by reducing the production lead times. Since the sublots exit from the system earlier in comparison to the case without LS, it also decreases the average WIP inventory and accordingly the associated WIP inventory costs. Finally, LS reduces the material handling capacity, interim storage and space requirements, since it handles smaller sized sublots instead of entire lot.

An LS problem can be described by a series of characteristics. In Section 2.1, a problem classification scheme incorporating these characteristics is introduced. In Section 2.2, a terminology is described to clarify the components of LS characteristics in detail. The dominance relations of LS problems based on the components given in Section 2.2 are summarized in Section 2.3. Finally, the common assumptions of LS problems are given in Section 2.4.

2.1 Classification

Table 2.1 gives comprehensive information on the characteristics of the LS problems. This table is adapted from Chang & Chiu (2005).

Table 2.1 Classification of LS problems in terms of main characteristics

Characteristic		**Notation**	**Component**
Production Type	α_1	F	Flow shop
		J	Job shop
		O	Open shop
Number of Machines		2	Two machines
		3	Three machines
		M	Multi machines
Product Type	α_2	1	Single product
		N	Multi products
Number of Sublots	β_1	fix	Fixed
		max	Maximum
Sublot Type	β_2	E	Equal
		C	Consistent
		V	Variable
Divisibility of the Sublot Size	β_3	D	Discrete
		R	Continuous
Sequence of the Sublots	β_4	IS	Intermingling
		NI	Non-Intermingling
Operation Continuity	β_5	II	Idling
		I_{no}	No Idling
Transfer Timing	β_6	W	Wait schedules
		W_{no}	No wait schedules
Setups	β_7	S_{no}	No setup
		S_A	Attached setup
		S_D	Detached setup
Availability	β_8	A_S	Sublot availability
		A_I	Item availability
Performance Measures	γ	$C_{\max}$	Makespan
		$\overline{F}$	Mean flow time
		$\sum F$	Total flow time
		$\overline{T}$	Mean tardiness
		n_T	Number of tardy jobs
		$\sum \lvert C-d \rvert$	Total deviation from the due date
		TC	Total cost
		$TC(C_{\max})$	Total cost with makespan

The following scheme is constructed by adapting the configurations presented by Potts & VanWassenhove (1992) and Chang & Chiu (2005) in order to classify and define the LS problem types. They presented a $\boldsymbol{\alpha}|\boldsymbol{\beta}|\boldsymbol{\gamma}$ representation for the LS problems, where $\boldsymbol{\alpha}$ represents the production environment, $\boldsymbol{\beta}$ defines the product characteristics and $\boldsymbol{\gamma}$ gives the performance measure. The levels of α, β and γ are given in Table 2.1. The first field is divided into two groups as $\alpha = \{\alpha_1, \alpha_2\}$ where α_1 shows the production type with number of machines and α_2 shows the number of products. The second field β indicates the product characteristics with eight different components, $\beta = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8\}$. The last field, γ, only presents the performance measures. The symbol "**–**" denotes that this characteristic is not taken into account in the problem or not mentioned in the study.

For instance, the $\{F_m, L_n \mid fix, E, D, IS, I_{no}, W_{no}, S_A, \mathrm{A}_S \mid C_{max}\}$ representation implies a problem with multiple products in a multi machine flow shop environment, with fixed number of discrete sized equal sublots, also considering non-intermingling case, no idling, no wait schedule and attached setups which aims to minimize the makespan.

Another example can be given for an existing study made by Biskup & Feldmann (2006) which can be represented as $\{F_m, L_1 \mid max, C/V, R/D, \text{-}, II, W, S_{no}/S_A/S_B, \mathrm{A}_S \mid C_{max}\}$. Their study deals with a single product in multi machine flow shop environments where maximum number of sublots is given, the sublot type is consistent or variable, and the sublot sizes can be either continuous or discrete. Since there is only one product, the intermingling or non-intermingling are not the case. The idling case, wait schedules and sublot availability is taken into consideration with no setup, attached setup and detached setup cases to minimize the makespan objective.

This representation scheme presented in this chapter will be used throughout the thesis.

2.2 Terminology

In this sub-section, the characteristics given in Table 2.1 are explained in detail with their components. At first, the characteristics familiar with the classical scheduling problems are described. Then the ones related to LS problems are introduced.

In terms of production type, several production systems may be considered; however here we only introduce the main production environments: flow shops, job shops and open shops. If the routes of all products are identical, that is, all products visit the same machines in the same order; the environment is referred to as a flow shop. A special case of flow shops, named permutation flow shops, on the other hand, assumes that the sequence of the products is the same on all machines.

If the products have different routes, this environment is referred to as a job shop, which is a generalization of a flow shop. (A flow shop is a job shop in which each and every single job has the same route.) The job shop models assume that a product may be processed on a particular machine at most once or several times on its route through the system.

Finally, the open shop scheduling model is a generalized version of flow shop and consists of m machines and n products. Each product has m operations. These operations are not necessarily performed in the same order for every product. Therefore, the routing for a product is the order of machines that the product visits (Sen & Benli, 1999). Open shops are similar with flow shops since it also requires m operations on m machines. However, all products in flow shops have to perform these operations in the same route, whereas the routes of products in open shops may differ. In this manner, open shop resembles the job shops where there may exist different routes for each product. However, note that, in job shops, each product does not need to have exactly m operations.

The number of machines is categorized into two components where two/three machine cases can be considered as smaller number of machines and the cases with more than three machines are classified as multi machine case. The studies concerning smaller number of machines occupy a wide area in the LS literature in comparison to the multi machine studies. This is probably caused by the growing complexity of problems by the increasing number of machines.

Product type component is categorized into single and multi product cases. Similar to other scheduling problems, the LS problem gets harder to solve with the increasing number of products. Therefore, in the literature, single product problems are studied more than multi product problems due to its simpler structure.

The performance criteria in LS problems can be either cost based or time based. The cost based LS studies generally aim to minimize total cost by determining the optimal sublot allocations. On the other hand, time based LS studies deal with makespan, mean flow time, mean tardiness, number of tardy jobs or total deviation from the due date, all of which are a function of time. Remember that one of the main benefits of LS is reduction in completion time of all products. Therefore, studies with the aim of minimizing makespan occupy a wide area in the literature. All research problems in this thesis also consider minimizing the makespan.

From the perspective of setup activity, in case of cost based performance measures, the only issue is the availability of a setup. However, for time based performance measures, the type of setups (i.e., attached or detached) also goes into the scheme additionally. The attached setup refers to the case when the setup of a product can be started only after the first sublot of that product has arrived at that machine. However, in detached setup case, the setup for a product on a machine can be performed even when the first sublot of that product is being processed on a previous machine. In detached setups, the setup on a machine is performed as soon as that machine finishes processing the previous product assigned to it. Figure 2.3 and Figure 2.4 illustrate the attached and detached setups, respectively. SU_{jm}

represents the setup operation of product *j* on machine *m* and S_{jsm} represents the processing of sublot *s* of product *j* on machine *m*.

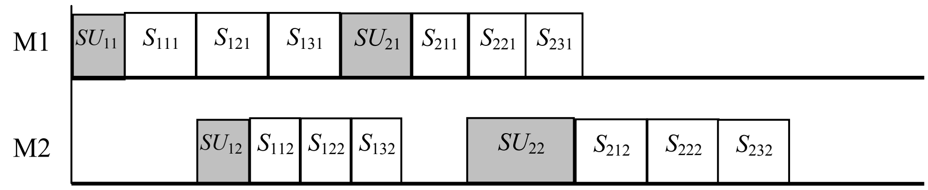

Figure 2.3 Attached setup case

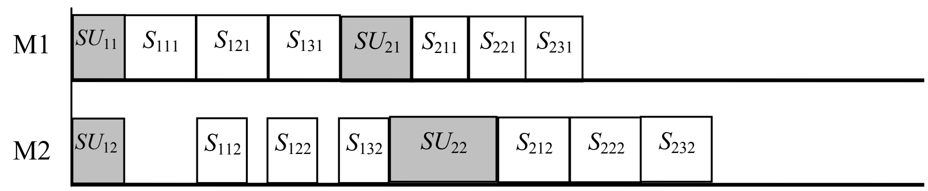

Figure 2.4 Detached setup case

The operation continuity characteristic is defined for the sublots of a product processed on the same machine; allowing the idle time between sublots or no-idle time between sublots. The no-idling case refers to a situation that the sublots of the same product are processed one after the other without any idle time on the same machine. For example, the production technology may dictate no idling if parts must be processed quickly to avoid cooling or chemical deterioration of machines (Baker & Jia, 1993). In case of idling (sometimes denoted as intermittent idling), there is no restriction and an idle time may exist between the sublots of the same product. The no-idling case requires an adjustment in modeling of the problem whereas this is not the case for idling. Idling case provides better makespan values than no-idling case. Figure 2.5 and Figure 2.6 illustrate the no-idling and idling cases, respectively.

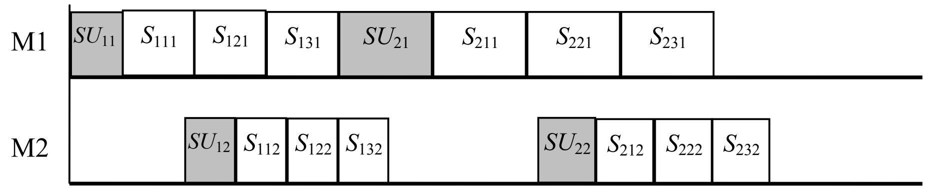

Figure 2.5 No-idling case

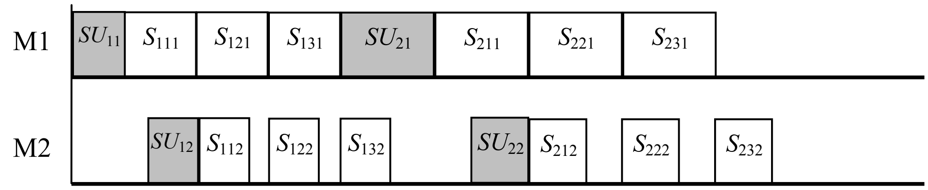

Figure 2.6 Idling case

The transfer timing is another characteristic that deals with no-idle time case in which idle time is not allowed through the consecutive machines of the same sublot. In no-wait schedules, the sublot of a product has to start its operation on the subsequent machine immediately after it finishes its operation on the current machine. This means each sublot has to be continuously processed on all machines. The studies including no-wait schedules have to specify this situation clearly, while the studies with wait schedules do not have to. Scheduling problems in no-wait flow shops arise in chemicals processing and petro-chemical production environments. Another example of the no-wait situation arises in hot metal rolling industries where the metals have to be processed continuously at high temperature (Sriskandarajah & Wagneur, 1999). The no-wait schedule is presented in Figure 2.7.

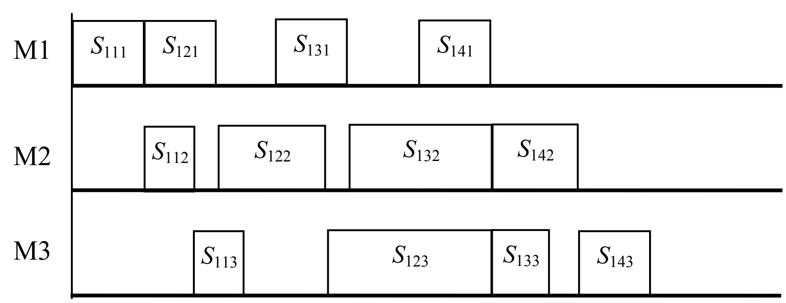

Figure 2.7 No-wait schedule

The other components, which are more related with LS (i.e., sublot types, sublot sizes, sequence of sublots, number of sublots and availability), are explained in detail in the following sub-sections.

2.2.1 Number of Sublots

The aim in LS problems is to determine the number of sublots and the sizes of each sublot according to some performance criteria. If a maximum level on the number of sublots is given as a parameter and then the optimal number of sublots is tried to be found within this restricted interval; this case is called “maximum number of sublots”. The number of sublots as well as sublot sizes has to be optimized in this case. However, some researchers assume that the number of sublots is fixed and known. In this case, there is no need to optimize the number of sublots, since the exact number of sublots is priori known and the entire lot has to be divided into this exact number. Therefore, the only remaining issue is the optimization of sublot sizes. This case is called “fixed number of sublots”. The main reasons of considering fixed number of sublots are twofold. The former one is that it quite simplifies the problem since the number of sublots is known and there is no need for extra computational effort to find the optimal number of sublots. The latter one is that the system on hand requires these restrictions (e.g., restriction on the capacity of the material handling equipment, fixed number of pallets, container associated with the moving of the sublots). In fixed number of sublots, the size of each sublot has to be at least one unit for the case of discrete sublots. For instance, the sublot sizes for the fixed number of sublots may be valued as 2-8-4-6-3. However, the sublot sizes for the “maximum number of sublots” case may contain one or more sublots with zero size such as 11-0-5-7-0. Note that, in this case, the maximum number of sublots is given as five, but the resulting number of sublots is three.

2.2.2 Sublot Types

The sublot types can be categorized into three groups; equal, consistent and variable.

All these sublot types have to satisfy Eq.(2.1). Let SS_{im} is the size of sublot i on machine m, L is the production lot size and S is the number of sublots. The sum of sublot sizes on the same machine has to be equal to the production lot size.

$$\sum_{i=1}^{S} SS_{im} = L \qquad m = 1,\ldots,M \tag{2.1}$$

Let us reconsider the example given at the beginning of this chapter. A single product is going to be processed on four machines with 2, 7, 6 and 3 minutes, respectively. The production lot size with 64 items is to be divided into four sublots. The data will be used in the following figures to illustrate different properties of sublot types.

Equal Sublots

The basic sublot type can be referred to as equal sublots, which denotes the case where all sublots of a product are of the same size. In addition, the sublot sizes are constant on all machines. Eq.(2.2) gives this relation in case of a fixed number of sublots. This relation may not be valid if the maximum number of sublots is predetermined.

$$SS_{im} = L / S \qquad i = 1,\ldots,S \qquad m = 1,\ldots,M \tag{2.2}$$

A Gantt chart characterizing equal sublots is given in Figure 2.2. A lot with 64 items is divided into four equal sublots each having 16 items. These sublots are scheduled among the machines and the completion time is resulted in 624 minutes. Remember that, the completion time without LS was 1152 minutes.

Consistent Sublots

In consistent sublot types, the size of sublots may vary within the same machine, however sublots have to stick their sizes through the consecutive machines. This situation is given in Eq.(2.3).

$$SS_{im} = SS_i \qquad i = 1,\ldots,S \qquad m = 1,\ldots,M \tag{2.3}$$

In addition, $SS_i \neq SS_{i+1}$ inequality relation has to be in order for at least a pair of sublots to produce a different sublot size configurations than equal sublots.

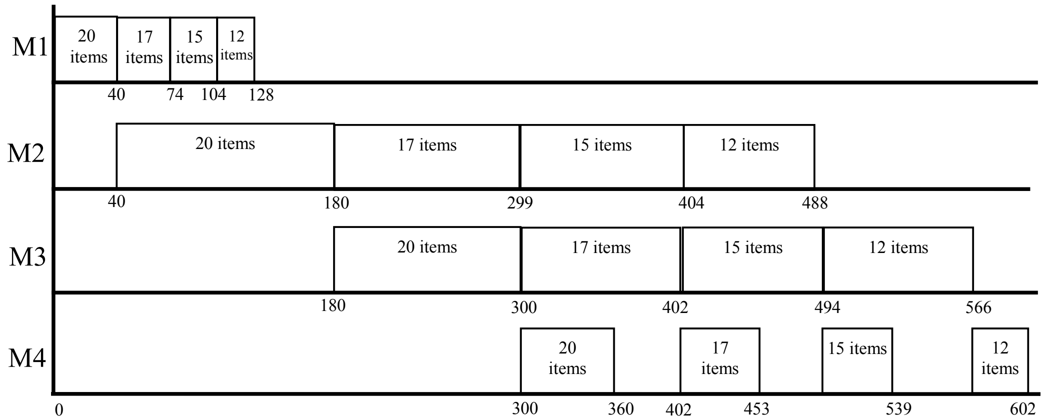

Figure 2.8 Consistent sublots

Figure 2.8 illustrates the case of consistent sublot types for four fixed sublots. The sublots include 20, 17, 15 and 12 items, respectively. In this case, the total completion time is decreased to 602 minutes, which is smaller than the total completion time of the case with equal sublots.

Variable Sublots

For the variable sublots, there is no restriction on the sublot size either within the same machine or on consecutive machines. In case of variable sublots, Eq.(2.4) should be in order for at least one pair of consecutive machines.

$$SS_{im} \neq SS_{i(m+1)} \qquad i = 1,...,S \qquad m = 1,...,M-1 \tag{2.4}$$

A schedule representing this sublot type is illustrated in Figure 2.9. The sublot sizes may vary within the sublots on the same machine and also within consecutive machines. Different from consistent sublots, size of the second sublot on the first machine is one and on the second machine is 19. The schedule ends at 589 minutes, which is smaller than the completion times of both cases with equal and consistent sublot types. Also it should be noted that, the reduction by the variable sublots is 48.8% when compared without LS case.

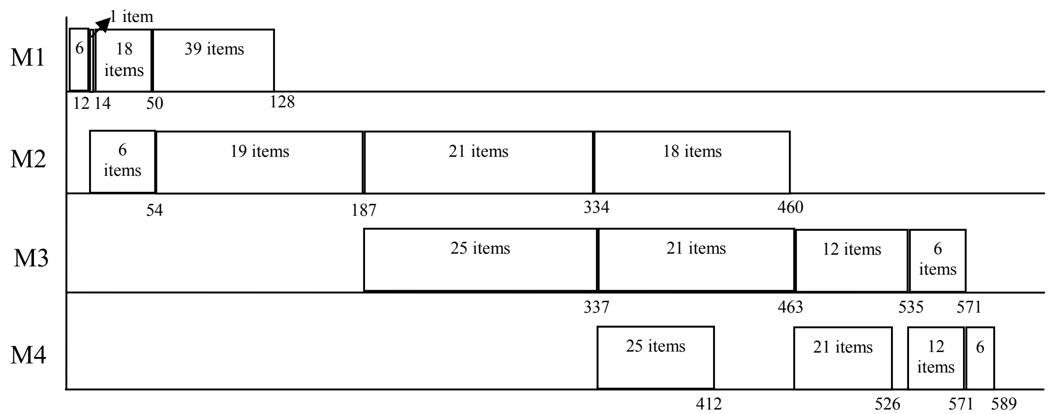

Figure 2.9 Variable sublots

2.2.3 Sublot Sizes

Another important component is the divisibility of the sublot sizes, i.e., discrete or continuous. In discrete version, the sublot size is to be integer (e.g., 12), while this is not the case for the continuous version (e.g., 12.33). The production systems that produce fluid based products such as gas, drinks or dye are the instances for continuous sublot sizes. On the other hand, the systems producing countable products such as machine or computer parts, and textiles (especially ready-to-wear clothing) are classical examples of discrete sublot sizes. Eq.(2.5) and Eq.(2.6) illustrate discrete and continuous sublot cases, respectively.

$$SS_{im} \in \mathrm{Z}^{+} \quad i = 1,\ldots,S \qquad m = 1,\ldots,M \tag{2.5}$$

$$SS_{im} \in \Re^{+} \quad i = 1,\ldots,S \qquad m = 1,\ldots,M \tag{2.6}$$

2.2.4 Intermingling/Non-intermingling Schedules

These schedules are the case for only multi product LS problems because these schedules deal with the sequence of sublots of the products. Non-intermingling schedules do not allow any interruption in the sequence of sublots of a product by the sublots of any other product(s). This means if a sublot of a product starts its operation on a machine, then the other sublots of that product have to follow this sublot on the sequence. In intermingling schedule cases, the sequence of sublots of any product can be interrupted by the sublots of other products. In this case, the sublots have to be handled as independent products. (Feldmann & Biskup, 2008)

These cases are illustrated in Figure 2.10 and Figure 2.11. An LS problem with three products and three sublots is presented for the product sequence 1-3-2. Remember that, the representation S_{jsm} corresponds to the sublot s of product j on machine m.

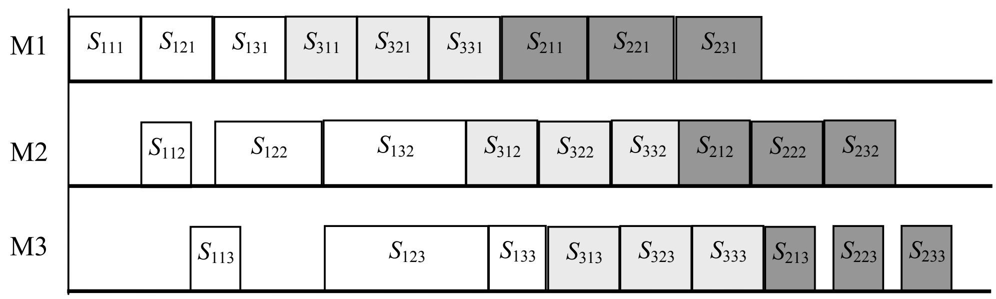

Figure 2.10 Non-intermingling schedule

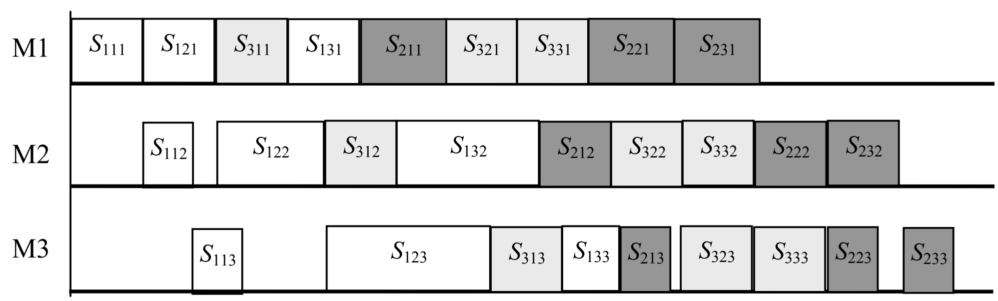

Figure 2.11 Intermingling schedule

2.2.5 Availability

Availability characteristic describes the situations when a new sublot can be configured for processing on a machine, after the items constituting that sublot have been processed on the preceding machine. (Sarin & Jaiprakash, 2007, p.47)

There are two cases for the availability component; the sublot availability and the item availability. The sublot availability does not allow a portion of a sublot to be transferred to the next operation to constitute a new sublot until all items in that sublot finish their operation on the current machine. In item availability, the items of a sublot, which finish their operations in the current machine, can be transferred to the next operation independently from the other items of this sublot. Item availability is meaningful for only variable sublots, since the sublot sizes in consistent or equal sublot types do not vary on the machines. Therefore, naturally, for equal and consistent sublots, sublot availability exists by default. Figure 2.9 and Figure 2.12 represents schedule instances for the sublot availability and item availability, respectively (For more information see Sarin and Jaiprakash, 2007, pg. 47).

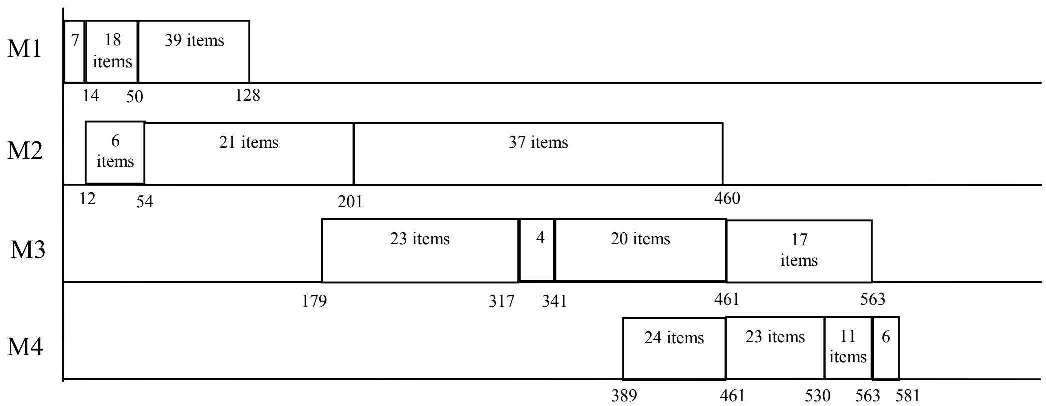

Figure 2.12 Item availability

2.3 Dominance Relations of Lot Streaming Problems

As mentioned earlier, LS problems have a number of characteristics. Some components of these characteristics dominate some other components in case of makespan objective. The dominance relations among some of the characteristics of LS problems can be summarized as follows. (Trietsch & Baker, 1993)

Related to the sublot sizes, variable sublot type (V) is dominant over consistent sublot type (C) which is dominant over equal sublot type (E). This means that a model with variable sublots should have shorter or equal makespan than the makespan of the same model with consistent or equal sublots. Any solution of equal sublot type will be an upper bound for consistent and variable sublot types for the minimization problems. Similarly, any solution of consistent sublot type will be an upper bound for variable sublot types.

$$C_{\max}(E) \geq C_{\max}(C) \geq C_{\max}(V)$$

It is clear that idling (II) dominates no idling (NI) case. The related dominance relations can be seen in Figure 2.13. The least restrictive case is V/II which means the minimal makespan will be achieved with variable sublots and idling case. There is no clear dominance between the models shown in the same level i.e. variable sublots with no idling (V/NI) and consistent sublots with idling (C/II). (Trietsch and Baker, 1993)

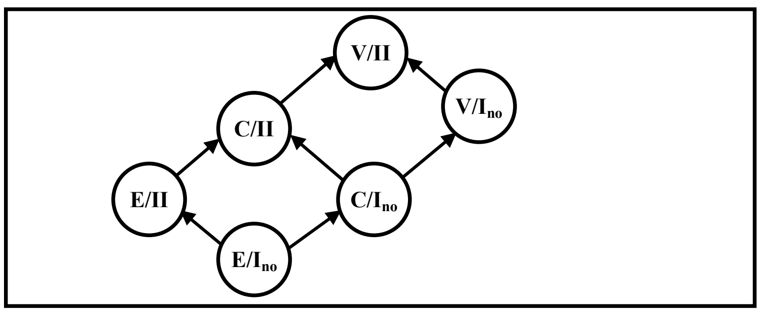

Figure 2.13 Dominance relationship of sublot types (E / C / V) and idling (II) / no-idling (I_{no}) cases

When divisibility of sublot sizes are taken into consideration, continuous (CV) sublot case dominates over discrete (DV) case. According to these dominance relationships, the least restrictive model is V/II/CV.

Another dominance relation exists between the intermingling and non-intermingling cases for multi product LS problems. Figure 2.13 can be adapted to Figure 2.14, to show this relation. Any non-intermingling schedule is dominated by intermingling schedules because non-intermingling schedules only consider the sequence of products while intermingling schedules consider sublots as well as products.

In case of maximum number of sublots, sublot sizes as well as the number of sublots have to be optimized. In fixed number of sublots case, on the other hand, there is no need to optimize the number of sublots, since the exact number of sublots is priori known and the entire lot has to be divided into this exact number. The only issue remains as the optimization of sublot sizes. Therefore, the fixed number of sublots case is a special version of maximum number of sublots and it eliminates the determination of number of sublots in LS problem.

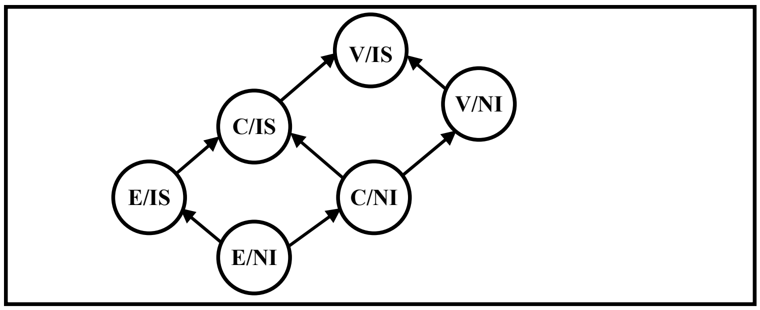

Figure 2.14 Dominance relationship of sublot types (E/C/V) and intermingling (IS) / non-intermingling (NI) schedules

In the literature, the complexities of single product LS problems are determined by Trietsch & Baker (1993). The single product LS problems with smaller number of machines are categorized as polynomial (P). The LS problems with m (m>3) number of machines get harder to solve and some of these problems especially with discrete sized sublots are categorized as non-deterministic polynomial (NP). The solution algorithms for continuous sublots are given as linear programming (LP) formulations and for discrete sublots as integer linear programming (ILP). Although the complexity of single product, multi-machine LS problem with variable sublot size, idling case, continuous sublot size is not described here, Biskup & Feldmann (2006) claim that this LS problem type is most probably NP hard, but no proof for this conjecture exists in the literature, that is, the complexity status of this problem is still open. However, the discrete sublot size version of this problem is exactly NP hard.

We can use the complexity of single product LS problems to define the complexity of multi product LS problems. The multi product LS problems in flow shops require scheduling the products through the machines as well as sublot allocation of the products. The first problem, scheduling products, is NP-complete for more than three machines (Garey, Johnson & Sethi, 1976). Surely, referring to the Table 2.2, discrete versions of multi product LS problems are NP. On the other hand, we cannot claim that all the continuous versions of multi product LS problems are NP. Nevertheless, the multi product LS problems are much harder to solve than the single product LS problems.

Table 2.2 Summary of solution status of LS Problems (Trietsch & Baker, 1993)

Number of Machines	**Consistent/ Variable**	**Idling/No-idling**	**Continuous/ Discrete**	**Complexity**	**Solution**
2	C	II	R	P	O(n)
2	V	II	R	P	O(n)
2	C	I_{no}	R	P	O(n)
2	V	I_{no}	R	P	O(n)
2	C	II	D	P	O(Un^2)
2	V	II	D	P	O(Un^2)
2	C	I_{no}	D	P	O(Un^2)
2	V	I_{no}	D	P	O(Un^2)
3	C	II	R	P	O(n)
3	V	II	R	P	O(n)
3	V	II	D	P	O(Un^2)
m	C	I_{no}	R	P	LP
m	C	I_{no}	D	NP	ILP
m	C	II	R	P	LP
m	C	II	D	NP	ILP
m	V	I_{no}	R	P	O(mn)
m	V	I_{no}	D	P	O(mUn^2)
m	V	II	R	?	-
m	V	II	D	NP	ILP

Considering the above dominance relations, the least complex LS problem can be described as $\{F_2, L_1 \mid fix, E, C, \text{-}, II, \text{-}, \text{-}, \mathrm{A}_S \mid C_{max}\}$ for the single product case and $\{F_2, L_n \mid fix, E, C, NI, II, \text{-}, \text{-}, \mathrm{A}_S \mid C_{max}\}$ for the multi product case.

2.4 Assumptions

The general assumptions used throughout the thesis are stated in the following. These assumptions are common for all investigated research problems. The additional assumptions of the research problems are going to be given in their respective chapters.

1. All product lots are available at time zero.
2. The production environment is limited to permutation flow shops. Recall that, permutation flow shop is a special case of flow shops where the sequence of the products is the same on all machines.
3. The flow shop is a multi machine one with number of machines being greater than three.(m>3)
4. The machine at each stage is continuously available. This means there is no uncontrolled idling such as machine breakdowns, unscheduled maintenance, etc.
5. Only one product lot can be processed on a machine at any time. Conversely, one machine cannot process more than one lot at a time.
6. Pre-emption of sublots is not allowed.
7. Once a machine starts a lot, it has to process the lot continuously until it is finished. This assumption indicates non-intermingling schedules in multi product cases.
8. The performance measure is to minimize the makespan.
9. Sublot transfer times are assumed negligible.
10. Neither attached nor detached setups are considered i.e., no setups.
11. The sublot availability case is taken into account for the variable sublot cases.

CHAPTER THREE
LITERATURE REVIEW

Lot streaming (LS) term was first introduced by Reiter in 1966. This concept has not received much attention until late 1980s and early 1990s; however, it has been a well-known research area since then with the introduction of optimized production technology concept (Sarin & Jaiprakash, 2007, p.20). Although several papers have studied LS problems since 1980's, the first comprehensive review is made by Chang and Chiu in 2005.

Since our research problems deal with flow shop environments and makespan objective, a comprehensive literature review is presented with respect to the most relevant work based on time models (especially minimizing makespan) in flow shops.

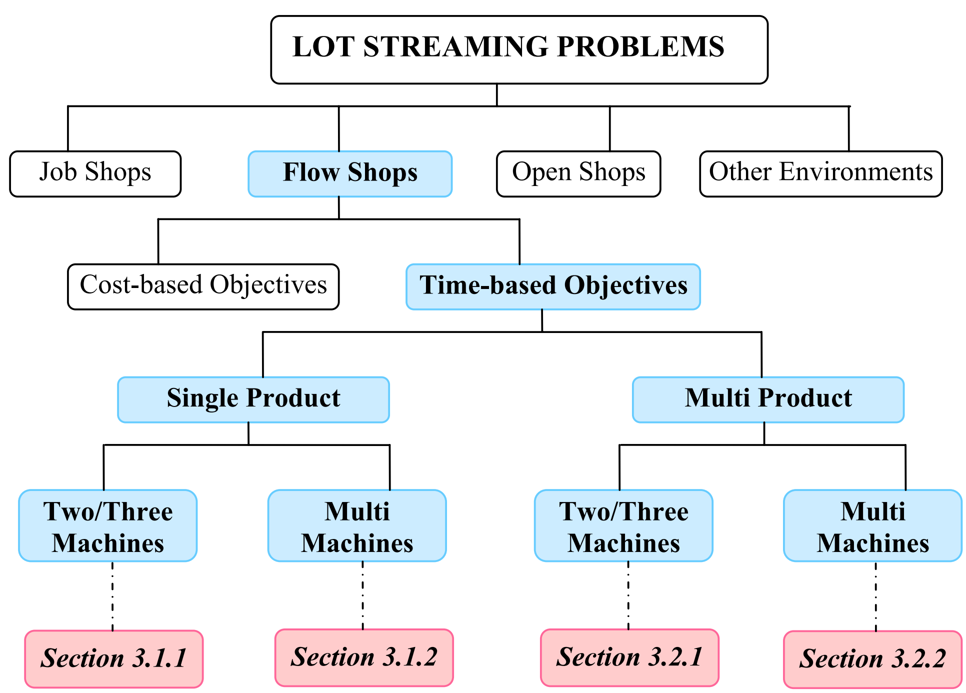

Figure 3.1 The organization of literature review

As mentioned before, a typical LS problem can be encountered in different production settings. The number of products and number of machines generally defines the production settings. Therefore, in this section, the studies are presented in four classes varying by the number of products and machines. The organization of this chapter with the problem characteristics are shown in Figure 3.1. The characteristics of LS problems are investigated under these sub-sections in detail. In the last section of this chapter, a summary of previous research is presented with respect to LS problem characteristics and the relations between the proposed research in this thesis and current literature are discussed.

3.1 Single Product Lot Streaming Problems

The aim in single product LS problems is to find the optimal number of sublots and the sizes of these sublots. Therefore, single product LS problems are naturally simpler than multi product LS problems. However, it still may be NP-hard due to the presence of some challenging LS characteristics. In terms of various LS characteristics, the complexities of single product LS problems have already been given in Table 2.2 in Section 2.3.

In a general study, Kalir & Sarin (2000) evaluate the potential benefits of LS in flow shops in terms of makespan, average flow time and average WIP level. They give the worst case performances of these objective functions with and without LS for the single product case.

3.1.1 Two/Three Machines

The problems with single product and smaller number of machines are the simplest ones and require less computational effort. Table 3.1 illustrates the characteristics of single product LS studies in two/three machine flow shops as well as applied solution approaches and their optimality.

A summary of the LS work on two and three machines from 1988 to 1993 can be found in Trietsch & Baker (1993).

Table 3.1 Single product LS studies in two/three machine flow shops

Author(s)	Year	Number of Products	Number of Machines	Number of Sublots	Sublot Type	Sublot Size	Sequence	Idling/ No	Wait/ No Wait	Availability	Setups	Objective Function	Solution Approach	Optimality
Potts and Baker	1989	Single	Two	Fix	Consistent	Continuous	-	No idling	-	-	No	Makespan	Exact	Optimal
		Single	Two	Fix	Equal	Continuous	-	No idling	-	-	No	Makespan	Worst case perform.	-
Trietsch and Baker	1993	Single	Two	Fix	Consistent	Continuous	-	No idling	-	-	No	Makespan	Dominance Relations	Optimal
		Single	Two	Fix	Consistent	Discrete	-	No idling	-	-	No	Makespan	Dominance Relations	Optimal
		Single	Three	Fix	Consistent	Continuous	-	No idling	-	-	No	Makespan	LP, Dominance Relations	Optimal, Near-Optimal
		Single	Three	Fix	Consistent	Discrete	-	No idling	-	-	No	Makespan	LP, Dominance Relations	Optimal, Near-Optimal
		Single	Three	Fix	Variable	Continuous	-	No idling	-	Sublot	No	Makespan	Dominance Relations	Optimal
		Single	Three	Fix	Variable	Continuous	-	Idling	No-wait	Sublot	No	Makespan	Dominance Relations	Near-Optimal
		Single	Three	Fix	Variable	Discrete	-	Idling	No-wait	Sublot	No	Makespan	Dominance Relations	Near-Optimal
Baker and Jia	1993	Single	Three	Fix	Equal	Continuous	-	No idling	-	-	No	Makespan	Worst case perform.	-
		Single	Three	Fix	Equal	Continuous	-	Idling	-	-	No	Makespan	Worst case perform.	-
		Single	Three	Fix	Consistent	Continuous	-	No idling	-	-	No	Makespan	Worst case perform.	-
		Single	Three	Fix	Consistent	Continuous	-	Idling	-	-	No	Makespan	Worst case perform.	-
		Single	Three	Fix	Variable	Continuous	-	Idling	-	Sublot	No	Makespan	-	Optimal
		Single	Three	Fix	Variable	Continuous	-	No idling	-	Sublot	No	Makespan	-	Optimal
Glass et al	1994	Single	Three	Fix	Consistent	Continuous	-	Idling	-	-	No	Makespan	Exact, Dominance Relations	Optimal
Chen and Steiner	1998	Single	Three	Fix	Consistent	Continuous	-	Idling	-	-	Attached	Makespan	Exact, Dominance Relations	Optimal/ Near-Optimal
Chen and Steiner	1996	Single	Three	Fix	Consistent	Continuous	-	Idling	-	-	Detached	Makespan	Exact, Dominance Relations	Optimal/ Near-Optimal
Sen et al.	1998	Single	Two	Fix	Equal	Continuous	-	-	-	Job	No	Makespan	Dominance Relations	Optimal
		Single	Two	Fix	Consistent	Continuous	-	-	-	Job	No	Makespan	Dominance Relations	Optimal
		Single	Two	Fix	Variable	Continuous	-	-	-	Job	No	Makespan	Dominance Relations	Optimal
		Single	Two	Fix	Equal	Continuous	-	-	-	Sublot	No	Mean Flow Time	Dominance Relations	Optimal
		Single	Two	Fix	Consistent	Continuous	-	-	-	Sublot	No	Mean Flow Time	Dominance Relations	Optimal
		Single	Two	Fix	Variable	Continuous	-	-	-	Sublot	No	Mean Flow Time	Dominance Relations	Optimal
		Single	Two	Fix	Equal	Continuous	-	-	-	Item	No	Mean Flow Time	Dominance Relations	Optimal
		Single	Two	Fix	Consistent	Continuous	-	-	-	Item	No	Mean Flow Time	Dominance Relations	Optimal
		Single	Two	Fix	Variable	Continuous	-	-	-	Item	No	Mean Flow Time	Dominance Relations	Optimal
Sriskandarajah and Wagneur	1999	Single	Two	Fix	Consistent	Continuous	-	-	No-wait	-	Detached	Makespan	LP, Exact	Optimal
		Single	Two	Fix	Consistent	Discrete	-	-	No-wait	-	Detached	Makespan	Heuristic	Near-Optimal
Bukchin et al.	2002	Single	Two	Max	Consistent	Continuous	-	Idling	-	Sublot	Attached	Mean Flow Time	Dominance Relations	Optimal, Near-Optimal
Liu	2008	Single	Two stage m-1 hybrid	Max	Consistent	Continuous	-	-	-	-	Sublot attached	Makespan	Exact, LP, Enumaration	Optimal
		Single	Two stage m-1 hybrid	Fix	Consistent	Continuous	-	-	-	-	Sublot attached	Makespan	Exact, LP	Optimal
		Single	Two stage m-1 hybrid	Max	Equal	Continuous	-	-	-	-	Sublot attached	Makespan	Exact	Optimal

In a flow shop environment, under the objective of minimizing makespan, the processing times of machines influence the sublot sizes. Therefore, two cases (i.e., $p_1 > p_2$ and $p_1 < p_2$, where p_1 and p_2 are the processing times of the first and the second machine, respectively) have to be analyzed in detail. Figure 3.2 and 3.3 illustrates these cases.

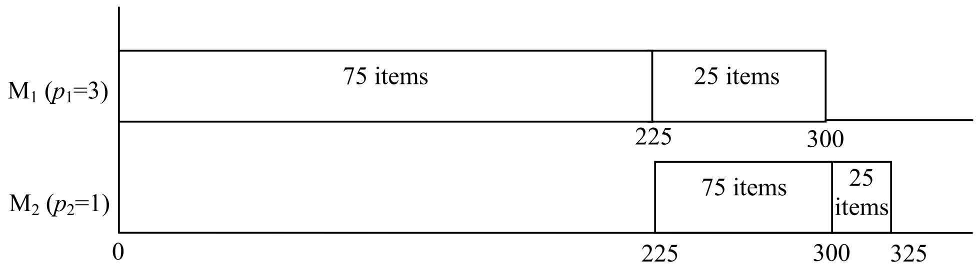

Figure 3.2 Lot streaming on two machines where $p_1 > p_2$

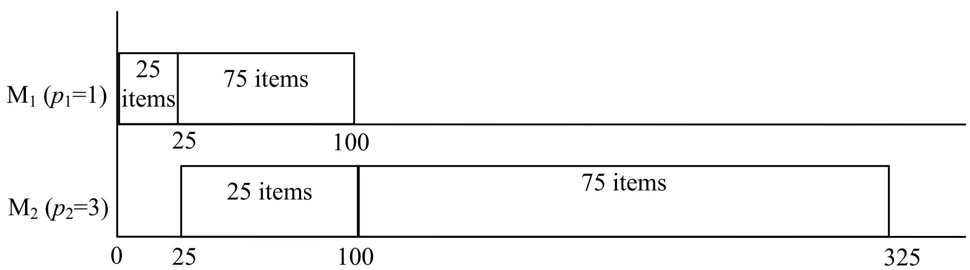

Figure 3.3 Lot streaming on two machines where $p_1 < p_2$

If $p_1 > p_2$, sublots can be processed on the first machine and accordingly on the second machine. In this case, the sublots are decreasing in size. If $p_1 < p_2$, the reverse problem can be handled in the same way (see Figure 3.2 and 3.3). It is proved by Potts & Baker (1989) that a LS problem and its inverse are equivalent. The reversibility property ensures that idling case is not necessary for two machines and optimal sublot sizes that minimize makespan on two machine flow shops can be found without idle time.

It is proved again by Potts & Baker (1989) that, for a given number of sublots, there exists an optimal schedule for the makespan criteria in which $SS_{i1} = SS_{i2}$ and $SS_{iM-1} = SS_{iM}$, where SS_{im} is the size of sublot i on machine m (m=1,…,M). Since there is only one transfer step between the first machine and second one in two machine flow shops, there is no need to consider variable sublots. Potts & Baker (1989) prove that, in two machine cases, all sublots are critical in an optimal solution and the optimal set of sublot sizes are geometric. Therefore, optimal sublot sizes can be obtained by consistent sublots (Trietsch & Baker, 1993). For two sublot cases, the optimal sublot sizes can be obtained by the ratio $q = p_2 / p_1$. For S sublots, the size of sublot i can be calculated as $SS_i = Lq^{i-1}/(1+q+...+q^{S-1})$ where L is the production lot size. This geometric sublot sizes are only valid for continuous sublots and do not hold for discrete sized ones. Some of the studies related with discrete sized sublots present rounding algorithms that first obtain continuous sizes and then convert these values to discrete ones (e.g., Chen & Steiner, 1997; Sriskandarajah & Wagneur, 1999; Trietsch & Baker, 1993). Trietsch & Baker (1993) develop an iterative algorithm which crosschecks the situation that the converted discrete sized sublots satisfy the given lower bound or not. The initial lower bound is equal to the makespan value of the continuous sized ones and it should be updated if it is not satisfied by the discrete sublot sizes. If the given lower bound is achieved by the discrete sizes then the algorithm stops, otherwise it continues on trials. Sriskandarajah & Wagneur (1999) propose a rounding and a generating algorithm to obtain near-optimal solutions for the no-wait schedules. The former converts continuous sized sublots to discrete ones; while the latter uses the property of equal sublots that first sizes all the sublots equally and then allocates the remaining items starting from the initial sublots. They obtain better results by rounding algorithm in comparison to the ones of generating algorithm.

For three machine cases, the relations of processing times of machines become more complex. Therefore, the following cases have to be analyzed for continuous sized sublots. For two consistent sublots, the resulting sublot sizes of each case are described by Baker (1988) as in the following.

Case 1. If $p_2^{\,2} > p_1 p_3$ and $p_1 \geq p_3$, then

$SS_1 = L \times p_1 / (p_1 + p_2)$ and $SS_2 = L \times p_2 / (p_1 + p_2)$, with no idling

Case 2. If $p_2^{\,2} > p_1 p_3$ and $p_1 < p_3$, then

$SS_1 = L \times p_2 / (p_2 + p_3)$ and $SS_2 = L \times p_3 / (p_2 + p_3)$, with no idling

Case 3. If ${p_2}^2 \le p_1 p_3$, then

$SS_1 = L \times (p_1 + p_2) / (p_1 + 2p_2 + p_3)$ and

$SS_2 = L \times (p_2 + p_3) / (p_1 + 2p_2 + p_3)$, with idling

Baker & Jia (1993) make computational analyses on three machine LS problems by comparing the results of equal and consistent sublots with the ones of variable sublots. They confirm the situations, also stated by Trietsch & Baker (1993), for more than two sublot cases.

- If ${p_2}^2 > p_1 p_3$,
  - Optimal sublot sizes can only be achieved by variable sublots. $C_{\max}(V) \le C_{\max}(C)$
  - No-idling case and idling case generate the optimal makespan independent of the sublot type. $C_{\max}(II) = C_{\max}(I_{no})$
- If ${p_2}^2 \le p_1 p_3$,
  - Optimal sublot sizes can be achieved by consistent sublots as well as variable sublots. $C_{\max}(V) = C_{\max}(C)$
  - Optimal makespan can only be achieved by idling case. $C_{\max}(II) \le C_{\max}(I_{no})$

When sublot sizes are variable and the no-idling constraint is enforced, the problem can be decomposed into two sub-problems consisting of first pair of machines (i.e., M_1 and M_2) and the second pair of machines (i.e., M_2 and M_3). For each pair of machines, the solution methodology for the two machine problem with no idling case can be used to obtain the continuous optimal sublot sizes. However, when variable sublots with idling case is considered, the dominance relations have to be analyzed. If ${p_2}^2 > p_1 p_3$, the problem can be solved optimally by decomposing it two-machine pairs and by solving each of these problems by using the two machine procedures with idling case. Otherwise, the consistent sublot sizes are optimal and geometric in the ratio $(p_2 + p_3)/(p_1 + p_2)$. A comprehensive analysis of three machine LS problem with continuous sized consistent sublots can be found in Glass, Gupta & Potts (1994).

In two/three machine LS studies, generally no setup case is considered whereas only a few studies deal with attached or detached setups. Chen & Steiner (1996, 1998) study the problem of Glass, Gupta & Potts (1994) and extend it to include setups. Chen & Steiner (1996) consider detached setups, while Chen & Steiner (1998) use attached setup type. In both studies, they investigate several cases to analyze the structural properties of three machine LS problems.

In two and three machine cases, equal sublot type is generally used to calculate the worst case performance by comparing its results with the makespan values of the optimal sublot types (Baker & Jia, 1993; Liu, 2008; Potts & Baker, 1989).

Different from makespan objective, Sen, Topaloglu & Benli (1998) and Bukchin, Tzur & Jaffe (2002) consider minimizing mean flow time in two machine flow shops. Sen, Topaloglu & Benli (1998) study the $\{F_2, L_1 \mid fix, E/C/V, R, -, -, -, A_S/A_I, S_{no} \mid \overline{F}\}$ problem to analyze the effect of processing times and sublot types under job availability, sublot availability and item availability cases. Since job availability case corresponds to makespan minimization problem, they only derive the results from the literature. For sublot availability case, they show that equal sublots generate the same results with the variable sublots when $p_1 \geq p_2$. They also derived some results from the literature for the item availability. As an overall result, they state that even when variable sublots are allowed, consistent sublots are optimal in all cases, except in sublot availability with $p_1 < p_2$. Their findings can be seen in Table 3.2. Referring to the last column of this table, they also suggest equal sublots to be used in practice due to its efficient worst case performance. Bukchin, Tzur & Jaffe (2002) evaluate the performance of average flow time and makespan for consistent sublots and sublot attached setups.

No-wait schedules are considered by Sriskandarajah & Wagneur (1999) and Trietsch & Baker (1993), for two and three machine LS problems, respectively. No-wait schedule case in two machines is quite simple, since there is only one step to consider this situation for each sublot. Sriskandarajah & Wagneur (1999) study this version of the problem for detached setups with consistent sublot type. Consistent sublots are optimal for two machines independent of the no-wait schedules. For three machine flow shops, Trietsch & Baker (1993) state that, in the presence of idling and variable sublots, the optimal schedule must be a no-wait schedule and also an optimal solution with consistent sublots can be obtained when $p_2^2 \leq p_1 p_3$. Otherwise, decomposition of the problem into two sub-problems is suggested where each sub-problem comprises two machines and solved by the two-machine procedures.

Table 3.2 Derived results of Sen, Topaloglu & Benli (1998)

		Consistent	**Variable**	**Equal/Optimal**
Job availability	$p_1 \geq p_2$	Geometric	Geometric	1.09
	$p_1 < p_2$	Geometric	Geometric	1.09
Sublot availability	$p_1 \geq p_2$	Equal	Equal*	1.00*
	$p_1 < p_2$	Algorithm 1	M1: Geometric* M2: Equal*	1.14*
Item availability	$p_1 \geq p_2$	Equal	Equal	1.00
	$p_1 < p_2$	Geometric	Geometric	1.18

* Conjectured

Liu (2008) considers an LS problem in a different production environment, i.e., a two stage hybrid flow shops with *m* machine at the first stage working parallel and only one machine at the second stage. The worst case performances of equal sublot case and the consistent sublot case (for fixed number of sublots) are evaluated by comparing their results with the optimal consistent ones.

In terms of solution approaches for the LS problems in this section, the dominance relations of processing times of machines play a significant role. Since the cases appearing in two and three machine are limited, each case is analyzed by the researchers individually. The conditions of the cases are determined where the optimal solutions can be obtained or not. The sublot sizes of optimal solutions are derived from theoretical formulations.

3.1.2 Multi Machines

The problem characteristics and solution approaches of the single product LS studies in multi machine flow shops are presented in Table 3.3.

3.1.2.1 Problem Characteristics

For single product multi machine flow shop LS problems, an early study is by Szendrovits (1975) with the objective of minimizing manufacturing cycle time as well as minimizing total cost under equal sublot types, continuous sublot sizes and no-idling case by using dominance relations of processing times of machines. Later, Ornek & Collier (1988) extend this problem to determine equal sublot sizes where the number of sublots may differ between machines.

As known from three machine case, in terms of processing times of machines, the number of cases to be analyzed increase with the increasing number of machines. For these types of LS problems, the number of alternatives quite increases and their relations get difficult to analyze.

Since multi machine LS problems are much harder to solve, most of the studies generally assume fixed number of sublots due to its simplicity. There are a number of studies for maximum number of sublots; however, most of these studies assume continuous sized equal sublots in which case the only remaining problem is to optimize the number of sublots (e.g. Bukchin & Masin, 2004; Kalir & Sarin, 2001a, 2003; Sarin, Kalir & Chen, 2008).

Due to the presence of single product, the intermingling and non-intermingling schedules are not the case.

Table 3.3 Single product LS studies in multi machine flow shops

Author(s)	Year	Number of Products	Number of Machines	Number of Sublots	Sublot Type	Sublot Size	Sequence	Idling/ No	Wait/ No Wait	Availability	Setups	Objective Function	Solution Approach	Optimality
Potts and Baker	1989	Single	Multi	Fix	Consistent	Continuous	-	No idling	-	-	No	Makespan	LP, Exact	Optimal
Szendrovits	1975	Single	Multi	Fix	Equal	Continuous	-	No idling	-	-	-	Makespan	Dominance Relations	-
Ornek and Collier	1988	Single	Multi	Fix	Equal	Continuous	-	No idling	-	-	-	Makespan	Dominance Relations	-
Truscott	1986	Single	Multi	Fix	Equal (Unit sized)	Discrete	-	No idling	-	-	Attached or Detached	Multi Objective	MIP, Exact	Optimal
Ramasesh et al	2000	Single	Multi	Fix	Equal	Continuous	-	No idling	-	-	Attached	Makespan	Dominance Relations	-
Kalir and Sarin	2001a	Single	Multi	Max	Equal	Continuous	-	-	-	-	Sublot attached	Makespan	Dominance Relations, Polynomial Time Alg.	Optimal
Sarin et al	2008	Single	Multi	Max	Equal	Continuous	-	-	-	-	Sublot Attached	Multi Objective	Polynomial Time Algorithm	Near-Optimal
Baker and Pyke	1990	Single	Multi	Fix	Consistent	Continuous	-	Idling	-	-	No	Makespan	Heuristic	Near-Optimal
Williams et al	1997	Single	Multi	Fix	Consistent	Continuous	-	Idling	-	-	No	Makespan	Dominance Relations	Optimal/ Near Optimal
Glass and Potts	1998	Single	Multi	Fix	Consistent	Continuous	-	-	-	-	No	Makespan	Exact, Dominance Relations	Optimal
Kropp and Smunt	1990	Single	Multi	Fix	Consistent	Continuous	-	Idling	-	-	No	Makespan	LP	Optimal
		Single	Multi	Fix	Consistent	Continuous	-	Idling	-	-	No	Mean Flow Time	Quadratic Programming	Optimal
		Single	Multi	Fix	Consistent	Continuous	-	Idling	-	-	Attached	Mean Flow Time	Heuristic	Near-Optimal
Bukchin and Masin	2004	Single	Multi	Max	Consistent	Discrete	-	-	-	-	Sublot attached	Multi Objective	Heuristic	Near-Optimal
Kumar et al	2000	Single	Multi	Fix	Consistent	Continuous	-	-	No-wait	-	Detached	Makespan	LP	Optimal
		Single	Multi	Fix	Consistent	Discrete	-	-	No-wait	-	Detached	Makespan	Heuristic	Near-Optimal
Chen and Steiner	1997	Single	Multi	Fix	Consistent	Discrete	-	Idling	-	-	No	Makespan	Heuristic	Near-Optimal
Chen and Steiner	2003	Single	Multi	Fix	Consistent	Discrete	-	Idling	No-wait	-	No	Makespan	LP	Optimal
		Single	Multi	Fix	Consistent	Discrete	-	Idling	-	-	No	Makespan	Heuristic	Near-Optimal
Liu	2003	Single	Multi	Fix	Variable	Continuous	-	-	-	Item	No	Makespan	Heuristic	Near-Optimal
		Single	Multi	Fix	Variable	Discrete	-	-	-	Item	No	Makespan	Heuristic	Near-Optimal
Chiu et al	2004	Single	Multi	Max	Variable	Discrete	-	No idling	-	Sublot	Attached or Detached	Multi Objective	LP, Heuristic	Optimal, Near-Optimal
Biskup and Feldmann	2006	Single	Multi	Max	Variable	Continuous	-	Idling	-	Sublot	Attached	Makespan	MIP	Optimal
		Single	Multi	Max	Variable	Continuous	-	Idling	-	Sublot	Detached	Makespan	MIP	Optimal
		Single	Multi	Max	Variable	Discrete	-	Idling	-	Sublot	Attached	Makespan	MIP	Optimal
		Single	Multi	Max	Variable	Discrete	-	Idling	-	Sublot	Detached	Makespan	MIP	Optimal
Huq et al	2004	Single	Multi	Fix	Consistent	Discrete	-	-	-	-	Sublot attached	Makespan	MIP	Near-Optimal
Kalir and Sarin	2003	Single	Multi	Max	Equal	Continuous	-	-	-	-	Sublot Attached	Makespan	Dominance Relations	Optimal

The LS studies on multi machines generally consider continuous sublot sizes. A few studies deal with discrete sized sublots. Most of these studies, except Biskup & Feldmann (2006), consider consistent sublot types. Biskup & Feldmann (2006) consider variable sublots with sublot availability case. This problem type is the hardest one in single product LS problems.

Recall that, in LS studies with two/three machines, a number of special cases arise with respect to idling or no-idling cases. For multi machine cases, on the other hand, there exist no such cases described for either idling or no-idling cases.

No-wait schedules are also considered by some studies (Chen & Steiner, 2003; Kumar, Bagchi & Sriskandarajah, 2000). These studies build LP models for the variants of no-wait cases and Kumar, Bagchi & Sriskandarajah (2000) proposed a heuristic approach that finds discrete sized consistent sublots.

As the number of machines increase, transportation activities between each machine pair become important. Ramasesh et al. (2000) use the relations of transportation, setup, waiting and processing times to develop manufacturing cycle time formulations. Truscott (1986) aims to minimize a multi objective function composed of makespan and number of transportations considering unit sized sublots. The transportation time, returning time, and capacity of transporter at each stage are given as parameters. Chiu, Chang & Lee (2004) consider transportation activities with limited number of capacitated transporters at each stage to minimize the total cost composed of makespan and number of transportations. They try to find the number and sizes of sublots at each machine as well as schedule of the transporters at each stage. For more information on transportation activities in LS problems, readers are referred to Edis, Ornek & Eliiyi (2007).

The setup operations are rarely considered in multi machine LS problems. Attached or detached setups are generally considered in the product basis; therefore, they occur only one time in the schedule. The sublot attached setups are required for each sublot, therefore the number of sublots becomes significant in this case since the number of setups increases with the increasing number of sublots. The “attached or detached” term in “Setups” column in Table 3.3 refers to a situation where both attached and detached setups are allowed.

For multi machine LS problems, minimizing the makespan, again, is the most popular time based objective. Only Kropp & Smunt (1990) consider a different performance measure, average flow time. A number of studies build multi objective functions in which more than one objective is aimed to be minimized simultaneously with makespan. Bukchin & Masin (2004) deal with a multi objective function containing two important objectives together, mean flow time and makespan. Due to significance of transportation activities mentioned earlier, Truscott (1986) and Chiu, Chang & Lee (2004) aim to minimize number of transportations and makespan together. With respect to a suggestion given by Kalir & Sarin (2001a), Sarin, Kalir & Chen (2008) use a unified objective function formed by giving weights to makespan, mean flow time, work in process inventory, sublot attached setup times and transfer times.

3.1.2.2 Solution Approaches

Most of multi machine versions of single product LS problems are quite complex to be analyzed by dominance relations. Therefore, researchers generally focus on analytical models and heuristics approaches. Even though structural properties have been identified for some versions of this problem, yet it is not uncommon to find heuristic approaches that have been proposed for its solution.

Since sublot sizes are known in case of continuous sized equal sublots, the only remaining decision variable is the optimal number of sublots. A number of studies consider these type of problems and proposed polynomial time algorithms (Kalir and Sarin, 2001a, 2003; Sarin, Kalir & Chen, 2008).

A rather difficult problem arises with continuous sized consistent sublots. A number of studies built mathematical programming models for this type of problems

(Kropp & Smunt, 1990; Kumar, Bagchi & Sriskandarajah, 2000; Potts & Baker, 1989). The limited number of sublots (e.g., two and three) is generally solved by considering dominance relations. For two sublot case, Chen & Steiner (2003) propose a polynomial time exact algorithm in no-wait schedules. For three sublot case, Williams, Tufekci & Akansel (1997) provide an exact algorithm using the network representation of the problem.

Solution approaches proposed for two sublot cases give upper bounds for multi sublot cases (e.g., Baker & Pyke, 1990; Williams, Tufekci & Akansel, 1997). Similarly, solution approaches proposed for equal sublots may give upper bounds for the consistent sublot cases (e.g., Baker & Pyke, 1990; Kropp & Smunt, 1990).

The most difficult problems in this section are the ones with variable sublots. Biskup & Feldmann (2006) give a MIP model formulation that easily obtains optimal solutions in continuous case but may fail to find optimal solutions in discrete case. Another MIP formulation for variable sublots is built by Chiu, Chang & Lee (2004) for discrete sized sublots but they could not obtain efficient results. Therefore, they propose two heuristic approaches in each of which decompose the entire problem to a series of two machine sub-problems. The first heuristic uses the MIP model and iteratively solves two machine problems in a cumulative manner, while the second one uses the processing times relations to apply forward or backward sub-algorithms of Trietsch & Baker (1993). Baker & Pyke (1990) propose a “two machine heuristic” that uses the structure of two machine cases which is similar to Campbell-Dudek-Smith (CDS) (1970) method in the solution of multi machines.

LS problem with discrete sized variable sublots is NP-hard (Liu, 2003). Computational complexity increases when the number of machines or sublots increases. It is unlikely to find optimal solutions based on the exhaustive search. Hence, researchers focus on the heuristic methods to obtain efficient solutions in a reasonable time. Liu (2003) propose a heuristic approach that first finds continuous sizes by considering the bottleneck machine and then rounds these values to discrete ones. A number of studies also use different rounding algorithms for various LS problem types (e.g., Chen & Steiner, 1997, 2003; Kumar, Bagchi & Sriskandarajah, 2003). However, these papers do not analyze the optimality gap between the case of optimal discrete sublot sizes and the case of discrete sized sublots obtained by rounding algorithms. Biskup & Feldmann (2006) also points out that this gap may be worthwhile to work on.

Another MIP model is built by Huq, Cutright & Martin (2004) considering given number of employees on each machine to minimize makespan. Employees, in this manner, are considered as multi processors. In this study, the sizes of the first and the last sublots are restricted to be equal, and the remaining intermediate sublots are to be equally sized within each other.

3.2 Multi Product Lot Streaming Problems

The multi product LS problems in flow shops require sequencing the products through the machines as well as sublot allocation of the products. Since the presence of sequencing decisions introduces a new dimension, which makes the problem much harder to solve, the studies in multi product cases mainly consider the simpler levels of LS characteristics. For instance, most of the papers study the non-intermingling case instead of intermingling case, which is much harder to handle. Similarly, in order to make the problem solvable, most of the researchers partition the whole problem into two sub-problems; the product sequencing problem and sublot allocation problem (see Figure 3.4). Note that, sublot allocation problem includes finding optimal number of sublots as well as sublot sizes. The following two sub sections review the multi product LS studies in two/three machine and multi machine cases, respectively.

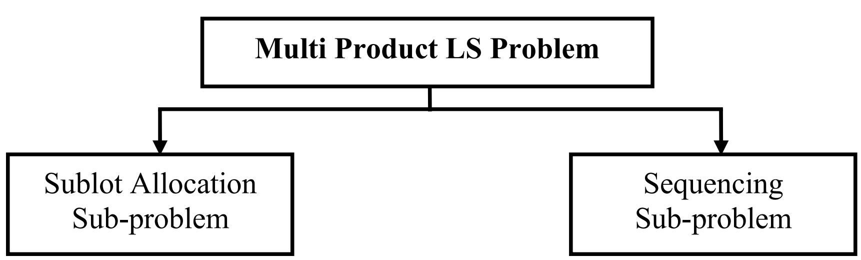

Figure 3.4 Sub-problems of a multi product LS problem

3.2.1 Two/Three Machines

Table 3.4 shows the problem characteristics and solution approaches of the multi product LS studies in two/three machine flow shops.

3.2.1.1 Problem Characteristics

Since the problem structure becomes more complex due to multi products, rather simple levels of sublot types and sublot sizes have commonly received attention. The simplest version of this type is the problem with unit-sized sublots and no setup. By assuming unit sized sublots, the sublot allocation sub-problem is eliminated and the underlying issue remains as the determination of the sequence of products or sublots.

The unit sized sublots are optimal for the multi product two machine flow shop LS problems for the makespan criteria in case no setup and transfer times occur and no restrictions on the transferring of the sublots or a limit on queue size on any machine exist. (Sarin & Jaiprakash, 2007, p.104)

The case of unit sized sublots with no setups is considered by Vickson & Alfredsson (1992) for two and three machine cases. Two machine case of this study is extended to detached setups by Cetinkaya & Kayaligil (1992) and to attached setups by Baker (1995), Ganapathy, Marimuthu & Ponnambalam (2004) and Marimuthu & Ponnambalam (2005).

If the assumption of unit sized sublots is relaxed in two machine LS problems, then the sublot allocation sub-problem appears in addition to the sequencing problem. For a given number of sublots, the sublot sizing problem and the product sequencing problem (i.e., non-intermingling schedule) are independent of each other. This means the sublot size of a product can be determined optimally independent of position of the product in the sequence. The sublot sizing and product sequencing problems keep their independence property in case of attached and detached setups (Sarin & Jaiprakash, 2007, p.153). These types of setups occur when the product type changes. The attached and detached setups in multi product cases may resemble the sublot related setups in single product cases. However, sublot related setup times are the same for all sublots in single product cases whereas the setup times for each product are different from each other in multi product case. The case of sublot attached setups in multi product LS problems does not satisfy the independence property of sublot sizing and product sequencing problems, since the sequence of sublots exists rather than sequence of products. Therefore, optimal schedules can only be obtained by intermingling cases.

Remember that, the non-intermingling or intermingling schedules appear only in multi product cases. All studies in this field consider non-intermingling case to utilize the independence property of sub-problems in two/three machine LS problems. Another reason for considering non-intermingling schedules may be the simplicity of this case in comparison to intermingling one. Since the intermingling schedule takes the sublots other than products into consideration on sequencing, it requires $(J*S)!$ sequence alternatives to be evaluated where this amount decreases to $J!$ in the non-intermingling case.

Recall that, optimal sublot sizes can be obtained by consistent sublots in no-idling case of single product two machine LS problems. Potts & Baker (1989) try to utilize this relation in multi product cases. They show that, for intermingling case, an optimal schedule cannot be found by a hierarchical procedure which firstly schedules the products without LS and then streams each product independently into optimal sublots. Their solution procedure produces optimal results in case of non-intermingling schedules due to independence property whereas it may not give optimal solutions for the intermingling schedule.

The only study that deals with no-wait schedules is made by Sriskandarajah & Wagneur (1999). The independence property still holds for the no-wait schedules in case of product detached setups.

Table 3.4 Multi product LS studies in two/three machine flow shops

Author(s)	Year	Number of Products	Number of Machines	Number of Sublots	Sublot Type	Sublot Size	Sequence	Idling/ No idling	Wait/ No Wait	Availability	Setups	Objective Function	Solution Approach	Optimality
Potts and Baker	1989	Multi	Two	Fix	Consistent	Continuous	Non-intermingling	No idling	-	-	No	Makespan	Heuristic	Near-Optimal
Sriskandarajah and Wagneur	1999	Multi	Two	Fix	Consistent	Continuous	Non-intermingling	-	No-wait	-	Detached	Makespan	Exact, Heuristic	Optimal/ Near-Optimal
		Multi	Two	Fix	Consistent	Discrete	Non-intermingling	-	No-wait	-	Detached	Makespan	Heuristic	Near-Optimal
		Multi	Two	Max	Consistent	Discrete	Non-intermingling	-	No-wait	-	Detached	Makespan	TS	Near-Optimal
Vickson and Alfredsson	1992	Multi	Two	Fix	Equal (Unit sized)	Discrete	Non-intermingling	-	-	-	No	Makespan	Dominance Relations- Johnson's Algorithm	Optimal
		Multi	Three	Fix	Equal (Unit sized)	Discrete	Non-intermingling	-	-	-	No	Makespan	Dominance Relations- Johnson's Algorithm	Optimal
Cetinkaya and Kayaligil	1992	Multi	Two	Fix	Equal (Unit sized)	Discrete	Non-intermingling	Idling	-	-	Detached	Makespan	Dominance Relations- Heuristic	Optimal
Baker	1995	Multi	Two	Fix	Equal (Unit sized)	Discrete	Non-intermingling	-	-	-	Attached	Makespan	Dominance Relations- Johnson's Algorithm	Near-Optimal
		Multi	Two	Fix	Equal (Unit sized)	Discrete	Non-intermingling	-	-	-	Detached	Makespan	Dominance Relations- Johnson's Algorithm	Optimal
Marimuthu and Ponnambalam	2005	Multi	Two	Fix	Equal (Unit sized)	Discrete	Non-intermingling	-	-	-	Attached	Makespan	SA, GA	Near-Optimal
Ganapaty et al	2004	Multi	Two	Fix	Equal (Unit sized)	Discrete	Non-intermingling	Idling	-	-	Attached	Makespan	TS, SA	Near-Optimal
		Multi	Two	Fix	Equal (Unit sized)	Discrete	Non-intermingling	Idling	-	-	Attached	Total Flow Time	TS, SA	Near-Optimal
Kalir and Sarin	2003	Multi	Two	Max	Equal	Discrete	Non-intermingling	-	-	-	Sublot Attached	Makespan	Johnson's Algorithm, Heuristic	Optimal, Near-Optimal
Cetinkaya	1994	Multi	Two	Fix	Equal	Continuous	Non-intermingling	-	-	-	Attached	Makespan	Exact	Optimal
		Multi	Two	Fix	Consistent	Continuous	Non-intermingling	-	-	-	Attached	Makespan	Exact	Optimal
		Multi	Two	Fix	Consistent	Discrete	Non-intermingling	-	-	-	Attached	Makespan	Exact	Optimal
Vickson	1995	Multi	Two	Max	Consistent	Continuous	Non-intermingling	Idling	-	-	Attached	Makespan	Exact	Optimal
		Multi	Two	Max	Consistent	Discrete	Non-intermingling	Idling	-	-	Detached	Makespan	Exact	Optimal
		Multi	Two	Max	Consistent	Continuous	Non-intermingling	Idling	-	-	Attached	Makespan	Johnson's Algorithm	Near-Optimal
		Multi	Two	Max	Consistent	Discrete	Non-intermingling	Idling	-	-	Detached	Makespan	Johnson's Algorithm	Near-Optimal
Zhang et al.	2005	Multi	Two stage m-1 hybrid	Max	Consistent	Continuous	Non-intermingling	-	-	-	Sublot Attached	Makespan	Heuristic	Near-Optimal

Multi product two machine discrete sized LS problems are easy to solve when the sublot sizes of all products are the same. The optimal solution can be obtained by enumerating all possible sublot sizes and finding the optimal sequence by using Johnson's algorithm. In the case of different sublot sizes for different product types, the possible combinations of sublot sizes of different products grow exponentially, even though for each combination, the optimal sequence of the products can be determined by using Johnson's algorithm. Therefore, there is no study dealing with variable sublots.

In case of discrete sublots, some of LS studies (e.g., Cetinkaya, 1994; Kalir & Sarin, 2003) build generating algorithms, whereas some others (e.g., Sriskandarajah & Wagneur, 1999; Vickson, 1995) propose algorithms which first obtain optimal continuous sublot sizes and then rounds them to discrete ones. .

The multi product version of the LS problem of Liu (2008) in a hybrid flow shop environment is presented by Zhang et al. (2005) to minimize the mean completion times of products for non-intermingling case.

Finally, in terms of performance criteria, almost all studies try to minimize makespan, since some sequencing heuristics, which can be adapted to LS problems, exist that minimizes makespan on classical flow shops. Only Ganapaty, Marimuthu & Ponnambalam (2004) consider minimizing total flow time as well as makespan in their study.

3.2.1.2 Solution Approaches

For two/three machine classical flow shop problems, there exist a number of exact (e.g. Johnson's algorithm) and heuristic algorithms. These algorithms have been adapted to the LS problems with some modifications.

As mentioned earlier, in two machine LS problems, the sublot allocation sub-problem can be eliminated by assuming unit sized sublots. The remaining sequencing problem can be solved optimally by applying the Johnson's algorithm, which is originally developed for classical two machine flow shops. In LS problems, each unit sized sublot can be considered as an individual product. Since all unit sized sublots of a product have the same processing times on the machines, all sublots of a product can be sequenced continuously. If processing times of products on machines are different from each other, the resulting sequence is a non-intermingling schedule. If any tie exists while sequencing the products because of having same processing time on the same machine, then only the sublots of these products can intermingle. Johnson's algorithm is the most popular solution approach in two machine LS problems, since it gives optimal schedules and/or can be modified for extra cases such as setups, equal sublots, transportation etc.

A number of LS studies apply Johnson's algorithm to unit sized sublot cases. Vickson & Alfredsson (1992) apply Johnson's algorithm without considering setups for two and three machine cases. Cetinkaya & Kayaligil (1992) consider detached setups and Baker (1995) consider both attached and detached setups.

Some LS studies consider unit sized sublots but apply meta-heuristic approaches instead of Johnson's algorithm. Ganapathy, Marimuthu & Ponnambalam (2004) consider attached setups and propose TS and SA based solution approaches. The same problem is studied by Marimuthu & Ponnambalam (2005) using a GA based approach.

Extensions of Johnson's algorithm are also utilized in some other cases. For example, a problem with fixed number of continuous sized equal sublots and attached setups is considered by Cetinkaya (1994) in two machine flow shops. This problem is no more difficult than the one with unit sized sublots, since the sublot sizes are known, the only difference occur at the sizes of sublots. Therefore, the processing time of a sublot should be calculated by multiplying the sublot size by the unit processing time of machine. The discrete sized version of this problem is studied by Kalir & Sarin (2003) for two cases of sublot sizes. The first one assumes the same sublot sizes for all products while the second one allows different sublot sizes. For the first case, they give equal sizes to the sublots of all products and construct an extra sublot for each product if any items remain, similar to flag heuristic of Kropp & Smunt (1990). Since the setup is sublot attached, the processing time of each sublot is obtained by adding the setup time to processing times of all items. The sublots are then considered as individual products which can be sequenced optimally by Johnson's algorithm on two machine flow shops. For the second case, at the first phase, they evaluate the performances of all sublot size alternatives of each product individually and select the best alternative with minimum makespan for each product. After finding sublot sizes for each product, the sequence of products through the machines are obtained via Johnson's algorithm. At the second phase, they try to improve the existing schedule to get minimum makespan by reducing the number of sublots for each product. Cetinkaya (1994), at first, proposes an optimal solution algorithm for consistent sublots which initially finds continuous sizes of consistent sublots and then sequences the products in two machine flow shops by Johnson's algorithm. Then, the author introduces a method to find discrete sized sublots and suggests the same solution approach again for the discrete sublot size version of the problem. Vickson (1995) builds closed form optimal solutions for continuous sized consistent sublots and proposes a fast polynomial algorithm for discrete sized sublots, under various setup types and transfer times.

Similar to the other types of LS problems, the discrete sized consistent sublots are generally handled by rounding algorithms which converts continuous sized sublots to discrete ones (Cetinkaya, 1994; Sriskandarajah & Wagneur, 1999; Vickson 1995).

The multi product version of the LS problem of Liu (2008) in a hybrid flow shop environment is presented by Zhang et al. (2005) to minimize the mean completion times of products for non-intermingling case. They try to find product sequence as well as the number and continuous sizes of consistent sublots for each product. Firstly, they build a MIP model of the problem, and then propose two heuristic algorithms named "whole job sequencing heuristic" and "aggregated-machine sequencing heuristic". The former one, at first, sequences the products without considering LS. Then, for the given sequence, it finds the number and size of sublots belonging to each product individually by an LP model. The non-optimality of this type of solution approach is proved by Potts & Baker (1989). The latter one works in a reverse manner. It first finds the number and size of each sublot belonging to each product individually by an LP model and obtains the total processing times of each product on the machines. The product sequence is obtained by using the given processing times, and the LS problem of each product is again solved by LP in order to improve the mean completion times of products. They present lower bounds and compare the results of heuristics with the best lower bound. The aggregated-machine sequencing heuristic performs better than the whole job sequencing heuristic.

3.2.2 Multi Machines

The problem characteristics and solution approaches of multi product LS studies on multi machine flow shops are given in Table 3.5.

3.2.2.1 Problem Characteristics

Due to the higher complexity of problems in this field, the related studies generally focus on simpler levels of LS characteristics.

The simplest versions of this type of LS problems are the ones with unit sized sublots or continuous sized equal sublots with fixed number of sublots. Both of these characteristics eliminate the sublot allocation sub-problem. Therefore, only sequencing sub-problem remains to be solved.

Marimuthu, Ponnambalam & Jawahar (2007, 2008) deal with unit sized sublots with attached setups. A number of studies (Kalir & Sarin, 2001b; Tseng & Liao, 2008; Yoon & Ventura 2002a, 2002b) consider continuous sized equal sublots with no setups. The extended version of this problem with attached setups is studied by Marimuthu, Ponnambalam & Jawahar (2009) whereas the case of sequence dependent setups is studied by Huang & Yang (2009). Kalir & Sarin (2003) study the same problem of Kalir & Sarin (2001b) for discrete sized equal sublots.

Table 3.5 Multi product LS studies in multi machine flow shops

Author(s)	Year	Number of Products	Number of Machines	Number of Sublots	Sublot Type	Sublot Size	Sequence	Idling/ No idling	Wait/ No Wait	Availability	Setups	Objective Function	Solution Approach	Optimality
Kumar et al	2000	Multi	Multi	Fix	Consistent	Continuous	Non-intermingling	-	No-wait	-	Detached	Makespan	Heuristic	Near-Optimal
		Multi	Multi	Fix	Consistent	Discrete	Non-intermingling	-	No-wait	-	Detached	Makespan	Heuristic	Near-Optimal
		Multi	Multi	Max	Consistent	Discrete	Non-intermingling	-	No-wait	-	Detached	Makespan	GA	Near-Optimal
Kalir and Sarin	2003	Multi	Multi	Max	Equal	Discrete	Intermingling	-	-	-	Sublot Attached	Makespan	Heuristic	Near-Optimal
Feldmann and Biskup	2008	Multi	Multi	Max	Consistent	Continuous	Non-intermingling	-	-	-	No	Makespan	MIP	Optimal
		Multi	Multi	Max	Consistent	Continuous	Intermingling	-	-	-	No	Makespan	MIP	Optimal
Kalir and Sarin	2001b	Multi	Multi	Fix	Equal	Continuous	Non-intermingling	Idling	-	-	No	Makespan	Heuristic	Near-Optimal
Marimuthu et al	2007	Multi	Multi	Fix	Equal (Unit sized)	Discrete	Non-intermingling	Idling	-	-	Attached	Total Flow time	TS, SA	Near-Optimal
Marimuthu et al	2008	Multi	Multi	Fix	Equal (Unit sized)	Discrete	Non-intermingling	-	-	-	Attached	Makespan	GA, HEA	Near-Optimal
		Multi	Multi	Fix	Equal (Unit sized)	Discrete	Non-intermingling	-	-	-	Attached	Total flow time	GA, HEA	Near-Optimal
Marimuthu et al	2009	Multi	Multi	Fix	Equal	Continuous	Non-intermingling	-	-	-	Attached	Makespan	TA, ACO	Near-Optimal
		Multi	Multi	Fix	Equal	Continuous	Non-intermingling	-	-	-	Attached	Total Flow time	TA, ACO	Near-Optimal
Hall et al	2003	Multi	Multi	Max	Consistent	Discrete	Non-intermingling	-	No-wait	-	Attached	Makespan	Heuristic	Near-Optimal
Kim and Jeong	2009	Multi	Multi	Fix	Consistent	Discrete	Non-intermingling	-	No-wait	-	Detached	Makespan	GA	Near-Optimal
Martin	2009	Multi	Multi	Fix	Consistent	Continuous	Intermingling	-	-	-	Job&sublot attached	Makespan	GA	Near-Optimal
		Multi	Multi	Fix	Consistent	Discrete	Intermingling	-	-	-	Job&sublot attached	Makespan	GA	Near-Optimal
		Multi	Multi	Max	Consistent	Discrete	Intermingling	-	-	-	Job&sublot attached	Makespan	GA	Near-Optimal
Liu et al	2006	Multi	Multi	Max	Variable	Continuous	Non-intermingling	-	-	Item	No	Makespan	Hybrid (TS+SA)	Near-Optimal
Yoon and ventura	2002a	Multi	Multi	Fix	Equal	Continuous	Non-intermingling	-	-	-	No	Mean weighted absolute deviation from due dates	Heuristic	Near-Optimal
		Multi	Multi	Fix	Equal	Continuous	Non-intermingling	-	No-wait	-	No	Mean weighted absolute deviation from due dates	Heuristic	Near-Optimal
		Multi	Multi	Fix	Consistent	Continuous	Non-intermingling	-	-	-	No	Mean weighted absolute deviation from due dates	Heuristic	Near-Optimal
Yoon and ventura	2002b	Multi	Multi	Fix	Equal	Continuous	Non-intermingling	-	-	-	No	Mean weighted absolute deviation from due dates	HGA	Near-Optimal
Tseng and Liao	2008	Multi	Multi	Fix	Equal	Continuous	Non-intermingling	-	-	-	No	Total weighted earliness and tardiness	Heuristic	Near-Optimal
Huang and Yang	2009	Multi	Multi	Fix	Equal	Continuous	Non-intermingling	-	-	-	Sequence dependent	Multi Objective	ACO	Near-Optimal
Smunt et al	1996	Multi	Multi	Fix	Equal/ Consistent	Discrete	Non-intermingling	-	-	-	Attached	Mean Flow Time, Standart deviation of flow time	Heuristic	Near-Optimal

Some of the studies (e.g., Kumar, Bagchi & Sriskandarajah, 2000; Martin, 2009) consider fixed number of sublots, at earlier stages of their studies and then extend their work to optimize the number of sublots.

A comprehensive study is made by Martin (2009) which investigates the problem of $\{F_m, L_n \,|\, fix/max, C, R/D, IS, \text{-}, \text{-}, \text{-}, S_A \,|\, C_{max}\}$. This study analyzes some important issues such as consistency of makespan and mean flow time objectives, the gain obtained by allowing intermingling, the performance of rounding algorithms and the difference between good and bad sequences when the sublot sizes are optimized by continuous sizes. The conclusions of these analyses can be listed as follows.

- makespan and mean flow time are not likely to be compatible objectives unless there is a high consistency in processing times of products in-between machines,
- intermingling can provide potentially useful advantages even with major setups,
- although it is important to determine good discrete sublot sizes, using a rounded LP solution provides excellent results,
- even with optimal sublot sizes, the sequence used is very important.

The multi product multi machine LS problems with variable sublots are the most challenging problems in the LS literature. Only Liu, Chen & Liu (2006) study this problem type to minimize makespan. They consider item availability case, which is also difficult to handle, but simplify the problem by considering non-intermingling schedules.

Although most of the studies focus on makespan objective, there exist a number of papers which deals with other time based objectives such as total flow time (Marimuthu, Ponnambalam & Jawahar, 2007, 2008, 2009), mean weighted absolute deviation from the due dates (Yoon & Ventura, 2002a, 2002b), total weighted earliness and tardiness (Tseng & Liao, 2008). A significant point is that, only Huang & Yang (2009) consider a multi objective function which includes machine idle time, product wait time and tardiness for continuous sized equal sublots.

The only study dealing with stochastic systems is by Smunt, Buss & Kropp (1996) with the objective of minimizing mean flow time and the standard deviation of flow time. They consider various levels of attached setup times, operation time variance, job size and shop load for a flow shop environment with five machines and 10 products. They use equal sublot types and flag concept of Kropp & Smunt (1990), and model their system via simulation. Finally, they show that the performance of LS techniques may differ with the stochastic nature of the system.

3.2.2.2 Solution Approaches

The only study that uses pure MIP formulation is made by Feldmann & Biskup (2008). They develop a MIP model for LS problems in permutation flow shops with continuous sized consistent sublots for both intermingling and non-intermingling schedules. They show that MIP model is efficient for two/three product, five/six sublot cases. However, they address heuristics/meta-heuristics approaches for discrete sublots and larger sized problems.

Due to the complexity of multi product multi machine LS problems, researchers generally focus on heuristic and meta-heuristic approaches. This type of LS problems is generally divided into a series of more tractable sub-problems: finding the number of sublots, obtaining the sublot sizes and sequencing the products or sublots. Some of LS studies eliminate one or two of these sub-problems by assumption. For instance, if the number of sublots is assumed to be fixed, then the first sub-problem is eliminated, similarly, if the unit sublot sizes are assumed, then the second sub-problem is removed.

Kalir & Sarin (2001b) eliminate the sublot sizing sub-problem by considering continuous sized equal sublots with fixed number of sublots. They propose a heuristic method, namely "bottleneck minimal idleness (BMI)", which aims to sequence the products on the bottleneck machine by not allowing idle time. They compare the results of BMI heuristic with the optimal results and the ones of Nawaz, Encore and Ham (NEH) (1983) heuristic which is known to be the best heuristic to sequence the products in classical flow shops. Their computational analysis states that BMI heuristic gives near-optimal (1.1%) makespan results and generates better values than the ones of NEH heuristic. Kalir & Sarin (2003) consider discrete sized version of this problem and apply the same heuristic to evaluate the possible sublot size alternatives.

Another heuristic approach, namely “global flow” which is based on a generalized TSP, is due to Hall et al. (2003) for discrete sized consistent sublots and no-wait schedule.

The most popular meta-heuristic approaches in this type of LS problems are the ones of evolutionary algorithms (EA) (e.g., GA, hybrid GA or hybrid EA) probably due to their popularity in scheduling problems. These EAs have been used in all sub-problems of multi product multi machine LS problems. Kumar, Bagchi & Sriskandarajah (2000) evaluate the performances of GA based approaches in all types of sub-problems. They consider fixed number of sublots in almost all problems. In only one problem, they tried to optimize the number of sublots by GA but their proposed method is able to solve up to five machine five product LS problem in a reasonable time. Although the solution quality of GA is good, its computational requirement is reported to be high. In addition, Martin (2009) uses GA in optimizing number of sublots. They used LP to obtain sublot sizes and again GA to sequence the sublots.

Most of the studies consider fixed number of sublots and simpler sublot types and tried to optimize the sequencing problem by meta-heuristics. The studies that use evolutionary based algorithms in sequencing problems are Kim & Jeong (2009), Kumar, Bagchi & Sriskandarajah (2000), Marimuthu, Ponnambalam & Jawahar (2008), Martin (2009), Yoon & Ventura (2002b).

Other than GA, TS and SA approaches are used for the sequencing sub-problem. Marimuthu, Ponnambalam & Jawahar (2007) applied TS and SA approaches individually and compared their performances under the unit sized sublots assumption to minimize total flow time. This problem is also studied by Marimuthu, Ponnambalam & Jawahar (2008) using GA and hybrid EA approaches. TS and SA are also used by Liu, Chen & Liu (2006) in a hybrid manner. They applied this solution procedure to optimize each sub-problem independently.

Ant Colony Optimization (ACO) is another meta-heuristic approach which is particularly preferred for sequencing sub-problem. Huang & Yang (2009) and Marimuthu, Ponnambalam & Jawahar (2009) consider continuous sized equal sublots with fixed number of sublots and applied ACO only to optimize the sequence of products for different objective functions.

A few LS studies make experimental analysis by evaluating several scenarios. Yoon & Ventura (2002a) use four initial job sequence rules (i.e., earliest due date, smallest slack time on the last machine, smallest overall slack time(OSL), smallest overall weighted slack time) and four job sequence generation rules (i.e., adjacent pairwise interchange, non-adjacent pairwise interchange(NAPI), extraction and forward shifted reinsertion, extraction and backward shifted reinsertion) to minimize mean weighted absolute deviation from due dates. They evaluate the performances of these rule pairs for the equal sublots with infinite buffer sizes, equal sublots with no-wait schedules and consistent sublots with infinite buffers. They state that OSL initial sequence rule with NAPI sequence generation rule gives better performance than the others. Smunt, Buss & Kropp (1996) consider various levels of setup times, operation time variance, job size and shop load for a stochastic flow shop environment with five machines and 10 products. They use equal sublot types and flag concept of Kropp & Smunt (1990), and model their system via simulation. They compare their results with the optimal results of Kropp & Smunt (1990) and show that the performance of LS techniques may differ with the stochastic nature of the system.

Finally, in an interesting study currently published, Glass & Herer (2009) prove that the LS problem and the small batch assembly line balancing problem have the same mathematical structure and suggest that the solution approaches for both problems can be used for each other.

3.3 Summary of Previous Research and Discussion

In this section, a summary of the LS literature and the limitations of the papers reviewed are presented with respect to different LS characteristics. The distinguishing features of the proposed research are then represented.

For smaller number of machines, dominance relations of processing times as well as the exact or heuristic algorithms of classical flow shop literature (e.g. Johnson's algorithm) are generally used to find optimal or near-optimal solutions. On the other hand, extensions of classical flow shop algorithms as well as heuristic and meta-heuristic approaches are mainly considered for multi machine problems.

Although the aim in single product LS problems is to determine optimal number of sublots and the corresponding optimal sizes of these sublots, in addition, another problem, optimal sequence of products, arises in the optimization of multi product LS problems. In order to make the multi product LS problems solvable, most of the solution approaches partition the whole problem into two sub-problems; the product sequencing problem and sublot allocation problem.

Most of the researchers assume that the number of sublots is fixed and known probably due to some restrictions caused by the system (e.g., fixed number of pallets). This assumption is also considered in some papers to reduce the complex structure of maximum number of sublots, since, in maximum number of sublots, the number of sublots should be incorporated into the problem as an additional decision variable.

Some of the studies consider continuous sized equal sublots with fixed number of sublots or unit sized sublots. In these cases, since number of sublots and their sizes are known, LS problems get simpler. These situations reduce the multi product LS problem to a product sequencing problem. On the other hand, especially multi product LS studies except Liu, Chen & Liu (2006) consider only equal and/or consistent sublot types instead of variable sublots, since the multi product LS problems are hard enough to solve even with consistent sublots.

A significant decision in LS problem is whether to use continuous or discrete values for the sublot sizes. The real life problems may require discrete values. However, the IP formulations developed for LS problems are generally capable of producing optimal results in a reasonable time for only continuous sized sublots. To obtain discrete sublot sizes, most of LS studies generally use rounding or simple generating algorithms which are generally lack of producing optimal results.

Almost all problems in multi product LS literature prefer non-intermingling schedules to intermingling ones, since intermingling schedules enlarge the solution space significantly in terms of sequencing alternatives.

Since, LS techniques provide a natural advantage in reducing makespan in flow shops; the papers dealing with makespan objective occupy a wide area in the time based LS literature. Nevertheless, a few papers also consider other time based objectives as well as multi objective ones.

In the view of solution approaches, for single product two/three machine cases especially the dominance relations of processing times are analyzed to get optimal solutions. Single product multi machine cases generally apply LP formulations and heuristic techniques. For multi product, two/three machine LS problems; the adapted heuristics from classical flow shop literature are generally used. Finally, for multi product multi machine cases, meta-heuristic techniques, especially GA, are widely used particularly to sequence the products.

A final remark is that, although the real life LS problems may have a stochastic structure, most of the studies consider only deterministic cases.

In the light of above inferences on LS literature, the proposed research in this thesis differs from the other studies with the collection of the following respects:

- One of the main goals of this thesis is to develop solution methods to the LS problems which may appear in real life environments. Therefore, the multi product multi machine LS problems are studied.
- Another issue widely encountered in real life LS problems is the stochastic behavior which is rarely studied in LS literature. The stochastic version of the single product multi machine LS problem in flow shops is also considered and analyzed in one of the research problems of this thesis.
- Rather than analyzing the performance of only pre-determined experimental sublot sizes in stochastic LS studies, a hybrid approach that integrates tabu-search and simulation is considered in optimizing the sublot sizes.
- Solution approaches proposed for large sized multi product multi machine problems are rather a few in the literature. Therefore, to solve large sized problems, a number of simple and efficient sequencing heuristics developed for pure flow shops are modified according to the requirements of LS for the sequencing sub-problem.
- Most of the studies in the multi product LS literature develop heuristic or meta-heuristic approaches. The studies that present MIP models of more complex LS problems are rather new. Hybrid methods that utilize the complementary strengths of heuristic/meta-heuristic algorithms and MIP models may produce more efficient results. Therefore, our solution approaches utilize the benefit of heuristic/meta-heuristic approaches in sequencing and of MIP models in sublot sizing.

The following three chapters introduce the investigated research problems and the relevant solution approaches.

CHAPTER FOUR
A TABU SEARCH-BASED HEURISTIC FOR SINGLE PRODUCT LOT STREAMING PROBLEMS IN FLOW SHOPS

4.1 Introduction

The single product LS problem considered in this chapter aims to minimize makespan in multi machine stochastic flow shops with discrete sized consistent sublots. This problem can be denoted as $\{F_m, L_1 \mid fix, C, D, -, II, -, -, - \mid C_{max}\}$. Remember that, in consistent sublots, the size of sublots may vary within the same machine; however, the sublots have to keep their sizes through the consecutive machines. Since the sizes of sublots are assumed to be integer and the number of sublots is fixed and known, the aim here is to find only optimal or near-optimal integer sublot sizes which minimize the makespan objective. The stochastic structure of the problem is due to the stochastic processing times of machines. Since the stochastic behavior makes the problem much harder to solve, this class of problems are rarely studied in the LS literature in comparison to the deterministic cases.

For the investigated class of deterministic LS problems, two/three machine cases are generally tried to be optimized by exact algorithms. On the other hand, for multi machine cases, the LP models as wells as heuristic approaches are widely studied. Especially for continuous sized sublots, the optimum sublot sizes are obtained by LP. For the discrete case, these continuous sublot sizes are rounded to integer ones by several heuristics (see, Chen and Steiner 1997, 2003; Kumar, Bagchi & Sriskandarajah, 2000; Liu, 2003; Sriskandarajah & Wagneur 1999). Other than rounding algorithms, Kropp & Smunt (1990) propose a "flag" heuristic to minimize makespan and generate integer sized sublots directly. They compare the performance of this heuristic with the equal sized sublots and state that it gives better results as the ratio of setup time to processing time grows.

Liu (2003) and Biskup & Feldmann (2006) study variable sublot cases. Liu (2003) propose a heuristic approach which first finds continuous sizes by taking the bottleneck machine into account and then rounds these values to discrete ones. Biskup & Feldmann (2006) build a MIP formulation considering modifications for all types of sublots (i.e., equal, consistent, variable), attached and detached setup cases. As expected, in terms of computational time, the performance of the proposed MIP model in optimizing discrete sublot sizes is not as efficient as in continuous sized sublots.

Compared with the deterministic cases, the number of papers is quite a few on stochastic systems. Jacobs & Bragg (1988) consider a multi product stochastic job shop problem with repetitive lots. The concept of repetitive lot is a kind of LS strategy. They use equal sublot types with discrete sublot sizes to minimize the mean flow time. They analyze the effects of various LS strategies using simulation. Smunt, Buss & Kropp (1996) focus on both stochastic job shop and flow shop LS problems with the objective of minimizing mean flow time and the standard deviation of flow time. They consider various levels of setup times, operation time variance, job size and shop load. They use equal sublot types and flag concept of Kropp & Smunt (1990) and model their system via simulation. They compare the results of their stochastic system by the optimal results of Kropp & Smunt (1990) and show that the LS techniques differ with the stochastic nature of the system.

Consequently, a vast number of studies are available on LS problems. However, only a few of them are concerned with stochastic shop environments. Furthermore, in these studies, only the performances of existing sublot size alternatives are evaluated instead of finding the optimal sizes of discrete sublots.

On the other hand, the techniques (e.g. LP, MIP) used for deterministic systems may not perform well for stochastic structures due to the variability of the system characteristics. In this study, a tabu search-based approach is proposed for the single product LS problems to be used in stochastic flow shops. This chapter is based on a paper by Edis & Ornek (2009a).

In Section 4.2, firstly, the concept of TS is briefly introduced and then the proposed tabu search based heuristic procedure is presented. The proposed heuristic is applied to the test problems and the computational results are presented and discussed in Section 4.3. Finally, conclusions and further directions are provided.

4.2 Proposed Tabu Search Procedure

The stochastic nature of the LS problem requires the evaluation of the system by the techniques designed for stochastic systems. Simulation is one of the efficient tools to be used in modeling and analyzing the stochastic systems. Therefore, we use simulation to handle the stochastic behavior of the system.

The sublots are assumed to be consistent with discrete sizes. The deterministic studies generally handle these problems by firstly finding the continuous sized sublots and then converting them to discrete ones. In some studies, these types of solution procedures give rather efficient results (Martin, 2009). However, as already stated, the stochastic nature of the problem does not allow the use of existing solution approaches proposed for deterministic problems. Moreover, the forms such as the flag heuristic (Kropp & Smunt, 1990), which performed well in a deterministic flow shop, seem to have little or no advantage when there is even a moderate amount of variability or congestion (Smunt, Buss & Kropp, 1996).

The meta-heuristic techniques do not receive so much attention in single product deterministic LS problems, because finding the continuous sized sublots and then rounding them to discrete ones is a good approximation method (Martin, 2009). However, meta-heuristic approaches, which are not preferred in deterministic cases, may be candidate solution techniques for stochastic systems. Since the sublot sizes have to be integer, searching the sublot size alternatives in the feasible solution region can be an appropriate solution alternative. To evaluate only feasible set of sublot sizes, one must ensure that the sum of sublot sizes has to be equal to the production lot size.

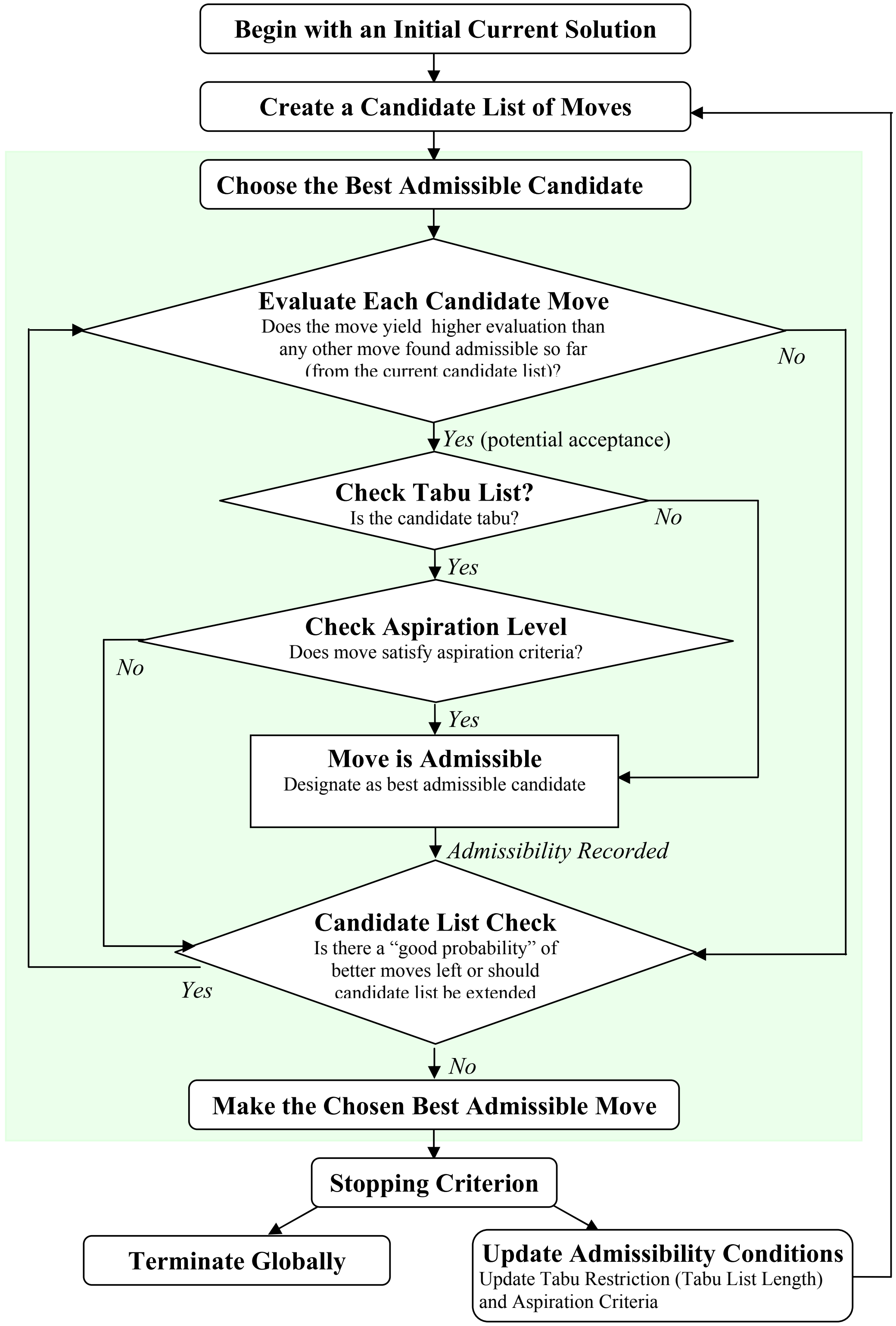

Figure 4.1 Tabu Search Procedure (Glover, 1986)

Tabu search is a meta-heuristic technique that is widely used in scheduling problems. The general TS procedure is given in Figure 4.1.

The efficiency of tabu search algorithms is mainly based on following factors:

- **The initial (starting) point or population**

A tabu search has to start from an initial point. This point can be generated in two ways: randomly or by an algorithm which takes the advantage of problem specific structure. If it is randomly generated, then it may produce different objective values at each run. In this case, the search procedure should be replicated several times to get an average value. In the latter, starting from a given initial point generates a single objective value. Starting from a good point (e.g., a point close to the optimal solution) increases the probability of getting good (optimal) results.

- **Alternative generation mechanism**

From a reference point, a number of alternatives can be generated to evaluate their performances and then the best one is selected as the new reference point in the next step. Neighborhood generation mechanism can be considered as the alternative generation mechanisms. These can be;

  - *Adjacent pairwise interchange:* Each product can only be interchanged with the adjacent ones. For instance, if the sequence is 1-2-3, then alternative sequences obtained by adjacent pairwise interchange are 2-1-3, 1-3-2.
  - *Insertion:* Generate alternatives by taking one product and inserting it into a different position. For instance, if the sequence is 1-2-3, then alternative sequences obtained by insertion are 2-1-3, 2-3-1, 1-3-2 and 3-1-2
  - *All pairwise interchange:* Generate all possible alternatives by changing all product pairs within each other. For instance, if the sequence is 1-2-3, then alternative sequences obtained by all pairwise interchange are 2-1-3, 3-2-1, and 1-3-2.

- *Random:* Generate a number of random neighbors. A random neighbor can be generated by two random numbers where the first one presents which product will be re-positioned and the second one denotes its new position. For instance, if the sequence is 1-3-2, and the generated numbers are three and one, the product at the third position (i.e., 2) is re-positioned at the first position. Then randomly generated alternative sequence is 2-1-3.

- **Termination criteria**

Termination criteria determine the length of the search. A search procedure has to be completed at some point. This point can be determined as

  - maximum number of iterations,
  - the number of non-improved iterations, or
  - the gap between the current result and the current best one.

- **Tabu list length**

The restricted moves are put into the tabu list. They have to wait in the tabu list for a number of iterations (i.e., tabu list length). This is again an important factor, since the tabu list directs the search. The length of tabu list can be either fixed or variable. The fixed one is usually preferred in the literature.

TS basically uses neighborhood search mechanisms to generate alternative solutions. This structure may be utilized to generate alternative integer sublot sizes in LS problems. While generating neighborhood alternatives, the equivalence relation between sublot sizes and production lot size can be automatically satisfied. One way is to increase the size of a sublot by a few units and accordingly decrease the size of another sublot by the same amount while the remaining sublots keep their sizes. Notice that such a generation mechanism does not alter the sum of sublot sizes. The sublot size alternatives can be generated in this manner and embedded into a tabu search scheme. For the stochastic case of this study, the evaluation of these alternatives can be performed via simulation. The general framework of the proposed tabu search based heuristic is illustrated in Figure 4.2. The additional notation and the steps of the proposed tabu search based heuristic are detailed as follows.

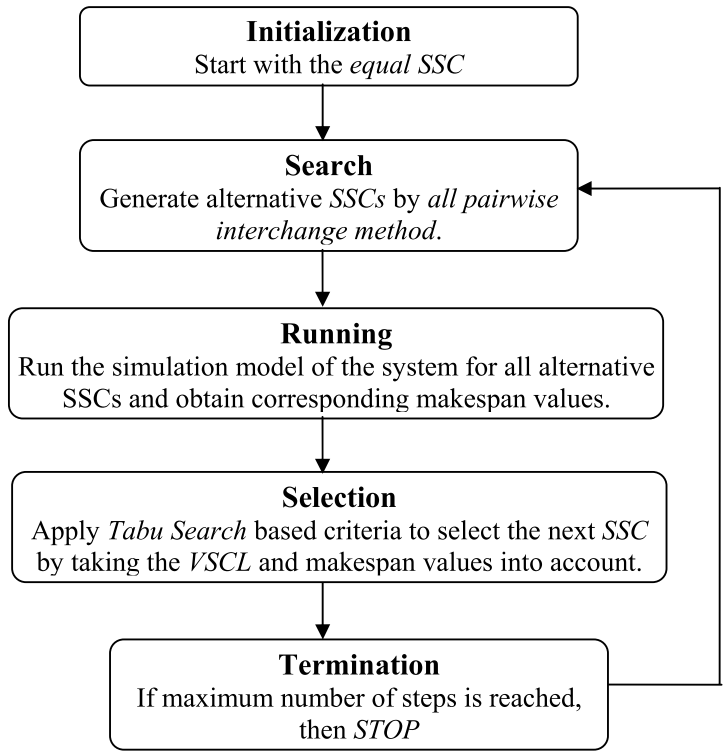

Figure 4.2 The framework of the proposed tabu search based heuristic

Notation:

S : number of sublots

s : sublot index, $s = 1,2,..,S$

L : production lot size

i : index of alternative sublot size configurations, $i = 1,2,..,I$

SS_s : size of sublot s, $SS_s \in Z^+$

SSC : sublot size configuration, $SSC = \{SS_1, SS_2, ..., SS_S\}$

$ASSC_i$: sublot size configuration of alternative i.

R_i : response value of alternative i.

$R_{\min}$: minimum response value among responses of alternative sublot configurations of current SSC.

R_{best} : best response value

R_{set} : set of alternative sublot configurations responses, $R_{set} = \{R_1, R_2, ..., R_I\}$

$SSC_{\min}$: corresponding sublot size configuration of $R_{\min}$

SSC_{best} : corresponding sublot size configuration of R_{best}

VSCL : paired set of currently visited sublot size configurations and their corresponding responses, $(SSC, R) \in VSCL$

sc : step counter , $sc \in \mathrm{Z}^+$

maxsc : maximum number of steps, $maxsc \in \mathrm{Z}^+$

Steps of the Proposed Tabu Search Approach

Step 1: Initialization:

Specify an integer value for maxsc. Set *sc* = 0 and $SS_s = L / S$ for all *s*=1,2,..,*S*. $SSC = \{SS_1, SS_2, ..., SS_S\}$. Run the simulation model of the system for *SSC* and get its response, *R*.

Set $SSC_{best} = SSC$, $R_{best} = R$. Add this (SSC_{best}, R_{best}) to VSCL.

Step 2: **Termination**

If *sc* > *maxsc* then STOP, otherwise go to Step 3.

Step 3: Search

Generate alternative sublot sizes of current *SSC* by all pairwise interchange method.

$$ASSC_1 = \{SS_1 - 1, SS_2 + 1, SS_3, ..., SS_{S-1}, SS_S\}$$

$$ASSC_2 = \{SS_1 + 1, SS_2 - 1, SS_3, ..., SS_{S-1}, SS_S\}$$

$$ASSC_3 = \{SS_1 - 1, SS_2, SS_3 + 1, ..., SS_{S-1}, SS_S\}$$

$$ASSC_4 = \{SS_1 + 1, SS_2, SS_3 - 1, ..., SS_{S-1}, SS_S\}$$

$$\vdots$$

$$ASSC_{I-1} = \{SS_1, SS_2, SS_3, ..., SS_{S-1} - 1, SS_S + 1\}$$

$$ASSC_I = \{SS_1, SS_2, SS_3, ..., SS_{S-1} + 1, SS_S - 1\}$$

Run the simulation model of the system for all alternatives $(ASSC_1, ASSC_2, ..., ASSC_I)$ and set $R_{set} = \{R_1, R_2, ..., R_I\}$, $R_{\min} = \min R_{set}$, $SSC_{\min} = \{\text{ASSC}_i \mid R_i = R_{\min}, \forall i\}$.

(If there is more than one sublot size configuration with the same $R_{\min}$ value, choose one of them arbitrarily.)

Step 4: Selection

If $R_{\min} \le R_{best}$ and $(SSC_{\min}, R_{\min}) \notin \text{VSCL}$, then $R_{best} = R_{\min}, SSC_{best} = SSC_{\min}$, add (SSC_{best}, R_{best}) to VSCL,

If $sc > 0$, then $sc = sc + 1$, go to Step 2.

Else, go to Step 2.

Else, search next minimum response, $(SSC, R) = \{(ASSC_i, R_i) \mid R_i = \min R'_{set}, \forall i\}$, where $R'_{set} = \{R_i \mid (ASSC_i, R_i) \notin VSCL, \forall i\}$

If $R'_{set} = \varnothing$, then STOP

Else, add this (SSC, R) to VSCL, $sc = sc + 1$, go to Step 2.

As an initial seed, we consider equal size sublot configurations by dividing the production lot size by the number of sublots. If the result is an integer number, then it gives the sublot sizes, otherwise the remainder is shared to sublots starting from the first sublot. For example, if the production lot size is 70 units and the number of sublots is eight, then the sublots would have sizes of 9, 9, 9, 9, 9, 9, 8, and 8.

In the tabu search based procedure defined above; first, a neighborhood based search procedure (all pairwise interchange method) is applied and the sublot size configurations are obtained. The alternative sublot size configurations are generated from an initial sublot size configuration in the following manner. For the first (second) alternative, the sublot size of the first sublot is decreased (increased) by one unit and accordingly the second one is increased (decreased) by the one unit so that the sum of sublot sizes remains fixed. The third and fourth alternative sublot size configurations are generated by applying this procedure to first and third sublots. This procedure goes on in this manner until all pairs of sublots are evaluated. For each sublot size configuration alternative, the corresponding makespan values are obtained. The best response value and its sublot size configuration are recorded in the visited sublot configurations list (VSCL). Normally, neighborhood based search procedure is terminated when there is no improvement in the response of two consecutive iterations. However, tabu search allows continuing on with another solution even if it is relatively worse. Similarly, in our tabu search-based procedure, the second best response is allowed to be visited in order to generate better results. By applying all pairwise interchange method, alternative sublot configurations of the current point are generated. Then the algorithm confirms that whether the best response among these alternatives is in VSCL or not. If it has already been in VSCL, then the next best response, which is not in VSCL, is selected to be used as the starting sublot configuration of the next step. If it is not in VSCL, we add this configuration and its response value to the VSCL. Then, the procedure iterates by generating and evaluating alternative sublot configurations of this new point. Adding the visited sublot configurations to VSCL helps to avoid looping. The procedure iterates until it reaches a pre-determined number of iterations.

Note that, in classical TS procedures, the forbidden moves remain in the tabu list by a pre-determined list length. However, in the proposed TS procedure, once a sublot size configuration is placed in the tabu list, then it remains in the list forever. This is due to the structure of sublot size configurations since a sublot size configuration is constituted by the exact values. Returning to same sublot size configuration would produce same alternatives and cause looping.

4.3 Computational Results and Comparisons

The data for the experimental design of LS problem are given in Table 4.1. We consider three different levels for production lot size, number of machines and number of sublots. This results in 27 different problem sets.

Since the considered LS problem has a stochastic behavior and there is no solution approach that gives the optimum makespan values, the proposed tabu search based approach is first evaluated on deterministic systems and then applied to the stochastic version of the problem. The impact of stochastic behavior on the system performance is also analyzed.

Table 4.1 Data for the single product LS problems

Production lot size, *L*	**# of sublots, *S***	**# of machines, *M***
50	5	5
100	8	7
150	10	10

4.3.1 Deterministic Case

Biskup & Feldmann (2006) study the deterministic version of this problem and build a MIP model which gives optimal sublot allocations. Therefore, the efficiency of the proposed heuristic for the deterministic version of the problem can be evaluated by comparing its results with the results of this MIP model. The MIP model is given below.

Parameters

S : number of sublots

M : number of machines

L : production lot size

t_m : processing time of one item on machine m

Indices

s : sublot index, $s = 1,2,..,S$

m : machine index, $m = 1,2,..,M$

Decision Variables

SS_s : size of sublot s

C_{sm} : completion time of sublot s on machine m

$$\text{Minimize } C_{SM}$$

subject to

$$C_{1m} \geq t_m SS_1 , \qquad m = 1,...,M \qquad (4.1)$$

$$C_{sm} - t_m SS_s \geq C_{s-1,m} , \qquad s = 2,...,S ,\ m = 1,...,M \qquad (4.2)$$

$$C_{sm} - t_m SS_s \geq C_{s,m-1} , \qquad s = 1,...,S ,\ m = 2,...,M \qquad (4.3)$$

$$\sum_{s=1}^{S} SS_s = L \qquad (4.4)$$

$$SS_s \geq 1 \qquad s = 1,...,S \qquad (4.5)$$

$$SS_s \in Z^+ \qquad s = 1,...,S \qquad (4.6)$$

$$C_{sm} \geq 0 \qquad s = 1,...,S ,\ m = 2,...,M \qquad (4.7)$$

The first constraints define the smallest possible completion time of the first sublot on each machine. By Constraints (4.2), the sublot *s* on machine *m* is allowed to start only after sublot *s*-1 on machine *m* has been finished. Similarly, Constraints (4.3) ensure that the sublot *s* on machine *m* is allowed to start only after sublot *s* on machine *m-1* has been finished. The sum of sublot sizes has to be equal to the production lot size is given in Equations (4.4). Since the LS problem on hand assumes fixed number of sublots, Constraints (4.5) are added to the model in order to enforce all sublot sizes to get non-zero values due to fixed number of sublots. Since the investigated LS problem allows only discrete sized sublots, the domain of SS_s variables are restricted in Constraints (4.6) to have only integer values. Finally, Constraints (4.7) are non-negativity constraints.

By varying the processing times of machines, five problem instances are generated for each combination of parameters given in Table 4.1. In total, it makes 135 test problems. The MIP model of the problem is built in OPL Studio 3.7 optimization package and solved on a Centrino 1.73 GHz processor with 1.5 GB RAM.

Table 4.2 Comparison of tabu search-based procedures in terms of production lot size in deterministic LS problems

	Number of Optimums				Average Deviation (min)				Average Proportional Deviation (%)				Average Computation Time (sec)			
	50	**100**	**150**	**Overall**	**50**	**100**	**150**	**Overall**	**50**	**100**	**150**	**Overall**	**50**	**100**	**150**	**Overall**
TS_10	36	25	18	79	3.80	11.62	29.60	15.01	0.36	0.54	0.88	0.59	3.73	3.78	3.85	3.79
TS_20	39	32	24	95	2.80	8.73	22.51	11.35	0.27	0.41	0.69	0.46	3.83	3.87	3.91	3.87
TS_50	40	35	30	105	2.44	7.49	18.47	9.47	0.24	0.36	0.57	0.39	3.86	3.89	3.94	3.9

Table 4.3 Comparison of tabu search-based procedures in terms of number of sublots in deterministic LS problems

	Number of Optimums				Average Deviation (min)				Average Proportional Deviation (%)				Average Computation Time (sec)			
	5	**8**	**10**	**Overall**	**5**	**8**	**10**	**Overall**	**5**	**8**	**10**	**Overall**	**5**	**8**	**10**	**Overall**
TS_10	34	29	16	79	5.11	12.09	27.82	15.01	0.15	0.51	1.11	0.59	2.49	4	4.87	3.79
TS_20	38	34	23	95	2.78	8.40	22.87	11.35	0.10	0.36	0.91	0.46	2.51	4.04	5.07	3.87
TS_50	38	37	30	105	2.78	6.69	18.93	9.47	0.10	0.31	0.76	0.39	2.53	4.09	5.09	3.9

Table 4.4 Comparison of tabu search-based procedures in terms of number of machines in deterministic LS problems

	Number of Optimums				Average Deviation (min)				Average Proportional Deviation (%)				Average Computation Time (sec)			
	5	**7**	**10**	**Overall**	**5**	**7**	**10**	**Overall**	**5**	**7**	**10**	**Overall**	**5**	**7**	**10**	**Overall**
TS_10	18	31	30	79	29.42	9.84	5.76	15.01	1.28	0.34	0.16	0.59	3.69	3.82	3.85	3.79
TS_20	25	35	35	95	23.51	6.47	4.07	11.35	1.03	0.23	0.12	0.46	3.78	3.91	3.93	3.87
TS_50	27	40	38	105	19.84	5.53	3.02	9.47	0.90	0.18	0.09	0.39	3.81	3.93	3.97	3.9

For the deterministic test problems, comparison of the proposed heuristic and the optimum results are given in Tables 4.2, 4.3, and 4.4. The detailed computational results of tabu search-based heuristic are presented in Appendix A1, A2 and A3 with respect to production lot sizes.

The average deviation is the average of the difference between the proposed heuristic solutions and the optimal ones. Similarly, the average proportional deviation is the proportion of the deviation from the optimum results. The results of tabu search-based heuristic are recorded when the step size reaches to 10, 20, and 50.

As shown in Tables 4.2, 4.3, and 4.4, tabu search with 10 steps (TS_10) gives 79 optimum results out of 135 test problems. Tabu search with 20 steps (TS_20) and 50 steps (TS_50) give 95 and 105 optimum results, respectively. Indeed, this is an expected result, since an increase in number of steps accordingly may increase the number of optimum results. The detailed results are given in three categories: production lot size (Table 4.2), number of sublots (Table 4.3), and number of machines (Table 4.4).

As the production lot size increases, the number of optimal solutions decreases in all solution alternatives. This is due to the increase in the number of feasible sublot configurations. As expected, as the number of steps of tabu search increases, the number of optimum values also increases, while the average deviation and average proportional deviation decrease. Similar results are observed in terms of number of sublots and number of machines. The increase in number of sublots also increases the number of alternative sublot configurations which makes it harder to get optimal solutions. This is also indicated in Table 4.3.

As the number of sublots and production lot sizes increase, number of optimums decreases; the average deviation and average proportional deviation also increase. However, the average proportional deviation is less than 1% in most cases. We observe diminishing improvements in makespan reduction for the increasing number of sublots. This result complies with that of Baker & Jia (1993), see Table 4.5.

Table 4.5 Diminishing improvements for makespan, $L = 50$

Number of Sublots (S)	Number of Machines (M)	Makespan	% decrease
5	5	1032	
8	5	912	13.20
10	5	884	3.16
5	7	1655	
8	7	1377	20.30
10	7	1284	7.20
5	10	1508	
8	10	1233	22.30
10	10	1146	7.60

Moreover, when the number of machines (stages) increases, makespan reduction becomes larger. The average deviation from optimum and average proportional deviation decline as the number of machines increase.

Note that, the computation times in deterministic cases are so small and do not vary in terms of production lot size and number of machines whereas they slightly increase with the increasing number of sublots.

As expected, the results indicate that TS_50 has generated the best results over other step sizes. However, running 50 steps requires so much time for the stochastic version of the problem, especially in 10-machine and 10-sublot case. Preliminary computational results show that we are able to obtain almost similar results with a step size of 30; hence, we assume 30 as the maximum number of steps in stochastic cases. The trade-off between number of steps and makespan values also support this choice. For instance, when the step size increases from 10 to 20, the number of optimum solutions increases from 79 to 95; whereas increasing the step size from 20 to 50 provides only 10 extra optimum solutions.

4.3.2 Stochastic Case

Since the results obtained by applying the proposed tabu search heuristic to deterministic LS problems are very promising and encouraging, i.e., the average proportional deviation is less than 0.4% compared to the optimum values, it is extended to the LS problems in stochastic flow shops. The same data is used for the stochastic cases by replacing deterministic processing times with stochastic ones. The processing times are assumed to be normally distributed with a standard deviation of 0.25 times of the means. The stochastic processing times make the LS problem much more difficult to solve. Since simulation is an efficient tool to model stochastic environments, makespan values of the consistent sublot configurations are obtained by simulation. The simulation model is built in ARENA 10.0 simulation software package (Rockwell Software, 2005) and the tabu search algorithm is coded in Visual Basic Applications (VBA) for Arena 10.0. The verification and validation of the simulation model is made and the runs are replicated 10 times for each sublot configuration. All the simulation runs are performed on Centrino 1.73 GHz processor with 1.5 GB RAM.

Since there is no available study in literature to compare the results of the proposed heuristic for the stochastic LS problem, the results are compared with the OptQuest (Rockwell Software, 2004) results of the ARENA software package. OptQuest is an optimization tool to be used in simulation models in ARENA. We use the same initial sublot configurations (equal sublots) for the OptQuest and select the “auto stop” criterion i.e., the procedure ends if there is no improvement within the last 100 sublot configurations. The results of the OptQuest are recorded at two different moments in time. The first one is the moment when the proposed heuristic obtained its best results. The other one is the moment when OptQuest finds its best results. The results of the proposed heuristic and of OptQuest are given in Appendix A4. Note that, the asterisk symbol refers to the situation where OptQuest finds its best result before the solution time of the proposed heuristic. The heuristic obtains the “minimum” results by iterating from only the best results obtained at each iteration, i.e., the best result before visiting the first worse result than the best one, sc=0.

Table 4.6 summarizes Appendix A4 and compares the results of proposed heuristic and OptQuest from two aspects. The first one is the comparison of results obtained at the same computational time. It can be said that the proposed heuristic gives better or at least the same makespan values in all problems. The second one is the comparison of the number of best results and average computational times. OptQuest obtains the same results in only two out of 27 problems at the time of minimum. Similarly, TS_10 gives three same and 24 better results while TS_20 obtains five same and 22 better results than OptQuest. In addition, TS_30 finds six same and 21 better results. The OptQuest obtains nine same results among 27 problems at the end of its computational time. For the second perspective, although the average computational time of minimum is smaller than the OptQuest's, it gives 22 best results. The average computational times of TS_10 and OptQuest are very close to each other; however, OptQuest can only find nine same results, while TS_10 finds 24 best results among 27 test problems.

Table 4.6 Comparison of the results of proposed TS heuristic and OptQuest for the stochastic LS problems

At time of	Average computational time (min)	No. of better results at the same computational time			No. of best results
		Heuristic	OptQuest	Same	
Minimum	10.97	25	-	2	22
TS_10	21.60	24	-	3	24
TS_20	33.22	22	-	5	25
TS_30	43.92	21	-	6	27
OptQuest	19.65	-	-	9	9

We conclude that the results of TS based approach combined with simulation are more efficient than those of the OptQuest's. Adding more step sizes not only reduces the average makespan but also increases the number of best results. It should also be noted that the average computational time of the heuristic for the selected step sizes is not too much to get better results. For example, the average time for obtaining the results of minimum is smaller than the others and it gives 22 better results among 27 test problems. This can be a choice for the decision maker. If we wait 11 additional minutes on the average, we obtain extra two better results. If we wait additional 22 min on the average, we can obtain three better results. That is, extra five better results can be obtained by waiting 33 min on the average.

The behavior of the stochastic LS problem may be evaluated by inputting the optimum sublot sizes obtained by deterministic case. If these sublot sizes are given to the simulation model of the stochastic LS problem, the corresponding makespan values can be obtained for the stochastic case. These results can be compared with the ones of proposed tabu search based procedure. This comparison is given in Appendix A5. In all test problems, the proposed tabu search procedure outperforms the optimal deterministic results in stochastic case. Furthermore, use of optimal sublots obtained by MIP model gives approximately 1% deviation in average from the proposed tabu search procedure. One should notice that using deterministic system results in stochastic systems generates 1% worse results than optimized sublot sizes in stochastic systems. It can be also seen in Appendix A2 that this deviation grows with the problem size. In addition, this deviation may vary with the increasing variance of the processing times which may be suggested to be analyzed in further studies.

4.4 Conclusions

Since even deterministic LS problems are NP-hard, it is rather difficult to mathematically model and solve stochastic systems. In this respect, the objective of this chapter is to develop a heuristic procedure for LS problems in stochastic flow shops. Hence, we propose a rather simple yet quite efficient heuristic algorithm to obtain good solutions for single product, multi machine LS problems with discrete-sized consistent sublots.

The proposed algorithm is first tested on deterministic problems to see how well it performs and is, therefore, compared against the optimum values obtained by a MIP model developed by Biskup & Feldmann (2006). Since the results are very promising, i.e., the heuristic obtains results close to optimal values less than 1%, it is then applied to stochastic flow shops. The proposed heuristic algorithm is a combination of simulation and TS. The TS tries to explore the neighborhood for better solutions, whereas the simulation handles the stochastic behavior of the system and calculates the necessary values. The results thus obtained are further compared with those of OptQuest's and we observe that the proposed heuristic outperforms OptQuest with respect to number of best results. Therefore, it is concluded that the proposed heuristic could be easily used to solve stochastic as well as deterministic LS problems in flow shop settings.

The TS based solution procedure described in this paper could be extended in many directions. It can be extended to a multi product case in a job shop environment. In this case, the sequence of products through the stages becomes very important. In addition, setups and transportation activities play an important role to determine lot sizes and sequences. Another extension may be improving the tabu search itself and hybridizing it with other meta-heuristics. Hybrid systems, especially GA with TS produce rather efficient results for production systems (see Tasan & Tunali, 2006).

CHAPTER FIVE
SEQUENCING AND LOT STREAMING IN MULTI PRODUCT PERMUTATION FLOW SHOPS

5.1 Introduction

Multi product LS problems in flow shops require not only sequencing the products through the machines but at the same time allocating sublots of these products. The first problem, namely sequencing the products, is NP-complete for more than three machines (Garey, Johnson & Sethi, 1976). The complexities of single product LS problems, which handle only sublot allocation, were already given in Table 2.2. Therefore, with regard to the complexities of both problems, almost all multi product multi machine (m>3) LS problems are strongly NP-hard.

In order to make the problem solvable, most of the researchers partition the whole problem into two sub-problems; the product sequencing problem and sublot allocation problem. Note that, sublot allocation problem also has two further sub-problems: finding optimal number of sublots and optimizing sublot sizes (see Figure 5.1).

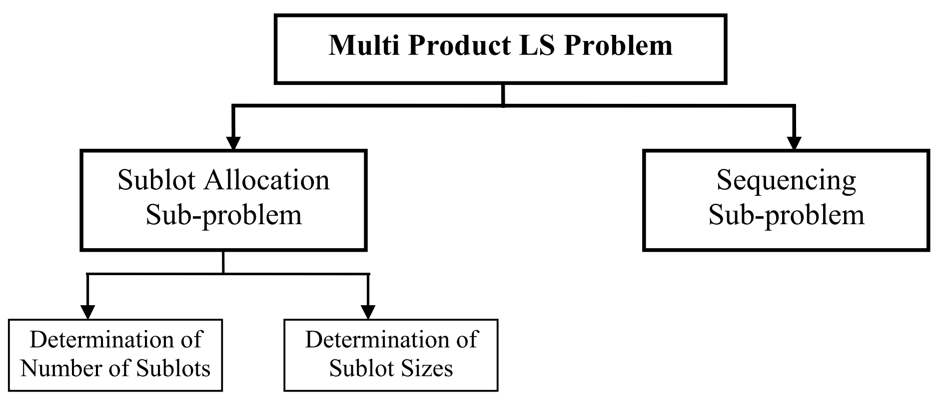

Figure 5.1 Sub-problems of a multi product LS problem

As in the other types of scheduling problems, the multi product, multi machine cases are the hardest ones in LS problems. To simplify this class of problems, most of studies assume that number of sublots is fixed, therefore the number of sublot determination is eliminated. Moreover, most of the researchers widely studied equal sublot types under fixed number of sublots (see Huang & Yang, 2009; Kalir & Sarin 2001b, 2003; Marimuthu, Ponnambalam & Jawahar, 2007, 2008, 2009; Tseng & Liao, 2008; Yoon & Ventura, 2002a, 2002b) which means sublot sizes are priori known and sublot allocation problem is eliminated. Hence, the only remaining issue is to determine the sequence of products. For instance, Kalir & Sarin (2001b) propose a heuristic named "Bottleneck Minimal Idleness (BMI)" to construct the sequence of products.

Most of the studies, except Feldmann & Biskup (2008) and Martin (2009), assume that the number of sublots are fixed and known. This assumption eliminates the determination of number of sublots and simplifies the problem. On the other hand, in the case of maximum number of sublots, the number of sublots is restricted by an upper bound i.e., maximum number of sublots, and one should also find the optimal number of sublots which may take value up to this upper bound. In maximum number of sublots, the domain of sublot size variables may increase (i.e., these variables may also take zero values). For instance, assume that the lot size is 12 and the number of discrete sublots is five. Then, an alternative sublot size vector for the fixed number of sublots may be $SS_i = (3,2,4,2,1)$ where $i = 1,\ldots,5$. If the maximum number of sublots is the case, then it can either be $SS_i = (3,2,4,2,1)$ with five sublots or $SS_i = (5,0,5,2,0)$ with three sublots.

Certainly, as the sublot types in LS problems vary from equal to consistent and consistent to variable, the problems become harder to solve. The studies that consider the consistent and variable sublot cases in this class of LS problems generally use meta-heuristic approaches especially the GA (Kim & Jeong, 2009; Kumar, Bagchi & Sriskandarajah, 2000; Marimuthu, Ponnambalam & Jawahar, 2008, 2009; Martin, 2009; Yoon & Ventura, 2002b). The only study that uses the pure mixed integer programming (MIP) formulation for continuous sized consistent sublots is made by Feldmann & Biskup (2008). They show that MIP model is efficient for small problem sizes such as two/three product, five/six sublot cases. However, they address heuristics/meta-heuristics approaches for discrete sublots and larger sized problems.

The only study concerning the variable sublot type is by Liu, Chen & Liu (2006). They decompose the problem into three sub-problems: product sequence determination, lot streaming reallocation machine determination, and lot streaming range determination. They develop a heuristic procedure that uses TS and SA approaches.

In this chapter, we consider three different multi product multi machine LS problems with the objective of minimizing makespan. In addition to the assumptions given in Section 2.4, other common characteristics and assumptions of these three problems are listed below.

- Maximum number of sublots is considered.
- The production environment is a permutation flow shop.
- Non-intermingling schedule is assumed among the sublots of the products.

These three investigated problems differ from each other by means of sublot types and the divisibility of sublot sizes. The first problem type considers consistent sublot type with continuous sized sublots whereas the second problem type deals with discrete sized sublots. The last investigated problem considers variable sublot type with continuous sized sublots. Throughout the thesis, these three problems will be identified in the following titles.

- Continuous sized consistent sublots, $\{F_m, L_n \mid max, C, R, NI, II, -, -, - \mid C_{max}\}$
- Discrete sized consistent sublots, $\{F_m, L_n \mid max, C, D, NI, II, -, -, - \mid C_{max}\}$
- Continuous sized variable sublots $\{F_m, L_n \mid max, V, R, NI, II, -, -, A_S \mid C_{max}\}$

Since all three investigated problems deal with multi product LS problems, the sequencing sub-problem has to be solved. Furthermore, since the investigated problems in this chapter assume maximum number of sublots with consistent and variable sublot types, the sublot allocation problem with its two sub-problems remains to be solved. Therefore, all sub-problems illustrated in Figure 5.1 have to be considered in all three investigated problems. Hence the proposed solution approaches handles the whole LS problem in two sub-problems: sequencing sub-problem and sublot allocation sub-problem.

The sequencing sub-problem in LS is indeed equivalent to the product sequencing problem in classical flow shops. In classical scheduling problems, the computational effort required to solve a problem grows remarkably fast as the number of products increases. Thus, if the number of products is large, it might not be practical to solve the problem exactly. In such cases, it is reasonable to consider heuristic sequencing procedures (e.g., Johnson's rule for two machine cases) which can provide an optimal/near optimal solution in a reasonable time. Therefore, the sequencing sub-problem of the LS problems can also be solved by applying efficient sequencing heuristics. The performance of the sequencing heuristics becomes important at this point. The sequencing heuristics used for multi machine permutation flow shops to minimize makespan may be considered as the sequencing heuristic alternatives to be used for the investigated LS problems.

Surely, by efficient sequencing heuristics, the sequencing part of the whole problem can be solved. However, the sublot allocation sub-problem still remains to be solved. This sub-problem may be handled by corresponding MIP models and MIP based solution approaches, since MIP models are able to handle two-sub problems of sublot allocation problem together. MIP models may easily solve small sized problems but probably fail to solve for medium and large sized problems. However, by utilizing the relaxed versions of MIP models, efficient solution procedures may be proposed to solve the sublot allocation sub-problem.

The framework of the proposed solution approach is illustrated in Figure 5.2. The entire multi product LS problem partitioned into two sub-problems as mentioned earlier. The sequencing is at first handled by the proposed sequencing heuristics, and then the sublot allocation problem is solved by the MIP based proposed solution approaches. The proposed solution approach incorporates the sequencing heuristics and MIP based solution approaches in that order.

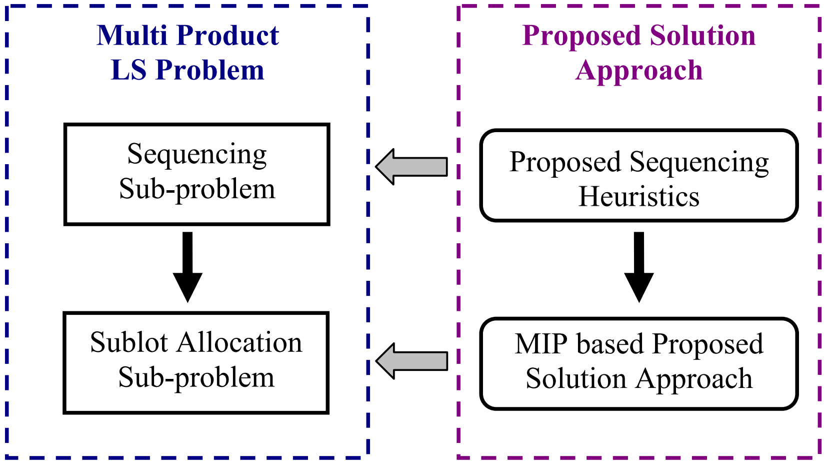

Figure 5.2 The framework of the proposed solution approach for multi product LS problems

Surely, handling the sub-problems in the reverse order may be considered as an alternative solution procedure. Such a solution approach has been applied to the research problems however, it has not performed as efficient as the proposed solution approach given in the thesis in terms of both solution quality and computation time.

In the light of above discussion, the aim of this section is threefold.

- The first one is to propose simple and efficient sequencing heuristics to be used in solving sequencing part of this class of LS problems.
- Second one is to propose to MIP based solution schemes to be used in solving the sublot allocation sub-problem for a given sequence.
- The third one is to propose a hybrid solution method (by combining above two solution procedures) to be used in solving medium and large sized LS problems.

The remainder of this chapter is organized as follows. In Section 5.2 the details of sequencing heuristics modified for the LS problems are given. Next, in Section 5.3, implementation issues to obtain the makespan values for three different multi product multi machine LS problem types are discussed. In Section 5.4, experiments are presented to demonstrate the efficiency of the sequencing heuristics on the makespan for each problem type. Finally, the work is summarized in Section 5.5 along with the directions for future research.

5.2 Sequencing Rules

The sequencing sub-problem in LS is indeed equivalent to the product sequencing problem in classical flow shops. There exist a number of efficient sequencing heuristics to minimize makespan in classical flow shops. These sequencing heuristics may be utilized to solve the sequencing part of the LS problem.

Ruiz & Maroto (2005) present a comprehensive review on permutation flow shop sequencing heuristics and evaluate their performances on the makespan objective without considering LS. They categorize the heuristics in two groups; constructive and improvement heuristics.

Another review is by Hejazi & Saghafian (2005) for general flow shop scheduling problems with the makespan criterion. They classify the heuristics by exact methods, constructive methods and meta-heuristics.

Referring to the above articles, we have selected simple yet efficient constructive sequencing heuristics, which can be directly or adaptively used for the LS problems in flow shops. Since meta-heuristics and exact algorithms may require enormous amount of time especially for medium and large sized problems, these approaches are out of the scope of this chapter. The selected sequencing heuristics are

- Shortest processing time (SPT)
- Longest processing time (LPT)
- Palmer's Algorithm (Palmer, 1965)
- Gupta's Algorithm (Gupta, 1971)
- Campbell, Dudek, and Smith (CDS) Algorithm (Campbell, Dudek & Smith, 1970)
- Nawaz, Encore and Ham (NEH) Algorithm (Nawaz, Encore & Ham, 1983)
- Bottleneck Minimal Idleness (BMI) (Kalir & Sarin, 2001b)

Some of these heuristics are used in their original form. These heuristics, except BMI, were originally developed for pure flow shop problems and do not consider any LS based criteria on sequencing. To add the LS effect, some of these heuristics are modified considering the production lot size (L) or total processing time weighted with production lot size (TPLS). The related additional notation is given below.

t_j : processing time of one unit of product j, j=1,.., J

L_j : production lot size of product j

S_j : slope index of product j ,

t_{jm} : processing time of one unit of product j on machine m, m=1,..,M

TPT_j : total processing time of one unit of product j on all machines,

$$TPT_j = \sum_{m=1}^{M} t_{jm}$$

$TPLS_j$: total processing time of one unit of product j multiplied by production lot size of product j, $TPLS_j = L_j \sum_{m=1}^{M} t_{jm}$

5.2.1 Modified SPT Rule

Shortest processing time (SPT) rule is a well known heuristic for sequencing products in flow shops. The SPT rule is modified according to the requirements of LS problem and two alternative SPT rules are obtained. TPT and TPLS are used to sort the products instead of t_j in the original SPT rule. If TPT is used for sorting, then the SPT rule is named as SPT(TPT), and if TPLS is used then it is named SPT(TPLS).

Steps of SPT(TPT) / SPT(TPLS) rules

Sort the products in increasing order of

- TPT_j's;
- $TPLS_j$'s;

(Break ties by giving priority to the product with smaller product number.)

5.2.2 Modified LPT Rule

Longest processing time (LPT) rule is another well known heuristic for the sequencing and scheduling problems. As in SPT, again the LPT rule is modified according to requirements of LS problem. Similarly, two criteria (TPT, TPLS) are used to sort the products instead of t_j in the original LPT rule. LPT sorting with TPT is named LPT(TPT), and LPT sorting with TPLS is named LPT(TPLS).

Steps of Modified LPT rule

Sort the products in decreasing order of

- TPT_j's;
- $TPLS_j$'s;

(Break ties by giving priority to the product with smaller product number)

5.2.3 Modified Palmer Algorithm

Palmer (1965) proposes a heuristic to schedule products for more than two machine flow shops. This heuristic gives priority to the products which have smaller processing times in earlier stages and it increases with the number of stages. He calculates the slope indexes for each product and then constructs the sequence by scheduling the products in descending order of slope indexes. The original Palmer Algorithm is denoted as Palmer(ORJ) and the modified Palmer algorithm including the effect of production lot sizes of products is called as Palmer(PLS). The steps of the two alternative Palmer Algorithms are given below.

Steps of Modified Palmer Algorithms

Step1. Calculate the slope indices of all the products by the formula given below.

- (ORJ) $S_j = -\sum_{m=1}^{M}[(M-(2m-1))t_{jm}]$
- (PLS) $S_j = -\sum_{m=1}^{M}[(M-(2m-1))(t_{jm}L_j)]$

Step 2. Sort the products in descending order of S_j 's.

(Break ties by giving priority to the product with smaller product number.)

5.2.4 Modified Gupta Algorithm

Gupta (1971) proposes an algorithm, which uses a slope index for flow shops with more than two machines, similar to the Palmer,'s heuristic. The original Gupta algorithm is denoted as Gupta(ORJ) and the modified Gupta's algorithm including the effect of production lot sizes of products is called as Gupta(PLS). The steps of the two alternative Gupta Algorithms are given as follows.

<u>*Steps of Modified Gupta Algorithms*</u>

Step1. Calculate the product indexes of all the products by the formula given below.

- (ORJ) $$SI_j = \frac{e_j}{\min\limits_{1 \le m \le M-1} \{t_{jm} + t_{j(m+1)}\}}$$
- (PLS) $$SI_j = \frac{e_j}{\min\limits_{1 \le m \le M-1} \{t_{jm} L_j + t_{j(m+1)} L_j\}}$$

where $e_j = \begin{cases} 1 & if \quad t_{j1} < t_{jm} \\ -1 & if \quad t_{j1} \ge t_{jm} \end{cases}$

Step 2. Sort the products in descending order of SI_j 's. (Break ties by giving priority to the product with smaller product number.)

5.2.5 Modified Campbell, Dudek, and Smith (CDS) Algorithm

Campbell, Dudek & Smith (1970) develop a constructive heuristic method for flow shop problems with makespan criterion. This procedure uses Johnson's rule (Johnson, 1954) in a heuristic way and creates several alternative schedules the best of which should be chosen. Johnson (1954) proposes a heuristic approach for two machine flow shop problems that gives optimal solutions. In this manner, CDS algorithm decomposes the multi machine flow shop problem into (M-1) number of alternative two machine problems, and then applies Johnson's rule to each alternative and selects the best one among them as the resulting sequence of the procedure. The original CDS Algorithm is named CDS(ORJ). Alternatively, the modified CDS algorithm including the effect of production lot sizes of products is denoted as CDS(PLS). Another modification is performed on the evaluation of the alternative sequences by assuming that the sublot sizes are equal and continuous. This assumption is due to several reasons. First, this assumption removes the sublot allocation sub-problem since number of sublots is assumed to be fixed and the sublot sizes are priori known. Second, it becomes easy to handle several alternatives quickly in comparison to the case of consistent/variable sublot types. Third, assuming equal sublots provides better upper bounds than the case without considering LS, since it adds the effect of LS to the solution procedure. This assumption is only used at the evaluation procedure of alternative sequences. Once, the best sequence is obtained, then the optimal number of sublots and their sizes are going to be find for this given sequence. These procedures are going to be given in the following section for each investigated problem type.

The steps of the two alternative CDS Algorithms are given below.

Steps of Modified CDS Algorithms

Step 1. Set alternative counter $z = 1$.

Step 2. Calculate the following formulas for each product that will be used while applying Johnson's rule.

- (ORJ) $t_{j1}'(z) = \sum_{m=1}^{z} t_{jm}$, and $t_{j2}'(z) = \sum_{m=M-z+1}^{M} t_{jm}$
- (PLS) $t_{j1}'(z) = L_j \sum_{m=1}^{z} t_{jm}$, and $t_{j2}'(z) = L_j \sum_{m=M-z+1}^{M} t_{jm}$

Step 3. To obtain the sequence of products, apply Johnson's rule for the two machine flow shop problem where $t_{j1}'(z)$ represents the processing time of product j

on the first machine in alternative sequence z, and $t_{j2}'(z)$ represents the processing time of product j on the second machine in alternative sequence z.

Johnson's rule

Step 3.1. Form the set U of products whose processing times are shorter on the first machine than on the second.

Step 3.2. Form the set V of products whose processing times are longer on the first machine than on the second.

Step 3.3. Arrange products in U in non-decreasing order by their processing times on the first machine. (Break ties by giving priority to the product which has smaller product number.)

Step 3.4. Arrange products in V in non-increasing order by their processing time on the second machine. Break ties giving priority to the product which has a smaller product number.)

Step 3.5. Concatenate U and V and that is the processing order for both machines.

Step 4. Assume that the sublot sizes of products are equal and continuous. Schedule the products on the machines in the sorted order and get the objective function value of the alternative sequence z, OFV(z).

Step 5. Set $z = z + 1$. If $z < M$, then go to Step 2,
else, go to Step 6.

Step 6. Select the best objective function value among the M-1 alternatives, $OFV = \min_{1 \le z \le M-1} OFV(z)$.

5.2.6 Modified Nawaz, Encore and Ham (NEH) Algorithm

Weng (2000) concludes that the NEH algorithm of Nawaz, Encore & Ham (1983) appears to be the best heuristic for flow shops in minimizing the makespan referring to Taillard (1993) and the mean flow time referring to Ho & Chang (1995). Ruiz & Maroto (2005) evaluate a number of sequencing heuristics and address the NEH algorithm as the best heuristic giving better makespan values among the others for the permutation flow shops.

NEH algorithm builds the final sequence in a constructive way, adding one product at a time. In this study, the NEH algorithm is modified with respect to the requirements of the LS problems. In the original NEH algorithm, the products are sorted in decreasing order of total processing times on the machines for each product. This type of NEH is denoted as NEH(D,TPT). If the products are sorted in increasing order of total processing times on the machines, then it is denoted as NEH(I,TPT). The decreasing and increasing versions of the NEH algorithm which use TPLS values of the products are represented as NEH(D,TPLS) and NEH(I,TPLS), respectively. Another modification is made on the evaluation of the partial solutions by assuming the sublot sizes are equal and continuous and the number of sublot sizes is fixed.

<u>*Steps of Modified NEH Algorithms*</u>

Step 1. Arrange the products in

- (D) decreasing order of
    - (TPT) TPT_j 's;
    - (TPLS) $TPLS_j$'s;
- (I) increasing order of
    - (TPT) TPT_j 's;
    - (TPLS) $TPLS_j$'s;

Step 2. Set counter c = 2. Pick the first two products from the arranged product list and schedule them in order to minimize the makespan as if there are only these two products. (Assume that the sublot sizes of products are equal and continuous.) Set the better one as the current partial solution.

Step 3. $c = c + 1$. Generate c candidate sequences by inserting the first product in the remaining product list into each slot of the current partial solution. (Assume that the sublot sizes of products are equal and continuous and the number of sublots is fixed.) Among these candidates, select the best one with the least makespan. Update the selected partial solution as the new current solution.

Step 4. If $c = J$ (number of products), a schedule (the current solution) has been found and stop. Otherwise, go to Step 3.

5.2.7 Bottleneck Minimal Idleness (BMI) Heuristic

The BMI heuristic is proposed by Kalir & Sarin (2001b) to minimize makespan by minimizing the idle time on the bottleneck machine for multi product, multi machine LS problems. However, it assumes equal sublot types with continuous size. BMI heuristic constructs the product sequence by determining the bottleneck machine and minimizing the idle time on that machine. Since, sizes of sublots are known and given for each product due to the equal sublot property, the only aim is to optimize the product sequence.

The bottleneck dominance theorem plays a key role in the BMI heuristic. It states that for a product j, if the $t_{j,BN} - \max_{1 \le m < BN} t_{jm} \ge 0$ inequality is satisfied, then under lot streaming, there would be no idle time created between the sublots of product j on the bottleneck machine (BN) where $BN \equiv \arg\max_{1 \le m \le M} \left\{ \sum_{j=1}^{J} L_j t_{jm} \right\}$. Kalir & Sarin (2001b) define a product as "bottleneck dominant" if it meets the bottleneck dominance property and as "bottleneck dominated" otherwise. Next, they try to sequence the products by minimizing the bottleneck idleness in-between the products and maximize the time buffer between machines BN and BN-1. They proposed a lexicographic type of rule, which sequences the products in decreasing order of closeness of their secondary bottleneck machine to the primary bottleneck machine. Secondary bottleneck machine means the upstream machine with the next largest unit processing time after the bottleneck machine. By utilizing this approach, some of the idle time that might have been created on the machines closer to the bottleneck machine is, in fact, absorbed because it overlaps with the processing of previous products. The bottleneck dominant products in the sequence built by lexicographic rule are immediately scheduled and the bottleneck dominated products, which does not create bottleneck idleness, are also sequenced in their order. However, some bottleneck dominated products may not satisfy this in many cases. In this situation, bottleneck dominated product is pushed forward in the sequence. (Kalir & Sarin, 2001b)

They compare BMI results with the results of NEH algorithm and state that their results are better than the NEH results. For more information, refer to Kalir and Sarin (2001b).

To obtain an alternative sequence, BMI heuristic is used in its original form and its results are compared with the proposed heuristics in Section 5.4.

5.3 Proposed Solution Approaches

The modified sequencing heuristics are described in the previous section. Although giving the sequence of products solves the sequencing part of the whole problem, sublot allocation sub-problem still remains to be solved. This sub-problem may also be NP-hard due to some problem characteristics. Therefore, some extra work may be required to obtain makespan values of the sequencing heuristics in reasonable computation times. The following sub-sections propose solution schemes for each investigated LS problem.

5.3.1 Continuous Sized Consistent Sublot

Only three studies (Feldmann & Biskup, 2008; Kumar, Bagchi & Sriskandarajah, 2000; Martin, 2009) consider this problem type. Feldmann & Biskup (2008) propose a MIP model for the problem which considers intermingling and non-intermingling schedules together. Their MIP model is able to solve LS problems up to only three products and seven sublots. Kumar, Bagchi & Sriskandarajah (2000) also study a similar LS problem but with no-wait schedules. They propose an algorithm which first determines the continuous sublot sizes for each individual product using LP, then sequence the products utilizing TSP. They especially focus on fixed number of sublots. However, they do not give any computational results. Martin (2009) considers continuous sized consistent sublots and intermingling case to minimize the makespan. The author uses GA to determine the number of sublots and the sequence of the products, and a MIP model to determine the size of the sublots. The results are given for 20 products and 10 machines LS problems.

In this section, the resulting sequences obtained by the proposed sequencing heuristics given in Section 5.2 are inputted to the MIP model of the Feldmann & Biskup (2008), and the corresponding makespan values are obtained. Notice that these resulting makespan values are optimal for the given sequence. Note that, the MIP model of Feldmann & Biskup (2008) includes the constraints for both intermingling and non-intermingling cases. In the MIP model, only the constraints related to non-intermingling case are presented. The notation and MIP model arranged for our investigated problem is given below.

Parameters:

S_j number of sublots of product j

M number of machines

J number of products

t_{jm} processing time for one item of product j on machine m

L_j number of identical items of product j to be produced (production lot size of product j)

R sufficiently large number

Indices:

s,t indices for the sublots, $s, t = 1, \ldots, S_j$

m index for the machines, $m = 1, \ldots, M$

j,k indices for the products, $j, k = 1, ..., J$

Decision Variables:

SS_{js} number of units produced in sublot s of product j

p_{jsm} processing time of sublot s of product j on machine m

b_{jsm} starting time of the sublot s of product j on machine m

y_{jk} binary variable, which takes 1 if product j is sequenced prior to product k, 0 otherwise.

$$\text{Minimize } C_{\max}$$

subject to

$$\sum_{s=1}^{S_j} SS_{js} = L_j \qquad j = 1,\dots,J \tag{5.1}$$

$$p_{jsm} = SS_{js} t_{jm} \qquad j = 1,\dots,J,\ s = 1,\dots,S_j,\ m = 1,\dots,M \tag{5.2}$$

$$b_{jsm} \ge b_{j,s-1,m} + p_{j,s-1,m} \qquad j = 1,\dots,J,\ s = 2,\dots,S_j,\ m = 1,\dots,M \tag{5.3}$$

$$b_{jsm} \ge b_{j,s,m-1} + p_{js,m-1} \qquad j = 1,\dots,J,\ s = 1,\dots,S_j,\ m = 2,\dots,M \tag{5.4}$$

$$b_{jS_jm} + p_{jS_jm} \le b_{k1m} + (1 - y_{jk})R \qquad j,k = 1,\dots,J,\ j \ne k,\ m = 1,\dots,M \tag{5.5}$$

$$b_{kS_jm} + p_{kS_jm} \le b_{j1m} + y_{jk}R \qquad j,k = 1,\dots,J,\ j \ne k,\ m = 1,\dots,M \tag{5.6}$$

$$C_{\max} \ge b_{jS_jM} + P_{jS_jM} \qquad j = 1,\dots,J \tag{5.7}$$

$$y_{jk} \in \{0,1\} \qquad j,k = 1,\dots,J,\ j \ne k \tag{5.8}$$

$$b_{jsm} \ge 0 \qquad j = 1,\dots,J,\ s = 1,\dots,S_j,\ m = 1,\dots,M \tag{5.9}$$

$$SS_{js} \ge 0 \qquad j = 1,\dots,J,\ s = 1,\dots,S_j \tag{5.10}$$

Restrictions (5.1) ensure that the sum of sublot sizes of product j has to be equal to production lot size of that product. With (5.2) the processing times of the sublots are calculated. The Constraints (5.3) and (5.4) ensure that the sublots of the same product do not overlap. Constraints (5.3) prevent two sublots, s and s-1, being processed simultaneously on one machine. With Constraints (5.4), sublot s on machine m is not allowed to start before sublot s on machine m-1 has been finished. Constraints (5.5) and (5.6) determine the sequence of sublots. Since it is a permutation flow shop, no machine index is needed for y. Constraints (5.5) are binding if (and only if) y_{jk} takes the value 1. In this case, product j is scheduled prior to product k on machine m and the processing of first sublot of product k is forced to start after last sublot of product j has been finished. If, on the other hand, y_{jk} takes the value zero, (5.5) are not binding, as R is added on the right-hand side. The disjunctive counterpart is reflected by Constraints (5.6). These constraints are only binding if y_{jk} takes the value zero. In (5.7), the completion time of the last sublot S_j on the last machine M are used to define the makespan $C_{\max}$. Constraints

(5.8) define the binary variables. Finally, Constraints (5.9) and (5.10) are non-negativity constraints.

The steps of proposed solution approach for this problem type are given below.

<u>The Steps of Solution Approach for Continuous sized Consistent Sublots</u>

Step 0. Sequencing

Find the sequence using one of the investigated sequencing heuristics.

Step 1. **Initialization**.

Give the resulting sequence to the MIP model considering continuous sized consistent sublots.

Step 2. **Running**

Run the MIP model and obtain the continuous sizes of consistent sublots and optimal makespan value for the given sequence.

5.3.2 Discrete Sized Consistent Sublots

This section considers the discrete version of the problem described in the previous section.

The studies concerning discrete sized sublots generally obtain continuous sized sublots at first, then use rounding algorithms to convert them to discrete ones (e.g., Chen & Steiner, 2003; Hall et al., 2003; Kumar, Bagchi & Sriskandarajah, 2000; Sriskandarajah & Wagneur, 1999; Vickson, 1995).

Hall et al. (2003), Kim & Jeong (2009), Kumar, Bagchi & Sriskandarajah (2000) and Martin (2009) consider this problem type. All studies, except Hall et al. (2003), use GA approach either to sequence the products or to get discrete sublot sizes. Meta-heuristic approaches such as GA may generate better results than simple sequencing heuristics; however, they require much computation time. For instance, Kumar, Bagchi & Sriskandarajah (2000) propose a heuristic, named MHEU, and a

GA based approach. GA based approach obtains better results than MHEU, but requires up to one order of magnitude computation time. Moreover, their GA based approach is able to solve only five machine five product LS problem with maximum number of sublots in a reasonable time. MHEU is also used by Hall et al. (2003) to compare the results of their algorithm named "global flow" which is based on a generalized TSP. Again, the computation time of MHEU is negligible when compared with the global flow algorithm. Martin (2009) deals with intermingling schedules and his/her findings are worthwhile to mention. The author confirms that use of rounding procedures (to obtain integer sized sublots from continuous sized sublots) provides acceptable and excellent results. The analysis on the product sequence show that even if optimal sublot sizes are used, there is a significant difference between the best and the worst sequences which indicate that sequencing is much more important than sublot sizing.

The proposed solution approach for discrete sized consistent sublots first applies all the steps of the solution procedure given in the previous section. Then, the continuous sized sublots are converted to the discrete ones by a rounding algorithm. Note that, due to the rounded values, the resulting makespan may not be optimal for the given sequence. However, this does not constitute a problem in comparing the performances of the sequencing heuristics, since makespan results of all sequencing heuristics are determined with the same approach.

The steps of the solution approach for the discrete sized consistent sublots are given below.

The Steps of Solution Approach for Discrete sized Consistent Sublots

Step 0. Sequencing

Find the sequence using one of the investigated sequencing heuristics.

Step 1. **Initialization**.

Give the resulting sequence to the MIP model considering continuous sized consistent sublots.

Step 2. **Running**

Run the MIP model and obtain the continuous sizes of consistent sublots and optimal makespan value for the given sequence.

Step 3. Rounding

For each product, apply a rounding algorithm to get discrete sublot sizes.

Step 4. Termination

Calculate the corresponding makespan value with respect to resulting sequence and discrete sublot sizes.

In Step 3, two different rounding algorithms are evaluated to get discrete sublot sizes:

- The forward rounding algorithm of Chen & Steiner (1997) and
- Rounding algorithm of Sriskandarajah & Wagneur (1999).

The notation and the steps of both rounding algorithms are given below.

s: sublot index, $(s = 1,\ldots,S_j)$

x_s^c :continuous sized sublot s ,

x_s^d :discrete sized sublot s

$\lfloor x_s^c \rfloor$: the largest integer less than or equal to x_s^c

$\lceil x_s^c \rceil$: the smallest integer greater than or equal to x_s^c

L : production lot size

The forward rounding algorithm, at first, rounds down the continuous values. Notice that, there would be a remainder value to be portioned. This amount is shared between sublots starting from the first one. The detailed steps are given below.

Steps of Forward Rounding Algorithm of Chen & Steiner (1997)

Step 1. Define $u = L - \sum_{s=1}^{S_j} \lfloor x_s^c \rfloor$.

Step 2. If x_s^c is integer, then set $x_s^d = x_s^c$.

For the first u sublots which are not integer, set $x_s^d = \lceil x_s^c \rceil$,

For the rest of the sublots, set $x_s^d = \lfloor x_s^c \rfloor$.

On the other hand, the rounding algorithm of Sriskandarajah & Wagneur (1999), at first, rounds up the continuous values. Notice that, there would be an extra amount to be removed. This amount is iteratively removed by subtracting one unit from the size of a sublot which has maximum deviation between its discrete and continuous value. The detailed steps are given below.

Steps of Rounding Algorithm of Sriskandarajah & Wagneur (1999)

Step 1. Set $W_0 = 0$, $W_1 = L$ and $\Gamma = \varnothing$

Step 2. For $s = 1$ to S do

{ $x_s^d = \lfloor x_s^c \rfloor + 1$

$W_0 = W_0 + x_s^d$ }

Step 3. $W_0 = W_0 - W_1$

find the product set Γ for which $x_s^d > 1$

Step 4. While $W_0 > 0$ do

{find $d_s = x_s^d - x_s^c$, $s \in \Gamma$

find r such that $d_r = \max_{s \in \Gamma} \{d_s\}$

$x_r^d = x_r^d - 1$

if $x_r^d = 1$, then $\Gamma = \Gamma - \{r\}$

$W_0 = W_0 - 1$ }

An example is given in Appendix B1 to demonstrate the steps of both rounding algorithms. The evaluation of both rounding algorithms results in favor of Sriskandarajah & Wagneur (1999). The results of two rounding algorithms are given in Appendix B2 and B3. Rounding algorithm of Sriskandarajah & Wagneur (1999) gives better results in 37 out of 40 test problems (see Section 5.4 for details of the test problems), while Chen & Steiner's forward rounding algorithm performs better results in only three test problems. Moreover, the rounding algorithm of Sriskandarajah & Wagneur (1999) provides 0.66 % (in average) better makespan values than those of Chen & Steiner's. Therefore, at Step 3 of the proposed solution approach, the rounding algorithm of Sriskandarajah & Wagneur (1999) is going to be used for further comparisons presented in Section 5.4.

5.3.3 Continuous Sized Variable Sublots

Due to the complexity of this problem type, there is almost no study in this field. Only Liu, Chen & Liu (2006) deals with investigated problem with fixed number of sublots. Our problem differs from this problem by the presence of maximum number of sublots. They divide the whole problem into three sequential sub-problems (product sequence determination, lot streaming reallocation machine determination, and lot streaming range determination) each of which applies TS and SA approaches. They give computational results up to 15 products, 10 machines and 4 sublots.

In addition to the common characteristics given in Section 5.2, the sublot availability case is considered for this problem due to the presence of variable sublots.

At first, the following MIP model for variable sublot types in permutation flow shops is developed based on the MIP models of Biskup & Feldmann (2006) and Feldmann & Biskup (2008). This MIP model is going to be used in the proposed solution approach.

<u>Additional Decision Variables:</u>

SS_{jsm} number of units produced in sublot s of product j on machine m

x_{jsmt} binary variable, which takes 1 if the s^{th} sublot of product j on machine m is not started before the t^{th} sublot of product j on machine m-1 has been finished, 0 otherwise.

minimize $C_{\max}$

$$\sum_{s=1}^{S_j} SS_{jsm} = L_j \qquad j = 1, ..., J,\ m=1,\ldots,M \qquad (5.11)$$

$$p_{jsm} = SS_{jsm} t_{jm} \qquad j = 1, ..., J,\ s = 1,\ldots,S_j,\ m =1,\ldots,M \qquad (5.12)$$

$$b_{jsm} \geq b_{js,m-1} + p_{js,m-1} \qquad j = 1, ..., J,\ s = 1,\ldots,S_j,\ m=2,\ldots,M \qquad (5.13)$$

$$b_{jsm} \geq b_{j,s-1,m} + p_{j,s-1,m} \qquad j = 1, ..., J,\ s = 2,\ldots,S_j,\ m =1,\ldots,M \qquad (5.14)$$

$$b_{jS_jm} + p_{jS_jm} \leq b_{k1m} + (1 - y_{jk})R \qquad j,k = 1,...,J,\ j \neq k,\ m =1,\ldots,M \qquad (5.15)$$

$$b_{kS_jm} + p_{kS_jm} \leq b_{j1m} + y_{jk}R \qquad j,k = 1,...,J,\ j \neq k,\ m =1,\ldots,M \qquad (5.16)$$

$$C_{\max} \geq b_{jS_jm} + p_{jS_jm} \qquad j = 1, ..., J \qquad (5.17)$$

$$b_{jsm} \geq b_{jt,m-1} + p_{jt,m-1} + (1 - x_{jsmt})R$$

$$j = 1,...,J,\ s,t = 1,\ldots,S_j,\ m =2,\ldots,M \qquad (5.18)$$

$$SS_{jsm} \leq \sum_{z=1}^{t} SS_{jz,m-1} - \sum_{z=1}^{s-1} SS_{jzm} + (1 - x_{jsmt} + x_{jsm,t+1})R$$

$$j = 1,...,J,\ s = 1,\ldots,S_j,\ t = 1,\ldots,S_j\text{-}1,\ m = 2,\ldots,M \qquad (5.19)$$

$$SS_{j1m} \leq \sum_{z=1}^{t} SS_{jz,m-1} + (1 - x_{j1mt} + x_{j1m,t1})R$$

$$j = 1,...,J,\ t = 1,\ldots,S_j\text{-}1,\ m = 2,\ldots,M \qquad (5.20)$$

$$x_{jsm1} = 1 \qquad j = 1,...,J,\ s = 1,\ldots,S_j,\ m = 2,\ldots,M \qquad (5.21)$$

$$x_{jS_jmt} = 1 \qquad j = 1,...,J,\ t = 1,\ldots,S_j,\ m = 2,\ldots,M \qquad (5.22)$$

$$x_{jsmt} = 1 \qquad j = 1,...,J,\ s,\ t = 1,\ldots,S_j,\ s = t,\ m = 2,\ldots,M \qquad (5.23)$$

$$x_{jsmt} \geq x_{jsm,t+1} \qquad j = 1,...,J,\ s = 2,\ldots,S_j,\ t = 1,\ldots,S_j\text{-}1,\ m = 2,\ldots,M \qquad (5.24)$$

$$x_{jsmt} \leq x_{j,s+1,mt} \qquad j = 1,...,J,\ s = 2,\ldots,S_j\text{-}1,\ t = 1,\ldots,S_j,\ m = 2,\ldots,M \qquad (5.25)$$

$$SS_{jsm} \geq 0,\ b_{jsm} \geq 0 \qquad j = 1,\ldots,J,\ s = 1,\ldots,S_j,\ m = 1,\ldots,M \qquad (5.26)$$

$$y_{jk} \in \{0,1\}, x_{jsmt} \in \{0,1\} \qquad j,k = 1,...,J,\ ,\ j \neq k,\ s,t = 1,\ldots,S_j,\ m = 2,\ldots,M \qquad (5.27)$$

Constraints (5.11) - (5.17) are similar constraints to consistent sublot case. The only difference is that the sublot size variable has three indices instead of two. The set of Constraints (5.18) is only binding if x_{jsmt} takes the value zero. If (and only if) x_{jsmt} takes the value one, sublot s of product j on machine m is not allowed to start before sublot t of the same product on the preceding machine (m-1) has been finished. In general, the size of a sublot, SS_{jsm}, cannot exceed the sum of all sublot sizes on machine m-1 that have been completed before SS_{jsm} starts, minus the sum of all sublots on machine m processed prior to SS_{jsm}. Constraints (5.19) and (5.20) ensure this statement and restrict the size of the variable sublots which are only binding for x_{jsmt} = 1 and $x_{jsm,t+1}$= 0. Equations (5.21) ensure that the start of the processing of a sublot s of product j on machine m must wait until at least the first sublot of that product on machine m-1 has been finished. Equations (5.22) restrict the start of processing the last sublot of product j on machine m until the processing of all sublots of that product on machine m-1 has been finished. Equations (5.23) ensure that a sublot of product j on machine m has to start after the processing of the same sublot of that product on machine m-1. Constraints (5.24) relate sublot s to sublots t and t+1 whereas Constraints (5.25) relate sublot t to sublots s and s+1. Equations (5.26) and (5.27) are non-negativity and binary constraints.

This MIP model includes two groups of binary decision variables. The first one is the same as in consistent sublot types, i.e., the decision variable y_{jk} that gives the sequence of products. The other set of decision variables x_{jsmt} is required to satisfy the relation between the sizes of the sublots processed at the previous machine and the sizes of the sublots going to be processed at the current machine (see Figure 5.3).

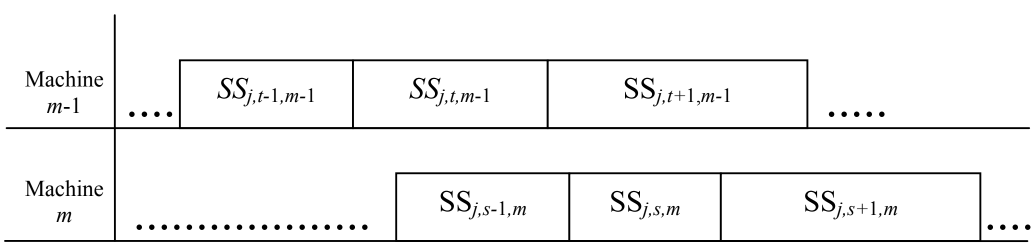

Figure 5.3 The relation of sublot sizes in variable sublot types with sublot availability

As given in Constraints (5.19) and (5.20), the size of sublot s on machine m, SS_{jsm}, should be less than or equal to the difference between the sum of all sublot sizes on machine m-1 that have been completed before SS_{jsm} starts, and the sum of all sublots on machine m processed prior to SS_{jsm}. This situation only occurs in case of variable sublots; remember that in both cases of equal and consistent sublots, the sublot sizes remain their sizes along the machines. These formulations are valid only for sublot availability case. In case of item availability, these formulations have to be modified.

The model presented above is rather difficult to solve due to huge number of binary variables y_{jk} and x_{jsmt}. Even if a pre-determined sequence of products which eliminates y_{jk} decision variables, it may still be impossible to obtain makespan in a reasonable amount of time due to enormous number of x_{jsmt} binary decision variables. One way to overcome this problem is to utilize the continuous relaxation of these x_{jsmt} variables. Under a given sequence, by relaxing x_{jsmt} decision variables, we observe that values of most of x_{jsmt} variables are so close to either zero or one. Using this observation, we may round x_{jsmt} values which are smaller than a pre-specified tolerance value ε to zero and greater than (1- ε) to one. For the given sequence and fixed zero-one x_{jsmt} values, the makespan can be determined by solving the MIP model iteratively until all x_{jsmt} get binary values. Of course, an infeasible solution might appear in some iterations of such a rounding method; however, these infeasibilities may be eliminated by embedding a feasibility checking procedure into the scheme. Finally, it should be noted that, the proposed solution procedure give near-optimal results. For further analysis, the performances of these results are compared with the original MIP model results.

The additional notation and the proposed algorithm are presented below.

$\mathrm{C} = \left\{ x_{jsmt} \mid 0 < x_{jsmt} < 1, x_{jsmt} \in \Re^{+} \right\}$

$\mathrm{D} = \left\{ x_{jsmt} \mid x_{jsmt} \in \{0,1\} \right\}$

T : a temporary set of x_{jsmt} which are candidates to be moved from set C to D.

ε : a predetermined small value, i.e., $0 < \varepsilon \leq 0.5$ and $\varepsilon \in \Re^{+}$

nox : a control variable, if $A = \varnothing$, it takes zero; otherwise, it takes one; where $A = \left\{ x_{jsmt} \in C \mid x_{jsmt} \leq \varepsilon \text{ and } x_{jsmt} \geq 1 - \varepsilon \right\}$.

The Steps of Solution Approach for Continuous sized Variable Sublots

Step 0. Sequencing

Determine the sequence of products by using one of the investigated sequencing heuristics.

Step 1. Initialization

Give the resulting sequence to the MIP model in which all x_{jsmt} values are relaxed

(i.e., $0 \le x_{jsmt} \le 1$)

Set *nox* = 0, *minvalue* = 1, *value* = 1, $\varepsilon = 0.1$

Step 2. Partitioning

Run the relaxed MIP model and obtain optimal x_{jsmt} values, say x^*_{jsmt}.

Partition the set of x^*_{jsmt} into two disjoint sets where

$C = \{x_{jsmt} \mid 0 < x^*_{jsmt} < 1\}$

$D = \{x_{jsmt} \mid x^*_{jsmt} \in \{0,1\}\}$

Step3. Termination

If all $x_{jsmt} \in D$, then STOP,

Else, *nox* = 0, *minvalue* = 1, $T = \varnothing$.

Step 4. Variable Fixing

For each $x_{jsmt} \in C$ apply the following steps.

If $x^*_{jsmt} \le \varepsilon$, then set $x_{jsmt} = 0$,

Else if $x^*_{jsmt} \ge 1 - \varepsilon$, then set $x_{jsmt} = 1$,

nox = 1, move x_{jsmt} from set C to set T.

Else if *nox* = 0, then set $value = \min\{x^*_{jsmt}, 1 - x^*_{jsmt}\}$

If *minvalue* ≥ *value*, then set *minvalue* = *value*, store corresponding x^*_{jsmt} value to be used in Step 5.

Step 5. Controlling

If *nox* = 1, then go to Step 6.

Else if *nox* = 0, then use the previously stored x^*_{jsmt} in Step 4.

If $x^*_{jsmt} \le 0.5$, then set $x_{jsmt} = 0$,

Else set $x_{jsmt} = 1$,

Move x_{jsmt} from set C to set T.

Step 6. Running

Set $D = D \cup T$. Add all $x_{jsmt} \in D$ with their corresponding 0 or 1 values to the MIP model as fixed equations and run the model again.

Step 7. Feasibility Checking

If the solution is infeasible then set $D = D - T$. For all $x_{jsmt} \in T$ (which get binary values in Step 4 and 5), reassign their previous real values (i.e., x^*_{jsmt}) and apply the following steps.

Step 7.1 Take the element in set T which has closest value to 0 or 1;

If $x^*_{jsmt} \le 0.5$, then set $x_{jsmt} = 0$,

Else $x_{jsmt} = 1$.

Step 7.2 Run the MIP model by adding only this x_{jsmt} as a fixed equation.

If the solution is feasible, then fix the value of $x_{jsmt} = 0$ (or $x_{jsmt} = 1$),

move x_{jsmt} from set T to set D, go to Step 7.1.

Else if the solution is infeasible, then fix the value of $x_{jsmt} = 1$ (or $x_{jsmt} = 0$),

and run the MIP model by adding only this x_{jsmt} as a fixed equation,

move x_{jsmt} from set T to set D and go to Step 3.

Else, if the solution is feasible, go to Step 3.

5.4 Computational Results

To evaluate the performances of sequencing heuristics, we use the problem instances generated by LSGen (Feldmann, 2005)

Table 5.1 Experimental problem types

Number of Products, (*J*)	Number of Sublots, (*S*)	Number of Machines, (*M*)
5	5	5
5	5	10
5	10	5
5	10	10
10	5	5
10	5	10
10	10	5
10	10	10

Eight problem types (see Table 5.1) each consisting of five problem instances are generated. These 40 problem instances are used to compare the performances of sequencing heuristics described in Section 5.2 by utilizing the solution schemes presented in Section 5.3. The MIP model part of the solution approaches built in OPL Studio 3.7 optimization package and solved on a Centrino 1.73 GHz processor with 1.5 GB RAM. Detailed computational results for three investigated problems are given in Appendix B4, B3 and B5. To evaluate the performances of proposed sequencing heuristics, the best makespan value found for each problem instance is used as a benchmark value. Notice that, the makespan values of sequencing heuristics which gives the best makespan are presented in bold. Table 5.2 summarizes the results for each problem type.

As seen from Table 5.2, for continuous sized consistent sublots, the best heuristic is NEH(D,TPLS). It gives 22 best results out of 40 problem instances and its average proportional deviation is 0.84%. It can be stated that, NEH(D,TPLS) returns either best results or very close results to the best ones. In addition, NEH(D,TPT) gives promising results with 19 best results; however, its average proportional deviation is 2.04%. Consequently, NEH based heuristics, except NEH(I,TPLS), produce better results than the other sequencing heuristics. Note that, the number of best results may not add up to 40 because different sequencing heuristics may obtain the same makespan value. The best solutions of sequencing heuristic are given in detail in terms of sequence and sublot sizes in Appendix B6. In Appendix B7, the detailed results of optimal solutions obtained from MIP model are given.

Table 5.2 Comparison of sequencing heuristics performances for each problem type

	Continuous Sized Consistent Sublots		**Discrete Sized Consistent Sublots**		**Continuous Sized Variable Sublots**	
	# of Best Results	**Avg. Prop. Dev. (%)**	**# of Best Results**	**Avg. Prop. Dev. (%)**	**# of Best Results**	**Avg. Prop. Dev. (%)**
LPT (TPT)	0	13.45	0	13.46	0	13.46
SPT (TPT)	1	10.20	1	10.56	1	10.54
LPT (TPLS)	1	12.65	1	12.62	1	12.65
SPT (TPLS)	0	12.31	1	12.37	0	12.32
NEH (D,TPT)	19	2.04	19	2.02	19	2.04
NEH (I,TPT)	16	1.63	14	1.73	16	1.63
NEH (D,TPLS)	**22**	**0.84**	**20**	**0.91**	**22**	**0.85**
NEH (I,TPLS)	7	2.55	7	2.54	7	2.54
CDS (ORJ)	5	2.74	4	2.69	5	2.76
CDS (PLS)	10	2.50	10	2.44	10	2.49
PALMER (ORJ)	6	5.07	6	5.09	6	5.06
PALMER (PLS)	7	5.02	7	5.01	7	5.01
GUPTA (ORJ)	0	9.27	0	9.30	0	9.28
GUPTA (PLS)	2	7.83	1	7.88	2	7.83
BMI (ORJ)	2	9.78	0	9.76	2	11.23

* $\text{Prop. Dev.}(\%) = 100\left(\frac{\text{Makespan}(X) - \text{Benchmark}}{\text{Benchmark}}\right)$ $\quad \text{Avg. Prop. Dev.}(\%) = \left(\sum_{i=1}^{40} \text{Prop.Dev.}(\%)\right) \Big/ 40$

For discrete sized consistent sublots, we observe that the best results are obtained again by NEH(D,TPLS). In fact, it is an expected outcome, because this solution procedure uses the same sequence and steps with only one additional stage which converts the continuous values obtained by MIP model to discrete ones. This sequencing heuristic gives best results in half of the instances with an average proportional deviation of 0.91 %.

Similar conclusions can be drawn for the continuous sized variable sublot case. Once more, the best sequencing heuristic is NEH(D,TPLS) with 22 best makespan and 0.85 % average proportional deviation.

Remember that NEH is stated as the best sequencing heuristic for the general flow shop scheduling problems (Ruiz & Maroto, 2005; Weng, 2000). Similarly, one of the modified version of NEH heuristic which also considers LS requirements, named NEH(D,TPLS), gives quite better results compared to other sequencing heuristics.

Table 5.3 Data and sublot sizes of problem instance 5-5-5-2

	Processing Times (min/item/machine)					**Sublot Sizes** (item/sublot)					**Lot Size**
Products	**M1**	**M2**	**M3**	**M4**	**M5**	SS_1	SS_2	SS_3	SS_4	SS_5	
1	11	3	6	5	11	0	0	10	6	4	20
2	10	8	2	9	12	0	1	5	6	8	20
3	2	6	4	6	4	0	4	0	7	0	11
4	3	4	6	7	2	5	8	9	10	5	37
5	4	9	11	7	8	3	3	4	4	3	17

Finally, the data for the second instance of the LS problem with five products, five sublots and five machines (5-5-5-2) is reported in Table 5.3 along with the results including the discrete sized consistent sublots. The first five columns after "products" column give the processing times of products per machine. The number of items in each sublot of products are given in subsequent columns and in the last column, the lot sizes of each product is presented. As can be seen from the sublot sizes, the first product is divided into three sublots, the second product into four sublots, the third product into two sublots and both of the products four and five into five sublots. The maximum number of sublots is selected as five sublots but some sublot sizes valued zero which means no extra sublot is required. The sequence of products for this problem instance is obtained as 5-2-3-1-4 from NEH(D,TPLS). Figure 5.4 gives the corresponding schedule. The representation *j*-*i* denotes sublot *i* of product *j*, for instance 5-1 means that the first sublot of product 5.

The performance of best sequencing heuristic, NEH(D,TPLS), can also be evaluated with the results of original MIP model. For this purpose, LS problems with 15 products are added to the problem instances. Total number of test problems is now 60.

The original MIP models are solved on a Centrino 1.73 GHz processor with 1.5 GB RAM and terminated by a 1000 seconds run time limit. The small sized problems are able to reach optimal solutions.

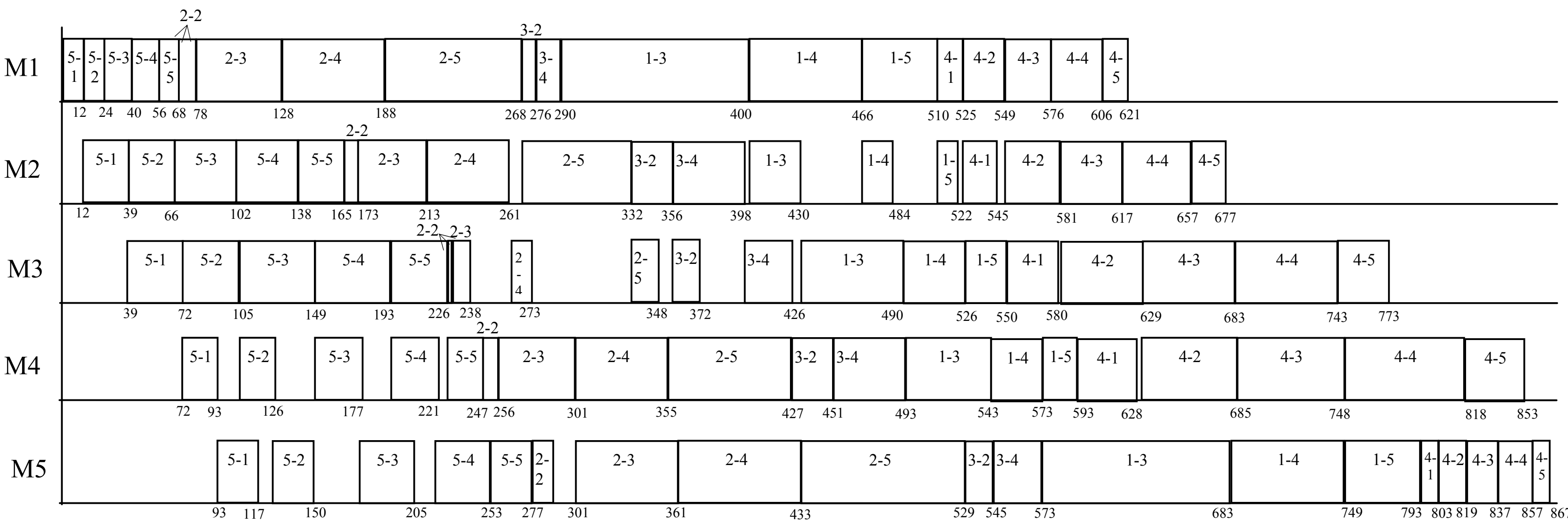

Figure 5.4 Gantt chart of the problem instance 5-5-5-2 for discrete sized consistent sublot

Since most of the large size problems time out due to 1000 second time limit, only lower and upper bounds are recorded. The NEH(D,TPLS) results are compared with the upper bounds of MIP models. Note that, the computational time for the NEH(D,TPLS) varies between 2 and 10 seconds with respect to the problem size. The results belonging to three investigated problem types are given in Appendix B8, B9 and B10 in detail. Table 5.4 summarizes these results.

For continuous sized consistent sublots, the NEH(D,TPLS) results are very close (0.69%) to the MIP results. The number of best results obtained from MIP and NEH(D,TPLS) are 32 and 12, respectively. Deeper analysis may be performed with respect to different levels of experimental points i.e., number of products, number of sublots and number of machines. The performance of NEH(D,TPLS) on finding number of better results improves at higher levels of experimental points.

Again, for discrete sized consistent sublots, the NEH(D,TPLS) results are very close (0.69%) to the MIP results. However, the MIP model gives 51 best results while the NEH(D,TPLS) gives only 9 best results. This is due to the fact that the proposed solution procedure suffers from optimum solution in two aspects. The first one is the given sequence may not be the optimal sequence. Even if the given sequence is optimal, the rounding procedure may not give optimal but a bit worse than optimal results. Similar to the previous case, as the problem scales up, the performance of proposed procedure improves. In higher level of all experimental points, the proposed method performs better than MIP in terms of average deviation.

The most obvious advantage of NEH(D,TPLS) and proposed solution procedure appears in case of variable sublots. In 39 out of 60 problem instances, the NEH(D,TPLS) outperforms the MIP model. The average proportional deviation favors NEH(D,TPLS) giving 1.59% better results than the MIP model. For this problem type, the MIP model fails to find better results than the NEH(D,TPLS) heuristic due to its huge number of 0-1 decision variables.

Table 5.4 Comparison of NEH(D,TPLS) heuristic results with the MIP results (within 1000 sec)

		Continuous Sized Consistent Sublots				Discrete Sized Consistent Sublots				Continuous Sized Variable Sublots			
		# of Better Results		# of Same Results	Avg. Prop. Dev (%)	# of Better Results		# of Same Results	Avg. Prop. Dev (%)	# of Better Results		# of Same Results	Avg. Prop. Dev (%)
		MIP	NEH (D,TPLS)			MIP	NEH (D,TPLS)			MIP	NEH (D,TPLS)		
# of products	5	10	0	10	0.36	20	0	0	0.91	8	11	1	-0.57
	10	15	1	4	2.39	20	0	0	2.31	6	12	2	-1.55
	15	7	11	2	-0.83	11	9	0	-1.27	3	16	1	-3.58
# of sublots	5	18	5	7	0.49	26	4	0	0.87	13	14	3	-1.75
	10	14	7	9	0.49	25	5	0	-0.18	4	25	1	-2.94
# of machines	5	13	4	13	0.83	28	2	0	0.95	12	14	4	0.30
	10	19	8	3	0.21	23	7	0	-0.13	5	25	0	-4.41
Overall		32	12	16	0.69	51	9	0	0.69	17	39	4	-1.59

Prop. Dev. (%) = 100[NEH(D,TPLS) –MIP]/MIP

A significant relative improvement occurs again in the higher levels of experimental points. For instance, in 10-machine test problems, NEH(D,TPLS) performs better than MIP model with 4.41% deviation in average.

It should also be noted that another advantage of proposed algorithms arises in terms of computation time. For all problem instances, NEH(D,TPLS) obtains the results up to only 10 seconds while MIP requires 1000 seconds especially in large sized problem instances.

In view of analysis given above, a common outcome of computational results belonging to three investigated problems is that proposed solution approach performs relatively better than MIP model in higher levels of all experimental points. At this moment, a question may arise: Up to which higher levels of experimental points the proposed procedures may provide solutions. To clarify this question, 18 problem types (see Table 5.5) each having five instances are generated. The NEH(D,TPLS) results of three investigated problem types are given in Appendix B11 and B12 for 30 and 50 products, respectively.

Table 5.5 Experimental problem types for large sized problems

Problem type	**# of products**	**# of sublots**	**# of machines**
1	30	5	5
2	30	5	10
3	30	5	15
4	30	10	5
5	30	10	10
6	30	10	15
7	50	5	5
8	50	5	10
9	50	5	15
10	50	5	20
11	50	10	5
12	50	10	10
13	50	10	15
14	50	10	20
15	50	20	5
16	50	20	10
17	50	20	15
18	50	20	20

As can be seen from the results, the computation time of each investigated problem increases as the problem scales up. The proposed solution procedures of consistent sublots are able to generate results in all problem instances. However, the proposed solution procedure of variable sublots cannot generate results in some of the instances due to memory requirements of the MIP model. These instances are the ones with higher levels of experimental points. By this analysis on variable sublots, the limits on the levels of experimental points are determined as 50-product, 20-sublot problems. It should be noted that, there is no study in the literature that gives results for these levels of problem.

5.5 Chapter Summary

Multi product LS problems in flow shops require sequencing the products through the machines as well as sublot allocation of the products.

In this study, three multi product, multi machine LS problems with non-intermingling case are investigated to minimize the makespan in permutation flow shops. These problems differ by the following characteristics:

- Continuous sized consistent sublots,
- Discrete sized consistent sublots and
- Continuous sized variable sublots.

The aims of this chapter may be listed as:

- To propose simple and efficient sequencing heuristics to be used for the sequencing subproblem of the entire LS problem.
- To analyze the performances of these sequencing heuristics and to determine the best ones with the aim of minimizing makespan.
- To propose solution scheme for each investigated problem to determine the sublot allocations for the given sequence.

In the solution approaches, the entire problem is partitioned into sequencing and sublot allocation sub-problems. For the sequencing sub-problem, a number of simple and efficient sequencing heuristics developed for general flow shops are modified according to LS requirements. For the sublot allocation sub-problem, mixed integer programming (MIP) based solution approaches are proposed. For the entire problem, a hybrid solution approach which uses sequencing heuristic in sequencing sub-problem and applies MIP based approaches for the sublot allocation sub-problem, is proposed. For all investigated problem types, NEH(D,TPLS) sequencing heuristic returns not only more number of best results, but also gives rather close results to the best ones. The performance of this heuristic is evaluated against the MIP model results. NEH(D,TPLS) generates rather close results for the consistent sublot cases and outperforms the MIP model for continuous sized variable sublot case.

This study can be further carried to the multi product LS problems with intermingling cases as well as non-intermingling case for discrete sized variable sublots by modifying the solution procedures.

The NEH(D,TPLS) can be used as the starting point of a search based solution approach or can be included into the initial population in evolutionary based algorithms to get better results in smaller times.

The work in this study presents an efficient and easily applicable sequencing heuristic, NEH(D,TPLS), embedded into the proposed solution approaches. The further studies may take the proposed solution schemes as a benchmark method.

CHAPTER SIX
A TABU SEARCH BASED HEURISTIC FOR MULTI PRODUCT LOT STREAMING PROBLEMS

6.1 Introduction

In previous section, the multi product, multi machine LS problems have been studied to obtain simple and efficient results in a reasonable time. Simple sequencing heuristics have been modified with respect to LS requirements and solution approaches for three different multi product, multi machine LS problems have been proposed. The performances of these sequencing heuristic are evaluated and the best one is suggested to be used in these types of LS problems. However, each of these sequencing heuristics generates only one sequence and evaluates its performance. However, there may yet remain a number of sequence alternatives to be able to generate better makespan values. To obtain optimal or near-optimal results, the other sequence alternatives in the solution space can be searched. For small number of products, the number of sequence alternatives is rather limited and full enumeration can be used to obtain optimal sequence. For instance, for two products, the number of alternatives is two: 2-1 and 1-2. The number of alternatives increases to six and 24 for three and four products, respectively. These numbers of alternatives are small enough to obtain optimal sequence by full enumeration. However, the number of sequence alternatives increase in exponential manner with respect to number of products (see Table 6.1). Evaluating only one sequence alternative may be a good approximation method for large sized LS problems; however, it may not be an efficient approximation method for small to medium sized LS problems.

Table 6.1 The number of products and corresponding number of sequencing alternatives

Number of products (J)	2	3	4	5	6	7	8	9	10
Sequencing alternatives (J!)	2	6	24	120	720	5040	40320	362880	3628800

From this point of view, optimal or near optimal results may be obtained by introducing an efficient search procedure for the sequencing part of the multi product LS problems.

Remember that, in the previous chapter, the proposed solution approaches utilize MIP models to determine the sublot sizes of the products under a given sequence. However, MIP solver requires a considerable computation time. Thus, the number of sequence alternatives to be evaluated should be restricted. The main purpose, in this manner, is to find the most efficient results in rather less number of iterations. For that reason, neighborhood-search algorithms are more appropriate than evolutionary algorithms (e.g., genetic algorithms) which require the evaluation of all alternatives within a population. Moreover, we have an efficient sequence heuristic, i.e., NEH(D,TPLS), at hand, to be used to obtain a good initial point.

In this section, a TS based solution approach is proposed for three investigated multi product LS problems introduced in the previous section. The framework of this solution approach is given in Figure 6.1. Different from the solution approach presented in previous chapter, this solution approach searches the alternative product sequences via TS in order to obtain better makespan values.

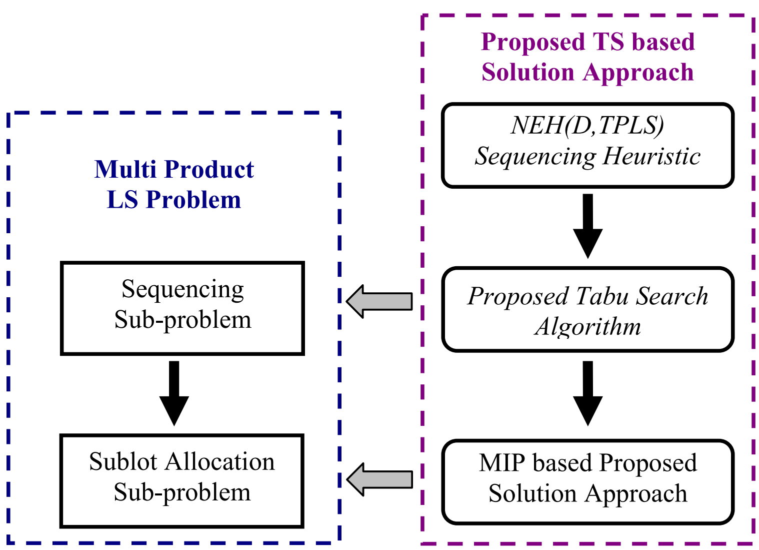

Figure 6.1 The framework of the proposed TS based solution approach for multi product LS problems

The main reasons of preferring TS are;

- a good sequencing heuristic (i.e., NEH(D,TPLS)) has already proposed in the previous chapter and it can be used as an initial sequence of a search algorithm.
- TS avoids from local optimum traps.
- TS requires less number of iterations (accordingly less computation time) to obtain efficient results when compared with some other types of meta-heuristics such as GA.
- TS utilizes the short term memory process.

6.2 The Proposed Tabu Search Algorithm

For the investigated LS problems, a TS based heuristic algorithm is developed to sequence the products. Additional notation used in the proposed algorithms is as follows:

TL	set of current pair of products in the tabu list
LM	last best makespan
BM	best makespan
BS	best sequence
NS	new sequence
R	very big number
N_A	number of alternative sequences generated from a NS
A_j	j^{th} alternative sequence ($j = 1,\ldots, N_A$)
M_j	corresponding makespan of sequence A_j
$move_j$	move (a pair of products) that generates the alternative sequence A_j
S_p	number of iterations that move p wait in TL ($p \in TL$)
$maxlength$	maximum number of iterations that a move should wait in TL
IC	iteration counter that counts the consecutive number of non-improved solutions
NIS	a limit on the consecutive number of non-improved solutions (termination criterion)

The Steps of Proposed Tabu Search based Heuristic Algorithm

Step 0. Initialization. Set $TL = \emptyset$, $IC = 0$, $BM = R$, $LM = R$, $BS = \emptyset$, $NS=\emptyset$.

Step 1. Initial Sequence. Apply an algorithm to get the initial sequence of products.

Step 2. Running. Give this initial sequence to the MIP model as an input and obtain the corresponding makespan value. Set BS and NS to the initial sequence, BM to the makespan value of the initial sequence.

Step 3. Alternative Sequence Generation and Evaluation

Step 3.1 **Generation**. Generate alternative sequences from NS by using an alternative generation method.

Step 3.2 **Running**. For j=1 to N_A

Succeed the neighborhood move, $move_j$ and obtain the corresponding sequence A_j. Give A_j to the MIP model as an input to obtain corresponding makespan value M_j

Step 3.3 **Sorting**. Sort A_j in non-decreasing order of M_j, $SL = [A_{[1]}, A_{[2]}, ..., A_{[N_A]}]$

(note that $[i]$ corresponds to the alternative at the position i in the sorted list)

Step 4. Selection of the new sequence by tabu search.

Set $i = 1$, $NS = \emptyset$

While $NS = \emptyset$

Do{

If $M_{[i]} < BM$ then,

If $move_{[i]} \notin TL$ then,

{set $NS = A_{[i]}$, $BS = A_{[i]}$, $BM = M_{[i]}$, $S_p = S_p$-1 $\forall p \in TL$,

add $move_{[i]}$ to the TL, $S_{move_{[i]}} = maxlength$}

Else {set $NS = A_{[i]}$, $BS = A_{[i]}$, $BM = M_{[i]}$, $S_p = S_p$-1 $\forall p \in TL$,

$S_{move_{[i]}} = maxlength$}

Else If $M_{[i]} = BM$ then,

If $move_{[i]} \notin TL$ then,

{set $NS = A_{[i]}$, $BS = A_{[i]}$, $BM = M_{[i]}$, $S_p = S_p$-1 $\forall p \in TL$,

add $move_{[i]}$ to the TL, $S_{move_{[i]}} = maxlength$}

Else If $M_{[i]} > BM$

If $move_{[i]} \notin TL$ then,

{set $NS = A_{[i]}$, $S_p = S_p$-1 $\forall p \in TL$,

add $move_{[i]}$ to the TL, $S_{move_{[i]}} = maxlength$}

$i = i$+1 }

For all $p \in TL$, if $S_p = 0$ remove $move_p$ from TL.

If $LM \leq BM$ then, set $IC = IC$+1,

Else {set $LM = BM$, $IC = 0$}

Step 5. Termination. If $IC \leq NIS$ then go to Step 3, Else STOP.

This TS based heuristic is applied to three investigated LS problems defined in the previous section. The general structure of the algorithm is the same for all investigated problems. However, it differs in terms of some parameter levels and the running steps of the algorithm (i.e., way of obtaining the makespan values of a given sequence). These differences are given in detail in following sections for each investigated problem. Remember that, the important factors that affect the efficiency of TS is described in Section 4.2. The common issues of the algorithm based on these factors are listed below.

- **Initial point**

The analysis and results of Chapter 5 clarify that NEH(D,TPLS) gives most efficient results in very small computation time requirements. Since NEH(D,TPLS) is the best one for all investigated problems, it can be used as an initial sequence in common.

- **Tabu list length**

The alternative tabu list lengths are selected to be three, five, seven and nine. After a number of preliminary trials, the length of tabu list is selected to be seven, *maxlength* = 7, which is generally advised in most of the studies (Glover, 1986). Detailed results are given in Table 6.2.

6.2.1 Continuous Sized Consistent Sublots

The different components for the first problem type (the continuous sized consistent sublots) are given below.

- **<u>Alternative Sequence Generation</u>**

If the computation time for each alternative is not so much, all pairwise interchange method is generally preferred, since it searches all possible alternatives within the neighborhood which increases the efficiency of the search procedure. Since MIP sub-problems with continuous sized consistent sublots can be solved in a few seconds, all-pairwise interchange method can be used for this problem type. Notice that, the number of alternatives generated from each new seed is $N_A = \frac{J(J-1)}{2}$.

- **<u>Termination Criteria</u>**

As mentioned before, an increase in the number of products causes an increase in the total number of alternative sequences (see Table 6.1). Thus, the termination criteria may be related with the number of products. It is considered as the termination criteria that if the number of consecutive non-improved solutions reaches to half of the number of products, i.e., $NIS = \left\lceil \frac{J}{2} \right\rceil$. By this choice, results can be obtained in a reasonable time.

- **<u>Running Steps</u>**

In Step 2 and Step 3.2 of the proposed algorithm, apply the steps of procedure, except Step 0, developed for continuous sized consistent sublots and given in Section 5.3.1.

Table 6.2 The effect of tabu list length on makespan values for discrete sized consistent sublots

Instance	**Tabu List Length=3**	**Tabu List Length=5**	**Tabu List Length=7**	**Tabu List Length=9**
5-5-5-1	889	889	889	889
5-5-5-2	866	866	866	866
5-5-5-3	1419	1419	1419	1419
5-5-5-4	753	753	753	753
5-5-5-5	1364	1364	1364	1364
5-5-10-1	1670	1670	1670	1670
5-5-10-2	1692	1692	1691	1691
5-5-10-3	1272	1272	1272	1272
5-5-10-4	1354	1354	1354	1354
5-5-10-5	1312	1312	1312	1312
5-10-5-1	839	839	839	839
5-10-5-2	819	819	819	819
5-10-5-3	1332	1332	1332	1332
5-10-5-4	741	741	741	741
5-10-5-5	1328	1328	1328	1328
5-10-10-1	1494	1494	1494	1494
5-10-10-2	1485	1485	1485	1485
5-10-10-3	1163	1163	1163	1163
5-10-10-4	1237	1237	1237	1237
5-10-10-5	1147	1147	1147	1147
10-5-5-1	1951	1951	1951	1951
10-5-5-2	2070	2070	2070	2070
10-5-5-3	2120	2120	2120	2120
10-5-5-4	1913	1913	1913	1913
10-5-5-5	1941	1941	1941	1941
10-5-10-1	3038	3038	3038	3038
10-5-10-2	2483	2483	2482	2482
10-5-10-3	2807	2807	2819	2807
10-5-10-4	2448	2448	2448	2448
10-5-10-5	2190	2190	2190	2173
10-10-5-1	1950	1950	1950	1950
10-10-5-2	2052	2052	2052	2052
10-10-5-3	2048	2048	2048	2048
10-10-5-4	1899	1899	1899	1899
10-10-5-5	1926	1926	1926	1926
10-10-10-1	2940	2940	2940	2940
10-10-10-2	2354	2338	2322	2354
10-10-10-3	2671	2671	2671	2671
10-10-10-4	2267	2267	2267	2267
10-10-10-5	2065	2065	2065	2065
Average	1732.73	1732.33	1732.18	1732.25

6.2.2 Discrete Sized Consistent Sublots

The different components for the second problem type (the discrete sized consistent sublots) are given below.

- **Alternative Sequence Generation**

In this problem type, the evaluation procedure of alternative sequences requires an extra rounding algorithm which necessitates an additional computation time. Therefore, adjacent pairwise interchange method instead of all-pairwise is considered for this problem type. By this choice, the number of alternatives generated from a new seed decreases to $N_A = J - 1$.

- **Termination Criteria**

The disadvantage of adjacent pairwise interchange method can be eliminated to a degree by termination criteria. If the number of consecutive non-improved solutions reaches to the number of products, $NIS = J$, the search terminates.

- **Running Steps**

In Step 2 and Step 3.2 of the proposed algorithm, apply the steps of procedure, except Step 0, developed for discrete sized consistent sublots and given in Section 5.3.2 (Apply the rounding algorithm of Sriskandarajah & Wagneur (1999))

6.2.3 Continuous Sized Variable Sublots

The different components for the third problem type (the continuous sized variable sublots) are given below.

- **Alternative Sequence Generation**

In this problem type, the evaluation procedure of alternative sequences requires an extra computational effort to determine x_{jsmt} values of MIP model given in Section 5.3.3. Therefore, adjacent pairwise interchange method instead of all-pairwise is considered for this problem type. By this choice, the number of alternatives generated from a new seed decreases to $N_A = J - 1$.

- **Termination Criteria**

If the number of consecutive non-improved solutions reaches to half of the number of products, $NIS = \left\lceil \frac{J}{2} \right\rceil$, the search is terminated. By this choice, results can be obtained in a reasonable time.

- **Running Steps**

In Step 2 and Step 3.2 of the proposed algorithm, apply the steps of procedure, except Step 0, developed for continuous sized variable sublots and given in Section 5.3.3.

6.3 Computational Results

The set of test problems given in Chapter 5 is also considered in the evaluation of the proposed TS based heuristic so that a common comparison scheme is obtained. The MIP model part of the solution approaches built in OPL Studio 3.7 optimization package and solved on a Centrino 1.73 GHz processor with 1.5 GB RAM.

The TS based heuristic for the investigated LS problems are evaluated and the computational results are given in Appendix C. In order to provide a fair comparison between MIP and TS based heuristic, MIP solver is run at least the computational time of TS based heuristic. More clearly, a 1000 second run time limit is set to all instances where TS based heuristic terminates under 1000 seconds; whereas, if TS based heuristic is able to give results in more than 1000 seconds, an adjusted time limit greater than the computation time of TS based heuristic is set to MIP.

The computational results belonging to continuous sized consistent sublots are given in Appendix C1. This table also includes the results of MIP and initial sequence, NEH(D,TPLS). MIP model could reach to optimal results in only five product cases. When all test problems are considered, TS based heuristic performs better than MIP model.

The computational results for the discrete sized consistent sublots are given in Appendix C2. TS based heuristic results are better than the ones of MIP under solution time limit and NEH(D,TPLS). MIP model gives optimal results in 17 out of 60 test problems all of which belongs to five product cases.

The computational results for the continuous sized variable sublots are given in Appendix C3. MIP model could obtain optimal solution in only two instances (5-5-5-3 and 5-5-5-5) out of 60 instances. The other instances could only reach feasible results in computation time limits.

The minimum makespan value of three solution approaches is considered as the benchmark value. The percentage deviation of MIP, NEH(D,TPLS) and TS based heuristic from the benchmark value are obtained for each investigated problem type.

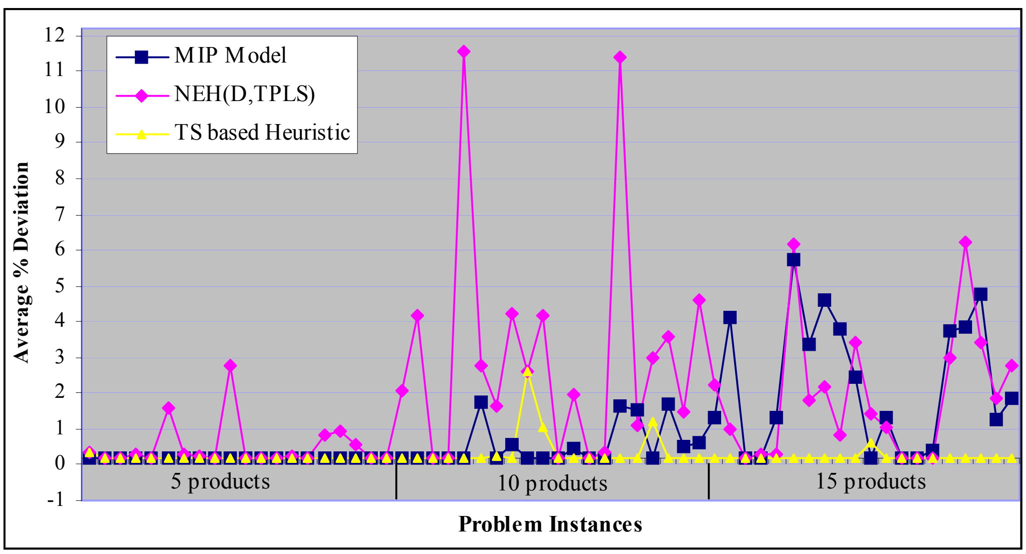

Figure 6.2 Percentage deviations of solution approaches from the minimum results for continuous sized consistent sublots.

As seen from Figure 6.2, the smaller problem instances can be optimally solved by the MIP model. TS based heuristic produces very close solutions to the optimal ones, whereas NEH(D,TPLS) results in small deviations in a number of test problems. For 10 and 15 product test instances, the percent deviations increase for both MIP and NEH(D,TPLS). For 10 product problems, the deviation of

NEH(D,TPLS) is greater than MIP, while for 15 product instances, the percent deviations are similar. For 10 and 15 product instances, TS based heuristic produces better results than the other ones.

As can be seen in Figure 6.3, for discrete sublot sizes, MIP gives better results than both NEH(D,TPLS) and TS based heuristic for five product cases as well as for some instances of 10 product cases. For larger problems, the deviations significantly changes in favor of TS based heuristic. Furthermore, the solutions of NEH(D,TPLS) which are worse than MIP and TS based heuristic in five and 10 product instances become competitive with MIP results in 15-product test instances confirming the inferences related to NEH(D,TPLS) given in Chapter 5.

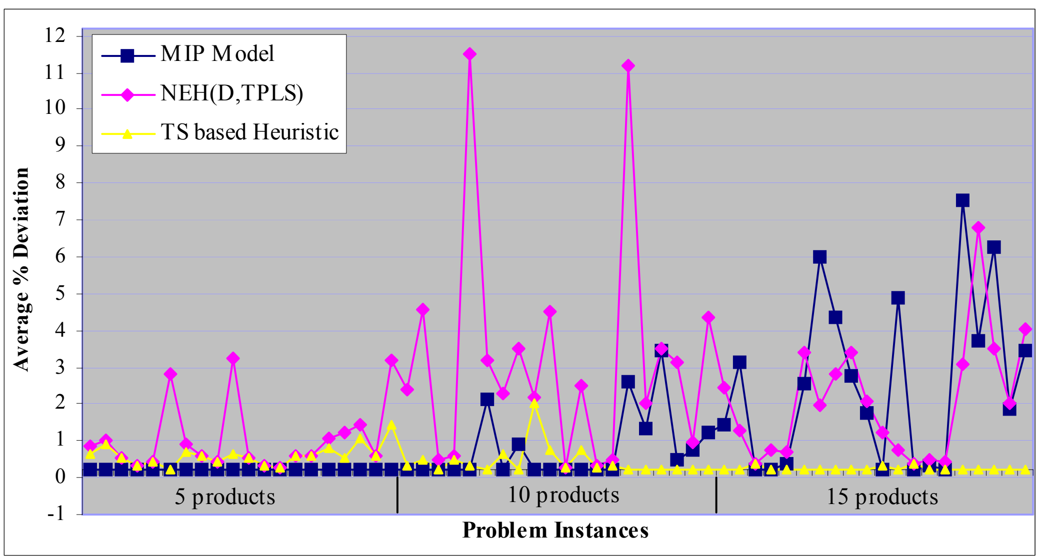

Figure 6.3 Percentage deviations of solution approaches from the minimum results for discrete sized consistent sublots.

For continuous sized variable sublot problems, the superiority of TS based heuristic results can be seen in Figure 6.4. The MIP could not perform better even than NEH(D,TPLS) in most instances. Especially for 15 product cases, the percent deviations of MIP are significantly higher than NEH(D,TPLS) and TS based heuristic.

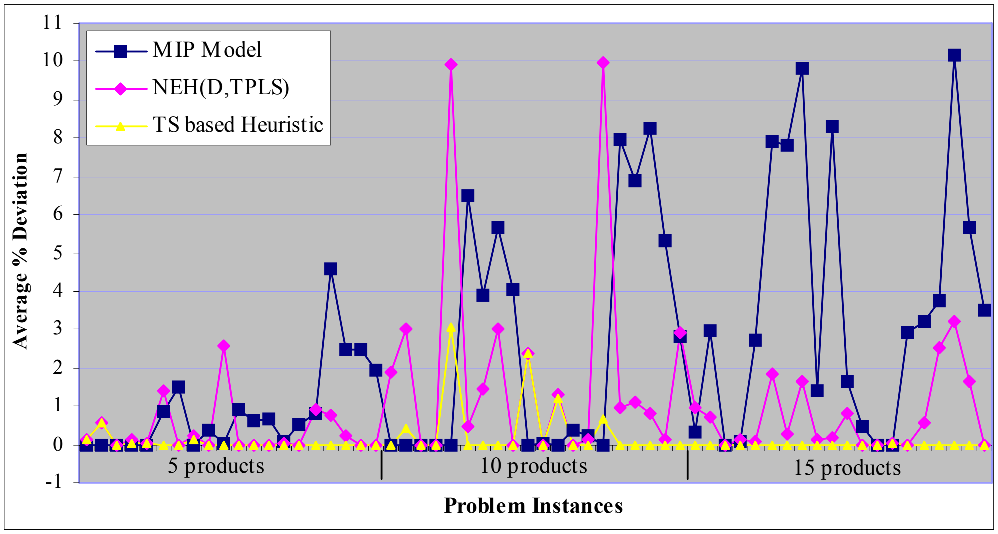

Figure 6.4 Percentage deviations of solution approaches from the minimum results for continuous sized variable sublots.

Figure 6.5 presents a comparison of three solution approaches in terms of overall results for all investigated problem types.

The minimum makespan value of three solution approaches is considered as the benchmark value. The average percent deviations of each solution approach from these benchmark values are given in Figure 6.5 for each investigated problem type. In all three problem types, TS based heuristic obviously gives better results than the other two approaches.

For consistent sublots, MIP model generates better results than initial sequence, NEH(D,TPLS). However, for variable sublots, NEH(D,TPLS) gets significantly better than MIP. This is most probably due to the fact that the MIP model of consistent sublot case is simpler than the variable sublot case. This case can also be seen in the comparison of MIP and TS based heuristic. TS based heuristic generates 0.14% deviation while MIP generates 2.37 % deviation from benchmark values in average.

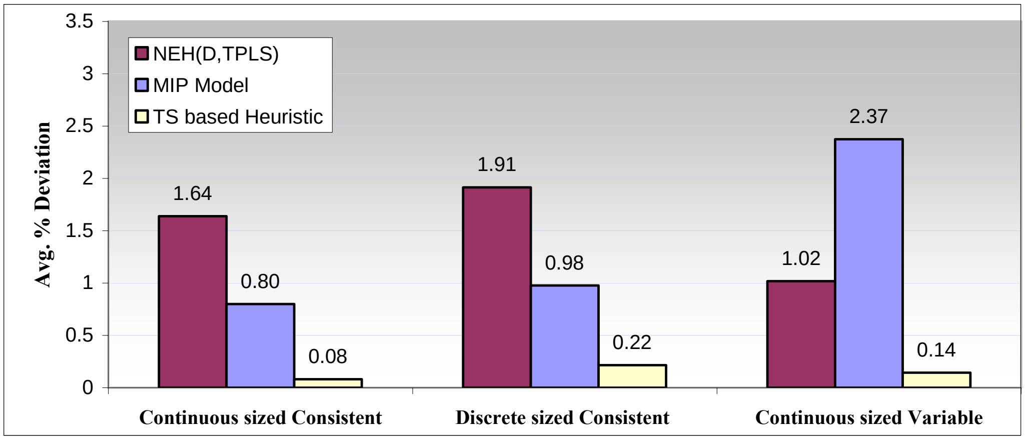

Figure 6.5 Comparison of solution approaches for the investigated problems

Since the TS based heuristic starts with the initial sequence NEH(D,TPLS), surely, it provides better results than NEH(D,TPLS). Figure 6.5 also shows that the performance of NEH(D,TPLS) for variable sublots is relatively better than the performance of it for consistent sublots.

It is also important, particularly for practical cases, to compare the computation time of solution approaches. The computation time of NEH(D,TPLS) is negligible when compared with the computation time of other solution approaches. On the other hand, MIP and TS based heuristic spend comparable times to obtain the results. The TS based heuristic generally spends less computation time than MIP, in average. The TS based heuristic reaches its results in 90.82 %, 81.18 % and 131.96 % less time than MIP for the three investigated problems, respectively. Therefore, it can be said that TS based heuristic not only generates better results but also gives these results in less computation times than MIP model.

One may expect that, starting from NEH(D,TPLS) and applying TS based heuristic procedure should significantly improve the makespan rather than relatively small improvements (1%-1.5%). This is probably caused by two reasons:

- The results of NEH(D,TPLS) are so good that TS based heuristic requires small improvements to get better results.

- TS based heuristic is quite inefficient in finding better results.

To evaluate these two cases, the LPT(TPT) sequencing heuristic, which gives the worst performance on the makespan, is selected as the initial sequence and TS based heuristic is then applied to obtain the makespan results. If TS based heuristic with LPT(TPT) initial sequence does not improve the makespan so much from the makespan values of LPT(TPT) sequence, then it can be said that TS based heuristic is quite inefficient in finding better results. Otherwise, we shall infer that NEH(D,TPLS) generates relatively better results and these results can be improved in only small proportions by TS based heuristic.

In the light of above discussion, TS based heuristic is applied starting from these two initial sequences for continuous sized consistent sublots. The detailed computational results are given in Appendix C4. Table 6.3 summarizes these results.

TS based heuristic improves NEH(D,TPLS) results by 1.74%, whereas improves LPT(TPT) sequence by 15.41%. Therefore, TS based heuristic works efficiently in finding better results. This also confirms that the tabu search parameters are selected appropriately. In addition, it can be said that the results of NEH(D,TPLS) are so good that TS based heuristic requires small improvements to get better results.

Table 6.3 Comparison of performance of TS based heuristic starting from NEH(D,TPLS) and LPT(TPT)

	Initial Sequence	**TS based Heuristic**		
	Avg. Makespan	Avg. Makespan	% improvement	Avg. Comp. Time
NEH (TPLS)	2196.23	2158.11	1.74	594.42
LPT(TPT)	2553.93	2160.34	15.41	776.11

Another discussion can be made on the results of TS based heuristic of two initial sequences. As seen from Table 6.3, TS based heuristic starting from both initial sequences generates almost the same makespan values in average. At this point, the computation times become significant. Since resulting average makespan value of NEH(D,TPLS), i.e., 2196.23, is better than the one of LPT(TPT), i.e., 2553.93; TS based heuristic with NEH(D,TPLS) requires less computation effort than with LPT(TPT). More specifically, applying TS based heuristic starting from

NEH(D,TPLS) sequence requires 30.57 % less computation time than starting from LPT(TPT).

6.4 Chapter Summary

This chapter has extended the work of Chapter 5 by introducing a TS based solution approach. The solution of NEH(D,TPLS) has been taken as initial sequence of TS based heuristic, since it generates better results for all types of investigated problems. However, TS based heuristic differs in terms of some parameter levels and the running steps of the algorithm (i.e., way of obtaining the makespan values of a given sequence) for each investigated problem.

To evaluate the proposed TS based heuristics, the results of NEH(D,TPLS) and MIP model have also been recorded. The following outcomes have been obtained.

For consistent sublots, the smaller problem instances can be optimally solved by MIP model under time limit whereas it fails to give optimal results as the problem size scales up. For large sized problems (i.e., 15 products), NEH(D,TPLS) gives comparable results with MIP model. When compared with the MIP model under time limit, the performance of TS based heuristic improves as the problem scales up. Furthermore, it gives these results in less computation times than MIP model.

For variable sublots, the efficiency of TS based heuristic becomes obvious in almost all problem instances. Moreover, the results of NEH(D,TPLS) becomes comparable with the ones of MIP model (under time limit) in not only for large sized but also small and medium sized test problems.

A final analysis has been performed to evaluate the efficiency of TS based heuristic. From this analysis, it has been concluded that;

- The results of NEH(D,TPLS) are so good that TS based heuristic requires small improvements to get better results.
- TS based heuristic is quite efficient in finding better results.

CHAPTER SEVEN
CONCLUSION

7.1 Summary of the Thesis

In this thesis, a class of LS problems in flow shops, which has not received much attention in literature, has been investigated. The main purpose of this thesis is to develop efficient solution algorithms for the investigated problems.

An LS problem has a number of characteristics varying in the production environment, i.e., sublot types, schedule structures etc. Comprehensive information on these characteristics has been given in the early stages of the thesis. Then, a comprehensive literature review on flow shop LS problems with time based objectives has been given. The research gaps and the LS problems, which have not received much attention in literature, are explored. In the light of this review, a single product multi machine LS problem in stochastic flow shops and three multi product multi machine LS problems in deterministic flow shops have been investigated. The common properties of these problems are the production environment (i.e., permutation flow shops) and objective function (i.e., minimizing makespan).

The single product multi machine LS problem is composed of discrete sized consistent sublots. Since even deterministic LS problems are NP-hard, it is rather difficult to model and solve stochastic systems. In this respect, a heuristic procedure, which tries to optimize the sublot sizes in stochastic flow shops, has been developed. The proposed algorithm is first evaluated on deterministic problems to see how well it performs and is, therefore, compared against the optimum values obtained by a MIP model developed by Biskup & Feldmann (2006). Since the results are very promising, i.e., the results of the heuristic are very close to optimal values, the proposed approach which is a combination of simulation and tabu search has then been applied to stochastic flow shops. The tabu search tries to explore the neighborhood for better solutions, whereas the simulation handles the stochastic behavior of the system and computes the necessary values. The results thus obtained have further been compared with those of OptQuest's which is a built-in optimization tool in ARENA simulation software. The proposed heuristic outperforms OptQuest. Therefore, it could easily be used to solve stochastic as well as deterministic LS problems in flow shop settings.

Three research problems in multi product, multi machine LS problems with non-intermingling case are then investigated. These problems differ from each other by the following characteristics:

- Continuous sized consistent sublots,
- Discrete sized consistent sublots and
- Continuous sized variable sublots.

The investigated LS problems are decomposed into a sequencing problem and a sublot allocation problem. For the sequencing problem, seven different sequencing heuristics widely used in the general flow shop scheduling have been selected. These sequencing heuristics have been modified according to LS properties and totally 15 different sequencing rules have been constructed and evaluated. If the sequence is given, only sublot allocation sub-problem remains. However, even with the given sequence, it is still difficult to find optimal sublot sizes in multi product LS problems. Therefore, solution approaches to get makespan values under a given sequence have been proposed. Particularly, the proposed solution approach for continuous sized variable sublots is novel in the LS literature.

For all investigated problem types, NEH(D,TPLS) heuristic gives not only more number of best results, but also produces rather close results to the best ones. The performance of this heuristic has been evaluated against the MIP model results. NEH(D,TPLS) generates rather close results for the consistent sublot cases and outperforms the MIP model for continuous sized variable sublot case. Further studies may consider NEH(D,TPLS) as a benchmark method to evaluate the performances of their approaches.

Evaluating only one sequence alternative may not be a good approximation method for small to medium sized LS problems. However, the benefit of the best sequencing heuristic can be carried out to a search procedure. Therefore, a TS based solution approach starting from the sequence of NEH(D,TPLS) has been proposed for three investigated multi product LS problems. The computational results show that TS based approach gives rather efficient results when compared with the ones of MIP models for all problem types.

7.2 Contributions

The research proposed in this thesis provides several contributions. This section presents the contributions with respect to problem types.

The contributions of single product LS problem are given in the following.

- A research problem of this thesis handles a single product LS problem in stochastic flow shops which is rarely studied in the LS literature although widely encountered in real life applications.

- The stochastic LS studies in the literature only analyze the performance of pre-determined experimental sublot sizes instead of optimizing the sublot sizes. As far as we know, no study so far, has proposed a heuristic search algorithm that finds discrete sublot sizes in stochastic flow shops. In this thesis, a hybrid heuristic approach that integrates TS and simulation is proposed. The stochastic behavior of the system is handled by simulation and the sublot size configurations are searched by tabu search meta-heuristic.

The contributions of multi product LS problem are given in the following.

- A research problem of this thesis deals with multi product LS problems with variable sublot types, which is one of the hardest cases in the LS literature. To the best of our knowledge, there exists only one study (i.e., Liu, Chen & Liu, 2006) for this class of problems.

- In this thesis, medium to large sized test instances of investigated problems are aimed to be solved. Most of real life applications require quite large problems to be solved. However, LS studies in the literature generally able to solve small to medium size problems.

- In this thesis, a number of simple but efficient sequencing heuristics developed for pure flow shops are modified according to the requirements of LS to handle the sequencing part of the multi product LS problem. The best one of these sequencing heuristics is proposed to be used in multi product LS problems. This proposed sequencing heuristic not only solves large sized LS problems in reasonable times but also get efficient results for small and medium sized LS problems in small computation times.

- Even with the given sequence, it is still difficult to find optimal sublot sizes in multi product LS problems due to sublot allocation sub-problem. The studies in the literature generally assume unit or equal sized sublots to eliminate the sublot allocation part of the multi product LS problem. In this thesis, solution approaches are proposed for each investigated research problem to obtain makespan values under a given sequence. Particularly, the proposed solution approach for continuous sized variable sublots is novel in the LS literature.

- Hybrid methods that utilize the complementary strengths of heuristic/meta-heuristic algorithms and MIP models may produce more efficient results. Therefore, proposed solution approaches in this thesis utilize the benefit of heuristic/meta-heuristic approaches in sequencing and of MIP models in sublot allocation. In addition, for variable sublot types, an alternative MIP model formulation is proposed based on the MIP models of Biskup & Feldmann (2006) and Feldmann & Biskup (2008).

7.3 Directions for Further Studies

Since, the LS problems have a number of characteristics; any change in these characteristics describes a different LS problem. Therefore, LS problems with complex characteristics are worth to study on.

Some of the future directions can be stated as follows.

- Other versions of research problems with either attached or detached setups can be studied.

- The proposed TS based solution approach for the single product LS problem in stochastic flow shops considers consistent sublots. The solution procedure may be extended to variable sublot types with some modifications.

- The proposed solution approaches proposed for the multi product LS problems deals with non-intermingling schedules. These approaches may be extended to intermingling schedules.

- The solution approach proposed for continuous sized variable sublots can be extended to solve the discrete sized variable sublots.

- The research problems can be extended to include transportation activities. At this point, the sequence of sublots in the transporter queue (i.e., the decision that which sublot is going to be transported) may become important in addition to the product sequencing decisions. Various transporter queue disciplines (e.g., first in first out, sublot with small size is first) can be generated to analyze their performances (see Edis & Ornek, 2009b).

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APPENDICES

APPENDIX A1. Comparison of tabu search based heuristic and optimum results, L = 50

L	***M***	***S***	**Ins. No**	**Processing times**										**Optimal Makespan (MIP)**	**TABU SEARCH - 10 STEP (TS_10)** Sublot Sizes												**TABU SEARCH - 20 STEP (TS_20)** Sublot Sizes												**TABU SEARCH - 50 STEP (TS_50)** Sublot Sizes											
				1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Makespan**	**Prop. Dev. (%)**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Makespan**	**Prop. Dev. (%)**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Makespan**	**Prop. Dev. (%)**
50	5	5	1	2	4	16	14	8						1032	5	14	12	11	8						1032	0.00	5	15	13	11	6						1032	0.00	5	15	13	11	6						1032	0.00
50	5	5	2	14	8	2	15	19						1262	8	9	10	11	12						1262	0.00	8	9	10	11	12						1262	0.00	8	9	10	11	12						1262	0.00
50	5	5	3	12	20	13	9	11						1342	9	14	11	9	7						1342	0.00	9	14	11	9	7						1342	0.00	9	14	11	9	7						1342	0.00
50	5	5	4	15	2	11	13	16						1210	10	12	10	10	8						1210	0.00	10	12	10	10	8						1210	0.00	10	12	10	10	8						1210	0.00
50	5	5	5	10	5	1	18	8						1052	6	9	13	15	7						1052	0.00	5	8	12	17	8						1052	0.00	5	8	12	17	8						1052	0.00
50	5	8	1	2	4	16	14	8						912	3	10	8	8	7	6	5	3			912	0.00	3	10	8	8	7	6	5	3			912	0.00	3	10	8	8	7	6	5	3			912	0.00
50	5	8	2	14	8	2	15	19						1125	5	5	6	7	6	7	9	5			1145	1.78	5	5	6	7	7	7	3	10			1145	1.78	5	5	6	7	7	3	7	10			1145	1.78
50	5	8	3	12	20	13	9	11						1173	7	11	9	6	5	5	4	3			1189	1.36	5	9	10	8	6	5	4	3			1173	0.00	5	9	10	8	6	5	4	3			1173	0.00
50	5	8	4	15	2	11	13	16						1051	6	6	6	6	6	6	7	7			1051	0.00	6	6	6	6	6	6	7	7			1051	0.00	6	6	6	6	6	6	7	7			1051	0.00
50	5	8	5	10	5	1	18	8						964	5	7	10	6	13	5	3	1			994	3.11	5	7	10	14	7	4	2	1			988	2.49	5	7	11	11	9	4	2	1			988	2.49
50	5	10	1	2	4	16	14	8						884	3	8	8	7	6	5	4	4	3	2	884	0.00	3	8	8	7	6	5	4	4	3	2	884	0.00	1	3	9	8	7	6	5	5	4	2	884	0.00
50	5	10	2	14	8	2	15	19						1076	4	4	5	5	6	7	6	8	4	1	1106	2.79	4	4	5	5	6	7	7	6	5	1	1106	2.79	4	4	5	5	6	7	7	2	5	5	1106	2.79
50	5	10	3	12	20	13	9	11						1117	4	6	5	9	7	6	4	4	3	2	1135	1.61	4	6	10	8	6	5	4	3	2	2	1117	0.00	4	6	10	8	6	5	4	3	2	2	1117	0.00
50	5	10	4	15	2	11	13	16						1002	4	5	5	5	5	5	5	5	5	6	1002	0.00	4	5	5	5	5	5	5	5	5	6	1002	0.00	4	5	5	5	5	5	5	5	5	6	1002	0.00
50	5	10	5	10	5	1	18	8						940	4	6	8	5	5	9	7	3	2	1	972	3.40	4	6	8	5	6	12	5	2	1	1	972	3.40	4	6	8	5	9	10	3	2	2	1	972	3.40
50	7	5	1	8	16	3	18	19	13	16				1655	9	10	10	11	10						1655	0.00	9	10	10	11	10						1655	0.00	9	10	10	11	10						1655	0.00
50	7	5	2	16	4	17	2	11	18	14				1526	10	10	10	11	9						1526	0.00	10	10	10	11	9						1526	0.00	10	10	10	11	9						1526	0.00
50	7	5	3	1	11	18	13	10	11	12				1410	8	12	11	10	9						1410	0.00	8	12	11	10	9						1410	0.00	8	12	11	10	9						1410	0.00
50	7	5	4	17	3	11	15	14	2	3				1284	11	11	10	9	9						1284	0.00	11	11	10	9	9						1284	0.00	11	11	10	9	9						1284	0.00
50	7	5	5	11	14	3	15	14	14	18				1547	9	9	10	11	11						1547	0.00	9	9	10	11	11						1547	0.00	9	9	10	11	11						1547	0.00
50	7	8	1	8	16	3	18	19	13	16				1377	5	6	6	7	7	7	6	6			1377	0.00	5	6	6	7	7	7	6	6			1377	0.00	5	6	6	7	7	7	6	6			1377	0.00
50	7	8	2	16	4	17	2	11	18	14				1284	6	6	6	6	6	7	7	6			1284	0.00	6	6	6	6	6	7	7	6			1284	0.00	6	6	6	6	6	7	7	6			1284	0.00
50	7	8	3	1	11	18	13	10	11	12				1178	4	6	9	8	7	6	5	5			1178	0.00	4	6	9	8	7	6	5	5			1178	0.00	4	6	9	8	7	6	5	5			1178	0.00
50	7	8	4	17	3	11	15	14	2	3				1103	8	7	7	6	6	6	5	5			1103	0.00	8	7	7	6	6	6	5	5			1103	0.00	8	7	7	6	6	6	5	5			1103	0.00
50	7	8	5	11	14	3	15	14	14	18				1275	5	5	6	6	6	7	7	8			1275	0.00	5	5	6	6	6	7	7	8			1275	0.00	5	5	6	6	6	7	7	8			1275	0.00
50	7	10	1	8	16	3	18	19	13	16				1284	4	5	5	6	6	6	5	5	4	4	1284	0.00	4	5	5	6	6	6	5	5	4	4	1284	0.00	4	5	5	6	6	6	5	5	4	4	1284	0.00
50	7	10	2	16	4	17	2	11	18	14				1203	4	5	5	5	5	5	6	6	5	4	1203	0.00	4	5	5	5	5	5	6	6	5	4	1203	0.00	4	5	5	5	5	5	6	6	5	4	1203	0.00
50	7	10	3	1	11	18	13	10	11	12				1103	5	8	7	6	5	5	4	4	3	3	1108	0.45	3	5	8	7	6	5	5	4	4	3	1103	0.00	3	5	8	7	6	5	5	4	4	3	1103	0.00
50	7	10	4	17	3	11	15	14	2	3				1043	7	6	6	5	5	5	4	4	4	4	1043	0.00	7	6	6	5	5	5	4	4	4	4	1043	0.00	7	6	6	5	5	5	4	4	4	4	1043	0.00
50	7	10	5	11	14	3	15	14	14	18				1191	3	4	5	5	5	6	6	5	4	7	1207	1.34	3	4	5	5	5	6	5	4	6	7	1207	1.34	3	4	4	5	5	5	5	6	6	7	1191	0.00
50	10	5	1	3	15	16	3	8	10	11	1	9	13	1508	10	11	10	10	9						1508	0.00	10	11	10	10	9						1508	0.00	10	11	10	10	9						1508	0.00
50	10	5	2	8	19	6	3	20	4	13	15	8	7	1778	10	11	11	10	8						1778	0.00	10	11	11	10	8						1778	0.00	10	11	11	10	8						1778	0.00
50	10	5	3	4	1	16	10	7	12	2	1	6	20	1531	9	9	10	11	11						1531	0.00	9	9	10	11	11						1531	0.00	9	9	10	11	11						1531	0.00
50	10	5	4	6	7	2	20	12	11	8	15	1	11	1643	8	12	11	10	9						1643	0.00	8	12	11	10	9						1643	0.00	8	12	11	10	9						1643	0.00
50	10	5	5	9	14	2	15	20	17	9	1	11	9	1758	9	11	12	10	8						1758	0.00	9	11	12	10	8						1758	0.00	9	11	12	10	8						1758	0.00
50	10	8	1	3	15	16	3	8	10	11	1	9	13	1233	6	7	7	7	6	6	6	5			1233	0.00	6	7	7	7	6	6	6	5			1233	0.00	6	7	7	7	6	6	6	5			1233	0.00
50	10	8	2	8	19	6	3	20	4	13	15	8	7	1461	4	8	8	8	7	6	5	4			1461	0.00	4	8	8	8	7	6	5	4			1461	0.00	4	8	8	8	7	6	5	4			1461	0.00
50	10	8	3	4	1	16	10	7	12	2	1	6	20	1309	5	5	6	6	6	7	7	8			1309	0.00	5	5	6	6	6	7	7	8			1309	0.00	5	5	6	6	6	7	7	8			1309	0.00
50	10	8	4	6	7	2	20	12	11	8	15	1	11	1357	5	9	8	7	6	6	5	4			1357	0.00	5	9	8	7	6	6	5	4			1357	0.00	5	9	8	7	6	6	5	4			1357	0.00
50	10	8	5	9	14	2	15	20	17	9	1	11	9	1427	5	6	7	9	8	6	5	4			1427	0.00	5	6	7	9	8	6	5	4			1427	0.00	5	6	7	9	8	6	5	4			1427	0.00
50	10	10	1	3	15	16	3	8	10	11	1	9	13	1146	5	5	6	6	5	5	5	5	4	4	1146	0.00	5	5	6	6	5	5	5	5	4	4	1146	0.00	5	5	6	6	5	5	5	5	4	4	1146	0.00
50	10	10	2	8	19	6	3	20	4	13	15	8	7	1354	4	6	6	6	6	6	5	4	4	3	1354	0.00	4	6	6	6	6	6	5	4	4	3	1354	0.00	4	6	6	6	6	6	5	4	4	3	1354	0.00
50	10	10	3	4	1	16	10	7	12	2	1	6	20	1236	4	4	4	4	5	5	5	6	6	7	1236	0.00	4	4	4	4	5	5	5	6	6	7	1236	0.00	4	4	4	4	5	5	5	6	6	7	1236	0.00
50	10	10	4	6	7	2	20	12	11	8	15	1	11	1267	3	6	7	7	6	5	5	4	4	3	1271	0.32	3	6	7	7	6	5	5	4	4	3	1271	0.32	3	6	7	7	6	5	5	4	4	3	1271	0.32
50	10	10	5	9	14	2	15	20	17	9	1	11	9	1303	3	4	5	6	7	7	6	5	4	3	1303	0.00	3	4	5	6	7	7	6	5	4	3	1303	0.00	3	4	5	6	7	7	6	5	4	3	1303	0.00

APPENDIX A2. Comparison of tabu search based heuristic and optimum results, L = 100

L	*M*	*S*	Ins. No	Processing times										Optimal Makespan (MIP)	TABU SEARCH - 10 STEP (TS_10)												TABU SEARCH - 20 STEP (TS_20)												TABU SEARCH - 50 STEP (TS_50)											
															Sublot Sizes										Makespan	Prop. Dev. (%)	Sublot Sizes										Makespan	Prop. Dev. (%)	Sublot Sizes										Makespan	Prop. Dev. (%)
				1	2	3	4	5	6	7	8	9	10		1	2	3	4	5	6	7	8	9	10			1	2	3	4	5	6	7	8	9	10			1	2	3	4	5	6	7	8	9	10		
100	5	5	1	2	4	16	14	8						2056	23	25	22	19	11						2074	0.88	23	25	22	19	11						2074	0.88	23	25	22	19	11						2074	0.88
100	5	5	2	14	8	2	15	19						2524	16	17	20	22	25						2524	0.00	16	17	20	22	25						2524	0.00	16	17	20	22	25						2524	0.00
100	5	5	3	12	20	13	9	11						2675	17	28	23	18	14						2675	0.00	17	28	23	18	14						2675	0.00	17	28	23	18	14						2675	0.00
100	5	5	4	15	2	11	13	16						2401	19	20	20	20	21						2401	0.00	19	20	20	20	21						2401	0.00	19	20	20	20	21						2401	0.00
100	5	5	5	10	5	1	18	8						2098	15	22	33	21	9						2118	0.95	15	22	33	21	9						2118	0.95	15	22	33	21	9						2118	0.95
100	5	8	1	2	4	16	14	8						1822	6	19	17	16	14	12	10	6			1824	0.11	6	20	18	16	14	12	9	5			1822	0.00	6	20	18	16	14	12	9	5			1822	0.00
100	5	8	2	14	8	2	15	19						2230	8	9	10	12	13	14	16	18			2233	0.13	8	9	10	11	13	15	16	18			2230	0.00	8	9	10	11	13	15	16	18			2230	0.00
100	5	8	3	12	20	13	9	11						2327	11	18	17	17	13	10	8	6			2360	1.42	10	16	21	16	13	10	8	6			2336	0.39	10	16	21	16	13	10	8	6			2336	0.39
100	5	8	4	15	2	11	13	16						2088	11	12	12	12	13	13	13	14			2088	0.00	11	12	12	12	13	13	13	14			2088	0.00	11	12	12	12	13	13	13	14			2088	0.00
100	5	8	5	10	5	1	18	8						1912	10	14	22	13	17	14	7	3			1986	3.87	10	14	22	14	22	11	5	2			1980	3.56	10	14	22	28	14	6	4	2			1976	3.35
100	5	10	1	2	4	16	14	8						1756	5	16	15	14	12	10	9	8	7	4	1758	0.11	5	17	15	13	12	10	9	8	7	4	1756	0.00	5	17	15	13	12	10	9	8	7	4	1756	0.00
100	5	10	2	14	8	2	15	19						2134	8	9	10	11	12	11	10	11	16	2	2212	3.66	8	9	10	11	12	11	10	16	11	2	2212	3.66	8	9	10	11	12	14	16	13	5	2	2212	3.66
100	5	10	3	12	20	13	9	11						2220	8	13	10	10	11	14	12	9	7	6	2294	3.33	8	13	10	15	15	12	9	7	6	5	2261	1.85	7	12	20	15	13	10	8	6	5	4	2220	0.00
100	5	10	4	15	2	11	13	16						1984	9	9	9	10	10	10	10	11	11	11	1984	0.00	9	9	9	10	10	10	10	11	11	11	1984	0.00	9	9	9	10	10	10	10	11	11	11	1984	0.00
100	5	10	5	10	5	1	18	8						1868	7	10	15	10	10	10	13	15	7	3	1938	3.75	7	10	15	10	10	10	21	10	5	2	1932	3.43	7	10	15	10	10	23	14	7	3	1	1928	3.21
100	7	5	1	8	16	3	18	19	13	16				3298	18	20	22	21	19						3298	0.00	19	21	22	20	18						3298	0.00	19	21	22	20	18						3298	0.00
100	7	5	2	16	4	17	2	11	18	14				3033	19	20	21	22	18						3033	0.00	19	20	21	22	18						3033	0.00	19	20	21	22	18						3033	0.00
100	7	5	3	1	11	18	13	10	11	12				2806	17	25	22	19	17						2806	0.00	17	25	22	19	17						2806	0.00	17	25	22	19	17						2806	0.00
100	7	5	4	17	3	11	15	14	2	3				2551	23	21	20	19	17						2551	0.00	23	21	20	19	17						2551	0.00	23	21	20	19	17						2551	0.00
100	7	5	5	11	14	3	15	14	14	18				3059	17	19	20	21	23						3059	0.00	17	19	20	21	23						3059	0.00	17	19	20	21	23						3059	0.00
100	7	8	1	8	16	3	18	19	13	16				2732	11	12	13	14	14	13	12	11			2732	0.00	11	12	13	14	14	13	12	11			2732	0.00	11	12	13	14	14	13	12	11			2732	0.00
100	7	8	2	16	4	17	2	11	18	14				2542	12	12	13	13	13	14	13	10			2548	0.24	12	12	13	13	13	14	13	10			2548	0.24	11	12	12	13	13	14	14	11			2542	0.00
100	7	8	3	1	11	18	13	10	11	12				2344	11	18	16	14	12	11	10	8			2344	0.00	11	18	16	14	12	11	10	8			2344	0.00	11	18	16	14	12	11	10	8			2344	0.00
100	7	8	4	17	3	11	15	14	2	3				2183	16	15	14	13	12	11	10	9			2183	0.00	16	15	14	13	12	11	10	9			2183	0.00	16	15	14	13	12	11	10	9			2183	0.00
100	7	8	5	11	14	3	15	14	14	18				2523	9	10	11	12	13	14	15	16			2523	0.00	9	10	11	12	13	14	15	16			2523	0.00	9	10	11	12	13	14	15	16			2523	0.00
100	7	10	1	8	16	3	18	19	13	16				2537	5	9	10	11	11	12	12	11	10	9	2539	0.08	5	9	10	11	11	12	12	11	10	9	2539	0.08	5	9	10	11	11	12	12	11	10	9	2539	0.08
100	7	10	2	16	4	17	2	11	18	14				2384	9	9	10	10	10	11	11	12	10	8	2384	0.00	9	9	10	10	10	11	11	12	10	8	2384	0.00	9	9	10	10	10	11	11	12	10	8	2384	0.00
100	7	10	3	1	11	18	13	10	11	12				2197	8	13	10	13	12	11	10	8	8	7	2222	1.14	9	14	14	12	11	10	9	8	7	6	2198	0.05	5	9	14	14	13	11	10	9	8	7	2197	0.00
100	7	10	4	17	3	11	15	14	2	3				2068	13	13	12	11	10	10	9	8	8	6	2068	0.00	13	13	12	11	10	10	9	8	8	6	2068	0.00	13	13	12	11	10	10	9	8	8	6	2068	0.00
100	7	10	5	11	14	3	15	14	14	18				2349	7	9	10	10	11	12	12	13	11	5	2414	2.77	7	9	10	10	11	12	12	12	3	14	2414	2.77	7	9	10	10	11	12	12	9	13	7	2414	2.77
100	10	5	1	3	15	16	3	8	10	11	1	9	13	2987	22	21	20	19	18						2987	0.00	22	21	20	19	18						2987	0.00	22	21	20	19	18						2987	0.00
100	10	5	2	8	19	6	3	20	4	13	15	8	7	3532	21	22	22	19	16						3532	0.00	21	22	22	19	16						3532	0.00	21	22	22	19	16						3532	0.00
100	10	5	3	4	1	16	10	7	12	2	1	6	20	3043	17	19	20	21	23						3043	0.00	17	19	20	21	23						3043	0.00	17	19	20	21	23						3043	0.00
100	10	5	4	6	7	2	20	12	11	8	15	1	11	3286	20	23	21	19	17						3293	0.21	20	23	21	19	17						3293	0.21	20	23	21	19	17						3293	0.21
100	10	5	5	9	14	2	15	20	17	9	1	11	9	3516	18	22	24	20	16						3516	0.00	18	22	24	20	16						3516	0.00	18	22	24	20	16						3516	0.00
100	10	8	1	3	15	16	3	8	10	11	1	9	13	2441	13	14	14	13	12	12	11	11			2441	0.00	13	14	14	13	12	12	11	11			2441	0.00	13	14	14	13	12	12	11	11			2441	0.00
100	10	8	2	8	19	6	3	20	4	13	15	8	7	2901	13	14	14	15	14	12	10	8			2901	0.00	13	14	14	15	14	12	10	8			2901	0.00	13	14	14	15	14	12	10	8			2901	0.00
100	10	8	3	4	1	16	10	7	12	2	1	6	20	2581	9	10	11	12	13	14	15	16			2581	0.00	9	10	11	12	13	14	15	16			2581	0.00	9	10	11	12	13	14	15	16			2581	0.00
100	10	8	4	6	7	2	20	12	11	8	15	1	11	2700	8	15	16	15	13	12	11	10			2700	0.00	8	15	16	15	13	12	11	10			2700	0.00	8	15	16	15	13	12	11	10			2700	0.00
100	10	8	5	9	14	2	15	20	17	9	1	11	9	2827	12	14	17	16	14	11	9	7			2851	0.85	10	12	14	17	16	13	10	8			2827	0.00	10	12	14	17	16	13	10	8			2827	0.00
100	10	10	1	3	15	16	3	8	10	11	1	9	13	2257	10	11	11	11	11	10	10	9	9	8	2257	0.00	10	11	11	11	11	10	10	9	9	8	2257	0.00	10	11	11	11	11	10	10	9	9	8	2257	0.00
100	10	10	2	8	19	6	3	20	4	13	15	8	7	2692	6	12	12	12	13	12	10	9	8	6	2694	0.07	5	12	12	13	13	12	10	9	8	6	2692	0.00	5	12	12	13	13	12	10	9	8	6	2692	0.00
100	10	10	3	4	1	16	10	7	12	2	1	6	20	2441	6	8	8	9	9	10	11	12	13	14	2442	0.04	7	8	8	9	9	10	11	12	13	13	2441	0.00	7	8	8	9	9	10	11	12	13	13	2441	0.00
100	10	10	4	6	7	2	20	12	11	8	15	1	11	2504	6	10	15	13	12	11	10	9	8	6	2507	0.12	5	10	15	13	12	11	10	9	8	7	2504	0.00	5	10	15	13	12	11	10	9	8	7	2504	0.00
100	10	10	5	9	14	2	15	20	17	9	1	11	9	2606	8	10	12	14	15	12	10	8	6	5	2620	0.54	8	10	12	14	15	12	10	8	6	5	2620	0.54	8	10	12	14	15	12	10	8	6	5	2620	0.54

APPENDIX A3. Comparison of tabu search based heuristic and optimum results, L = 150

L	*M*	*S*	Ins. No	Processing times										Optimal Makespan (MIP)	TABU SEARCH - 10 STEP (TS_10)												TABU SEARCH - 20 STEP (TS_20)												TABU SEARCH - 50 STEP (TS_50)											
				1	2	3	4	5	6	7	8	9	10		Sublot Sizes										Makespan	Prop. Dev. (%)	Sublot Sizes										Makespan	Prop. Dev. (%)	Sublot Sizes										Makespan	Prop. Dev. (%)
															1	2	3	4	5	6	7	8	9	10			1	2	3	4	5	6	7	8	9	10			1	2	3	4	5	6	7	8	9	10		
150	5	5	1	2	4	16	14	8						3080	38	36	32	28	16						3116	1.17	32	36	32	28	22						3116	1.17	32	36	32	28	22						3116	1.17
150	5	5	2	14	8	2	15	19						3775	24	27	30	32	37						3786	0.29	23	26	30	33	38						3775	0.00	23	26	30	33	38						3775	0.00
150	5	5	3	12	20	13	9	11						4011	23	38	37	29	23						4035	0.60	26	43	34	26	21						4011	0.00	26	43	34	26	21						4011	0.00
150	5	5	4	15	2	11	13	16						3589	29	29	30	31	31						3589	0.00	29	29	30	31	31						3589	0.00	29	29	30	31	31						3589	0.00
150	5	5	5	10	5	1	18	8						3140	23	34	51	29	13						3172	1.02	23	34	51	29	13						3172	1.02	23	34	51	29	13						3172	1.02
150	5	8	1	2	4	16	14	8						2730	9	30	26	23	20	18	15	9			2730	0.00	9	30	26	23	20	18	15	9			2730	0.00	9	30	26	23	20	18	15	9			2730	0.00
150	5	8	2	14	8	2	15	19						3332	14	16	18	20	20	19	22	21			3404	2.16	14	16	18	20	20	23	28	11			3404	2.16	14	16	18	20	22	24	10	26			3404	2.16
150	5	8	3	12	20	13	9	11						3483	17	28	19	23	22	16	14	11			3567	2.41	17	28	24	25	19	15	12	10			3534	1.46	17	29	29	23	18	14	11	9			3509	0.75
150	5	8	4	15	2	11	13	16						3126	17	18	18	19	19	19	20	20			3126	0.00	17	18	18	19	19	19	20	20			3126	0.00	17	18	18	19	19	19	20	20			3126	0.00
150	5	8	5	10	5	1	18	8						2868	14	21	30	19	20	28	12	6			2972	3.63	14	21	30	19	28	22	11	5			2964	3.35	14	21	30	26	34	15	7	3			2950	2.86
150	5	10	1	2	4	16	14	8						2632	8	26	23	20	17	15	13	12	10	6	2632	0.00	8	26	23	20	17	15	13	12	10	6	2632	0.00	7	22	24	21	18	16	14	12	10	6	2632	0.00
150	5	10	2	14	8	2	15	19						3194	12	13	15	17	16	15	15	15	24	8	3318	3.88	12	13	15	17	16	15	15	20	25	2	3318	3.88	12	13	15	17	16	16	23	26	10	2	3318	3.88
150	5	10	3	12	20	13	9	11						3322	10	16	15	15	15	15	20	18	15	11	3516	5.84	10	16	15	15	15	21	20	16	12	10	3450	3.85	10	16	15	20	25	20	15	11	10	8	3384	1.87
150	5	10	4	15	2	11	13	16						2971	14	14	14	15	15	14	16	16	16	16	2974	0.10	14	14	14	14	15	14	16	16	16	17	2974	0.10	13	14	14	14	15	15	16	16	16	17	2971	0.00
150	5	10	5	10	5	1	18	8						2796	11	16	23	15	15	15	17	23	10	5	2916	4.29	11	16	23	15	15	15	24	19	8	4	2908	4.01	11	16	23	15	15	24	26	11	6	3	2900	3.72
150	7	5	1	8	16	3	18	19	13	16				4942	27	30	32	32	29						4942	0.00	27	30	32	32	29						4942	0.00	27	30	32	32	29						4942	0.00
150	7	5	2	16	4	17	2	11	18	14				4543	29	30	31	32	28						4543	0.00	29	30	31	32	28						4543	0.00	29	30	31	32	28						4543	0.00
150	7	5	3	1	11	18	13	10	11	12				4196	29	36	32	28	25						4202	0.14	29	36	32	28	25						4202	0.14	29	36	32	28	25						4202	0.14
150	7	5	4	17	3	11	15	14	2	3				3808	34	32	30	28	26						3808	0.00	34	32	30	28	26						3808	0.00	34	32	30	28	26						3808	0.00
150	7	5	5	11	14	3	15	14	14	18				4570	23	29	30	33	35						4605	0.77	26	28	30	32	34						4570	0.00	26	28	30	32	34						4570	0.00
150	7	8	1	8	16	3	18	19	13	16				4089	15	18	20	21	22	20	18	16			4089	0.00	15	18	20	21	22	20	18	16			4089	0.00	15	18	20	21	22	20	18	16			4089	0.00
150	7	8	2	16	4	17	2	11	18	14				3806	18	19	19	20	21	21	18	14			3812	0.16	17	18	19	19	20	21	20	16			3806	0.00	17	18	19	19	20	21	20	16			3806	0.00
150	7	8	3	1	11	18	13	10	11	12				3504	13	21	23	24	21	18	16	14			3546	1.20	13	20	26	23	20	18	16	14			3508	0.11	13	20	26	23	20	18	16	14			3508	0.11
150	7	8	4	17	3	11	15	14	2	3				3270	24	22	21	19	18	17	16	13			3270	0.00	24	22	21	19	18	17	16	13			3270	0.00	24	22	21	19	18	17	16	13			3270	0.00
150	7	8	5	11	14	3	15	14	14	18				3765	15	16	17	18	19	20	22	23			3765	0.00	15	16	17	18	19	20	22	23			3765	0.00	15	16	17	18	19	20	22	23			3765	0.00
150	7	10	1	8	16	3	18	19	13	16				3807	12	14	15	16	17	18	17	15	14	12	3808	0.03	12	14	15	16	17	18	17	15	14	12	3808	0.03	11	13	14	15	16	17	18	17	15	14	3807	0.00
150	7	10	2	16	4	17	2	11	18	14				3562	13	14	15	15	16	16	17	17	15	12	3564	0.06	13	14	14	15	15	16	16	17	17	13	3562	0.00	13	14	14	15	15	16	16	17	17	13	3562	0.00
150	7	10	3	1	11	18	13	10	11	12				3280	15	24	15	16	17	16	14	12	11	10	3340	1.83	13	22	21	19	17	15	13	11	10	9	3298	0.55	10	16	23	20	18	16	14	12	11	10	3280	0.00
150	7	10	4	17	3	11	15	14	2	3				3090	20	19	18	17	16	15	14	13	12	6	3090	0.00	20	19	18	17	16	15	14	13	12	6	3090	0.00	20	19	18	17	16	15	14	13	12	6	3090	0.00
150	7	10	5	11	14	3	15	14	14	18				3509	11	14	16	17	16	15	15	18	22	6	3681	4.90	11	14	16	17	16	15	19	21	19	2	3681	4.90	11	14	16	17	18	19	20	21	4	10	3681	4.90
150	10	5	1	3	15	16	3	8	10	11	1	9	13	4480	30	32	31	29	28						4480	0.00	30	32	31	29	28						4480	0.00	30	32	31	29	28						4480	0.00
150	10	5	2	8	19	6	3	20	4	13	15	8	7	5294	31	32	33	29	25						5294	0.00	31	32	33	29	25						5294	0.00	31	32	33	29	25						5294	0.00
150	10	5	3	4	1	16	10	7	12	2	1	6	20	4538	26	28	30	32	34						4538	0.00	26	28	30	32	34						4538	0.00	26	28	30	32	34						4538	0.00
150	10	5	4	6	7	2	20	12	11	8	15	1	11	4915	23	37	33	30	27						4921	0.12	23	37	33	30	27						4921	0.12	23	37	33	30	27						4921	0.12
150	10	5	5	9	14	2	15	20	17	9	1	11	9	5251	29	35	34	29	23						5286	0.67	26	31	37	31	25						5251	0.00	26	31	37	31	25						5251	0.00
150	10	8	1	3	15	16	3	8	10	11	1	9	13	3634	17	18	20	21	20	19	18	17			3656	0.61	17	18	20	21	20	19	18	17			3656	0.61	19	20	21	20	19	18	17	16			3634	0.00
150	10	8	2	8	19	6	3	20	4	13	15	8	7	4341	19	20	21	22	22	19	16	11			4347	0.14	19	20	21	22	22	19	16	11			4347	0.14	20	21	22	23	20	17	15	12			4341	0.00
150	10	8	3	4	1	16	10	7	12	2	1	6	20	3872	14	15	17	18	19	21	22	24			3872	0.00	14	15	17	18	19	21	22	24			3872	0.00	14	15	17	18	19	21	22	24			3872	0.00
150	10	8	4	6	7	2	20	12	11	8	15	1	11	4029	13	25	24	22	20	18	16	12			4029	0.00	13	25	24	22	20	18	16	12			4029	0.00	13	25	24	22	20	18	16	12			4029	0.00
150	10	8	5	9	14	2	15	20	17	9	1	11	9	4232	15	18	22	26	23	19	15	12			4232	0.00	15	18	22	26	23	19	15	12			4232	0.00	15	18	22	26	23	19	15	12			4232	0.00
150	10	10	1	3	15	16	3	8	10	11	1	9	13	3374	16	17	17	17	16	15	14	13	13	12	3374	0.00	16	17	17	17	16	15	14	13	13	12	3374	0.00	16	17	17	17	16	15	14	13	13	12	3374	0.00
150	10	10	2	8	19	6	3	20	4	13	15	8	7	4023	8	16	17	18	18	19	17	15	13	9	4033	0.25	8	16	17	18	18	19	17	15	13	9	4033	0.25	11	17	17	18	19	18	16	14	12	8	4025	0.05
150	10	10	3	4	1	16	10	7	12	2	1	6	20	3659	5	14	15	16	17	18	19	20	22	4	3761	2.79	5	14	15	16	17	18	19	21	13	12	3761	2.79	5	14	15	16	17	18	19	20	21	5	3761	2.79
150	10	10	4	6	7	2	20	12	11	8	15	1	11	3752	11	21	21	19	17	15	14	12	11	9	3757	0.13	11	21	21	19	17	15	14	12	11	9	3757	0.13	9	17	22	20	18	16	14	13	12	9	3753	0.03
150	10	10	5	9	14	2	15	20	17	9	1	11	9	3897	11	15	18	21	21	18	15	13	10	8	3915	0.46	11	14	17	20	22	19	16	13	10	8	3904	0.18	11	13	15	18	22	20	17	14	11	9	3897	0.00

APPENDIX A4. Comparison of the TS based results and OptQuest results in stochastic LS problems

L	*S*	*M*	Processing Times										At time of Minimum			At time of Tabu-10 Steps			At time of Tabu-20 Steps			At time of Tabu-30 Steps			At time of OptQuest	
													Completion Times		Time	Completion Times		Time	Completion Times		Time	Completion Times		Time	Comp. Time	Time
			1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Tabu**	**OptQuest**	**(min)**	**Tabu**	**OptQuest**	**(min)**	**Tabu**	**OptQuest**	**(min)**	**Tabu**	**OptQuest**	**(min)**	**OptQuest**	**(min)**
50	5	5	12	20	13	9	11						**1380.63**	1423.40	1.68	**1380.63**	1416.73	4.93	**1380.63**	1385.25	8.17	**1380.63**	1385.25	11.41	1385.25	13.67
100	5	5	12	20	13	9	11						2759.64	2829.28	2.62	2759.64	2787.19	5.87	2759.64	2761.12	9.11	**2759.19**	2759.64	12.35	**2759.19**	17.08
150	5	5	12	20	13	9	11						**4136.73**	4172.42	3.79	**4136.73**	4164.27	6.98	**4136.73**	4155.75	10.17	**4136.73**	4142.48	13.37	**4136.73**	26.14
50	8	5	12	20	13	9	11						1234.51	1284.34	8.29	**1233.21**	1257.53	17.27	**1233.21**	1244.15	26.37	**1233.21**	**1233.21**	35.41	**1233.21**	45.03
100	8	5	12	20	13	9	11						**2456.32**	2487.14	19.38	**2456.32**	2476.83	28.53	**2456.32**	2467.32	37.64	**2456.32**	2462.28	46.76	2462.28	51.38
150	8	5	12	20	13	9	11						**3680.94**	*	30.36	**3680.94**	*	39.33	**3680.94**	*	48.34	**3680.94**	*	57.34	3700.73	13.78
50	10	5	12	20	13	9	11						**1185.82**	*	20.49	**1185.82**	*	35.18	**1185.82**	*	49.87	**1185.82**	*	65.04	1195.54	19.98
100	10	5	12	20	13	9	11						2389.55	2392.81	21.32	2368.70	2381.84	36.56	**2361.51**	2373.69	51.76	**2361.51**	*	66.97	2373.69	55.76
150	10	5	12	20	13	9	11						3568.32	3580.22	36.14	3555.56	3569.90	50.66	3545.48	3568.31	65.20	**3542.08**	*	79.72	3568.31	66.00
50	5	7	8	16	3	18	19	13	16				**1745.52**	**1745.52**	0.81	**1745.52**	*	4.24	**1745.52**	*	7.77	**1745.52**	*	11.41	**1745.52**	3.50
100	5	7	8	16	3	18	19	13	16				**3486.22**	3491.04	0.79	**3486.22**	3487.76	4.49	**3486.22**	**3486.22**	8.22	**3486.22**	*	11.97	**3486.22**	8.82
150	5	7	8	16	3	18	19	13	16				**5224.62**	5236.55	1.17	**5224.62**	*	4.93	**5224.62**	*	8.69	**5224.62**	*	12.44	5236.55	3.34
50	8	7	8	16	3	18	19	13	16				**1423.37**	1427.05	4.24	**1423.37**	*	14.78	**1423.37**	*	51.34	**1423.37**	*	61.63	1427.05	7.20
100	8	7	8	16	3	18	19	13	16				**2840.83**	2864.51	6.32	**2840.83**	*	16.81	**2840.83**	*	27.34	**2840.83**	*	37.89	2864.51	9.45
150	8	7	8	16	3	18	19	13	16				**4259.55**	*	7.43	**4259.55**	*	18.02	**4259.55**	*	28.61	**4259.55**	*	39.22	4297.33	3.23
50	10	7	8	16	3	18	19	13	16				**1330.49**	*	5.21	**1330.49**	*	22.38	**1330.49**	*	39.53	**1330.49**	*	56.68	1343.09	3.23
100	10	7	8	16	3	18	19	13	16				**2650.38**	2662.21	8.63	**2650.38**	*	25.79	**2650.38**	*	42.95	**2650.38**	*	60.11	**2650.38**	18.64
150	10	7	8	16	3	18	19	13	16				**3975.72**	3983.16	13.75	**3975.72**	*	30.88	**3975.72**	*	48.02	**3975.72**	*	65.15	3979.74	18.66
50	5	10	3	15	16	3	8	10	11	1	9	13	**1415.08**	1436.92	1.55	**1415.08**	*	5.83	**1415.08**	*	10.18	**1415.08**	*	14.53	1436.92	3.24
100	5	10	3	15	16	3	8	10	11	1	9	13	**2825.55**	2873.84	2.66	**2825.55**	2827.55	7.04	**2825.55**	**2825.55**	11.46	**2825.55**	*	16.05	**2825.55**	12.81
150	5	10	3	15	16	3	8	10	11	1	9	13	4239.62	4308.65	3.14	**4236.48**	4265.05	7.59	**4236.48**	4238.40	12.03	**4236.48**	4238.40	16.54	**4236.48**	19.18
50	8	10	3	15	16	3	8	10	11	1	9	13	**1159.99**	1188.90	5.06	**1159.99**	*	17.61	**1159.99**	*	30.23	**1159.99**	*	42.90	1188.90	9.58
100	8	10	3	15	16	3	8	10	11	1	9	13	**2310.59**	2319.63	10.19	**2310.59**	*	22.85	**2310.59**	*	35.53	**2310.59**	*	48.23	2319.63	10.58
150	8	10	3	15	16	3	8	10	11	1	9	13	**3461.37**	3481.90	13.96	**3461.37**	*	26.64	**3461.37**	*	39.30	**3461.37**	*	52.01	3461.91	23.73
50	10	10	3	15	16	3	8	10	11	1	9	13	**1082.53**	1095.21	12.31	**1082.53**	*	32.77	**1082.53**	*	53.20	**1082.53**	*	73.67	1095.21	12.43
100	10	10	3	15	16	3	8	10	11	1	9	13	**2148.17**	2163.48	22.48	**2148.17**	*	42.85	**2148.17**	*	63.20	**2148.17**	*	83.54	2163.48	23.75
150	10	10	3	15	16	3	8	10	11	1	9	13	**3223.37**	*	32.29	**3223.37**	*	52.38	**3223.37**	*	72.83	**3223.37**	*	93.47	**3223.37**	30.30
															10.97			21.60			33.22			43.92		19.65

APPENDIX A5 Comparison of the performances of TS_30 and optimum deterministic solutions applied in stochastic case

L	*S*	*M*	**Processing Times**										**Deterministic Case**													**Stochastic Case**		
													Optimum Sublot Sizes										**Optimum**	**TS_50**	**Prop.Dev (%)**	**Optimum Deterministic**	**TS_30**	**Prop.Dev (%)**
			1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**						
50	5	5	12	20	13	9	11						9	14	11	9	7						1342	1342	0.00	1386.00	1380.63	0.39
50	5	7	8	16	3	18	19	13	16				9	10	10	11	10						1655	1655	0.00	1746.92	1745.52	0.08
50	5	10	3	15	16	3	8	10	11	1	9	13	10	11	10	10	9						1508	1508	0.00	1423.47	1415.08	0.59
50	8	5	12	20	13	9	11						5	9	10	8	6	5	4	3			1173	1173	0.00	1236.36	1233.21	0.26
50	8	7	8	16	3	18	19	13	16				5	6	6	7	7	7	6	6			1377	1377	0.00	1425.82	1423.37	0.17
50	8	10	3	15	16	3	8	10	11	1	9	13	6	7	7	7	6	6	6	5			1233	1233	0.00	1172.76	1159.99	1.10
50	10	5	12	20	13	9	11						4	6	10	8	6	5	4	3	2	2	1117	1117	0.00	1193.24	1185.82	0.63
50	10	7	8	16	3	18	19	13	16				4	5	5	6	6	6	5	5	4	4	1284	1284	0.00	1333.25	1330.49	0.21
50	10	10	3	15	16	3	8	10	11	1	9	13	5	5	6	6	5	5	5	5	4	4	1146	1146	0.00	1100.83	1082.53	1.69
100	5	5	12	20	13	9	11						17	28	23	18	14						2675	2675	0.00	2777.10	2759.19	0.65
100	5	7	8	16	3	18	19	13	16				19	21	22	20	18						3298	3298	0.00	3528.93	3486.22	1.23
100	5	10	3	15	16	3	8	10	11	1	9	13	22	21	20	19	18						2987	2987	0.00	2845.79	2825.55	0.72
100	8	5	12	20	13	9	11						8	13	22	17	14	11	8	7			2327	2336	0.39	2490.82	2456.32	1.40
100	8	7	8	16	3	18	19	13	16				11	12	13	14	14	13	12	11			2732	2732	0.00	2849.98	2840.83	0.32
100	8	10	3	15	16	3	8	10	11	1	9	13	13	14	14	13	12	12	11	11			2441	2441	0.00	2359.75	2310.59	2.13
100	10	5	12	20	13	9	11						7	12	20	15	13	10	8	6	5	4	2220	2220	0.00	2380.59	2361.51	0.81
100	10	7	8	16	3	18	19	13	16				8	9	10	11	12	12	11	10	9	8	2537	2539	0.08	2658.89	2650.38	0.32
100	10	10	3	15	16	3	8	10	11	1	9	13	10	11	11	11	11	10	10	9	9	8	2257	2257	0.00	2195.03	2148.17	2.18
150	5	5	12	20	13	9	11						26	43	34	26	21						4011	4011	0.00	4178.79	4136.73	1.02
150	5	7	8	16	3	18	19	13	16				27	30	32	32	29						4942	4942	0.00	5264.70	5224.62	0.77
150	5	10	3	15	16	3	8	10	11	1	9	13	30	32	31	29	28						4480	4480	0.00	4280.27	4236.48	1.03
150	8	5	12	20	13	9	11						12	20	33	26	20	16	13	10			3483	3509	0.75	3734.87	3680.94	1.47
150	8	7	8	16	3	18	19	13	16				15	18	20	21	22	20	18	16			4089	4089	0.00	4288.17	4259.55	0.67
150	8	10	3	15	16	3	8	10	11	1	9	13	19	20	21	20	19	18	17	16			3634	3634	0.00	3523.17	3461.37	1.79
150	10	5	12	20	13	9	11						7	12	20	29	23	18	14	11	9	7	3322	3384	1.87	3592.48	3542.08	1.42
150	10	7	8	16	3	18	19	13	16				11	13	14	15	16	17	18	17	15	14	3807	3807	0.00	4018.20	3975.72	1.07
150	10	10	3	15	16	3	8	10	11	1	9	13	16	17	17	17	16	15	14	13	13	12	3374	3374	0.00	3291.10	3223.37	2.10
													Average										2609.30	2612.96	0.11	2676.94	2649.49	0.97

APPENDIX B1 An Example for the Rounding Algorithms

Suppose that, production lot size L = 16, number of sublots S = 4 and the resulting continuous sizes of sublots are $x^c = \{2.8, 3.2, 5.4, 4.6\}$.

Steps of Forward Rounding Algorithm of Chen and Steiner (1997)

Step 1. $u = L - \sum_{s=1}^{S} \lfloor x_s^c \rfloor$, $u = 16 - (2+3+5+4) = 2$.

Step 2. For the first u sublots which are not integer, set $x_s^d = \lceil x_s^c \rceil$, then for the first two sublots, $x_1^d = \lceil 2.8 \rceil = 3$ and $x_2^d = \lceil 3.2 \rceil = 4$.

For the rest of the sublots, set $x_s^d = \lfloor x_s^c \rfloor$, then for the remaining two sublots, $x_3^d = \lfloor 5.4 \rfloor = 5$ and $x_4^d = \lfloor 4.6 \rfloor = 4$.

The resulting discrete sized sublots are $x^d = \{3, 4, 5, 4\}$.

Steps of Rounding Algorithm of Sriskandarajah and Wagneur (1999)

Step 1. Set $W_0 = 0$, $W_1 = L$ and $\Gamma = \emptyset$, then $W_1 = 16$.

Step 2. For s =1 to S do

{ $x_s^d = \lfloor x_s^c \rfloor + 1$

$W_0 = W_0 + x_s^d$ }

For s =1, $x_1^d = \lfloor 2.8 \rfloor + 1 = 3$, $W_0 = 0 + 3 = 3$

For s = 2, $x_2^d = \lfloor 3.2 \rfloor + 1 = 4$, $W_0 = 3 + 4 = 7$

For s = 3, $x_3^d = \lfloor 5.4 \rfloor + 1 = 6$, $W_0 = 7 + 6 = 13$

For s = 4, $x_4^d = \lfloor 4.6 \rfloor + 1 = 5$, $W_0 = 13 + 5 = 18$

Step 3. $W_0 = W_0 - W_1$

find the product set Γ for which $x_s^d > 1$

Then, $W_0 = 18 - 16 = 2$ and $\Gamma = \{1,2,3,4\}$

Step 4. While $W_0 > 0$ do

{find $d_s = x_s^d - x_s^c$, $s \in \Gamma$

find r such that $d_r = \max_{s \in \Gamma} \{d_s\}$

$x_r^d = x_r^d$ -1

if x_r^d =1, then $\Gamma = \Gamma$-{r}

$W_0 = W_0$ -1}

Since W_0 =2 > 0,

$d_1 = 3 - 2.8 = 0.2$; $d_2 = 4 – 3.2 = 0.8$; $d_3 = 6 – 5.4 = 0.6$; $d_4 = 5 – 4.6 = 0.4$

$d_r = \max\{0.2, 0.8, 0.6, 0.4\} = 0.8 = d_2$, then $r = 2$, x_2^d = 4 -1=3, W_0 = 2-1 = 1.

Since W_0 =1 > 0 now,

$d_1 = 3 - 2.8 = 0.2$; $d_2 = 3 – 3.2 = -0.2$; $d_3 = 6 – 5.4 = 0.6$; $d_4 = 5 – 4.6 = 0.4$

$d_r = \max\{0.2, -0.2, 0.6, 0.4\} = 0.6 = d_3$, then $r = 3$, x_3^d = 6 -1=5, W_0 = 1-1 = 0.

Since W_0 = 0 > 0 does not hold, STOP.

The resulting discrete sized sublots are $x^d = \{3, 3, 5, 5\}$.

APPENDIX B2 Computational results of sequencing heuristics for discrete sized consistent sublots with the rounding algorithm of Chen and Steiner (1997)

# of products	Maximum # of sublots	# of machines	Instance No	LPT (TPT)	SPT (TPT)	LPT (TPLS)	SPT (TPLS)	NEH (D,TPT)	NEH (I,TPT)	NEH (D,TPLS)	NEH (I,TPLS)	CDS (ORJ)	CDS (PLS)	PALMER (ORJ)	PALMER (PLS)	GUPTA (ORJ)	GUPTA (PLS)	BMI (ORJ)	Best Makespan
5	5	5	1	1023	**888**	1105	1202	983	943	901	915	937	988	1205	1116	1137	1032	1087	888
5	5	5	2	925	997	969	964	933	**864**	**864**	875	877	877	996	996	992	995	999	864
5	5	5	3	1592	1503	1573	1547	**1430**	1482	**1430**	1482	1473	1473	1546	1480	1592	1570	**1430**	1430
5	5	5	4	967	852	880	853	**763**	**763**	**763**	847	803	803	1012	1012	919	887	808	763
5	5	5	5	1725	1498	1694	1570	1400	1388	**1365**	1388	1454	1457	1826	1826	1744	1739	1454	1365
5	5	10	1	1812	1775	1694	1841	1718	1694	1718	1694	1703	**1687**	1868	1868	1750	1728	1721	1687
5	5	10	2	1924	1699	1924	**1698**	1703	1703	1703	1703	1719	1703	1858	1858	1936	1892	1747	1698
5	5	10	3	1438	1436	1433	1451	**1286**	**1286**	**1286**	1310	**1286**	1351	1545	1448	1518	1518	1376	1286
5	5	10	4	1575	1584	1590	1569	**1368**	1395	**1368**	1387	1389	**1368**	1704	1640	1547	1625	1511	1368
5	5	10	5	1476	1540	1527	1492	**1319**	1326	1361	1383	**1319**	**1319**	1549	1496	1449	1480	1527	1319
5	10	5	1	1000	860	1064	1171	957	**850**	**850**	863	904	958	1173	1085	1102	993	1028	850
5	10	5	2	882	956	916	913	890	829	**828**	840	**828**	**828**	937	937	937	957	948	828
5	10	5	3	1492	1416	1474	1457	**1351**	**1351**	**1351**	**1351**	1386	1386	1465	1422	1492	1471	**1351**	1351
5	10	5	4	936	841	856	808	**749**	**749**	**749**	808	777	777	985	985	870	854	802	749
5	10	5	5	1683	1433	1652	1506	1361	1432	**1325**	1356	1421	1412	1741	1741	1694	1705	1421	1325
5	10	10	1	1679	1618	1518	1676	1522	1529	1522	1512	**1509**	1529	1704	1704	1584	1564	1512	1509
5	10	10	2	1688	1544	1694	1536	**1508**	**1508**	**1508**	**1508**	1562	**1508**	1659	1659	1704	1667	1587	1508
5	10	10	3	1279	1322	1312	1308	**1189**	**1189**	**1189**	1203	**1189**	1248	1377	1321	1348	1348	1251	1189
5	10	10	4	1426	1451	1450	1471	**1241**	1326	**1241**	**1241**	**1241**	1245	1535	1492	1409	1455	1392	1241
5	10	10	5	1308	1350	1307	1314	1178	**1176**	**1176**	1199	1178	1178	1351	1305	1213	1297	1307	1176
10	5	5	1	2203	2322	2210	2226	**1970**	2002	1996	2002	1977	1977	2434	2453	2430	2377	2103	1970
10	5	5	2	2850	2371	2682	2703	2229	**2139**	2159	2168	2212	2198	2810	2788	2620	2641	2723	2139
10	5	5	3	2590	2325	2380	2296	**2135**	2185	2138	2235	2158	2182	2637	2602	2559	2554	2461	2135
10	5	5	4	2177	2148	2223	2181	1919	1962	**1916**	1957	1939	**1916**	2549	2556	2565	2567	2134	1916
10	5	5	5	2328	2397	2253	2582	2083	2037	2163	2106	2140	**2012**	2819	2678	2671	2770	2315	2012
10	5	10	1	3490	3465	3811	3613	3142	3212	**3131**	3247	3314	3252	3698	3715	3499	3569	3587	3131
10	5	10	2	3073	3064	2861	3130	2540	2619	2546	2619	2587	**2505**	3113	3098	3035	3037	2690	2505
10	5	10	3	3293	3395	3263	3222	2932	**2841**	2924	2965	2969	2955	3405	3460	3420	3373	3253	2841
10	5	10	4	2842	2996	2982	2815	**2443**	2616	2458	2458	2568	2566	2980	2922	2922	2988	2861	2443
10	5	10	5	2660	2521	2585	2541	2284	2302	**2274**	2336	2306	2282	2605	2611	2753	2656	2551	2274
10	10	5	1	2157	2283	2145	2179	**1959**	1969	1963	1977	1976	1976	2350	2391	2382	2315	2091	1959
10	10	5	2	2808	2334	2627	2650	2250	2125	2100	**2058**	2189	2124	2723	2713	2597	2596	2630	2058
10	10	5	3	2553	2256	2345	2256	**2060**	2132	2061	2134	2118	2118	2551	2535	2468	2469	2391	2060
10	10	5	4	2162	2068	2172	2095	**1915**	1934	1919	1916	**1915**	1919	2501	2515	2512	2512	2058	1915
10	10	5	5	2287	2354	2214	2533	2087	2048	2151	2195	2076	**1995**	2763	2653	2648	2739	2251	1995
10	10	10	1	3367	3321	3543	3489	3012	3056	**3011**	3152	3164	3108	3496	3512	3318	3354	3418	3011
10	10	10	2	2876	2786	2685	2852	2488	2436	2429	2520	2436	**2386**	2913	2919	2841	2803	2646	2386
10	10	10	3	3153	3187	3131	3033	2826	**2770**	2779	**2770**	2867	2846	3229	3270	3203	3182	3139	2770
10	10	10	4	2678	2816	2745	2636	2400	2381	**2291**	**2291**	2384	2396	2772	2684	2698	2732	2663	2291
10	10	10	5	2515	2365	2465	2404	2134	**2109**	2161	2197	2205	2127	2498	2457	2578	2495	2455	2109

APPENDIX B3 Computational results of sequencing heuristics for discrete sized consistent sublots (with rounding algorithm of Sriskandarajah and Wagneur, 1999)

# of products	Maximum # of sublots	# of machines	Instance No	LPT (TPT)	SPT (TPT)	LPT (TPLS)	SPT (TPLS)	NEH (D,TPT)	NEH (I,TPT)	NEH (D,TPLS)	NEH (I,TPLS)	CDS (ORJ)	CDS (PLS)	PALMER (ORJ)	PALMER (PLS)	GUPTA (ORJ)	GUPTA (PLS)	BMI (ORJ)	Best Makespan
5	5	5	1	1000	**889**	1097	1186	973	932	891	905	929	971	1040	1048	1040	929	1076	889
5	5	5	2	917	995	961	954	932	**867**	**867**	871	870	870	**867**	**867**	881	984	988	867
5	5	5	3	1580	1491	1567	1527	**1419**	1483	**1419**	1483	1461	1461	1544	1492	1491	1494	**1419**	1419
5	5	5	4	952	844	873	849	**753**	**753**	**753**	838	795	795	**753**	**753**	915	870	803	753
5	5	5	5	1718	1498	1690	1563	1401	1385	**1364**	1385	1452	1453	**1364**	**1364**	1553	1554	1452	1364
5	5	10	1	1797	1762	**1670**	1830	1714	**1670**	1714	**1670**	1673	1690	1734	1734	1770	1775	1698	1670
5	5	10	2	1910	1699	1910	**1695**	**1695**	**1695**	**1695**	**1695**	1713	**1695**	1739	1739	1713	1699	1746	1695
5	5	10	3	1419	1411	1402	1427	**1272**	**1272**	**1272**	1302	**1272**	1337	1347	1381	1337	1337	1356	1272
5	5	10	4	1571	1584	1596	1552	**1354**	1390	**1354**	1371	1372	**1354**	1466	1492	1625	1575	1501	1354
5	5	10	5	1459	1525	1513	1486	**1312**	1320	1346	1377	**1312**	**1312**	1356	1384	1418	1449	1513	1312
5	10	5	1	975	859	1052	1153	940	**839**	**839**	850	895	944	1027	1026	1027	895	1015	839
5	10	5	2	866	951	907	906	889	822	**819**	836	828	828	**819**	**819**	823	926	937	819
5	10	5	3	1472	1412	1455	1443	**1332**	**1332**	**1332**	**1332**	1380	1382	1447	1395	1412	1415	**1332**	1332
5	10	5	4	923	828	842	798	**741**	**741**	**741**	798	772	772	**741**	**741**	897	852	793	741
5	10	5	5	1679	1423	1640	1495	1365	1423	**1328**	1356	1408	1410	**1328**	**1328**	1494	1495	1408	1328
5	10	10	1	1640	1593	1500	1658	**1498**	1511	**1498**	1499	1499	1511	1561	1561	1620	1613	1499	1498
5	10	10	2	1667	1534	1666	1529	**1495**	**1495**	**1495**	**1495**	1531	**1495**	1573	1573	1516	1513	1574	1495
5	10	10	3	1263	1302	1281	1285	1167	**1163**	1167	1188	**1163**	1219	1246	1247	1225	1225	1237	1163
5	10	10	4	1406	1444	1427	1442	**1237**	1304	**1237**	**1237**	**1237**	1240	1308	1347	1496	1416	1377	1237
5	10	10	5	1287	1329	1275	1296	1171	**1167**	**1167**	1191	1171	1171	1221	1221	1248	1267	1275	1167
10	5	5	1	2196	2299	2207	2207	**1957**	1997	1992	1993	1970	1970	1982	2027	2027	1970	2100	1957
10	5	5	2	2829	2363	2673	2685	2227	**2129**	2154	2166	2189	2192	2329	2329	2332	2242	2711	2129
10	5	5	3	2581	2312	2375	2285	2127	2175	2126	2220	2148	2171	2158	**2120**	2184	2213	2448	2120
10	5	5	4	2176	2132	2223	2171	**1915**	1955	**1915**	1950	1934	**1915**	1989	1989	1934	**1915**	2113	1915
10	5	5	5	2313	2380	2244	2562	2077	2027	2158	2098	2123	**2007**	2082	2039	2203	2123	2304	2007
10	5	10	1	3479	3453	3795	3595	**3112**	3214	3128	3233	3297	3233	3397	3400	3575	3529	3576	3112
10	5	10	2	3047	3053	2845	3119	2516	2609	2522	2604	2569	**2488**	2645	2636	3019	2838	2682	2488
10	5	10	3	3288	3367	3256	3207	2919	**2836**	2912	2958	2946	2936	3000	2990	3352	3230	3233	2836
10	5	10	4	2830	2982	2965	2797	**2430**	2593	2451	2433	2555	2556	2629	2629	2824	2744	2840	2430
10	5	10	5	2639	2517	2546	2528	**2262**	2298	2272	2307	2311	2276	2406	2425	2589	2514	2534	2262
10	10	5	1	2155	2255	2141	2165	**1950**	1969	**1950**	1965	1966	1966	1967	2014	2014	1966	2086	1950
10	10	5	2	2791	2310	2619	2621	2239	2107	2088	**2049**	2166	2113	2314	2314	2262	2191	2617	2049
10	10	5	3	2541	2246	2327	2246	2052	2127	**2050**	2127	2100	2112	2101	2076	2130	2162	2377	2050
10	10	5	4	2154	2066	2170	2078	**1899**	1921	1902	1905	1903	**1899**	1925	1925	1903	1904	2040	1899
10	10	5	5	2268	2335	2192	2510	2070	2038	2138	2195	2052	**1983**	2065	2032	2202	2091	2240	1983
10	10	10	1	3324	3322	3550	3461	**2982**	3042	2994	3140	3138	3085	3203	3207	3359	3334	3373	2982
10	10	10	2	2843	2783	2654	2849	2452	2404	2399	2504	2420	**2365**	2496	2462	2817	2739	2616	2365
10	10	10	3	3127	3150	3115	3016	2812	2760	**2749**	2759	2846	2822	2858	2801	3185	3106	3115	2749
10	10	10	4	2657	2803	2719	2633	2394	2360	2284	**2264**	2375	2364	2481	2491	2651	2611	2641	2264
10	10	10	5	2492	2339	2443	2370	2128	**2089**	2151	2185	2197	2117	2307	2279	2458	2373	2418	2089

APPENDIX B4 Computational results of sequencing heuristics for continuous sized consistent sublots

# of products	Maximum # of sublots	# of machines	Instance No	LPT (TPT)	SPT (TPT)	LPT (TPLS)	SPT (TPLS)	NEH (D,TPT)	NEH (I,TPT)	NEH (D,TPLS)	NEH (I,TPLS)	CDS (ORJ)	CDS (PLS)	PALMER (ORJ)	PALMER (PLS)	GUPTA (ORJ)	GUPTA (PLS)	BMI (ORJ)	Best Makespan
5	5	5	1	991.91	**878.39**	1087.42	1172.18	961.24	921.46	879.46	893.46	917.56	963.91	1024.20	1035.17	1024.20	917.56	1064.88	**878.39**
5	5	5	2	906.37	992.14	950.33	943.10	922.89	**856.17**	**856.17**	861.04	856.68	856.68	**856.17**	**856.17**	873.07	973.07	977.412	**856.17**
5	5	5	3	1569.36	1484.07	1552.00	1515.63	**1403.69**	1460.16	**1403.69**	1460.16	1454.07	1454.07	1534.28	1482.28	1484.07	1484.07	**1403.69**	**1403.69**
5	5	5	4	937.17	829.20	860.92	842.12	**746.98**	**746.98**	**746.98**	831.48	791.63	791.63	**746.98**	**746.98**	901.05	860.94	795.926	**746.98**
5	5	5	5	1705.24	1484.61	1673.67	1550.14	1391.35	1375.71	**1355.67**	1375.71	1440.86	1443.49	**1355.67**	**1355.67**	1542.10	1542.10	1440.86	**1355.67**
5	5	10	1	1780.21	1741.05	**1653.07**	1800.16	1676.41	**1653.07**	1676.41	**1653.07**	**1653.07**	1659.26	1712.65	1712.65	1750.13	1756.17	1678	**1653.07**
5	5	10	2	1885.50	1679.98	1885.41	1676.68	**1675.07**	**1675.07**	**1675.07**	**1675.07**	1693.68	**1675.07**	1724.64	1724.64	1695.43	1677.17	1732.16	**1675.07**
5	5	10	3	1396.30	1391.86	1384.83	1405.32	**1254.63**	**1254.63**	**1254.63**	1287.00	**1254.63**	1310.12	1327.50	1362.08	1310.12	1310.12	1340.18	**1254.63**
5	5	10	4	1550.57	1559.25	1572.30	1537.85	**1341.58**	1368.81	**1341.58**	1358.30	1353.39	**1341.58**	1448.13	1475.20	1605.20	1552.05	1483.42	**1341.58**
5	5	10	5	1448.93	1413.35	1505.87	1456.14	**1291.99**	1293.60	1325.46	1355.69	**1291.99**	**1291.99**	1330.41	1346.82	1391.34	1412.96	1505.87	**1291.99**
5	10	5	1	961.47	846.16	1039.90	1137.54	928.99	**825.13**	**825.13**	837.51	880.58	933.47	1011.65	1015.46	1011.65	880.58	1001.36	**825.13**
5	10	5	2	852.56	943.28	891.80	892.77	876.74	**809.73**	**809.73**	823.24	813.42	813.42	**809.73**	**809.73**	813.51	914.04	922.329	**809.73**
5	10	5	3	1457.06	1400.67	1439.55	1425.64	**1317.62**	**1317.62**	**1317.62**	**1317.62**	1366.69	1370.67	1431.11	1379.11	1400.67	1400.67	**1317.62**	**1317.62**
5	10	5	4	906.14	811.47	828.79	790.08	**730.17**	**730.17**	**730.17**	790.08	761.72	761.72	**730.17**	**730.17**	882.12	836.97	780.928	**730.17**
5	10	5	5	1660.43	1414.28	1624.45	1477.74	1348.80	1408.55	**1311.21**	1342.95	1393.12	1396.91	**1311.21**	**1311.21**	1478.49	1478.49	1393.12	**1311.21**
5	10	10	1	1621.25	1564.61	1477.11	1619.51	**1473.27**	1482.97	**1473.27**	1476.08	1481.45	1482.97	1526.98	1526.98	1589.24	1583.23	1476.08	**1473.27**
5	10	10	2	1642.93	1510.10	1640.24	1503.41	**1472.85**	**1472.85**	**1472.85**	**1472.85**	1521.44	**1472.85**	1539.35	1539.35	1493.08	1486.41	1540	**1472.85**
5	10	10	3	1234.07	1272.53	1252.27	1253.21	1145.36	**1141.40**	1145.36	1173.29	**1141.40**	1199.10	1218.50	1225.31	1199.86	1199.86	1211.41	**1141.40**
5	10	10	4	1381.66	1414.81	1402.40	1413.78	**1213.56**	1281.27	**1213.56**	**1213.56**	**1213.56**	1219.16	1283.33	1325.82	1466.91	1391.59	1353.95	**1213.56**
5	10	10	5	1250.65	1188.70	1260.78	1263.94	1146.66	**1118.63**	**1118.63**	1167.43	1146.66	1146.66	1187.33	1188.18	1216.48	1240.32	1260.78	**1118.63**
10	5	5	1	2182.50	2285.70	2199.25	2198.73	**1942.18**	1985.09	1978.46	1978.76	1960.66	1960.66	1969.60	2019.94	2019.94	1960.66	2088.73	**1942.18**
10	5	5	2	2816.93	2353.72	2657.52	2674.06	2214.74	**2113.04**	2142.97	2156.50	2176.94	2180.97	2321.42	2321.42	2316.74	2232.87	2695.99	**2113.04**
10	5	5	3	2563.31	2298.48	2356.31	2271.15	**2115.01**	2163.25	**2115.01**	2204.12	2135.54	2159.14	2149.14	**2115.01**	2175.75	2203.78	2433.54	**2115.01**
10	5	5	4	2164.40	2100.94	2216.46	2158.37	**1906.97**	1947.21	**1906.97**	1943.28	1927.60	**1906.97**	1980.62	1980.62	1927.60	**1906.97**	2100.94	**1906.97**
10	5	5	5	2305.03	2369.41	2227.40	2553.48	2071.30	2019.90	2151.83	2087.97	2117.66	**1996.83**	2073.72	2029.18	2192.53	2111.08	2293.96	**1996.83**
10	5	10	1	3457.67	3430.73	3768.00	3564.68	**3097.67**	3182.02	3100.85	3207.75	3281.17	3210.71	3381.64	3381.64	3553.35	3502.69	3544.96	**3097.67**
10	5	10	2	3017.74	3027.05	2813.59	3093.05	2496.24	2590.18	2496.76	2576.50	2540.28	**2466.55**	2615.51	2615.52	2995.23	2809.73	2660.55	**2466.55**
10	5	10	3	3256.86	3367.30	3225.94	3188.19	2899.25	**2817.50**	2895.98	2936.06	2931.07	2916.72	2982.22	2972.82	3327.93	3207.30	3210.07	**2817.50**
10	5	10	4	2797.79	2950.58	2950.06	2759.49	**2412.75**	2566.54	2426.89	2413.42	2531.24	2533.69	2606.28	2602.07	2795.41	2720.76	2814.31	**2412.75**
10	5	10	5	2613.88	2496.60	2522.63	2503.27	2252.48	2278.79	**2245.58**	2287.91	2288.51	2250.81	2384.28	2403.94	2560.23	2485.32	2503.51	**2245.58**
10	10	5	1	2135.97	2239.09	2124.06	2151.98	**1938.07**	1951.81	1938.07	1950.63	1955.59	1955.59	1957.70	2004.51	2004.51	1955.59	2069.03	**1938.07**
10	10	5	2	2777.24	2295.47	2599.43	2605.72	2227.08	2093.28	2076.08	**2040.19**	2149.88	2101.70	2302.04	2302.04	2249.85	2181.02	2602.53	**2040.19**
10	10	5	3	2516.85	2230.30	2309.99	2226.75	**2035.52**	2107.56	**2035.52**	2107.56	2084.59	2100.47	2086.34	2065.93	2115.50	2149.27	2355.31	**2035.52**
10	10	5	4	2142.45	2023.86	2156.44	2064.53	**1895.85**	1912.49	**1895.85**	1897.89	1895.87	**1895.85**	1916.16	1916.16	1895.87	**1895.85**	2023.86	**1895.85**
10	10	5	5	2258.59	2316.71	2181.35	2498.56	2061.37	2022.13	2129.57	2183.85	2036.96	**1970.63**	2052.60	2018.22	2181.30	2077.51	2225.77	**1970.63**
10	10	10	1	3293.30	3282.31	3502.93	3426.11	2953.94	3010.79	**2953.83**	3099.36	3099.97	3058.69	3178.08	3178.08	3325.37	3295.38	3347.06	**2953.83**
10	10	10	2	2815.49	2750.55	2632.06	2816.55	2427.20	2378.54	2367.28	2474.49	2394.51	**2341.74**	2469.82	2434.56	2786.42	2697.98	2577.79	**2341.74**
10	10	10	3	3092.93	3145.61	3072.14	2984.07	2780.63	**2729.10**	2731.79	2729.20	2813.87	2790.27	2825.24	2770.27	3152.97	3075.09	3083.84	**2729.10**
10	10	10	4	2626.51	2762.49	2696.56	2586.26	2353.66	2326.04	2249.50	**2231.76**	2342.79	2338.60	2454.79	2460.57	2616.63	2573.42	2616.16	**2231.76**
10	10	10	5	2459.77	2319.26	2408.42	2342.93	2094.86	**2065.25**	2130.75	2153.41	2175.46	2096.43	2281.61	2253.16	2424.72	2348.79	2378.79	**2065.25**

APPENDIX B5 Computational results of sequencing heuristics for continuous sized variable sublots (ε=0.1)

# of products	Maximum # of sublots	# of machines	Instance No	LPT (TPT)	SPT (TPT)	LPT (TPLS)	SPT (TPLS)	NEH (D,TPT)	NEH (I,TPT)	NEH (D,TPLS)	NEH (I,TPLS)	CDS (ORJ)	CDS (PLS)	PALMER (ORJ)	PALMER (PLS)	GUPTA (ORJ)	GUPTA (PLS)	BMI (ORJ)	Best Makespan
5	5	5	1	991.91	**877.51**	1087.42	1172.18	961.24	921.46	879.46	893.46	916.68	963.91	1024.20	1035.17	1024.20	916.68	1064.88	877.51
5	5	5	2	906.37	988.96	950.28	943.10	922.89	**856.17**	**856.17**	861.03	856.68	856.68	**856.17**	**856.17**	873.07	973.07	977.14	856.17
5	5	5	3	1569.36	1484.07	1552.00	1515.63	**1403.69**	1460.16	**1403.69**	1460.16	1454.07	1454.07	1534.28	1482.28	1484.07	1484.07	**1403.69**	1403.69
5	5	5	4	937.17	829.20	860.92	842.12	**746.98**	**746.98**	**746.98**	831.48	791.63	791.63	**746.98**	**746.98**	901.05	860.94	795.93	746.98
5	5	5	5	1704.97	1484.01	1673.63	1550.14	1391.35	1375.71	**1355.67**	1375.71	1440.86	1443.49	**1355.67**	**1355.67**	1542.10	1542.10	1440.86	1355.67
5	5	10	1	1780.21	1741.02	**1653.07**	1800.16	1676.41	**1653.07**	1676.41	**1653.07**	**1653.07**	1655.97	1702.90	1702.90	1749.72	1754.85	1677.58	1653.07
5	5	10	2	1882.91	1679.98	1882.91	1676.68	**1675.01**	**1675.01**	**1675.01**	**1675.01**	1693.24	**1675.01**	1724.64	1724.64	1695.41	1675.22	1728.43	1675.01
5	5	10	3	1396.30	1391.86	1384.05	1405.32	**1254.34**	**1254.34**	**1254.34**	1284.70	**1254.34**	1309.55	1326.27	1361.37	1309.55	1309.55	1339.10	1254.34
5	5	10	4	1550.57	1559.12	1572.30	1535.28	**1341.51**	1368.75	**1341.51**	1356.33	1352.43	**1341.51**	1448.12	1473.22	1602.62	1552.05	1483.12	1341.51
5	5	10	5	1448.93	1500.52	1505.87	1456.14	**1291.99**	1293.60	1325.46	1355.69	**1291.99**	**1291.99**	1330.41	1346.82	1391.34	1412.96	1505.87	1291.99
5	10	5	1	961.47	845.89	1039.90	1137.54	928.99	**824.94**	**824.94**	837.47	880.38	933.47	1011.65	1015.46	1011.65	880.38	1001.36	824.94
5	10	5	2	852.56	940.66	891.77	892.77	876.69	**809.73**	**809.73**	823.24	813.42	813.42	**809.73**	**809.73**	813.39	914.04	922.26	809.73
5	10	5	3	1457.06	1400.67	1439.55	1425.64	**1317.62**	**1317.62**	**1317.62**	**1317.62**	1366.69	1370.67	1431.11	1379.11	1400.67	1400.67	**1317.62**	1317.62
5	10	5	4	906.14	811.47	828.79	790.08	**730.17**	**730.17**	**730.17**	790.08	761.72	761.72	**730.17**	**730.17**	882.12	836.97	780.93	730.17
5	10	5	5	1660.38	1413.77	1624.41	1477.74	1348.80	1408.55	**1311.21**	1342.95	1393.12	1396.91	**1311.21**	**1311.21**	1478.49	1478.49	1393.12	1311.21
5	10	10	1	1621.25	1564.20	1475.44	1619.51	**1473.27**	1481.21	**1473.27**	1475.12	1480.22	1481.21	1520.86	1520.86	1588.77	1583.23	1475.12	1473.27
5	10	10	2	1640.91	1510.10	1638.73	1503.41	**1472.81**	**1472.81**	**1472.81**	**1472.81**	1521.15	**1472.81**	1539.28	1539.28	1493.07	1484.87	1539.00	1472.81
5	10	10	3	1233.76	1272.52	1252.27	1253.21	1143.92	**1141.36**	1143.92	1173.20	**1141.36**	1199.07	1217.85	1224.68	1199.86	1199.86	1211.19	1141.36
5	10	10	4	1381.66	1413.69	1402.39	1413.78	**1213.53**	1281.04	**1213.53**	**1213.53**	**1213.53**	1219.03	1281.40	1325.72	1465.91	1391.59	1353.86	1213.53
5	10	10	5	1250.65	1299.45	1260.78	1263.94	1146.66	**1118.35**	**1118.35**	1163.45	1146.66	1146.66	1187.33	1188.18	1216.48	1240.32	1260.78	1118.35
10	5	5	1	2182.50	2285.70	2199.25	2198.73	**1942.18**	1985.09	1978.46	1978.76	1960.66	1960.66	1969.60	2019.94	2019.94	1960.66	2088.73	1942.18
10	5	5	2	2816.93	2353.72	2657.52	2674.06	2214.74	**2113.04**	2142.97	2156.50	2176.94	2180.97	2321.42	2321.42	2316.74	2232.87	2695.99	2113.04
10	5	5	3	2563.31	2298.48	2356.31	2270.23	**2115.01**	2163.10	**2115.01**	2204.12	2135.54	2159.14	2149.14	**2115.01**	2175.75	2203.78	2431.97	2115.01
10	5	5	4	2164.40	2121.87	2216.46	2158.37	**1906.97**	1947.21	**1906.97**	1943.28	1927.60	**1906.97**	1980.62	1980.62	1927.60	**1906.97**	2100.94	1906.97
10	5	5	5	2305.03	2369.41	2227.40	2553.48	2071.30	2019.90	2151.83	2085.66	2117.66	**1996.83**	2073.72	2029.18	2192.53	2111.08	2293.96	1996.83
10	5	10	1	3455.87	3430.73	3763.87	3561.44	3094.85	3182.02	**3093.18**	3204.81	3280.88	3207.77	3381.64	3381.64	3553.35	3502.69	3544.96	3093.18
10	5	10	2	3017.74	3027.05	2813.49	3093.05	2494.68	2590.18	2496.39	2575.76	2540.24	**2463.45**	2612.78	2611.96	2995.23	2807.77	2660.46	2463.45
10	5	10	3	3254.68	3339.30	3225.74	3188.19	2899.14	**2816.47**	2895.98	2936.06	2931.07	2916.72	2982.22	2972.82	3327.92	3207.22	3209.87	2816.47
10	5	10	4	2797.79	2947.72	2950.06	2759.19	**2408.05**	2564.61	2426.89	2413.42	2531.24	2533.69	2605.90	2601.61	2795.41	2720.76	2814.31	2408.05
10	5	10	5	2613.88	2496.60	2521.98	2503.27	2247.57	2278.10	**2245.31**	2286.84	2288.51	2249.79	2384.28	2403.94	2560.23	2485.32	2503.40	2245.31
10	10	5	1	2135.97	2239.09	2124.06	2151.98	**1938.07**	1951.81	1938.07	1950.63	1955.59	1955.59	1957.70	2004.51	2004.51	1955.59	2069.03	1938.07
10	10	5	2	2777.24	2295.47	2599.43	2605.72	2227.08	2093.28	2076.08	**2040.19**	2149.88	2101.70	2302.04	2302.04	2249.85	2181.02	2602.53	2040.19
10	10	5	3	2516.85	2230.30	2309.99	2226.59	**2035.52**	2107.56	**2035.52**	2107.56	2084.59	2100.47	2086.34	2065.93	2115.50	2149.27	2355.19	2035.52
10	10	5	4	2142.45	2050.44	2156.44	2064.53	**1895.85**	1912.49	**1895.85**	1897.89	1895.87	**1895.85**	1916.16	1916.16	1895.87	**1895.85**	2023.86	1895.85
10	10	5	5	2258.59	2316.64	2181.35	2498.56	2061.37	2022.13	2129.57	2183.85	2036.96	**1970.63**	2052.60	2018.22	2181.30	2077.51	2225.77	1970.63
10	10	10	1	3292.95	3282.31	3501.36	3425.26	**2952.88**	3009.25	2953.83	3098.61	3099.95	3052.79	3178.08	3178.08	3325.37	3295.38	3346.75	2952.88
10	10	10	2	2815.49	2749.57	2632.06	2815.57	2427.20	2377.54	2366.52	2474.43	2394.51	**2339.64**	2469.75	2434.29	2786.42	2697.76	2577.79	2339.64
10	10	10	3	3092.93	3117.61	3071.97	2984.07	2780.37	**2729.02**	2731.79	2729.05	2813.87	2790.27	2825.08	2770.11	3152.97	3074.92	3083.84	2729.02
10	10	10	4	2626.51	2762.24	2696.56	2586.26	2353.55	2326.04	2248.97	**2230.84**	2342.71	2337.18	2454.66	2460.57	2616.63	2573.42	2615.92	2230.84
10	10	10	5	2459.75	2319.26	2408.35	2342.93	2093.60	**2065.02**	2126.66	2148.18	2175.46	2096.43	2281.61	2253.08	2424.72	2348.64	2378.61	2065.02

APPENDIX B6 Detailed benchmark results of five product instances for continuous sized consistent sublots

Inst. No	*L*	*S*=5						*S*=10										
		Best Solution	Sublot Sizes					Best Solution	Sublot Sizes									
			1	2	3	4	5		1	2	3	4	5	6	7	8	9	10
5-5-1	14	[3,2,1,4,5]	6.13	5.11	4.26	3.55	2.96	[3,4,5,1,2]	0	0	0	0	0	1.92	2.36	2.75	3.21	3.75
	22		2.62	4.66	8.29	7.67	5.75		4.37	3.64	3.04	2.53	2.11	1.76	1.46	1.22	1.02	0.85
	29		1.05	2.1	4.19	8.38	10.3		0.42	0.74	1.32	2.35	4.18	7.43	7.16	3.25	1.48	0.67
	26	878.39	2.69	3.58	4.77	6.37	1.59	825.13	1.68	3.64	3.36	3.1	2.87	2.65	2.44	2.25	2.08	1.92
	19		2.87	1.03	3.52	4.11	0		0.39	0.56	0.74	0.99	1.32	1.76	2.35	3.04	3.76	4.08
5-5-2	20	[5,2,3,1,4]	0	0	9.51	6.39	4.11	[5,2,3,1,4]	0	0	0	0	0.78	3.3	6.6	4.54	2.92	1.87
	20		0	1.3	4.56	6.08	8.05		0	0	0.01	0.03	0.15	0.69	3.1	4.35	5.78	5.88
	11		0	3.87	0	7.13	0		0	0	0	0	0	0	0	0	4.54	6.46
	37	856.17	5.28	7.55	8.81	10.3	5.08	809.73	2.41	3.44	4.92	5.74	6.69	7.81	4.3	1.23	0.35	0.1
	17		2.68	3.28	4	4.3	2.74		0.51	1.14	1.4	1.71	2.09	2.55	3.12	2.19	1.39	0.89
5-5-3	26	[1,2,5,4,3]	0.21	0.64	1.93	5.8	17.4	[1,2,5,4,3]	0.01	0.02	0.06	0.18	0.54	1.62	3.64	4.85	6.46	8.62
	26		0	2.2	7.25	7.91	8.63		0	0	0	1.4	3.26	3.56	3.88	4.23	4.62	5.04
	10		2.57	2.25	1.96	1.72	1.5		1.72	1.5	1.32	1.15	1.01	0.88	0.77	0.65	0.55	0.45
	32	1403.69	8.1	7.43	6.81	6.24	3.42	1317.62	4.59	4.21	3.86	3.53	3.24	2.97	2.72	2.5	2.29	2.1
	37		8.63	8.24	7.87	6.69	5.57		5.04	4.81	4.59	4.38	4.18	3.9	3.25	2.71	2.26	1.88
5-5-4	17	[1,5,3,4,2]	1.29	1.93	2.9	4.35	6.53	[1,5,3,4,2]	0.15	0.23	0.34	0.51	0.76	1.14	1.71	2.56	3.84	5.77
	14		2.31	2.89	3.61	2.89	2.31		0.95	1.18	1.48	1.85	2.31	1.85	1.48	1.18	0.95	0.76
	30		8.75	6.72	9.97	3.58	0.98		2.66	8.59	7.17	5.74	1.64	3.06	0.84	0.23	0.06	0.02
	20	746.98	0.98	2.6	6.94	6.4	3.08	730.17	0.02	0.05	0.12	0.32	0.86	2.29	6.1	5.62	3.37	1.26
	14		0	2	4	8	0		0	0	0	0	0	0	0	3.67	5.98	4.35
5-5-5	38	[4,2,3,1,5]	0	6.73	14.8	13	3.47	[4,2,3,1,5]	0	0	0	0	0	0	4.32	12.8	11.2	9.77
	31		2.67	4	6	9	9.32		0.3	0.44	0.66	1	1.49	2.24	3.36	5.04	7.56	8.9
	20		5.79	5.61	3.93	2.75	1.92		0	0	0	0	1.49	6.67	4.67	3.27	2.29	1.6
	39	1355.67	1.2	3.2	8.52	19	7.11	1311.21	0.07	0.19	0.52	1.37	3.67	9.78	14.9	5.6	2.1	0.79
	38		6.31	7.57	8.75	8.02	7.35		2.11	2.53	3.04	3.65	4.38	5.25	4.83	4.43	4.06	3.72
5-10-1	21	[4,5,3,1,2]	0	3.05	6.35	5.97	5.62	[3,5,4,1,2]	0.73	0.94	1.21	1.55	2	2.57	3.28	3.09	2.9	2.73
	29		2.33	5.82	7.66	6.51	6.68		0	2.4	5.19	4.41	4.02	3.45	2.96	2.53	2.17	1.86
	30		5.34	6.17	6.17	6.17	6.17		3	3	3	3	3	3	3	3	3	3
	22	1653.07	2.57	3.33	4.83	6.28	5	1473.27	0.15	1.85	3.49	4.77	3.58	2.68	2.01	1.51	1.13	0.84
	37		6.66	7.01	7.38	7.77	8.18		3.22	3.39	3.57	3.76	3.95	4.16	4.38	4.57	3.43	2.57
5-10-2	38	[4,2,1,3,5]	7.87	7.87	7.87	7.87	6.52	[4,2,1,3,5]	2.97	4.11	4.14	4.14	4.14	4.14	4.14	4.14	3.36	2.73
	23		0.8	2.12	5.66	6.97	7.46		0.23	0.41	0.74	1.33	2.4	3.35	3.59	3.85	4.12	2.97
	37		6.8	7.55	7.55	7.55	7.55		3.14	3.49	3.88	3.88	3.88	3.88	3.88	3.88	3.85	3.21
	21	1675.07	6.68	7.34	4.33	1.86	0.8	1472.85	1.96	2.16	2.37	2.61	2.87	3.16	3.47	1.49	0.64	0.27
	18		0	5.45	4.96	4.14	3.45		3.58	2.98	2.48	2.07	1.73	1.44	1.2	1	0.83	0.69
5-10-3	26	[1,4,3,5,2]	3.18	6	6	5.6	5.23	[1,4,5,3,2]	0.85	1.6	3.02	3.02	3.02	3.02	3.02	3.02	2.82	2.63
	11		4.22	2.82	1.88	1.25	0.83		0	0	0	0	0.12	0.68	1.13	1.89	3.15	4.02
	25		4.84	6.9	6.59	4.86	1.82		0.61	0.87	1.24	1.77	2.52	3.6	5.15	6.66	2	0.6
	30	1254.63	5.1	5.67	6.3	6.46	6.46	1141.40	2.37	2.63	2.92	3.15	3.15	3.15	3.15	3.15	3.15	3.15
	14		2.42	3.32	3.02	2.74	2.49		0	0	0	1.01	2.78	2.52	2.29	2.09	1.9	1.41
5-10-4	16	[3,1,4,5,2]	5.24	3.93	2.95	2.21	1.66	[1,4,3,5,2]	0.3	0.49	0.79	1.3	2.12	2.6	2.72	2.85	1.75	1.08
	28		6.83	7.18	5.88	4.81	3.31		1.52	4.22	4.1	3.89	3.7	3.1	2.54	2.08	1.7	1.17
	19		0.7	1.86	4.97	5.68	5.79		0	0	0	0	0.01	0.97	2.58	5.06	5.78	4.61
	28	1341.58	2.49	5.12	8.05	7.99	4.36	1213.56	1.48	3.33	3.63	3.44	3.26	3.09	2.93	2.77	2.63	1.43
	33		5.72	6.24	6.8	7.42	6.83		1.58	1.93	2.36	2.89	3.53	3.85	4.2	4.58	5	3.08
5-10-5	24	[3,1,2,4,5]	0	0	5.24	9.16	9.6	[1,4,2,3,5]	1	1.33	1.77	2.36	2.66	2.78	2.92	3.05	3.2	2.93
	39		6.85	7.71	8.68	8.13	7.63		3.1	3.58	4.13	4.21	4.21	4.21	4.21	4.21	4.21	2.96
	12		0.8	2.09	2.5	3	3.6		0	0	0	0	0	1.14	1.9	3.17	3	2.78
	13	1291.99	0	0	0.93	5.35	6.72	1118.63	0	0.17	1.19	1.34	1.46	1.59	1.74	1.9	2.07	1.55
	11		6.93	3.07	0.77	0.19	0.05		2.49	1.99	1.6	1.28	1.02	0.82	0.65	0.52	0.42	0.21

APPENDIX B7 Detailed optimal MIP results of five product instances for continuous sized consistent sublots

Inst. No	L	S=5 Best Solution	S=5 Sublot Sizes 1	2	3	4	5	S=10 Best Solution	S=10 Sublot Sizes 1	2	3	4	5	6	7	8	9	10
	14	[3,2,1,4,5]	2.46	2.87	3.35	3.91	1.4	[3,4,5,1,2]	2.04	0.01	0.06	0.33	1.99	2.78	0.56	3.33	2.92	0
	22		6.13	5.11	4.26	3.55	2.96		4.37	3.64	3.04	2.53	2.11	1.76	1.46	1.22	1.02	0.85
5-5-1	29		2.62	4.66	8.29	7.67	5.75		0.42	0.74	1.32	2.35	4.18	7.43	7.16	3.25	1.48	0.67
	26	878.39	1.05	2.1	4.19	8.38	10.3	825.13	1.68	3.64	3.36	3.1	2.87	2.65	2.44	2.25	2.08	1.92
	19		2.69	3.58	4.77	6.37	1.59		0.43	0.57	0.76	1.01	1.35	1.8	2.4	2.96	3.66	4.08
	20	[5,2,3,1,4]	0	9.52	6.38	4.09	0	[5,2,3,1,4]	5.63	6.34	3.02	0.59	1.18	2.36	0.64	0.18	0.05	0.01
	20		0.82	3.7	5.64	1.79	8.05		0.01	0	0.04	0	0.16	0.74	3.34	4.05	5.78	5.88
5-5-2	11		0	3.86	7.14	0	0		0	0	0	0	0	0	0	0	4.54	6.46
	37	856.17	5.28	7.55	8.81	10.3	5.08	809.73	2.41	3.44	4.92	5.74	6.69	7.81	4.3	1.23	0.35	0.1
	17		2.68	3.28	4	4.3	2.74		0.51	1.14	1.4	1.71	2.09	2.55	3.12	2.19	1.39	0.89
	26	[1,2,5,4,3]	0.21	0.64	1.93	5.8	17.4	[1,2,5,4,3]	0.01	0.02	0.06	0.18	0.54	1.62	3.64	4.85	6.46	8.62
	26		2.81	3.41	4.69	6.45	8.63		3.13	0.6	0.82	1.13	1.55	2.13	2.94	4.04	4.62	5.04
5-5-3	10		2.57	2.25	1.96	1.72	1.5		1.72	1.5	1.32	1.15	1.01	0.88	0.77	0.65	0.55	0.45
	32	1403.69	8.1	7.43	6.81	6.24	3.42	1317.62	4.59	4.21	3.86	3.53	3.24	2.97	2.72	2.5	2.29	2.1
	37		8.63	8.24	7.87	6.69	5.57		5.04	4.81	4.59	4.38	4.18	3.9	3.25	2.71	2.26	1.88
	17	[5,1,3,4,2]	9.53	5.37	1.54	0.44	0.13	[5,1,3,4,2]	0	0	8.62	5.99	1.71	0.49	0.14	0.04	0.01	0
	14		2.31	2.89	3.61	2.89	2.31		0.95	1.18	1.48	1.85	2.31	1.85	1.48	1.18	0.95	0.76
5-5-4	30		1.63	13.1	10.8	3.58	0.98		0.06	0.49	3.96	11.8	9.46	3.06	0.84	0.23	0.06	0.02
	20	746.23	0.98	2.6	6.94	6.4	3.08	729.93	0.02	0.05	0.12	0.32	0.86	2.29	6.1	5.62	3.37	1.26
	14		0.46	0.91	1.83	3.66	7.15		0.01	0.03	0.06	0.12	0.24	0.47	0.94	1.89	3.78	6.46
	38	[4,2,3,1,5]	0	1.13	10.1	14.3	12.5	[4,2,3,1,5]	0	0	0	0	0	4.32	0	12.8	11.2	9.77
	31		2.67	4	6	9	9.32		0.3	0.44	0.66	1	1.49	2.24	3.36	5.04	7.56	8.9
5-5-5	20		7.46	5.22	3.66	1.58	2.09		6.86	4.8	3.36	2.35	1.38	0	0	0	0	1.23
	39	1355.67	1.2	3.2	8.52	19	7.11	1311.21	0.07	0.19	0.52	1.37	3.67	9.78	14.9	5.6	2.1	0.79
	38		6.31	7.57	8.75	8.02	7.35		2.11	2.53	3.04	3.65	4.38	5.25	4.83	4.43	4.06	3.72
	21	[4,5,3,2,1]	0	5.46	5.35	5.25	4.94	[5,3,4,1,2]	0.89	1.45	2.36	2.77	2.61	2.46	2.31	2.17	2.05	1.93
	29		5.92	6.66	6.99	5.35	4.09		3.47	2.56	3.07	3.68	3.84	3.27	2.83	2.42	2.08	1.78
5-10-1	30		5.34	6.17	6.17	6.17	6.17		1.51	2.38	3.74	3.74	3.74	3.74	3.74	3.74	2.38	1.3
	22	1653.07	2.19	3.71	4.83	6.28	5	1464.32	1.56	3.74	1.56	3.13	1.06	1.73	2.97	3.04	2.19	1.02
	37		6.66	7.01	7.38	7.77	8.18		3.12	3.29	3.46	3.64	3.83	4.04	4.25	4.47	4.02	2.88
	38	[4,1,3,2,5]	8.06	8.06	8	7.27	6.61	[2,4,3,1,5]	3.09	4.12	4.12	4.12	4.12	4.12	4.12	4.12	3.35	2.72
	23		5.4	5.81	5.77	3.67	2.34		0.03	0.09	0.24	0.65	1.72	4.6	5.29	5.15	3.22	2.01
5-10-2	37		7.13	7.92	7.92	7.66	6.38		3.11	3.58	4.03	4.48	4.82	4.49	4.18	3.48	2.9	1.93
	21	1673.27	3.44	3.78	4.16	4.58	5.04	1462.00	0	0	0.01	0.09	0.79	4.74	4.74	2.57	3.92	4.14
	18		3.27	5.18	4.31	3.6	1.64		0	0	0	0	0.14	3.92	2.71	3.92	3.92	3.4
	26	[1,4,3,2,5]	3.11	5.88	5.88	5.75	5.37	[1,4,5,3,2]	0.85	1.6	3.02	3.02	3.02	3.02	3.02	3.02	2.82	2.63
	11		0.09	0.78	0	3.8	6.33		0.6	1	0.34	1.89	0	0	0	3.15	4.02	0
5-10-3	25		4.69	6.71	5.99	4.39	3.22		0.61	0.87	1.24	1.77	2.52	3.6	5.15	6.66	2	0.6
	30	1254.11	5.28	5.87	6.28	6.28	6.28	1141.40	2.37	2.63	2.92	3.15	3.15	3.15	3.15	3.15	3.15	3.15
	14		4.42	3.4	2.62	2.01	1.55		0	0	2.63	2.39	0	2.17	1.97	1.79	1.63	1.41
	16	[3,1,4,5,2]	5.24	3.93	2.95	2.21	1.66	[1,4,3,5,2]	0.3	0.49	0.79	1.3	2.12	2.6	2.72	2.85	1.75	1.08
	28		6.83	7.18	5.88	4.81	3.31		2.98	4.1	3.89	3.7	3.51	3.01	2.46	2.01	1.38	0.95
5-10-4	19		0.7	1.86	4.97	5.68	5.79		0.03	0.09	0	0.24	0.63	1.57	3.66	2.74	5.48	4.57
	28	1341.58	2.49	5.12	8.05	7.99	4.36	1213.56	1.48	3.33	3.63	3.44	3.26	3.09	2.93	2.77	2.63	1.43
	33		5.72	6.24	6.8	7.42	6.83		1.72	2.1	2.56	3.13	3.42	3.73	4.07	4.44	4.84	2.98
	24	[3,1,2,4,5]	0.94	1.42	4.25	8.5	8.9	[1,4,2,3,5]	1.31	1.75	2.33	2.44	2.56	2.68	2.81	2.94	2.7	2.47
	39		6.85	7.71	8.68	8.13	7.63		3.1	3.58	4.13	4.21	4.21	4.21	4.21	4.21	4.21	2.96
5-10-5	12		0.8	2.09	2.5	3	3.6		0.63	0	0	1.05	0	0	1.75	2.92	2.92	2.71
	13	1291.99	0	0	0.5	3.49	9.01	1118.63	0.19	0.19	1.32	1.44	1.57	1.71	1.87	2.04	1.53	1.15
	11		6.93	3.07	0.77	0.19	0.05		2.49	1.99	1.6	1.28	1.02	0.82	0.65	0.52	0.42	0.21

APPENDIX B8 Computational results of NEH(D,TPLS) heuristic and MIP (within 1000 seconds) for 5 product instances

				Continuous Sized Consistent Sublots				Discrete Sized Consistent Sublots				Continuous Sized Variable Sublots			
# of products	**Maximum # of sublots**	**# of machines**	**Instance No**	**Lower Bound**	**Upper Bound (MIP)**	**CPU Time (Sec)**	**NEH (D,TPLS)**	**Lower Bound**	**Upper Bound (MIP)**	**CPU Time (Sec)**	**NEH (D,TPLS)**	**Lower Bound**	**Upper Bound (MIP)**	**CPU Time (Sec)**	**NEH (D,TPLS)**
5	5	5	1	878.39	**878.39**	1.63	879.46	885.00	**885**	2.34	891	853.78	**878.36**	1000.00	879.46
5	5	5	2	856.17	**856.17**	1.50	**856.17**	860.00	**860**	3.14	867	831.80	**851.33**	1000.00	856.17
5	5	5	3	1403.69	**1403.69**	1.47	**1403.69**	1414.00	**1414**	2.02	1419	1403.69	**1403.69**	411.94	**1403.69**
5	5	5	4	746.23	**746.23**	1.49	746.98	752.00	**752**	6.06	753	735.34	**745.89**	1000.00	746.98
5	5	5	5	1355.67	**1355.67**	1.49	**1355.67**	1361.00	**1361**	1.86	1364	1355.30	**1355.30**	36.03	1355.67
5	5	10	1	1653.07	**1653.07**	2.47	1676.41	1670.00	**1670**	6.00	1714	1401.76	**1667.34**	1000.00	1676.41
5	5	10	2	1673.27	**1673.27**	2.45	1675.07	1683.00	**1683**	6.69	1695	1470.13	1700.21	1000.00	**1675.01**
5	5	10	3	1254.11	**1254.11**	2.03	1254.63	1267.00	**1267**	8.92	1272	1113.08	**1251.52**	1000.00	1254.34
5	5	10	4	1341.58	**1341.58**	1.92	**1341.58**	1351.00	**1351**	7.25	1354	1162.00	1346.43	1000.00	**1341.51**
5	5	10	5	1291.99	**1291.99**	1.81	1325.46	1306.00	**1306**	24.00	1346	1173.64	**1292.23**	1000.00	1325.46
5	10	5	1	825.13	**825.13**	2.25	**825.13**	836.00	**836**	167.61	839	818.00	832.25	1000.00	**824.94**
5	10	5	2	809.73	**809.73**	1.95	**809.73**	818.00	**818**	633.83	819	792.00	814.71	1000.00	**809.73**
5	10	5	3	1317.62	**1317.62**	2.26	**1317.62**	1331.00	**1331**	29.50	1332	1292.00	1326.20	1000.00	**1317.62**
5	10	5	4	729.93	**729.93**	2.14	730.17	735.58	**738**	1000.00	741	723.00	730.63	1000.00	**730.17**
5	10	5	5	1311.21	**1311.21**	2.11	**1311.21**	1322.11	**1323**	1000.00	1328	1298.00	1317.71	1000.00	**1311.21**
5	10	10	1	1464.32	**1464.32**	5.50	1473.27	1485.00	**1485**	748.86	1498	1327.00	**1471.42**	1000.00	1473.27
5	10	10	2	1462.00	**1462.00**	6.99	1472.85	1480.00	**1480**	113.56	1495	1213.00	1528.66	1000.00	**1472.81**
5	10	10	3	1141.40	**1141.40**	4.47	1145.36	1153.00	**1153**	131.75	1167	1045.00	1169.43	1000.00	**1143.92**
5	10	10	4	1213.56	**1213.56**	4.92	**1213.56**	1225.81	**1232**	1000.00	1237	1144.00	1243.49	1000.00	**1213.53**
5	10	10	5	1118.63	**1118.63**	5.03	**1118.63**	1133.00	**1133**	100.63	1167	1013.00	1139.85	1000.00	**1118.35**
Average				1192.39	1192.39	2.79	1196.63	1203.43	1203.90	249.70	1214.90	1108.28	1203.33	922.40	1196.51

APPENDIX B9 Computational results of NEH(D,TPLS) heuristic and MIP (within 1000 seconds) for 10 product instances

				Continuous Sized Consistent Sublots				**Discrete Sized Consistent Sublots**				**Continuous Sized Variable Sublots**			
# of products	**Maximum # of sublots**	**# of machines**	**Instance No**	**Lower Bound**	**Upper Bound (MIP)**	**CPU Time (Sec)**	**NEH (D,TPLS)**	**Lower Bound**	**Upper Bound (MIP)**	**CPU Time (Sec)**	**NEH (D,TPLS)**	**Lower Bound**	**Upper Bound (MIP)**	**CPU Time (Sec)**	**NEH (D,TPLS)**
10	5	5	1	1714.54	**1942.18**	1000.00	1978.46	1614.40	**1949**	1000.00	1992	1380.000	**1942.18**	1000.00	1978.46
10	5	5	2	2057.50	**2061.00**	1000.00	2142.97	1929.77	**2064**	1000.00	2154	1650.000	**2080.51**	1000.00	2142.97
10	5	5	3	1945.61	**2115.02**	1000.00	**2115.02**	1938.01	**2120**	1000.00	2126	1476.667	**2115.02**	1000.00	**2115.02**
10	5	5	4	1865.97	**1906.97**	1000.00	**1906.97**	1842.28	**1908**	1000.00	1915	1484.000	**1906.97**	1000.00	**1906.97**
10	5	5	5	1584.48	**1932.48**	1000.00	2151.83	1691.32	**1939**	1000.00	2158	1512.000	**1957.84**	1000.00	2151.83
10	5	10	1	2407.50	**3069.35**	1000.00	3100.85	2374.00	**3097**	1000.00	3128	1266.028	3278.98	1000.00	**3093.18**
10	5	10	2	1893.52	**2460.70**	1000.00	2496.76	1784.30	**2471**	1000.00	2522	1169.131	2556.73	1000.00	**2496.39**
10	5	10	3	2241.46	**2794.87**	1000.00	2895.98	1875.41	**2839**	1000.00	2912	1106.838	2970.21	1000.00	**2895.98**
10	5	10	4	1833.65	**2369.71**	1000.00	2426.89	1797.60	**2404**	1000.00	2451	1114.200	2524.62	1000.00	**2426.89**
10	5	10	5	1588.80	**2160.16**	1000.00	2245.58	1699.99	**2178**	1000.00	2272	1021.655	**2192.66**	1000.00	2245.31
10	10	5	1	1555.19	**1938.07**	1000.00	**1938.07**	1489.62	**1949**	1000.00	1950	1107.000	1938.21	1000.00	**1938.07**
10	10	5	2	1704.57	**2045.64**	1000.00	2076.08	1483.85	**2041**	1000.00	2088	1497.000	**2049.12**	1000.00	2076.08
10	10	5	3	1674.52	**2035.52**	1000.00	**2035.52**	1542.80	**2047**	1000.00	2050	1061.000	2043.45	1000.00	**2035.52**
10	10	5	4	1621.48	**1893.48**	1000.00	1895.85	1410.90	**1897**	1000.00	1902	1174.000	1898.11	1000.00	**1895.85**
10	10	5	5	1483.37	**1942.68**	1000.00	2129.57	1281.91	**1972**	1000.00	2138	1380.025	**1936.84**	1000.00	2129.57
10	10	10	1	2035.80	2966.44	1000.00	**2953.83**	1908.91	**2973**	1000.00	2994	1209.396	3172.52	1000.00	**2953.83**
10	10	10	2	1515.47	**2302.76**	1000.00	2367.28	1205.50	**2397**	1000.00	2399	609.000	2681.41	1000.00	**2366.52**
10	10	10	3	1604.04	**2682.22**	1000.00	2731.79	1316.93	**2679**	1000.00	2749	1090.000	2827.41	1000.00	**2731.79**
10	10	10	4	1414.48	**2227.74**	1000.00	2249.50	1207.95	**2280**	1000.00	2284	720.888	2423.08	1000.00	**2248.97**
10	10	10	5	1346.46	**2049.25**	1000.00	2130.75	1040.92	**2086**	1000.00	2151	813.000	2183.04	1000.00	**2126.66**
Average				1754.42	2244.81	1000.00	2298.48	1621.82	2264.50	1000.00	2316.75	1192.09	2333.94	1000.00	2297.79

APPENDIX B10 Computational results of NEH(D,TPLS) heuristic and MIP (within 1000 seconds) for 15 product instances

				Continuous Sized Consistent Sublots				Discrete Sized Consistent Sublots				Continuous Sized Variable Sublots			
# of products	**Maximum # of sublots**	**# of machines**	**Instance No**	**Lower Bound**	**Upper Bound (MIP)**	**CPU Time (Sec)**	**NEH (D,TPLS)**	**Lower Bound**	**Upper Bound (MIP)**	**CPU Time (Sec)**	**NEH (D,TPLS)**	**Lower Bound**	**Upper Bound (MIP)**	**CPU Time (Sec)**	**NEH (D,TPLS)**
15	5	5	1	1619.40	**2658.69**	1000.00	2683.42	1258.26	**2668**	1000.00	2695	1156.000	**2667.16**	1000.00	2683.42
15	5	5	2	1277.99	2787.95	1000.00	**2703.82**	1219.17	2770	1000.00	**2720**	1165.000	2765.50	1000.00	**2703.82**
15	5	5	3	1508.61	**3172.41**	1000.00	**3172.41**	1481.19	**3177**	1000.00	3182	1286.000	3172.47	1000.00	**3172.41**
15	5	5	4	1296.03	**2951.60**	1000.00	2955.06	1305.13	**2955**	1000.00	2971	1129.000	**2953.66**	1000.00	2955.06
15	5	5	5	1398.52	2991.60	1000.00	**2961.32**	1446.07	**2971**	1000.00	2979	1421.000	3039.48	1000.00	**2961.32**
15	5	10	1	1248.80	**2870.46**	1000.00	2882.87	1225.53	**2880**	1000.00	2904	879.795	3055.34	1000.00	**2882.85**
15	5	10	2	1554.63	3491.30	1000.00	**3336.62**	1523.68	3493	1000.00	**3360**	1031.701	3579.71	1000.00	**3329.09**
15	5	10	3	1540.09	3496.34	1000.00	**3413.50**	1622.64	3492	1000.00	**3431**	1076.678	3686.09	1000.00	**3411.10**
15	5	10	4	1514.39	3758.19	1000.00	**3650.63**	1544.18	**3652**	1000.00	3673	1029.988	3696.92	1000.00	**3649.61**
15	5	10	5	1484.07	**3393.87**	1000.00	3427.50	1385.40	3510	1000.00	**3449**	1006.433	3696.09	1000.00	**3418.49**
15	10	5	1	1095.45	**2619.14**	1000.00	2650.82	1133.14	**2640**	1000.00	2667	1174.000	**2637.06**	1000.00	2650.82
15	10	5	2	1007.05	2682.58	1000.00	**2675.72**	987.00	2800	1000.00	**2690**	985.000	2800.27	1000.00	**2675.72**
15	10	5	3	1242.74	**3171.01**	1000.00	**3171.01**	1068.01	**3177**	1000.00	3182	1059.000	**3171.01**	1000.00	**3171.01**
15	10	5	4	1058.41	**2936.42**	1000.00	2936.44	1000.41	**2947**	1000.00	2951	1028.000	2938.71	1000.00	**2936.44**
15	10	5	5	1259.08	2930.25	1000.00	**2924.49**	1100.40	**2934**	1000.00	2941	1344.000	2990.56	1000.00	**2924.49**
15	10	10	1	1031.59	2827.20	1000.00	**2778.27**	882.77	2956	1000.00	**2801**	905.000	2884.82	1000.00	**2778.26**
15	10	10	2	1188.25	3427.84	1000.00	**3403.33**	1035.59	3539	1000.00	**3431**	732.000	3575.73	1000.00	**3403.27**
15	10	10	3	1128.93	3304.29	1000.00	**3261.67**	1212.67	3496	1000.00	**3290**	817.000	3540.68	1000.00	**3261.04**
15	10	10	4	1273.85	3620.86	1000.00	**3567.27**	1274.39	3657	1000.00	**3597**	900.000	3822.28	1000.00	**3566.92**
15	10	10	5	1076.83	**3294.99**	1000.00	3315.36	861.11	**3342**	1000.00	3343	804.000	3469.07	1000.00	**3312.92**
Average				1290.23	3119.35	1000.00	3093.58	1228.34	3152.80	1000.00	3112.85	1046.48	3207.13	1000.00	3092.40

APPENDIX B11 Computational results of NEH(D,TPLS) heuristic for 30 product problems

# of products	Maximum # of sublots	# of machines	Instance No	Continuous Sized Consistent Sublots		Discrete Sized Consistent Sublots		Continuous Sized Variable Sublots	
				NEH (D,TPLS)	CPU Time (sec)	NEH(D,TPLS)	CPU Time (sec)	NEH (D,TPLS)	CPU Time (sec)
30	5	5	1	4659.68	15.73	4668	29.75	4659.68	30.97
30	5	5	2	6032.89	16.91	6039	33.00	6032.89	30.70
30	5	5	3	5644.49	17.83	5655	34.08	5644.49	33.00
30	5	5	4	5544.26	17.14	5553	32.83	5544.26	33.28
30	5	5	5	5146.18	17.42	5156	34.83	5146.18	33.61
30	5	10	1	5953.07	30.19	5976	61.28	5942.29	68.02
30	5	10	2	5679.77	29.89	5700	67.06	5679.30	72.66
30	5	10	3	6447.61	30.53	6486	69.17	6447.61	78.52
30	5	10	4	5484.67	32.49	5503	69.06	5482.60	74.91
30	5	10	5	6268.57	32.94	6288	67.70	6268.57	74.47
30	5	15	1	6553.03	52.04	6596	98.45	6548.89	121.55
30	5	15	2	6431.42	55.56	6465	107.24	6429.43	143.72
30	5	15	3	5934.10	55.35	5964	105.20	5930.89	136.71
30	5	15	4	6177.08	56.00	6217	106.84	6174.10	144.88
30	5	15	5	6304.68	56.56	6322	107.24	6304.68	139.88
30	10	5	1	4653.41	30.14	4665	47.23	4653.41	54.49
30	10	5	2	6035.76	27.06	6044	50.88	6035.76	59.05
30	10	5	3	5643.01	25.99	5655	51.57	5643.01	60.56
30	10	5	4	5503.81	26.28	5517	51.46	5503.81	60.01
30	10	5	5	5131.79	26.72	5144	51.62	5131.79	61.97
30	10	10	1	5784.99	58.75	5813	105.78	5784.97	219.96
30	10	10	2	5454.56	60.14	5487	108.66	5453.99	234.61
30	10	10	3	6207.25	60.21	6237	115.22	6202.35	287.86
30	10	10	4	5323.35	63.58	5353	115.51	5321.33	207.75
30	10	10	5	6270.53	59.30	6303	109.60	6270.51	185.69
30	10	15	1	6319.57	111.88	6364	180.37	6318.65	336.61
30	10	15	2	6411.68	115.24	6454	199.17	6408.36	641.70
30	10	15	3	5725.28	105.49	5763	190.51	5711.88	371.14
30	10	15	4	6191.22	109.50	6243	192.59	6188.34	402.89
30	10	15	5	6196.01	108.00	6225	193.70	6196.01	410.00

APPENDIX B12 Computational results of NEH(D,TPLS) heuristic for 50 product problems

# of products	Maximum # of sublots	# of machines	Instance No	Continuous Sized Consistent Sublots		Discrete Sized Consistent Sublots		Continuous Sized Variable Sublots	
				NEH (D,TPLS)	CPU Time (sec)	NEH (D,TPLS)	CPU Time (sec)	NEH (D,TPLS)	CPU Time (sec)
50	5	5	1	8236.18	101.94	8252.00	207.19	8236.18	212.79
50	5	5	2	7935.05	110.28	7950.00	220.91	7935.05	227.20
50	5	5	3	9102.93	109.72	9108.00	221.61	9102.93	230.14
50	5	5	4	8186.08	109.98	8199.00	226.19	8186.08	234.74
50	5	5	5	8710.29	111.23	8721.00	228.22	8710.29	235.08
50	5	10	1	8926.48	230.28	8953.00	473.95	8926.48	472.54
50	5	10	2	9331.76	251.19	9354.00	496.61	9330.58	487.48
50	5	10	3	10086.35	255.31	10108.00	494.23	10085.46	490.90
50	5	10	4	9377.06	256.33	9413.00	498.78	9376.89	442.98
50	5	10	5	9648.34	252.85	9680.00	494.36	9646.22	476.69
50	5	15	1	9827.81	368.50	9857.00	731.70	9823.52	721.37
50	5	15	2	9908.06	386.82	9939.00	782.01	9894.96	761.82
50	5	15	3	10017.81	398.28	10048.00	784.25	10040.25	763.90
50	5	15	4	9295.62	401.16	9327.00	778.34	9286.79	797.00
50	5	15	5	9143.63	405.64	9186.00	773.62	9131.06	821.62
50	5	20	1	9623.18	548.86	9666.00	971.59	9601.70	1002.37
50	5	20	2	10475.24	573.80	10522.00	1021.03	10518.59	1216.79
50	5	20	3	11083.84	587.51	11144.00	1027.47	11089.31	1135.13
50	5	20	4	10590.43	597.31	10646.00	1060.03	10716.56	1245.14
50	5	20	5	10760.43	580.34	10809.00	1049.35	10754.04	1114.56
50	10	5	1	8153.27	152.15	8169.00	307.50	8153.27	333.92
50	10	5	2	7919.37	160.55	7930.00	327.80	7919.37	342.48
50	10	5	3	9096.45	162.68	9108.00	335.01	9096.45	343.16
50	10	5	4	8176.66	164.98	8189.00	334.87	8176.66	347.53
50	10	5	5	8697.03	165.06	8707.00	334.04	8697.03	343.43
50	10	10	1	8883.25	371.43	8917.00	698.59	8883.23	962.53
50	10	10	2	9231.01	368.35	9259.00	703.78	9230.56	750.58
50	10	10	3	10099.14	387.67	10132.00	703.17	10099.11	1029.23
50	10	10	4	9304.78	363.04	9336.00	702.76	9299.43	722.97
50	10	10	5	9625.69	369.65	9655.00	699.43	9624.40	704.98
50	10	15	1	9609.89	717.81	9655.00	1254.24	9609.81	2345.40
50	10	15	2	9649.39	756.75	9693.00	1225.21	9647.82	2198.09
50	10	15	3	9928.72	802.86	9977.00	1311.12	9928.72	4470.84
50	10	15	4	9225.07	720.89	9273.00	1315.80	9115.69	4470.84
50	10	15	5	9150.65	711.44	9197.00	1382.30	9104.21	3133.82
50	10	20	1	9604.11	1152.09	9658.00	2246.94	9603.91	7987.22
50	10	20	2	10115.44	1557.42	10180.00	1902.04	10269.72	11249.16
50	10	20	3	10910.81	1200.72	10981.00	1732.93	10888.77	4718.74
50	10	20	4	10215.43	1071.17	10283.00	1872.95	10328.46	2468.35
50	10	20	5	10608.62	998.51	10667.00	1921.45	10604.94	4921.92
50	20	5	1	8153.00	271.44	8167.00	499.45	8153.00	563.68
50	20	5	2	7922.24	276.96	7936.00	536.31	7922.24	687.94
50	20	5	3	9096.00	276.82	9107.00	544.42	9096.00	610.71
50	20	5	4	8176.00	274.75	8189.00	549.32	8176.00	696.45
50	20	5	5	8692.63	277.43	8708.00	543.86	8692.63	628.87
50	20	10	1	8819.13	692.64	8858.00	1160.46	8851.69	3540.18
50	20	10	2	9313.02	829.99	9344.00	1227.40	9271.87	2632.01
50	20	10	3	9995.44	726.83	10032.00	1213.51	***	***
50	20	10	4	9035.68	652.29	9084.00	1253.37	***	***
50	20	10	5	9625.12	635.38	9659.00	1230.14	9625.12	2922.73
50	20	15	1	9493.03	1573.10	9544.00	3266.42	***	***
50	20	15	2	9522.85	1755.05	9577.00	2441.56	***	***
50	20	15	3	9692.26	1284.32	9750.00	2546.02	***	***
50	20	15	4	9028.09	1472.78	9087.00	2595.35	***	***
50	20	15	5	9097.99	1694.81	9166.00	2318.55	***	***
50	20	20	1	9368.66	2491.94	9447.00	3991.06	***	***
50	20	20	2	10114.89	2813.81	10192.00	4659.04	***	***
50	20	20	3	10728.53	2848.58	10805.00	3273.47	***	***
50	20	20	4	10297.86	2332.38	10385.00	4078.22	***	***
50	20	20	5	10436.43	2520.28	10514.00	3466.85	***	***

*** Out of Memory

APPENDIX C1 Computational results for continuous sized consistent sublots

# of products	Maximum # of sublots	# of machines	Instance No	MIP Model Results			Tabu Search based Heuristic Results		
				Lower Bound	Upper Bound (MIP)	Computation Time (Sec)	Initial Makespan	Final Makespan	Computation Time (Sec)
5	5	5	1	878.39	878.39	1.63	879.46	879.46	0.66
5	5	5	2	856.17	856.17	1.50	856.17	856.17	0.62
5	5	5	3	1403.69	1403.69	1.47	1403.69	1403.69	0.55
5	5	5	4	746.23	746.23	1.49	746.98	746.23	0.75
5	5	5	5	1355.67	1355.67	1.49	1355.67	1355.67	0.66
5	5	10	1	1653.07	1653.07	2.47	1676.41	1653.07	3.90
5	5	10	2	1673.27	1673.27	2.45	1675.07	1673.27	3.05
5	5	10	3	1254.11	1254.11	2.03	1254.63	1254.11	1.91
5	5	10	4	1341.58	1341.58	1.92	1341.58	1341.58	1.68
5	5	10	5	1291.99	1291.99	1.81	1325.46	1291.99	1.99
5	10	5	1	825.13	825.13	2.25	825.13	825.13	1.91
5	10	5	2	809.73	809.73	1.95	809.73	809.73	1.82
5	10	5	3	1317.62	1317.62	2.26	1317.62	1317.62	1.58
5	10	5	4	729.93	729.93	2.14	730.17	729.93	2.27
5	10	5	5	1311.21	1311.21	2.11	1311.21	1311.21	1.72
5	10	10	1	1464.32	1464.32	5.50	1473.27	1464.32	7.33
5	10	10	2	1462.00	1462.00	6.99	1472.85	1462.00	15.41
5	10	10	3	1141.40	1141.40	4.47	1145.36	1141.40	8.22
5	10	10	4	1213.56	1213.56	4.92	1213.56	1213.56	6.85
5	10	10	5	1118.63	1118.63	5.03	1118.63	1118.63	5.80
10	5	5	1	1714.54	1942.18	1000.00	1978.46	1942.18	91.40
10	5	5	2	2057.50	2061.00	1000.00	2142.97	2061.03	25.85
10	5	5	3	1945.61	2115.02	1000.00	2115.01	2115.02	39.68
10	5	5	4	1865.97	1906.97	1000.00	1906.97	1906.97	33.98
10	5	5	5	1584.48	1932.48	1000.00	2151.83	1932.48	43.77
10	5	10	1	2407.50	3069.35	1000.00	3100.85	3022.80	69.80
10	5	10	2	1893.52	2460.70	1000.00	2496.76	2461.16	78.68
10	5	10	3	2241.46	2794.87	1000.00	2895.98	2784.37	93.49
10	5	10	4	1833.65	2369.71	1000.00	2426.89	2426.89	49.15
10	5	10	5	1588.80	2160.16	1000.00	2245.58	2178.05	79.80
10	10	5	1	1555.19	1938.07	1000.00	1938.07	1938.07	105.84
10	10	5	2	1704.57	2045.64	1000.00	2076.08	2040.19	73.02
10	10	5	3	1674.52	2035.52	1000.00	2035.52	2035.52	99.43
10	10	5	4	1621.48	1893.48	1000.00	1895.85	1893.48	55.91
10	10	5	5	1483.37	1942.68	1000.00	2129.57	1915.38	150.65
10	10	10	1	2035.80	2966.44	1000.00	2953.83	2927.09	460.86
10	10	10	2	1515.47	2302.76	1000.00	2367.28	2326.40	329.91
10	10	10	3	1604.04	2682.22	1000.00	2731.79	2642.64	504.06
10	10	10	4	1414.48	2227.74	1000.00	2249.50	2221.16	444.05
10	10	10	5	1346.46	2049.25	1000.00	2130.75	2041.22	494.61
15	5	5	1	1619.40	2658.69	1000.00	2683.42	2629.68	179.14
15	5	5	2	1277.99	2787.95	1000.00	2703.82	2682.60	133.02
15	5	5	3	1508.61	3172.41	1000.00	3172.42	3172.41	388.07
15	5	5	4	1296.03	2951.60	1000.00	2955.06	2951.60	112.72
15	5	5	5	1398.52	2991.60	1000.00	2961.32	2958.51	162.61
15	5	10	1	1280.07	2870.46	1500.00	2882.87	2720.35	1409.59
15	5	10	2	1657.10	3387.94	3000.00	3336.62	3284.29	2275.72
15	5	10	3	1540.09	3496.34	1000.00	3413.50	3348.03	891.65
15	5	10	4	1514.39	3758.19	1000.00	3650.63	3628.52	581.73
15	5	10	5	1484.07	3393.87	1000.00	3427.50	3319.89	1094.35
15	10	5	1	1095.45	2619.14	1000.00	2650.82	2630.06	506.52
15	10	5	2	1007.05	2682.58	1000.00	2675.72	2652.68	1042.05
15	10	5	3	1242.74	3171.01	1000.00	3171.01	3171.01	515.00
15	10	5	4	1089.77	2936.42	1500.00	2936.44	2936.42	1220.76
15	10	5	5	1259.08	2930.25	1000.00	2924.49	2924.46	538.02
15	10	10	1	1070.67	2798.11	6000.00	2778.27	2702.86	6089.70
15	10	10	2	1281.06	3327.38	6000.00	3403.33	3210.28	3671.42
15	10	10	3	1328.93	3304.29	6000.00	3261.67	3160.61	3101.94
15	10	10	4	1341.25	3547.96	6000.00	3567.27	3509.74	5542.04
15	10	10	5	1200.40	3285.44	6000.00	3315.36	3231.57	2816.46
Average				***1423.81***	***2180.26***	***1134.26***	***2196.23***	***2158.11***	***594.42***

APPENDIX C2 Computational results for discrete sized consistent sublots

# of products	Maximum # of sublots	# of machines	Instance No	MIP Model Results			Tabu Search based HeuristicResults		
				Lower Bound	Upper Bound	Computation Time (Sec)	Initial Makespan	Final Makespan	Computation Time (Sec)
5	5	5	1	885.00	885	2.34	891	889	1.85
5	5	5	2	860.00	860	3.14	867	866	1.17
5	5	5	3	1414.00	1414	2.02	1419	1419	0.97
5	5	5	4	752.00	752	6.06	753	753	1.05
5	5	5	5	1361.00	1361	1.86	1364	1364	1.03
5	5	10	1	1670.00	1670	6.00	1714	1670	3.38
5	5	10	2	1683.00	1683	6.69	1695	1691	5.94
5	5	10	3	1267.00	1267	8.92	1272	1272	2.39
5	5	10	4	1351.00	1351	7.25	1354	1354	2.53
5	5	10	5	1306.00	1306	24.00	1346	1312	4.39
5	10	5	1	836.00	836	167.61	839	839	2.73
5	10	5	2	818.00	818	633.83	819	819	2.72
5	10	5	3	1331.00	1331	29.50	1332	1332	2.41
5	10	5	4	735.58	738	1000.00	741	741	2.83
5	10	5	5	1322.11	1323	1000.00	1328	1328	2.58
5	10	10	1	1485.00	1485	748.86	1498	1494	9.84
5	10	10	2	1480.00	1480	113.56	1495	1485	12.02
5	10	10	3	1153.00	1153	131.75	1167	1163	11.13
5	10	10	4	1225.81	1232	1000.00	1237	1237	9.95
5	10	10	5	1133.00	1133	100.63	1167	1147	9.83
10	5	5	1	1614.40	1949	1000.00	1992	1951	27.99
10	5	5	2	1929.77	2064	1000.00	2154	2070	41.77
10	5	5	3	1938.01	2120	1000.00	2126	2120	30.03
10	5	5	4	1842.28	1908	1000.00	1915	1913	31.24
10	5	5	5	1691.32	1939	1000.00	2158	1941	46.52
10	5	10	1	2374.00	3097	1000.00	3128	3038	172.22
10	5	10	2	1784.30	2471	1000.00	2522	2482	214.78
10	5	10	3	1875.41	2839	1000.00	2912	2819	161.84
10	5	10	4	1797.60	2404	1000.00	2451	2448	103.55
10	5	10	5	1699.99	2178	1000.00	2272	2190	128.93
10	10	5	1	1489.62	1949	1000.00	1950	1950	72.62
10	10	5	2	1483.85	2041	1000.00	2088	2052	111.57
10	10	5	3	1542.80	2047	1000.00	2050	2048	90.80
10	10	5	4	1410.90	1897	1000.00	1902	1899	82.06
10	10	5	5	1281.91	1972	1000.00	2138	1926	158.36
10	10	10	1	1908.91	2973	1000.00	2994	2940	456.00
10	10	10	2	1205.50	2397	1200.00	2399	2322	1194.87
10	10	10	3	1316.93	2679	1000.00	2749	2671	713.05
10	10	10	4	1207.95	2280	1000.00	2284	2267	470.24
10	10	10	5	1040.92	2086	1000.00	2151	2065	604.80
15	5	5	1	1258.26	2668	1000.00	2695	2636	263.01
15	5	5	2	1219.17	2770	1000.00	2720	2691	414.65
15	5	5	3	1481.19	3177	1000.00	3182	3182	189.46
15	5	5	4	1305.13	2955	1000.00	2971	2954	258.12
15	5	5	5	1446.07	2971	1000.00	2979	2965	292.34
15	5	10	1	1288.61	2880	3000.00	2904	2814	2257.90
15	5	10	2	1577.97	3493	3000.00	3360	3302	1599.66
15	5	10	3	1677.64	3483	3000.00	3431	3344	2908.80
15	5	10	4	1615.70	3652	3000.00	3673	3560	2295.04
15	5	10	5	1452.60	3439	3000.00	3449	3386	1044.81
15	10	5	1	1133.14	2640	1000.00	2667	2643	826.56
15	10	5	2	987.00	2800	1000.00	2690	2675	669.18
15	10	5	3	1068.01	3177	1000.00	3182	3182	524.39
15	10	5	4	1000.41	2947	1000.00	2951	2943	615.16
15	10	5	5	1100.40	2934	1000.00	2941	2934	582.95
15	10	10	1	1009.45	2921	10000.00	2801	2722	6734.49
15	10	10	2	1078.53	3331	10000.00	3431	3218	8034.76
15	10	10	3	1303.16	3377	10000.00	3290	3184	5077.26
15	10	10	4	1398.14	3590	10000.00	3597	3532	4284.38
15	10	10	5	1020.70	3325	15000.00	3343	3220	14184.29
Average				***1365.44***	***2198.30***	***1753.23***	***2214.83***	***2172.90***	***967.69***

APPENDIX C3 Computational results for continuous sized variable sublots

# of products	Maximum # of sublots	# of machines	Instance No	MIP Model Results			Tabu Search based Heuristic Results		
				Lower Bound	Upper Bound	Computation Time (Sec)	Initial Makespan	Final Makespan	Computation Time (Sec)
5	5	5	1	853.78	878.36	1000.00	879.46	879.46	1.03
5	5	5	2	831.80	851.33	1000.00	856.17	856.17	1.41
5	5	5	3	1403.69	1403.69	411.94	1403.69	1403.69	1.64
5	5	5	4	735.34	745.89	1000.00	746.98	746.23	1.28
5	5	5	5	1355.30	1355.30	36.03	1355.67	1355.67	1.08
5	5	10	1	1401.76	1667.34	1000.00	1676.41	1653.07	9.43
5	5	10	2	1470.13	1700.21	1000.00	1675.01	1675.01	5.86
5	5	10	3	1113.08	1251.52	1000.00	1254.34	1253.37	6.88
5	5	10	4	1162.00	1346.43	1000.00	1341.51	1341.52	6.36
5	5	10	5	1173.64	1292.23	1000.00	1325.46	1291.99	8.98
5	10	5	1	818.00	832.25	1000.00	824.94	824.94	4.20
5	10	5	2	792.00	814.71	1000.00	809.73	809.73	6.70
5	10	5	3	1292.00	1326.20	1000.00	1317.62	1317.62	7.30
5	10	5	4	723.00	730.63	1000.00	730.17	729.93	6.61
5	10	5	5	1298.00	1317.71	1000.00	1311.21	1311.21	4.67
5	10	10	1	1327.00	1471.42	1000.00	1473.27	1459.87	63.25
5	10	10	2	1213.00	1528.66	1000.00	1472.81	1461.97	80.42
5	10	10	3	1045.00	1169.43	1000.00	1143.92	1141.36	79.27
5	10	10	4	1144.00	1243.49	1000.00	1213.53	1213.53	40.56
5	10	10	5	1013.00	1139.85	1000.00	1118.35	1118.35	57.70
10	5	5	1	1380.00	1942.18	1000.00	1978.46	1942.18	39.48
10	5	5	2	1650.00	2080.51	1000.00	2142.97	2089.23	24.58
10	5	5	3	1476.67	2115.02	1000.00	2115.01	2115.02	29.84
10	5	5	4	1484.00	1906.97	1000.00	1906.97	1906.97	18.77
10	5	5	5	1512.00	1957.84	1000.00	2151.83	2017.81	49.44
10	5	10	1	1266.03	3278.98	1000.00	3093.18	3079.26	89.67
10	5	10	2	1169.13	2556.73	1000.00	2496.39	2460.70	178.13
10	5	10	3	1106.84	2970.21	1000.00	2895.98	2811.34	289.71
10	5	10	4	1114.20	2524.62	1000.00	2426.89	2426.89	90.88
10	5	10	5	1021.66	2192.66	1000.00	2245.31	2244.96	130.63
10	10	5	1	1107.00	1938.21	1000.00	1938.07	1938.07	90.83
10	10	5	2	1497.00	2049.12	1000.00	2076.08	2074.19	73.00
10	10	5	3	1061.00	2043.45	1000.00	2035.52	2035.52	111.60
10	10	5	4	1174.00	1898.11	1000.00	1895.85	1893.48	129.77
10	10	5	5	1380.03	1936.84	1000.00	2129.57	1949.71	406.85
10	10	10	1	924.00	3157.69	2000.00	2953.83	2925.19	1971.85
10	10	10	2	915.00	2502.41	2000.00	2366.52	2340.99	1346.93
10	10	10	3	960.00	2932.62	2000.00	2731.79	2709.47	989.70
10	10	10	4	702.00	2365.44	2000.00	2248.97	2245.65	814.53
10	10	10	5	853.00	2124.85	2000.00	2126.66	2066.81	2456.93
15	5	5	1	1156.00	2667.16	1000.00	2683.42	2658.47	83.47
15	5	5	2	1165.00	2765.50	1000.00	2703.82	2685.22	69.43
15	5	5	3	1286.00	3172.47	1000.00	3172.42	3172.42	69.94
15	5	5	4	1129.00	2953.66	1000.00	2955.06	2951.60	74.04
15	5	5	5	1421.00	3039.48	1000.00	2961.32	2958.59	94.90
15	5	10	1	879.79	3055.34	1000.00	2882.85	2831.30	560.45
15	5	10	2	1031.70	3579.71	1000.00	3329.09	3320.50	451.11
15	5	10	3	1076.68	3686.09	1000.00	3411.10	3356.52	504.19
15	5	10	4	1029.99	3696.92	1000.00	3649.61	3645.78	692.16
15	5	10	5	1006.43	3696.09	1000.00	3418.49	3412.77	467.15
15	10	5	1	1068.00	2673.42	1500.00	2650.82	2630.06	1390.77
15	10	5	2	1058.00	2688.17	1500.00	2675.72	2675.72	663.72
15	10	5	3	921.00	3171.03	1500.00	3171.01	3171.01	360.77
15	10	5	4	967.00	2936.29	1500.00	2936.44	2936.44	411.79
15	10	5	5	1025.00	3010.25	1500.00	2924.49	2924.49	388.97
15	10	10	1	768.00	2851.63	11000.00	2778.26	2762.87	4405.02
15	10	10	2	1080.00	3445.52	11000.00	3403.27	3320.18	8528.91
15	10	10	3	829.00	3479.74	11000.00	3261.04	3159.06	6915.12
15	10	10	4	900.00	3708.58	11000.00	3566.92	3509.76	10889.39
15	10	10	5	1083.00	3429.87	11000.00	3312.92	3312.92	3236.45
Average				***1113.328***	***2237.468***	***1932.47***	***2195.570***	***2175.230***	***833.09***

APPENDIX C4 Computational results of TS based heuristic starting from NEH(D,TPLS) and LPT(TPT)

Number of products	Maximum # of sublots	Number of machines	Instance No	TS with NEH(D,TPLS)			TS with LPT(TPT)		
				Initial Makespan	Final Makespan	Computation Time (Sec)	Initial Makespan	Final Makespan	Computation Time (Sec)
5	5	5	1	879.46	879.46	0.66	991.91	879.46	0.91
5	5	5	2	856.17	856.17	0.62	906.37	856.17	0.88
5	5	5	3	1403.69	1403.69	0.55	1569.36	1403.69	1.00
5	5	5	4	746.98	746.23	0.75	937.17	746.23	0.81
5	5	5	5	1355.67	1355.67	0.66	1705.24	1355.67	1.28
5	5	10	1	1676.41	1653.07	3.90	1780.21	1653.07	2.12
5	5	10	2	1675.07	1673.27	3.05	1885.50	1676.68	3.09
5	5	10	3	1254.63	1254.11	1.91	1396.30	1254.11	2.38
5	5	10	4	1341.58	1341.58	1.68	1550.57	1341.58	3.17
5	5	10	5	1325.46	1291.99	1.99	1448.93	1291.99	2.11
5	10	5	1	825.13	825.13	1.91	961.47	825.13	2.91
5	10	5	2	809.73	809.73	1.82	852.56	809.73	2.86
5	10	5	3	1317.62	1317.62	1.58	1457.06	1317.62	2.11
5	10	5	4	730.17	729.93	2.27	906.14	729.93	2.45
5	10	5	5	1311.21	1311.21	1.72	1660.43	1311.21	3.83
5	10	10	1	1473.27	1464.32	7.33	1621.25	1464.32	9.92
5	10	10	2	1472.85	1462.00	15.41	1642.93	1462.00	9.53
5	10	10	3	1145.36	1141.40	8.22	1234.07	1141.40	10.34
5	10	10	4	1213.56	1213.56	6.85	1381.66	1213.56	10.99
5	10	10	5	1118.63	1118.63	5.80	1250.65	1118.63	9.80
10	5	5	1	1978.46	1942.18	91.40	2182.50	1942.18	19.94
10	5	5	2	2142.97	2061.03	25.85	2816.93	2061.03	47.71
10	5	5	3	2115.01	2115.02	39.68	2563.31	2115.01	27.62
10	5	5	4	1906.97	1906.97	33.98	2164.40	1906.97	35.62
10	5	5	5	2151.83	1932.48	43.77	2305.03	1932.47	25.67
10	5	10	1	3100.85	3022.80	69.80	3457.67	3066.48	139.63
10	5	10	2	2496.76	2461.16	78.68	3017.74	2460.70	215.39
10	5	10	3	2895.98	2784.37	93.49	3256.86	2817.50	87.98
10	5	10	4	2426.89	2426.89	49.15	2797.79	2380.33	120.38
10	5	10	5	2245.58	2178.05	79.80	2613.88	2132.62	220.58
10	10	5	1	1938.07	1938.07	105.84	2135.97	1938.07	91.02
10	10	5	2	2076.08	2040.19	73.02	2777.24	2091.46	90.41
10	10	5	3	2035.52	2035.52	99.43	2516.85	2035.52	75.05
10	10	5	4	1895.85	1893.48	55.91	2142.45	1893.47	60.81
10	10	5	5	2129.57	1915.38	150.65	2258.59	1915.38	117.03
10	10	10	1	2953.83	2927.09	460.86	3293.30	2992.99	515.42
10	10	10	2	2367.28	2326.40	329.91	2815.49	2300.91	481.68
10	10	10	3	2731.79	2642.64	504.06	3092.93	2635.81	709.68
10	10	10	4	2249.50	2221.16	444.05	2626.51	2227.48	493.54
10	10	10	5	2130.75	2041.22	494.61	2459.77	2051.73	561.01
15	5	5	1	2683.42	2629.68	179.14	3324.61	2666.86	195.18
15	5	5	2	2703.82	2682.60	133.02	3535.04	2742.93	256.82
15	5	5	3	3172.42	3172.41	388.07	3416.89	3172.41	297.83
15	5	5	4	2955.06	2951.60	112.72	3363.83	2951.60	388.99
15	5	5	5	2961.32	2958.51	162.61	3712.03	2958.51	257.47
15	5	10	1	2882.87	2720.35	1409.59	3326.84	2715.09	1084.43
15	5	10	2	3336.62	3284.29	2275.72	3875.91	3338.24	1078.98
15	5	10	3	3413.50	3348.03	891.65	3996.98	3340.82	1161.50
15	5	10	4	3650.63	3628.52	581.73	4341.46	3579.34	2095.20
15	5	10	5	3427.50	3319.89	1094.35	4005.58	3331.99	1571.01
15	10	5	1	2650.82	2630.06	506.52	3276.43	2587.73	1401.85
15	10	5	2	2675.72	2652.68	1042.05	3464.35	2679.44	572.70
15	10	5	3	3171.01	3171.01	515.00	3335.28	3171.01	615.38
15	10	5	4	2936.44	2936.42	1220.76	3291.78	2936.42	710.49
15	10	5	5	2924.49	2924.46	538.02	3666.56	2924.62	1378.84
15	10	10	1	2778.27	2702.86	6089.70	3207.40	2639.80	7141.85
15	10	10	2	3403.33	3210.28	3671.42	3841.03	3185.87	5007.56
15	10	10	3	3261.67	3160.61	3101.94	3865.31	3195.93	3834.19
15	10	10	4	3567.27	3509.74	5542.04	4170.98	3520.24	6720.49
15	10	10	5	3315.36	3231.57	2816.46	3782.78	3231.32	6577.00
Average				***2196.23***	***2158.11***	***594.42***	***2553.93***	***2160.34***	***776.11***