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ELASTO-PLASTIC STRESS ANALYSIS IN A COMPOSITE MATERIAL WITH A HOLE

A Dissertation Presented to the
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ABSTRACT

This study is about elasto-plastic stress analysis in a composite material with a hole.While the thesis we studied at two steps.Firstly we produced a composite material and found its properties.After this process, we begun theoretical study.It has two steps.Firstly we studied about elastic solution .Elastic solution gave us only elastic stresses at composite material.Then we started at elasto-plastic solving.Because real materials did not have only elastic deformation.If we know that when materials are in plastic region, we use a material safety.Elasto-plastic solution is made by finite element modelling. If the stress at a node is in the plastic region then, we calculated plastic stress.Some examples are added last pages.According to that solutions boundary of yielding points are symmetric to reinforcing directions

Difficulty of this study is fabricating.But we can make it successfully.

Necati Ataberk

1996 January/İzmir

ÖZET

Bu çalışma ortasında delik bulunan kompozit bir malzemede elasto-plastik gerilme analizi hakkındadır.Tez iki aşamalıdır.Birinci aşamada kompozit malzeme üretildi ve malzeme özellikleri bulundu.Bu aşamadan sonra iki aşamalı teorik çalışma başladı.Birinci aşamada elastik çözüm yapıldı.Elastik çözüm bize sadece kompozit malzemedeki elastik gerilmeleri verdi.Bundan sonra elasto-plastik gerilme analizine başlandı.Çünkü gerçek malzemeler sadece elastik şekil değişimine uğramazlar.Eğer biz bir malzemenin nezaman plastik bölgeye gireceğini bilirsek malzmeleri daha güvenli olarak kullanabiliriz.Elasto-plastik çözüm sonlu elemanlar modellemesi kullanılarak yapıldı.Eğer bir noktadaki gerilme plastik bölgenin içindeyse plastik gerilme analizi yapıldı.Bazı uygulamalar son sayfalara eklenmiştir.Buna göre akan(plastik şekil değişimine uğrayan) noktaların sınırları takviye doğrultusuna göre simetrik olmaktadır.

Bu çalışmanın zor yanı üretim aşamasıdır.Ama bu aşama da başarıyla tamamlanmıştır.

Necati Ataberk
Ocak 1996/İzmir

Symbol	Unit	Description
E_1	Mpa = N/mm^2	Modulus of elasticity at reinforcement direction
E_2	Mpa	Modulus of elasticity at 90° with reinforcement direction.
G_{12}	Mpa	Shearing modulus
σ	Mpa	Stress (tensile or compression)
τ	Mpa,	Shearing stress
ε	None	Strain (tensile or compression)
γ	None	Strain (angular)
ν_{12}	None	Poison's ratio
K	Mpa	Coefficient of plastic region
n	None	Superscript of plastic region
X	Mpa	Ultimate Tensile strength of material at reinforcement direction
Y	Mpa	Ultimate Tensile strength of material at 90° with reinforcement direction
S	Mpa	Ultimate Shearing strength

Chapter I

1.1 INTRODUCTION

The word "composite " in composite material signifies that two or more materials are combined on a macroscopic scale to form a useful key is the macroscopic examination of a material.Different materials can be combined on a microscopic scale , such as in alloying , but the resulting material is macroscopically homogeneous.The advantage of Composite is that they usually exhibit the best qualities that neither constituent possesses. The properties that can be improved by forming a composite material include :

* strength
* stiffness
* corrosion resistance
* wear resistance
* weight
* attractiveness
* fatigue life
* temperature-dependent behavior
* thermal insulation
* thermal conductivity
* acoustical insulation

Naturally , not all of the above properties are improved at the same time nor is there usually any requirement to do so.

1.2 CLASSIFICATION AND CHARACTERISTICS OF COMPOSITE MATERIALS

There are three commonly accepted types of composite materials :

1.Fibrous composites which consist of fibers in a matrix

2.Laminated composites which consist of layers of various materials

3.Particulate composites which are composed of particles in a matrix

1.2.1 Fibrous Composites

Long fibers in various forms are inherently much stiffer and stronger than the same material in bulk form.For example , ordinary plate glass fractures at stress of only a few thousand pounds per square inch, yet glass fibers have strengths of 400.000 to 700.000 psi in commercially available forms and about 1.000.000 psi in laboratory prepared forms.Obviously , then , the geometry of a fiber is somehow crucial to the evaluation of its strength and must be considered in structural applications.More properly the paradox of a fiber having different properties from the bulk form is due to more perfect structure of a fiber.The crystals are aligned in the fiber along the fiber axis.Moreover , there are fewer internal defects in fibers than

A fiber is characterized geometrically not only its very high length-to-diameter ratio but by its near crystal-sized diameter.Strengths and stiffnesses of a few selected fiber materials are shown in Table 1-1. Many common materials are listed for the purpose of comparison.

Table 1.1 Properties of Fibers

Fiber or wire	Density ρ (kN/m^3)	Tensile strength S (GN/m^2)	S/ρ $(10^3 m)$	Tensile stiffness E (GN/m^2)	E/ρ $(10^6 m)$
Aluminum	26.3	0 .62	24	73	2.8
Titanium	46.1	1.9	41	115	2.5
Steel	76.6	4.1	54	207	2.7
E-glass	25.0	3.4	136	72	2.9
S-glass	24.4	4.8	197	86	3.5
Carbon	13.8	1.7	123	190	14
Beryllium	18.2	1.7	93	300	16
Boron	25.2	3.4	137	400	16
Graphite	13.8	1.7	123	250	18

Consider a mixture of two metals , say A and B in Figure below, that are liquid at elevated temperatures.There is a composition , known as the eutectic composition, at which the mixture instantly freezes into a combination of two solid phases, α and A_3 B.Phase α is a solid solution of B in A has X percent of B.Since A_3B has 25 percent B, obviously α has less B than the eutectic composition.That is, some areas of the composite are poor in B where as others are rich in B.At the eutectic composition, the physical structure of the two zones of material is a near perfect array of fibers of α in a matrix of A_3B if the freezingtakes place in a special manner.For example, in Figure below the mixture of A and B is placed in a container and a heat source is moved up the container.The solidification that takes place after the heat source passes has fibers perpendicular to the solid-liquid interface.Thus, the fibers are vertical in Figure-B .If the composition is on either side of the eutectic composition, then other growths called dendrites occur in addition to the fibers of α.The resulting structure is both less perfect and weaker.Thus, the use of eutectic composites of two materials is limited to a fixed composition for those two materials.Examples of eutectic composites are Al-Al_3Ni, Cu-Cr and Ta-Ta_2C.

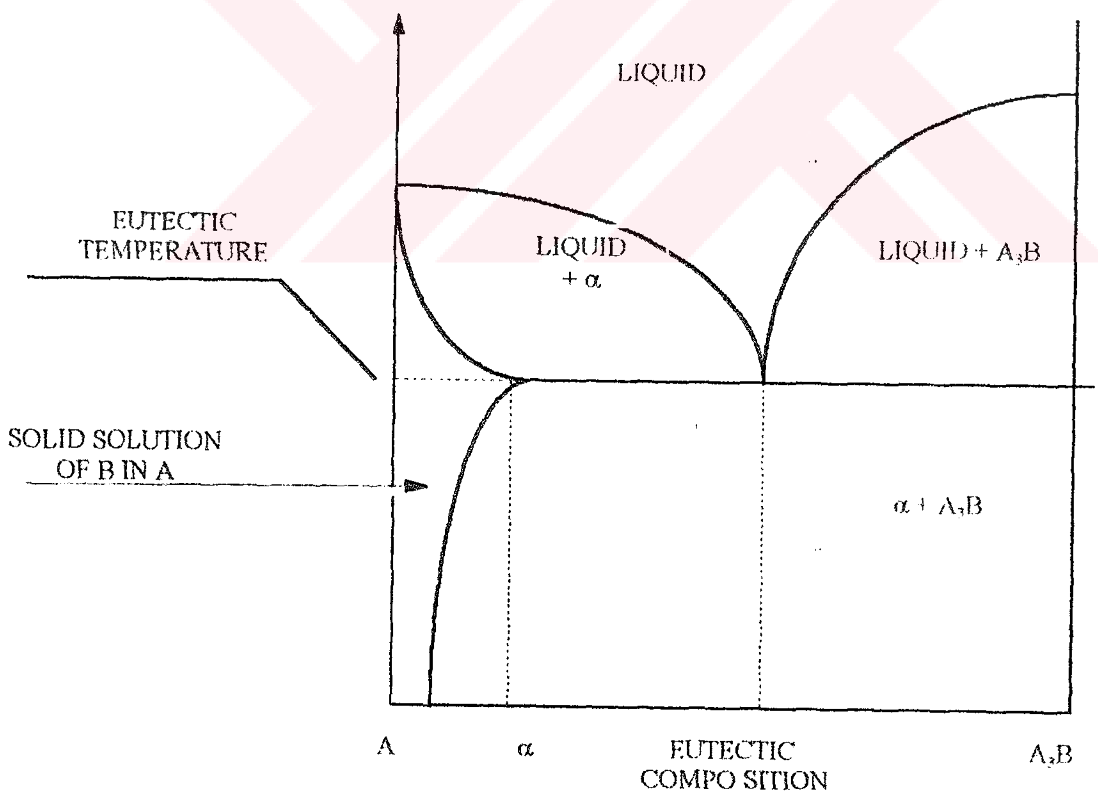


Figure 1.1 Phase diagram for mixture of metal A and metal B

Properties of Matrices

Naturally, fibers and whiskers are of the little use unless they are bound together to take the form of a structural element which can take loads.The binder material is usually called a matrix.The purpose of the matrix is manifold: support, protection, stress transfer, etc.Typically, the matrix is of considerably lower density, stiffness, and strength than fibers or whiskers.However, the combination of fibers or whiskers and a matrix can have very high strength and stiffness, yet still have low density.

1.2.2 Laminated Composites

Laminated composites consist of layers of at least two different materials that are bounded together.Lamination is used to combine the best aspects of the constituent layers in order to achieve a more useful material.The properties that can be emphasized by lamination are strength, stiffness, low weight, corrosion resistance, wear resistance, beauty or attractiveness, thermal insulation, acoustical insulation, etc. Such claims are best represented by the examples of the following paragraphs in which bimetals, clad metals, laminated glass, plastic-based laminates, and laminated fibrous composites are described.

Bimetals

Bimetals are laminates of two different metals with significantly different coefficients of thermal expansion.Under change in temperature, bimetals warp or deflect a predictable amount and are therefore well suited for use in temperature measuring devices.For example, a simple thermostat can be made from a cantilever strip of two metals bonded together as shown in figure below. There metal A with coefficient of thermal expansion α_A is bonded to metal B with α_B less than α_A .Thus, on increase of temperature over the temperature at which bonding was performed, metal A tends to expand more than metal B; hence, the strip bends down.If the temperature were decreased from the reference temperature, metal A would tend to shrink more

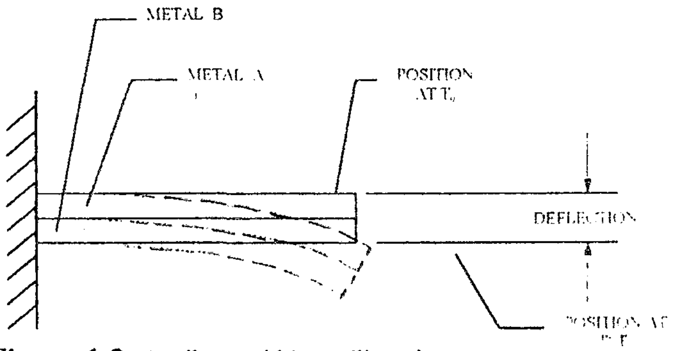


Figure 1.2 Cantilevered bimetallic strip

than metal B, and consequently, the strip would bend up.This is a simple example of coupling between bending and extension.Obviously application of uniform temperature to a single homogeneous metal causes only extension.However, application of uniform temperature to two bonded dissimilar metals was just shown to result in bending as well as extension.Coupling between bending and extension is a typical result when materials are laminated.

Clad Metals

The cladding or sheathing of one metal with another is done to obtain the best properties of both.For example, high-strength aluminum alloys do not resist corrosion; however pure aluminum and some aluminum alloys are very corrosion resistant.Thus, a high-strength aluminum alloy covered with a corrosion-resistant aluminum alloy is a composite material with unique and attractive advantages over its constituents.

Recently, aluminum wire, clad with about 10 percent copper, was introduced as a replacement for a copper wire in the electrical wiring market.Aluminum wire by itself is economical and lightweight, but over heats and is difficult to connect.On the other hand, wire is expensive and relatively heavy, but stays cool, and can be connected easily.The copper-clad aluminum wire lightweight and connectable, stays cool, and is cheaper than copper wire.Moreover, copper-clad aluminum wire is much less susceptible to the ever-present, construction site problem of theft because of lower salvage value than copper wire.Copper-clad aluminum wire comes 3/16 inch in diameter and can be drawn as fine as 0.005 inch in diameter without affecting the percentage copper cladding.An initial disadvantage of debonding during drawing has been substantially overcome, although careful control is still necessary.

Laminated Glass

The concept of protection of one layer of material by another as described under "clad metals" can be extended in a rather unique way to safety glass.Ordinary window glass is durable enough to retain its transparency under the extremes of weather.However, glass is quite brittle and is dangerous because it can break into many sharp-edged pieces.On the other hand, a plastic called polyvinyl butyral is very tough (deforms to high strains without fracture), but is very flexible and susceptible to scratching.Safety glass is a layer of polyvinyl butyral sandwiched between two layers of glass.The glass in the composite protects the plastic from scratching and gives it stiffness.The plastic provides the toughness of the composite.Thus, together, the glass and plastic protect each other in different ways and lead to a composite with properties that are vastly improved over those of its constituents.

Plastic Based Laminates

Many materials can be saturated with various plastics and subsequently treated for many purposes.The common product Formica is merely layers of heavy kraft paper impregnated with a phenolic resin overlaid by a plastic-saturated decorative sheet which, in turn, is overlaid with a plastic-saturated cellulose mat.Heat and pressure are used to bind the layers together.A useful variation on the theme is obtained when an aluminum layer is placed between the decorative layer and the craft paper layer to quickly dissipate the heat of, for example, a burning cigarette on a kitchen counter.

Layers of glass or asbestos fabrics can be impregnated with silicones to yield a composite with significant high-temperature properties.Glass or nylon fabrics can be laminated with various resins to yield an impact-and penetration-resistant composite that is uniquely suitable as lightweight personnel armor.The list of examples is seemingly endless, but the purpose of illustration is served by the preceding examples.

Laminated Fibrous Composites

Laminated fibrous composites are a hybrid class of composites involving both fibrous composites and lamination techniques.A more common name is laminated fiber-reinforced composites.Here, layers of fiber-reinforced material are built up with the fiber directions of each layer typically oriented in different directions to give different strengths and stiffnesses in the various directions.Thus, the strengths and stiffnesses of the laminated fiber-reinforced composite can be tailored to the specific design requirements of the structural element being built.Examples of laminated fiber-reinforced composites include Polaris missile cases, fiberglass boat hulls, aircraft wing panels and body sections, tennis rackets etc.

1.3 Mechanical Behavior of Composite Materials

Composite materials have many characteristics that are different from more conventional engineering materials.Some characteristics are merely modifications of conventional behavior; others are totally new and require new analytical and experimental procedures.Most common engineering materials are *homogeneous* and *isotropic* :

A *homogeneous* body has uniform properties throughout, i.e., the properties are not a function of *position* in the body.

An *isotropic* body has a material properties that are the same in every direction at a point in the body, i.e., the properties are not a function of *orientation* at a point in the body.

Bodies with temperature-dependent isotropic material properties are not homogeneous when subjected to a temperature gradient, but still are isotropic.

In contrast, composite materials are often both *inhomogeneous*(or heterogeneous - the two terms will be used interchangeably) and *nonisotropic* (orthotropic or, more generally, anisotropic):

An *inhomogeneous* body has nonuniform properties over the body, i.e., the properties are a function of *position* in the body.

An *orthotropic* body has material properties that are different in three mutually perpendicular directions at a point in the body and, further, have three mutually perpendicular planes of material symmetry.Thus the properties are a function of *orientation* at a point in the body.

An *isotropic* body has material properties that are different in alldirections at a point in the body.There are no planes of material property symmetry.Again, the properties are a function of *orientation* at a point in the body.

Some composite materials have very simple forms of inhomogeneity.For example. laminated safety glass has three layers each of which is homogeneous and isotropic: thus, the inhomogeneity of the composite is a step function in the direction perpendicular to the plane of the glass.Also, some particulate composites are inhomogeneous, yet isotropic although some are anisotropic.Other composite material are typically more complex.

Because of the inherent heterogeneous nature of composite materials, they are conveniently studied from two points of view: micromechanics and macromechanics:

Micromechanics is the study of composite material behavior wherein the *interaction* of the constituent materials is examined on a microscopic scale.

Macromechanics is the study of composite material behavior wherein the material is presumed *homogeneous* and the effects of the constituent materials are detected only as averaged apparent properties of the composite.

Use of both the concepts of macromechanics and micromechanics allows the *tailoring* of a composite material to meet particular structural requirement with little waste of material capability.The ability to tailor a composite material to its job is one of the most significant advantages of a composite over an ordinary material.Tailoring of a material yields only the stiffness and strength required in a given direction.In contrast, an isotropic material is, by definition, constrained to have excess strength and stiffness in any direction other than of the largest requirement.

The inherent anisotropy (most often only orthotropy) of composite materials leads to mechanical behavior characteristics that are quite different from those conventional isotropic materials.The behavior of isotropic, orthotropic, and anisotropic materials .Under loadings of normal stress and shear stress is shown in figure below and discussed in following paragraphs.

For isotropic materials, normal stress causes extension in the direction of the applied stress and contraction in the perpendicular direction. Also, shear stress causes only shearing deformation

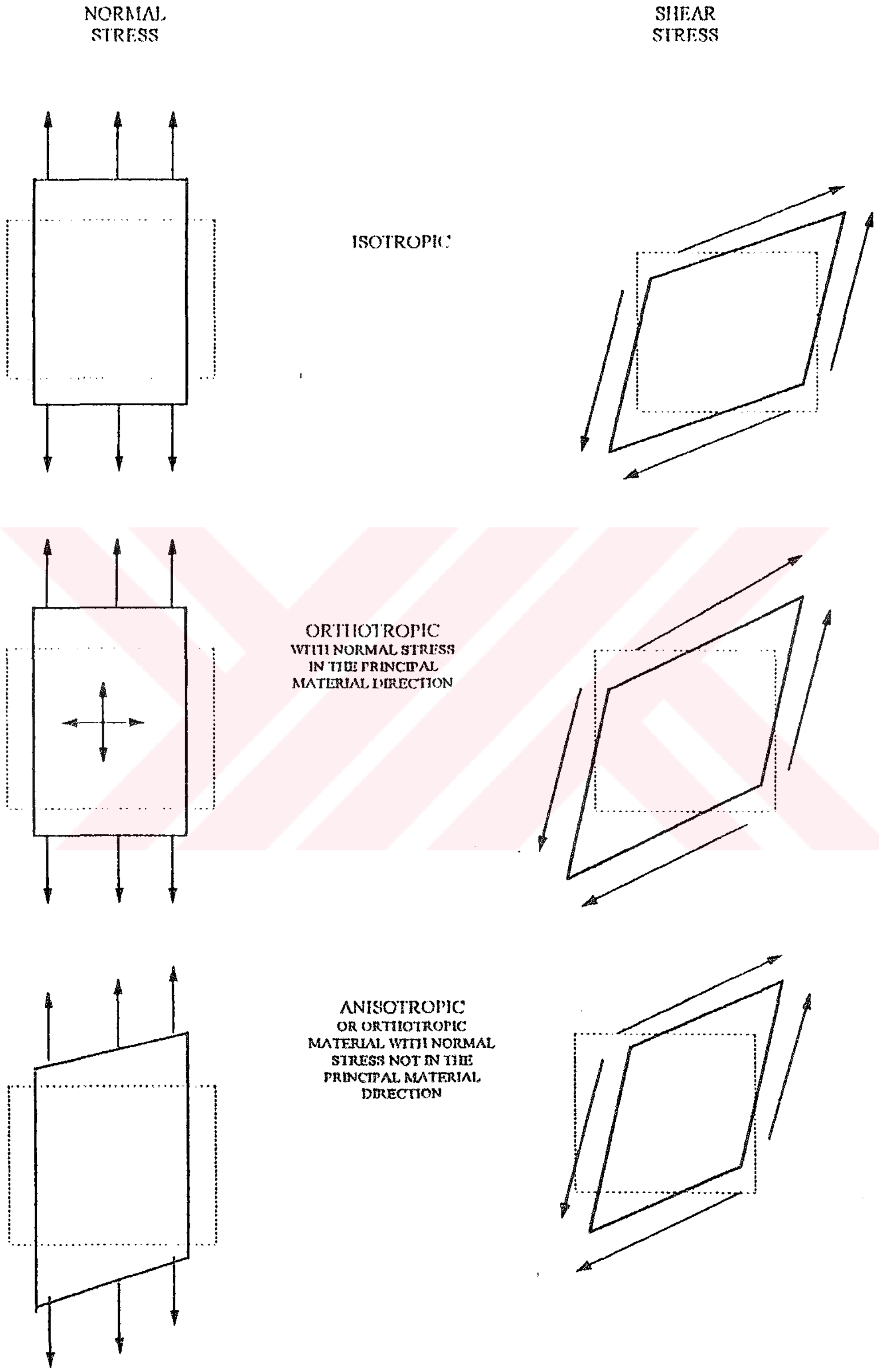


Figure 1.3 Mechanical behavior of anisotropic materials

For orthotropic materials, like isotropic materials, normal stress in a principal material direction (along one of the intersections of three orthogonal planes of material symmetry) results in extension in the direction of the applied stress and contraction perpendicular to the stress.However, due to different properties in the two principal material directions, the contraction can be either more or less than the contraction of a similarly loaded isotropic material with the same elastic modulus in the direction of the load.Shear stress causes shearing deformation, but the magnitude of the deformation is independent of the various Young's moduli and Poisson's ratios.That is the shear modulus of an orthotropic material is, unlike isotropic materials, not dependent on other material properties.

For anisotropic materials, application of a normal stress leads not only to extension in the direction of the stress and contraction perpendicular to it, but to shearing deformation.Conversely, shearing stress causes extension and contraction in addition to the distortion of shearing deformation.This coupling between both loading modes and both deformation modes is also characteristic of orthotropic materials subjected to normal stress in a nonprincipal material direction.For example, cloth is an orthotropic material composed of two sets of interwoven fibers at right angles to each other.If cloth is subjected to stress at 45 ° to a fiber direction, both stretching and distortion occur.

Coupling between deformation modes and loading modes creates problems that are not easily overcome and, the very least, cause a reorientation of thinking.For example the conventional dog-bone tensile specimen shown in figure below obviously can not be used to determine the tensile moduli of anisotropic materials.

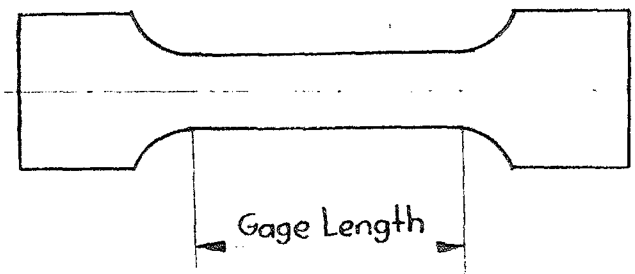


Figure 1.4 Dog-Bone tensile specimen

For an isotropic material, loading on a dog-bone is actually a prescribed lengthening which is only coincidentally a prescribed stress due to the symmetry of an isotropic material.However, for an isotropic material, only the prescribed lengthening occurs due to the lack of symmetry of the material and the clamped ends of the specimen.Accordingly, shearing stresses result in addition to normal stress.Furthermore, the specimen has a tendency to bend.Thus, the strain measured in specimen gage in figure above can not be used with the axial stiffness or modulus.Techniques more sophisticated than dog-bone test must be used to determine the mechanical properties of a composite material.

The foregoing characteristics of the mechanical behavior of composite materials have been presented without proof.

1.4 Basic Terminology of Laminated Fiber-Reinforced Composite Materials

For the remainder of this study, emphasis will be placed on fiber-reinforced composite laminates.The fibers are long and continuous as opposed to whiskers; hence, the name filamentary composite is often used.The concepts developed herein are applicable mainly to fiber-reinforced composite laminates, but are also valid for other laminates and whisker composites with some fairly obvious modifications.That is, fiber-reinforced composite laminates are used as a uniform example throughout this study, but concepts used to analyze their behavior are often applicable to other composite materials.In many instances, the applicability will be made clear as an example complementary to the principal example of fiber-reinforced composite laminates.

The basic terminology of fiber- reinforced composite laminates will be introduced in the following paragraphs.For a lamina, the configurations and functions of the constituent materials, fibers in matrix, will be described.The characteristics of the fibers and matrix are then discussed.Finally, a laminate is defined to round out this introductions to the characteristics of fiber- reinforced composite laminates.

1.4.1 Laminae

A lamina is a flat (some times curved as in a shell) arrangement of unidirectional fibers or woven fibers in a matrix.Two typical laminae are shown in figure below along with their principal material axes which are parallel and perpendicular to the fiber directions.The fibers, or filaments, are the principal reinforcing or load-carrying agent.They typically strong and stiff.The matrix can be organic, ceramic, or metallic.The function of the matrix is to support and protect the fibers and to provide a means of distributing load among and transmitting load between the fibers.

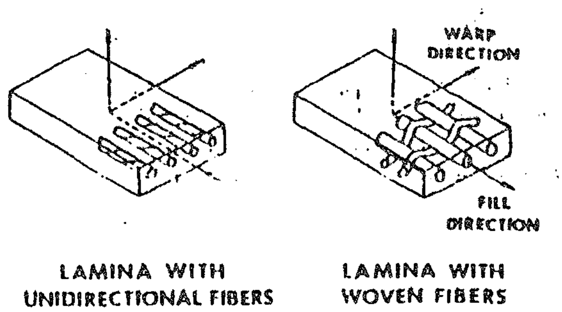


Figure 1.5 Two principal types of laminae

The latter function is especially important if a fiber breaks as figure below. There, load from one side of a broken fiber as well as to adjacent fibers. The mechanism for the load transfer is the shearing stress developed in the matrix; the shearing stress the pulling out of the broken fiber. This load-transfer mechanism is the means by which whisker- reinforced composites carry any load at all above the inherent matrix strength.

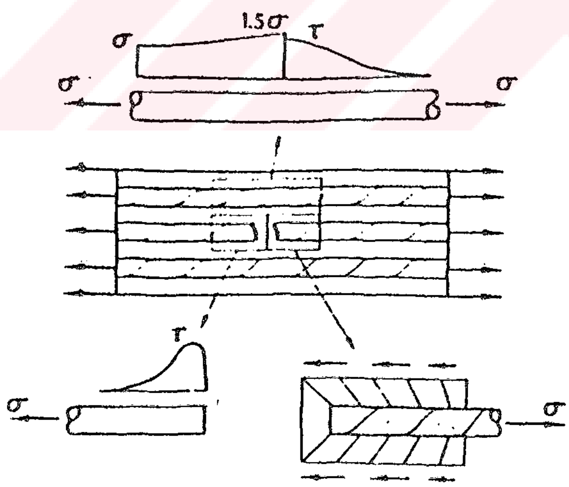


Figure 1.6 Effect of broken fiber on matrix and fiber stresses

The properties of the lamina constituents, the fibers and the matrix, have been only briefly discussed so far.Their stress-strain behavior is typified as one of the classes depicted in Figure below.Fibers generally exhibit linear elastic behavior, although reinforcing steel bars in concrete are more nearly elastic-perfectly plastic.Aluminum and some composites exhibit elastic-plastic behavior which is really nonlinear elastic behavior if there is no unloading.Commonly resinous matrix materials are viscoplastic.The various stress-strain relations are referred to as constitutive relations as they describe the mechanical constitution of the material.

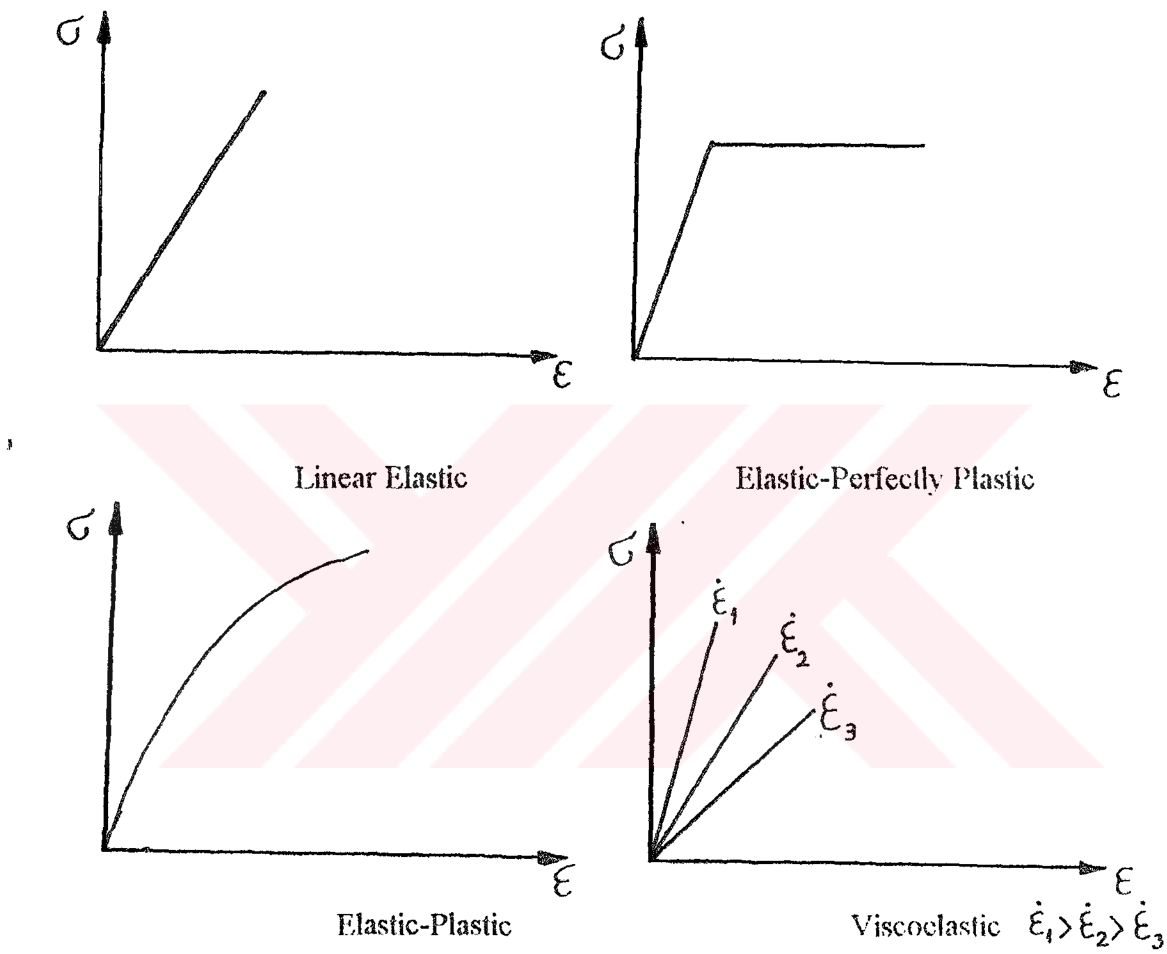


Figure 1.7 Various Stress-Strain behaviors

Fiber-reinforced composites such as boron-epoxy and graphite-epoxy are usually treated as linear elastic materials since the fibers provide the majority of the strength and stiffness.Refinement of that approximation requires consideration of some form of plasticity, viscoelasticty, or both (viscoplasticty).Very little work has been done to implement those idealizations of composite material behavior in structural applications.

1.4.2 Laminates

A *laminate* is a stack of laminae with various orientations of principal material directions in the laminae as in figure below.Note that the fiber orientation of the layers in this figure is not symmetric about the middle surface of the laminate.This situation will be discussed in following pages.The layers of a laminate are usually bound together by the same matrix material that is used in the laminae.Laminates can be composed of plates of different materials or, in the present context, layers of fiber-reinforced laminae.A laminated circular cylindrical shell can be constructed by winding resin-coated fibers on a mandrel first with one orientation to the shell axis, then another, and so until the desired thickness is built up.

A major purpose of lamination is to tailor the directional dependence of strength and stiffness of a material to match the loading environment of the structural element.Laminates are uniquely suited to this objective since the principal material directions of each layer can be oriented according to need.For example, six layers of a ten-layer laminate could be oriented in one direction and the other four at 90° to that direction; resulting laminate then has a strength and extensional stiffness roughly 50 percent higher in one direction than the other.The ratio of the extensional stiffness in the two directions is approximately 6/4, but the ratio of bending stiffness is unclear since the order of lamination is not specified in the example.Moreover, if the laminae are not arranged symmetrically about the middle surface of the laminate, stiffness exist that describe coupling between bending and extension.These characteristics will be discussed in following pages.

A potential problem in the construction of laminates is the introduction of shearing stresses between layers.The shearing stresses arise due to the tendency of each layer to deform independently of its neighbors because all may have different properties (at least from the standpoint of orientation of principal material directions).Such shearing stresses are largest at the edges of a laminate and may cause delamination there.As will be shown in following pages the transverse normal stress can also cause delamination.

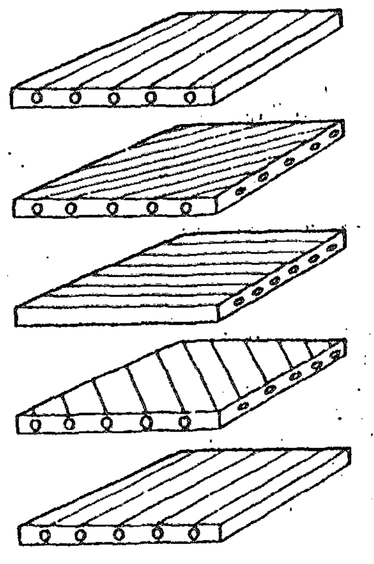


Figure 1.8 Laminate construction

MACROMECHANICAL BEHAVIOR OF A LAMINATE

2.1. Introduction

A laminate is two or more laminae bonded together to act as an integral structural element (Fig.2.1). The laminae principal material directions are oriented to produce a structural element capable of resisting load in several directions. The stiffness of such a composite material configuration is obtained from the properties of the constituent laminae. The procedures enable the analysis of laminates that have individual laminae with principal material directions oriented at arbitrary angles to the chosen or natural axes of the laminate. As a consequence of the arbitrary orientations, the laminate may not have definable principal directions.

2.2. Stress-Strain Relations For Plane Stress in An Orthotropic Material

For a lamina in the *1-2* plane as shown in Fig. 2.1, a plane stress state is defined by setting

$$\sigma_3 = 0 \qquad \tau_{23} = 0 \qquad \tau_{31} = 0 \tag{2.1}$$

For orthotropic materials, such a procedure results in implied strains of

$$\varepsilon_3 = S_{13}\sigma_1 + S_{23}\sigma_2 \qquad \gamma_{23} = \gamma_{31} = 0 \tag{2.2}$$

The stress-strain relations are

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \tag{2.3}$$

The preceding stress-strain and strain-stress relations are basis for the stiffness and stress analysis an individual lamina subjected to forces in its own plane. The relations are therefore indispensable in the analysis of laminates.

2.3. Stress-Strain Relations For a Lamina of Arbitrary Orientation

The stress and strains were defined in the principal material directions for an orthotropic material. However, the principal directions of orthotropy often do not coincide with coordinate directions that are geometrically natural to the solution of the problem. For example, consider the helically wound fiberglass-reinforced circular cylindrical shell in Fig. 2.2.a. There, the coordinates natural to the solution of the shell problem are shell coordinates x, y, z whereas the principal material coordinates are x', y', z'. The wrap angle is defined by $Cos(y',y)=Cos\alpha$; also $z'=z$. Other examples include laminated plates with different laminate at different orientations. Thus, a relation is needed between the stresses and strains in the principal material directions and those in the body coordinates. Then, a method of transforming stress-strain relations from one coordinate system to another is also needed. At this point, from elementary mechanics of materials, the transformation equations for expressing stresses in an *x-y* coordinate system in terms of stresses in a *1-2* coordinate system are

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2-n^2 \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2-n^2 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \qquad (2.7)$$

where $m=Cos\theta$ and $n=Sin\theta$ and θ is the angle from the x-axis to the 1-axis (Fig.2.2.b). This square matrix $[T]$ is transformation matrix. The stress-strain relations in xy coordinates are

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [\bar{Q}] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad (2.8)$$

supplemented by Eq. (2.2) where

$$S_{11}=\frac{1}{E_1},\qquad S_{22}=\frac{1}{E_2},\qquad S_{66}=\frac{1}{G_{12}}\qquad S_{12}=-\frac{\nu_{12}}{E_1}=-\frac{\nu_{21}}{E_2},\tag{2.4}$$

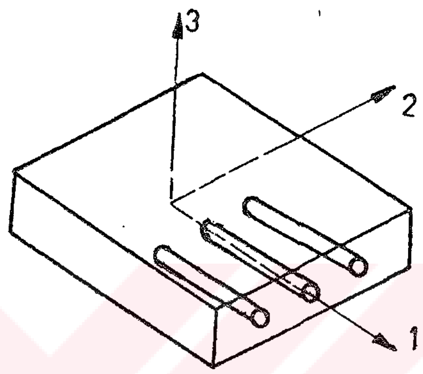


Fig.2.1. Unidirectionally reinforced lamina.

The strain-stress relations in Eq. (2.3) can be inverted to obtain the stress-strain relations:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix}\tag{2.5}$$

where the Q_{ij}, the called reduced stiffnesses are

$$Q_{11}=\frac{E_1}{1-\nu_{12}\nu_{21}},\quad Q_{22}=\frac{E_2}{1-\nu_{12}\nu_{21}},\quad Q_{12}=\frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}=\frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}},\quad Q_{66}=G_{12}\tag{2.6}$$

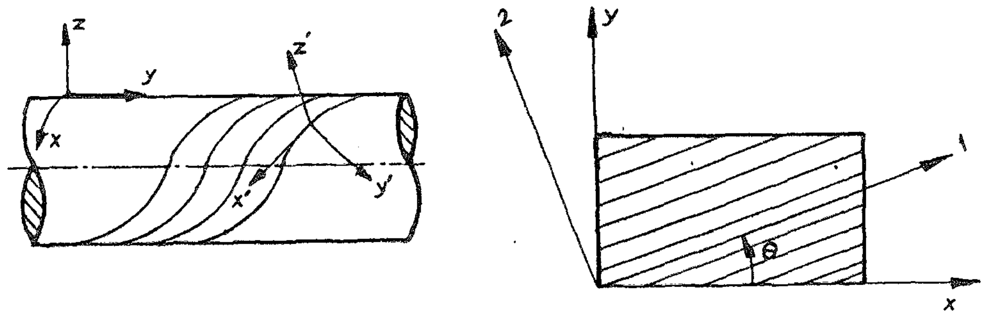


Fig. 2.2. a) Helically wound fiber-reinforced circular cylindrical shell **b)** Positive rotation of principal material axes from arbitrary *xy* axes

in which

$$
\begin{aligned}
\bar{Q}_{11} &= Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4 \\
\bar{Q}_{22} &= Q_{11}n^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}m^4 \\
\bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(n^4 + m^4) \\
\bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})m^3n + (Q_{12} - Q_{22} + Q_{66})n^3m \\
\bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})n^3m + (Q_{12} - Q_{22} + Q_{66})m^3n \\
\bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{66})m^2n^2 + Q_{66}(n^4 + m^4)
\end{aligned}
\tag{2.9}
$$

where the bar over the $\bar{Q}_{ij}$ matrix denotes that it is dealing with the transformed reduced stiffnesses instead of the reduced stiffnesses, Q_{ij}.

2.4. Laminated Composite Materials

Classical lamination theory embodies a collection of stress and deformation hypotheses. By use of this theory, it can be consistently proceed from the basic building block, the lamina, to the end result, a structural laminate. Actually, because of the stress and deformation hypotheses that are inseparable part of classical lamination theory, a more correct name would be classical thin lamination theory, or even classical laminated plate theory.

2.4.1. Lamina Stress-Strain Behavior

The stress-strain relations in principal material coordinates for a lamina of an orthotropic material under plane stress are given in Eq.(2.5).The stress-strain relations in arbitrary coordinates, Eq.(2.8), are useful in the definition of the arbitrary orientation of the constituent laminae. Both Eqs.(2.5) and (2.8) can be thought of as stress-strain relations for the k^{th} layer of a multilayered laminate. Thus, Eq. (2.8) can be written as

$$\{\sigma\}_k = [\overline{Q}]_k \{\varepsilon\}_k \tag{2.10}$$

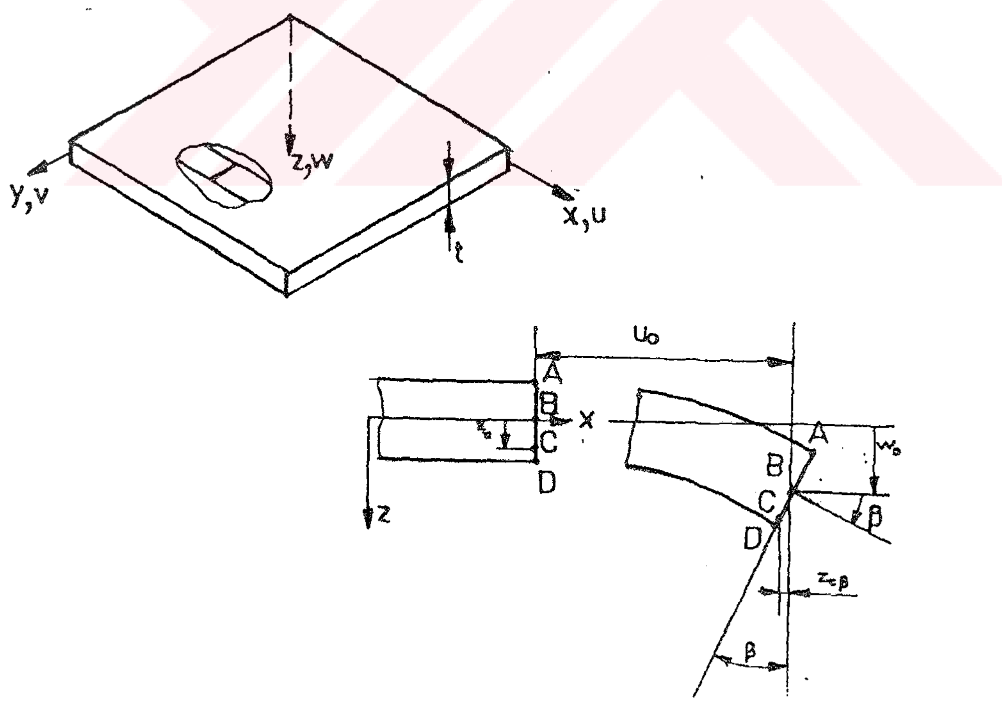


Fig. 2.2. Geometry of deformation in the *xz* plane

2.4.2. Strain and Stress Variation in a Laminate:

It will be proceeded in this section to define the strain and stress variations through the thickness of a laminate. The resultant forces and moments on a laminate will then be obtained by integrating Eq.(2.10) through the laminate thickness.

Knowledge of the variation of stress and strain through the laminate thickness is essential to the definition of the extensional and bending stiffnesses of a laminate. The laminate is presumed to consist of perfectly bonded laminae. Moreover, the bonds are presumed to be infinitesimally thin as well as non-shear deformable. That is, the displacements are continuous across lamina boundaries so that no lamina can slip relative to another. Thus, the laminate acts as a single layer with very special properties, but nevertheless acts as a single layer of material.

The implications of the Kirchhoff or the Kirchhoff -Love hypothesis on the laminate displacements u, v and w in the x, y and z directions are derived by use of the laminate cross section in the xz plane shown in Fig. 2.2. The displacements in the x-direction of point B from the undeformed to the deformed middle surface is u_o. Since line ABCD remains straight under deformation of the laminate ,

$$u_c = u_o - z_c \beta_x \qquad v_c = v_o - z_c \beta_y \tag{2.11}$$

But since, under deformation, line ABCD further remains perpendicular to the middle surface, β_x , β_y are the slopes of the laminate middle surface in the x-direction and y-direction, respectively, that is,

$$u_c = u_o - z_c \frac{\partial w_o}{\partial x} \qquad v_c = v_o - z_c \frac{\partial w_o}{\partial y} \tag{2.12}$$

The laminate strains have been reduced to ε_x, ε_y and γ_{xy} by virtue of the Kirchhoff hypothesis. For small strains, the remaining strains are defined in terms of displacements as

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \varepsilon_y = \frac{\partial v}{\partial y} \qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{2.13}$$

Thus, for the derived displacements u and v in Eq. (2.12), the strains are

$$\varepsilon_x = \frac{\partial u_o}{\partial x} - z\frac{\partial^2 w_o}{\partial x^2}, \quad \varepsilon_y = \frac{\partial v_o}{\partial y} - z\frac{\partial^2 w_o}{\partial y^2} \quad and \quad \gamma_{xy} = \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} - 2z\frac{\partial^2 w_o}{\partial x \partial y} \tag{2.14}$$

or in matrix form

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + z\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{2.15}$$

where the middle surface strains and the middle surface curvatures are respectively

$$\begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_o}{\partial x} \\ \dfrac{\partial v_o}{\partial y} \\ \dfrac{\partial u_o}{\partial y} + \dfrac{\partial v_o}{\partial x} \end{Bmatrix}, \qquad \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = -\begin{Bmatrix} \dfrac{\partial^2 w_o}{\partial x^2} \\ \dfrac{\partial^2 w_o}{\partial y^2} \\ 2\dfrac{\partial^2 w_o}{\partial x \partial y} \end{Bmatrix}. \tag{2.16}$$

Thus, the Kirchhoff hypothesis has been readily verified to imply a linear variation of strain through the laminate thickness. Because of the strain-displacement relations in Eq. (2.13), the foregoing strain analysis is valid only for plates.

By substitution of the strain variation through the thickness, Eq. (2.16), in the stress-strain relations, Eq.(2.10), the stresses in the k^{th} layer can be expressed in terms of the laminate middle surface strains and curvatures as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \left\{ \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \right\} \tag{2.17}$$

Since the $\bar{Q}_{ij}$ can be different for each layer of the laminate, the stress variation through the laminate thickness is not necessarily linear, even though the strain variation is linear. Instead, typical strain and stress variations are shown in Fig. 2.4.

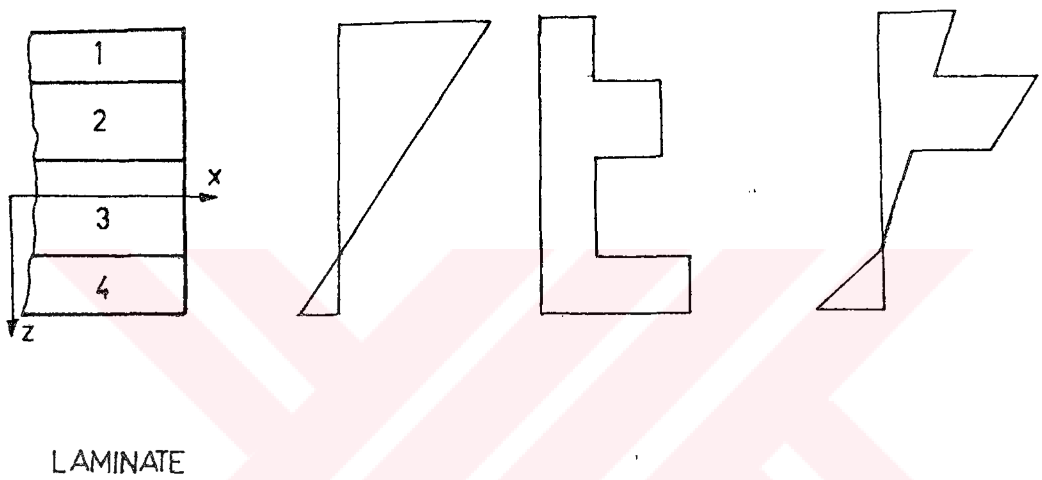


Fig. 2.4. Hypothetical variation of strain and stress through the laminate thickness.

2.4.3. Resultant Laminate Forces and Moments:

For a laminated plate thickness t, the stress resultants and stress couples are defined as

$$\begin{Bmatrix} N_x & M_x \\ N_y & M_y \\ N_{xy} & M_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k \{1, z\} dz, \qquad \begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix}_k dz \tag{2.18}$$

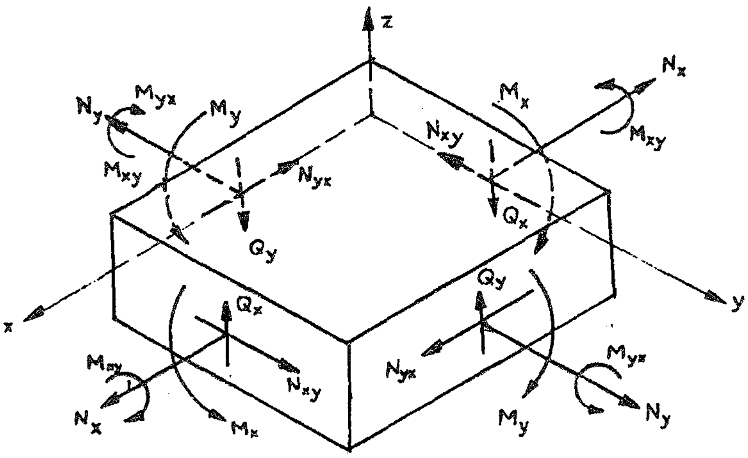


Fig. 2.5. Stress resultants and stress couples

Actually, in-plane stress resultants per unit length (width) of the cross section of the laminate and moment resultants per unit length for an N-layered laminate are depicted in Fig. 2.5 and are defined as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k dz = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k dz \tag{2.19}$$

and

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k zdz = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k zdz \tag{2.20}$$

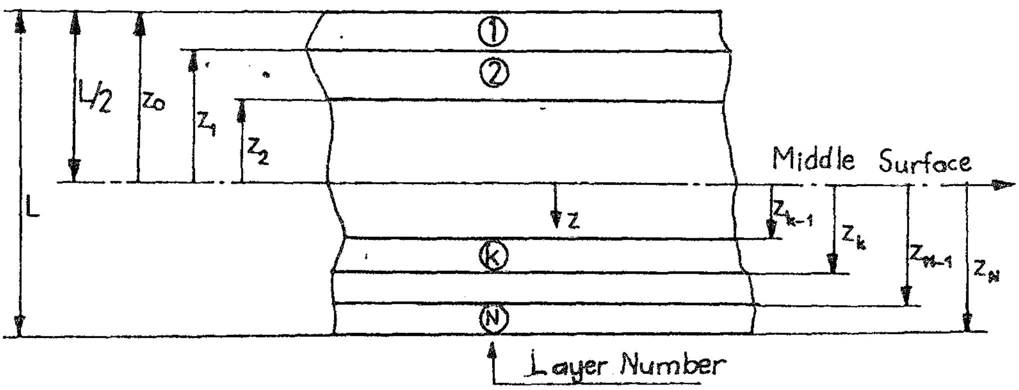


Fig. 2.6. Geometry of an n-layered laminate

where z_k and z_{k-1} are defined in Fig.2.6. It may be noted that z_o -$t/2$. These force and moment resultants do not depend on z after integration, but are functions of x and y, the coordinates in the plane of the laminate middle surface.The integration indicated in Eqs. (2.19) and (2.20) can be rearranged to take advantage of the fact that the stiffness matrix for a lamina is constant within the lamina. Thus, the stiffness matrix goes outside the integration over each layer, but is within the summation of force and moment resultants for each layer. When the lamina stress-strain relations, Eq.(2.17), are substituted

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \left\{ \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} dz + \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} z dz \right\} \quad (2.21.a)$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \left\{ \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} z dz + \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} z^2 dz \right\} \quad (2.21.b)$$

However, it should recalled that ε_x^o, ε_y^o, γ_{xy}^o, κ_x, κ_y and κ_{xy} are not functions of z but are middle surface values so can be removed from under the summation signs. Thus, Eqs. (2.21.a and b) can be written as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (2.22.a)$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (2.22.b)$$

where

$$A_{ij} = \sum_{k=1}^{N} (\overline{Q}_{ij})_k (z_k - z_{k-1}) \quad \text{or} \quad A_{ij} = \sum_{k=1}^{N} (\overline{Q}_{ij})_k t_k \tag{2.23.a}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{N} (\overline{Q}_{ij})_k (z_k^2 - z_{k-1}^2) \quad \text{or} \quad B_{ij} = \sum_{k=1}^{N} (\overline{Q}_{ij})_k t_k \bar{z}_k \tag{2.23.b}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{N} (\overline{Q}_{ij})_k (z_k^3 - z_{k-1}^3) \quad \text{or} \quad D_{ij} = \sum_{k=1}^{N} (\overline{Q}_{ij})_k (t_k \bar{z}_k^2 + \frac{t_k^3}{12}) \tag{2.23.c}$$

wherein t_k is the thickness and $\bar{z}_k$ is the distance to the centroid of the k^{th} layer. In Eqs. (2.23.a,b and c), the A_{ij} are called extensional stiffnesses, the B_{ij} are called coupling stiffnesses, and the D_{ij} are called bending stiffnesses.

2.4.4. Laminated Composites Accounting for Transverse Shear Deformation:

It is necessary to include transverse shear deformation in the analysis of most plate structures composed of composite materials. Hence, for a given in-plane modulus, the plate is very weak in transverse shear resistance, and the effects of transverse shear deformation are significant, and cannot be neglected. Therefore in this case, Eq. (2.5) is modified to be

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} \tag{2.24}$$

Here, Q_{11}, Q_{22}, Q_{12} and Q_{66} are given by Eq. (2.6). Q_{44} and Q_{55} are as follows

$$Q_{44}=G_{23} \quad \text{and} \quad Q_{55}=G_{31} \tag{2.25}$$

The stress-strain relations for a generally orthotropic lamina including transverse shear deformation are, referring to Eq. (2.8)

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & 0 & 0 & \overline{Q}_{16} \\ \overline{Q}_{21} & \overline{Q}_{22} & 0 & 0 & \overline{Q}_{26} \\ 0 & 0 & \overline{Q}_{44} & \overline{Q}_{45} & 0 \\ 0 & 0 & \overline{Q}_{45} & \overline{Q}_{55} & 0 \\ \overline{Q}_{16} & \overline{Q}_{26} & 0 & 0 & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{Bmatrix} \tag{2.26}$$

The majority of the $\overline{Q}$ quantities were defined as bellows

$$\overline{Q}_{44} = Q_{44}m^2 + Q_{55}n^2, \quad \overline{Q}_{55} = Q_{44}n^2 + Q_{55}m^2, \quad \text{and} \quad \overline{Q}_{45} = (Q_{55} - Q_{44})mn \tag{2.27}$$

With these changes, all stress resultantsand stress couples can be expressed for a laminate composed of k laminae.

2.5. Laminated Plates

In this section, single-layered configurations are treated first to provide a baseline for multiple-layered configurations. Next, laminates that are symmetric about their middle surface are classified. Then, laminates with laminae that are antisymmetrically disposed about their middle surface are described. Finally, laminates with complete lack of middle surface symmetry are discussed.

2.5.1. Single-Layered Configurations:

The special single-layered configurations treated in this section are isotropic, specially orthotropic, generally orthotropic, and anisotropic.

Single Isotropic Layer:

For a single isotropic layer with material properties, E and ν, and thickness, t the laminate stiffness reduce to, $B_{ij} = 0$,

$$A_{11} = \frac{Et}{1-\nu^2} = A, \qquad D_{11} = \frac{Et^3}{12(1-\nu^2)} = D, \qquad A_{66} = \frac{1-\nu}{2}A, \qquad D_{66} = \frac{1-\nu}{2}D$$

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A & \nu A & 0 \\ \nu A & A & 0 \\ 0 & 0 & \frac{1-\nu}{2}A \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix}, \qquad \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D & \nu D & 0 \\ \nu D & D & 0 \\ 0 & 0 & \frac{1-\nu}{2}D \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{2.28}$$

Single Specially Orthotropic Layer:

For a single specially orthotropic layer of thickness, t, and lamina stiffnesses, Q_{ij}, given by Eq. (2.6), the laminate stiffnesses are, $B_{ij}=0$,

$$(A_{11}, A_{12}, A_{22}, A_{66}) = (Q_{11}, Q_{12}, Q_{22}, Q_{66})t, \qquad (D_{11}, D_{12}, D_{22}, D_{66}) = (Q_{11}, Q_{12}, Q_{22}, Q_{66})\frac{t^3}{12}$$

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix}, \qquad \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{21} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{2.29}$$

Single Generally Orthotropic Layer:

For a single generally orthotropic layer of thickness, t, and lamina stiffnesses, Q_{ij}, given by Eq. (2.9), the laminate stiffnesses are

$$A_{ij} = \overline{Q}_{ij}t, \qquad D_{ij} = \overline{Q}_{ij}\frac{t^3}{12} \qquad B_{ij}=0$$

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix}, \qquad \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{21} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{2.30}$$

Single Anisotropic Layer:

The only difference in appearance between a single generally orthotropic layer and an anisotropic layer is that the latter has lamina stiffnesses, Q_{ij}, whereas the generally orthotropic layer has stiffnesses, $\overline{Q}_{ij}$. The laminate stiffnesses are

$$A_{ij} = \overline{Q}_{ij}t, \qquad D_{ij} = \overline{Q}_{ij}\frac{t^3}{12} \qquad B_{ij}=0$$

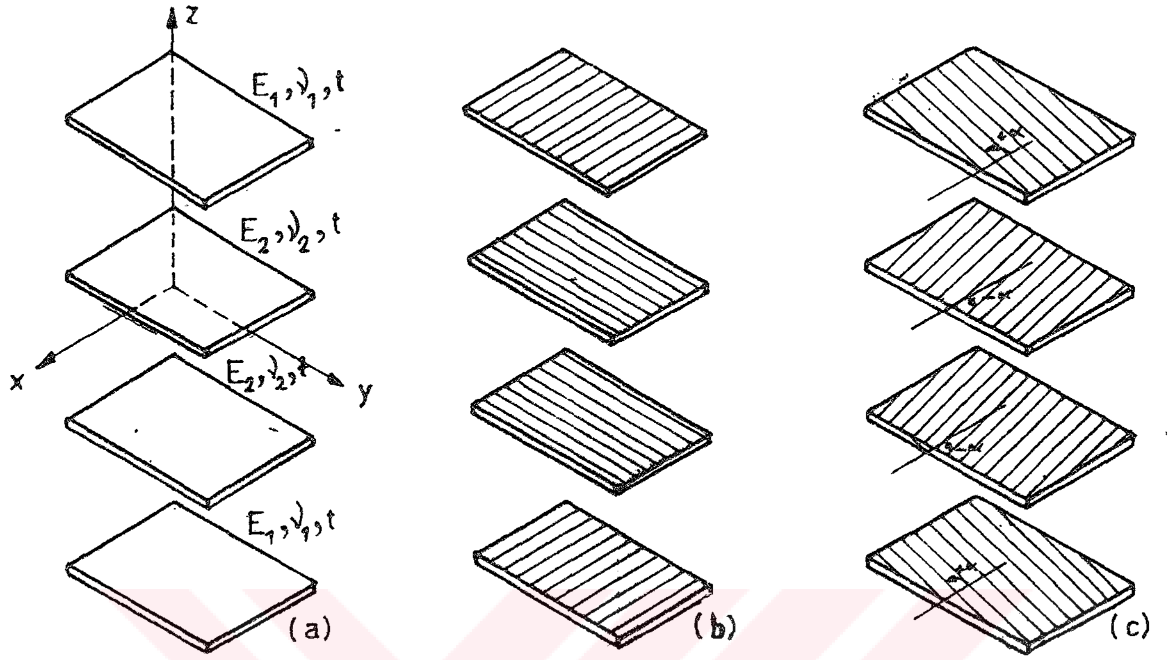


Fig. 2.7. Unbounded view of a four-layered **a)** symmetric laminate with isotropic layers **b)** regular symmetric cross-ply laminate **c)** regular symmetric angle-ply laminate

2.5.2. Symmetric Laminates:

For laminates that are symmetric in both geometry and material properties about the middle surface, the general stiffness equations, Eq.(2.23), simplify. In particular, because of the symmetry of the $\left(\bar{Q}_{ij}\right)_k$ and the thicknesses t_k, all the coupling stiffnesses, that is, the B_{ij}, can be shown to be zero. The elimination of coupling between bending and extension has two important practical ramifications. First, such laminates are usually much easier to analyze than laminates with coupling. Second, symmetric laminates do not have a tendency to twist from the inevitable thermally induced contractions that occur during cooling following the curing process. Consequently, symmetric laminates are commonly used unless special circumstances require an unsymmetric laminate.

Symmetric Laminates with multiple Isotropic Layers:

If multiple isotropic layers of various thicknesses are arranged symmetrically about a middle surface from both a geometric and a material property standpoint, the resulting laminate does not exhibit coupling between bending and extension. A simple example of a symmetric laminate with three isotropic layers is shown in Fig. 2.7.a. The extensional and bending stiffnesses for the general case are calculated from Eq.(2.23) wherein for the k^{th} layer

$$(\bar{Q}_{11})_k = (\bar{Q}_{22})_k = \frac{E_k}{1-\nu_k^2}, \qquad (\bar{Q}_{12})_k = \frac{\nu_k E_k}{1-\nu_k^2} \qquad (\bar{Q}_{66})_k = \frac{E_k}{2(1-\nu_k^2)}$$

The force and moment resultants take the form

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix}, \qquad \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{21} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \qquad (2.31)$$

Symmetric Laminates with multiple Specially Orthotropic Layers:

Because of the analytical complications involving the stiffnesses A_{16}, A_{26}, D_{16}, and D_{26}, a laminate is desired that does not have these stiffnesses. Laminates can be made with orthotropic layers that have principal material directions aligned with the laminate axes. If the thicknesses, locations, and material properties of the laminae are symmetric about the middle surface of the laminate, there is no coupling between bending and extension. The extensional and bending stiffnesses are calculated from Eq.(2.23) wherein the k^{th} layer

$$(\bar{Q}_{ii})_{i=1,2}^k = \frac{E_i^k}{1-\nu_{12}^k \nu_{21}^k}, \qquad (\bar{Q}_{12})_k = \frac{\nu_{12}^k E_1^k}{1-\nu_{12}^k \nu_{21}^k}, \quad (\bar{Q}_{66})_k = G_{12}^k, \quad (\bar{Q}_{16})_k = (\bar{Q}_{26})_k = 0$$

Because $(\overline{Q}_{16})_k$ and $(\overline{Q}_{26})_k$ are zero, the stiffnesses A_{16}, A_{26}, D_{16} and D_{26} vanish. Also, The stiffnesses B_{ij} are zero because of symmetry. The force and moment resultants take the form of Eqs.(2.31). A very common special case of symmetric laminates with multiple specially orthotropic layers occurs when the laminae are all of the same thickness and material properties, but have their major principal material directions alternating at $0°$ and $90°$ to the laminate axes, for example $0°/90°/90°/0°$. A simple example of a regular symmetric cross-ply laminate with four layers of equal thickness and properties is shown in Fig.2.7.b. Such laminates are called regular symmetric cross-ply laminates

Symmetric Laminates with multiple Generally Orthotropic Layers:

A laminate of multiple generally orthotropic layers that are symmetrically disposed about the middle surface exhibits no coupling between bending and extension; that is, B_{ij} are zero. Therefore, the force and moment resultants are represented by Eq. (2.30). There, all the A_{ij}, and D_{ij}, are required because of coupling between normal forces and shearing strain, shearing force and normal strains, normal moments and twist, and twisting moments and normal curvatures. Such coupling is evidenced by the A_{16}, A_{26}, D_{16} and D_{26} stiffnesses. A special subclass of this class of symmetric laminates is the regular symmetric angle-ply laminate. Such laminates have orthotropic laminae of equal thicknesses. The adjacent laminae have opposite signs of the angle of orientation of the principal material properties with respect to the laminate axes, for example, $+\alpha/-\alpha/-\alpha/+\alpha$, for four layers shown in Fig.2.7.c.

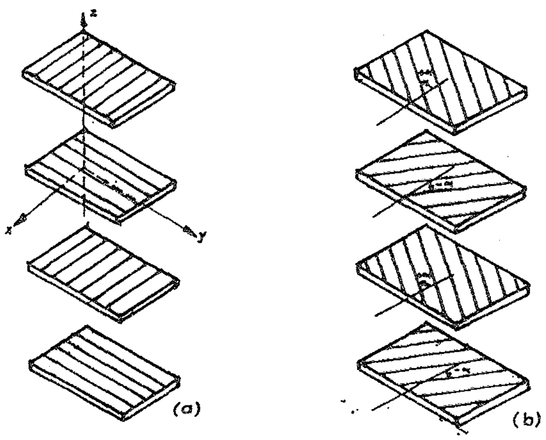


Fig. 2.8. Unbounded view of a four-layered regular antisymmetric **a)** cross-ply , **b)** angle-ply laminate

Symmetric Laminates with multiple Anisotropic Layers:

The general case of a laminate with multiple anisotropic layers symmetrically disposed about the middle surface does not have any stiffness simplifications other than the elimination of the B_{ij} by virtue of symmetry. The A_{16}, A_{26}, D_{16} and D_{26} stiffnesses all exist and do not necessarily. go to zero as the numbers of layers is increased. Thus, many of the stiffness simplifications possible for other laminates cannot be achieved for this class.

2.5.3. Antisymmetric Laminates:

Symmetry of a laminate about the middle surface is often desirable to avoid coupling between bending and extension. However, many physical applications of laminated composites require nonsymmetric laminates to achieve design requirements. For example, coupling is a necessary feature to make jet turbine fan blades with pretwist.

The orientations for an antisymmetric laminate with four-layers alternate from layer to layer, e.g., $+\alpha/-\alpha/+\alpha/-\alpha$. Therefore, symmetry about the middle surface is destroyed and the behavioral characteristics of the laminate can be substantially chanced from the symmetric case. Each pair of laminae must have the same thickness.

Antisymmetric Cross-Ply Laminates:

An antisymmetric cross-ply laminate consists of an even number of orthotropic laminae laid on each other with principal material directions alternating at 0° and 90° to the laminate axes as shown in Fig. 2.8.a. The force and moment resultants are calculated by using Eqs.(2.22.a and b) but in these equations:

$$A_{16}=A_{26}=B_{12}=B_{16}=B_{26}=B_{66}=D_{16}=D_{26}=0 \quad \text{and} \quad B_{22}=-B_{11} \qquad (2.32)$$

A regular antisymmetric cross-ply laminate is defined to have laminae all of equal thickness and is common because of simplicity of fabrication. As the number of layers increases, the coupling stiffness B_{11} can be shown to approach zero.

Antisymmetric Angle-Ply Laminates:

An antisymmetric angle-ply laminate has laminae oriented at $+\alpha$ degrees to the laminate coordinate axes on one side of the middle surface and corresponding equal thickness laminae oriented at $-\alpha$ degrees on the other side. A simple example of an antisymmetric angle-ply laminate with four layers is shown in Fig. 2.8.b. The force and moment resultants for this kind of laminates are calculated by using Eqs.(2.22.a and b) but in these equations:

$$A_{16}=A_{26}=B_{11}=B_{12}=B_{22}=B_{66}=D_{16}=D_{26}=0 \tag{2.33}$$

The coupling stiffnesses B_{16} and B_{26} can be shown to go to zero as the number of layers in the laminate increases for a fixed laminate thickness.

2.5.4. Nonsymmetric Laminates:

No special reduction of the stiffnesses is possible when t_k is arbitrary. That is, coupling between bending and extension can be obtained by unsymmetric arrangement about the middle surface of isotropic layers with different thicknesses. Thus, coupling between bending and extension is not a manifestation of material orthotropy but rather of laminate heterogeneity; that is, a combination of both geometric and material properties. The force and moment resultants are calculated by using Eqs.(2.22.a and b) but in these equations:

$$A_{16}=A_{26}=B_{16}=B_{26}=D_{16}=D_{26}=0 \tag{2.34}$$

Nonsymmetric laminates with multiple specially orthotropic layers can be shown to have the force and moment resultants in Eqs. (2.34.a and b) but with different A_{22}, B_{22} and D_{22}

from A_{ij}, B_{ij}, and D_{ij}, respectively. That is, there are no shear coupling terms, and therefore the solution of problems with this kind of lamination is about as easy as with isotropic layers.

Nonsymmetric laminates with multiple generally orthotropic layers or with multiple anisotropic layers have force and moment resultants no simpler than Eqs. (2.22.a and b) All stiffnesses are present. Hence, configurations with either of those two laminae are much more difficult to analyze than configurations with either multiple isotropic layers or multiple specially orthotropic layers.

Chapter III

3.1 Metal-Matrix Composite Materials

The technology of metal-matrix composite materials is being developed very rapidly.Compared to glass-fiber-reinforced plastics, metal-matrix composites are superior for their performance at elevated temperatures.The strength and elastic moduli of metal matrices are higher than those resin matrices over a wide range of temperature.As to deformation of the composites, metal matrices can greatly enhance the ductility of the composite.The stress concentrations induced by cracked fibers can be relaxed through the plastic deformation of matrix.As a result there is less chance of a brittle failure of the composite.The majority of the load applied to a metal matrix composite is carried by the reinforcing fibers .Since the metal matrices are strong in shear strength and, in general, well bonded to fibers, short fibers can be used effectively for the purpose of strengthening.An obvious disadvantage of this type of composites is their relatively high density.

3.2 Fabrication Methods for Metal-Matrix Composites

In this section the methods used for fabricating metal-matrix composites are briefly.These methods serve first to combine the fiber and matrix materials and then to consolidate the combination to form the desired shape of the end-product.

Powder Metallurgy Technique

The technique of *powder metallurgy* involves the compacting of solid materials in the form of powders.The powder process has been used for ceramic as well as metallic materials.The product resulting from the powder process is uniform in composition, in contrast to alloys produced by casting.In the latter case, segregation of the component phases often occurs during solidification, and homogenization of the alloy is needed.Since no melting or casting involved, the powder process is more economical than many other fabrication techniques.In this process, powders of ceramics or metals are first prepared and then fed into a mould of desired shape.Pressure is then applied to further compact the powder.In order to facilitate the bonding among powder particles, the compact is often heated to a temperature which is below the melting point but high enough to develop significant solid state diffusion.The use of heat to bond solid particles is known as *sintering* or *firing*. There is no separate bonding phase generated in the sintering process.Through diffusion, the point of contact between two neighboring particles develops into a surface and the bonding between them is hence strengthened.The driving force for sintering is the elimination of particle surface area.

Metallic materials such as copper, nickel, aluminum, cobalt, and steel are often used in the powder process as matrix materials.The metal matrices in the form of powders are first mixed with whiskers or copped fibers .The combination is then consolidated by pressing, sintering, hot extrusion, or rolling, in order to enhance the density and strength of the composite.The exposure to high temperature and pressure for long periods may be detrimental to some composite systems.

Liquid Metal Infiltration

Metal such as aluminum, magnesium, silver, and copper have been used as matrix materials in this process because of their relatively lower melting points.The method of liquid metal infiltration is desirable in producing relatively small size composite specimens. fibers collimated in a mould are infiltrated with liquid metal.By employing the idea of liquid metal infiltration, it is possible to cast composite structures such as rods and beams by passing a bundle of filaments through a liquid-metal bath in a continuous manner.Structures solidified in this manner have uniform cross-sections with uniaxial reinforcement, and need little additional work.The application of the liquid-metal infiltration process is limited by the available choice matrix and reinforcing materials.The degradation of many fibers at high temperatures rules out their use.Another consideration is the wetting of reinforcements by the liquid metal.This problem will not be discussed in this study.

Diffusion Bonding

Just as in the case of sintering, the diffusion bonding process is carried out under high pressure and elevated temperature.Filaments of stainless steel, boron, and silicon carbide have been used with matrices such as aluminum and titanium alloys.Unlike the powder process, the matrix metals used in most commercially available composites are in the form of metal foils.In order to fully develop the bonding strength among the foils and between the foil and the fiber, they all have to be thoroughly cleansed.The fibers are then laid on the metal sheets in predetermined spacing and orientation.Alternate layers of metal foils and reinforcing fibers can be arranged for the desired content of reinforcements.The lay-up is encased in a metal can which is sealed and evacuated.The whole assembly is subsequently heated and pressed to facilitate the development of diffusion bonding.The applied pressure and temperature, as well as their duration's for diffusion bonding to develop, vary with the composite systems.For instance, the boron - aluminum composite develops satisfactory bonding at 454°C under a pressure of 4.14×10^7 N/m^2 has been observed for 6061 Al reinforced with 48 volume per cent of boron fibers by diffusion bonding.Prolonged hot pressing may cause reduction in the composite strength.The change of microstructure accompanying the reduction in strength in figure below for a stainless steel-aluminum composite material.

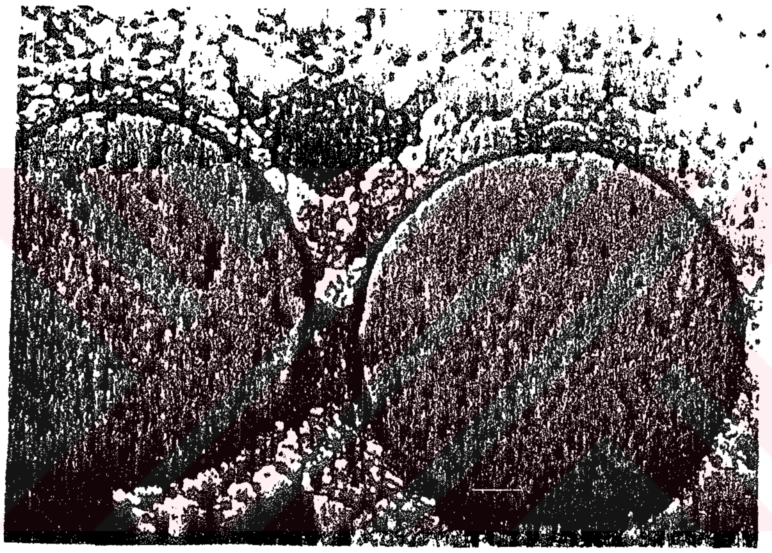


Figure 3.1 Microstructure of Aluminum-Stainless steel composite

The formation of an inter-metallic compound in the vicinity of the fiber-matrix interface is responsible for the weakening of the composite strength.The diffusion bonding process may also be used to consolidate tape forms produced by methods such as plasma spray, hot rolling, and vapor deposition.The tapes easy to handle and can be arranged in predetermined orientations .

Electroforming

Electroforming has the advantage of combining the fiber and matrix materials at low temperatures, and thus degradation of reinforcing materials can be avoided.The major apparatuses for electroforming consist of a plating bath a mandrel which serves as the cathode in the deposition process.A continuous filament is wound onto the mandrel while the metal matrix material is being deposited.The spacing filaments can be closely controlled in the winding process, and high-volume fractions of fiber content can be achieved.Monolayer tapes formed by process can be further consolidated into composite structure members by diffusion bonding.For multilayer composites formed in this manner, voids tend to form between fibers and between successively deposited layers.Filaments of boron, silicon carbide, and tungsten have been successively incorporated into a nickel matrix by electroforming.Other matrix metals, as well as alumina whiskers, also have been employed in this method of fabrication.

Vapor Deposition

The process of vapor deposition is carried out by decomposing a compound of metal matrix material and its subsequent deposition on the reinforcing materials.The reinforcements can be in the forms of continuous filaments or random whisker mats.A main advantage of this technique is that the chemical decomposition process can be accomplished at a relatively low temperature and the degradation of fibers can be minimized.High-volume fraction of fiber can be attained by this process.However, the slow and costly process of vapor deposition is its major disadvantage.Metals such as aluminum and nickel have been used for deposition.

Rolling

Both hot and cold rolling can be employed to incorporate filaments and metal strips into continuous tapes.In this processes, the fibers and metal strips are fed through rollers under applied pressure.The rollers are heated to a high temperature in the case of hot-rolling.Both pressure and temperature serve to accelerate diffusion bonding, although the contact time of the composite assembly with the applied pressure and temperature is relatively short.The metal strips can be grooved in order to provide precise alignment of the filaments.The sandwich construction of continuous tapes by the rolling process is restricted to a few layers in thickness.However, tapes fabricated in this manner can be laid up and further consolidated by diffusion bonding.The rolling process can also be used to consolidate continuous fibers coated with a metallic matrix material.

Extrusion

One method of extrusion is known as *co-extrusion*, which does not need the application of high temperature.Figure below indicates a design of extrusion tooling for making composite wires.The wire, consisting of a steels core surrounded by an aluminum alloy sheath, is produced by simultaneous feeding of the reinforcing filament and extruding of the matrix metal.Composite wires formed in this manner can be further rolled into tapes plates through diffusion bonding.There are other methods of extrusion in which fibers are first aligned in matrix powders, and this assembly is pressed into the form of bars.A single perform, or several of them sealed in can, are then extruded to the desired dimension.

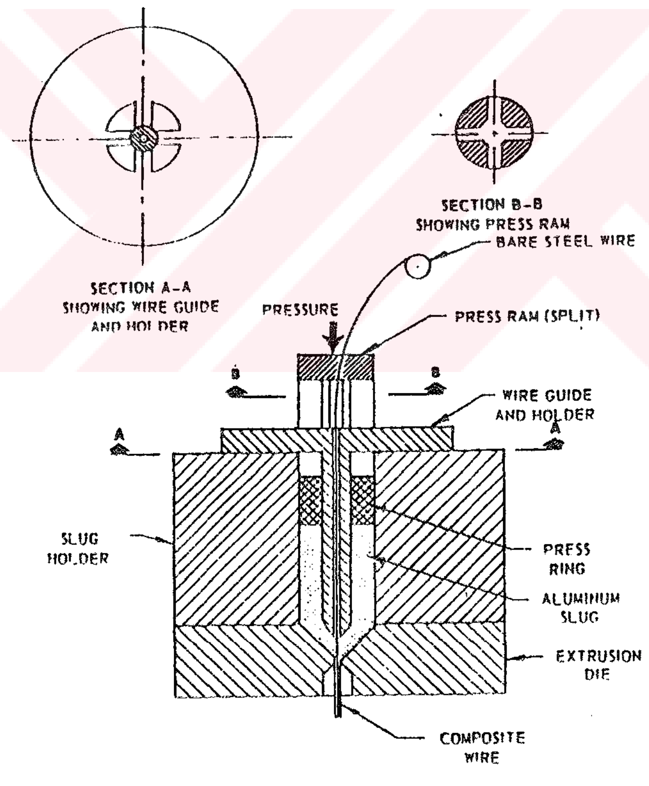


Figure 3.2 Design of extrusion tooling

Other Methods

Methods including *plasma spray,pneumatic impaction,* and the simultaneous growth of the reinforcing and matrix materials from a melt also have been used for producing metal-matrix composites.The method of plasma spray is suitable for low melting point metals.It employs a plasma torch which sprays matrix materials in the form of liquid droplets on to rotating mandrel covered with aligned fibers .The composite formed is then removed from the mandrel and hot-pressed to eliminate voids.As one example, aluminum has been successively sprayed on silicone carbide-coated boron fiber .Composite tapes formed in this manner can be further consolidated into structural parts by diffusion bonding.Pneumatic impaction has been used to consolidate the mixture of metal powders and reinforcing materials by applying high pressure impact.In view of its basic difference in the fabrication process relative to all the other methods.

An examination of the fabrication methods reviewed above indicates that the matrix used in the consolidation process may assume different forms.These include the metallic powders used in pneumatic impaction and the powder metallurgy technique; the liquid form used in liquid metal infiltration and plasma spray; the molecular form of matrix metals appearing in the electroforming and vapor deposition processes; and the metal foils employed in diffusion bonding and the rolling process.It is also noted that one form of matrix metal incorporated with reinforcements may be subjected to more than one fabrication process.For instance, composite materials produced by sintering are subsequently extruded and rolled.Monolayer tapes fabricated by various methods may also be further consolidated by rolling and diffusion bonding.

3.3 Properties of Aluminum-Stainless-steel Composites

Metallic materials have been reinforced with all kinds of fiber materials discussed before.Now we represent Aluminum-Stainless-steel composite which is manufactured by this thesis.In this process the diffusion bonding method is used.

First aluminum foils layed up in a mold then stainless-steel wires put it up.This layer is done replace three times.This layers are put in a mold.The mold heated until 550°C at that time pressed at 6.66 Mpa during 15 minutes.At the end of pressing mold is cooled at room temperature. After this process we start examining the composite. These tests are made :

1. Tensile test in elastic and plastic region

This test is made two steps. First 1-direction (reinforced direction) tensile test is made in elastic region. We prepaired a tensile test specimen that 0° angle with reinforce direction two stain-gages pasted to it. One of them measured elongation that tensile direction other measured 90° angle with tensile direction. Thus we obtain Modulus of Elasticity of material (E_1) , Poisson's ratio (ν_{12}) . After this loading tensile load is increased up to failure. Thus, we obtain plastic material properties (K , n).

2. Tensile test in elastic region

This test is made only elastic region. A tensile specimen is prepared 2-direction (at 90° angle with reinforced direction). And pasted one strain-gage only. This gage is measured elongation of tensile direction. Thus, we obtain Modulus of Elasticity of material (E_2). After this test we prepair a tensile test specimen that is at 45° angle with reinforced direction. Thus, we obtain Shearing Modulus of material (G_{12}) by calculation.

After these processes we obtain ultimate tensile strength and shearing strength of material (X , Y , S). Table below indicates these properties.

Property	Symbol	Value
Modulus of elasticity at 0°	E_1	83.000 Mpa
Modulus of elasticity at 90°	E_2	56.000 Mpa
Modulus of elasticity at 45°	$E_{45°}$	82.041 Mpa
Shearing modulus	G_{12}	50.482 Mpa
Ultimate tensile strength at 0°	X	130 Mpa
Ultimate tensile strength at 90°	Y	54 Mpa
Ultimate shearing strength	S	23.54 Mpa
Poisson's ratio	ν_{12}	0.37
Coefficient of plastic region	K	270 Mpa
Superscript of plastic region	n	0.168

Table 3.1 Properties of Aluminum-Stainless steel Composite

4.1 Biaxial Strength Theories For An Orthotropic Lamina

Our attention in this section will be restricted to biaxial loading.Some of the biaxial strength theories that have been studied are:

1.Maximum stress theory
2.Maximum strain theory
3.Tsai-hill theory
4.Tsai-Wu tensor theory

4.1.1 Maximum Stress Theory

In the maximum stress theory, the stresses in principal material directions must be less than the respective strengths, otherwise fracture is said to have occurred that is, for tensile stresses,

$$\begin{aligned} \sigma_1 &< X_t \\ \sigma_2 &< Y_t \\ \tau_{12} &< S \end{aligned} \qquad (4.1)$$

and for compressive stresses

$$\begin{aligned} \sigma_1 &> X_c \\ \sigma_2 &> Y_t \end{aligned} \qquad (4.2)$$

Note that the shear strength is independent of the sign of τ_{12} .If any one of the foregoing inequalities is not satisfied, Then the *assumption* is made that the material has failed by the failure mechanism associated with X_t , X_c , Y_t , Y_c , or S, respectively.Note that there is *no interaction* between modes of failure in this criterion - there are actually three subcriteria.

In applications of the maximum stress criterion, the stresses in the body under consideration must be transformed to stresses in the principal material directions.For example, Tsai considered a unidirectionally reinforced composite subjected to uniaxial load at angle θ to the fibers as shown in Figure below.

$$\begin{aligned} \sigma_1 &= \sigma_x \cos^2\theta \\ \sigma_2 &= \sigma_x \sin^2\theta \\ \tau_{12} &= \sigma_x \sin\theta \cos\theta \end{aligned} \qquad (4.3)$$

Then by inversion of equation (4.3) and substitution of equation (4.1) the maximum uniaxial stress, σ_x , is the smallest of

$$\begin{aligned} \sigma_x &< \frac{X}{\cos^2\theta} \\ \sigma_x &< \frac{Y}{\sin^2\theta} \\ \sigma_x &< \frac{S}{\sin\theta \cos\theta} \end{aligned} \qquad (4.4)$$

4.1.2 Maximum Strain Theory

The maximum strain theory is quite similar to the maximum stress theory.Here, strains are limited rather than stresses.Specifically, the material is said to have failed if one or more of the following inequalities is not satisfied :

$$\begin{aligned} \sigma_1 &< X_t \\ \sigma_2 &< Y_t \\ |\tau_{12}| &< S \end{aligned} \qquad (4.5)$$

Includes for materials with different strength in tension and compression

$$\sigma_1 > X_c$$
$$\sigma_2 > Y_c \tag{4.6}$$

where

X_t, X_c are maximum tensile and compressive normal strain in the 1-direction
Y_t, Y_t are maximum tensile and compressive normal strain in the 2-direction
S is maximum shear strain in the 1-2 plane

As with shear strength, the maximum shear strain is unaffected by the sign of the shear stress.The strains in principal material directions, σ_1 ,σ_2 , τ_{12} , must be found from the strains in body coordinates by transformation before the criterion can be applied.

For a unidirectionally reinforced composite subjected to uniaxial load at angle θ to the fibers (the example of problem in the section on maximum stress theory) the allowable stresses can be found from the allowable strains X_t , Y_t , etc. in the following manner.

Firs given that the stress-strain relations are

$$\varepsilon_1 = \frac{1}{E_1}(\sigma_1 - \nu_{12}\,\sigma_2)$$

$$\varepsilon_2 = \frac{1}{E_2}(\sigma_2 - \nu_{21}\,\sigma_1) \tag{4.7}$$

$$\gamma_{12} = \frac{\tau_{12}}{G_{12}}$$

upon substitution of the transformation equations

$$\sigma_1 = \sigma_x \cos^2\theta$$
$$\sigma_2 = \sigma_x \sin^2\theta \tag{4.8}$$
$$\tau_{12} = -\sigma_x \sin\theta\cos\theta$$

In the stress-strain relations Eq.(4.7) the strains can be expressed as

$$\varepsilon_1 = \frac{1}{E_1}(\cos^2\theta - \nu_{12}\sin^2\theta)\,\sigma_x$$

$$\varepsilon_2 = \frac{1}{E_2}(\sin^2\theta - \nu_{21}\cos^2\theta)\,\sigma_x \tag{4.10}$$

$$\gamma_{12} = \frac{1}{G_{12}}(\sin\theta\cos\theta)\,\sigma_x$$

Finally, if the usual restriction to linear elastic behavior to the failure is made.

$$X_{\varepsilon t} = \frac{X_t}{E_1}$$

$$Y_{\varepsilon t} = \frac{Y_t}{E_2}$$

$$S_\varepsilon = \frac{S}{G_{12}}$$

and

$$X_{\varepsilon c} = \frac{X_t}{E_1} \qquad (4.11)$$

$$Y_{\varepsilon c} = \frac{Y_c}{E_2}$$

(which could equally well come from measured values in an experiment), than the maximum strain criterion for this example can be expressed as

$$\sigma_x < \frac{X}{\cos^2\theta - \nu_{12}\sin^2\theta}$$

$$\sigma_x < \frac{Y}{\sin^2\theta - \nu_{21}\cos^2\theta} \qquad (4.12)$$

$$\sigma_x < \frac{S}{\sin\theta\cos\theta}$$

By comparison of the maximum strain criterion Eq.(4.4) with the maximum stress criterion Eq.(4.12) , it is obvious that the only difference is the inclusion of Poisson's ratio terms in the strain criterion.

4.3 Tsai-Hill Theory

Hill proposed a yield criterion for anisotropic materials :

$$(G+H)\sigma_1^2 + (F+H)\sigma_2^2 + (F+G)\sigma_3^2 - 2H\sigma_1\sigma_2 - 2G\sigma_1\sigma_3 - 2F\sigma_3\sigma_2 + 2L\tau_{23}^2 + 2M\tau_{13}^2 + 2N\tau_{12}^2 = 1 \qquad (4.13)$$

This anisotropic yield criterion will be used as an isotropic strength criterion in the spirit of both being limits of linear elastic behavior.Thus, Hill's yield strengths F, G, H, L, M, and N will be regarded as failure strengths.Hill's theory is an extension of von-Mises' isotropic yield criterion.The von-Mises criterion, in turn, can be related to the amount of energy that is used to distort the body rather than to change its volume.However, distortion cannot be separated from dilatation in orthotropic materials so Eq.(4.13) is not related to distortional energy failure theory.

The failure strength parameters F, G, H, L, M, and N were related to the usual failure strengths X, Y, and S for a lamina by Tsai.First, if only τ_{12} acts on the body then, since its maximum value is S:

$$2N = \frac{1}{S^2} \qquad (4.14)$$

Similarly, if only σ_1 acts on the body, then

$$G + H = \frac{1}{X^2} \qquad (4.15)$$

and if only σ_2 acts , then

$$F + H = \frac{1}{Y^2} \tag{4.16}$$

the strength in the 3-direction is denoted by Z and only σ_3 acts, then

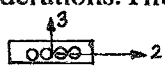
$$F + G = \frac{1}{Z^2} \tag{4.17}$$

$$2H = \frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}$$

$$2G = \frac{1}{X^2} + \frac{1}{Z^2} - \frac{1}{Y^2} \tag{4.18}$$

$$2F = \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}$$

For plane stress in the 1-2 plane of a unidirectional lamina with fibers in the 1-direction, $\sigma_3 = \tau_{13} = \tau_{23} = 0$.However, from the cross section of such a lamina in Figure below, $Y = Z$ from geometrical symmetry considerations.Thus, Eq.(4.13) leads to



$$\frac{\sigma_1^2}{X^2} - \frac{\sigma_1\sigma_2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\tau_{12}^2}{S^2} = 1 \tag{4.19}$$

as the governing failure criterion in terms of the familiar lamina strengths X, Y, and S.

Finally, for the off-axis composite example, substitution of the Eq.(4.8)(stress transformation equations) in Eq.(4.19) yields the Tsai-Hill failure criterion

$$\frac{\cos^4\theta}{X^2} + \left(\frac{1}{S^2} - \frac{1}{X^2}\right)\cos^2\theta\,\sin^2\theta + \frac{\sin^4\theta}{Y^2} = \frac{1}{\sigma_x^2} \qquad (4.20)$$

4.4 Tsai-Wu Tensor Theory

The preceding biaxial strength theories suffer from various inadequacies in their description of experimental data.One obvious way to improve the correlation between theory and experiment is to increase the number of terms in the prediction equation.This increase in curve fittingability plus the added feature of representing the various strengths in tensor form was used by Tsai and Wu In the processes, several new strength definitions are required, mainly having to do having to do with interaction between stresses in two directions.

Tsai and Wu postulated that a failure surface in stress space exists in the form

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j = 1 \quad i, j = 1,2,\ldots\ldots..6 \qquad (4.21)$$

Wherein F_i and F_{ij} are strength tensors of the second and forth rank, respectively and the usual contracted stress notation is used except that $\sigma_4 = \tau_{23}$, $\sigma_5 = \tau_{31}$, $\sigma_6 = \tau_{12}$.Equation (4.21) is obviously very complicated; we will restrict our attention to the reduction of Eq.(4.21) for an orthotropic lamina under plane stress conditions :

$$F_1\sigma_1 + F_2\sigma_2 + F_6\sigma_6 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 2F_{12}\sigma_{12} = 1 \qquad (4.22)$$

The terms that are linear in the stresses are useful in representing different strengths in tension and compression.The terms that are quadratic in the stresses are the more or less usual terms to represent an ellipsoid in stress space.However, the term involving F_{12} is intirely new to us and is used to represent the interaction between normal stresses in the 1- and 2- directions in amanner quite unlike the shear strength.

Some of the components of the strength tensors are defined interms of the enginnering strengths already discussed.For example, consider a uniaxial load on a specimen in teh 1-direction.Under tensile load, the engineering strength is X_t whereas under compressive load, it is X_c .Thus under tensile load,

$$F_1 X_t + F_{11} X_t^2 = 1 \tag{4.23}$$

and under compressive load,

$$F_1 X_c + F_{11} X_c^2 = 1 \tag{4.24}$$

Upon simultaneous solution of Eqs.(4.23) and (4.24)

$$F_1 = \frac{1}{X_t} + \frac{1}{X_c} \tag{4.25}$$

$$F_{11} = -\frac{1}{X_t X_c}$$

Similarly ,

$$F_2 = \frac{1}{Y_t} + \frac{1}{Y_c} \tag{4.26}$$

$$F_{22} = -\frac{1}{Y_t Y_c}$$

and

$$F_6 = 0$$

$$F_{66} = \frac{1}{S^2}$$

The determination of the forth rank tensor term F_{12} remains.Basically, F_{12} cannot be determined from any uniaxial test in the principal material directions.Instead , biaxial test must be used.This fact should be surprising sice F_{12} is the coeeficient of σ_1 and σ_2 in the failure criterion, Eq.(4.22).Thus, for example, we can impose a state of biaxial tension described by $\sigma_1 = \sigma_2 = \sigma$ and all stresses are zero.Accordingly, from Eq.(4.22)

$$(F_1 + F_2)\,\sigma + (F_{11} + F_{22} + 2F_{12})\,\sigma^2 = 1 \qquad (4.27)$$

Now solve for F_{12} after substituting the definations just derived for F_1 , F_2 , F_{11} , and F_{22} :

$$F_{12} = \frac{1}{2\sigma^2}\left(1 - \left(\frac{1}{X_t} + \frac{1}{X_c} + \frac{1}{Y_t} + \frac{1}{Y_c}\right)\sigma + \left(\frac{1}{X_tX_c} + \frac{1}{Y_tY_c}\right)\sigma^2\right) \qquad (4.28)$$

The value of F_{12} then depends on the various engineering strengths plus the biaxial tensile failure stress, σ.Tsai and Wu also discuss the use of off-axis uniaxial tests to determine the interaction strengths such as F_{12}.

At this point recallthat all interaction between normal stresses σ_1 and σ_2 in the Tsai-Hill theory is related to the strength in the 1-direction :

$$\frac{\sigma_1^2}{X^2} - \frac{\sigma_1\sigma_2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\tau_{12}^2}{S^2} = 1 \qquad (4.29)$$

The Tsai-Wu tensor failure theory ais obviously of more general character than Tsai-Hill theory.Specific advantages of Tsai-Wu theory include :

1. Invariance under rotation or redefination of coordinates,
2. Transformation via known tensor transformation laws,
3. Symmetry properties akin to those of the stiffness and compliances.

Accordingly, the mathematical operations with this tensor failure theory are well known and relatively straightforward.

1. Calculation of Elasto-Plastic Stresses

Various computational procedures have been used with success for a limit range of elasto-plastic problems utilizing the finite element approch.Two main formulation appear.In the first, during an increment of loading, the increase of plastic strain is computed and treated as an *initial strain* for which the elastic stress distribution is adjusted.This approach manifestly fails in ideal plasticity is postulated or if the hardening is small.The second approach is that in which the stress-strain relationship every load increment is adjusted to take into account plastic deformation.With properly specified elasto plastic matrix this incremental elasticity approach can successfully treat ideal as well as hardening plasticity.

From the computational point of view the *incremental elasticity* process has one serious disadvantage.At each step of computation the stiffness of the structure is changed and iterative process of solution are neccesary to avoid excesive computer times.The *initial stress* method
is developedby Zienkiewicz as an alternative approach to the incremental elasticity process.By using the fact that even in ideal plasticity increments of starin prescribe uniquely the stress system (while the reverse is not true for ideal plasticity)an adjustment process is derived in which initial stresses are distributed elastically through the structure.

This approach permits the advantage of initial process (in which the basic elasticity matrix remains unchaneged) to be retained.The process appears to be the most rapidly convergent.To start elasto-plastic stress analysis, this method uses one dimensional tensile specimen in elasto-plastic region, then moves on to the two and three dimensional strees case.For a tensile specimen loaded just over the elastic region ($\varepsilon_{total}= \varepsilon_1$).Stress σ_x is calculated linear elasticity, thus the stress σ_{f1} as shown in Figure below is given by the following form ;

$$\sigma_{01}=\sigma_x-\sigma_{f1}=\sigma_1-\sigma_{f1}$$

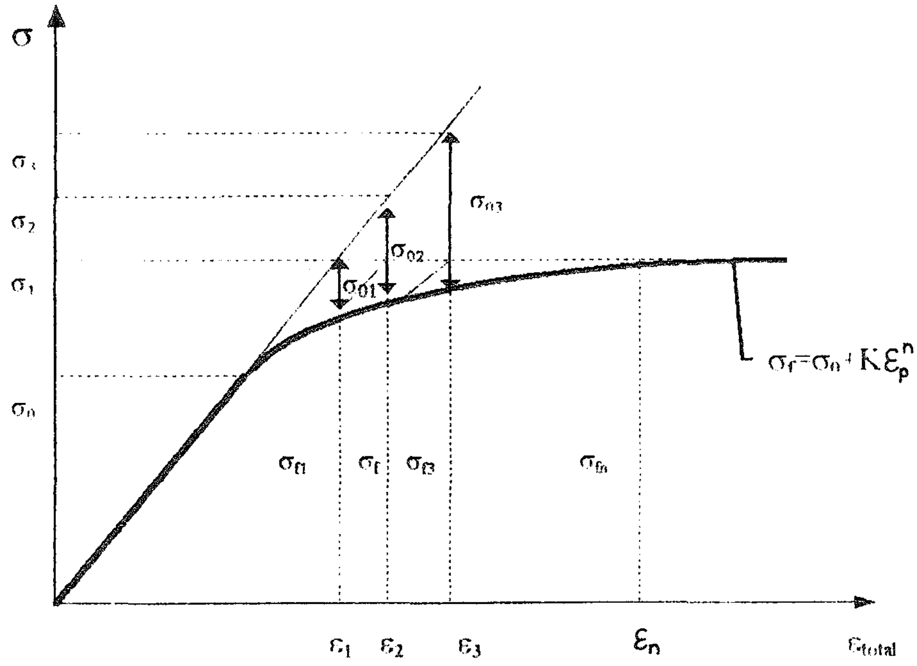


Figure 5.1 Tensile test diagram

By using σ_{01} one obtains increasing sress value

$$\sigma_2=\sigma_x+\sigma_{01} \tag{5.1}$$

Which corresponding to ε_2 .The stress difference between σ_2 and real stress at ε_2 gives σ_{02} in last equation.The follwing analog iteration steps lead to the point corresponding to the elesto-plastic strain ε_n and stress σ_x. Where $\sigma_{oi}$is the initial stress.

For calculation of stress in two dimensional cases equivalent stress is usually obtained according to *Von Misses* criterion (Distortion energy theory).The equivalent stress in plane stress case is

$$\sigma=\{0.5((\sigma_x-\sigma_y)^2+\sigma_x^2+\sigma_y^2)+3\tau_{xy}^2\}^{1/2} \tag{5.2}$$

Where σ_x , σ_y , and τ_{xy} are the stress components.

Therefore initial stress can be obtained for plastic region is one dimensional case

$$\sigma_0=\sigma-\sigma_f \tag{5.3}$$

Where σ_f is obtained from σ , ε_{total} diagram a uniaxially loaded tensile specimen.But the initial stress can not be exactly described as in Figure above in two dimensional case. it can be mathematically described as follows

$$\{\sigma_0\}=\{\sigma_{0x}\quad \sigma_{oy}\quad \tau_{oxy}\} \tag{5.4}$$

Where σ_{0x}, σ_{oy}, and τ_{oxy} are components of the initial stress in plane stress case.By using the following formal, one obtains

$$\{\sigma_0\}=\{\sigma\}\ \sigma_{oy}/\sigma \tag{5.5}$$

Where the components of $\{\sigma_0\}$ are proportional to elastically calculated stress.The related equivalent stress value is equal to $\{\sigma_0\}$ obtained in one dimensional case, according to Eq.(5.1) :

$$\sigma_0=\{0.5((\sigma_{ox}-\sigma_{oy})^2+\sigma_{ox}^{\ 2}+\sigma_{oy}^{\ 2})+3\tau_{oxy}^{\ 2}\}^{1/2} \tag{5.6}$$

The loading corresponding to the initial stress as follows;

$$\{F\}_{\sigma_o}=\ _v[B]^T\{\sigma_0\}\,dV \tag{5.7}$$

First the solution vector is calculated for $\{F\}_{\sigma_{01}}$, mechanical loading in the first iteration step

$$\{\delta\}_1=[K]^{-1}\{F\}_m \tag{5.8}$$

Where $\{F\}_m=\{F_s\}+\{F\}_{\sigma 0}$ then the following iteration steps $\{\delta\}_i$, i=1,2,.......n are calculated until there is no difference between $\{\delta\}_{i+1}$.Then the displacement vector is

$$\{\delta\}_n=[K]^{-1}\{F\}_m \tag{5.9}$$

Finally the stress σ_n corresponding to $\{\delta\}_n$ in elasto-plastic region is calculated as

$$\{\sigma\}_n=[C].[B]\{\delta\}_n \tag{5.10}$$

In elasto-plastic region, resudial stress are found at the end of iteration as follows.

In elasto-plastic region, resudial stress are found at the end of iteration as follows,

$$\{\sigma_{0i}\} = -(\{\sigma\}_L - \{\sigma\}_n) \qquad (5.11)$$

where $\{\sigma\}_L$ is the linear elastic stress obtained at the end of iteration.In the polar coordinates, resudial stresses are written as

$$\{\sigma_{0i}\}_{pc} = [\,T\,] . \{\sigma\}_{0i} \qquad (5.12)$$

Where

$$[\,T\,] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \sin^2\theta - \cos^2\theta \end{bmatrix}$$

is transformation matrix.

6.1 Solving the Problem

If we apply a force to a material we can create a few elongation.If loading or stress in the material is in elastic region and then we stand up it, elongation returns to begining of the loading.But stress is in the material greater than yielding point, material can not return its original form.Thus, we must do elasto-plastic stress analysis.Elasto-plastic stress analysis has two steps.First is elastic stress analysis in elastic region and other in plastic region at the tensile test diagram.In this study we use finite element modeling (FEM).Solution of elasto-plastic stress analysis is made by a computer program.

In this study we used Tsai-Hill criterion as we investigate before.Firstly we obtain the stress at a point by FEM.If this value is greater than the yielding point then plastic analysis is begun.We use this step

$$\sigma = \sigma_0 + K\,\varepsilon^n \qquad (6.1)$$

Equation (6.1) for using elongation.Elongation value is obtained by this equation. Exact value is computing by Newton method as we investigate before.Newton iteration method steps are continued until elongation value difference we want.If iteration steps stopped than we calculate plastic-stress at that node.

If we look at diagrams that presents following pages we can see that some curves near the hole.That curves indicate boundary of yielding.The nodes in this curves have residual stresses.But all other nodes are not in plastic region.Plastic deformation is not acceptable. But some cases it is not harmful.If among of plastic deformation is in acceptable limit we can load that material up to yield point (before first loading) safety.Because this case yielding point is increased.But this process is very limited.

6.2 Discussing of Results

We consider a composite plate that we product before.And consider that plate has a hole.Material properties of that plate are written in Chapter 2.And we solve this plate's strengths according to load, reinforcing direction and hole diameters.

We see that strength distribution and yield points are symmetric with reinforcing direction.We also see that strength distribution is different from isotropic material and laid up to reinforcing direction.In calculating we consider exact plate which. in loading-direction and material reinforce-direction are different but , a quarter plate these directions are not different.

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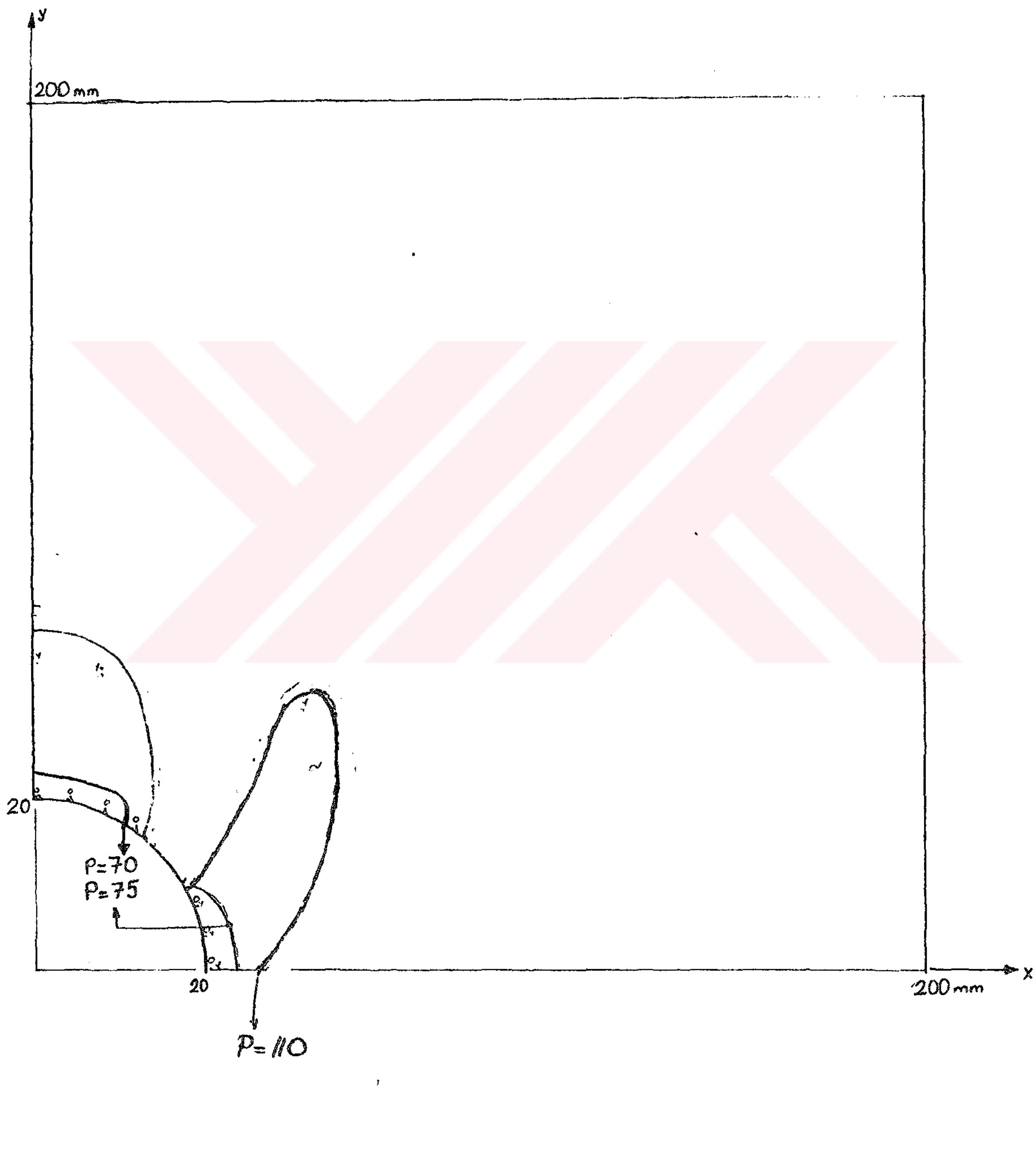
APPENDIX-1



Loading Direction = 0° Reinforcement Direction
Load = 70 Mpa

Yielded Nodes	X Coordinate	Y Coordinate	Residual Equvalent Stress
1	0.00	20.00	285.04
2	3.90	19.62	276.19
3	7.65	18.48	239.29
7	18.48	7.65	239.10
8	19.62	3.90	295.03
9	20.00	0.00	301.78

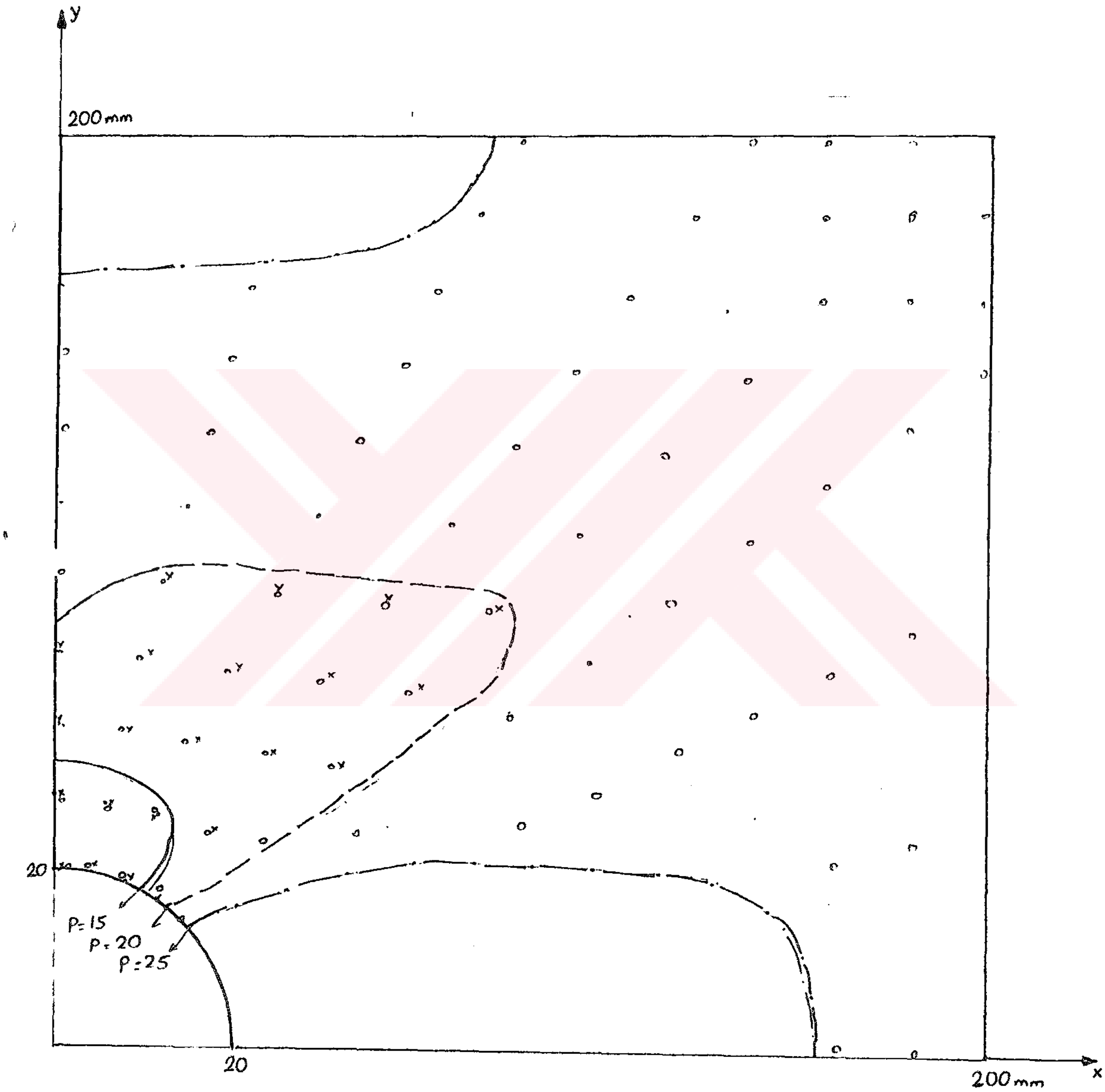
Loading Direction = 0° Reinforcement Direction
Load = 75 Mpa

Yielded Nodes	X Coordinate	Y Coordinate	Residual Equvalent Stress
1	0.00	20.00	305.40
2	3.90	19.62	295.92
3	7.65	18.48	256.39
7	18.48	7.65	256.18
8	19.62	3.90	316.10
9	20.00	0.00	323.34

Loading Direction = 0° Reinforcement Direction
Load = 110 Mpa

Yielded Nodes	X Coordinate	Y Coordinate	Residual Equvalent Stress
1	0.00	20.00	447.92
2	3.90	19.62	434.02
3	7.65	18.48	376.04
4	11.11	16.63	274.16
6	16.63	11.11	250.19
7	18.48	7.65	336.75
8	19.62	3.90	463.62
9	20.00	0.00	474.23
19	0.00	56.00	266.34
20	10.93	54.92	235.49
23	39.60	39.60	233.95

APPENDIX-2



Loading Direction = 90° Reinforcement Direction
Load = 12 Mpa

Yielded Nodes	X Coordinate	Y Coordinate	Residual Equvalent Stress
1	0.00	20.00	254.35
2	3.90	19.62	247.56

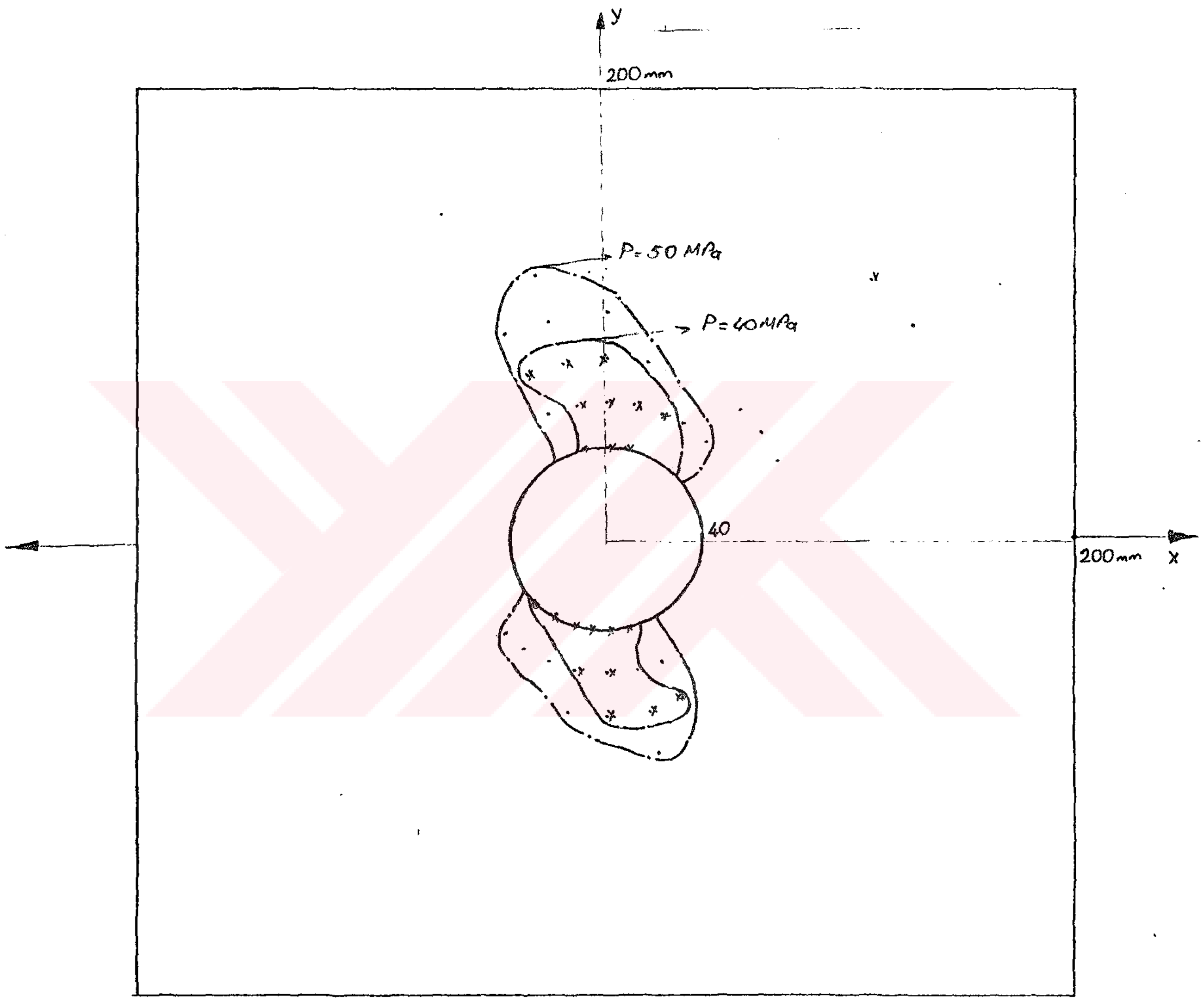
Loading Direction = 90° Reinforcement Direction
Load = 15 Mpa

Yielded Nodes	X Coordinate	Y Coordinate	Residual Equvalent Stress
1	0.00	20.00	317.94
2	3.90	19.62	309.45
3	7.65	18.48	281.36

Loading Direction = 90° Reinforcement Direction
Load = 20 Mpa

Yielded Nodes	X Coordinate	Y Coordinate	Residual Equvalent Stress
1	0.00	20.00	423.92
2	3.90	19.62	412.60
3	7.65	18.48	375.15
4	11.11	16.63	296.25
5	14.14	14.14	235.68
10	0.00	38.00	232.74
11	7.41	37.27	239.84
12	14.54	35.11	234.18
21	21.43	51.74	234.43
22	31.11	46.56	239.25
23	39.60	39.60	235.57

APPENDIX-3



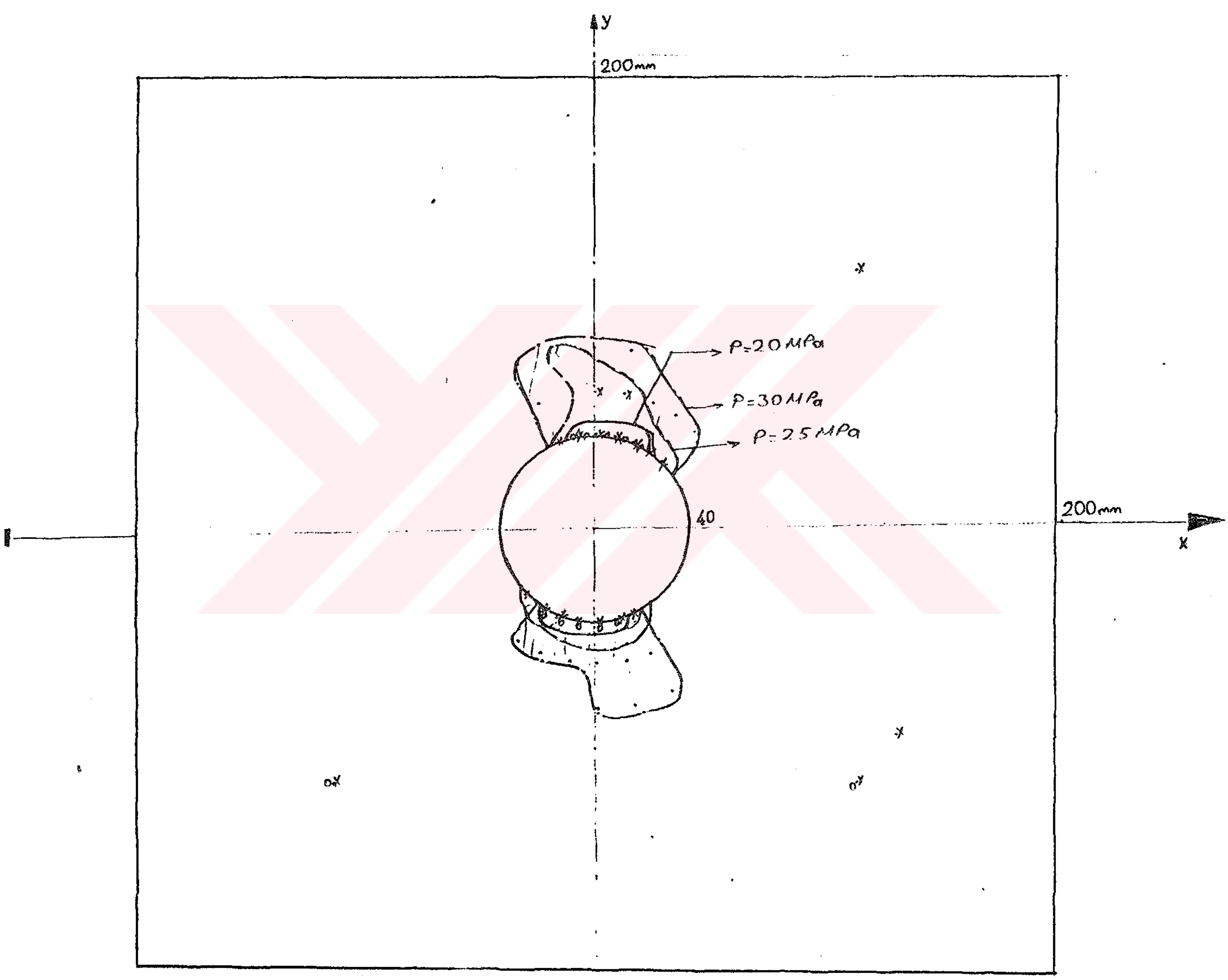
Loading Direction = 30° Reinforcement Direction
Load = 50 Mpa

Yielded Nodes	X Coordinate	Y Coordinate	Residual Equvalent Stress
1	-28.28427	-28.28427	291.90
5	-40.00000	0.00000	247.22
6	-39.23141	7.80361	249.47
11	-15.30734	36.95518	275.02
12	-7.80361	39.23141	380.52
13	0.00000	40.00000	477.20
14	7.80361	39.23141	590.58
15	15.30734	36.95518	599.80
16	22.22281	33.25879	520.33
17	28.28427	28.28427	398.98
21	40.00000	0.00000	261.33
22	39.23141	-7.80361	264.30
27	15.30734	-36.95518	275.86
28	7.80361	-39.23141	377.15
29	0.00000	-40.00000	467.27
30	-7.80361	-39.23141	569.93
31	-15.30734	-36.95518	572.57
32	-22.22281	-33.25878	489.83
33	-42.42640	-42.42641	231.20
43	-22.96101	55.43277	245.40
44	-11.70542	58.84712	293.42
45	0.00000	60.00000	311.38
46	11.70542	58.84712	318.77
47	22.96100	55.43277	289.98
48	33.33421	49.88818	277.75
49	42.42641	42.42641	252.39
59	22.96100	-55.43277	236.62
60	11.70542	-58.84712	281.05
61	0.00000	-60.00000	294.32
62	-11.70542	-58.84712	298.67
63	-22.96101	-55.43277	269.84
64	-33.33422	-49.88817	259.26
76	-15.60723	78.46282	343.28
77	0.00000	80.00000	328.20
78	15.60722	78.46282	266.34
81	56.56854	56.56854	230.25
82	66.51757	44.44562	230.78
83	73.91036	30.61468	230.16
91	30.61467	-73.91036	276.28
92	15.60722	-78.46282	324.56
93	0.00000	-80.00000	304.77
94	-15.60723	-78.46282	241.51
107	-38.26835	92.38795	230.21
108	-19.50904	98.07853	254.35
109	0.00000	100.00000	244.02
124	19.50903	-98.07853	233.25
140	-23.41084	117.69423	234.56

Loading Direction = 30° Reinforcement Direction
Load = 40 Mpa

Yielded Nodes	X Coordinate	Y Coordinate	Residual Equvalent Stress
1	-28.28427	-28.28427	233.52
12	-7.80361	39.23141	304.41
13	0.00000	40.00000	381.76
14	7.80361	39.23141	472.47
15	15.30734	36.95518	479.84
16	22.22281	33.25879	416.27
17	28.28427	28.28427	319.19
28	7.80361	-39.23141	301.72
29	0.00000	-40.00000	373.81
30	-7.80361	-39.23141	455.95
31	-15.30734	-36.95518	458.06
32	-22.22281	-33.25878	391.86
44	-11.70542	58.84712	234.74
45	0.00000	60.00000	249.11
46	11.70542	58.84712	255.01
47	22.96100	55.43277	231.98
61	0.00000	-60.00000	235.46
62	-11.70542	-58.84712	238.93
75	-30.61468	73.91036	230.18
76	-15.60723	78.46282	274.62
77	0.00000	80.00000	262.56
91	30.61467	-73.91036	240.41
92	15.60722	-78.46282	259.64
93	0.00000	-80.00000	243.81

APPENDIX-4



Loading Direction = 45° Reinforcement Direction
Load = 25 Mpa

Yielded Nodes	X Coordinate	Y Coordinate	Residual Equvalent Stress
1	-28.28427	-28.28427	248.67
11	-15.30734	36.95518	233.87
12	-7.80361	39.23141	305.09
13	0.00000	40.00000	364.31
14	7.80361	39.23141	421.39
15	15.30734	36.95518	413.70
16	22.22281	33.25879	344.62
17	28.28427	28.28427	265.09
27	15.30734	-36.95518	235.47
28	7.80361	-39.23141	304.13
29	0.00000	-40.00000	359.33
30	-7.80361	-39.23141	409.79
31	-15.30734	-36.95518	398.24
32	-22.22281	-33.25878	327.07
45	0.00000	60.00000	230.88
46	11.70542	58.84712	237.23
76	-15.60723	78.46282	235.94

Loading Direction = 45° Reinforcement Direction
Load = 20 Mpa

Yielded Nodes	X Coordinate	Y coordinate	Residual Equavalent Stress
12	-7.80361	39.23141	244.07
13	0.00000	40.00000	291.45
14	7.80361	39.23141	337.11
15	15.30734	36.95518	330.96
16	22.22281	33.25879	275.69
28	7.80361	-39.23141	243.30
29	0.00000	-40.00000	287.46
30	-7.80361	-39.23141	327.83
31	-15.30734	-36.95518	318.59
32	-22.22281	-33.25878	261.66

Loading Direction = 45° Reinforcement Direction
Load = 30 Mpa

Yielded Nodes	X Coordinate	Y Coordinate	Residual Equvalent Stress
1	-28.28427	-28.28427	238.87
11	-15.30734	36.95518	234.81
12	-7.80361	39.23141	366.11
13	0.00000	40.00000	437.17
14	7.80361	39.23141	505.66
15	15.30734	36.95518	496.44
16	22.22281	33.25879	413.54
17	28.28427	28.28427	318.11
27	15.30734	-36.95518	238.04
28	7.80361	-39.23141	364.96
29	0.00000	-40.00000	431.20
30	-7.80361	-39.23141	491.75
31	-15.30734	-36.95518	477.89
32	-22.22281	-33.25878	392.49
43	-22.96101	55.43277	236.52
44	-11.70542	58.84712	267.89
45	0.00000	60.00000	277.06
46	11.70542	58.84712	284.68
47	22.96100	55.43277	266.53
48	33.33421	49.88818	257.38
49	42.42641	42.42641	236.14
59	22.96100	-55.43277	231.11
60	11.70542	-58.84712	260.07
61	0.00000	-60.00000	266.20
62	-11.70542	-58.84712	271.93
63	-22.96101	-55.43277	253.45
64	-33.33422	-49.88817	244.65
75	-30.61468	73.91036	264.53
76	-15.60723	78.46282	283.12
77	0.00000	80.00000	264.82
78	15.60722	78.46282	231.97
91	30.61467	-73.91036	257.48
92	15.60722	-78.46282	270.79
93	0.00000	-80.00000	249.66

TÜRKÇE ABSTRAKT (en fazla 250 sözcük) :

(TÜBİTAK / TÜRDOK'un Abstrakt Hazırlama Kılavuzunu kullanınız.)

Özet

Bu çalışma ortasında delik bulunan kompozit bir malzemede elasto-plastik gerilme analizi hakkındadır.Tez iki aşamalıdır.Birinci aşamada kompozit malzeme üretildi ve malzeme özellikleri bulundu.Bu aşamadan sonra iki aşamalı teorik çalışma başladı.Birinci aşamada elastik çözüm yapıldı.Elastik çözüm bize sadece kompozit malzemedeki elastik gerilmeleri verdi.Bundan sonra elasto-plastik gerilme analizine başlandı.Çünkü gerçek malzemeler sadece elastik şekil değişimine uğramazlar.Eğer biz bir malzemenin ne zaman plastik bölgeye gireceğini bilirsek malzemeleri daha güvenli olarak kullanabiliriz.Elasto-plastik çözüm sonlu elemanlar modellemesi kullanılarak yapıldı.Bazı örnek problem çözümleri son sayfalara eklenmiştir.Buna göre akan (plastik şekil değişimine uğrayan) noktaların sınırları takviye doğrultusuna göre simetrik olmaktadır.

Bu çalışmanın zor yanı üretim aşamasıdır.Ama bu aşamada başarıyla tamamlanmıştır.

İNGİLİZCE ABSTRAKT (en fazla 250 sözcük) :

ABSTRACT

This study is about elasto-plastic stress analysis in a composite material with a hole.While the thesis we studied at two steps.Firstly we produced a composite material and found its properties.After this process, we begun theoretical study.It has two steps.Firstly we studied about elastic solution .Elastic solution gave us only elastic stresses at composite material.Then we started at elasto-plastic solving.Because real materials did not have only elastic deformation.If we know that when materials are in plastic region, we use a material safety.Elasto-plastic solution is made by finite element modelling. If the stress at a node is in the plastic region then, we calculated plastic stress.Some examples are added last pages.According to that solutions boundary of yielding points are symmetric to reinforcing directions

Difficulty of this study is fabricating.But we can make it successfully.