CAPACITATED TRANSPORTATION PROBLEMS AND AN APPLICATION

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ABSTRACT

The feasible (cost efficient) shipment of the products to wholesalers or to warehouses is a common problem for all companies. Such a problem is called a transportation problem, which is a special case of the linear programming problem. The general model, which corresponds to the classical transportation problem, comprises of the objective function, supply constraints, demand constraints, and nonnegativity constraints. However, if the decision variables which are the amounts of shipment have capacity constraints from various reasons such as capacity of trucks, warehouse capacity etc., then a capacitated transportation model is used. The objective is, generally, the minimization of cost.

In this research, capacitated transportation model and solution methods are studied and applied to an actual industrial problem. In application, the objective is defined as minimizing the total transportation cost, while satisfying the capacity constraints on the decision variables as well as the demand and supply constraints. The total number of decision variables and constraints in practice made the problem so huge that the model became to be beyond the capability of the available software packages. Thus, in order to obtain a solution, an approximate solution method is developed.

In the beginning, the problem is simplified to a smaller capacitated transportation problem and solved. With regards to the solution of the simplified model, sub problems are defined. The WinQSB software package is used and results are evaluated.

Keywords: Transportation Problem, Capacitated Transportation Problem

ÖZET

Ürünlerin depolara ya da satıcılara taşınması, bütün işletmeler için ortak bir problemdir. Bu tür problemler doğrusal programlama problemlerinin özel bir hali olan ulaştırma problemi olarak adlandırılır. Klasik ulaştırma modeline karşılık gelen genel model, amaç fonksiyonu, arz kısıtları, talep kısıtları ve negatif olmama kısıtlarından oluşur. Bununla birlikte, eğer taşınan malın miktarına karşılık gelen karar değişkenleri farklı sebeplerden dolayı kapasite kısıtlarına sahipse, kapasiteli ulaştırma modeli kullanılır. Amaç genelde maliyet enküçüklemesidir.

Bu araştırmada, kapasiteli ulaştırma modeli ve çözüm yöntemi çalışılarak, gerçek bir endüstri problemine uygulanmıştır. Uygulamada amaç, arz ve talep kısıtlarının yanında kapasite kısıtları da sağlanarak taşıma maliyetlerinin en küçüklenmesi olarak tanımlanmıştır. Pratikte, toplam karar değişkeni ve kısıt sayısının problemin boyutunu çok büyük bir hale getirmesi sebebiyle, mevcut bilgisayar paket programlarıyla çözülemediğinden, en iyi çözümü elde etmek için yaklaşık bir çözüm yöntemi geliştirilmiştir.

Öncelikle, problem basitleştirilerek çözüldü. Daha sonra, basitleştirilmiş problemin sonuçlarına dayanarak alt problemler tanımlandı. Problemleri çözmek için WinQSB paket programı kullanılarak sonuçlar yorumlandı.

Anahtar kelimeler: Ulaştırma Problemi, Kapasiteli Ulaştırma Problemi

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CHAPTER ONE
INTRODUCTION

Capacitated transportation model is a special cases of transportation model, which includes upper bound constraints.

The capacitated transportation model has been applied to obtain solutions to real life problems, and various articles describing these applications have been published. Some of these articles are summarized below.

Richards E. W. and Bhadury J., in 1995, describe a project that was done for Shad Valley Program, where it was required to assign students to seminars so as to maximize the satisfaction of the students with their assignments. In the paper, two models are proposed to determine optimal assignments. The first model is based on the Capacitated Transportation Problem and a network formulation is proposed to solve it. The second model is a two phase model whose first phase involves solving a Bottleneck Capacitated Transportation Problem and the second phase solving a Capacitated Transportation Problem. A simple search algorithm is proposed that solves the second model. Implementation of these models is described and the results obtained are discussed. Extensions to the two models are also proposed.

Hojati M., in 1996, describes for the given area and its population units, he wish to divide the area into *m* district has almost the same population of eligible voters(within a given tolerance), is contiguous, compact, and has a minimum number of split population units. This fair representation problem has been a great concern of the public for decades. The districting problem is also used in the design of sales territories. Redistricting occurs often because of population shifts or for political reasons.

For the political districting problem, he propose the following solution methodology:

a) Use Langrangian relaxation to determine the centres of the districts, then
b) Use the transportation technique to assign population units to centres, and finally
c) Resolve the splitting problem by solving a sequence of capacitated transportation problems.

Special solution method have been developed for capacitated transportation problem in the books which are Dantzig in 1966, and Dantzig and Thapa in 1997.

Although there are some studies about capacitated transportation problem, it is still a subject to be improved. Hence, we study this model and applied it to real life problem. Because the data was too large, we developed a solution method with two phases.

The formulations and the details of the solution methods for transportation model and capacitated transportation model is described in chapters two and three, respectively. In the last chapter, a real life problem is modeled with capacitated transportation model and the two phased solution procedure, which is developed because the total number of constraints is too large to handle, is described and applied to the data.

CHAPTER TWO
TRANSPORTATION PROBLEM

Linear programming is a widely used model type that can solve decision problems with many thousands of variables. The word "programming" is used here in the sense of "planning". Generally, the feasible values of the decisions are delimited by a set of constraints that are described by mathematical functions of the decision variables. The feasible decisions are compared using an objective function of the decision variables. For a linear program the constraints and the objective function are required to be linearly related to the variables of the problem.

A linear program (LP) is a problem that can be expressed as follows (the so-called standard form):

Minimize cx

subject to

$$Ax = b$$
$$x \geq 0$$

where x is the vector of variables to be solved for, A is a matrix, c and b are vectors of known coefficients. The expression "cx" is called the objective function, and the equations "Ax=b" are called the constraints. All these entities must have consistent dimensions, of course, and you can add "transpose" symbols to taste. The matrix A is generally not square, hence you don't solve an LP by just inverting A. Usually A has more columns than rows, and Ax=b is therefore quite likely to be under-determined, leaving great latitude in the choice of x with which to minimize cx.

The objective function of an LP must be a linear function of the decision variables and each of the LP constraints must be a linear inequality. The assumptions of LP, which are proportionality, additivity, divisibility and certainty, are summarized below.

1. Proportionality requires that the contribution of each variable in the objective function or its usage of the resources be directly proportional to the level or value of the variable.

2. The assumption of additivity concerns with the effect of conducting activities jointly. Additivity requires that the objective function be the direct sum of the individual contributions of the different variables. Similarly, the left side of each constraint must be the sum of the individual usages of each variable from the corresponding source.

3. The divisibility assumption is that activity units that can be divided into any fractional levels, so that non-integer values for the decision variables are permissible. Frequently, linear programming is still applied when an integer solution is required. If the solution obtained is non-integer variables are merely rounded to integer values.

4. The certainty assumption is that all the parameters of the model are known constants. In real problems, this assumption is seldom satisfied precisely. Linear programming models usually are formulated to select some future course of action. Therefore, the parameters used would be based on a prediction of future conditions, which inevitably introduces some degree of uncertainty.

The transportation model is a special type of the linear programming model concerning with selecting routes between manufacturing plants and distribution warehouses or between regional distribution warehouses and local distribution outlets.

As its name implies, the transportation method was first formulated as a special procedure for finding the minimum cost program for distributing homogenous units of a product from several points of supply (sources) to a number of points of demand (destinations).

The objective of the typical problem of this type is to minimize the cost of moving the resource. The simplex method can be used to solve this type of problem, although it is not the easiest method to use. A special algorithm (a computational procedure) called the transportation method or distribution method is available for solving transportation problems. The transportation method greatly simplifies the computation for a problem that can be expressed in the transportation-method format. In fact, the transportation method allows us to solve manually a problem that would require very lengthy calculations or a computer to solve by the simplex method.(Dilworth, 1993, p. 157)

2.1 Transportation Model

The transportation model is a special class of the linear programming problem. It deals with the situation in which a commodity is shipped from sources (e.g., plants) to destinations (e.g., warehouses). The objective is to determine the amounts of shipped from each source to each destination that minimize the total shipping cost while satisfying both the supply limits and the demand requirements. The model assumes that the shipping cost on a given route is directly proportional to the number of units shipped on that route.

The first article about the transportation model has been published by the Russian mathematician L.V. Kantrovich. The standard transportation model and the solution to it, has been stated by F.L. Hitchcock in 1941. In 1942 Kantrovich, and in 1947 T.C. Koopmans and G.B. Dantzig have contributed to develop the model.

2.1.1 Linear Programming Formulation for the Transportation Model

In transportation problem, there are m sources, supplying $a_1, a_{2,\ldots,} a_m$ of the product and n destinations, demanding $b_1, b_2, \ldots, b_n$ of the product, respectively. A unit transportation cost from source i to destination j is c_{ij}. The objective of the problem is to determine the x_{ij}'s which represent the amount to be transported from source i to destination j to minimize the total cost.

Transportation model has two important assumptions which are homogeneity and proportionality. Homogeneity is equality of the product types to be shipped and proportionality is the contribution of each variable in the objective function or its usage of the resources is directly proportional to the level or value of the variable.

The general problem is represented by the network in Figure 2.1.

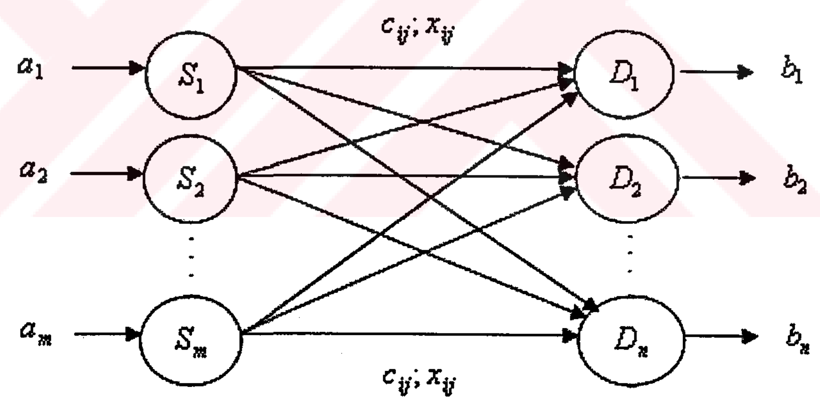

Figure 2.1 Graphical representation of general transportation model

The general transportation model is represented by the network in Figure 2.1. There are m sources and n destinations, each represented by a node. The arcs linking the sources and destinations represent the routes between the sources and the destinations. Arc (i,j) joining source i to destination j carries two pieces of information; (1) the unit transportation cost from source i to destination j, c_{ij}, and (2) number of units shipped from source i to destination j, x_{ij}. The amount of supply at source i is a_i and the amount of demand at destination j is b_j. The objective of the problem is to determine the unknowns x_{ij} that will minimize the total transportation cost while satisfying all the supply and demand restrictions.

Because of the transportation model is a special type of the linear programming model, the assumptions of linear programming model, which are defined in the beginning of this chapter are valid for the transportation model.

Linearity in transportation models implies that both the proportionality and additivity properties are satisfied. The additivity assumption is that, for each function the total function value can be obtained by adding the individual contributions from the respective shipments (or assignments). The divisibility assumption is that the decision variables which are number of units shipped from source i to destination j, can take non-integer values. The certainty assumption is that, all the parameters of the model such as objective function coefficients, demand of destination j, supply of source i and technological coefficients, are all known.

Decision variables and parameters of the transportation model are given below:

Decision variables:

x_{ij} = the number of units shipped from source i to destination j

Parameters:

c_{ij} = the unit transportation cost from source i to destination j

a_i = total supply of source i

b_j = total demand of destination j

A transportation model can be formulated as follows :

Objective function: $$\text{Min } z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \qquad (2.1)$$

Supply constraints:

The amount of product is shipped to destinations need to be at most that source's supply.

$$\sum_{j=1}^{n} x_{ij} \le a_i \qquad (i = 1, 2, 3, \ldots, m) \qquad (2.2)$$

Demand constraints:

The amount of product is received by destinations need to be at least that destination's capacity.

$$\sum_{i=1}^{m} x_{ij} \ge b_j \qquad (j = 1, 2, 3, \ldots, n) \qquad (2.3)$$

Nonnegativity condition : $$x_{ij} \ge 0 \qquad (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n) \qquad (2.4)$$

In general balanced transportation problem,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad (2.5)$$

is satisfied. It means that all the constraints must be binding.

And the transportation problem's table is shown in Table 2.1.

Table 2.1 A transportation table

	1	2	. . .	n	Supply
1	x_{11} c_{11}	x_{12} c_{12}	. . .	x_{1n} c_{1n}	a_1
2	x_{21} c_{21}	x_{22} c_{22}	. . .	x_{2n} c_{2n}	a_2
⋮	⋮	⋮	. . .	⋮	⋮
m	x_{m1} c_{m1}	x_{m2} c_{m2}	. . .	x_{mn} c_{mn}	a_m
Demand	b_1	b_2	. . .	b_n	

To solve the transportation problem, different methods are developed. Some of these methods are discussed in the next section.

2.2 Solution Methods for Transportation Models

Since the transportation problem is a special case of linear programming problem, simplex or revised simplex methods which are used to solve linear programming problems, can also be used to solve the transportation problem. Because of this, any software package which involves linear programming or transportation problem solution algorithms can be used when solving transportation problem.

The simplex algorithm can be improved to make the solution of transportation problem much easier, while improving the simplex algorithm the general procedure of simplex must be followed.

The algorithm starts with an initial basic solution and seeks the optimal solution by improving the basic solution in each step. When a better solution can not be found, the current solution is defined as the final or another words the optimal solution.

Various methods have been developed to obtain the initial basic feasible solution and to test whether a better solution exists in each step for the transportation problem.

The transportation problem which is a special class of the linear programming problem, can be solved by its own solution methods. *The transportation method is an iterative process. It begins with a feasible solution, then improves it every iteration until it can be improved no further. The objective function for a transportation problem can be expressed in terms of cost or profit, and the algorithm can be worked to reduce costs to a minimum value or to increase profit to a maximum. The costs considered in a minimization problem are not limited ot transportation costs, so the method has more versatility than the name implies.* (Dilworth, 1993, p. 157) The steps of this method are shown in below.

2.2.1 Properties of the Initial Basic Solution

A solution is said to be a feasible solution if it satisfies the equations (2.2), (2.3) and (2.4). Satisfying (2.2) and (2.3) yields to satisfying (2.5). In order to be feasible, a solution must have a balanced transportation model. But the unbalanced model can also be solved.

A solution can be a basic solution if, number of assignment is equal to $(m + n - 1)$ and the assigned cell don't form a loop in either direction. $(m + n - 1)$ cells are assigned the solution set, which consists of $(m + n - 1)$ variables x_{ij}, is called the basic. The assigned cells don't form a loop in either direction

Some examples of loops and nonloops are illustrated in Figure 2.2.

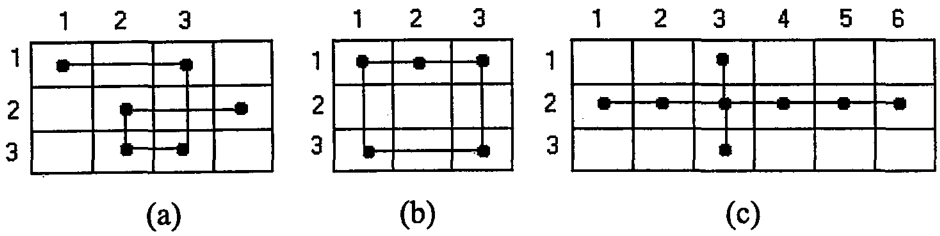

Figure 2.2 Examples of loops and nonloops: (a) not a loop; (b) loop, property is violated; (c) not a loop.

2.2.2 Solution Steps

There are various methods for solving the transportation model. But, to find the optimal solution, all of these methods need a "initial basic solution". So there are two steps for solving the model:

1. Finding an initial basic solution,
2. Finding the optimal solution.

First, we begin with explaining the initial basic solution.

2.2.3 Methods of Finding the Feasible Initial Basic Solution

Various techniques have been developed to find the feasible starting basic solution, with satisfying both the requirements of being feasible and being basic.

Northwest Corner Method, Least-cost Method, Vogel's Approximation Method, and Russel's Approximation Method are the four widely used methods for obtaining the initial basic solution for the transportation problem. The simplest one of these methods is Northwest Corner Method.

The difference between among the three method is "quality" of the starting basic solution they produce, in the sense that a better starting solution yields a smaller objective value. In general, the Vogel method yields the best starting basic solution, *and the northwest-corner method yields the worst. The trade-off is that the nortwest-corner method involves the least computations.*(Taha, 1997, p. 181)

We are now ready to discuss the Northwest Corner Method that can be used to find a basic feasible solution for a balanced transportation problem.

If a set of values for the x_{ij}'s satisfies all but one of the constraints of a balanced transportation problem, the values for the x_{ij}'s will automatically satisfy the other constraint.

- **Northwest-Corner Method**

In solving any linear programming problem, we should, in general, expect the total number of iterations required to depend on how close the value of the objective function for the first feasible solution is to the actual minimum. Since the nortwest_corner rule does not consider the size of the c_{ij}, *we cannot expect the corresponding value of the objective function to be close to the minimum.*(Gass, 1975, p. 267)

In this method, first assignment is made to the cell in the northwest corner of the table. Since a source will be consumed or a destination will be satisfied in each iteration, a row or column is crossed out. In each iteration, assignment is made to the cell in the northwest corner of the resulting table of the previous iteration. The iterations stop when all assignments are made.

1. Select the cell in the northwest corner of the table.
2. Assign the possible maximum value to this cell ($x_{ij} = \min[a_i, b_j]$).

Special case: If $a_i = 0$ or $b_j = 0$ (degenerate), assign ε, which stands for a very very small value.

3. Cross out the row or column with zero supply or demand to indicate that no further assignments can be made in that row or column. If both the row and the column net to zero simultaneously, cross out one only, and leave a zero supply (demand) in the uncrossed out row (column).
4. Subtract the assigned value from the supply and the demand values.
5. If all of the cells are crossed out, then stop; else go back to 1.

Although the Northwest Corner algorithm is easy to apply, it is not the best algorithm, because the c_{ij} values are not taken in consideration. Since the rest of the methods make use of these c_{ij} values, they provide a better initial basic solution which reduces the number of steps needed to obtain the optimal solution. Detailed descriptions of these method's algorithms can be found in every operations research related book such as Dantzig(1966), Wagner(1969), Harvey(1979), Lieberman(1990), Taha(1997), Winston(1994) etc.

Of the three methods we have discussed for finding a basic feasible solution, the northwest-corner method requires the least effort and Vogel's method requires the most efforts. Extensive research (Glover et al., 1974) has shown, however, that when Vogel's method is used to find an initial basic feasible solution, it usually takes substantially fewer pivots than if the northwest-corner method or the minimum cost method had been used. For this reason, the nortwest-corner and the minimum cost method are rarely used to find a basic feasible solution to a large transportation problem.(Winston, 1991, p. 344).

2.2.4 Methods for Obtaining the Optimal Solution

When the initial basic solution is found by any of these methods, to test whether this solution optimal or not, each nonbasic variable must be included in the solution and the total cost must be compared. If the total cost decreases, a better solution is found. On the other hand, If it increases, the nonbasic variables should not be a part of the solution. In order to find the minimum value of the objective function the procedure which is described above should be applied systematically. The two main methods to obtain optimal solution:

a. Stepping-Stone Method, and
b. MODI Method.

Stepping-Stone Method was developed by W.W.Cooper and A. Charnes in 1953, the Modified Distribution Method was developed by R.O. Ferguson in 1955 and Vogel's Approximation method was developed by W.R.Vogel and N.V.Reinfeld in the years of 1956-57.

To develop an optimal solution for a transportation problem means evaluating each unused cell to determine whether a shift into it is advantageous from a total cost standpoint. If it is, the shift is made, and the process is repeated. When all cells have been evaluated and appropriate shifts made, the problem is solved.

Although the general procedures, are the same for both of these methods they differ in the process of determining the entering and leaving variables. The stepping stone method uses the costs to determine the entering and leaving variables, which makes the decision process easy to explain, whereas the MODI method uses the dual variables to make his decision. But this difference doesn't affect the economic interpretations of the results of these two methods. Both of these methods will be explained below, however, simplex and MODI method will be used in the following chapters. The solution procedures of these two methods are described below.

a) Stepping-Stone Method

This method systematizes the procedure of finding an empty cell in the transportation table worth making an assignment, and if such a cell is found, transferring the value of an assigned cell to this cell by keeping the feasibility conditions satisfied.

The method has two main steps. The first one, determines if a cell is worth making an assignment; or, in other words, the entering variable. The second one explains how the assignment will be done and how much will be assigned, which determines the leaving variable.

While determining the entering variable;

Whether a cell is worth making an assignment or not, can be decided by the assignment's effect on the total cost. If the total cost decreases, then the cell is assigned a value. To investigate the effect of a cell to the total cost, one unit is assumed to be assigned to the cell.

The cost values of the assigned cells are called "stones" and the empty cells are called "water". The cell to be investigated is marked with the "+" sign. The objective is to loop back to the starting cell by only stepping on the "stone" cells. But, after each step, a 90 degree left or right turn must be made. Such a move is called "rectilinear". While stepping, the starting cell is marked with a "+", and the following cells are marked with "-" and "+", consecutively. The cost values of the cells on the loop are summed, taking the signs of the cells into consideration, and the value of the starting cell is found. For each unassigned cell, one and only one loop exists. In a loop, there's no unassigned cell, other than the starting cell.

In this method, there is no need to investigate all of the unassigned cells. The first unassigned cell with the negative result can be chosen as the entering variable. However, to choose the cell with the maximum absolute negative value as the entering variable decreases the number of the iterations in order to find the optimal solution.

While determining the leaving variable;

After determining the entering variable, the leaving variable is determined and the assigned value in this cell is shifted to the cell of the entering variable.

It is known that the number of assigned cells, or basic variables, must be equal to $(m + n - 1)$ in a basic solution. When a new cell is assigned, the number of basic variables becomes $(m + n)$, and the solution is, no longer, a basic solution. But, the assigned value is shifted from a pre-assigned cell in a way that does not violate the feasibility conditions, and the number of basic variables stays as $(m + n - 1)$.

The leaving variable is chosen as the cell with the minimum value among the "-" signed cells on the loop, which is formed by the stepping stone method in the previous step. The assigned value of this cell is subtracted from the values of the "-" signed cells and added to the values of the "+" signed cells. This way, a new variable is added to the basic solution without violating the feasibility conditions.

The steps of stepping-stone method which is the first way of the methods to determine optimal solution are described below.

A. Choose the entering variable by examining the unassigned cells on the basic solution table obtained in the last step

1. Pick an unassigned cell
2. By stepping on the assigned cells, form a loop. After each step, turn left or right.
3. Starting from the chosen cell, sign each cell on the loop with "+" and "-", consecutively.
4. Sum the cost values of the cells on the loop with their signs.
5. If the sum is negative, choose this cell as the entering variable; else go back to step 1.

6. If the sum is positive for all the unassigned cells, then the optimum solution is found; stop.

B. Determine the leaving variable and make the assignment.

1. Take the loop for the entering variable with its signs into consideration.
2. Find the "-" signed cell on the loop with the minimum value. This is the leaving variable.

Special case: If there are two or more cells with the minimum value, to prevent degeneration, add ε to the values of the cells, except the one with the maximum c_{ij} value.

3. Subtract the minimum assigned value from the values of the "+" signed cells, and add the value to the values of the "-" signed cells.
4. Calculate the total cost.

C. For a better basic solution go back to A.

As a brief summary, this determines whether the initial solution found by the least cost rule is optimum. We know from the simplex method that a given solution minimizes the objective function only if the relative cost coefficients of the nonbasic variables (net change in z per unit increase in the nonbasic variables) are greater than or equal to zero.

Similar to the inner product rule used in the simplex method for calculating the relative costs, there is a simple way to calculate all the c_{ij} coefficients directly.(Ravindran et al., 1987, p. 83)

b) MODI Method

There are two disadvantages of the Stepping-Stone Method. The first of these two is the criterion which is used to decide which nonbasic variable will enter the basic solution. In the Stepping-Stone Method, it is enough for a variable to result to a negative value in order to be considered as the entering variable. Because this method takes the first cell with negative value as the entering variable, another cell with a greater decreasing effect on the total cost can be discarded. But, choosing the cell with the absolute negative value as the entering variable decreases the number of iterations to obtain the optimal value. This situation, which is ignored in the Stepping-stone method, has taken into consideration in the MODI Method.

The second disadvantage of the Stepping-Stone Method is the necessity to form a loop for each of the empty cells. Although forming loops is respectively easy for small problems, it becomes quite hard when it comes to problems with large number of variables, whereas in the MODI Method, only one loop is formed for the entering variable.

i. Theoretical Basis of the MODI Method

Theorem 1: Assume that $u_1, u_2, \ldots, u_m$ and $v_1, v_2, \ldots, v_n$ arbitrarily constants; according to this every obtained solution for a (*m.n*) transportation problem with a_i, b_j side constraints. And c_{ij} cost values, is a solution to the transportation problem with the same side constraints and $c_{ij} - u_i - v_j$ cost values. The reverse case is also true. Furthermore, the optimal solution for one of the problems is also optimal for the other.

Proof: Let x_{ij} be a solution set for the first problem. According to this the total transportation cost for any feasible basic solution becomes

$$TC_1 = \sum_i \sum_j x_{ij} c_{ij}$$

On the other hand, the total cost for the same solution set for the second problem is

$$TC_2 = \sum_i \sum_j x_{ij}\left(c_{ij} - u_i - v_j\right) = \sum_i \sum_j x_{ij} c_{ij} - \sum_i \left(\sum_j x_{ij}\right) u_i - \sum_j \left(\sum_i x_{ij}\right) v_j$$

According to (2.2) the term in the first parenthesis is equal to a_i; and according to (2.3) the term in the second parenthesis is equal to b_j. By substituting these values is

$$TC_2 = \sum_i \sum_j x_{ij} c_{ij} - \left(\sum_i a_i u_i + \sum_j b_j v_j\right) = \sum_i \sum_j x_{ij} c_{ij} - K = TC_1 - K$$

obtained. Since a_i, u_i, v_j, b_j are all constants this expression will be equal to a constant *K*. Subtracting a constant from the objective function does not change the solution set which minimizes the function; this only changes the value of the function. Consequently, any set of x_{ij} which is a solution for the first problem with c_{ij} costs, is also a solution for the second problem with the $c_{ij} - u_i - v_j$ costs. An optimal solution for one problem is also optimal for the other.

With the help of Theorem 1, constants can be added to or subtracted from the cost matrix's rows or columns without changing the optimal solution set. This way, it is possible to make the cost values of the assigned cells to zero.

Theorem 2 : Let X be a feasible basic solution for a transportation problem :

1. If $c_{ij} - u_i - v_j$ values for the assigned cells are zero and nonnegative for the other cells then X is the optimal solution set.
2. If $c_{ij} - u_i - v_j$ values for the assigned cells are zero but negative for at least one of the other cells, then X is not the optimal solution set.

Proof : Duality theorem is used to prove this theorem. It can also be proved by making use of the simplex algorithm.

ii. Application of the Method

Application of the method is also mainly based on two steps. The first of this steps is to determine which of the empty cells is to be assigned, and the second one is to determine the leaving variable and the necessary shift of assignments. The MODI method differs from the stepping stone method by the application of the first step.

While determining the entering variable;

Whether a cell is worth making an assignment or not, can be decided by the assignment's effect on the total cost. If the total cost decreases, then the cell is assigned a value. In the MODI method, the entering variable is chosen among the empty cells worth assigning with the most decreasing effect on the total cost. This is done with the help of Theorem 1 and Theorem 2. u_i and v_j are subtracted from the cost matrix's rows and columns, respectively, in order to make the cost values of the assign cells of the starting solution table zero. If none of the $c_{ij} - u_i - v_j$ values of the unassigned cells are negative, then the optimal solution is obtained. The problem, here, is determination of the u_i and v_j values. Since one u_i for each row and one v_j for each column should be determined the total number of u_i and v_j is $(m + n)$. Since there are $(m + n - 1)$ basic variables, according to Theorem 2, to calculate $(m + n)u_i$ and v_j, $(m + n - 1)$ equations can be formed. In order to find the values of $(m + n)$ constants an arbitrarily value (usually 0) is given to u_i or v_j and the remaining $(m + n - 1)$ variables are calculated like above. The general approach is to assign zero to the values of u_i or v_j which belongs to the row or column of the starting solution table with the most assigned cells.

While determining leaving variable;

After determining the enter variable the leaving variable must be determined and the assignment of this cell must be shifted to the entering cell. The very same procedure of the stepping stone method is used in the MODI method.

The steps of MODI method which is one of the methods to determine the optimal solution are described below.

A. Determine the entering variable by evaluating the unassigned cells in the last obtained feasible basic solution table.

1. Determine the values of u_i and v_j for each row and column in the table.

    a. Make the value of u_i or v_j of the row or column with the most assigned cells, equal to zero.
    b. Use the calculated u_i or v_j to calculate the unknown u_i or v_j for each assigned cell, which satisfies the equation $c_{ij} - u_i - v_j = 0$. Repeat this until all values of *U* and *V* are calculated.

2. For each empty cell calculate $d_{ij} = c_{ij} - u_i - v_j$ and write it to the upper right corner of the cell.
3. If all d_{ij} are nonnegative then the optimal solution is obtained, stop. If at least one negative d_{ij} exist the current solution is not optimal; choose the cell with the absolute maximum d_{ij} among the negative ones as the entering variable.

B. Determine the leaving variable and necessary assignments

1. Form the loop for the entering cell and sign the cells on the loop.

a. Starting from the entering cell and using only the assigned cells form a loop. Each line on the loop must be either vertical of horizontal. After each line a 90 degree turn must be made.
b. Starting from the entering cell, sign each cell on the loop with "+" and "-" consecutively.

2. Choose the cell with the minimum assignment value among the cells sign with "-", as the leaving variable.

Special Case: If there are two or more assignments with the same minimum value, add ε to these cells, except the one which has the max c_{ij} value.

3. Subtract the minimum assignment value from the "-" signed cells' assignments; and add this value to the "+" signed cells' assignments.
4. Calculate the total cost.

C. For a better basic solution go to step A.

After illustrating the solution methods which are the methods to finding optimal solution, we will explain some special cases of transportation problem.

2.3 Special Cases of Transportation Problems

Degeneracy exists in a transportation problem when the number of filled cells is less than the number of rows plus the number of columns minus one $(m + n - 1)$. Degeneracy may be observed during the initial allocation when the first entry in a row or column satisfies both the row and column requirements. Degeneracy requires some adjustment in the matrix to evaluate the solution achieved. The form of this adjustment involves inserting some value in an empty cell so a closed path can be developed to evaluate other empty cells. This value may be thought of as an infinitely small amount, having no direct bearing on the cost of the solution.

2.3.1 Degeneration

Degeneration in a transportation problem occurs when the solution fails to be a basic solution, during the process of finding either the feasible starting basic solution or the optimal solution. Degeneration may also occur when any of the sources is equal to any of the demand.

$$\text{for} \quad I\{1,2,\ldots,m\} \text{ and } J\{1,2,\ldots,n\}, \quad \sum_{i\in I} a_i = \sum_{j\in J} b_j$$

A basic solution has to have $(m + n - 1)$ basic variables which don't form a loop. If the number of basic variables is different from $(m + n - 1)$ or a loop can be formed, then degeneration occurs. Since there are not enough assigned cells, for such a case, it is impossible to

1. Form the loops for evaluating the empty cells in the stepping stone method,
2. Calculate the u_i and v_j values in the MODI method

Since none of the methods describe above lets more than $(m + n - 1)$ basic variable, the important case of degeneration is the case with less than $(m + n - 1)$ basic variables.

a. Degeneration during the process of finding the starting solution

According to the common second step of the least cost method, northwest corner method and VAM, the chosen cell must be assigned a value until the demand is fully satisfied or the source is completely consumed. In other words, the cell (i,j) can be assigned the value $x_{ij} = \min(a_i, b_j)$. But if $a_i = b_j$ then degeneration occurs. Because by assigning $x_{ij} = a_i = b_j$ to cell (i,j) the i^{th} row for the source and j^{th} column for the demand must be crossed out. Since there will be less than $(m + n - 1)$ basic variables in the resulting solution table, this is a case of degeneration. All of the method above have prevented this case of degeneration.

b. Degeneration during the process of finding the optimal solution

In both of the MODI and Stepping Stone Methods, to make a variable enter the solution, the related cell is taken as a starting point and a loop is formed. The cells which form the loop are marked "+" and "-", consecutively. The minimum value among "-" signed cells is subtracted from the values of the other cells with "-" signs. If two or more cells with "-" signs have a minimum value, this is the case of degeneration. Because, when the assignment is subtracted from the cells with "-" signs, the assignment with the minimum value is equal to 0. Although two or more variables leave the basis, only one variable enter the basis. Because of having less than $(m + n - 1)$ variables, degeneration occurs.

2.3.2 Unbalanced Transportation Model

When the total source is equal to the total demand, the transportation model is called general transportation model or balanced transportation model. But in real life this is not generally, the case. The methods described above, are useful, only, for the balanced models. For this reason, an unbalanced problem can only be solved after it has been turned into a balanced one. This can be accomplished by adding a dummy source or destination. A model is unbalanced when the total source is different form the total demand.

Probably the most usual circumstance is for the availability capacity to exceed demand. Many companies try to maintain extra inventory or service capacity to ensure flexibility and prompt response to demand. Usually the choice of the supply location to be underutilized in such circumstances is an issue that affects only the operations function. Operations managers can obtain useful guidance in reaching capacity underutilization decisions by use of the transportation method.(Dilworth, 1979, p.167)

a. Total source is greater than the total demand

When the total source is greater than the total demand, a dummy destination is added to the problem, and the difference between these values is shipped to this destination.

$$\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$$

$$b_{n+1} = \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$$

Since the destination is dummy, shipping means the assigned source is held and that the cost is zero. After the dummy destination is added to the transportation table with zero cost and demand which is equal to the surplus, the problem can be solved with the methods above.

b. Total demand is greater than the total source

When the total demand is greater than the total source, although the total source is completely shipped, the total demand will not be fully satisfied.

$$\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$$

$$a_{m+1} = \sum_{j=1}^{n} b_j - \sum_{i=1}^{m} a_i$$

If this is the case, the problem can be balanced by adding a dummy source. The shipping cost is zero, again. The problem is now balanced and can be solved with the methods above.

2.3.3 Constrained Shipping

Up to now, in all the transportation models it is assumed that shipment can be made to all of the destinations from all of the sources. But, this may not always be the case. The reason for this, can either be the expensive shipping costs or the impossibility of connecting all nodes, i.e., the shipping from source *i* to destination *j* is prohibited for those reasons, x_{ij} must be equal to zero in the result. For this reason, c_{ij} which is placed the (*i,j*) cell is given M value which stands for a very very large number. So that, even one unit of the assignments of that cell's shipping cost very high, this cell's assignment is prevented.

2.3.4 Certain Shipping

If certain shipping wants to be transport, because of the many reasons such as the close destination between the source and the destination, technological requirement i.e., then c_{ij} value is taken zero related with the source and the demand.

2.3.5 Alternative Optimal Solution

The equivalence of $d_{ij} = 0$ means that to assigning (*i,j*) cell is not change the optimal transportation cost. However, in the optimal solution table with doesn't exist $-d_{ij}$ value, if one or more nonbasic variable's d_{ij} value is equal to zero, there are other optimal solution with the same total transportation cost.

2.3.6 Maximization in the Transportation Problem

Suppose that the objective function of a transportation method problem were written in terms of profit rather than cost. Optimization of such a problem would require the maximization of the objective function. The only change in the transportation algorithm is to move material into the unoccupied cell with the largest positive value on each iteration. Cells in a dummy row or column represent material that actually will not be shipped or sold, so they still contribute zero to the objective function and should have 0's in their upper corner blocks.

"Nothing else should be changed in the procedure that was outlined previously. A closed path is evaluated with a plus sign in the unoccupied cell where the path begins, just as before. The amount that should be shifted in a path is still the smallest amount in the negative corners."(Dilworth, 1993, p.167)

The only important difference between the procedure for solving a minimization problem and for a maximization problem lies in the rule for improvement based on the empty cell evaluations. In maximization, the sign rule is reversed and an optimal solution is obtained when the cell evaluations give all negative values; a positive sign indicates additional benefit can be obtained by transferring units into the corresponding cell location. As in minimization, the cell that is introduced into the solution is the one that offers the greatest benefit per unit.(Tersine, 1985, p.136).

2.4 Sensitivity Analysis for Transportation Problem

Sensitivity analysis is an effect of parameters on the optimal solution. Up to now, we explained how the optimal solution is obtained for the transportation problem with the constant data. But it is normal that the unit shipping cost , amount of source and, amount of demands changes in the time. Changing these variables can change the optimal solution. Sensitivity analysis helps us to obtain a better interpretation and to determine how the optimal solution changes related to the differences on the parameters. Two aspects of sensitivity analysis for the transportation problems are :

- Sensitivity analysis of the cost
- Sensitivity analysis of supply and demand

In this section, the analysis that determines the range of a given parameter for which the solution, as originally stated, remains optimal will be discussed.

2.4.1 Sensitivity Analysis of the Cost

When the unit cost of the basic and nonbasic variables change, optimal solution can be affected by these variations. Sensitivity analysis investigates how much optimal solution is influenced by the variations on the basic and nonbasic variables.

a. To change the objective function coefficient of a nonbasic variable

The values of u_i and v_j don't change. If the unit transportation cost changes, x_{ij} which is affected by this change, is the test amount of nonbasic variable. As long as $c_{ij} - u_i - v_j \geq 0$, the optimum solution remains the same. It means each c_{ij} value is at least, equal to $u_i + v_j$. Since we are not changing $c_B B^{-1}$, the u_i's and the v_j's remain unchanged. In objective coefficient row, only the coefficient of x_{ij} will change. Thus, as long as the coefficient of x_{ij} in the optimal row 0 is nonpositive, the current basis remains optimal.

b. To change the objective function coefficient of a basic variable

To determine the sensitivity of the cost coefficient of the basic variables;

When we are changing $c_B B^{-1}$, the coefficient of each nonbasic variable in objective function row may change, and to determine whether the current basis remains optimal, the new u_i's and v_j's must be found and these values are used to price out all nonbasic variables. As long as all nonbasic variables price out nonpositive, the current basis remains optimal.

2.4.2 Sensitivity Analysis on the Supply and Demand

In the case of increasing or decreasing the amounts of supply and demand, sensitivity analysis is used to determine whether the current basis remains optimal. This change is observed in the balanced transportation problem.

The u_i's and v_j's may be thought of as the negative of each constraint's shadow prices. If the increase in the amount of production or the amount of demand are shown as Δa_i and Δb_j, respectively, then the value of objective function is equal to

$$\text{New z value} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij} + \Delta a_i(u_i) + \Delta b_j(v_j)$$

1. If x_{ij} is a basic variable in the optimal solution, Δ is added to x_{ij}.
2. If x_{ij} is a nonbasic variable in the optimal solution, the loop is found which involves x_{ij} and some of the basic variables is found. An odd cell is found in the loop that is in row *i*. Starting from this odd cell, Δ is added and subtracted from the values of the cells forming the loop, consecutively.

CHAPTER THREE
CAPACITATED TRANSPORTATION MODEL

Consider a linear optimization model having the form of a transportation model with the addition of upper bound constraints $x_{ij} \leq u_{ij}$ on the amounts to be shipped over the various routes. A model of this form is said to be a capacitated transportation model. Such models occur frequently in applications and it is important to be able to efficiently handle the capacity constraints which may be far more numerous than the ordinary constraints.

A capacitated transportation model can be analyzed by a modification of the transportation algorithm analogous to the modification of the simplex algorithm for upper bound constraints or by the out of kilter algorithm for capacitated network models. Discussions of these algorithms may be found in the books by Spivey and Thrall (1970) and Poots and Oliver(1972), respectively (Harvey, 1979, pp. 205, 206).

3.1 Linear Programming Formulation for the Capacitated Transportation Model

The capacitated transportation model satisfies all of the assumptions of the classical transportation model, and consequently the assumptions of the linear programming model. The general model is represented by the network in Figure 3.1.

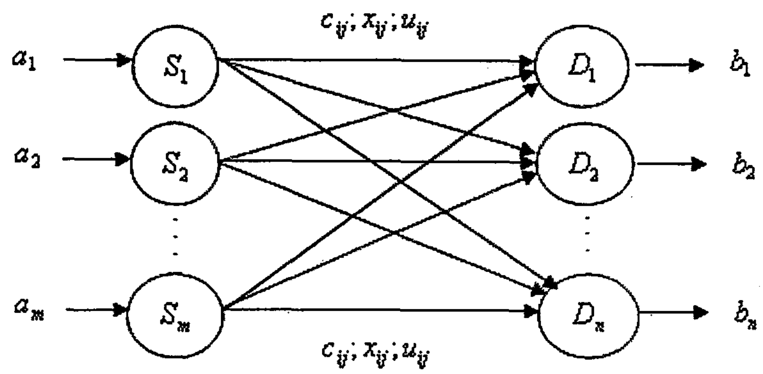

Figure 3.1 Graphical representation of a capacitated transportation model

The general capacitated transportation model is represented by the network in Figure 3.1. A unit transportation cost from source i to destination j is denoted by c_{ij}. The objective of the model is to determine the x_{ij}'s which represent the number of units shipped from source i to destination j, total supply of source i is denoted by a_i, total demand of destination j is denoted by b_j, the u_{ij} values, used in the upper bound constraints, denote the maximum amount which can be shipped from source i to destination j. These upper bound constraints are the only difference between capacitated and classical transportation models.

The decision variables, parameters, model and the constraints for a capacitated transportation model can be described as follows :

Decision variables:

$x_{ij} =$ the number of units shipped from source i to destination j

Parameters:

$i = 1, \ldots, m$: sources
$j = 1, \ldots, n$: destinations
$c_{ij} =$ the unit transportation cost from source i to desination j

u_{ij} = the maximum amount which can be shipped from source i to destination j

a_i = total supply of source i

b_j = total demand of destination j

Objective function : $$\text{Min } z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \qquad (3.2)$$

Supply constraints : $$\sum_{j=1}^{n} x_{ij} = a_i \qquad (i = 1, 2, 3, \ldots, m) \qquad (3.3)$$

Demand constraints: $$\sum_{i=1}^{m} x_{ij} = b_j \qquad (j = 1, 2, 3, \ldots, n) \qquad (3.4)$$

Capacity constraints: $$x_{ij} \leq u_{ij} \qquad (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n) \qquad (3.5)$$

Nonnegativity condition : $$x_{ij} \geq 0 \qquad (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n) \qquad (3.6)$$

The table of the capacitated transportation problem is shown in Table 3.1.

Table 3.1 A capacitated transportation table

	1	2	. . .	n	Supply
1	x_{11} u_{11} c_{11}	x_{12} u_{12} c_{12}	. . .	x_{1n} u_{1n} c_{1n}	a_1
2	x_{21} u_{21} c_{21}	x_{22} u_{22} c_{22}	. . .	x_{2n} u_{2n} c_{2n}	a_2
⋮	⋮	⋮	. . .	⋮	⋮
m	x_{m1} u_{m1} c_{m1}	x_{m2} u_{m2} c_{m2}	. . .	x_{mn} u_{mn} c_{mn}	a_m
Demand	b_1	b_2	. . .	b_n	

3.2 Solution of Capacitated Transportation Problem

As in the classical transportation problem, an initial basic solution is found and then the solution is improved to obtain the optimal solution.

3.2.1 Finding an Initial Basic Feasible Solution

While simple rules have been devised for finding an initial solution in an uncapacitated transportation problem, it does not appear possible to construct such a rule in the capacitated case.

First, the model is checked to determine whether it is balanced or not. If it is not balanced, a dummy row or column is added. After this modification, the solution phase begins.

1. The cell with the minimum cost in the table is selected and assigned the maximum value possible. If this assignment fully satisfies either the row's supply or the column's demand then the variable is called a basic variable. Otherwise if the assigned value is limited with the upper bound value of the cell then it is called a bounded variable. This step is repeated until there exists no possible assignment.

2. Next, the table is checked to determine if all of the demands and supplies are fully satisfied. If this is not the case, then

   a. Two cells, u_0 and v_0, with suitable assignment are added to the row and the column which are not fully satisfied.
   b. A new table is formed, in which u_0 and v_0 have costs equal to 1, and the others have costs equal to 0.
   c. The assigned values of u_0 and v_0 are then moved to the appropriate cells.
   d. A new table, in which the costs are equal to the first table, is formed. This new table is a basic feasible solution.

We will show how the solution steps, described above are adapted efficiently to the capacitated transportation problem by solving a simple numerical example given in Table 3.3 below.

Table 3.3 The capacitated transportation problem

	1	2	3	4	Supply
1	$u_{11}=12$ $c_{11}=10$	$u_{12}=13$ $c_{12}=5$	$u_{13}=5$ $c_{13}=6$	$u_{14}=20$ $c_{14}=7$	25
2	$u_{21}=14$ $c_{21}=8$	$u_{22}=20$ $c_{22}=2$	$u_{23}=10$ $c_{23}=7$	$u_{24}=9$ $c_{24}=6$	25
3	$u_{31}=18$ $c_{31}=9$	$u_{32}=4$ $c_{32}=3$	$u_{33}=25$ $c_{33}=4$	$u_{34}=7$ $c_{34}=8$	50
Demand	15	20	30	35	

First, the cell with the minimum c_{ij} in Table 3.4 is selected which is c_{22} with a value of 2. Then the maximum possible value is assigned to this cell, which sets $x_{22}=$ 20. If the size of this assignment is finally limited by a row or column equation, it is considered as a basic variable and make no more assignments in that row or column. If, on the other hand, the value of the assignment is limited by its upper bound restriction, then the variable is considered as a non-basic variable at its upper bound and a bar is placed above the variable. In case of a tie between the two types of limitations, the row or column is always considered as limiting and the variable is taken as basic. Then the same procedure is repeated with the remaining cells.

Applied to Table 3.4, this routine yields the following assignments, in order : $x_{13}=$ 5(basic), $x_{14}=$ 20(basic), $x_{22}=$ 20(basic), $x_{24}=$ 5(basic), $x_{31}=$ 15(basic), $x_{33}=$ 25(bounded), $x_{34}=$ 7(bounded).

Table 3.4 The solution table

12 10	13 5	**5** 5 6	**20** 20 7	**25**	-1
14 8	**20** 20 2	10 7	**5** 9 6	**25**	-1
15 18 9	4 3	**$\overline{25}$** 25 4	$\overline{7}$ 7 8	**50**	1
15	**20**	**30**	**35**		u_i
-1	1	1	1	v_j	

Since the third row and fourth column still have 3 units unassigned, the solution is not feasible. Extra "short" cells are added to the array: an $i = 0$ row and $j = 0$ column, and $c'_{ij} = 0$ replaces the original c_{ij}, and $c'_{ij} = 1$ in the shortage cells. This is summarized in Table 3.5.

Note that $c_{30} = c_{04} = 1$ must equal u_3 and v_4 respectively, since the slack rows and columns can be regarded as having prices u_0 and v_0 equal to zero.

Proceeding now with minimizing the sum of the artificial variables, in particular, $x_{04} + x_{30}$, a feasible solution can be achieved with only a single iteration, as given by Table 3.6.

Table 3.5 The capacitated transportation table with shortage cells

				$3-\theta_0$ 1		
	12 $c'_{11}=0$	13 0	5 5 0	20 20 0	25	-1
	14 0	$20-\theta_0$ 20 0	10 0	$5+\theta_0$ 9 0	25	-1
$3-\theta_0$ 1	15 18 0	θ_0^* 4 0	$\overline{25}$ 25 0	$\overline{7}$ 7	50	1
	15	20	30	35		u_i
	-1	1	1	1	v_j	

To understand Table 3.6, The optimality conditions must be known.

3.2.2 Finding Optimal Solution

After achieving the initial basic feasible solution, this is checked to see if it is the optimal solution, as in the classical transportation problem. The optimality conditions are as follows:

$$0 < x'_{ij} < u_{ij} \quad \Rightarrow \quad d_{ij} = 0$$

$$x'_{ij} = 0 \quad \Rightarrow \quad d_{ij} \geq 0$$

$$x'_{ij} = u_{ij} \quad \Rightarrow \quad d_{ij} \leq 0$$

where,

x'_{ij} = the value of the x_{ij} which is the number of units shipped from source i to destination j

u_{ij} = the maximum amount can be shipped from source i to destination j

d_{ij} = the relative cost factor which is equal to $c_{ij} - u_i - v_j$.

If the initial basic feasible solution does not satisfy the optimality conditions, a closed loop is constructed starting with the cell which violates the optimality condition. Each corner of the resulting loop, must coincide with a current basic variable. Next, we assign the amount θ to the cell with violates the optimality.

1. If this cell is a bounded one, θ is alternately added to or substracted from the value of cells, starting with substracting θ from the value of this cell.
2. If this cell is a nonbasic one, θ is alternately added to or substracted from the value of cells, starting with adding θ to the value of this cell.

Maximum value of θ is determined based on three conditions;

1. The capacity values of the cells is not violated,
2. The supply limits and the demand requirements remain satisfied,
3. No negative assignments are allowed through any of the routes.

Table 3.6 Basic feasible solution

12 10	13 5	5 5 6	20 20 7	25	0
14 8	17 20 2	10 7	8 9 6	25	-1
15 18 9	3 4 3	$\overline{25}$ 25 4	$\overline{7}$ 7 8	50	0
15	20	30	35		u_i
9	3	6	7	v_j	

The original cost factors, c_{ij}, are now restored. However, this solution is not optimal, because x_{34} is a non-basic variable at its upper bound, whose relative cost factor should be nonpositive, while in reality, $\bar{c}_{34} = c_{34} - u_3 - v_4 = 8 - 0 - 7 = +1$. Thus, it pays to decrease x_{34} from its upper bound value, keeping the other non-basic variables fixed and adjusting the basic variables.

Table 3.7 First iteration in solving the problem

12 10	13 5	5 5 6	20 20 7	25	0
14 8	17-θ 20 2	10 7	8+θ 9 6	25	-1
15 18 9	3+θ 4 3	$\overline{25}$ 25 4	$\overline{7}-\theta^*$ 7 8	50	0
15	20	30	35		u_i
9	3	6	7	v_j	

The greatest decrease, θ, that maintains feasibility is $\theta = 1$, and at this value it is stopped by the upper bounding restriction,

$x_{24} = 8 + \theta \leq 9.$

Table 3.8 Optimal solution

12 10	13 5	**5** 5 6	**20** 20 7	25	-1
14 8	**16** 20 2	10 7	**$\overline{9}$** 9 6	25	-1
15 18 9	**4** 4 3	**$2\overline{5}$** 25 4	**6** 7 8	50	0
15	20	30	35		u_i
9	3	7	8	v_j	

The new array, given in Table 3.6, is optimal. Optimal assignments are $x_{13} = 5$ (basic), $x_{14} = 20$ (basic), $x_{22} = 16$ (basic), $x_{24} = 9$ (bounded), $x_{31} = 15$ (basic), $x_{32} = 4$ (basic), $x_{33} = 25$ (bounded), $x_{34} = 6$ (basic). Optimum value of the objective function is 551.

3.3 Special Cases in Capacitated Transportation Problem

Degeneracy exist in a a capacitated transportation problem when the number of filled cells is less than the number of rows plus the number of columns minus one ($m + n - 1$). Degeneracy may be observed during the initial allocation when the first assignment in a row or column satisfies both the row and column requirements. Degeneracy requires some adjustment in the matrix to evaluate the solution achieved. The form of this adjustment involves inserting some value in an empty cell so a closed path can be developed to evaluate other empty cells. This value may be thought of as an infinitely small amount, having no direct bearing on the cost of the solution.

The special cases, which are degeneration, unbalanced transportation problem, certain shipping etc. for the capaciated and the classical transportation problems, and their standardization procedures are the same.

3.4 Sensitivity Analysis for Capacitated Transportation Problem

Sensitivity analysis is important for several reasons. In many applications, the values of parameters may change. If a parameter changes, sensitivity anlaysis often makes it unnecessary to solve the problem again. Knowledge of sensitivity analysis often enables the analyst to determine from the original solution how changes in parameters change its optimal solution. Two aspects of sensitivity analysis for the capacitated transportation problems are:

a. Sensitivity analysis of the cost
b. Sensitivity analysis of supply and demand

Sensitivity analysis will be applied to the optimal solution which is given in Table 3.8 of the problem.

3.4.1 Sensitivity Analysis of the Cost

When the unit cost of either the basic or the nonbasic variables change, optimal solution can be affected. Sensitivity analysis investigates how much the optimal solution is influenced by these variations. To determine the effect of changing the model's parameters on the solution, sensitivity analysis can be carried out. The sensitivity anlaysis process is described below using the solution of the example.

a) Changing the objective function coefficient of a nonbasic variable

Since we are not changing $c_B B^{-1}$, the u_i's and the v_j's remain unchanged. In objective coefficient row, only the coefficient of x_{ij} will change. Thus, as long as the coefficient of x_{ij} in the optimal row 0 is nonpositive, the current basis remains optimal. The value of $c_{ij} - u_i - v_j$ for the nonbasic variable x_{ij} is used as the test criterion, to check if the optimal solution remains the same.

If the nonbasic variable x_{ij} is not bounded then the optimum solution remains the same as long as $c_{ij} - u_i - v_j \geq 0$. When c_{ij} takes a value less than $u_i + v_j$, d_{ij} becomes negative and the optimality conditions are violated.

If the nonbasic variable is bounded, then the optimum solution does not change as long as $c_{ij} - u_i - v_j \leq 0$. When c_{ij} takes a value greater than $u_i + v_j$, d_{ij} becomes positive and the optimality conditions are violated.

In the optimal solution of the example, $x_{11}, x_{12}, x_{21}, x_{23}, x_{24}$ are nonbasic variables. To keep the optimal solution unchanged, the intervals of Δ are, relatively, as follows:

$$d_{11} = 10 + \Delta - (-1) - 9 \geq 0 \quad \Rightarrow \quad \Delta \geq -2 \quad \Rightarrow c_{11} \geq 8$$
$$d_{12} = 5 + \Delta - (-1) - 3 \geq 0 \quad \Rightarrow \quad \Delta \geq -3 \quad \Rightarrow c_{12} \geq 2$$
$$d_{21} = 8 + \Delta - (-1) - 9 \geq 0 \quad \Rightarrow \quad \Delta \geq 0 \quad \Rightarrow c_{21} \geq 8$$
$$d_{23} = 7 + \Delta - (-1) - 7 \geq 0 \quad \Rightarrow \quad \Delta \geq -1 \quad \Rightarrow c_{23} \geq 6$$

Since x_{24} and x_{33} are bounded variables, to keep the optimal solution unchanged, the intervals of Δ are, relatively, as follows:

$$d_{24} = 6 + \Delta - (-1) - 8 \leq 0 \quad \Rightarrow \quad \Delta \leq 1 \quad \Rightarrow c_{24} \leq 7$$
$$d_{33} = 4 + \Delta - 0 - 7 \leq 0 \quad \Rightarrow \quad \Delta \leq 3 \quad \Rightarrow c_{33} \leq 7$$

b) Changing the objective function coefficient of a basic variable

To determine the sensitivity of the cost coefficient of the basic variables;

When we are changing $c_B B^{-1}$, the coefficient of each nonbasic variable in objective function row may change, and to determine whether the current basis remains optimal, the new u_i's and v_j's must be calculated and these values are used to price out all nonbasic variables and bounded variables. As long as all d_{ij} values for the nonbasic variables are positive and d_{ij} values for the bounded variables are negative, the current basis remains optimal.

If c_{13} changes from 6 to $(6+\Delta)$, then the new values for u_i and b_j are calculated as follows:

$$
\begin{array}{ll}
u_1 = -1 & v_1 = 9 \\
u_2 = -1 & v_2 = 3 \\
u_3 = 0 & v_3 = 7+\Delta \\
 & v_4 = 8
\end{array}
$$

Then the relative intervals of Δ are calculated as:

$$
\begin{array}{lll}
d_{11} = 10-(-1)-9 \geq 0 & \Rightarrow & 2 \geq 0 \\
d_{12} = 5-(-1)-3 \geq 0 & \Rightarrow & 3 \geq 0 \\
d_{21} = 8-(-1)-9 \geq 0 & \Rightarrow & 0 \geq 0 \\
d_{23} = 7-(-1)-(7+\Delta) \geq 0 & \Rightarrow & \Delta \leq 1 \\
d_{24} = 6-(-1)-8 \leq 0 & \Rightarrow & -1 \leq 0 \\
d_{33} = 4-0-(7+\Delta) \leq 0 & \Rightarrow & \Delta \geq -3
\end{array}
$$

When c_{13} changes from 6 to $(6+\Delta)$, to keep the optimal solution unchanged, the value of Δ must be between –3 and +1.

If c_{14} changes from 7 to $(7+\Delta)$, then the new values of u_i and b_j are calculated as follows:

$$
\begin{array}{ll}
u_1 = -1 & v_1 = 9 \\
u_2 = -1 & v_2 = 3 \\
u_3 = 0 & v_3 = 7+\Delta \\
 & v_4 = 8
\end{array}
$$

Then the relative intervals of Δ are calculated as:

$$
\begin{array}{ll}
d_{11} = 10-(\Delta-1)-9 \geq 0 & \Rightarrow \ \Delta \leq 2 \\
d_{12} = 5-(\Delta-1)-3 \geq 0 & \Rightarrow \ \Delta \leq 3 \\
d_{21} = 8-(-1)-9 \geq 0 & \Rightarrow \ 0 \geq 0 \\
d_{23} = 7-(-1)-(7-\Delta) \geq 0 & \Rightarrow \ \Delta \geq -1 \\
d_{24} = 6-(-1)-8 \leq 0 & \Rightarrow \ -1 \leq 0 \\
d_{33} = 4-0-(7-\Delta) \leq 0 & \Rightarrow \ \Delta \leq 3
\end{array}
$$

When c_{14} changes from 7 to $(7+\Delta)$, to keep the optimal solution unchanged, the value of Δ must be between -1 and 2.

If c_{22} changes from 2 to $2+\Delta$, then the new values u_i and b_j are calculated as follows:

$$
\begin{array}{ll}
u_1 = -1 & v_1 = 9 \\
u_2 = \Delta-1 & v_2 = 3 \\
u_3 = 0 & v_3 = 7 \\
 & v_4 = 8
\end{array}
$$

Then the relative intervals of Δ are calculated as:

$$
\begin{array}{ll}
\mathrm{d}_{11} = 10-(-1)-9 \geq 0 & \Rightarrow \ 2 \geq 0 \\
\mathrm{d}_{12} = 5-(-1)-3 \geq 0 & \Rightarrow \ 3 \geq 0
\end{array}
$$

$d_{21} = 8-(\Delta-1)-9 \geq 0 \quad \Rightarrow \quad \Delta \leq 0$
$d_{23} = 7-(\Delta-1)-7 \geq 0 \quad \Rightarrow \quad \Delta \leq 1$
$d_{24} = 6-(\Delta-1)-8 \leq 0 \quad \Rightarrow \quad \Delta \geq -1$
$d_{33} = 4-0-7 \leq 0 \quad \Rightarrow \quad -3 \leq 0$

When c_{22} changes from to $2+\Delta$, to keep the optimal solution unchanged, the value of Δ must be between -1 and 0.

If c_{31} changes from 9 to $9+\Delta$, then the new values of u_i and b_j are calculated as follows:

$u_1 = -1 \quad v_1 = 9+\Delta$
$u_2 = -1 \quad v_2 = 3$
$u_3 = 0 \quad v_3 = 7$
$v_4 = 8$

Then the relative intervals of Δ are calculated as:

$d_{11} = 10-(-1)-(9+\Delta) \geq 0 \quad \Rightarrow \quad \Delta \leq 2$
$d_{12} = 5-(-1)-3 \geq 0 \quad \Rightarrow \quad 3 \geq 0$
$d_{21} = 8-(-1)-(9+\Delta) \geq 0 \quad \Rightarrow \quad \Delta \leq 0$
$d_{23} = 7-(-1)-7 \geq 0 \quad \Rightarrow \quad 1 \geq 0$
$d_{24} = 6-(-1)-8 \leq 0 \quad \Rightarrow \quad -1 \leq 0$
$d_{33} = 4-0-7 \leq 0 \quad \Rightarrow \quad -3 \leq 0$

When c_{31} changes from 9 to $9+\Delta$, to keep the optimal solution unchanged, the value of Δ must be less than 0.

If c_{32} changes from 3 to $3+\Delta$, then the new values of u_i and b_j are calculated as follows:

$$
\begin{array}{ll}
u_1 = -1 & v_1 = 9 \\
u_2 = -\Delta - 1 & v_2 = 3 + \Delta \\
u_3 = 0 & v_3 = 7 \\
 & v_4 = 8
\end{array}
$$

Then the relative intervals of Δ are calculated as:

$$
\begin{array}{lll}
d_{11} = 10 - (-1) - 9 \geq 0 & \Rightarrow & 2 \geq 0 \\
d_{12} = 5 - (-1) - (3 + \Delta) \geq 0 & \Rightarrow & \Delta \leq 3 \\
d_{21} = 8 - (-\Delta - 1) - 9 \geq 0 & \Rightarrow & \Delta \geq 0 \\
d_{23} = 7 - (-\Delta - 1) - 7 \geq 0 & \Rightarrow & \Delta \geq -1 \\
d_{24} = 6 - (-\Delta - 1) - 8 \leq 0 & \Rightarrow & \Delta \leq 1 \\
d_{33} = 4 - 0 - 7 \leq 0 & \Rightarrow & -3 \leq 0
\end{array}
$$

When c_{32} changes from 3 to $3 + \Delta$, to keep the optimal solution unchanged, the value of Δ must be between 0 and 1.

If c_{34} changes from 8 to $8 + \Delta$, then the new values of u_i and b_j are calculated as follows:

$$
\begin{array}{ll}
u_1 = -\Delta - 1 & v_1 = 9 \\
u_2 = -1 & v_2 = 3 \\
u_3 = 0 & v_3 = 7 + \Delta \\
 & v_4 = 8 + \Delta
\end{array}
$$

Then the relative intervals of Δ are calculated as:

$$
\begin{array}{lll}
d_{11} = 10 - (-\Delta - 1) - 9 \geq 0 & \Rightarrow & \Delta \geq 0 \\
d_{12} = 5 - (-\Delta - 1) - 3 \geq 0 & \Rightarrow & \Delta \leq -3 \\
d_{21} = 8 - (-1) - 9 \geq 0 & \Rightarrow & 0 \geq 0 \\
d_{23} = 7 - (-1) - (7 + \Delta) \geq 0 & \Rightarrow & \Delta \leq 1 \\
d_{24} = 6 - (-1) - (8 + \Delta) \leq 0 & \Rightarrow & \Delta \geq -1 \\
d_{33} = 4 - 0 - (7 + \Delta) \leq 0 & \Rightarrow & \Delta \geq -3
\end{array}
$$

When c_{34} changes from 8 to $8+\Delta$, to keep the optimal solution unchanged, the value of Δ must be between 0 and 1.

3.4.2 Sensitivity Analysis on the Supply and Demand

In the case of increasing or decreasing the amounts of supply and demand, sensitivity analysis is used to determine whether the current basis remains optimal. This change is observed in the balanced capacitated transportation problem.

The u_i's and v_j's may be thought of as the negative of each constraint's shadow prices. If the increase in the amount of production or the amount of demand is shown as Δa_i and Δb_j, respectively, then the value of objective function becomes

$$\text{New } z \text{ value} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \Delta a_i (u_i) + \Delta b_j (v_j)$$

To arrange the current optimal solution, the procedure which is described below is used.

a. If x_{ij} is a basic variable in the optimal solution, Δ is added to x_{ij}.
b. If x_{ij} is a nonbasic variable or bounded variable in the optimal solution, the loop, which involves x_{ij} and some of the basic variables, is found. An odd cell is found in the loop that is in row *i*. Starting from this odd cell Δ is added and subtracted from the values of the cells forming the loop, consequtively.

Here, the capacity constraints are the RHS of the inequalities and the interval of Δ is computed by these values. Thus, the value of Δ is determined according to the related cell capacity, because no variable can be assigned a value that is greater than the capacity of the related cell.

For basic variables, As long as $x_{ij} + \Delta \leq u_{ij}$, the optimal solution remains the same.

$x_{13} = 5$ and $u_{13} = 5$, $5 + \Delta \leq 5 \Rightarrow \Delta \leq 0$

$x_{14} = 20$ and $u_{14} = 20$, $20 + \Delta \leq 2 \Rightarrow \Delta \leq 0$

$x_{22} = 16$ and $u_{22} = 20$, $16 + \Delta \leq 20 \Rightarrow \Delta \leq 4$

$x_{31} = 15$ and $u_{31} = 18$, $15 + \Delta \leq 18 \Rightarrow \Delta \leq 3$

$x_{32} = 4$ and $u_{32} = 4$, $4 + \Delta \leq 4 \Rightarrow \Delta \leq 0$

$x_{34} = 6$ and $u_{34} = 7$, $6 + \Delta \leq 7 \Rightarrow \Delta \leq 1$

For the nonbasic variables, a closed loop that starts and ends at the cell, of which the sensitivity is investigated, is constructed.

To find out how many units can be assigned to x_{11}, without changing the optimal solution, a loop is formed as in Table 3.9 and the following inequalities are solved.

Table 3.9 Senisitivity anaylysis for x_{11}

+Δ 12	13	5 5	20-Δ 20	25	-1
10	5	6	7		
14	16 20	10	$\bar{9}$ 9	25	-1
8	2	7	6		
15-Δ 18	4 4	$\bar{25}$ 25	6+Δ 7	50	0
9	3	4	8		
15	20	30	35		u_i
9	3	7	8	v_j	

$$\left.\begin{array}{l} \Delta \leq 12 \\ 20-\Delta \leq 20 \\ 6+\Delta \leq 7 \\ 15-\Delta \leq 18 \end{array}\right\} \quad -3 \leq \Delta \leq 0$$

With respect to the inequality, for x_{11}, as long as Δ is between -3 and 0, the optimal solution remains the same.

To find out how many units can be assigned to x_{12}, without changing the optimal solution, a loop is formed as in Table 3.10 and the following inequalities are solved.

Table 3.10 Sensitivity analysis for x_{12}

	12	$+\Delta$	13	5	5	$20-\Delta$	20	25	-1
	10		5		6		7		
	14	16	20		10	$\bar{9}$	9	25	-1
	8		2		7		6		
15	18	$4-\Delta$	4	$2\bar{5}$	25	$6+\Delta$	7	50	0
	9		3		4		8		
15		20		30		35			u_i
9		3		7		8		v_j	

$$\left.\begin{array}{l}\Delta \le 13 \\ 20-\Delta \le 20 \\ 6+\Delta \le 7 \\ 4-\Delta \le 4\end{array}\right\} 0 \le \Delta \le 1$$

According to the inequality, for x_{12}, as long as Δ is between 0 and 1, the optimal solution remains the same.

As for the variables x_{21} and x_{23}, no loop can be formed to determine the values of the related Δ, so the value of Δ is 0 for these variables.

As for the variables x_{24} and x_{33}, since these are bounded variables with maximum assigned values, no change can be made on these variables, and the value of Δ is also taken 0 for these bounded variables.

3.5 On the Equivalence of Capacitated and Classical Transportation Problems

It can be noted that each variable x_{ij} appears in three equations with non-zero coefficients; not only in (3.3) and (3.4), which are the row and column equations used in the classical problem, but in addition in the upper bounding inequality (3.5), which may be rewritten as

$$x_{ij} + y_{ij} = u_{ij} \qquad (y_{ij} \geq 0)$$

where variable, y_{ij} represents slack. Consider the problem of finding $x_{ij} \geq 0$ and min z satisfying

Objective function:

$$\text{Min z} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Supply constraints:

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad (i = 1, \ldots, m)$$

Demand constraints:

$$-\sum_{i=1}^{m} x'_{ij} = -b_j \qquad (j = 1, \ldots, n)$$

Capacity constraints:

$$-x_{ij} - y_{ij} = -u_{ij} \qquad (\text{all } i, j)$$

$$y_{ij} + x'_{ij} = u_{ij} \qquad (\text{all } i, j)$$

The system can, however, be replaced by an obviously equivalent one in which each variable enters only two equations just as in the classical transportation form.

CHAPTER FOUR
APPLICATION

The data, which will be used for the application, is taken from a company which produces and distributes beverages. Since the information is confidential, neither the name of the company nor the name of the brand will be included in this context. Due to various reasons, such as competition with other companies and increasing costs, the net income of the company has decreased in the last years. To overcome this problem, the management decided to analyze the main processes with high costs. Shipment of the products from the production plants to different locations is one of the main items in this list. The objective of this application is to develop a shipping plan, which consists of the amount of products to be shipped from the production plants to the locations, with the minimum cost. To achieve this objective, supply, demand and capacity constraints are formed for a one year period, and the capacitated transportation model, which is described in chapter 4, is applied to the data.

4.1 Definition of the Problem

The company has six production plants, which are located in Ankara, Bursa, Çorlu, İstanbul, İzmir and Mersin, and the products are shipped from these plants to 78 cities across Turkey. When formulating the model, the 6 production plants should be treated as sources and the 78 cities should be treated as destinations. But, this will lead to a model with 468 variables and 552 constraints, of which 6 are supply constraints, 78 are demand constraints and 468 are capacity constraints. In the following tables, supply and demand amounts are given as a box. One box is equal to eight kilograms. For example, supply for Ankara is 8.257.576 boxes which correspond $8.257.576 \times 8$ kilograms.

The original data which consists unit transportation costs of the shipments from plant *i* to city *j*, and capacity of the cities and supply of the plants, demands of the cities are given in Table 4.1 and Table 4.2, respectively.

The general model of the capacitated transportation problem, the definitions of the variables and parameters are defined as follows:

Decision variables:

x_{ij} = the number of boxes shipped from source i to destination j

Parameters:

$i = 1, \ldots, 6$: plants

$j = 1, \ldots, 78$: cities

c_{ij} = the unit transportation cost from plant i to city j

u_{ij} = the maximum amount which can be shipped from plant i to city j

a_i = total supply of plant i

b_j = total demand of city j

Objective function:

$$\text{Min } z = \sum_{i=1}^{6} \sum_{j=1}^{78} c_{ij} x_{ij}$$

Supply constraints:

$$\sum_{j=1}^{78} x_{ij} \le a_i \qquad (i = 1, 2, 3, \ldots, 6)$$

Demand constraints :

$$\sum_{i=1}^{6} x_{ij} \ge b_j \qquad (j = 1, 2, 3, \ldots, 78)$$

Capacity constraints: $x_{ij} \leq u_{ij}$ $(i = 1, 2, \ldots, 6; j = 1, 2, \ldots, 78)$

Nonnegativity condition : $x_{ij} \geq 0$ $(i = 1, 2, \ldots, 6; j = 1, 2, \ldots, 78)$

Since, such a problem is beyond the capability of the available software packages, to obtain an approximate optimal solution; the method described in the next section is developed.

4.2 Solution Method

To find an approximate optimal solution, the problem is simplified and, with regards to the solutions of the simplified model, sub problems are defined.

The solution process is as follows:

First, the definition of the original problem is stated. Then destination regions are formed by grouping the cities with respect to their shipping costs, the amount of demands and regional neighborhood, regions are formed. Treating these regions, instead of cities, as destinations a new and simplified model is formed, which can be handled by software packages.

Next, the simplified problem is solved. If, in the optimal solution of the simplified problem, only one plant satisfies a region's demand then it is accepted that the demands of the individual cities, forming this specific region, are shipped from the related plant. When the demand of a region is satisfied by two or more plants, then a new sub problem is formed, which consists of the cities in the region as destinations and the plants with assignments as sources. The sub problems are solved.

Finally, the optimal solution table of the original problem is then formed, using the results obtained from the simplified and sub problem's solutions, and an approximate optimal solution is found.

4.3 Solution

In the beginning, the cities are grouped with respect to their shipping costs, the amount of demands and regional neighborhood, and destination regions are formed. These regions are presented on the map in Appendix 1, and can be listed as follows.

Region 1: Aksaray, Çankırı, Kırıkkale, Kırşehir, Konya, Nevşehir
Region 2: Edirne, Kırklareli, Tekirdağ
Region 3: Adapazarı, Balıkesir, Bilecik, Bolu, Bursa, Çanakkale, İzmit, Yalova
Region 4: Ağrı, Ardahan, Batman, Bitlis, Hakkari, Iğdır, Kars, Mardin, Muş, Siirt, Şırnak, Van,
Region 5: Bartın, Karabük, Kastamonu, Zonguldak,
Region 6: Amasya, Çorum, Samsun, Tokat, Yozgat
Region 7: Artvin, Bayburt, Giresun, Gümüşhane, Ordu, Rize, Trabzon
Region 8: Afyon, Eskişehir, Kütahya, Uşak
Region 9: Aydın, Denizli, Muğla
Region 10: İzmir, Manisa
Region 11: Burdur, Isparta
Region 12: Karaman, Mersin
Region 13: Bingöl, Diyarbakır, Elazığ, Erzincan, Malatya, Tunceli
Region 14: Adana, Kayseri, Niğde, Osmaniye, Sivas
Region 15: Adıyaman, Gaziantep, Hatay, K.Maraş, Urfa
Region 16: İstanbul
Region 17: Sinop
Region 18: Ankara
Region 19: Antalya

Simplified problem is formulated with 6 plants and 19 regions. Definitions of the decision variables, the objective function and the constraints are given below.

Decision variables:

x_{ik} = the number of boxes shipped from plant i to region k

Parameters:

$i = 1, 2, \ldots, 6$: plants

$k = 1, 2, \ldots, 19$: regions

c_{ik} = the unit transportation cost from plant i to region k

u_{ik} = the maximum amount which can be shipped from plant i to region k

a_i = total supply of plant i

b_k = total demand of region k

Objective function:

$$\text{Min } z = \sum_{i=1}^{6} \sum_{j=1}^{19} c_{ik} x_{ik}$$

Supply constraints:

$$\sum_{j=1}^{19} x_{ik} \leq a_i \qquad (i = 1, 2, 3, \ldots, 6)$$

Demand constraints :

$$\sum_{i=1}^{6} x_{ik} \geq b_k \qquad (k = 1, 2, 3, \ldots, 19)$$

Capacity constraints : $x_{ik} \leq u_{ik}$ $\quad (i = 1, 2, \ldots, 6;\ k = 1, 2, \ldots, 19)$

Nonnegativity condition : $x_{ik} \geq 0$ $\quad (i = 1, 2, \ldots, 6;\ k = 1, 2, \ldots, 19)$

The unit transportation costs from plant *i* to region *k* are calculated by finding the mean of the cities' costs for that region, the maximum amount which can be shipped from plant *i* to region *k* are calculated as sum of the cities' u_{ik} values for that region.

The data which consists unit transportation costs of the shipments from plant *i* to region *j*, and capacity of the regions and supply of the plants, demands of the regions are given in Table 4.3 and Table 4.4., respectively. In this simpler model, each region is considered as if it is one destination. The shipping cost for a region is taken as the mean of the costs of the cities in the region. When the shipping costs are calculated, the costs of the empty cells are given as a 500.000 TL. Whereas, the demand and the capacity of the region is taken as the sum of the demands/capacities of the related cities. A minus sign (-) in any row denotes no shipment is possible from the related source.

In capacity table, for the sake of simplicity, the capacity values are given as a thousand boxes. An empty row in the table shows that there is no shipment between the plant and the region.

The model for the data in Table 4.3 and Table 4.4 is given follows;

The Objective function:

$$
\begin{aligned}
\text{Min } z = {} & 1321x_{11} + 500000x_{12} + 250734x_{13} + 46.561x_{14} + 2297x_{15} + 1775x_{16} \\
& + 3908x_{17} + 126087x_{18} + 334094x_{19} + 251066x_{110} + 1875x_{111} + 250938x_{112} \\
& + 169359x_{113} + 1964x_{114} + 102575x_{115} + 500000x_{116} + 5414x_{117} + 654x_{118} \\
& + 2438x_{119} + 3039x_{21} + 2485x_{22} + 1055x_{23} + 213181x_{24} + 2535x_{25} + 3735x_{26} \\
& + 76786x_{27} + 1479x_{28} + 3055x_{29} + 1797x_{210} + 2409x_{211} + 251865x_{212} \\
& + 7117x_{213} + 4303x_{214} + 203781x_{215} + 1353x_{216} + 4250x_{217} + 2195x_{218} \\
& + 3519x_{219} + 3073x_{31} + 902x_{32} + 64263x_{33} + 48948x_{34} + 2781x_{35} + 3713x_{36} \\
& + 5750x_{37} + 2609x_{38} + 3534x_{39} + 2875x_{310} + 3063x_{311} + 3469x_{312} + 171292x_{313} \\
& + 3863x_{314} + 5525x_{315} + 1222x_{316} + 8125x_{317} + 2500x_{318} + 3500x_{319} + 2885x_{41} \\
& + 167396x_{42} + 64180x_{43} + 49109x_{44} + 2609x_{45} + 3525x_{46} + 5545x_{47} + 2438x_{48} \\
& + 3375x_{49} + 2688x_{410} + 2969x_{411} + 3281x_{412} + 171156x_{413} + 3800x_{414} \\
& + 104200x_{415} + 625x_{416} + 3500x_{417} + 2375x_{418} + 3375x_{419} + 500000x_{51} \\
& + 500000x_{52} + 313180x_{53} + 500000x_{54} + 500000x_{55} + 500000x_{56} + 500000x_{57} \\
& + 1895x_{58} + 1562x_{59} + 1071x_{510} + 500000x_{511} + 251635x_{512} + 500000x_{513} \\
& + 500000x_{514} + 500000x_{515} + 500000x_{516} + 500000x_{517} + 500000x_{518} \\
& + 2755x_{519} + 417064x_{61} + 500000x_{62} + 500000x_{63} + 293606x_{64} + 375875x_{65} \\
& + 500000x_{66} + 500000x_{67} + 375708x_{68} + 500000x_{69} + 3563x_{610} + 3375x_{611} \\
& + 250324x_{612} + 86440x_{613} + 201038x_{614} + 2137x_{615} + 3375x_{616} + 500000x_{617} \\
& + 500000x_{618} + 3363x_{619}
\end{aligned}
$$

Supply constraints:

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{110} + x_{111} + x_{112} + x_{113}$$
$$+ x_{114} + x_{115} + x_{116} + x_{117} + x_{118} + x_{119} \leq 8.257.576$$

$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} + x_{28} + x_{29} + x_{210} + x_{211} + x_{212} + x_{213}$$
$$+ x_{214} + x_{215} + x_{216} + x_{217} + x_{218} + x_{219} \leq 6.742.424$$

$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} + x_{37} + x_{38} + x_{39} + x_{310} + x_{311} + x_{312} + x_{313}$$
$$+ x_{314} + x_{315} + x_{316} + x_{317} + x_{318} + x_{319} \leq 41.818.180$$

$$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} + x_{47} + x_{48} + x_{49} + x_{410} + x_{411} + x_{412} + x_{413}$$
$$+ x_{414} + x_{415} + x_{416} + x_{417} + x_{418} + x_{419} \leq 10.682.000$$

$$x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} + x_{57} + x_{58} + x_{59} + x_{510} + x_{511} + x_{512} + x_{513}$$
$$+ x_{514} + x_{515} + x_{516} + x_{517} + x_{518} + x_{519} \leq 8.711.861$$

$$x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} + x_{67} + x_{68} + x_{69} + x_{610} + x_{611} + x_{612} + x_{613}$$
$$+ x_{614} + x_{615} + x_{616} + x_{617} + x_{618} + x_{519} \leq 48.218.540$$

Demand constraints:

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} \geq 3.227.376$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} \geq 3.156.329$$

$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} \geq 6.925.481$$

$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{64} \geq 3.158.174$$

$$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} \geq 2.222.917$$

$$x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} \geq 2.167.244$$

$$x_{17} + x_{27} + x_{37} + x_{47} + x_{57} + x_{67} \geq 3.873.035$$

$$x_{18} + x_{28} + x_{38} + x_{48} + x_{58} + x_{68} \geq 3.234.085$$

$$x_{19} + x_{29} + x_{39} + x_{49} + x_{59} + x_{69} \geq 4.394.265$$

$$x_{110} + x_{210} + x_{310} + x_{410} + x_{510} + x_{610} \geq 5.378.981$$

$$x_{111} + x_{211} + x_{311} + x_{411} + x_{511} + x_{611} \geq 3.419.120$$

$$x_{112} + x_{212} + x_{312} + x_{412} + x_{512} + x_{612} \geq 3.720.871$$

$$x_{113} + x_{213} + x_{313} + x_{413} + x_{513} + x_{613} \geq 4.527.576$$

$$x_{114} + x_{214} + x_{314} + x_{414} + x_{514} + x_{614} \geq 5.538.689$$

$$x_{115} + x_{215} + x_{315} + x_{415} + x_{515} + x_{615} \geq 8.546.712$$

$$x_{116} + x_{216} + x_{316} + x_{416} + x_{516} + x_{616} \geq 3.443.627$$

$$x_{117} + x_{217} + x_{317} + x_{417} + x_{517} + x_{617} \geq 617.709$$

$$x_{118} + x_{218} + x_{318} + x_{418} + x_{518} + x_{618} \geq 1.998.966$$

$$x_{119} + x_{219} + x_{319} + x_{419} + x_{519} + x_{619} \geq 12.573.030$$

Capacity constraints:

$x_{11} \leq 28.512.000$
$x_{12} \leq 0$
$x_{13} \leq 8.316.000$
$x_{14} \leq 32.670.000$
$x_{15} \leq 11.880.000$
$x_{16} \leq 18.414.000$
$x_{17} \leq 20.790.000$
$x_{18} \leq 11.880.000$
$x_{19} \leq 3.564.000$
$x_{110} \leq 3.564.000$
$x_{111} \leq 5.940.000$
$x_{112} \leq 4.752.000$
$x_{113} \leq 11.880.000$
$x_{114} \leq 20.196.000$
$x_{115} \leq 11.880.000$
$x_{116} \leq 0$
$x_{117} \leq 2.970.000$
$x_{118} \leq 14.256.000$
$x_{119} \leq 2.970.000$

$x_{21} \leq 17.820.000$
$x_{22} \leq 17.820.000$
$x_{23} \leq 55.242.000$
$x_{24} \leq 22.572.000$
$x_{25} \leq 7.128.000$
$x_{26} \leq 12.474.000$
$x_{27} \leq 10.692.000$
$x_{28} \leq 8.316.000$
$x_{29} \leq 5.346.000$
$x_{210} \leq 3.564.000$
$x_{211} \leq 5.940.000$
$x_{212} \leq 2.970.000$
$x_{213} \leq 17.820.000$
$x_{214} \leq 14.850.000$
$x_{215} \leq 8.910.000$
$x_{216} \leq 5.940.000$
$x_{217} \leq 1.782.000$
$x_{218} \leq 2.970.000$
$x_{219} \leq 2.970.000$

$x_{31} \leq 17.820.000$
$x_{32} \leq 17.820.000$
$x_{33} \leq 38.610.000$
$x_{34} \leq 32.670.000$
$x_{35} \leq 11.880.000$
$x_{36} \leq 14.850.000$
$x_{37} \leq 20.790.000$
$x_{38} \leq 24.354.000$
$x_{39} \leq 21.384.000$
$x_{310} \leq 14.256.000$
$x_{311} \leq 5.940.000$
$x_{312} \leq 5.940.000$
$x_{313} \leq 11.880.000$
$x_{314} \leq 14.850.000$
$x_{315} \leq 14.850.000$
$x_{316} \leq 17.820.000$
$x_{317} \leq 2.970.000$
$x_{318} \leq 2.970.000$
$x_{319} \leq 2.970.000$

$x_{41} \leq 17.820.000$	$x_{51} \leq 0$	$x_{61} \leq 3.564.000$
$x_{42} \leq 11.880.000$	$x_{52} \leq 0$	$x_{62} \leq 0$
$x_{43} \leq 38.610.000$	$x_{53} \leq 21.384.000$	$x_{63} \leq 0$
$x_{44} \leq 32.670.000$	$x_{54} \leq 0$	$x_{64} \leq 14.850.000$
$x_{45} \leq 11.880.000$	$x_{55} \leq 0$	$x_{65} \leq 2.970.000$
$x_{46} \leq 14.850.000$	$x_{56} \leq 0$	$x_{66} \leq 0$
$x_{47} \leq 20.790.000$	$x_{57} \leq 0$	$x_{67} \leq 0$
$x_{48} \leq 24.354.000$	$x_{58} \leq 8.316.000$	$x_{68} \leq 2.970.000$
$x_{49} \leq 21.384.000$	$x_{59} \leq 5.346.000$	$x_{69} \leq 0$
$x_{410} \leq 14.256.000$	$x_{510} \leq 7.128.000$	$x_{610} \leq 5.940.000$
$x_{411} \leq 5.940.000$	$x_{511} \leq 0$	$x_{611} \leq 8.316.000$
$x_{412} \leq 5.940.000$	$x_{512} \leq 2.970.000$	$x_{612} \leq 12.474.000$
$x_{413} \leq 11.880.000$	$x_{513} \leq 0$	$x_{613} \leq 14.850.000$
$x_{414} \leq 14.850.000$	$x_{514} \leq 0$	$x_{614} \leq 11.880.000$
$x_{415} \leq 11.880.000$	$x_{515} \leq 0$	$x_{615} \leq 17.226.000$
$x_{416} \leq 17.820.000$	$x_{516} \leq 0$	$x_{616} \leq 3.564.000$
$x_{417} \leq 2.970.000$	$x_{517} \leq 0$	$x_{617} \leq 0$
$x_{418} \leq 2.970.000$	$x_{518} \leq 0$	$x_{618} \leq 0$
$x_{419} \leq 2.970.000$	$x_{519} \leq 2.970.000$	$x_{619} \leq 4.158.000$

The above model has been solved with WinQSB and outputs are given in Appendix 2. The transportation solution table with optimal assignments is given in Table 4.5.

Table 4.5 Optimal solution for simplified model (box)

	Cities																			
Plants	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	R15	R16	R17	R18	R19	Supply
Ankara	-	-	-	3.158.174	-	2.167.244	-	-	-	-	-	-	-	457.127	-	-	-	-	2.475.031	8.257.576
Bursa	-	-	2.214.848	-	-	-	-	-	-	-	-	-	4.527.576	-	-	-	-	-	-	6.742.424
Çorlu	3.227.376	3.156.329	4.710.633	-	2.222.917	-	222.367	3.234.085	-	1.061.385	3.419.120	3.720.871	-	5.081.562	-	-	-	1.998.966	2.970.000	41.818.180
Dudullu	-	-	-	-	-	-	3.650.668	-	-	-	-	-	-	-	-	3.443.627	617.709	-	2.970.000	10.682.000
İzmir	-	-	-	-	-	-	-	-	4.394.265	4.317.596	-	-	-	-	-	-	-	-	-	8.711.861
Mersin	-	-	-	-	-	-	-	-	-	-	-	-	-	-	8.546.712	-	-	-	4.158.000	48.218.540
Demand	3.227.376	3.156.329	6.925.481	3.158.174	2.222.917	2.167.244	3.873.035	3.234.085	4.394.265	5.378.981	3.419.120	3.720.871	4.527.576	5.538.689	8.546.712	3.443.627	617.709	1.998.966	12.573.031	

The assignments in the optimal table show the total amount of shipment to be made from plants to the regions:

From Ankara:

to 4th region , $x_{14} = 3.158.174$

to 6th region, $x_{16} = 2.167.244$

to 14th region, $x_{114} = 457.127$

to 19th region, $x_{119} = 2.475.031$

From Bursa:

to 3rd region, $x_{23} = 2.214.848$

to 13th region, $x_{213} = 4.527.576$

From Çorlu:

to 1st region, $x_{31} = 3.227.376$

to 2nd region, $x_{32} = 3.156.329$

to 3rd region, $x_{33} = 4.710.633$

to 5th region, $x_{35} = 2.222.917$

to 7th region, $x_{37} = 222.367$

to 8th region, $x_{38} = 3.234.085$

to 10th region, $x_{310} = 1.061.385$

to 11th region, $x_{311} = 3.419.120$

to 12th region, $x_{312} = 3.720.871$

to 14th region, $x_{314} = 5.081.562$

to 18th region, $x_{318} = 1.998.966$

to 19th region, $x_{319} = 2.970.000$

From Dudullu:

to 7th region, $x_{47} = 3.650.668$

to 16^{th} region, $x_{416} = 3.443.627$

to 17^{th} region, $x_{417} = 617.709$

to 19^{th} region, $x_{419} = 2.970.000$

From İzmir:

to 9^{th} region, $x_{59} = 4.394.265$

to 10^{th} region, $x_{510} = 4.317.596$

From Mersin :

to 15^{th} region, $x_{615} = 8.546.712$

to 19^{th} region, $x_{619} = 4.158.000$

The figures of the positive assignments are given in Figure 4.1.

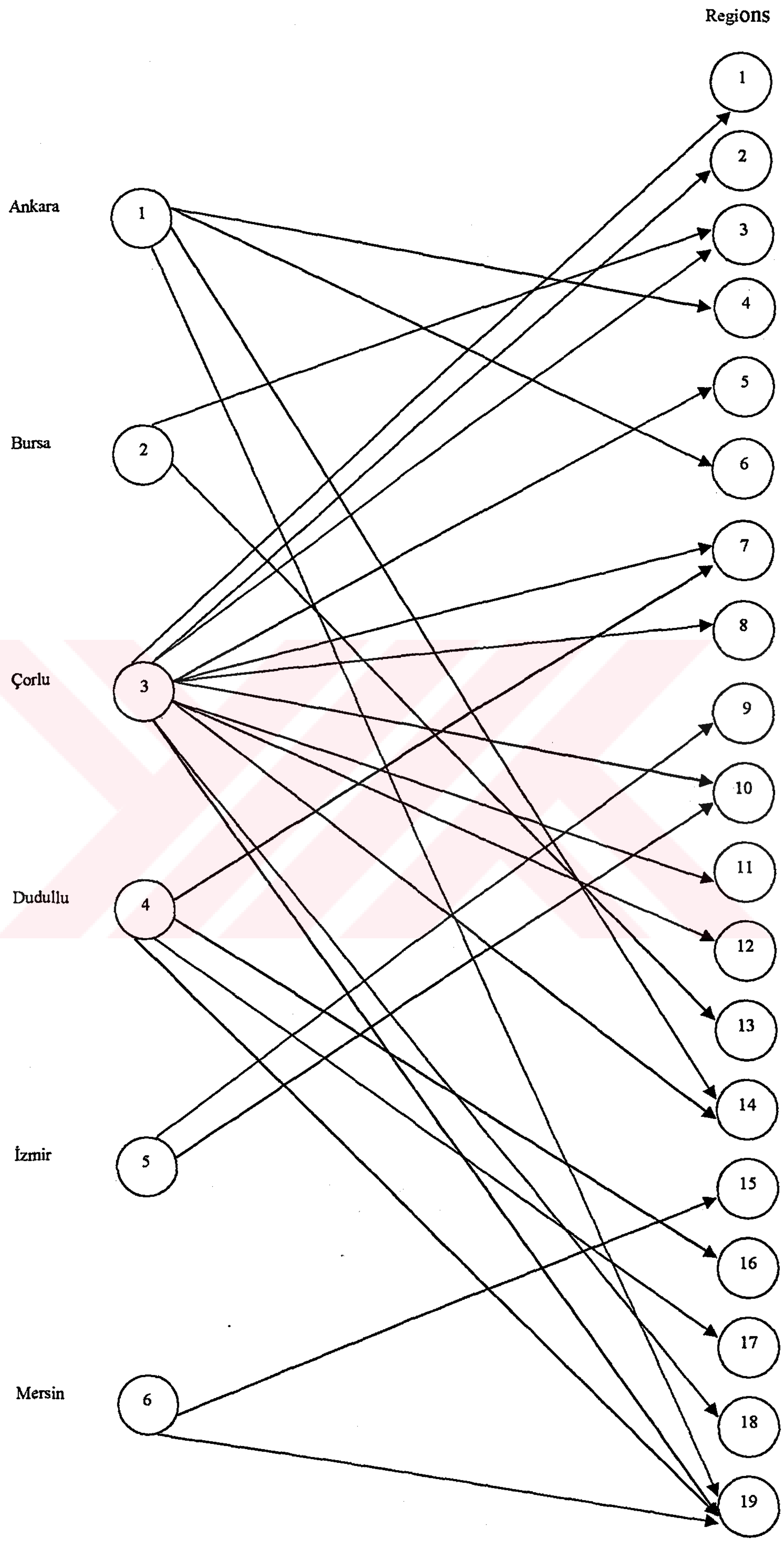

Figure 4.1 Graphical representation of the sub problems

As seen in the Figure 4.1., the sub problems for region 3, 7, 10, 14, 19 must be defined. Because, the assignments to 3rd region from Bursa is 2.214.848 and from Çorlu is 4.710.633, to 7th region from Çorlu is 222.367 and from Dudullu is 3.650.668, to 10th region from Çorlu is 1.061.385 and from İzmir is 4.317.596, to 14th region from Ankara is 457.127 and from Çorlu is 5.081.562 and finally, to 19th region from Ankara is 2.475.031, from Çorlu is 2.970.000, from Dudullu is 2.970.000 and from Mersin is 4.158.000.

- **Phase II**

When there are two or more assignments for a region in the optimal table of the simplified model, the optimal assignments to the cities in the region, can be found by forming a new model for these plants and cities and solving the model. The right hand side (RHS) values for these models are obtained from the assignments of the first model's optimal table. The data for these sub-models, which will be used when solving them, are as follows:

Sub problem 1:

Because the demand of the 3rd region is shipped from plants Bursa and Çorlu the related sub problem consists of Adapazarı, Balıkesir, Bilecik, Bursa, Çanakkale, İzmit and Yalova as destinations and Bursa and Çorlu as sources.

Table 4.6 Unit transportation cost for sub problem 1(TL. per box)

	Cities							
Plants	Adapazarı (1)	Balıkesir (2)	Bilecik (3)	Bolu (4)	Bursa (5)	Çanakkale (6)	İzmit (7)	Yalova (8)
Bursa	1.068	1.183	775	1.563	654	1.574	938	688
Çorlu	1.750	2.491	2.750	1.875	1.863	1.750	1.625	-

Table 4.7 Shipment capacities for sub problem 1(1000 boxes)

Plants	Adapazarı (1)	Balıkesir (2)	Bilecik (3)	Bolu (4)	Bursa (5)	Çanakkale (6)	İzmit (5)	Yalova (6)	Supply
	Cities								
Bursa	5.940	5.940	5.940	1.782	17.820	5.940	5.940	5.940	2.214.848
Çorlu	5.940	5.940	5.940	2.970	5.940	5.940	5.940	-	4.710.633
Demand	509.987	1.245.810	301.961	757.830	2.163.547	1.106.038	741.926	98.382	

Model formulation for sub problem 1:

Objective function:

$$\text{Min } z = 1068x_{11} + 1183x_{12} + 775x_{13} + 1563x_{14} + 654x_{15} + 1574x_{16} + 938x_{17}$$
$$+ 688x_{18} + 1750x_{21} + 2491x_{22} + 2750x_{23} + 1875x_{24} + 1863x_{25}$$
$$+ 1750x_{26} + 1625x_{27} + 500000x_{28}$$

Supply constraints:

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} \leq 2.214.848$$
$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} + x_{28} \leq 4.710.633$$

Demand constraints:

$$x_{11} + x_{21} \geq 509.987$$
$$x_{12} + x_{22} \geq 1.245.810$$
$$x_{13} + x_{23} \geq 301.961$$
$$x_{14} + x_{24} \geq 757.830$$
$$x_{15} + x_{25} \geq 2.163.547$$
$$x_{16} + x_{26} \geq 1.106.038$$
$$x_{17} + x_{27} \geq 741.926$$
$$x_{18} + x_{28} \geq 98.382$$

Capacity constraints:

$$x_{11} \leq 5.940.000$$
$$x_{12} \leq 5.940.000$$
$$x_{21} \leq 5.940.000$$
$$x_{22} \leq 5.940.000$$

$x_{13} \leq 5.940.000$ $\quad$ $x_{23} \leq 5.940.000$

$x_{14} \leq 1.782.000$ $\quad$ $x_{24} \leq 2.970.000$

$x_{15} \leq 17.820.000$ $\quad$ $x_{25} \leq 5.940.000$

$x_{16} \leq 5.940.000$ $\quad$ $x_{26} \leq 5.940.000$

$x_{17} \leq 5.940.000$ $\quad$ $x_{27} \leq 5.940.000$

$x_{18} \leq 5.940.000$ $\quad$ $x_{28} \leq 0$

The above models have been solved with WinQSB and output is given in Appendix 3. The optimal solution table is given in Table 4.8.

Table 4.8 Optimal solution table for sub problem 1(box)

Plants	Cities								Supply
	Adapazarı (1)	Balıkesir (2)	Bilecik (3)	Bolu (4)	Bursa (5)	Çanakkale (6)	İzmit (5)	Yalova (6)	
Bursa	-	1.245.810	301.961	-	568.695	-	-	98.382	2.214.848
Çorlu	509.987	-	-	757.830	1.594.852	1.106.038	741.926	-	4.710.633
Demand	509.987	1.245.810	301.961	757.830	2.163.547	1.106.038	741.926	98.382	

The assignments in the optimal table show the total amount of shipment to be made from plants to the cities.

Sub problem 2:

Because the demand of the 7^{th} region is shipped from plants Çorlu and Dudullu, the related sub problem consists of Artvin, Bayburt, Giresun, Gümüşhane, Ordu, Rize, Trabzon as destinations and Ankara and Dudullu as sources.

Table 4.9 Unit transportation cost for sub problem 2 (TL. per box)

Plants	Cities						
	Artvin (1)	Bayburt (2)	Giresun (3)	Gümüşhane (4)	Ordu (5)	Rize (6)	Trabzon (7)
Çorlu	8.875	5.500	4.250	5.313	4.500	6.500	5.313
Dudullu	8.625	5.250	4.063	5.125	4.313	6.313	5.125

Table 4.10 Shipments capacities for sub problem 2(1000 boxes)

Plants	Cities							Supply
	Artvin (1)	Bayburt (2)	Giresun (3)	Gümüşhane (4)	Ordu (5)	Rize (6)	Trabzon (7)	
Çorlu	2.970	2.970	2.970	2.970	2.970	2.970	2.970	222.367
Dudullu	2.970	2.970	2.970	2.970	2.970	2.970	2.970	3.650.668
Demand	226.043	287.714	408.193	164.689	356.123	289.916	2.140.357	

Model formulation for sub problem 2:

Objective function:

$$\text{Min z} = 8875x_{11} + 5500x_{12} + 4250x_{13} + 5313x_{14} + 4500x_{15} + 6500x_{16} + 5313x_{17} + 8625x_{21} + 5250x_{22} + 4063x_{23} + 5125x_{24} + 4313x_{25} + 6313x_{26} + 5125x_{27}$$

Supply constraints:

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} \leq 222.367$$

$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} \leq 3.650.668$$

Demand constraints:

$$x_{11} + x_{21} \geq 226.043$$

$$x_{12} + x_{22} \geq 287.714$$

$$x_{13} + x_{23} \geq 408.193$$

$$x_{14} + x_{24} \geq 164.689$$

$$x_{15} + x_{25} \geq 356.123$$

$$x_{16} + x_{26} \geq 289.916$$

$$x_{17} + x_{27} \geq 2.140.357$$

Capacity constraints:

$$x_{11} \leq 2.970.000 \qquad x_{21} \leq 2.970.000$$
$$x_{12} \leq 2.970.000 \qquad x_{22} \leq 2.970.000$$
$$x_{13} \leq 2.970.000 \qquad x_{23} \leq 2.970.000$$
$$x_{14} \leq 2.970.000 \qquad x_{24} \leq 2.970.000$$
$$x_{15} \leq 2.970.000 \qquad x_{25} \leq 2.970.000$$
$$x_{16} \leq 2.970.000 \qquad x_{26} \leq 2.970.000$$
$$x_{17} \leq 2.970.000 \qquad x_{27} \leq 2.970.000$$

The above models have been solved with WinQSB and output is given Appendix 4. The optimal solution table is given in Table 4.11.

Table 4.11 Optimal solution table for sub problem 2 (box)

Plants	Cities: Artvin (1)	Bayburt (2)	Giresun (3)	Gümüşhane (4)	Ordu (5)	Rize (6)	Trabzon (7)	Supply
Çorlu	-	-	222.367	-	-	-	-	222.367
Dudullu	226.043	287.714	185.826	164.689	356.123	289.916	2.140.357	3.650.668
Demand	226.043	287.714	408.193	164.689	356.123	289.916	2.140.357	

Sub Problem 3:

Because the demand of the 10th region is shipped from plants Çorlu and İzmir, the related sub problem consists of İzmir, Manisa as destinations and Bursa and İzmir as sources.

Table 4.12 Unit transportation cost for sub problem 3 (TL. per box)

Plants	Cities: İzmir (1)	Manisa (2)
Çorlu	2.875	2.875
İzmir	1.114	1.027

Table 4.13 Shipment capacities for sub problem 3 (1000 boxes)

Plants	Cities İzmir (1)	Manisa (2)	Supply
Çorlu	1.782	1.782	1.061.385
İzmir	5.346	1.782	4.317.596
Demand	2.700.079	2.678.902	

Model formulation for sub problem 3:

Objective function:

$$\text{Min } z = 2875x_{11} + 2875x_{12} + 1114x_{21} + 1027x_{22}$$

Supply constraints:

$$x_{11} + x_{12} \leq 1.061.385$$

$$x_{21} + x_{22} \leq 4.317.596$$

Demand constraints:

$$x_{11} + x_{21} \geq 2.700.079$$

$$x_{12} + x_{22} \geq 2.678.902$$

Capacity constraints:

$$x_{11} \leq 1.782.000 \qquad x_{21} \leq 5.346.000$$

$$x_{12} \leq 1.782.000 \qquad x_{22} \leq 1.782.000$$

The above models have been solved with WinQSB and output is given Appendix 5. The optimal solution table is given in Table 4.14.

Table 4.14 Optimal solution table for sub problem 3 (box)

Plants	Cities İzmir (1)	Manisa (2)	Supply
Çorlu	164.483	896.902	1.061.385
İzmir	2.535.596	1.782.000	4.317.596
Demand	2.700.079	2.678.902	

Sub problem 4:

Because the demand of the 14th region is shipped from plants Ankara and Çorlu, the related sub problem consists of Adana, Kayseri, Niğde, Osmaniye, Sivas as destinations and Çorlu and Dudullu as sources.

Table 4.15 Unit transportation cost for sub problem 4 (TL. per box)

Plants	Cities Adana (1)	Kayseri (2)	Niğde (3)	Osmaniye (4)	Sivas (5)
Ankara	1.884	1.563	1.563	1.938	2.875
Çorlu	3.750	3.875	3.875	4.125	3.688

Table 4.16 Shipment capacities for sub problem 4 (1000 boxes)

Plants	Cities Adana (1)	Kayseri (2)	Niğde (3)	Osmaniye (4)	Sivas (5)	Supply
Ankara	2.970	4.752	4.752	2.970	4.752	457.127
Çorlu	2.970	2.970	2.970	2.970	2.970	5.081.562
Demand	2.931.496	576.608	654.925	835.108	540.552	

Model formulation for sub problem 4:

Objective function:

$$\text{Min } z = 1884x_{11} + 1563x_{12} + 1563x_{13} + 1938x_{14} + 2875x_{15} + 3750x_{21} + 3875x_{22} + 3875x_{23} + 4125x_{24} + 3688x_{25}$$

Supply constraints:

$$x_{11}+x_{12}+x_{13}+x_{14}+x_{15}\leq 457.127$$

$$x_{21}+x_{22}+x_{23}+x_{24}+x_{25}\leq 5.081.562$$

Demand constraints:

$$x_{11}+x_{21}\geq 2.931.496$$

$$x_{12}+x_{22}\geq 576.608$$

$$x_{13}+x_{23}\geq 654.925$$

$$x_{14}+x_{24}\geq 835.108$$

$$x_{15}+x_{25}\geq 540.552$$

Capacity constraints:

$$x_{11}\leq 2.970.000 \qquad x_{21}\leq 2.970.000$$

$$x_{12}\leq 4.752.000 \qquad x_{22}\leq 2.970.000$$

$$x_{13}\leq 4.752.000 \qquad x_{23}\leq 2.970.000$$

$$x_{14}\leq 2.970.000 \qquad x_{24}\leq 2.970.000$$

$$x_{15}\leq 4.752.000 \qquad x_{25}\leq 2.970.000$$

The above models have been solved with WinQSB and output is given Appendix 6. The optimal solution table is given in Table 4.17.

Table 4.17 Optimal solution table for sub problem 4 (box)

Plants	Cities Adana (1)	Kayseri (2)	Niğde (3)	Osmaniye (4)	Sivas (5)	Supply
Ankara			457.127			457.127
Çorlu	2.931.496	576.608	197.798	835.108	540.552	5.081.562
Demand	2.931.496	576.608	654.925	835.108	540.552	

Sub problem 5:

Because the 19. region consists of only one city there is no need o define a sub problem for this region.

In the optimal solution, the shipments to Antalya, from Ankara is 2.475.031, from Çorlu is 2.970.000, from Dudullu 2.970.000, from Mersin is 4.158.000.

4.4 Results

In this thesis, the solution method with two steps was developed for the capacitated transportation problem with too large data. The solution of phase 1 and phase 2 were combined. Finally, we have been obtained approximate optimal solution for the problem.

Approximate optimal solution for original data is represented in Table 4.18. The problem with 6 plants and 78 cities must have 78 + 6 – 1 = 83 basic variables for feasibility. As seen in the Table 4.18, 86 assignments have been occurs. 83 of these assignments are basic variables and 3 of them bounded variables. So, we obtained an approximate optimal solution to the original problem with 468 variables and 552 constraints, this way.

CONCLUSIONS

The general solution method of a capacitated transportation problem can be described as follows: First, capacitated transportation model is formulated for the problem. If the total number of the variables and the constraints is small enough to handle with the available software packages, the problem is solved with a software package. But if the number is large, either the number of destinations or the number of sources is decreased in order to obtain a simplified model. However, the process of decreasing the number of sources or destinations must rely on experience or some experts' opinions. Next, the obtained simplified model is solved by using a software package.

If in the optimal solution of the simplified model, the demand of a grouped destination is shipped from two or more sources or the supply of a grouped source is shipped to two or more destinations then a subproblem is defined. This sub problem is modeled with each of the original destinations as demand centers or each of the original sources as supply centers. These sub problems are solved and by combining these solutions with the solution of the simplified problem, the solution table for the original problem is constructed.

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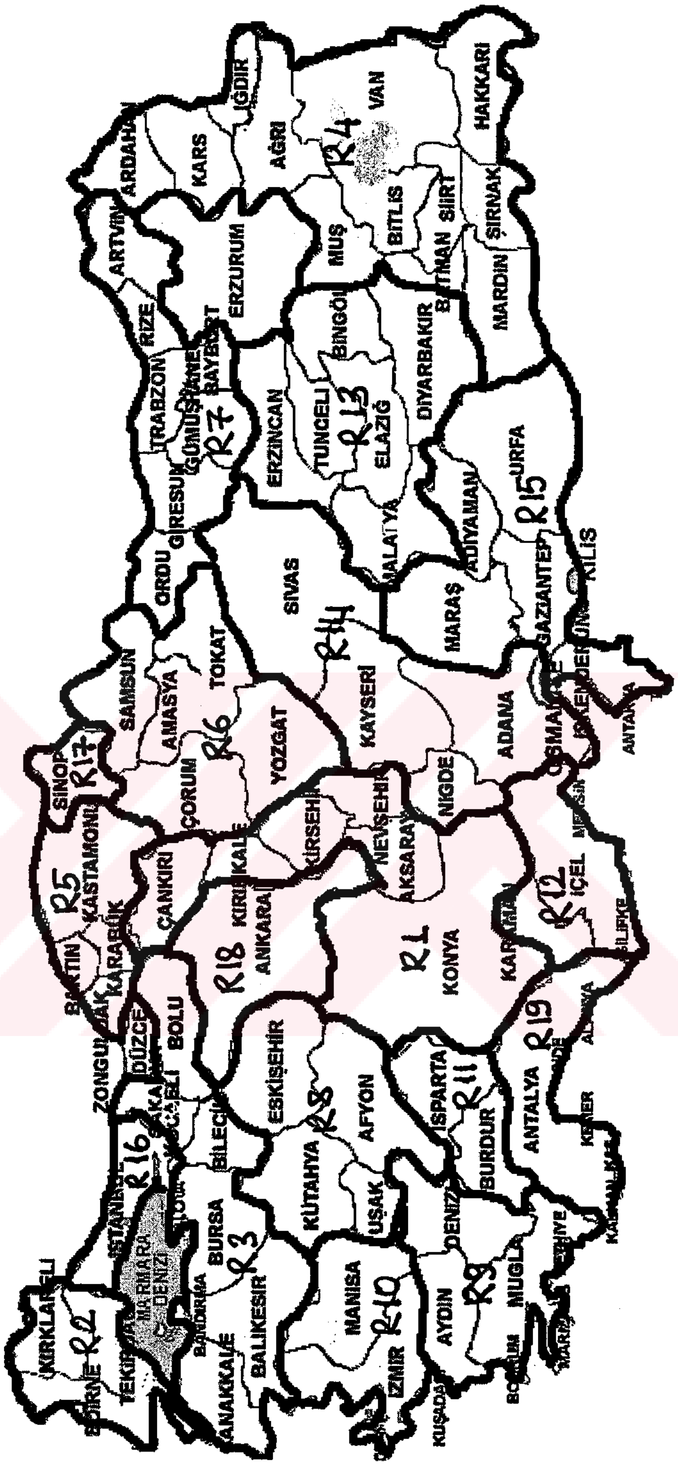
APPENDICES

A 1. The Regions



A 2. Optimal solution table for simplified problem

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status
1	X11	0	1.321,0000	0	147,0000	at bound
2	X12	0	500.000,0000	0	500.997,0000	at bound
3	X13	0	250.734,0000	0	188.370,0000	at bound
4	X14	3.158.174,0000	46.561,0000	147.047.700.000,0000	0	basic
5	X15	0	2.297,0000	0	1.415,0000	at bound
6	X16	2.167.244,0000	1.775,0000	3.846.858.000,0000	0	basic
7	X17	0	3.908,0000	0	57,0000	at bound
8	X18	0	126.087,0000	0	125.377,0000	at bound
9	X19	0	334.094,0000	0	332.627,0000	at bound
10	X110	0	251.066,0000	0	250.090,0000	at bound
11	X111	0	1.875,0000	0	711,0000	at bound
12	X112	0	250.938,0000	0	249.368,0000	at bound
13	X113	0	169.359,0000	0	100.933,0000	at bound
14	X114	457.127,0000	1.964,0000	897.797.400,0000	0	basic
15	X115	0	102.575,0000	0	102.337,0000	at bound
16	X116	0	500.000,0000	0	501.069,0000	at bound
17	X117	0	5.414,0000	0	3.608,0000	at bound
18	X118	0	654,0000	0	53,0000	at bound
19	X119	2.475.031,0000	2.438,0000	6.034.126.000,0000	0	basic
20	X21	0	3.039,0000	0	63.174,0000	at bound
21	X22	0	2.485,0000	0	64.791,0000	at bound
22	X23	2.214.848,0000	1.055,0000	2.336.665.000,0000	0	basic
23	X24	0	213.181,0000	0	227.929,0000	at bound
24	X25	0	2.535,0000	0	62.962,0000	at bound
25	X26	0	3.735,0000	0	63.269,0000	at bound
26	X27	0	76.786,0000	0	134.244,0000	at bound
27	X28	0	1.479,0000	0	62.078,0000	at bound
28	X29	0	3.055,0000	0	62.897,0000	at bound
29	X210	0	1.797,0000	0	62.130,0000	at bound
30	X211	0	2.409,0000	0	62.554,0000	at bound
31	X212	0	251.865,0000	0	311.604,0000	at bound
32	X213	4.527.576,0000	7.117,0000	32.222.760.000,0000	0	basic
33	X214	0	4.303,0000	0	63.648,0000	at bound
34	X215	0	203.781,0000	0	264.852,0000	at bound
35	X216	0	1.353,0000	0	63.731,0000	at bound
36	X217	0	4.250,0000	0	63.753,0000	at bound
37	X218	0	2.195,0000	0	62.903,0000	at bound
38	X219	0	3.519,0000	0	62.390,0000	at bound
39	X31	3.227.376,0000	3.073,0000	9.917.727.000,0000	0	basic
40	X32	3.156.329,0000	902,0000	2.847.009.000,0000	0	basic
41	X33	4.710.633,0000	64.263,0000	302.719.400.000,0000	0	basic
42	X34	0	48.948,0000	0	488,0000	at bound
43	X35	2.222.917,0000	2.781,0000	6.181.932.000,0000	0	basic

A 2. Optimal solution table for simplified problem(continued)

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status
44	X36	0	3.713,0000	0	39,0000	at bound
45	X37	222.367,0000	5.750,0000	1.278.610.000,0000	0	basic
46	X38	3.234.085,0000	2.609,0000	8.437.728.000,0000	0	basic
47	X39	0	3.534,0000	0	168,0000	at bound
48	X310	1.061.385,0000	2.875,0000	3.051.482.000,0000	0	basic
49	X311	3.419.120,0000	3.063,0000	10.472.760.000,0000	0	basic
50	X312	3.720.871,0000	3.469,0000	12.907.700.000,0000	0	basic
51	X313	0	171.292,0000	0	100.967,0000	at bound
52	X314	5.081.562,0000	3.863,0000	19.630.070.000,0000	0	basic
53	X315	0	5.525,0000	0	3.388,0000	at bound
54	X316	0	1.222,0000	0	392,0000	at bound
55	X317	0	8.125,0000	0	4.420,0000	at bound
56	X318	1.998.966,0000	2.500,0000	4.997.415.000,0000	0	basic
57	X319	2.970.000,0000	3.500,0000	10.395.000.000,0000	0	basic
58	X41	0	2.885,0000	0	17,0000	at bound
59	X42	0	167.396,0000	0	166.699,0000	at bound
60	X43	0	64.180,0000	0	122,0000	at bound
61	X44	0	49.109,0000	0	854,0000	at bound
62	X45	0	2.609,0000	0	33,0000	at bound
63	X46	0	3.525,0000	0	56,0000	at bound
64	X47	3.650.668,0000	5.545,0000	20.242.950.000,0000	0	basic
65	X48	0	2.438,0000	0	34,0000	at bound
66	X49	0	3.375,0000	0	214,0000	at bound
67	X410	0	2.688,0000	0	18,0000	at bound
68	X411	0	2.969,0000	0	111,0000	at bound
69	X412	0	3.281,0000	0	17,0000	at bound
70	X413	0	171.156,0000	0	101.036,0000	at bound
71	X414	0	3.800,0000	0	142,0000	at bound
72	X415	0	104.200,0000	0	102.268,0000	at bound
73	X416	3.443.627,0000	625,0000	2.152.267.000,0000	0	basic
74	X417	617.709,0000	3.500,0000	2.161.981.000,0000	0	basic
75	X418	0	2.375,0000	0	80,0000	at bound
76	X419	2.970.000,0000	3.375,0000	10.023.750.000,0000	0	basic
77	X51	0	500.000,0000	0	498.731,0000	at bound
78	X52	0	500.000,0000	0	500.902,0000	at bound
79	X53	0	313.180,0000	0	250.721,0000	at bound
80	X54	0	500.000,0000	0	453.344,0000	at bound
81	X55	0	500.000,0000	0	499.023,0000	at bound
82	X56	0	500.000,0000	0	498.130,0000	at bound
83	X57	0	500.000,0000	0	496.054,0000	at bound
84	X58	0	1.895,0000	0	1.090,0000	at bound
85	X59	4.394.265,0000	1.562,0000	6.863.842.000,0000	0	basic
86	X510	4.317.596,0000	1.071,0000	4.624.145.000,0000	0	basic

A 2. Optimal solution table for simplified problem(continued)

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status
87	X511	0	500.000,0000	0	498.741,0000	at bound
88	X512	0	251.635,0000	0	249.970,0000	at bound
89	X513	0	500.000,0000	0	431.479,0000	at bound
90	X514	0	500.000,0000	0	497.941,0000	at bound
91	X515	0	500.000,0000	0	499.667,0000	at bound
92	X516	0	500.000,0000	0	500.974,0000	at bound
93	X517	0	500.000,0000	0	498.099,0000	at bound
94	X518	0	500.000,0000	0	499.304,0000	at bound
95	X519	0	2.755,0000	0	222,0000	at bound
96	X61	0	417.064,0000	0	413.991,0000	at bound
97	X62	0	500.000,0000	0	499.098,0000	at bound
98	X63	0	500.000,0000	0	435.737,0000	at bound
99	X64	0	293.606,0000	0	245.146,0000	at bound
100	X65	0	375.875,0000	0	373.094,0000	at bound
101	X66	0	500.000,0000	0	496.326,0000	at bound
102	X67	0	500.000,0000	0	494.250,0000	at bound
103	X68	0	375.708,0000	0	373.099,0000	at bound
104	X69	0	500.000,0000	0	496.634,0000	at bound
105	X610	0	3.563,0000	0	688,0000	at bound
106	X611	0	3.375,0000	0	312,0000	at bound
107	X612	0	250.324,0000	0	246.855,0000	at bound
108	X613	0	86.440,0000	0	16.115,0000	at bound
109	X614	0	201.038,0000	0	197.175,0000	at bound
110	X615	8.546.712,0000	2.137,0000	18.264.320.000,0000	0	basic
111	X616	0	3.375,0000	0	2.545,0000	at bound
112	X617	0	500.000,0000	0	496.295,0000	at bound
113	X618	0	500.000,0000	0	497.500,0000	at bound
114	X619	4.158.000,0000	3.363,0000	13.983.350.000,0000	0	basic
	Objective	Function	(Min.) =	663.539.400.000,0000		

A 2. Optimal solution table for simplified problem(continued)

	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price
1	Ankara	8.257.576,0000	<=	8.257.576,0000	0	-1.899,0000
2	Bursa	6.742.424,0000	<=	6.742.424,0000	0	-63.208,0000
3	Corlu	35.025.610,0000	<=	41.818.180,0000	6.792.573,0000	0
4	Dudullu	10.682.000,0000	<=	10.682.000,0000	0	-205,0000
5	Izmir	8.711.861,0000	<=	8.711.861,0000	0	-1.804,0000
6	Mersin	12.704.710,0000	<=	48.218.540,0000	35.513.830,0000	0
7	G1	3.227.376,0000	>=	3.227.376,0000	0	3.073,0000
8	G2	3.156.329,0000	>=	3.156.329,0000	0	902,0000
9	G3	6.925.481,0000	>=	6.925.481,0000	0	64.263,0000
10	G4	3.158.174,0000	>=	3.158.174,0000	0	48.460,0000
11	G5	2.222.917,0000	>=	2.222.917,0000	0	2.781,0000
12	G6	2.167.244,0000	>=	2.167.244,0000	0	3.674,0000
13	G7	3.873.035,0000	>=	3.873.035,0000	0	5.750,0000
14	G8	3.234.085,0000	>=	3.234.085,0000	0	2.609,0000
15	G9	4.394.265,0000	>=	4.394.265,0000	0	3.366,0000
16	G10	5.378.981,0000	>=	5.378.981,0000	0	2.875,0000
17	G11	3.419.120,0000	>=	3.419.120,0000	0	3.063,0000
18	G12	3.720.871,0000	>=	3.720.871,0000	0	3.469,0000
19	G13	4.527.576,0000	>=	4.527.576,0000	0	70.325,0000
20	G14	5.538.689,0000	>=	5.538.689,0000	0	3.863,0000
21	G15	8.546.712,0000	>=	8.546.712,0000	0	2.137,0000
22	G16	3.443.627,0000	>=	3.443.627,0000	0	830,0000
23	G17	617.709,0000	>=	617.709,0000	0	3.705,0000
24	G18	1.998.966,0000	>=	1.998.966,0000	0	2.500,0000
25	G19	12.573.030,0000	>=	12.573.030,0000	0	4.337,0000
26	U11	0	<=	28.512.000,0000	28.512.000,0000	0
27	U12	0	<=	0	0	0
28	U13	0	<=	8.316.000,0000	8.316.000,0000	0
29	U14	3.158.174,0000	<=	32.670.000,0000	29.511.830,0000	0
30	U15	0	<=	11.880.000,0000	11.880.000,0000	0
31	U16	2.167.244,0000	<=	18.414.000,0000	16.246.760,0000	0
32	U17	0	<=	20.790.000,0000	20.790.000,0000	0
33	U18	0	<=	11.880.000,0000	11.880.000,0000	0
34	U19	0	<=	3.564.000,0000	3.564.000,0000	0
35	U110	0	<=	3.564.000,0000	3.564.000,0000	0
36	U111	0	<=	5.940.000,0000	5.940.000,0000	0
37	U112	0	<=	4.752.000,0000	4.752.000,0000	0
38	U113	0	<=	11.880.000,0000	11.880.000,0000	0
39	U114	457.127,0000	<=	20.196.000,0000	19.738.870,0000	0
40	U115	0	<=	11.880.000,0000	11.880.000,0000	0
41	U116	0	<=	0	0	0
42	U117	0	<=	2.970.000,0000	2.970.000,0000	0
43	U118	0	<=	14.256.000,0000	14.256.000,0000	0

A 2. Optimal solution table for simplified problem(continued)

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status
44	U119	2.475.031,0000	<=	2.970.000,0000	494.969,0000	0
45	U21	0	<=	17.820.000,0000	17.820.000,0000	0
46	U22	0	<=	17.820.000,0000	17.820.000,0000	0
47	U23	2.214.848,0000	<=	55.242.000,0000	53.027.150,0000	0
48	U24	0	<=	22.572.000,0000	22.572.000,0000	0
49	U25	0	<=	7.128.000,0000	7.128.000,0000	0
50	U26	0	<=	12.474.000,0000	12.474.000,0000	0
51	U27	0	<=	10.692.000,0000	10.692.000,0000	0
52	U28	0	<=	8.316.000,0000	8.316.000,0000	0
53	U29	0	<=	5.346.000,0000	5.346.000,0000	0
54	U210	0	<=	3.564.000,0000	3.564.000,0000	0
55	U211	0	<=	5.940.000,0000	5.940.000,0000	0
56	U212	0	<=	2.970.000,0000	2.970.000,0000	0
57	U213	4.527.576,0000	<=	17.820.000,0000	13.292.420,0000	0
58	U214	0	<=	14.850.000,0000	14.850.000,0000	0
59	U215	0	<=	8.910.000,0000	8.910.000,0000	0
60	U216	0	<=	5.940.000,0000	5.940.000,0000	0
61	U217	0	<=	1.782.000,0000	1.782.000,0000	0
62	U218	0	<=	2.970.000,0000	2.970.000,0000	0
63	U219	0	<=	2.970.000,0000	2.970.000,0000	0
64	U31	3.227.376,0000	<=	17.820.000,0000	14.592.620,0000	0
65	U32	3.156.329,0000	<=	17.820.000,0000	14.663.670,0000	0
66	U33	4.710.633,0000	<=	38.610.000,0000	33.899.370,0000	0
67	U34	0	<=	32.670.000,0000	32.670.000,0000	0
68	U35	2.222.917,0000	<=	11.880.000,0000	9.657.083,0000	0
69	U36	0	<=	14.850.000,0000	14.850.000,0000	0
70	U37	222.367,0000	<=	20.790.000,0000	20.567.630,0000	0
71	U38	3.234.085,0000	<=	24.354.000,0000	21.119.920,0000	0
72	U39	0	<=	21.384.000,0000	21.384.000,0000	0
73	U310	1.061.385,0000	<=	14.256.000,0000	13.194.620,0000	0
74	U311	3.419.120,0000	<=	5.940.000,0000	2.520.880,0000	0
75	U312	3.720.871,0000	<=	5.940.000,0000	2.219.129,0000	0
76	U313	0	<=	11.880.000,0000	11.880.000,0000	0
77	U314	5.081.562,0000	<=	14.850.000,0000	9.768.438,0000	0
78	U315	0	<=	14.850.000,0000	14.850.000,0000	0
79	U316	0	<=	17.820.000,0000	17.820.000,0000	0
80	U317	0	<=	2.970.000,0000	2.970.000,0000	0
81	U318	1.998.966,0000	<=	2.970.000,0000	971.034,0000	0
82	U319	2.970.000,0000	<=	2.970.000,0000	0	-837,0000
83	U41	0	<=	17.820.000,0000	17.820.000,0000	0
84	U42	0	<=	11.880.000,0000	11.880.000,0000	0
85	U43	0	<=	38.610.000,0000	38.610.000,0000	0
86	U44	0	<=	32.670.000,0000	32.670.000,0000	0

A 2. Optimal solution table for simplified problem(continued)

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status
87	U45	0	<=	11.880.000,0000	11.880.000,0000	0
88	U46	0	<=	14.850.000,0000	14.850.000,0000	0
89	U47	3.650.668,0000	<=	20.790.000,0000	17.139.330,0000	0
90	U48	0	<=	24.354.000,0000	24.354.000,0000	0
91	U49	0	<=	21.384.000,0000	21.384.000,0000	0
92	U410	0	<=	14.256.000,0000	14.256.000,0000	0
93	U411	0	<=	5.940.000,0000	5.940.000,0000	0
94	U412	0	<=	5.940.000,0000	5.940.000,0000	0
95	U413	0	<=	11.880.000,0000	11.880.000,0000	0
96	U414	0	<=	14.850.000,0000	14.850.000,0000	0
97	U415	0	<=	11.880.000,0000	11.880.000,0000	0
98	U416	3.443.627,0000	<=	17.820.000,0000	14.376.370,0000	0
99	U417	617.709,0000	<=	2.970.000,0000	2.352.291,0000	0
100	U418	0	<=	2.970.000,0000	2.970.000,0000	0
101	U419	2.970.000,0000	<=	2.970.000,0000	0	-757,0000
102	U51	0	<=	0	0	0
103	U52	0	<=	0	0	0
104	U53	0	<=	21.384.000,0000	21.384.000,0000	0
105	U54	0	<=	0	0	0
106	U55	0	<=	0	0	0
107	U56	0	<=	0	0	0
108	U57	0	<=	0	0	0
109	U58	0	<=	8.316.000,0000	8.316.000,0000	0
110	U59	4.394.265,0000	<=	5.346.000,0000	951.735,0000	0
111	U510	4.317.596,0000	<=	7.128.000,0000	2.810.404,0000	0
112	U511	0	<=	0	0	0
113	U512	0	<=	2.970.000,0000	2.970.000,0000	0
114	U513	0	<=	0	0	0
115	U514	0	<=	0	0	0
116	U515	0	<=	0	0	0
117	U516	0	<=	0	0	0
118	U517	0	<=	0	0	0
119	U518	0	<=	0	0	0
120	U519	0	<=	2.970.000,0000	2.970.000,0000	0
121	U61	0	<=	3.564.000,0000	3.564.000,0000	0
122	U62	0	<=	0	0	0
123	U63	0	<=	0	0	0
124	U64	0	<=	14.850.000,0000	14.850.000,0000	0
125	U65	0	<=	2.970.000,0000	2.970.000,0000	0
126	U66	0	<=	0	0	0
127	U67	0	<=	0	0	0
128	U68	0	<=	2.970.000,0000	2.970.000,0000	0
129	U69	0	<=	0	0	0

A 2. Optimal solution table for simplified problem(continued)

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status
130	U610	0	<=	5.940.000,0000	5.940.000,0000	0
131	U611	0	<=	8.316.000,0000	8.316.000,0000	0
132	U612	0	<=	12.474.000,0000	12.474.000,0000	0
133	U613	0	<=	14.850.000,0000	14.850.000,0000	0
134	U614	0	<=	11.880.000,0000	11.880.000,0000	0
135	U615	8.546.712,0000	<=	17.226.000,0000	8.679.288,0000	0
136	U616	0	<=	3.564.000,0000	3.564.000,0000	0
137	U617	0	<=	0	0	0
138	U618	0	<=	0	0	0
139	U619	4.158.000,0000	<=	4.158.000,0000	0	-974,0000

A 3. Optimal solution table for sub problem 1

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status
1	X11	0	1.068,0000	0	527,0000	at bound
2	X12	1.245.810,0000	1.183,0000	1.473.793.000,0000	0	basic
3	X13	301.961,0000	775,0000	234.019.800,0000	0	basic
4	X14	0	1.563,0000	0	897,0000	at bound
5	X15	568.695,0000	654,0000	371.926.500,0000	0	basic
6	X16	0	1.574,0000	0	1.033,0000	at bound
7	X17	0	938,0000	0	522,0000	at bound
8	X18	98.382,0000	688,0000	67.686.820,0000	0	basic
9	X21	509.987,0000	1.750,0000	892.477.200,0000	0	basic
10	X22	0	2.491,0000	0	99,0000	at bound
11	X23	0	2.750,0000	0	766,0000	at bound
12	X24	757.830,0000	1.875,0000	1.420.931.000,0000	0	basic
13	X25	1.594.852,0000	1.863,0000	2.971.209.000,0000	0	basic
14	X26	1.106.038,0000	1.750,0000	1.935.566.000,0000	0	basic
15	X27	741.926,0000	1.625,0000	1.205.630.000,0000	0	basic
16	X28	0	500.000,0000	0	498.103,0000	at bound
	Objective	Function	(Min.) =	10.573.240.000,0000		

A **3.** Optimal solution table for sub problem 1(continued)

	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price
1	Bursa	2.214.848,0000	<=	2.214.848,0000	0	-1.209,0000
2	Corlu	4.710.633,0000	<=	4.710.633,0000	0	0
3	Adapazari	509.987,0000	>=	509.987,0000	0	1.750,0000
4	Balikesir	1.245.810,0000	>=	1.245.810,0000	0	2.392,0000
5	Bilecik	301.961,0000	>=	301.961,0000	0	1.984,0000
6	Bolu	757.830,0000	>=	757.830,0000	0	1.875,0000
7	Bursa	2.163.547,0000	>=	2.163.547,0000	0	1.863,0000
8	Canakkale	1.106.038,0000	>=	1.106.038,0000	0	1.750,0000
9	Izmit	741.926,0000	>=	741.926,0000	0	1.625,0000
10	Yalova	98.382,0000	>=	98.382,0000	0	1.897,0000
11	U11	0	<=	5.940.000,0000	5.940.000,0000	0
12	U12	1.245.810,0000	<=	5.940.000,0000	4.694.190,0000	0
13	U13	301.961,0000	<=	5.940.000,0000	5.638.039,0000	0
14	U14	0	<=	1.782.000,0000	1.782.000,0000	0
15	U15	568.695,0000	<=	17.820.000,0000	17.251.300,0000	0
16	U16	0	<=	5.940.000,0000	5.940.000,0000	0
17	U17	0	<=	5.940.000,0000	5.940.000,0000	0
18	U18	98.382,0000	<=	5.940.000,0000	5.841.618,0000	0
19	U21	509.987,0000	<=	5.940.000,0000	5.430.013,0000	0
20	U22	0	<=	5.940.000,0000	5.940.000,0000	0
21	U23	0	<=	5.940.000,0000	5.940.000,0000	0
22	U24	757.830,0000	<=	2.970.000,0000	2.212.170,0000	0
23	U25	1.594.852,0000	<=	5.940.000,0000	4.345.148,0000	0
24	U26	1.106.038,0000	<=	5.940.000,0000	4.833.962,0000	0
25	U27	741.926,0000	<=	5.940.000,0000	5.198.074,0000	0
26	U28	0	<=	0	0	0

A 4. Optimal solution table for sub problem 2

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status
1	X11	0	8.875,0000	0	63,0000	at bound
2	X12	0	5.500,0000	0	63,0000	at bound
3	X13	222.367,0000	4.250,0000	945.059.800,0000	0	basic
4	X14	0	5.313,0000	0	1,0000	at bound
5	X15	0	4.500,0000	0	0	at bound
6	X16	0	6.500,0000	0	0	at bound
7	X17	0	5.313,0000	0	1,0000	at bound
8	X21	226.043,0000	8.625,0000	1.949.621.000,0000	0	basic
9	X22	287.714,0000	5.250,0000	1.510.499.000,0000	0	basic
10	X23	185.826,0000	4.063,0000	755.011.000,0000	0	basic
11	X24	164.689,0000	5.125,0000	844.031.100,0000	0	basic
12	X25	356.123,0000	4.313,0000	1.535.959.000,0000	0	basic
13	X26	289.916,0000	6.313,0000	1.830.240.000,0000	0	basic
14	X27	2.140.357,0000	5.125,0000	10.969.330.000,0000	0	basic
	Objective	Function	(Min.) =	20.339.750.000,0000		

A 4. Optimal solution table for sub problem 2(continued)

	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price
1	Corlu	222.367,0000	<=	222.367,0000	0	0
2	Dudullu	3.650.668,0000	<=	3.650.668,0000	0	-187,0000
3	Artvin	226.043,0000	>=	226.043,0000	0	8.812,0000
4	Bayburt	287.714,0000	>=	287.714,0000	0	5.437,0000
5	Giresun	408.193,0000	>=	408.193,0000	0	4.250,0000
6	Gümüshane	164.689,0000	>=	164.689,0000	0	5.312,0000
7	Ordu	356.123,0000	>=	356.123,0000	0	4.500,0000
8	Rize	289.916,0000	>=	289.916,0000	0	6.500,0000
9	Trabzon	2.140.357,0000	>=	2.140.357,0000	0	5.312,0000
10	U11	0	<=	2.970.000,0000	2.970.000,0000	0
11	U12	0	<=	2.970.000,0000	2.970.000,0000	0
12	U13	222.367,0000	<=	2.970.000,0000	2.747.633,0000	0
13	U14	0	<=	2.970.000,0000	2.970.000,0000	0
14	U15	0	<=	2.970.000,0000	2.970.000,0000	0
15	U16	0	<=	2.970.000,0000	2.970.000,0000	0
16	U17	0	<=	2.970.000,0000	2.970.000,0000	0
17	U21	226.043,0000	<=	2.970.000,0000	2.743.957,0000	0
18	U22	287.714,0000	<=	2.970.000,0000	2.682.286,0000	0
19	U23	185.826,0000	<=	2.970.000,0000	2.784.174,0000	0
20	U24	164.689,0000	<=	2.970.000,0000	2.805.311,0000	0
21	U25	356.123,0000	<=	2.970.000,0000	2.613.877,0000	0
22	U26	289.916,0000	<=	2.970.000,0000	2.680.084,0000	0
23	U27	2.140.357,0000	<=	2.970.000,0000	829.643,0000	0

A 5. Optimal solution table for sub problem 3

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status
1	X11	164.483,0000	2.875,0000	472.888.600,0000	0	basic
2	X12	896.902,0000	2.875,0000	2.578.593.000,0000	0	basic
3	X21	2.535.596,0000	1.114,0000	2.824.654.000,0000	0	basic
4	X22	1.782.000,0000	1.027,0000	1.830.114.000,0000	0	basic
	Objective	Function	(Min.) =	7.706.250.000,0000		
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price
1	Corlu	1.061.385,0000	<=	1.061.385,0000	0	0
2	Izmir	4.317.596,0000	<=	4.317.596,0000	0	-1.761,0000
3	Izmir	2.700.079,0000	>=	2.700.079,0000	0	2.875,0000
4	Manisa	2.678.902,0000	>=	2.678.902,0000	0	2.875,0000
5	C11	164.483,0000	<=	7.128.000,0000	6.963.517,0000	0
6	C12	896.902,0000	<=	7.128.000,0000	6.231.098,0000	0
7	C21	2.535.596,0000	<=	5.346.000,0000	2.810.404,0000	0
8	C22	1.782.000,0000	<=	1.782.000,0000	0	-87,0000

A 6. Optimal solution table for sub problem 4

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status
1	X11	0	1.884,0000	0	446,0000	at bound
2	X12	0	1.563,0000	0	0	at bound
3	X13	457.127,0000	1.563,0000	714.489.500,0000	0	basic
4	X14	0	1.938,0000	0	125,0000	at bound
5	X15	0	2.875,0000	0	1.499,0000	at bound
6	X21	2.931.496,0000	3.750,0000	10.993.110.000,0000	0	basic
7	X22	576.608,0000	3.875,0000	2.234.356.000,0000	0	basic
8	X23	197.798,0000	3.875,0000	766.467.300,0000	0	basic
9	X24	835.108,0000	4.125,0000	3.444.820.000,0000	0	basic
10	X25	540.552,0000	3.688,0000	1.993.556.000,0000	0	basic
	Objective	Function	(Min.) =	20.146.800.000,0000		
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price
1	Ankara	457.127,0000	<=	457.127,0000	0	-2.312,0000
2	Corlu	5.081.562,0000	<=	5.081.562,0000	0	0
3	Adana	2.931.496,0000	>=	2.931.496,0000	0	3.750,0000
4	Kayseri	576.608,0000	>=	576.608,0000	0	3.875,0000
5	Nigde	654.925,0000	>=	654.925,0000	0	3.875,0000
6	Osmaniye	835.108,0000	>=	835.108,0000	0	4.125,0000
7	Sivas	540.552,0000	>=	540.552,0000	0	3.688,0000
8	U11	2.931.496,0000	<=	2.970.000,0000	38.504,0000	0
9	U12	0	<=	4.752.000,0000	4.752.000,0000	0
10	U13	457.127,0000	<=	4.752.000,0000	4.294.873,0000	0
11	U14	0	<=	2.970.000,0000	2.970.000,0000	0
12	U15	0	<=	4.752.000,0000	4.752.000,0000	0
13	U21	2.931.496,0000	<=	2.970.000,0000	38.504,0000	0
14	U22	576.608,0000	<=	2.970.000,0000	2.393.392,0000	0
15	U23	197.798,0000	<=	2.970.000,0000	2.772.202,0000	0
16	U24	835.108,0000	<=	2.970.000,0000	2.134.892,0000	0
17	U25	540.552,0000	<=	2.970.000,0000	2.429.448,0000	0