DOKUZ EYLÜL UNIVERSITY

GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

COMPARISON OF NO CLAIM DISCOUNT (NCD) SYSTEMS

by

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İZMİR

COMPARISON OF NO CLAIM DISCOUNT (NCD) SYSTEMS

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We have read the thesis entitled **"COMPARISON OF NO CLAIM DISCOUNT (NCD) SYSTEMS"** completed by **MÜGE TOYDEMİR** under supervision of **ASSOC. PROF. DR. GÜÇKAN YAPAR** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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COMPARISON OF NO CLAIM DISCOUNT (NCD) SYSTEMS

ABSTRACT

No claim discount system (NCD) is used in non-life insurance and also it is widely used in motor insurance. NCD systems penalize policyholders at fault in accidents by premium increase, and reward claim-free years by premium discount. By doing this, policyholders are subdivided into homogeneous classes according to their typical variables and are encouraged to drive carefully.

In this thesis, first, some information about situation of motor insurance and the use of NCD systems in both Turkey and the World is given. The claim number distributions and use the method of simulation for deciding the optimum NCD systems are explained in detail. In addition, generating random variable methods from claim number and claim amount distributions that are used in the simulation are explained.

In the application, a NCD system that is widely used for motor insurance in Turkey is defined, and by using the motor insurance data of year 2008 an optimal system is tried to reach with the simulation method. Simulation program is written in MATLAB. As a result of the simulation program, for each year number of claims, amount of claims, number of claimants, number of policyholders in each discount class, premium income and claim outgo year is obtained. In order to reach optimum NCD systems on the bases of the long-term results obtained from the simulation program, changes are made on the transition rules and premiums. Finally, by increasing the premiums, optimal system was reached.

KeyWords: No claim discount system, motor insurance, motor third-liability insurance, generate random number, claim number distribution, simulation, MATLAB

HASARSIZLIK İNDİRİMİ SİSTEMLERİNİN KARŞILAŞTIRILMASI

ÖZ

Hasarsızlık indirimi sistemi (HİS) birçok hayat dışı sigortacılık alanında kullanıldığı gibi yaygın olarak otomobil sigortalarında kullanım alanına sahiptir. HİS'i kazada cezalı olan poliçe sahiplerini prim artırımı ile cezalandırır, hasarsız bir yıl geçirenleri ise prim indirimi ile ödüllendirir. Bu sayede tipik değişkenlerine göre poliçe sahipleri homojen sınıflara ayrılır ve dikkatli araba kullanmaları konusunda teşvik edilirler.

Bu çalışmada öncelikle Türkiye'de ve Dünya'daki motor sigortalarının durumu ve kullanılan HİS'leri hakkında bilgiler verilmiştir. Hasar sayısı dağılımları ve optimum hasarsızlık indirimi sistemine karar vermek için kullanılan simülasyon metodu hakkında ayrıntılı bilgi verilmiştir. Ek olarak, simülasyonda kullanılan hasar sayısı ve hasar miktarı dağılımlarından rassal değişken türetme metodları anlatılmıştır.

Uygulamada Türkiye'de kasko sigortalarında yaygın olarak kullanılan bir HİS'i tanımlanmıştır ve 2008 yılı kasko sigortası verileri kullanılarak optimum sisteme simülasyon metodu ile ulaşılmaya çalışılmıştır. Simülasyon programı MATLAB' da yazılmıştır. Simülasyon programının sonucunda her bir yıl için hasar sayısı, hasar miktarı, hasar beyan edenlerin sayısı, her bir indirim sınıfında yer alan poliçe sahibi sayısı, hasar gideri ve prim geliri elde edilmektedir. Simülasyon programdan elde edilen uzun dönem sonuçlar temel alınarak optimum HİS'ne ulaşabilmek için, geçiş kurallarında ve primde değişiklik yapılmıştır. Sonuç olarak primlerde artırım yaparak optimum HİS'ne ulaşılmıştır.

Anahtar Kelimeler: Hasarsızlık indirimi sistemi, motor sigortaları, üçüncü dereceden zorunlu trafik sigortaları, rassal sayı türetme, hasar sayısı türetme, simülasyon, MATLAB

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CHAPTER ONE

NO CLAIM DISCOUNT SYSTEM IN AUTOMOBILE INSURANCE

1.1 Introduction

The story of people and their motor vehicles is one of the great love affairs of this century. Around the world, there were also about 806 million cars and light trucks on the road in 2007. By 2020, that number will reach 1 billion (Cars Emit Carbon Dioxide. Global Warming, Focus on the Future, 1997).

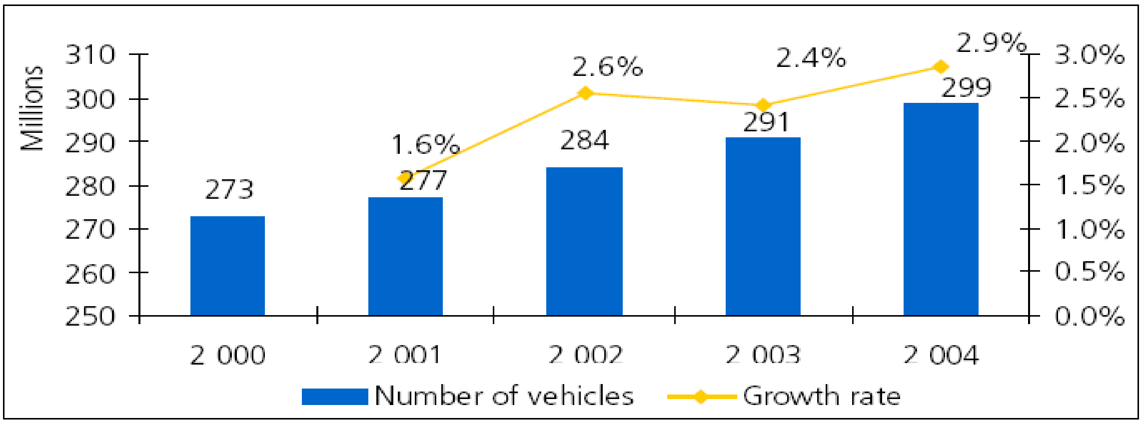

Figure 1.1 Number of vehicles in Europe

With almost 300 million vehicles on the road, Europe has the largest motor insurance market in the world, just ahead of the North American (US and Canada) market, which numbers approximately 250 million vehicles. In Turkey, there were also about 6.5 million cars on the road in 2007. (Turkish Statistical Institute, 2007) The European market has experienced a steady growth rate of 1.5% to 3% between 2000 and 2004, to reach a total of 299 million vehicles. On average over the last five years, the market has grown by 2.4%. About 78% of these vehicles are personal and commercial four-wheeled vehicles, 10% are utility vehicles and 12% are motorcycles.

The motor market is also the largest sector in non-life insurance business. There are around 1,000 motor insurance companies active in Europe. In 2006, they generated a total premium income of €127.2bn, which corresponds to a 1% decrease (inflation adjusted) as compared to 2005. This fall is mainly driven by a reduction in policyholders' motor liability premiums.

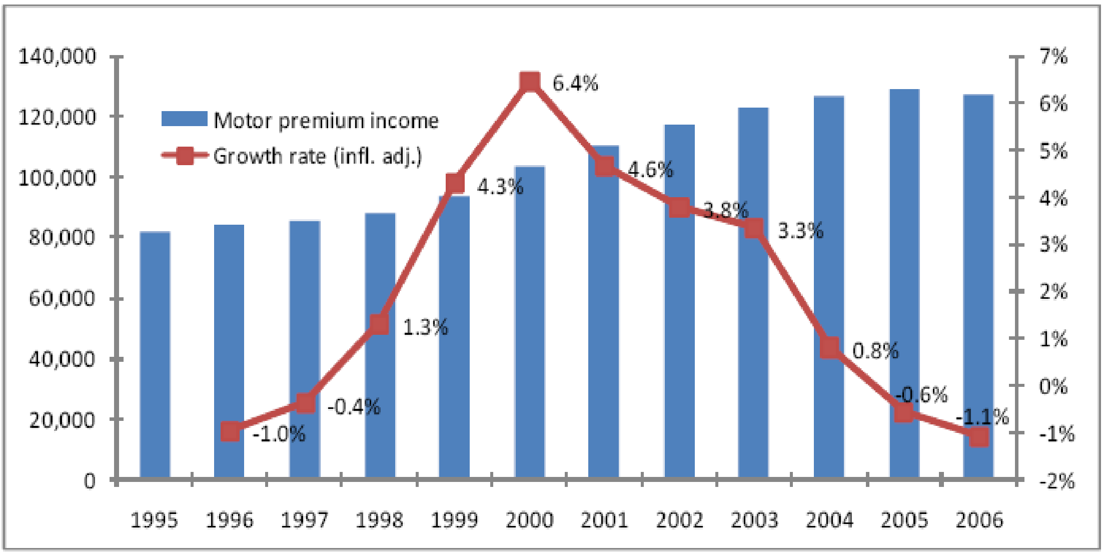

Figure 1.2 Motor premium income (euro million)

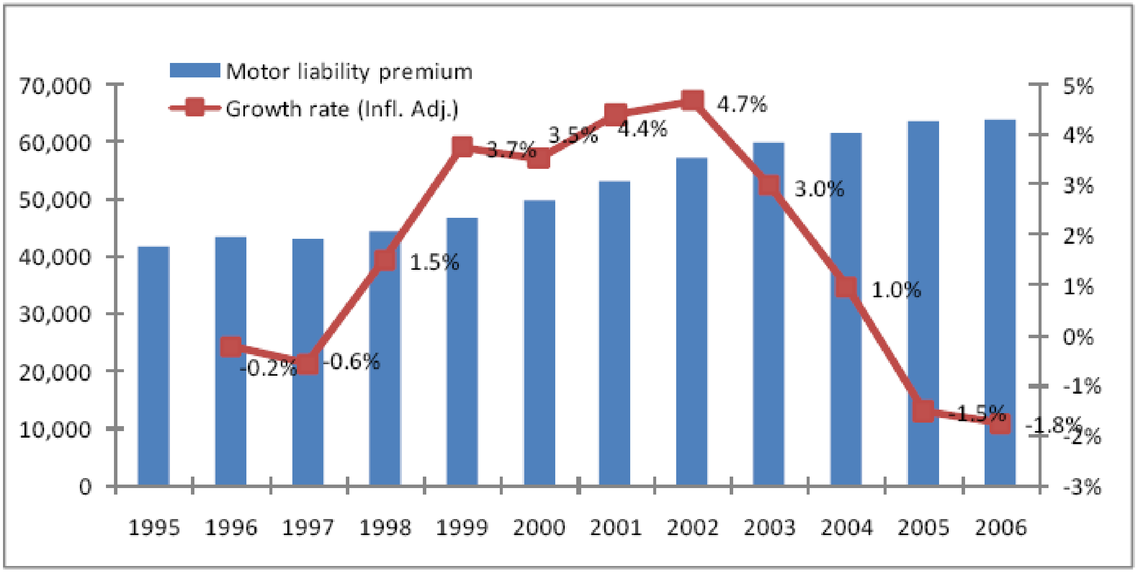

Figure 1.3 Motor liability premium (euro million)

The premium income in motor liability represents a total of €64,000 and is in decline for the second consecutive year: -1.8% in 2006 against -1.5% in 2005. Growth rates are steadily fallen down in recently year. This reflects the intense competition between insurers to increase their market shares.

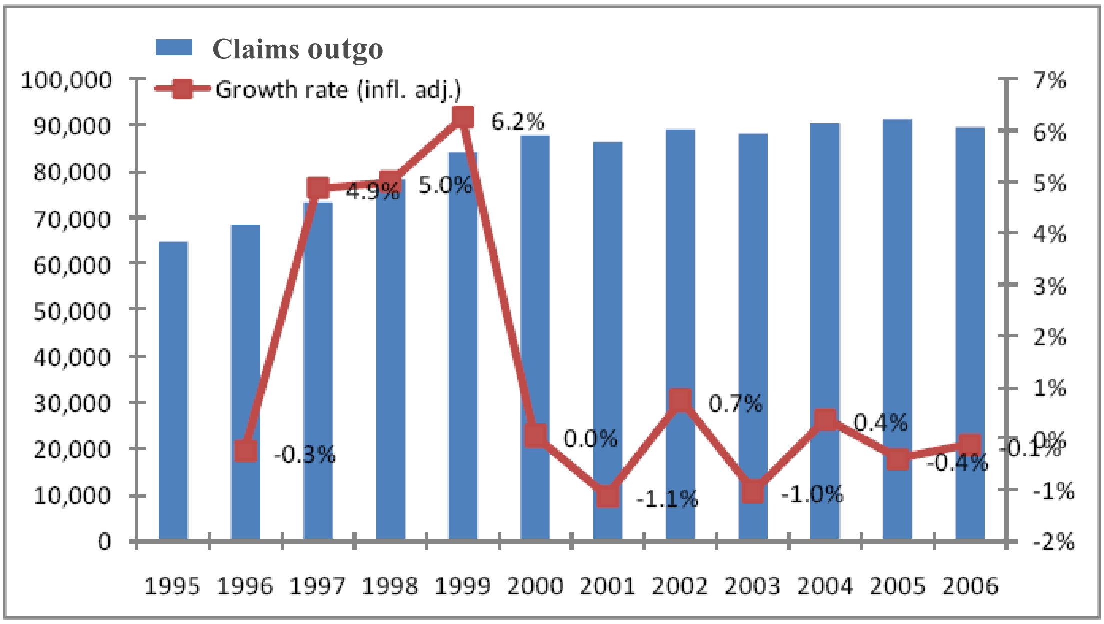

Figure 1.4 Motor claims outgo (euro million)

After a substantial increase between 1994 and 1999, claims outgo has remained more or less stable between 2000 and 2006. This stability has not been directly reflected in the premium income, in order to restore the profitability of this business line which experienced significant losses at the start of this decade.

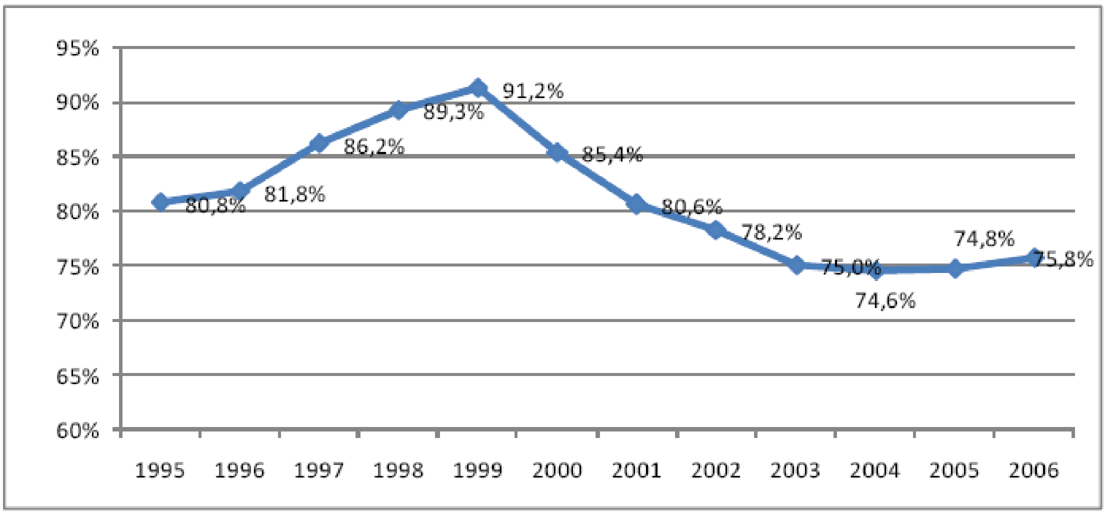

Figure 1.5 Claims ratio in motor (euro million)

The claims ratio embodies the growth in both claims and premiums. The years 1995 to 1999 are characterized by a continuous rise of the claims ratio indicating a faster increase of claims compared to premium income. This trend has led to an unsustainable situation and substantial underwriting losses for insurance companies. From 1999, the stability in claims outgo and a slight increase in premium income have allowed insurers to restore their profitability and to absorb the reduction in investment income that occurred during the financial crisis of 2001.

The slight increases observed in 2005 and 2006 lead to the assumption that the insurance sector has entered into the growth phase of a new cycle that is likely to seriously reduce its profitability (CEA Statistics, 2007).

Table 1.1 Profitability and Loss premium ratio in Turkey

TURKEY	Motor Liability Insurance		Motor Insurance	
	Profitability (TL)	Loss Premium Ratio	Profitability (TL)	Loss Premium Ratio
2006	-118,637	81.55%	-250,073	91.50%
2007	-244,646,329	90.01%	4,574,092	75.86%
2008	-467,280,599	110.45%	75,117,367	81.62%

As it is seen from the Table 1.1 there are loss premium ratio and profitability in motor insurance and motor liability insurance belonging to the years 2006, 2007 and 2008 in Turkey. Decreases in premiums cause by competition affected the sector adversely in addition to high increases in spare part prices occurred in the branch of motor vehicle physical damage insurance, technical profit targets couldn’t be achieved throughout the sector and the year 2006 was closed with loss. Along with correct and careful pricing methods initiated by the companies especially in the branch of motor vehicles physical damage insurance as of the end of 2006; studies are including decreasing damage, controlling risk and controlling damage costs started to be concluded and the years 2007 and 2008 became a successful year for entire sector in this branch.

Harmful results couldn’t be corrected in consideration of these positive developments seen in technical profitability in automobile branch. After “Regulation on Tariff Application in Highway Motor Vehicle Compulsory Liability Insurance” came into force, companies get the opportunity to determine tariff premiums partially; it is a bit difficult for this branch to gain technical profit without a full freedom in 2007 and 2008.

Other important factors for the increase in damages in automobile branches are forgery and corruption events. Estimations to the fact that more than 20 % of loss claim paid in the sector is constituted by false damages show presence of a serious and systematic corruption (Association of Insurance and Reinsurance Companies of Turkey, 2008).

An unfortunate consequence has been the parallel growth of accidents and casualties, with over 100.000 deaths annually in the World. Motor third-part liability insurance has consequently been made compulsory in most development countries, and actuaries from all over the world face the problem of designing tariff structures that will fairly distribute the burden of claims among policyholders.

1.2 Definition of a No Claim System

Most development countries use several classification variables to differentiate premiums among automobile third-party liability policyholders. Typical variables include age, sex, and occupation of the main driver, the town where he resides, and the type and use of his car. More exotic variables, such as the driver's marital status and smoking behaviour, or even the colour of his car, have been introduced in some countries. Such variables are often called *a priori* rating variables, as their values can be determined before the policyholder starts to drive. The main purpose for their use is to subdivide policyholders into homogeneous classes (Lemaire, 1995).

While life insurance premiums are set with a fairly universal approach, such is not the case in automobile insurance. Despite the use of many a *priori* variables, very heterogeneous driving behaviours are still observed in each tariff cell.

Individual abilities of each driver, such as accuracy of judgment, aggressiveness at the wheel, knowledge of the highway code and drinking pattern, are also tremendously important in influencing the number of accidents (but not measurable in a cost-effective way). Indeed, several studies performed around the world (Lemaire, 1977b and 1985) have shown that these factors are the most important: the best predictor of the future number of claims is not the driver's age, sex or occupation, but his past claims behaviour. Hence the idea came in the mid-1950s to allow for premium adjustment *a posteriori*, after having observed the claims history of each policyholder. Such practices, called experience rating, merit-rating, no-claims discount (NCD), or bonus-malus systems, penalize the insureds responsible for one or more accidents by an additional premium or malus, and reward claim-free policyholders, by awarding a discount or bonus. Their main purpose-besides encouraging insureds to drive carefully- is to better assess individual risks so that everyone will pay, in the long run, a premium corresponding to his own claim frequency (Lemaire, 1995).

NCD system is used in automobile third-party liability insurance. Some countries also use NCD system in collision and comprehensive coverage, and sometimes the NCD system used in collision is not the same as the one used in third-party.

For insurance carriers, NCD systems are also a response to adverse selection, the asymmetry of information about policyholder behaviour. A good example of adverse selection is the purchase of collision insurance. It is well known that the drivers who buy optional collision coverage have a much higher claim frequency than those who purchase only compulsory third party liability -proof that insureds known more about their driving behaviour than the insurance company (Lemaire, 1985). NCD systems are a way to partially correct this lack of knowledge about policyholders' driving patterns. It is, for instance, intuitive that annual mileage has to be positively correlated with claim frequencies. Yet most countries consider that this variable cannot be measured accurately or inexpensively. NCD systems are a way to partially compensate for this lack of knowledge about driving patterns. Annual mileage is measured indirectly, through the more numerous claims of those who spend more time on the road.

Nevertheless, the very idea of a NCD system has several drawbacks. Some actuaries have rejected the idea of a posteriori rating by terming the idea of a rebate of part of the premium to good (or simply lucky) policyholders, contrary to the very notion of insurance, as it goes against some of its fundamental principles (Lemaire, 1995):

- Economic stability guaranteed to the insureds. The policyholder is protected against all third-party liability claims in return for the payment of a fixed premium, small in comparison with the possible amount of a claim.

- Cooperation and solidarity. Policyholders with no claims come to the help of unfortunate ones.

- Law of large numbers. A policy by itself is lost in the mass of the portfolio. Such systems can be extremely different, from county to country.

As stated in Lemaire (1995), a system employed in a automobile insurance is called a no claim discount system when

- all policyholders of a given tariff group can be partitioned into a finite number of classes, denoted C_i (i = 1,2,…,s), so that their premium depends only on the class they belong to (the number of classes is denoted by "s"), and

- the class of a policyholder for a given period (usually a year) is determined uniquely by the class in the preceding period and the number of claims reported in that period.

Such a system is defined by the initial class C_{io}, premium scale $\bar{b}=(b_1,\ldots,b_s)$ where b_i is the premium level in class C_i, as well as transition rules i.e. rules governing the transfer of a policyholder from one class to another when the number of his or her claims is known.

The transition rules are represented by means of $s \times s$ matrices

$$T_k = (t_{ij}^{(k)}) \tag{1.1}$$

$$t_{ij}^{(k)} = \begin{cases} 1 & if \quad T_k(i) = j \\ 0 & if \quad T_k(i) \neq j \end{cases} \tag{1.2}$$

$T_k(i) = j$ denotes the transfer of a policyholder reporting k claims C_i from class C_j in the next period. The probability of moving from C_i to C_j for a policyholder with claim frequency λ is given by

$$P_{ij}(\lambda) = \sum_{k=0}^{\infty} p_k(\lambda)\ t_{ij}^{(k)} \tag{1.3}$$

where $p_k(\lambda)$ is the probability that a driver with claim frequency λ has k claims in one period. Obviously $p_{ij}(\lambda) \geq 0$ and

$$\sum_{j=1}^{s} p_{ij}(\lambda) = 1 \tag{1.4}$$

$$M(\lambda) = (p_{ij}(\lambda)) = \sum_{k=0}^{\infty} p_k(\lambda) T_k \tag{1.5}$$

(1.5) is the transition matrix of this Markov chain. As we shall assume that the claim frequency is stationary in time (no improvement in the policyholder's driving ability), the chain is homogeneous.

NCD system forms a Markov chain process. A (first-order) Markov chain is a stochastic process in which the future development depends only on the present state but not on the history of the process or the manner in which the present state was reached. It is a process without memory, such that the state of the chain is the different NCD system classes. The knowledge of the present class and the number of claims for the year suffice to determine next year's class. It is not necessary to know how the policyholder reached his current class.

1.3 Models for the Claim Number Distribution

Four different probability models are developed to represent the distribution of the number of claims in an insurance portfolio.

1.3.1 Poisson Model – Homogeneous Portfolio

In this model we assume that all policyholders have the same underlying risk; the occurrence of a claim constitutes a random event, and there is no reason for penalizing the insureds responsible for a claim.

Let us formulate the three following assumptions. $N(t,t+\Delta t)$ denote the number of claims in the time interval $(t,t+\Delta t)$;

1. $P[N(t,t+\Delta t)=1]=\lambda\Delta t+o(\Delta t)$ (1.6)
2. $P[N(t,t+\Delta t)>1]=o(\Delta t)$ (1.7)
3. Let τ and τ' be two disjoint time intervals. Then

$$P[N(\tau)=k \quad and \quad N(\tau')=k']=P[N(\tau)=k].P[N(\tau')=k']. \tag{1.8}$$

(A function f(x) is $o(h) \quad if \quad \lim_{h\to 0} f(h)/h=0$)

The first assumption implies that the probability of an accident during a small interval $(t,t+\Delta t)$ is, ignoring higher-order terms, proportional to the duration of the interval. In particular, it does not depend on the start of the interval. The second assumption requires the probability of two or more accidents in this time interval to be negligible. The third demands the number of accidents relating to disjoint time intervals to be independent.

It is well known that these three assumptions imply that the distribution $\{p_k, k=0,1,2,\ldots\}$ of the number of claims in a given year is Poisson distributed with parameter λ. Indeed, if $p_k(t)=P[N(0,t)=k]$, we have (Lemaire, 1995)

$$p_k(t+\Delta t) = p_k(t).P[N(t,t+\Delta t)=0] + p_{k-1}(t).P[N(t,t+\Delta t)=1] + \sum_{i=2}^{k} p_{k-i}(t).P[N(t,t+\Delta t)=i] \quad (1.9.)$$

$$= p_k(t).[1-\lambda\Delta t + o(\Delta t)] + p_{k-1}(t)[\lambda\Delta t + o(\Delta t)] + \sum_{i=2}^{k} p_{k-i}(t).o(\Delta t) \quad (1.10)$$

$$= p_k(t)(1-\lambda\Delta t) + p_{k-1}(t)\lambda\Delta t + o(\Delta t) \quad (1.11)$$

for $k=0,1,\ldots$ (setting $p_{-1}(t)=0$) Consequently,

$$\frac{p_k(t+\Delta t)-p_k(t)}{\Delta t} = -\lambda p_k(t) + \lambda p_{k-1}(t) + \frac{o(\Delta t)}{\Delta t}. \quad (1.12)$$

By taking the limit for $\Delta t \to 0$,

$$p_k^{'}(t) = -\lambda p_k(t) + \lambda p_{k-1}(t) \qquad k=1,2,\ldots \quad (1.13)$$

$$p_0^{'}(t) = -\lambda p_0(t) \qquad k=0 \quad (1.14)$$

By recursively solving this set of differential equations with the initial conditions $p_0(0)=1$ and $p_k(0)=0$ if $k>0$, we obtain

$$p_k(t) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}. \quad (1.15)$$

Thus the Poisson is the only distribution that verifies the three properties. Consequently, if we consider that the accident pattern of automobile drivers conforms to the three properties, we have no another choice than to adopt the Poisson to model the distribution of the number of claims of individual policyholders.

We recall that the mean and the variance of this distribution are equal to λ. Can we use to Poisson model the distribution of the number of claims in a portfolio? We assume a homogeneous portfolio, where each policyholder's claim number has a Poisson distribution with the same parameter λ.If this model is not compatible with statistical testing, claims in a portfolio can be assumed to occur randomly. All insureds of the same tariff class should pay the same premium, independently of their past claims behaviour.

Table 1.2.Observed Distribution of Number Claims in a Portfolio

k	n_k
0	96,978
1	9,240
2	704
3	43
4	9
>4	0
Total	106,974

Table 1.2 shows the distribution of the number of claims n_k in the automobile third-party liability portfolio of a Belgian company. It contains $n = 106,974$ observations and has a mean $\overline{x} = 0.1011$ and a variance $s^2 = 0.1074$. The portfolio was observed in 1976 (Lemaire, 1977b, 1979a) but figures are quite similar today, as claim frequencies in Belgium have remained very stable in 1991 was 0.1033 (Lemaire, 1995).

1.3.2 Negative Binomial Model – Heterogeneous Portfolio

We assume that the policyholders do not all have the same underlying risk such as behavior of policyholders is heterogeneous. We need a model that reflects the different underlying risks. We suppose that the distribution $\{p_k(\lambda),\ k = 0,1,2,\ldots\}$ of the number of claims for each policyholder is a Poisson distribution,

$$p_k(\lambda) = \frac{e^{-\lambda}\lambda^k}{k!} \qquad k = 0,1,2,\ldots \tag{1.16}$$

whose parameter λ varies from one individual to another. Each policyholder is characterized according to the value of his parameter λ.In this approach, λ is considered to be the observed value of a random variable Λ.The negative binomial model and Poisson-inverse Gaussian models considers continuous distributions for Λ.For large portfolios, it seems natural to use a continuous approach. We will therefore assume in the remaining chapters that Λ has a continuous distribution on the segment $[0,\infty)$. The density function of Λ will be denoted $u(\lambda)$. It is called the *structure function*. The resulting distribution of the number of claims in the portfolio

$$p_k = \int_0^\infty \frac{e^{-\lambda}\lambda^k}{k!} u(\lambda) d\lambda \qquad k = 0,1,2,\ldots \tag{1.17}$$

is called a *mixed Poisson* distribution. Let us choose for the distribution of Λ, called the *mixing distribution*, a Gamma with parameters a and τ.

$$u(\lambda) = \frac{\tau^a e^{-\tau\lambda}\lambda^{a-1}}{\Gamma(a)} \qquad a, \tau > 0. \tag{1.18}$$

The Gamma is also known as the Pearson Type III distribution. Its mean is a/τ, its variance a/τ^2, its skewness coefficient $2/\sqrt{a}$, and its moment-generating function

$$M(t)=\left(\frac{\tau}{\tau-t}\right)^{a} \qquad 0\le t<\tau. \tag{1.19}$$

When $a=1$, the Gamma reduces to the exponential distribution.

Recall some properties of the Γ function:

$$\Gamma(a)=\int_0^\infty t^{a-1}e^{-t}dt; \tag{1.20}$$

$$\Gamma(a+1)=a\Gamma(a); \tag{1.21}$$

If a is an integer. $\Gamma(a+1)=a!$ (1.22)

The distribution $\{p_k;\ k=0,1,2,\ldots\}$ of the number of claims in the portfolio is obtained by integration;

$$p_k=\int_0^\infty p_k(\lambda)u(\lambda)d\lambda=\int_0^\infty \frac{e^{-\lambda}\lambda^k}{k!}\frac{\tau^a e^{-\tau\lambda}\lambda^{a-1}}{\Gamma(a)}d\lambda \tag{1.23}$$

$$=\frac{\tau^a}{k!\Gamma(a)(1+\tau)^{k+a}}\int_0^\infty e^{-\lambda(1+\tau)}\left[\lambda(1+\tau)\right]^{k+a-1}d\left[\lambda(1+\tau)\right] \tag{1.24}$$

$$=\frac{\Gamma(k+a)}{\Gamma(k+1)\Gamma(a)}\frac{\tau^a}{(1+\tau)^{k+a}} \tag{1.25}$$

$$=\binom{k+a-1}{k}\left(\frac{\tau}{1+\tau}\right)^a\left(\frac{1}{1+\tau}\right)^k \tag{1.26}$$

$$=\binom{k+a-1}{k}p^a q^k \tag{1.27}$$

$$p=\frac{\tau}{1+\tau} \qquad and \qquad q=1-p=\frac{1}{1+\tau} \tag{1.28}$$

and defining, as generalized combinatorial coefficient,

$$\binom{k+a-1}{k} = \frac{\Gamma(k+a)}{\Gamma(k+1)\Gamma(a)}. \tag{1.29}$$

We obtain a negative binomial distribution, of mean;

$$m = \frac{a}{\tau} \tag{1.30}$$

and variance;

$$\sigma^2 = \frac{a}{\tau}\left(1+\frac{1}{\tau}\right). \tag{1.31}$$

The variance of negative binomial is calculated great than its mean. The skewness coefficient is

$$\frac{2-\frac{\tau}{1+\tau}}{\sqrt{\frac{a}{1+\tau}}}. \tag{1.32}$$

The moment-generating function is

$$\left(\frac{\tau}{1+\tau-e^t}\right)^a \qquad t < \ln(1+\tau). \tag{1.33}$$

A table of the Gamma function does not require for calculation of negative binomial probabilities. The easiest way to compute negative binomial probabilities is to use the recursion

$$p_{k-1} = \frac{k+a}{(k+1)(1+\tau)} p_k \tag{1.34}$$

starting from (Lemaire, 1995)

$$p_0 = \left(\frac{\tau}{1+\tau} \right)^a . \tag{1.35}$$

1.3.3 Poisson-Inverse Gaussian Model

Mixed Poisson distribution is widely used for modelling claim counts when the portfolio is heterogeneous. The mixing distribution represents a measure of this heterogeneity. Recent papers by Willmot (1986, 1987), Venter (1991a), Besson and Partrat (1992), Tremblay (1992), and Lemaire (1992) have suggested an alternative to the negative binomial: the Poisson-inverse Gaussian distribution. In this model, the distribution of Λ is an inverse Gaussian IG(g,h) (see Holla 1967, and Sichel, 1971):

$$U(\lambda) = \frac{g}{\sqrt{2\pi h \lambda^{3/2}}} e^{-\frac{1}{2h\lambda}(\lambda-g)^2} \qquad g,h > 0. \tag{1.36}$$

Then the resulting mixed Poisson is called the Poisson-inverse Gaussian. Its mean is m=g, while its variance is $\sigma^2 = g(1+h)$. The probabilities p_k can be calculated recursively

$$p_0 = e^{\frac{g}{h}\left[1-(1+2h)^{1/2}\right]} \tag{1.37}$$

$$p_1 = g p_0 (1+2h)^{-1/2} \tag{1.38}$$

$$(1+2h)k(k-1)p_k = h(k-1)(2k-3)p_{k-1} + g^2 p_{k-2} \qquad k = 2,3,\ldots \tag{1.39}$$

We obtain estimators of g and h (providing $s^2 > x$) (Lemaire 1995)

$$\hat{g} = x \tag{1.40}$$

$$\hat{h} = (s^2 / x) - 1 \tag{1.41}$$

1.3.4 Good-Risk/Bad-Risk Model

In this mixed Poisson process, the mixing structure function is a simple function of two point discrete distribution. The portfolio consists of only two categories of drivers: a fraction a_1 of "good" drivers (Poisson parameter λ_1) and a fraction $a_2 = 1 - a_1$ of "bad" drivers (parameter λ_2):

$$p_k = a_1 \frac{\lambda_1 e^{-\lambda_1}}{k!} + a_2 \frac{\lambda_2 e^{-\lambda_2}}{k!}, \qquad (1.42)$$

with $a_1, a_2, \lambda_1, \lambda_2 > 0, a_1 + a_2 = 1$. Its means is $a_1\lambda_1 + a_2\lambda_2$, its variance $\alpha_2 - m^2$, where $\alpha_2 = a_1\lambda_1^2 + a_1\lambda_1 + a_2\lambda_2^2 + a_2\lambda_2$. The third central moment is $\mu_3 = \alpha_3 - 3m\alpha_2 + 2m^3$, where $\alpha_3 = a_1\lambda_1^3 + a_2\lambda_2^3 + 3(a_1\lambda_1^2 + a_2\lambda_2^2) + a_1\lambda_1 + a_2\lambda_2$.

The moment estimators of the parameters are

$$\hat{a}_1 = \frac{a - \hat{\lambda}_2}{\hat{\lambda}_1 - \hat{\lambda}_2} \qquad (1.43)$$

$$\hat{\lambda}_1, \hat{\lambda}_2 = \frac{S \pm \sqrt{S^2 - 4P}}{2} \qquad (1.44)$$

$$S = \frac{c - ab}{b - a^2} \qquad P = \frac{ac - b^2}{b - a^2} \qquad (1.45)$$

$$a = \overline{x} \qquad b = \alpha_2^* - \overline{x} \qquad c = \alpha_3^* - 3\alpha_2^* + 2\overline{x}\,. \qquad (1.46)$$

α_2^* and α_3^* are, the moments around the origin of order 2 and 3 of the observed distribution (Lemaire, 1995).

1.4 Description of All No Claim Discount System

For each NCD system used in automobile third-party liability insurance, we provide the number of classes, all premium levels, the starting levels, and a short description of the transition rules: the number of classes decreased following a claim-free year, and the number of classes increased following claims. Special rules and assumptions are mentioned.

In most other countries, the best class is 0 or 1, but in a few cases 1 is the worst class. The top-discount class is always 1, except for the two countries (Belgium and Switzerland) where it is officially 0.

1.4.1 Turkish System

NCD system had to be applied in 1996 by Turkey. TRAMER to take effect as a result of de facto, NCD system had to be applied completely in 2004. Following the centralized supervision provided by TRAMER (Motor TPL Insurance Information Centre), the tendency in the decrease of non-insured motor vehicles continued in 2007 and declined to 17%. System has been revised several times until today. Most recent version is listed below.

Table 1.3 Turkish system

Class						
1	2	3	4	5	6	7
160	140	120	100	90	85	70
Premium Level (%)						

No claim premium discount and premium increase due to the damage are made according to the transition rules which this appears in that Table 1.3. For the people who will drive for the first time with the name of vehicle operator forth class which does not include premium increase and discount is applied. Starting level is 100.

For the insurance holders whose insurance policy ended on the date of 01.01.2009 and on the following dates, in the case that no claim for indemnification is made in the terminated insurance period, premium discount ratio which will be applied in the insurance policy that will be renewed is determined through one upper level according to the terminated policy. On the other hand, for each indemnification claim which is made within the time of terminated insurance policy, premium increase that will be applied in the renewal is determined according to one lower level of the terminated policy.

After the determination of premium level, according to the current situation the policy in operation is rearranged as no premium in the case that notification of claim about the terminated period is realized within the period in which the policy is in operation and this case is taken into consideration in order to make operation in the renewed policy over one level below.

In the case that a vehicle operator owns more than one vehicle or when more than one vehicle are operated with the title of an enterprise or foundation or with the tickets sold by this enterprise, a separate policy is made for each vehicle and the premium level to be applied is determined separately for each vehicle.

Premium increase which is made due to premium discount or indemnification claim is followed by the operator.

The policyholder is responsible for presenting the copy of vehicle license for the vehicles registered under his name; the notary bill of sale or the invoice or the document required by the insurance company for the vehicles are not registered under his name yet.

In the case that the latest policy information about the insurance holder is not controlled by the insurance man through TRAMER data base, the insurance holder presents the latest policy copy to the insurance company.

The insurance company determines the discount and increase ratios in parallel with the documents presented by the insurance holder and the damage certificate it will obtain from TRAMER. Premium level is calculated according to the premium ratio in the step of highest increase in the case that the document of vehicle sale or licence copy and the latest policy copy are not presented with the aim of determining premium level.

Insurance companies have to write the date and number on the insurance policy and to keep the damage certificate as document or electronically at least for three years, if they apply premium increase due to the damage discount and indemnification payment.

Discount and increase ratios are applied in succession by accumulating in premium and not being applied.

Damage discount is not applied in the case that the new insurance policy following the short term insurance and short term insurance policy is annual. However, premium discount is applied for these policies due to damage (TRAMER, 2009).

1.4.2 Belgium System

Table 1.4 Old Belgium system (1971)

Class																	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
60	65	70	75	80	85	90	95	100	100	105	110	115	120	130	140	160	200
Premium Level (%)																	

Starting level: 85 for pleasure use and commuting, 100 for business use

Claim-free: -1. Cannot be above level 100 after 4 consecutive claim-free years.

First claim: +2. Subsequent claims: + 3

Table 1.5 New Belgium system (1992)

Class																						
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
54	54	54	57	60	63	66	69	73	77	81	85	90	95	100	105	111	117	123	130	140	160	200
Premium Level (%)																						

Starting level: 85 for pleasure use and commuting, l00 for business use

Claim-free: -1. Cannot be above level 100 after 4 consecutive claim-free years.

First claim: +4. Subsequent claims: +5

1.4.3 Brazilian System

Table 1.6 Brazilian system

Class						
1	2	3	4	5	6	7
65	70	75	80	85	90	100
Premium Level (%)						

Starting level: 100

Claim-free: -1

Each claim: +1

1.4.4 Denmark System

Table 1.7 Denmark system

Class									
1	2	3	4	5	6	7	8	9	10
30	40	50	60	70	80	90	100	120	150
Premium Level (%)									

Starting level: 100

Claim-free: -1

Each claim: +2

1.4.5 Finnish System

Table 1.8 Old Finnish system

Class													
1	2	3	4	5	6	7	8	9	10	11	12	13	14
40	50	50	50	50	60	60	70	80	100	110	120	130	150
Premium Level (%)													

Starting level: 120

Claim-free: -l

First claim: from +6 (lowest classes) to +1 (highest classes). Subsequent claims: +3

Table 1.9 New Finnish system

Class																
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	100	100
Premium Level (%)																

Starting level: lowest 100

Claim-free: -1

First claim: +3 or +4. Subsequent claim: +4 or +5

1.4.6 French System

Number of classes: 351

Premium Levels: all integers from 50 to 350

Starting level: 100

Claim-free: 5% reduction. Cannot be above level 100 after 2 consecutive claim-free years.

Each claim: 25% increase, 12.5% if shared responsibility.

The lowest level is 50; it is reached after thirteen consecutive claim-free years from the starting class. A recent modification is that the first claim of a policyholder who was at the lowest level for at least 3 years is not penalized.

1.4.7 German System

Table 1.10 Old German system (Early 1980s)

Class																	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
40	40	40	40	40	45	50	55	60	65	70	85	100	125	175	175	200	200
Premium Level (%)																	

Starting level: 175, or 125 if driver's licensed for at least three years

Claim-free: -1 or to level 100, if more favourable

Each claim: from +1 or +2 (highest levels) to +4 or +5 (lowest levels)

Table 1.11 New German system

Class																					
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
30	35	35	35	40	40	40	40	40	45	45	50	55	60	65	70	85	100	125	155	175	200
Premium Level (%)																					

Starting level: 175 or 125, depending on experience and other cars in the same household.

Claim-free: -1, except in the upper classes.

Each claim: from +1 (upper classes) to +9 (lowest class)

1.4.8 Hong Kong System

Table 1.12 Hong Kong system

Class					
1	2	3	4	5	6
40	50	60	70	80	100
Premium Level (%)					

Starting level: 100

Claim-free: -1

First claim: +2 or +3. Subsequent claims: all discounts lost

1.4.9 Italian System

Table 1.13 Old Italian system

Class												
1	2	3	4	5	6	7	8	9	10	11	12	13
70	70	70	75	80	85	92	100	115	132	152	175	200
Premium Level (%)												

Starting level: 115

Claim-free: -1

Each claim: +1

Table 1.14 New Italian system (1991)

Class																	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
50	53	56	59	62	66	70	74	78	82	88	94	100	115	130	150	175	200
Premium Level (%)																	

Starting level: 115

Claim-free: -1

First claim: +2. Subsequent claim: +3

1.4.10 Japanese System

Table 1.15 Japanese system (1984)

Class															
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
40	40	40	42	45	50	60	70	80	90	100	110	120	130	140	150
Premium Level (%)															

Starting level: 100

Claim-free: -1

Old System (1984)

Each claim: +2 Property Damage, + 4 Bodily Injury

New System (1993)

Each claim: +3

* 12.5% of all claims have bodily injury implications.

1.4.11 Kenyan System

Table 1.16 Kenyan system

Class						
1	2	3	4	5	6	7
40	50	60	70	80	90	100
Premium Level (%)						

Starting level: 100

Claim-free: -1

Each claim: all discounts loss

1.4.12 Korean System

Number of classes: 37

Premium Levels: 40, 45, 50, 55, 60 … 210, 215, 220

Starting level: 100

Claim-free: The premium level generally decreases by 10. However, moving down is only allowed after 3 claim-free years. The policy cannot be above level 100 after 3 claim-free years.

Each claim: After each accident, at-fault policyholders receive a specified number of penalty points. Property damage claims are penalised by 0.5 or 1 penalty point, depending on the cost. Bodily injury claims are penalised by 1 to 4 points, depending on the type of injury. Serious offenses, like hit-and-run and drunk driving, are assessed supplementary points, up to 3. The premium increase is 10 levels per penalty point, with a few exceptions.

1.4.13 Luxembourg System

Table 1.17 Luxembourg system

Class																					
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
50	50	60	65	70	75	80	85	90	100	100	105	110	115	120	130	140	160	180	200	225	250
Premium Level (%)																					

Starting level: 100

Claim-free: -1. Cannot be above level 100 after 4 consecutive claim-free years

Each claim: +2

New system

Two new classes, at levels 47.5 and 45, have been added.

Each claim: +3

1.4.14 Malaysian – Singaporean System

Table 1.18 Malaysian – Singaporean system

Class					
1	2	3	4	5	6
45	55	61.67	70	75	100
Premium Level (%)					

Starting level: 100

Claim-free: -1

Each claim: all discounts lost

1.4.15 Dutch System

Table 1.19 Dutch system (1981)

Class													
1	2	3	4	5	6	7	8	9	10	11	12	13	14
30	32.5	35	37.5	40	45	50	55	60	70	80	90	100	120
Premium Level (%)													

Starting level: 70 to 100, depending on age of policyholder and annual mileage

Claim-free: -1

Each claim: +3 to +5

1.4.16 Norwegian System

Table 1.20 Norwegian system

Class																						
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	…
30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	…
Premium Level (%)																						

Starting level: 100

Claim-free: -1 or level 120, if more favourable

First claim: +2 (highest levels) or +3 (3 lowest levels). Subsequent claims: +2

New system: Several NCD system currently coexist. The following system was launched in 1987 by a leading company (Neuhaus, 1988).

Number of classes: infinite

Premium Levels: all integers from 25 up

Starting level: 80, for drivers aged at least 25 insuring their privately owned vehicle. 100 for all others.

Claim-free: 13 % discount.

Each claim: fixed amount premium increase (NOK 2.500 in 1988). The penalty cannot however exceed 50% of the basic premium. The penalty is reduced by half for the drivers who have had between five and nine consecutive claim-free years at level 25, for their first claim. It is waived for drivers who have had at least ten consecutive years at the 25 level, for their first claim. An extra deductible is enforced if the claimant is at a higher level than 80, prior to the claim.

1.4.17 Portuguese System

Table 1.21 Portuguese system

Class					
1	2	3	4	5	6
70	100	115	130	145	200
Premium Level (%)					

Starting level: 100

Claim-free: -1 after two-consecutive claim-free years

Each claim: +1

1.4.18 Spanish System

Table 1.22 Spanish system

Class				
1	2	3	4	5
70	80	90	100	100
Premium Level (%)				

Starting level: highest 100

Claim-free: -1

Each claim: all discounts lost

The use of this NCD system has now been discontinued by most insurers, as complete rating freedom now exists.

1.4.19 Swedish System

Table 1.23 Swedish system

Class						
1	2	3	4	5	6	7
25	40	50	60	70	80	100
Premium Level (%)						

Starting level: 100

Claim-free: -1. Level 25 is only awarded after 6 consecutive claim-free years.
Each claim: +2

A fixed premium of SEK 100 (about 10% of the average premium) is not affected by the NCD system.

1.4.20 Swiss System

Table 1.24 Swiss system

Class																					
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
45	50	55	60	65	70	75	80	90	100	110	120	130	140	155	170	185	200	215	230	250	270
Premium Level (%)																					

Starting level: 100
Claim-free: -1

Old system
Each claim: +3

New system (1990)
Each claim: +4

1.4.21 Taiwanese System

Table 1.25 Taiwanese system

Class								
1	2	3	4	5	6	7	8	9
50	65	80	100	110	120	130	140	150
Premium Level (%)								

Starting level: 100
Claim-free: -1 or to level 80, if more favourable
Claims: if k claims, to level 100+ 10k

1.4.22 Thai System

Table 1.26 Thai system

Class						
1	2	3	4	5	6	7
60	70	80	100	120	130	140
Premium Level (%)						

Starting level: 100

Claim-free: - 1 or to level 80, if more favourable

First claim: to level 100. Two or more claims: to level 120 or +1 (least favourable)

1.4.23 A Typical British System

Table 1.27 A Typical British system

Class						
1	2	3	4	5	6	7
33	40	45	55	65	75	100
Premium Level (%)						

Starting level: 75

Claim-free: -1

First claim: +3 (level 33), +2 (levels 40 and 45), +1. Subsequent claims: +2

As British insurers enjoy complete tariff structure freedom, many NCD system coexist. Many insurers have recently introduced "protected discount schemes" policyholders who have reached the maximum discount may elect to pay a surcharge, usually in the [10%-20%] range, to have their entitlement to discount preserved in case of a claim. More than two claims in five years result in disqualification from the protected discount scheme. Both the protected and unprotected forms are analysed (Lemaire, & Zi, 1994).

CHAPTER TWO
SIMULATION

2.1 Introduction

In different parts of the world within the automobile third-party liability insurance has consequently been made compulsory in most development countries, NCD system is used very different from each other. As described in 31 different countries in the first chapter. Therefore to different problem occur when deciding the most appropriate NCD system. The first evaluation of the system and its effects are compared to the second problem is the definition of the most appropriate NCD system (Lemaire, 1988).

We will consider the second as a problem. To identify the most appropriate NCD system we can use one of the optimization or simulation methods. Simulation can be a useful technique for solving quite difficult problems in general insurance. To solve this problem, we will use the method of simulation.

Use the method of simulation to assess the effects of to the change on the long-term claim outgo and premium income of the company and comment on the results.

2.2 Basic of Simulation

Simulation has had on-again, off-again history in actuarial practice. For example, in the 1970s, aggregate loss calculations were commonly done by simulation because the analytical methods available at the time were not adequate. However, the typical simulation often took a full day on the company's mainframe computer, a serious drag on resources. In the 1980s analytic methods such as Heckman-Meyers and the recursive formula were developed and were found to be significantly faster and more accurate. Today, desktop computers have sufficient power to run complex simulations that allow for the analysis of models not suitable for current analytic approaches (Klugman et all, 2004).

Simulation can be a very useful and powerful tool for studying problems in general insurance. It should not, however, be used indiscriminately. There are two main situations in which it may well be the best approach:

1. when the problem cannot be solved exactly mathematically;
2. when an exact mathematical solution is possible but extremely difficult and/or tedious, and adequate approximate results can be obtained quickly and simply by simulation.

Simulation should never be used when a simple, exact mathematical method is available (Hossack et all, 1983).

2.3 Generating Random Variable

The building block of a simulation study is the ability to generate random numbers, where a random number represents the value of a random variable uniformly distributed on (0, 1).

Whereas random numbers were originally either manually or mechanically generated, by using such techniques as spinning wheels, or dice rolling, or card shuffling, the modern approach is to use a computer to successively generate pseudo-random numbers. These pseudo-random numbers constitute a sequence of values, which, although they are deterministically generated, have all the appearances of being independent uniform (0, 1) random variables (Ross, 2003).

2.3.1 Generating Discrete Random Variables

However, there are several methods for generating discrete random variables, one method is presented because we will use only Poisson random variables as discrete random variable in ours application.

2.3.1.1 The Inverse Transform Method

Suppose we want to generate the value of a discrete random variable X having probability mass function.

$$P\{X = x_j\} = p_j, \qquad j = 0,1,\ldots, \quad \sum_j p_j = 1 \tag{2.1}$$

To accomplish this, we generate a random number U —that is, U is uniformly distributed over (0,1)— and set

$$X = \begin{cases} x_0 & if \quad U < p_0 \\ x_1 & if \quad p_0 \le U < p_0 + p_1 \\ \vdots & \\ x_j & if \quad \sum_{i=1}^{j-1} p_i \le U < \sum_{i=1}^{j} p_i \\ \vdots & \end{cases} \tag{2.2}$$

Since, for $0 < a < b < 1$, $P\{a \le U < b\} = b - a$, we have that

$$P\{X = x_j\} = P\left\{\sum_{i=1}^{j-1} p_i \le U < \sum_{i=1}^{j} p_i\right\} = p_j \tag{2.3}$$

where $x_1, x_2, x_3 \ldots$ are the possible values X can be take on

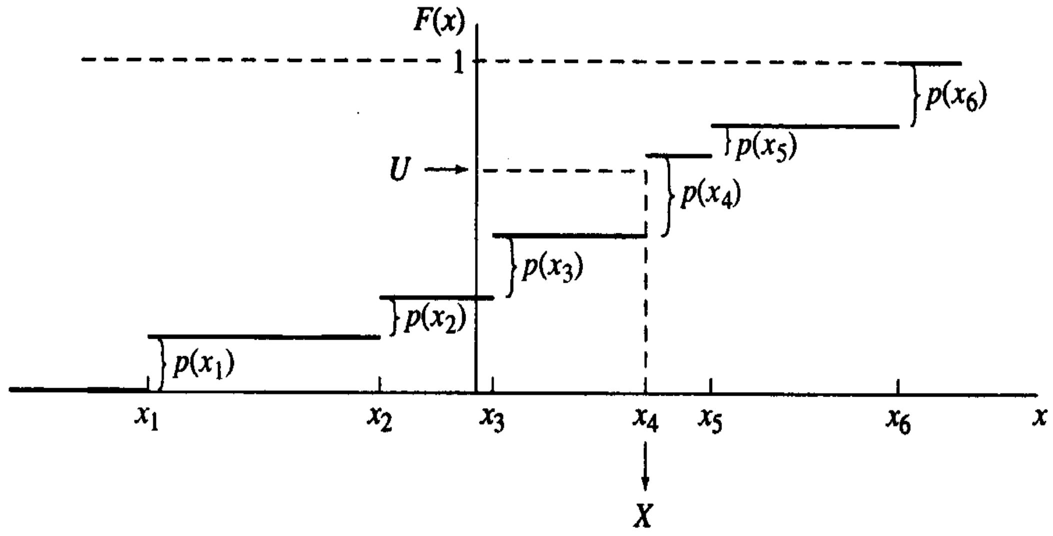

Figure 2.1 Cumulative distribution functions

and so X has the desired distribution.

The above can be written algorithmically as;

1. Generate a random number U
2. If $U < p_0$ set $X = x_0$ and stop
3. If $U < p_0 + p_1$ set $X = x_1$ and stop
4. If $U < p_0 + p_1 + p_2$ set $X = x_2$ and stop

   ⋮

If the x_i, $i \geq 0$, are ordered so that $x_0 < x_1 < x_2 \cdots$ and if we let F denote the distribution function of X, then $F(x_k) = \sum_{i=0}^{k} p_i$ and so X will equal x_j *if* $F(x_{j-1}) \leq U < F(x_j)$

In order words, after generating a random number U we determine the value of X by finding the interval $(F(x_{j-1}), F(x_j))$ in which U lies. It is for this reason that the above is called the discrete inverse transform method for generating X (Ross, 1990).

2.3.1.1.1. Generating a Poisson Random Variable: The random variable X is Poisson with mean λ if

$$p_i = P\{X = i\} = e^{-\lambda}\frac{\lambda^i}{i!} \qquad i = 0,1,\ldots \tag{2.4}$$

The key to using the inverse transform method to generate such a random variable is the following identity

$$p_{i+1} = \frac{\lambda}{i+1}p_i, \qquad i \geq 0 \tag{2.5}$$

Upon using the above recursion to compute the Poisson probabilities as they become needed, the inverse transform algorithm for generating a Poisson random variable with mean λ can be expressed as follows steps.

1. Generate a random number *U*.
2. $i = 0, \quad p = e^{-\lambda}, \quad F = p$.
3. If $U < F$, set $X = i$ and stop.
4. $p = \lambda p/(i+1)$, $F = F + p$, $i = i+1$.
5. Go to step 3.

The above algorithm successively checks whether the Poisson value is 0 then whether it is 1, then 2, and so on. Thus the number of comparisons needed will be 1 greater than the generated value of the Poisson (Ross, 1990).

2.3.2 Generating Continuous Random Variables

Two methods are presented for normal random variables and lognormal random variables.

2.3.2.1 Generating Normal Random Variables

2.3.2.1.1 Box – Muller Method: Let X and Y be independent unit normal random variables and let R and θ denote the polar coordinates of the vector (X, Y). That is (see Figure 2.2),

$$R^2 = X^2 + Y^2 \tag{2.6}$$

$$\tan\theta = \frac{Y}{X} \tag{2.7}$$

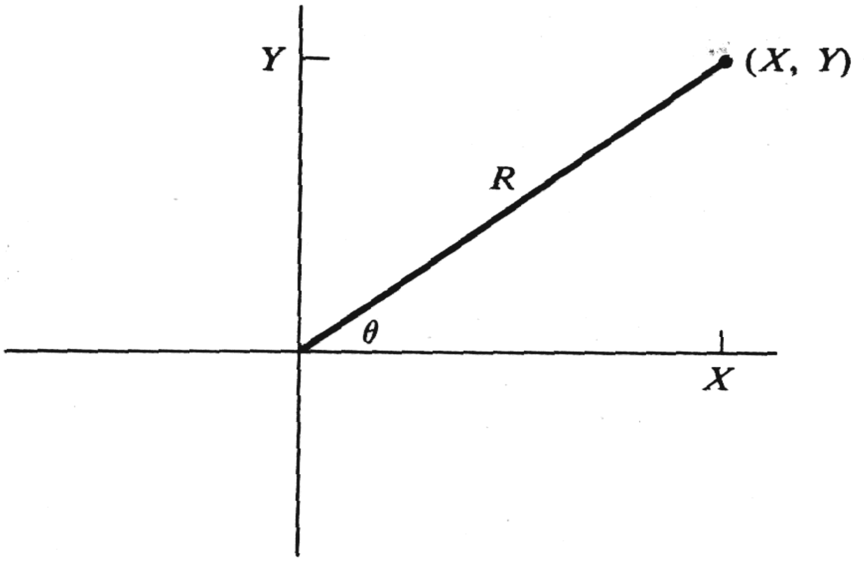

Figure 2.2 The polar coordinates of the vector *(X, Y)*.

Since *X* and *Y* are independent, their joint density is the product of their individual densities and is thus given by

$$f(x,y) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \quad = \frac{1}{2\pi} e^{-(x^2+y^2)/2} \tag{2.8}$$

To determine the joint density of R^2 and θ— call it $f_{R^2,\theta}(d,\theta)$— we make the change of variables

$$d = x^2 + y^2 \quad , \quad \theta = \tan^{-1}\left(\frac{y}{x}\right) \tag{2.9}$$

As the Jacobian of this transformation —that is, the determinant of partial derivatives of d and θ with respect to x and y— is easily shown to equal 2, it follows from Equation (2.8) that the joint density function of R^2 and θ is given by

$$f_{R^2,\theta}(d,\theta) = \frac{1}{2}\frac{1}{2\pi}e^{-d/2}, \qquad 0 < d < \infty, \quad 0 < \theta < 2\pi \tag{2.10}$$

However, as this equal to the product of an exponential density having mean 2 (namely, $\frac{1}{2}e^{-d/2}$) and the uniform density on (0, 2π) (namely, 1/2π), it follows that

R^2 and θ are independent, with R^2 being exponential with mean 2 and θ being uniformly distributed over (0, 2π) (2.11)

We can now generate a pair of independent normal random variables X and Y by using (2.11) to first generate their polar coordinates and then transforming back to rectangular coordinates. This is accomplished as follows:

STEP 1: Generate random numbers U_1 and U_2.

STEP 2: $R^2 = -2\log U_1$ (and thus R^2 is exponential with mean 2).

STEP 3: Now let

$$\begin{aligned} X &= R\cos\theta = \sqrt{-2\log U_1}\,\cos(2\pi U_2) \\ Y &= R\sin\theta = \sqrt{-2\log U_1}\,\sin(2\pi U_2) \end{aligned} \tag{2.12}$$

The transformations given by Equations (2.12) are known as the Box–Muller transformations (Ross, 1990).

2.3.2.1.2 Polar Method: The use of the Box–Muller transformations (2.12) to generate a pair of independent unit normal is computationally not very efficient: the reason for this being the need to compute the sine and cosine trigonometric functions. There is, however, fortuitously a way to get around this time-consuming difficulty by an indirect computation of the sine and cosine of a random angle (as opposed to a direct computation which generates U and then computes the sine and cosine of 2πU). To begin, note that if U is uniform on (0, 1) then 2U is uniform on (0, 2) and so 2U – 1 is uniform on (-1, 1). Thus, if we generate random numbers U1 and U2 and set,

$$\begin{aligned} V_1 &= 2U_1 - 1 \\ V_2 &= 2U_2 - 1 \end{aligned} \tag{2.13}$$

then (V_1, V_2) is uniformly distributed in the square of area 4 centred at (0, 0) — see Figure 2.3.

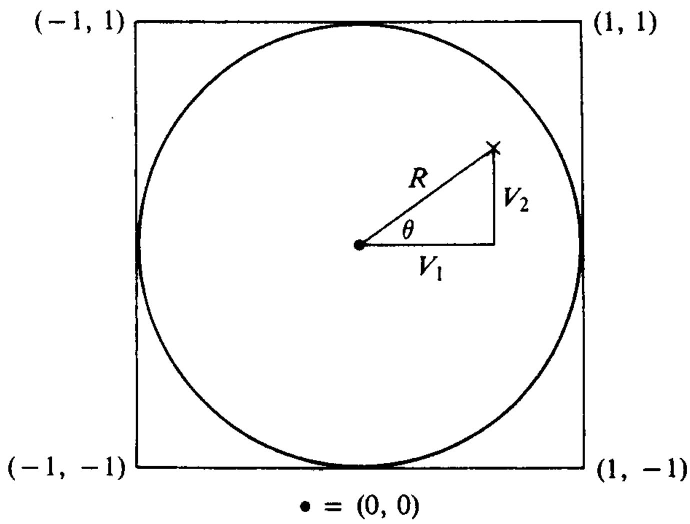

Figure 2.3.The circle of radius 1 centred at (0, 0)

Suppose now that we continually such pairs (V_1, V_2) until we obtain one that is contained in the circle of radius 1 centred at (0, 0)— that is, until (V_1, V_2) is such that $V_1^2 + V_2^2 \le 1$. It now follows that such a pair (V_1, V_2) is uniformly distributed in the circle. If we let R and θ denote the polar coordinates of this pair then it is not difficult to verify that R and θ are independent, with R^2 being uniformly distributed on (0, 1) and with θ being uniformly distributed over (0, 2π). Since θ is thus a random angle, it follows that we can generate the sine and cosine of a random angle θ by generating a random point (V_1, V_2) in the circle and then setting

$$\sin\theta = \frac{V_2}{R} = \frac{V_2}{\left(V_1^2 + V_2^2\right)^{1/2}} \tag{2.14}$$

$$\cos\theta = \frac{V_1}{R} = \frac{V_1}{\left(V_1^2 + V_2^2\right)^{1/2}} \tag{2.15}$$

It now follows from the Box-Muller transformation (2.12) that we can generate independent unit normals by generating a random number U and setting

$$X = \left(-2\log U\right)^{1/2} \frac{V_1}{\left(V_1^2 + V_2^2\right)^{1/2}} \tag{2.16}$$

$$Y = \left(-2\log U\right)^{1/2} \frac{V_2}{\left(V_1^2 + V_2^2\right)^{1/2}} \tag{2.17}$$

In fact, since $R^2 = V_1^2 + V_2^2$ is itself uniformly distributed over (0,1) and is independent of the random angle θ, we can use it as the random number U needed in Equations (2.16) and (2.17). Therefore, letting $S = R^2$, we obtain that

$$X = \left(-2\log S\right)^{1/2} \frac{V_1}{S^{1/2}} = V_1 \left(\frac{-2\log S}{S}\right)^{1/2} \tag{2.18}$$

$$Y = \left(-2\log S\right)^{1/2} \frac{V_2}{S^{1/2}} = V_2 \left(\frac{-2\log S}{S}\right)^{1/2} \tag{2.19}$$

are independent unit normals when (V_1, V_2) is a randomly chosen point in the circle of radius 1 centred at the origin, and $S = V_1^2 + V_2^2$.

Summing up we thus have the following approach to generating a pair of independent unit normals:

STEP 1: Generate random numbers U_1 and U_2.

STEP 2: Set $V_1 = 2U_1 - 1$, $V_2 = 2U_2 - 1$, $S = V_1^2 + V_2^2$

STEP 3: If S>1 return to step 1.

STEP 4: Return the independent unit normals

$$X = \sqrt{\frac{-2\log S}{S}}\, V_1 \tag{2.20}$$

$$Y = \sqrt{\frac{-2\log S}{S}}\, V_2 \tag{2.21}$$

The above is called the polar method. Since the probability that a random point in the square will fall within the circle is equal to $\pi/4$ (the area of the circle divided by the area of the square), it follows that, on average and the polar method will require $4/\pi = 1.273$ iterations of Step 1. Hence it will, on average require 2.546 random numbers, 1 logarithm, 1 square root, 1 division, and 4.546 multiplications to generate two independent unit normals.

CHAPTER THREE
APPLICATION

3.1 Introduction

Motor insurance and motor liability insurance constitutes an important part of non-life insurances and the number of vehicles is also increasing with years. While the premium income decreases in Turkey and in the world, as opposed to this the increase in claim outgo causes the insurance companies lose money in motor insurance and motor liability insurance. If measures are not taken, at the end of the long-term insurance companies in the sector is expected to damage.

In our application, by using the simulation methods we will obtain the long-term results of the No Claim Discount (NCD) system similar to NCD system in Turkey, which is the most preferred motor insurance system. According to obtained results, we will perform various simulations for deciding the most appropriate NCD system.

By using the simulation method, we will obtain the total claim outgo and the premium income by the policyholders for the desired insured period. Due to the claim number distribution models that were described in first chapter, we will assume that the policyholders in our system are homogeneous with regards to risk, and in the light of this assumption we will use the poisson model. We will also assume that for any given claim, distribution of amount of claims is lognormal. From the poisson distribution, the poisson random variables for number of claims; and from the lognormal distribution, the lognormal random variables for amount of claims will be generated for the desired insured period as described in the second chapter.

In our system, for each policyholder and for each insured period we can separately generate number of claims and amount of claims. For any policyholder according to the generated amount of claims, the generated number of claims can be declared to the insurance company or cannot. We will assume that when deciding the minimum claim sizes policyholders are take into account according to NCD premium level.

According to this statement at the end of the insured period, the policyholders that declared a claim and that did not declared a claim take place in discount classes under the framework of the transition rules of the NCD system and in the next period they pay the amount of the new premium level.

For the desired period of time, number of claims and amount of claims of policyholders, number of claimants, number of policyholders in each discount class, premium income and claim outgo for each period can be derived from the simulation program that is written in MATLAB. By running our simulation program with different variables and by comparing the obtained results, we can choose the optimum NCD system for the policyholders.

3.2 Definition of the No Claim Discount System (NCD)

In this application, we will use the data of the motor insurance holders in Turkey during the year 2008. The total premium income, the total policy numbers, and the average premium amounts between 01.01.2008 - 31.06.2008 period in Turkey can be seen in Table 3.1. Between the specified dates, the total number of treated personal automobile insurance policies is 2,087,624, the total premium amount is 1,613,789,088 TL, and the average premium is 773 TL. In our application we will only use data of the personal automobiles and we will assume that the average amount of the increase in the premium amount will approximately be %1 for the next year and the amount of average premium is considered to reach 850 TL (Association of Insurance and Reinsurance Companies of Turkey, 2008).

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Table 3.1 Total premium income, the total policy numbers and average premium according to motor vehicle types in Turkey between the 01/01/2008 – 31/12/2008

Motor vehicles	**Policy numbers**	**Premium income**	**Average premium**
Personal Automobile	2,087,624	1,613,789,088	773
Taxi	9,732	12,032,585	1,236
Minibus	102,089	76,076,017	745
Small bus	50,896	44,423,901	873
Big bus	18,852	69,747,816	3,700
Van	789,003	533,540,854	676
Truck	177,629	267,446,858	1,506
Business machine	4,047	5,189,293	1,282
Tractor	75,189	21,408,516	285
Trailers	45,828	43,426,485	948
Motorcycle	10,368	7,922,278	764
Tanker	9,317	12,732,284	1,367
Tow truck	54,733	109,732,362	2,005
Special-purpose automobile	1,813	2,068,766	1,141
Others	17,165	22,493,965	1,310

In this application we will assume that all the policyholders will have the same exposure of risk and by this way our system will have a homogenous portfolio poisson model. We will start the system with 1000 policyholders and any intervention from outside of the system will not be accepted during or at the end of the insurance periods. There will not be any insured entry outside the system and there will not be any decrease in the number of insured available in the system. We will ignore the problem of expenses, and assume that all the 1000 policyholders remain with the company. Insurance period will be considered as a year.

Let's consider the NCD system which has 4 discount classes. In the system, the initial class is 0 and the best class is 4. In the table 3.2 below are the discount classes and the amount of the premium. In class 1, the policyholder pays only 80% of the full premium, in class 2 only 60% and in class 3 only 40% of the full premium. During insurance period, policyholders take place in discount classes under the framework of transition rules of NCD system.

Table 3.2 NCD System discount classes

Class	Premium level (%)	Premium
0	0	850 TL
1	20	680 TL
2	40	510 TL
3	60	340 TL

The NCD system of transition rules;

i. if a policyholder makes no claim in a year, he or she moves one position forward to the next higher discount class (or stays in class 3)

ii. if a policyholder makes one or more claim in a year, he moves one position back to the lower discount class (or stays at initial class) the following year.

We assume that all policyholders will be logged into the NCD system from initial class 0 and they will pay 850 TL for first year. At the end of one year, policyholders who make one or more claim will stay at initial class and will still pay 850 TL; policyholders who make no claim will move from class 0 to class 1 and will pay 680 TL.

3.2.1 Number of Accident and Amount of Damage

We assume that 20 of every 100 policyholders that are homogeneous with regards to risk make a claim in a year. By this way, to this the number of claims in a given year is poisson distributed with parameter $\lambda = 0.2$. We will assume that the average amount of claims for the accidents of each policyholder is $\overline{X} = 2,000\,TL$ and the standard deviation of the claims is $V(X) = 4,000\,TL$ and they have a lognormal distribution. Using this information, the mean and the variance of lognormal distribution can be calculated as below.

$$\overline{X} = \exp(\mu + \frac{1}{2}\sigma) = 2.000 \tag{3.1}$$

$$V(X)^2 = \exp(2\mu + \sigma^2)\,[\exp(\sigma^2) - 1] = (4.000)^2 \tag{3.2}$$

$$\overline{X} = \exp(\sigma^2) - 1 = (4.000/2.000)^2 \tag{3.3}$$

$$\sigma^2 = 1,61 \tag{3.4}$$

$$\mu = 6,795 \tag{3.5}$$

With parameters $\mu = 6.795$ and $\sigma^2 = 1.61$, the amount of claims is obtained with Lognormal distribution.

3.2.2 Minimum Claim Sizes According to NCD System Level

If a policyholder in the class 0 makes a claim, his premium for the following year will be 850 TL instead of 680 TL in the first year. On the other hand, if his claim amount is less than 170 TL, it is for his own interests to make a claim. Next year the policyholders decide to make a claim if the amount involved is less than 340 TL that is 170 TL for the first year and 170 TL for the second year.

By thinking the following years discount amount of the insurance premium and the discount percentage of his class, the insured, who made an accident, may not declare the accident. We assume that the policyholders decide to make a claim or not according to minimum claim sizes. If the amount of the claim is less than the minimum claim size, the policyholders will make no a claim. If the amount of claim is greater than the minimum claim size, then the policyholders will make a claim. The measured minimum claim sizes for this system are given in Table 3.3.

Table 3.3 Minimum claim sizes

Premium level (%)	Premium	Minimum claim sizes		
		Year 1	Year 2	Year 3
0	850TL	170TL	340TL	510TL
20	680TL	340TL	680TL	850TL
40	510TL	340TL	510TL	510TL
60	340TL	170TL	170TL	170TL

We firstly assume that the policyholder has one-year time horizon. He will compare the size of the loss with the increase in premium the following year if a claim is made, and only make a claim if the latter is smaller than the former (Hossack, Pollard, & Zehnwirth, 1983). A policyholder in class 0 (no discount) will make a claim if the loss exceeds 170 TL. For discount class 1, class 2 and class 3, the minimum losses to make claiming worthwhile are 340 TL, 340 TL and 170 TL. These figures are shown in Figure 3.1. For 3 years minimum claim sizes of NCD system for 4 classes is calculated. Minimum claim sizes in subsequent years are calculated the same as third year. The minimum claim sizes for NCD system discount level has been circled in the Figure 3.1.

Determining minimum claim sizes we assume that the policyholders will make no claim for subsequent year and no influence of inflation.

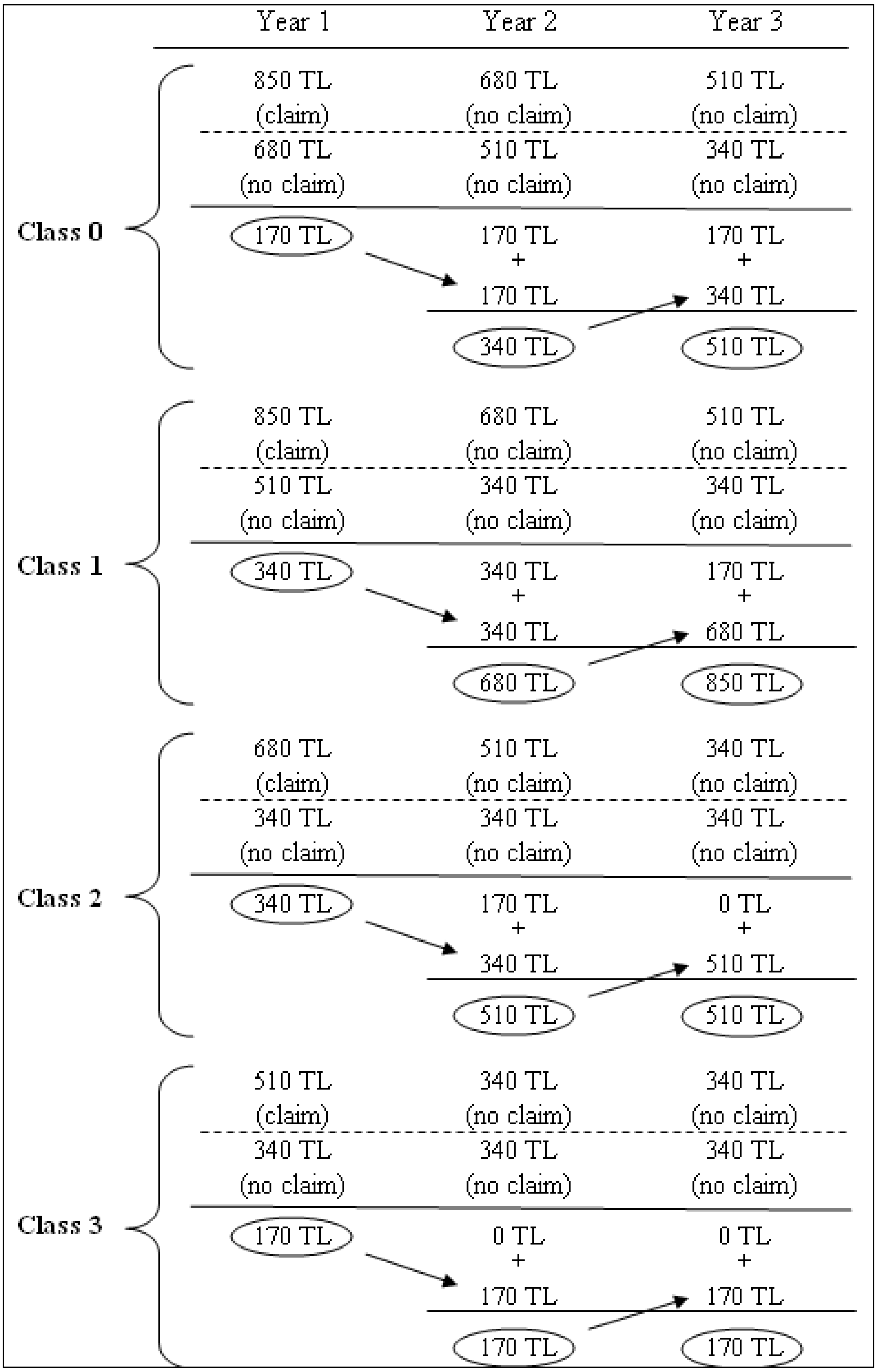

Figure 3.1 Calculation of minimum claim sizes

3.3 Simulation Program of the NCD System

Simulation program of the NCD system was written in MATLAB and is 450 lines. Besides, nine-line programs were written for each graphical illustration. When the simulation program is initiated, it asks for the user to enter the number of policyholders that will take place in the system, the amount of years that the simulation will process, the claim frequency λ and the parameters of the amount of claim distribution (μ,σ). Number of claims of each insured is generated by the poisson distribution with "poissrnd(λ)" that is one of the MATLAB functions written by us. According to these generated claim numbers, with the lognormal distribution that has the parameters μ and σ, the amount of claim is calculated separately for each claim and this is done by the written MATLAB function, "lognormal (μ , σ)". For each year a random variables for the number of claims and the amount of claims are generated separately. In the second section, the methods to generate random data are mentioned. (Matlab Programming Version 7, 2006)

Actions are repeated separately for many years as the number of discount classes on the system. Since each discount category can be reached in years equal to the number of discount classes, system becomes stable and for the subsequent years a recursion is generated. This recursion provides the program to continue until the desired years.

The minimum claim sizes that are calculated according to the transition rules of the NCD system are compared with the claim amount of each policyholder, who made an accident, and the decision of whether to declare the claim of the insured or not is given. At the end of each year, the base on the transition rules, the insured takes his place in a discount class considering whether he declared the claim or not. When the program ends for each year, the number of all the claims, the number of declared claims, the number of undeclared claims, the total of the premium incomes, the total amount of all the claims, the amount of declared claims, the amount of undeclared claims and the distribution of the policyholders in the discount classes are obtained.

Simulation program runs throughout the amount of desired years. The data obtained as a result of the simulation are assessed and used to decide whether the system is appropriate. By using different variables, such as claim frequency λ, parameter of amount of claims distribution (μ, σ) and transition rules, we try to reach the optimum NCD system. We can see the simulation and graphics programs in the appendix.

The output obtained from simulation program is demonstrated in Figure 3.2 for 1^{st}, 10^{th} and 40^{th} years. We are written simulation program in Turkish for the sake of simplicity.

```
Simulasyon  Programı...
Simulasyonda Kac Kisi Yer Alsin: 1000
Simulasyon Kac Yil Sursun: 40
Poisson Dagiliminin Ortalamasi Kac Olsun: 0.2
Log.normal Dagilimin Ortalamasi Kac Olsun: 6.975
Log-normal Dagilimin Standart Sapmasi Kac Olsun: 1.268


1. Yil Sonuçlari
-----------------------
Tüm hasarlar toplami: 199
Beyan edilen hasar sayisi: 190
Beyan edilmeyen hasar sayisi: 810
Prim gelirleri toplami: 850000
Tüm hasarlarin toplam miktari: 384670
Beyan edilen hasarlarin miktari: 383638
Beyan edilmeyen hasarlarin miktari: 1032
______________________________________________________________
%0. Grup KS    %20. Grup KS    %40. Grup KS    %60.Grup KS
..............................................................
     190            810              0               0
______________________________________________________________


10. Yil Sonuçlari
-----------------------
Tüm hasarlar toplami: 202
Beyan edilen hasar sayisi: 178
Beyan edilmeyen hasar sayisi: 822
Prim gelirleri toplami: 388620
Tüm hasarlarin toplam miktari: 513962
```

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Beyan edilen hasarlarin miktari: 508587
Beyan edilmeyen hasarlarin miktari: 5375
_______________________________________________________________
%0. Grup KS      %20. Grup KS      %40. Grup KS      %60.Grup KS
...............................................................
     8                32                179               781
_______________________________________________________________


40. Yil Sonuçlari
----------------------
Tüm hasarlar toplami: 202
Beyan edilen hasar sayisi: 185
Beyan edilmeyen hasar sayisi: 815
Prim gelirleri toplami: 379440
Tüm hasarlarin toplam miktari: 513962
Beyan edilen hasarlarin miktari: 489397
Beyan edilmeyen hasarlarin miktari: 4306
_______________________________________________________________
%0. Grup KS      %20. Grup KS      %40. Grup KS      %60.Grup KS
...............................................................
     4                31                176               789
_______________________________________________________________
```

Figure 3.2 Output

3.4 NCD System Simulation

Distribution of every policyholder in NCD system to discount classes takes 5 years because we assume that 1000 policyholders in the NCD system mentioned above enters into the system from beginning class. We should analyze long term system results in order to decide optimum NCD system. We will make 40 years of simulation for this and will ignore the inflation effect for 40 years.

We will examine 40 years of simulation that is obtained from NCD system in the light of existing assumptions and decide whether our NCD system is an optimum NCD system. We will constantly make simulations in the system by doing necessary editions until reaching optimum NCD system.

Simulation results obtained according to existing NCD system are demonstrated in the table and graphics below.

Table 3.4 Number of claims and number of claimants for the first five years

Year	Number of claims	Number of claimants
1	199	190
2	218	146
3	190	125
4	198	164
5	194	163

In Table 3.4, we see that 199 people from 1000 people have accident, 190 of those people make a claim because their claim amount is more than 170TL which is the minimum claim size calculated for 1 year. 190 people who make a claim stay in the same category in the following year. 810 people who do not make a claim have the right to move to one class high discount category.

In the second year, 190 people in first discount class, 218 of 810 people in second discount class had accident and 146 people made a claim. At the end of second year, from 218 people ones who take place in the first category class and have claim amount above 340 TL make a claim to insurance company and stay in the first class. People of second discount class who have more than 680TL claim amount make a claim and falls from second discount class to the first. This system goes on by years. During 40 years, we can obtain number of claims and number of claimants of all years.

In Figure 3.3, we see that claim numbers vary between 175 and 235. 18th and 33rd years are years of highest claim numbers. Also, number of claims is in maximum level between these years. For a great number of claims was made.

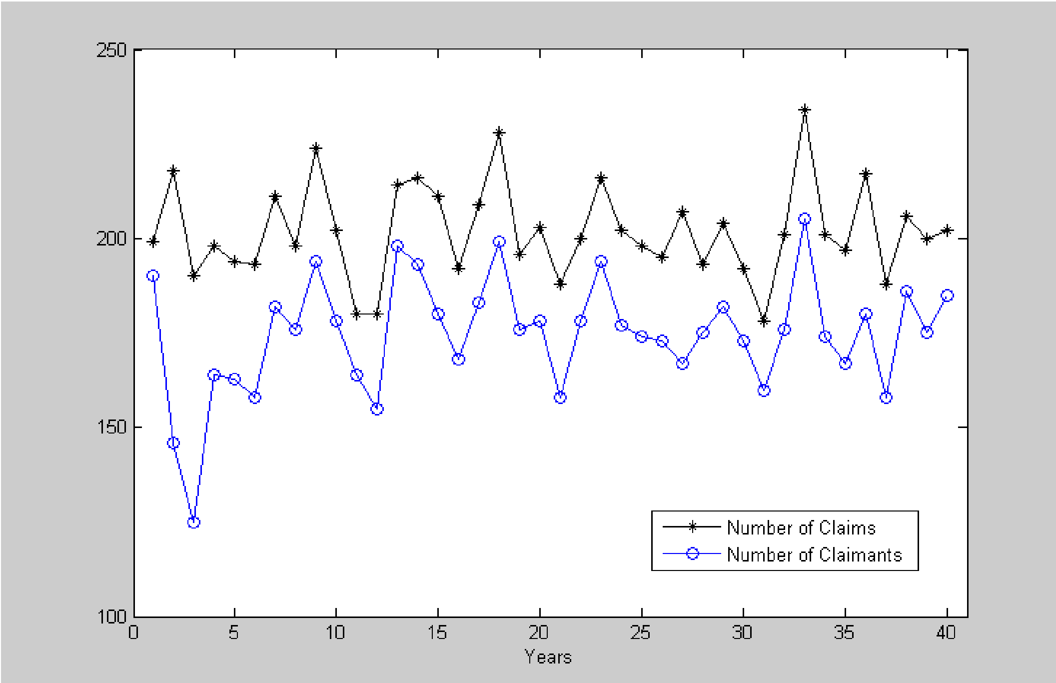

Figure 3.3 Number of claims and number of claimants for the 40 years

In Table 3.5, we can see the five years distribution of policyholders in discount categories. Our simulation program can show 40 years distribution of policyholders in discount categories. Total number of policyholders in all discount categories does not change at the end of every year because there will be no entrance or exit in the system.

Table 3.5 The distribution of policyholders in the various discount classes for the first five years

Discount classes	Year 1	Year 2	Year 3	Year 4	Year 5
0%	190	146	36	28	8
20%	810	159	215	50	74
40%	0	695	143	309	151
60%	0	0	606	613	767

Total claim amount of 199 who had an accident in the first year is 384,670 TL. Total claim amount of 190 people who made a claim is 383,638 TL. This is our one year claim outgo. First year premium income of 1000 people will be 850.000 TL with the calculation of 1000x850TL. Total claim amount, claim outgo and premium incomes of first five years are demonstrated in Table 3.6; claim outgo and premium incomes of other years are demonstrated in Figure 3.4.

Table 3.6 Amount of claims, claim outgo and premium income for the first five years (TL)

Years	Amount of claims	Claim outgo	Premium income
1	384,670	383,638	850,000
2	476,726	452,956	712,300
3	373,530	350,404	586,670
4	445,122	369,788	455,770
5	376,815	369,560	423,810

In Table 3.6 and Figure 3.3, it is observed that premium incomes decrease by years and it varies between 375,000 and 400,000 after first five years. Claim outgo shows a decrease and increase between 350,000 and 650,000. They show an unstable fluctuation as opposed to premium incomes.

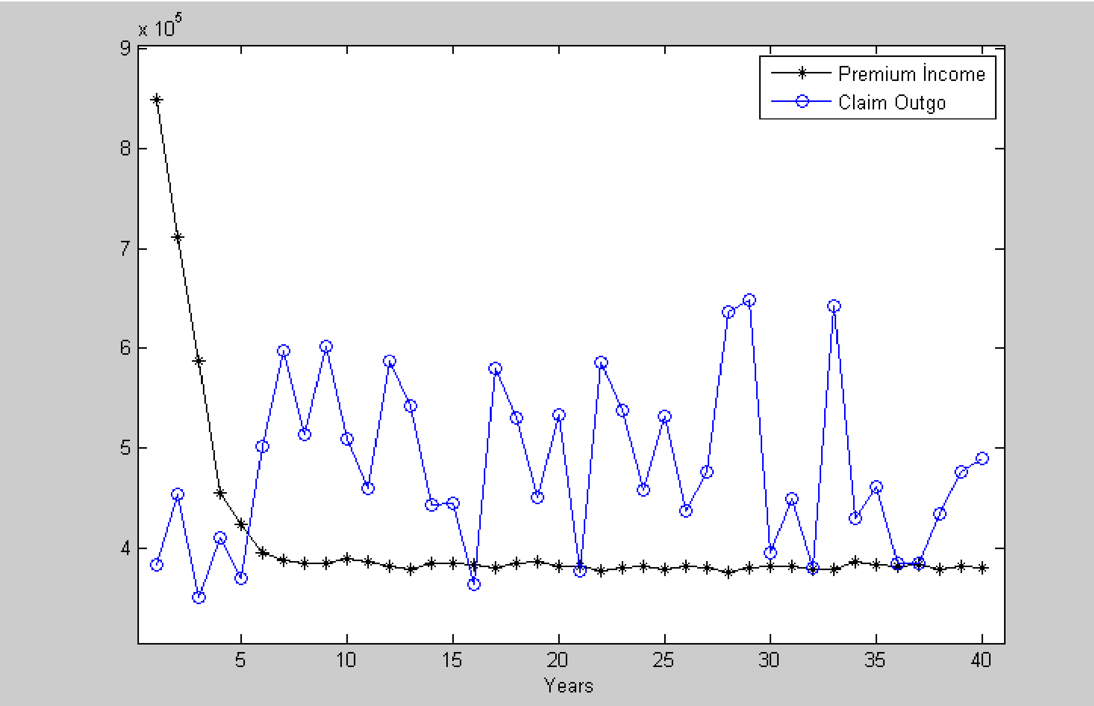

Figure 3.4 Claim outgo and premium income for the 40 years (TL)

Almost every year, claim outgo is more than premium incomes except first five years. It is clear that this system will make a loss.

Loss premium ratio, which comprises claims outgo divided by premium income. Profitability is calculated by subtracting claim outgo from premium income. While profitability and loss premium ratio are calculated, operational expenses are not taken into account. Taking 40 years total premium incomes and total claim outgo into account profitability and average loss premium ratio of these 40 years are found. As a result, profitability is -2,830,655 TL and loss premium ratio is 117%. Loss premium ratio is quite high. This NCD system is not suitable for the existing portfolio.

In order to reach optimum NCD system, we can change current transition rules in the system or increase premium amount. Firstly, let’s try to reach the optimum system by changing transition rules in the system. This change will provide that policyholders make fewer claims and consequently claim outgo diminish.

3.5 NCD System Simulation with Different Transition Rules

We have changed transition rules to be more deterrent and to encourage more careful driving.

The NCD system of new transition rules;

i. If a policyholder makes one or more claim in a year, he moves back to the initial discount class instead of moving one position back to the lower discount class on the discount classes (or stay at initial class) the following year.

ii. In the same way again if a policyholder makes no claim in a year, he or she moves one position forward to the next higher discount class (or stay in class 3).

According to this new transition rule, there will be change in minimum claim sizes. Minimum claim sizes that are calculated according to new transition rules are as in Table 3.7.

Table 3.7 Minimum claim sizes

Premium level discount (%)	Premium	Minimum claim sizes		
		Year 1	Year 2	Year 3
0	850TL	170TL	340TL	510TL
20	680TL	340TL	680TL	850TL
40	510TL	510TL	850TL	1020TL
60	340TL	510TL	850TL	1020TL

Minimum claim sizes in Table 3.7 are higher than the minimum claim sizes of previous NCD system which appears in Table 3.3. Policyholders are more deterrent in terms of making a claim and more encouraging in the subject of careful driving.

Let's make simulation with same claim amount parameters in the portfolio of previous NCD system which has same features according to new transition rules. We changed minimum claim sizes in the existing simulation program and arranged the transitions between discount classes according to new transition rules. Total claim number and claimant's number of first five years are demonstrated in Table 3.8. By years, total claim number and claimant's number are in Figure 3.4.

Table 3.8 Number of claims and number of claimants for the first five years

Year	Number of claims	Number of claimants
1	187	173
2	184	138
3	185	107
4	192	102
5	197	102

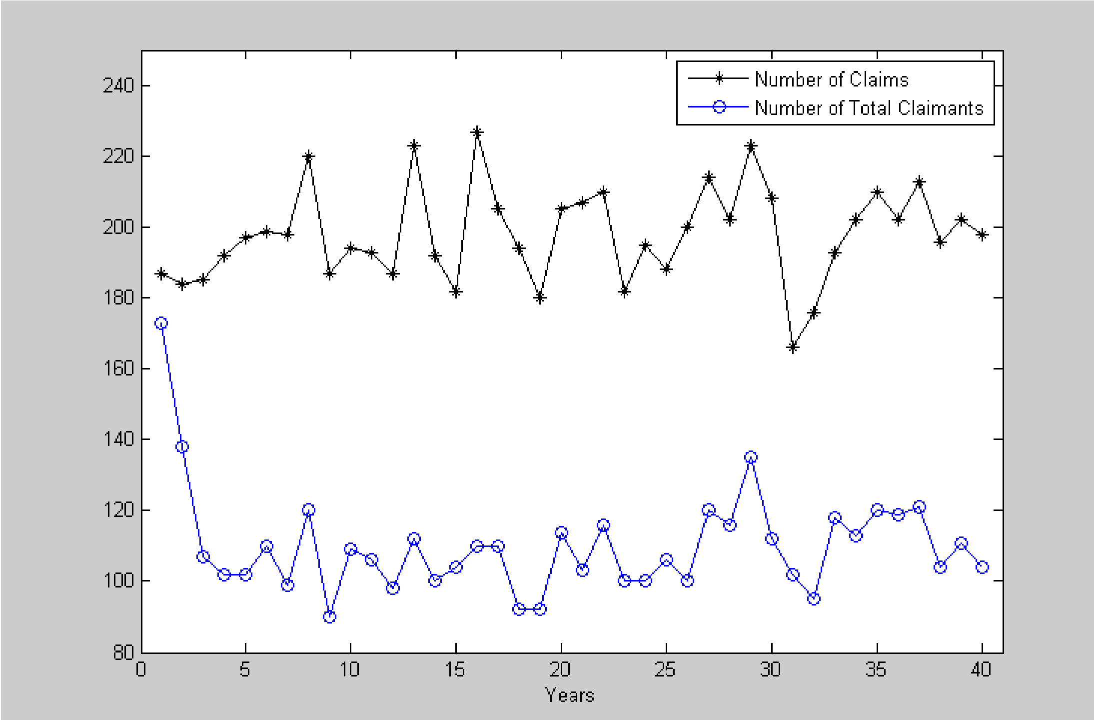

Figure 3.5 Number of claims and number of claimants for the 40 years

Influence of changes made in transition rules on number of people who make claim is observed in Figure 3.5. While claim numbers almost stay same, number of policyholders who make claim decreased.

Table 3.9 The distribution of policyholders in the various discount classes for the first five years

Discount classes	Year 1	Year 2	Year 3	Year 4	Year 5
0%	173	138	107	102	102
20%	827	140	116	107	90
40%	0	722	125	95	95
60%	0	0	652	696	713

This change in transition rules encourages policyholders not to make a claim. Distribution of policyholders among discount classes in the first five years is demonstrated in Table 3.9.

Table 3.10 Amount of claims, claim outgo and premium income for the first five years (TL)

Years	Amount of claims	Claim outgo	Premium income
1	419,226	417,720	850,000
2	493,258	477,374	709,410
3	395,694	347,254	580,720
4	400,250	366,152	455,260
5	480,059	439,450	444,550

In Table 3.10, total claim number and claim number for which claim is made in the first five years is shown; in Figure 3.6 claim outgo and premium incomes of all years is demonstrated. While profitability is -2,830,655 TL in the first NCD system, it is 1,147,419 TL in the system whose transition rules were changed. Loss premium ratio is 94%. There is a decrease in claim outgo but increase in premium incomes is not enough.

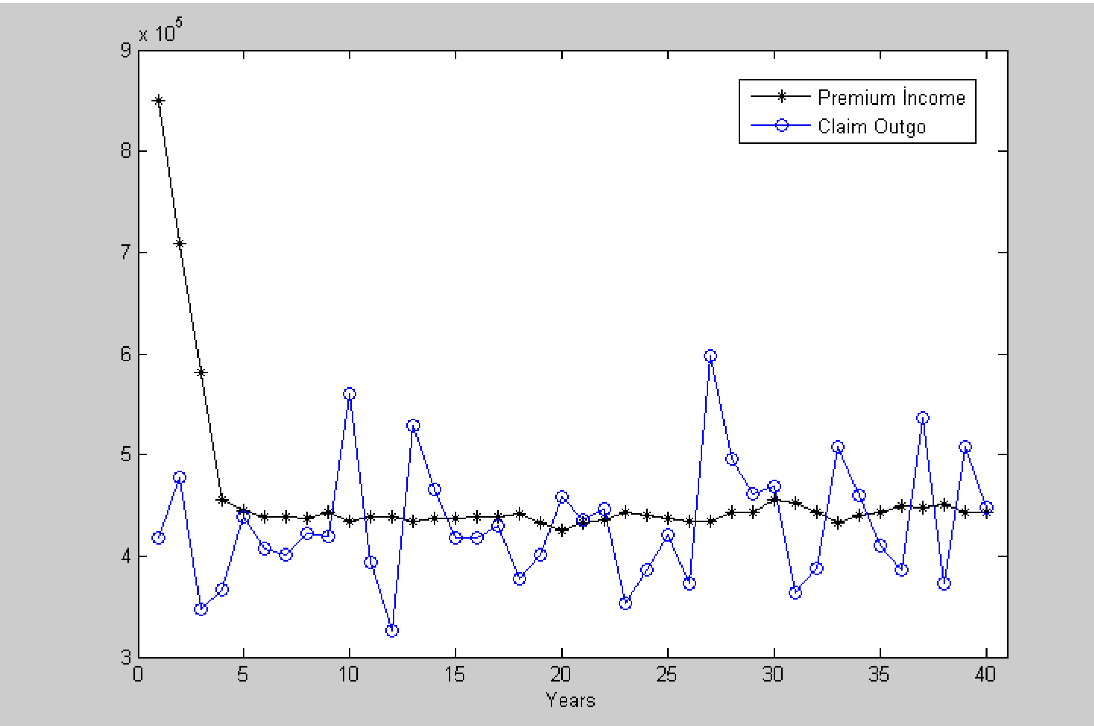

Figure 3.6 Claim outgo and premium income for the 40 years

Optimum system is not reached. We never want claim outgo to be higher than premium income. 15 years claim outgo is higher than premium incomes even though for almost half of the claims were made due to the changes in transition rules. Since the premiums are still very low.

3.6 The NCD System Simulation of Increased Premium

Optimum NCD system is not reached after the changes in transition rules in existing NCD system. As mentioned in the first chapter, in Turkey after 2006, sectoral improvement was enabled by increasing premiums in motor insurance. Let's examine 40 years system results by increasing premiums in our NCD system in order to reach optimum system. We will increase 850 TL initial premium amount to 1.500 TL. In Table 3.11, new minimum claim sizes which are calculated according to the increased premium amount.

Table 3.11 Minimum claim sizes

Premium level discount (%)	Premium	Minimum claim sizes		
		Year 1	Year 2	Year 3
0	1,500 TL	300 TL	600 TL	900 TL
20	1,200 TL	600 TL	1,200 TL	1,500 TL
40	900 TL	600 TL	900 TL	900 TL
60	600 TL	300 TL	300 TL	300 TL

We up date simulation program in parallel with new minimum claim sizes which were calculated according to increased premium amount. Simulation results obtained at the end of 40 years are demonstrated in the tables and figures below.

Table 3.12 Number of claims and number of claimants for the first five years

Year	Number of claims	Number of claimants
1	199	157
2	220	93
3	196	84
4	199	134
5	197	148

In Table 3.12 and Figure 3.7, number of claims and number of people who make a claim take place. Increase in premium amount decreased the number of claims made by policyholders although it does not decrease the number as much as the changes in transition rules.

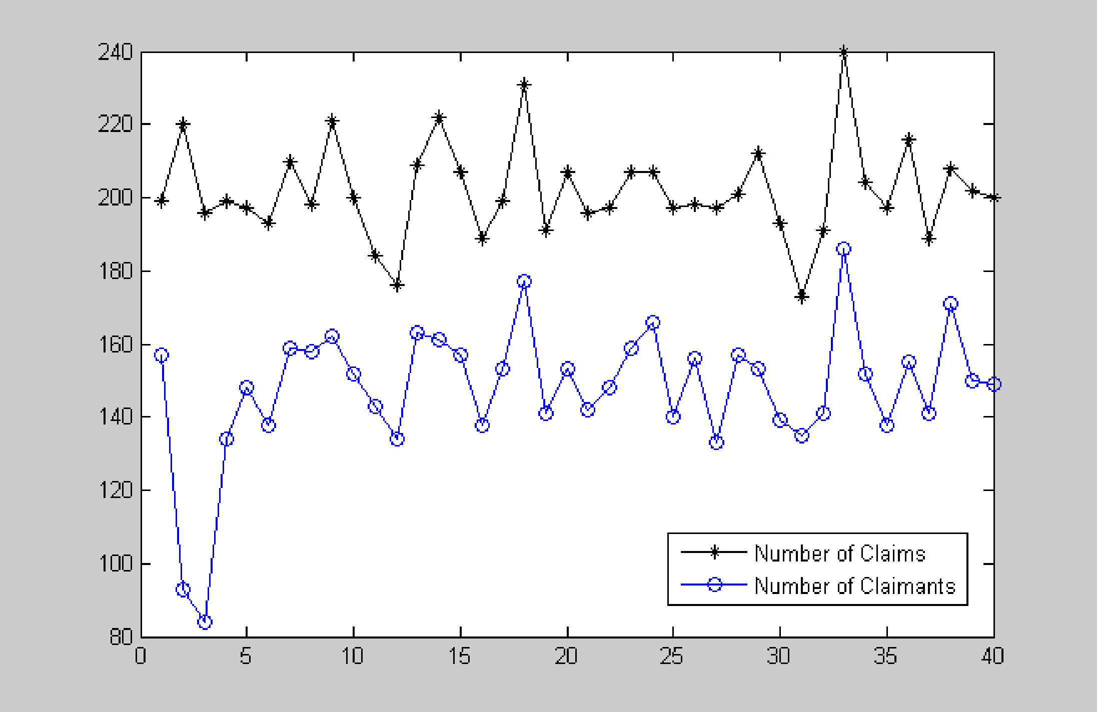

Figure 3.7 Number of claims and number of claimants for the 40 years

Table 3.13 The distribution of policyholders in the various discount classes for the first five years

Discount Classes	Year 1	Year 2	Year 3	Year 4	Year 5
0%	157	93	16	7	1
20%	843	138	149	29	42
40%	0	769	134	256	140
60%	0	0	701	708	817

Table 3.14 Amount of claims, claim outgo and premium income for the first five years (TL)

Years	Amount of claims	Claim outgo	Premium income
1	321,304	313,404	1,500,000
2	404,312	341,329	1,247,100
3	317,217	268,514	997,200
4	347,022	320,925	744,000
5	320,850	307,671	700,500

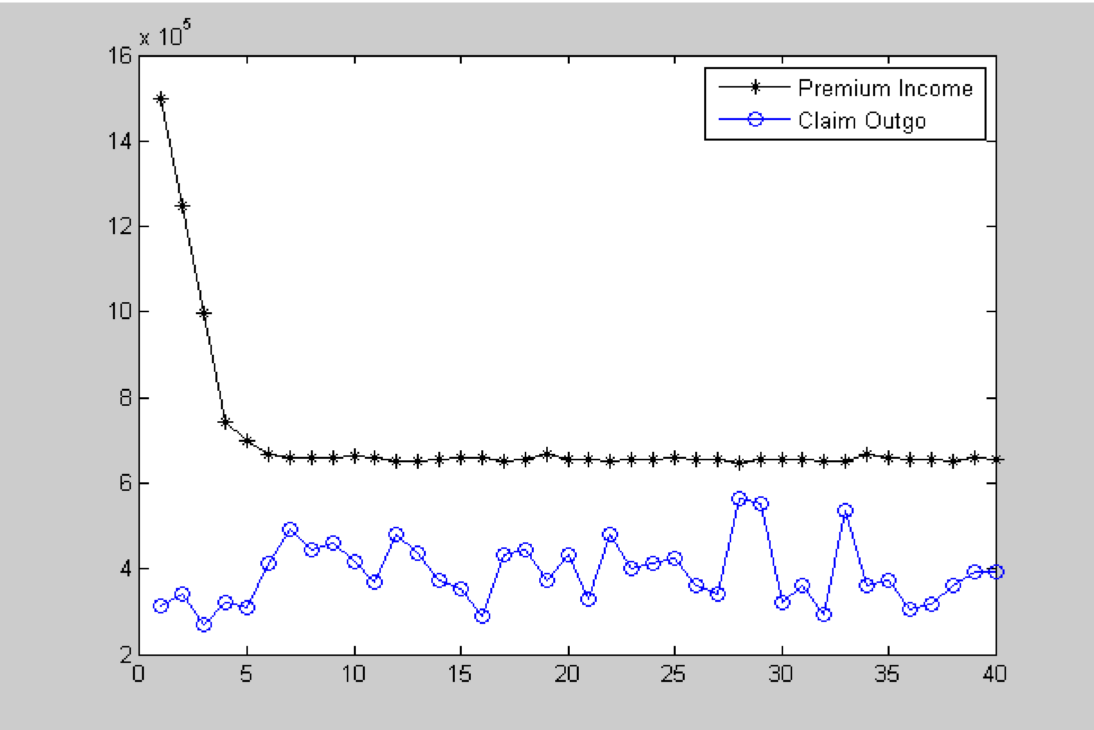

Figure 3.8 Claim outgo and premium income for the 40 years

As a result of profitability is 12,544,499 TL and loss premium ratio is 55%. By means of simulation realized by increasing premiums, profit is obtained at the end of 40 years. In Figure 3.8, we can see that total premium incomes are more than claim outgo throughout 40 years. By increasing premiums, we attained optimum NCD system.

CHAPTER FOUR
CONCLUSIONS

As describe in this thesis, there are many different NCD systems are used in motor insurance in many countries around the world. NCD system is used also in motor third-party liability insurance. In obvious that motor third-part liability insurance has consequently been made compulsory in most development countries, and actuaries from all over the world face the problem of designing tariff structures that will fairly distribute the burden of claims among policyholders. After solving these problems, optimum NCD system can be attained by means of simulation.

In the application chapter of this thesis, we have used simulation method deciding the optimum NCD system for the existing portfolio. A system which is similar to the NCD system in Turkey is used. We made changes in transition rules in order to provide that policyholders make fewer claims with the purpose of attaining optimum NCD system, but it was not enough. By taking simulation and results in Turkey into account, we decided that optimum NCD system can be reached by increasing premiums. After simulation realized by increasing premiums, we reached optimum system.

As a result, in Turkey and all over the world, there is loss in motor insurance in the long-term. Insurance companies can make simulations in their existing NCD system in order to see the future and to take necessary measurements for preventing the loss. Based on data obtained from simulation results, companies can try to reach optimum NCD system by updating transition rules, premiums, and discount classes.

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APPENDICES

APPENDIX A

SIMULATION PROGRAM

```
>> fprintf('\n\nSimulasyon Programı...');
KisiSayisi=input('\nSimulasyonda Kac Kisi Yer Alsin: ' );
yil=input('Simulasyon Kac Yil Sursun: ' );
lambda=input('Poisson Dagiliminin Ortalamasi Kac Olsun: ');
mu=input('Log.normal Dagilimin Ortalamasi Kac Olsun: ');
sigma=input('Log-normal Dagilimin Standart Sapmasi Kac Olsun: ');

   s(1:yil)=struct('hasar',zeros(4,3), 'beyanvar', zeros(1,4),'mbeyanvar', zeros(1,4), ...
      'ths', zeros(1,4), 'thm', zeros(1,4), 'beyanyok', zeros(1,4), 'mbeyanyok',
zeros(1,4),...

'stprim',0,'sthm',0,'sths',0,'smbeyanvar',0,'smbeyanyok',0,'sbeyanvar',0,'sbeyanyok',0,..
.
      'hs',0,'hm',0,'g',zeros(1,4));
  ... for i=1:yil
  ...    s(i).hasar=zeros(4,3);
  ...    s(i).beyanvar=zeros(1,4);
  ...    s(i).mbeyanvar=zeros(1,4);
  ...    s(i).ths=zeros(1,4);
  ...    s(i).thm=zeros(1,4);
  ...    s(i).beyanyok=zeros(1,4);
  ...    s(i).mbeyanyok=zeros(1,4);
  ...    s(i).stprim=0;
  ...    s(i).sthm=0;
  ...    s(i).sths=0;
  ...    s(i).smbryanvar=0;
  ...    s(i).smbeyanyok=0;
  ...    s(i).sbeyanvar=0;
  ...    s(i).sbeyanyok=0;
  ...    s(i).hs=0;
  ...    s(i).hm=0;
  ...    s(i).g=zeros(1,4);
  ... end

fprintf('\n\nSimulasyon Basliyor....');

for i=1:yil
   if i==1 %1. yil ise
      s(i).hs=poissrnd(lambda,[1,KisiSayisi]);
      for a=1:KisiSayisi
         % s(i).hs(a)=poissrnd(lambda)
```

```
        if s(i).hs(a)==0
            s(i).hasar(1,1)=s(i).hasar(1,1)+1;
        elseif s(i).hs(a)==1
            s(i).hasar(1,2)=s(i).hasar(1,2)+1;
        else s(i).hasar(1,3)=s(i).hasar(1,3)+1;
        end
    end

    s(i).ths(1)=s(i).hasar(1,2)+(2*s(i).hasar(1,3));

    %s(i).hm=lognrnd(mu,sigma,1,s(i).ths(1));
    for a=1:s(i).ths(1)
        s(i).hm(a)=lognrnd(mu,sigma);
        if s(i).hm(a)>170
            s(i).beyanvar(1)=s(i).beyanvar(1)+1;
            s(i).mbeyanvar(1)=s(i).mbeyanvar(1)+s(i).hm(a);
        end
    end
    s(i).thm(1)=sum(s(i).hm);
    s(i).mbeyanyok(1)=s(i).thm(1)-s(i).mbeyanvar(1);
    s(i).beyanyok(1)=KisiSayisi-s(i).beyanvar(1);

    % 1.yil sonunda genel toplam %
    s(i).stprim=KisiSayisi*850;
    s(i).sthm=s(i).thm(1);
    s(i).sths= s(i).ths(1);
    s(i).smbeyanvar=s(i).mbeyanvar(1);
    s(i).smbeyanyok=s(i).mbeyanyok(1);
    s(i).sbeyanvar=s(i).beyanvar(1);
    s(i).sbeyanyok=s(i).beyanyok(1);
    s(i).g(1)=s(i).beyanvar(1);
    s(i).g(2)=s(i).beyanyok(1);
    fprintf('\n\n%d.yil tamamlandi...',i);

elseif i==2 %2.yil ise
% g(1)= %0 grubunda yer alanlar %
    if s(i-1).g(1)>0
        s(i).hs=poissrnd(lambda, [1 s(i-1).g(1)]);
        for a=1:s(i-1).g(1)
            %s(i).hs(a)=poissrnd(lambda)
            if s(i).hs(a)==0
                s(i).hasar(1,1)=s(i).hasar(1,1) + 1;
            elseif s(i).hs(a)==1
                s(i).hasar(1,2)=s(i).hasar(1,2)+1;
            else s(i).hasar(1,3)=s(i).hasar(1,3)+1;
            end
        end
        s(i).ths(1)=s(i).hasar(1,2)+(2*s(i).hasar(1,3));
```

```
        for a=1:s(i).ths(1)
            s(i).hm(a)=lognrnd(mu,sigma);
            if s(i).hm(a)>340
                s(i).mbeyanvar(1)=s(i).mbeyanvar(1)+s(i).hm(a);
                s(i).beyanvar(1)=s(i).beyanvar(1)+1;
            end
        end
        s(i).thm(1)=sum(s(i).hm);
        s(i).mbeyanyok(1)=s(i).thm(1)-s(i).mbeyanvar(1);
        s(i).beyanyok(1)=s(i-1).g(1)-s(i).beyanvar(1);
    end

% g(2)= %20 grubunda yer alanlar %
    if s(i-1).g(2)>0
        s(i).hs=poissrnd(lambda, [1 s(i-1).g(2)]);
        for a=1:s(i-1).g(2)
            %s(i).hs(a)=poissrnd(lambda)
            if s(i).hs(a)==0
                s(i).hasar(2,1)=s(i).hasar(2,1)+1;
            elseif s(i).hs(a)==1
                s(i).hasar(2,2)=s(i).hasar(2,2)+1;
            else s(i).hasar(2,3)=s(i).hasar(2,3)+1;
            end
        end
        s(i).ths(2)=s(i).hasar(2,2)+(2*s(i).hasar(2,3));

        for a=1:s(i).ths(2)
            s(i).hm(a)=lognrnd(mu,sigma);
            if s(i).hm(a)>680
                s(i).mbeyanvar(2)=s(i).mbeyanvar(2)+s(i).hm(a);
                s(i).beyanvar(2)=s(i).beyanvar(2)+1;
            end
        end
        s(i).thm(2)=sum(s(i).hm);
        s(i).mbeyanyok(2)=s(i).thm(2)-s(i).mbeyanvar(2);
        s(i).beyanyok(2)=s(i-1).g(2)-s(i).beyanvar(2);
    end

    % 2.yil sonu genel toplam %
    s(i).stprim=(s(i-1).g(1)*850)+(s(i-1).g(2)*680);
    s(i).sthm=s(i).thm(1)+s(i).thm(2);
    s(i).sths=s(i).ths(1)+s(i).ths(2);
    s(i).smbeyanvar=s(i).mbeyanvar(1)+s(i).mbeyanvar(2);
    s(i).smbeyanyok=s(i).mbeyanyok(1)+s(i).mbeyanyok(2);
    s(i).sbeyanvar=s(i).beyanvar(1)+s(i).beyanvar(2);
    s(i).sbeyanyok=s(i).beyanyok(1)+s(i).beyanyok(2);
    s(i).g(1)=s(i).beyanvar(1)+s(i).beyanvar(2);
```

```
    s(i).g(2)=s(i).beyanyok(1);
    s(i).g(3)=s(i).beyanyok(2);
    fprintf('\n\n%d.yil tamamlandi...',i);

elseif i==3
    % 3.yil basi %
    % g(1)= %0 grubunda yer alanlar %

    if s(i-1).g(1)>0
        s(i).hs=poissrnd(lambda, [1 s(i-1).g(1)]);
        for a=1:s(i-1).g(1)
            %s(i).hs(a)=poissrnd(lambda)
            if s(i).hs(a)==0
                s(i).hasar(1,1)=s(i).hasar(1,1)+1;
            elseif s(i).hs(a)==1
                s(i).hasar(1,2)=s(i).hasar(1,2)+1;
            else s(i).hasar(1,3)=s(i).hasar(1,3)+1;
            end
        end
        s(i).ths(1)=s(i).hasar(1,2)+(2*s(i).hasar(1,3));

        for a=1:s(i).ths(1)
            s(i).hm(a)=lognrnd(mu,sigma);
            if s(i).hm(a)>510
                s(i).mbeyanvar(1)=s(i).mbeyanvar(1)+s(i).hm(a);
                s(i).beyanvar(1)=s(i).beyanvar(1)+1;
            end
        end
        s(i).thm(1)=sum(s(i).hm);
        s(i).mbeyanyok(1)=s(i).thm(1)-s(i).mbeyanvar(1);
        s(i).beyanyok(1)=s(i-1).g(1)-s(i).beyanvar(1);
    end

    % g(2)= %20 grubunda yer alanlar %

    if s(i-1).g(2)>0
        s(i).hs=poissrnd(lambda,[1 s(i-1).g(2)]);
        for a=1:s(i-1).g(2)
            %s(i).hs(i)=poissrnd(lambda)
            if s(i).hs(a)==0
                s(i).hasar(2,1)=s(i).hasar(2,1)+1;
            elseif s(i).hs(a)==1
                s(i).hasar(2,2)=s(i).hasar(2,2)+1;
            else s(i).hasar(2,3)=s(i).hasar(2,3)+1;
            end
        end
        s(i).ths(2)=s(i).hasar(2,2)+(2*s(i).hasar(2,3));
```

```
    for a=1:s(i).ths(2)
        s(i).hm(a)=lognrnd(mu,sigma);
        if s(i).hm(a)>850
            s(i).mbeyanvar(2)=s(i).mbeyanvar(2)+s(i).hm(a);
            s(i).beyanvar(2)=s(i).beyanvar(2)+1;
        end
    end
    s(i).thm(2)=sum(s(i).hm);
    s(i).mbeyanyok(2)=s(i).thm(2)-s(i).mbeyanvar(2);
    s(i).beyanyok(2)=s(i-1).g(2)-s(i).beyanvar(2);
end

% g(3)= %40 grubunda yer alanlar %
if s(i-1).g(3)>0
    s(i).hs=poissrnd(lambda,[1 s(i-1).g(3)]);
    for a=1:s(i-1).g(3)
        %s(i).hs(a)=poissrnd(lambda);
        if s(i).hs(a)==0
            s(i).hasar(3,1)=s(i).hasar(3,1)+1;
        elseif s(i).hs(a)==1
            s(i).hasar(3,2)=s(i).hasar(3,2)+1;
        else s(i).hasar(3,3)=s(i).hasar(3,3)+1;
        end
    end
    s(i).ths(3)=s(i).hasar(3,2)+(2*s(i).hasar(3,3));

    for a=1:s(i).ths(3)
        s(i).hm(a)=lognrnd(mu,sigma);
        if s(i).hm(a)>510
            s(i).mbeyanvar(3)=s(i).mbeyanvar(3)+s(i).hm(a);
            s(i).beyanvar(3)=s(i).beyanvar(3)+1;
        end
    end
    s(i).thm(3)=sum(s(i).hm);
    s(i).mbeyanyok(3)=s(i).thm(3)-s(i).mbeyanvar(3);
    s(i).beyanyok(3)=s(i-1).g(3)-s(i).beyanvar(3);
end

% 3.yil sonu genel toplam %
s(i).stprim=(s(i-1).g(1)*850)+(s(i-1).g(2)*680)+(s(i-1).g(3)*510);
s(i).sthm=s(i).thm(1)+s(i).thm(2)+s(i).thm(3);
s(i).sths=s(i).ths(1)+s(i).ths(2)+s(i).ths(3);
s(i).smbeyanvar=s(i).mbeyanvar(1)+s(i).mbeyanvar(2)+s(i).mbeyanvar(3);
s(i).smbeyanyok=s(i).mbeyanyok(1)+s(i).mbeyanyok(2)+s(i).mbeyanyok(3);
s(i).sbeyanvar=s(i).beyanvar(1)+s(i).beyanvar(2)+s(i).beyanvar(3);
s(i).sbeyanyok=s(i).beyanyok(1)+s(i).beyanyok(2)+s(i).beyanyok(3);
s(i).g(1)=s(i).beyanvar(1)+s(i).beyanvar(2);
s(i).g(2)=s(i).beyanyok(1)+s(i).beyanvar(3);
```

```
    s(i).g(3)=s(i).beyanyok(2);
    s(i).g(4)=s(i).beyanyok(3);
    fprintf('\n\n%d.yil tamamlandi...',i);

elseif i==4
% 4.yil basi %
% g(1)= %0 grubunda yer alanla
    if s(i-1).g(1)>0
        s(i).hs=poissrnd(lambda, [1 s(i-1).g(1)]);
        for a=1:s(i-1).g(1)
            %s(i).hs(a)=poissrnd(lambda);
            if s(i).hs(a)==0
                s(i).hasar(1,1)=s(i).hasar(1,1)+1;
            elseif s(i).hs(a)==1
                s(i).hasar(1,2)=s(i).hasar(1,2)+1;
            else s(i).hasar(1,3)=s(i).hasar(1,3)+1;
            end
        end
        s(i).ths(1)=s(i).hasar(1,2)+(2*s(i).hasar(1,3));

        for a=1:s(i).ths(1)
            s(i).hm(a)=lognrnd(mu,sigma);
            if s(i).hm(a)>510
                s(i).mbeyanvar(1)=s(i).mbeyanvar(1)+s(i).hm(a);
                s(i).beyanvar(4)=s(i).beyanvar(1)+1;
            end
        end
        s(i).thm(1)=sum(s(i).hm);
        s(i).mbeyanyok(1)=s(i).thm(1)-s(i).mbeyanvar(1);
        s(i).beyanyok(1)=s(i-1).g(1)-s(i).beyanvar(1);
    end

    % g(2)= %20 grubunda yer alanlar %
    if s(i-1).g(2)>0
        s(i).hs=poissrnd(lambda, [1 s(i-1).g(2)]);
        for a=1:s(i-1).g(2)
            %s(i).hs(a)=poissrnd(lambda);
            if s(i).hs(a)==0
                s(i).hasar(2,1)=s(i).hasar(2,1)+1;
            elseif s(i).hs(a)==1
                s(i).hasar(2,2)=s(i).hasar(2,2)+1;
            else s(i).hasar(2,3)=s(i).hasar(2,3)+1;
            end
        end
        s(i).ths(2)=s(i).hasar(2,2)+(2*s(i).hasar(2,3));

        for a=1:s(i).ths(2)
            s(i).hm(a)=lognrnd(mu,sigma);
```

```
      if s(i).hm(a)>850
         s(i).mbeyanvar(2)=s(i).mbeyanvar(2)+s(i).hm(a);
         s(i).beyanvar(2)=s(i).beyanvar(2)+1;
      end
   end
   s(i).thm(2)=sum(s(i).hm);
   s(i).mbeyanyok(2)=s(i).thm(2)-s(i).mbeyanvar(2);
   s(i).beyanyok(2)=s(i-1).g(2)-s(i).beyanvar(2);

end

% g(3)= %40 grubunda yer alanlar %
if s(i-1).g(3)>0
   s(i).hs=poissrnd(lambda, [1 s(i-1).g(3)]);
   for a=1:s(i-1).g(3)
      %s(i).hs(a)=poissrnd(lambda);
      if s(i).hs(a)==0
         s(i).hasar(3,1)=s(i).hasar(3,1)+1;
      elseif s(i).hs(a)==1
         s(i).hasar(3,2)=s(i).hasar(3,2)+1;
      else s(i).hasar(3,3)=s(i).hasar(3,3)+1;
      end
   end
   s(i).ths(3)=s(i).hasar(3,2)+(2*s(i).hasar(3,3));

   for a=1:s(i).ths(3)
      s(i).hm(a)=lognrnd(mu,sigma);
      if s(i).hm(a)>510
         s(i).mbeyanvar(3)=s(i).mbeyanvar(3)+s(i).hm(a);
         s(i).beyanvar(3)=s(i).beyanvar(3)+1;
      end
   end

   s(i).thm(3)=sum(s(i).hm);
   s(i).mbeyanyok(3)=s(i).thm(3)-s(i).mbeyanvar(3);
   s(i).beyanyok(3)=s(i-1).g(3)-s(i).beyanvar(3);
end

% g(4)= %60 grubunda yer alanlar %

if s(i-1).g(4)>0
   s(i).hs=poissrnd(lambda, [1 s(i-1).g(4)]);
   for a=1:s(i-1).g(4)
      %s(i).hs(a)=poissrnd(lambda);
      if s(i).hs(a)==0
         s(i).hasar(4,1)=s(i).hasar(4,1)+1;
      elseif s(i).hs(a)==1
         s(i).hasar(4,2)=s(i).hasar(4,2)+1;
```

```
            else s(i).hasar(4,3)=s(i).hasar(4,3)+1;
            end
         end
         s(i).ths(4)=s(i).hasar(4,2)+(2*s(i).hasar(4,3));

         for a=1:s(i).ths(4)
            s(i).hm(a)=lognrnd(mu,sigma);
            if s(i).hm(a)>170
               s(i).mbeyanvar(4)=s(i).mbeyanvar(4)+s(i).hm(a);
               s(i).beyanvar(4)=s(i).beyanvar(4)+1;
            end
         end
         s(i).thm(4)=sum(s(i).hm);
         s(i).mbeyanyok(4)=s(i).thm(4)-s(i).mbeyanvar(4);
         s(i).beyanyok(4)=s(i-1).g(4)-s(i).beyanvar(4);
      end

      % 4.yil sonu genel toplam %
      s(i).stprim=(s(i-1).g(1)*850)+(s(i-1).g(2)*680)+(s(i-1).g(3)*510)+(s(i-
1).g(4)*340);
      s(i).sthm=s(i).thm(1)+s(i).thm(2)+s(i).thm(3)+s(i).thm(4);
      s(i).sths=s(i).ths(1)+s(i).ths(2)+s(i).ths(3)+s(i).ths(4);

s(i).smbeyanvar=s(i).mbeyanvar(1)+s(i).mbeyanvar(2)+s(i).mbeyanvar(3)+s(i).mbey
anvar(4);

s(i).smbeyanyok=s(i).mbeyanyok(1)+s(i).mbeyanyok(2)+s(i).mbeyanyok(3)+s(i).mbe
yanyok(4);

s(i).sbeyanvar=s(i).beyanvar(1)+s(i).beyanvar(2)+s(i).beyanvar(3)+s(i).beyanvar(4);

s(i).sbeyanyok=s(i).beyanyok(1)+s(i).beyanyok(2)+s(i).beyanyok(3)+s(i).beyanyok(4
);
      s(i).g(1)=s(i).beyanvar(1)+s(i).beyanvar(2);
      s(i).g(2)=s(i).beyanyok(1)+s(i).beyanvar(3);
      s(i).g(3)=s(i).beyanyok(2)+s(i).beyanvar(4);
      s(i).g(4)=s(i).beyanyok(3)+s(i).beyanyok(4);
      fprintf('\n\n%d.yil tamamlandi...',i);


   else   % 4.yildan sonra %

      A=[510,850,510,170];
      for k=1:4   % k=1 için %0 grubunda yer alanlar %
            % k=2 için %20 grubunda yer alanlar %
            % k=3 için %40 grubunda yer alanlar %
            % k=4 için %60 grubunda yer alanlar %
         if s(i-1).g(k)>0
```

```
            s(i).hs=poissrnd(lambda, [1 s(i-1).g(k)]);
            for a=1:s(i-1).g(k)
               %s(i).hs(a)=poissrnd(lambda);
               if s(i).hs(a)==0
                  s(i).hasar(k,1)=s(i).hasar(k,1)+1;
               elseif s(i).hs(a)==1
                  s(i).hasar(k,2)=s(i).hasar(k,2)+1;
               else s(i).hasar(k,3)=s(i).hasar(k,3)+1;
               end
            end

            s(i).ths(k)=s(i).hasar(k,2)+(2*s(i).hasar(k,3));

            for a=1:s(i).ths(k)
               s(i).hm(a)=lognrnd(mu,sigma);
               if s(i).hm(a)>A(k)
                  s(i).mbeyanvar(k)=s(i).mbeyanvar(k)+s(i).hm(a);
                  s(i).beyanvar(k)=s(i).beyanvar(k)+1;
               end
            end
            s(i).thm(k)=sum(s(i).hm);
            s(i).mbeyanyok(k)=s(i).thm(k)-s(i).mbeyanvar(k);
            s(i).beyanyok(k)=s(i-1).g(k)-s(i).beyanvar(k);
         end
      end

      % n.yil sonu genel toplam %
      s(i).stprim=( s(i-1).g(1)*850)+( s(i-1).g(2)*680)+( s(i-1).g(3)*510)+( s(i-
1).g(4)*340);
      s(i).sthm= s(i).thm(1)+ s(i).thm(2)+ s(i).thm(3)+ s(i).thm(4);
      s(i).sths= s(i).ths(1)+ s(i).ths(2)+ s(i).ths(3)+ s(i).ths(4);

s(i).smbeyanvar=s(i).mbeyanvar(1)+s(i).mbeyanvar(2)+s(i).mbeyanvar(3)+s(i).mbey
anvar(4);

s(i).smbeyanyok=s(i).mbeyanyok(1)+s(i).mbeyanyok(2)+s(i).mbeyanyok(3)+s(i).mbe
yanyok(4);

s(i).sbeyanvar=s(i).beyanvar(1)+s(i).beyanvar(2)+s(i).beyanvar(3)+s(i).beyanvar(4);

s(i).sbeyanyok=s(i).beyanyok(1)+s(i).beyanyok(2)+s(i).beyanyok(3)+s(i).beyanyok(4
);
      s(i).g(1)=s(i).beyanvar(1)+s(i).beyanvar(2);
      s(i).g(2)=s(i).beyanyok(1)+s(i).beyanvar(3);
      s(i).g(3)=s(i).beyanyok(2)+s(i).beyanvar(4);
      s(i).g(4)=s(i).beyanyok(3)+s(i).beyanyok(4);
      fprintf('\n\n%d.yil tamamlandi...',i);
```

```
   end
end

%sonuclar %
%for i=1:yil
%
yillar=[s(i).sths,s(i).sbeyanvar,s(i).sbeyanyok,s(i).stprim,s(i).sthm,s(i).smbeyanvar,s(i
).smbeyanyok]
%    kisiler=[s(i).g(1),s(i).g(2),s(i).g(3),s(i).g(4)]
%end

fprintf('\n\nSimulasyon Bitti');
for i=1:yil
   fprintf('\n\n\n%d. Yil Sonuçlari',i);
   fprintf('\n----------------------');
   fprintf('\nTüm hasarlar toplami: %d',s(i).sths);
   fprintf('\nBeyan edilen hasar sayisi: %d',s(i).sbeyanvar);
   fprintf('\nBeyan edilmeyen hasar sayisi: %d',s(i).sbeyanyok);
   fprintf('\nPrim gelirleri toplami: %d',s(i).stprim);
   fprintf('\nTüm hasarlarin toplam miktari: %0.0f',s(i).sthm);
   fprintf('\nBeyan edilen hasarlarin miktari: %0.0f',s(i).smbeyanvar);
   fprintf('\nBeyan edilmeyen hasarlarin miktari: %0.0f',s(i).smbeyanyok);

fprintf('\n\t_______________________________________________________________')
;
   fprintf('\n\t%%0. Grup KS\t\t%%20. Grup KS\t%%40. Grup KS\t%%60.Grup
KS');
   fprintf('\n\t...........\t............\t............\t..........');
   fprintf('\n\t\t %d\t\t\t\t %d\t\t\t\t %d\t\t\t\t%d',s(i).g(1),s(i).g(2),s(i).g(3),s(i).g(4));

fprintf('\n\t_______________________________________________________________')
;

   %fprintf('\n20 grubunda yer alan kisi sayisi: %d',s(i).g(2));
   %fprintf('\n40 grubunda yer alan kisi sayisi: %d',s(i).g(3));
   %fprintf('\n60 grubunda yer alan kisi sayisi :%d',s(i).g(4));

end
   fprintf('\n\n\n\t\t\tPrim Geliri\t\t\tHasar Gideri');
   fprintf('\n\t\t\t_____________\t\t_____________');
   for i=1:yil
   fprintf('\n\t%d. Yil\t\t%d\t\t\t\t%0.0f',i,s(i).stprim,s(i).smbeyanvar);
end
fprintf('\n\nTHE END!!!');
```

APPENDIX B

GRAPHICS OF NUMBER OF CLAIMS AND CLAIMANTS

```
for i=1:yil
x=1:yil;
y1(i) =s(i).sths;
y2(i)=s(i).sbeyanvar;
disp(y1)
disp(y2)
disp(x)
end
plot(x,y1(:),'-*k',x,y2(:),'-o')
```

APPENDIX C

GRAPHICS OF PREMIUM INCOME AND CLAIM OUTGO

```
for i=1:yil
x=1:yil;
y1(i) =s(i).stprim;
y2(i)=s(i).smbeyanvar;
disp(y1)
disp(y2)
disp(x)
end
plot(x,y1(:),'-*k',x,y2(:),'-o')
```