DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

DYNAMIC ANALYSIS OF NON LUBRICATED, MULTI STAGE PISTON AIR COMPRESSORS

by
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İZMİR

DYNAMIC ANALYSIS OF NON LUBRICATED, MULTI STAGE PISTON AIR COMPRESSORS

A Thesis Submitted to the
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In Partial Fulfillment of the Requirements for the Degree of Master of Science
in
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by
Bayram Ufuk ÜSTÜN

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M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled "**DYNAMIC ANALYSIS OF NON LUBRICATED, MULTI STAGE PISTON AIR COMPRESSORS**" completed by **BAYRAM UFUK ÜSTÜN** under supervision of **PROF.DR. MUSTAFA SABUNCU** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science

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Finally, this thesis would not have been possible unless the support of my parents. Without my parents encouragement, I would not have finished the degree.

Bayram Ufuk ÜSTÜN

DYNAMIC ANALYSIS OF NON LUBRICATED, MULTI STAGE PISTON AIR COMPRESSORS

ABSTRACT

The primary purpose of this study is to explore the design and basic calculations of the Non Lubricated, Multistage Piston Air Compressor. These type of compressors are special machines in the industry because of the working conditions. These kind of machines are used where the pure air is required, such as PET blowing industry, Food and Medical Sectors, Electronic industry etc.

In this study we designed all the parts with using Pro-Engineer CAD program and made all of the calculations with Ansys Finite Element Program. For the optimum compressor design there are some specific calculations such as, strength analysis of the moving parts, balancing of the rotating and reciprocating parts, flywheel selection etc. Shortly in these study we examined all of these specific calculations and designs.

This project was the goverment supported R&D project of the MAKSAS MAKİNA SANAYİİ A.Ş. so at the end of the project we had a chance to compare our calculation with the test results. On the other hand, during the project we could follow all production and test processes and controlled the design instantly.

Keywords: Non-Lubricated, Piston Air Compressor, finite element, balancing, flywheel, kinematic and dynamic calculations, strength

YAĞSIZ, ÇOK KADEMELİ, PİSTONLU HAVA KOMPRESÖRLERİNİN DİNAMİK ANALİZİ

ÖZ

Bu çalışmanın ilk amacı, yağsız, çok kademeli, pistonlu hava kompresörlerinin tasarım ve temel hesaplamalarını incelemektir. Çalışma koşulları nedeniyle bu tip kompresörler endüstride özel makinalardir. Bu makinalar, saf hava ihtiyacının olduğu PET şişirme endüstrisinde, ilaç ve gıda endüstrilerinde, elektronik endüstrisinde kullanılmaktadır.

Bu çalışmada bütün parçalar Pro-Engineer CAD programı kullanılarak tasarlanmıştır ve bütün hesaplamalar ANSYS sonlu elemanlar programı kullanılarak yapılmıştır. En uygun kompresör tasarımı için hareketli parçaların mukavemet analizleri, dönen ve gidip gelme hareketi yapan parçaların dengelenmesi, volan seçimi gibi özel hesaplamalar vardır. Bu çalışmada bütün bu özel hesaplamalar incelenmiştir.

Bu proje MAKSAŞ MAKİNA SANAYİİ A.Ş.’ nin devlet destekli Ar&Ge projesidir. Bu nedenle proje sonunda, yapılan hesaplamalar ile test sonuçlarını karşılaştırma şansına sahibiz. Yanı sıra proje süresince bütün üretim ve test aşamaları takip edilerek tasarım sürekli olarak kontrol edilmiştir.

Anahtar Sözcükler: Yağsız, Pistonlu hava kompresörü, sonlu elemanlar methodu, dengeleme, volan, kinematik ve dinamik hesaplamalar, muhavemet

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CHAPTER ONE
INTRODUCTION OF COMPRESSOR

1.1 History Of Reciprocating Compressor

Before looking at the design details of the oil-free reciprocating compressor, it may be of interest to review what has been produced in the past and to examine the present " state of art"

Oil-free cylinder designs were created in the early 1930s. These cylinder designs used water for lubrication and saw service in brewery applications. Soap and water for lubrication was used for compressors pumping oxygen.

In about the mid 1930s, the first high-pressure , 2000 psi non-lubricated air compressor was made using carbon rings. In subsequent years, many single and multi stage compressors ware made using carbon as the wearing material fort he piston and rider bands. The carbon piston ring construction is shown in Figure 1.1

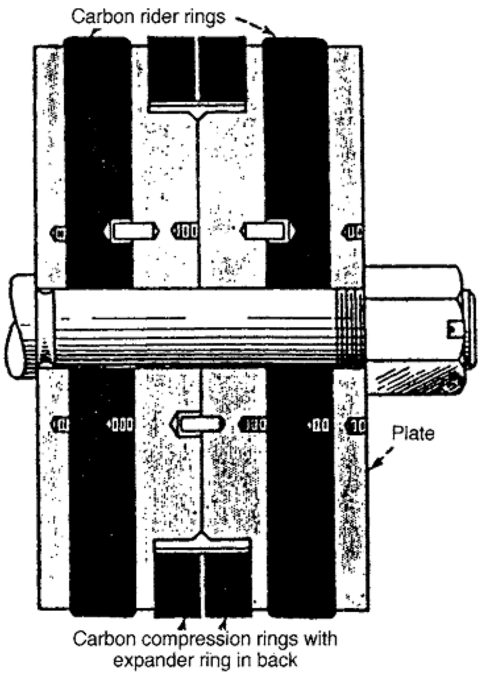


Figure 1.1 Piston with Carbon Rings

This was a “non-floating” type piston, which meant that the carbon rings transferred the weight and load of the iron piston on to the cylinder bore. Piston rings with expanders were used to seal the gas.

Another type of construction was a “floating” piston, in which a tail rod used with a small auxiliary crosshead. The tail rod supported the piston and prevented it from touching the bore. Carbon-rider rings were not used.

The pressure packing was either a soft braided asbestos yarn, sometimes filled with animal fat lubricant, or rings made of graphite or segmented carbon. Carbon has a great disadvantage; it is an extremely brittle material and requires extreme care when installing to prevent chipping and breakage.

Prior to the advent of high performance polymers, the process industry had adopted a standard of ordinary teflon construction. Piston and packing rings are often fabricated from a group of materials based on DuPont’s polytetrafluroethylene (PTFE). Various fillers are used such as glass (fibre), carbon, bronze or graphite.

1.2 Methods Of Compression

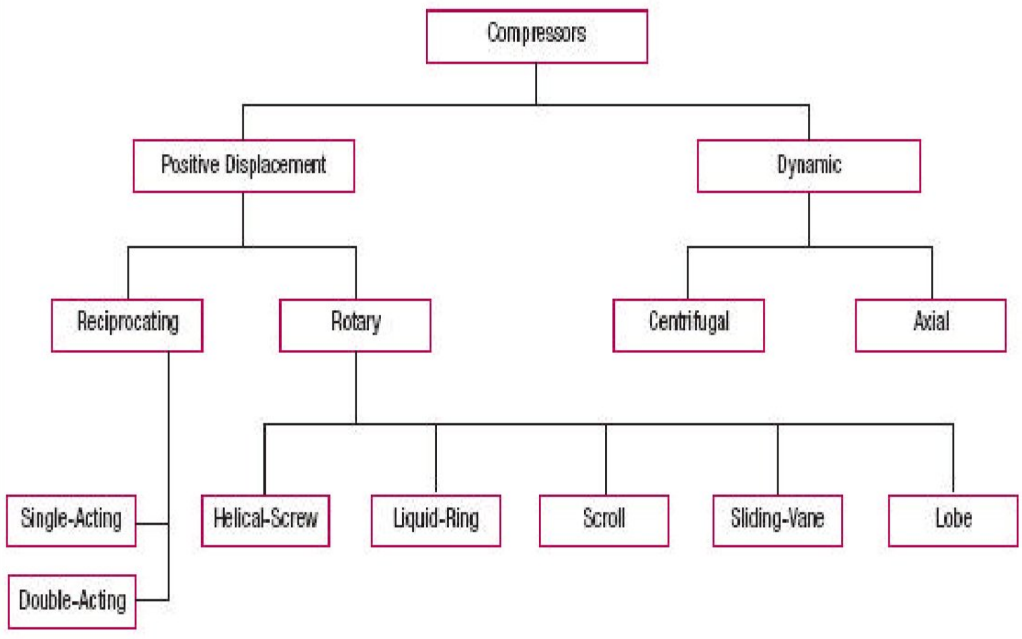
Four methods are used to compress gas. Two are in the intermittent class, and two are in the continuous flow class.

1. Trap consecutive quantities of gas in some type of enclosure, reduce the volume (thus increasing the pressure), then push the compressed gas out of the enclosure.
2. Trap consecutive quantities of the gas in some type of enclosure, carry it without volume change to the discharge opening, compress the gas by backflow from the discharge system, then push the compressed gas out of the enclosure.
3. Compress the gas by the mechanical action of rapidly rotating impellers or bladed rotors that impart velocity and pressure to the flowing gas.

4. Entrain the gas in a high velocity jet of the same or another gas and convert the high velocity of the mixture into pressure in a diffuser.

Compressors using methods 1 and 2 are in the intermittent class and are known as positive displacement compressors. Those using method 3 are known as dynamic compressors. Compressors using method 4 are known as ejectors and normally operate with an intake below atmospheric pressure.

Table 1.1 Compression methods



Compressors change mechanical energy into gas energy. This is in accordance with the First Law of Thermodynamics, which states that energy can not be created or destroyed during a process, although the process may change mechanical energy into gas energy. Some of the energy is also converted into non-usable forms such as heat loss.

Mechanical energy can be converted into gas energy in one of two ways;

1. By positive displacement of the gas into a smaller volume. Flow is directly proportional to speed of the compressor, but the pressure ratio is determined by pressure in the system into which the compressor is pumping.
2. By dynamic action imparting velocity to the gas. This velocity is then converted into pressure. Flow rate and pressure ratio both very as a function of speed, but only within a very limited range and then only with properly designed control systems.

1.3 Air Compressors Basic Operation

An air compressor operates by converting mechanical energy into pneumatic energy via compression. The input energy could come from a drive motor, gasoline engine or power takeoff.

Modern compressors use pistons, vanes and other pumping mechanisms to draw air from atmosphare, compress it and discharge it into a receiver or pressure system. The most basic types of air compressors are designated as " Positive Displacement " and " Non-positive Displacement ". The characteristic action of a positive displacement compressor is thus a distinct volumetric change a literal displacement action by which successive volumes of air are confined within a closed chamber of fixed volume and the pressure as gradually increased by reducing the volume of the space.

The forces are static that is, the pumping rate is essentially constant, given a fixed operating speed. The principle is the same as the action of a piston/cylinder assambly in a simple hand pump.

Gas compression has been one of the anchor points of the industrial revolution, beginning with low pressure air supply for iron and steel refining, through higher pressure air supply for drilling and plant operating equipment, to high pressure as required for chemical synthesis, storage and pipeline deliveries of fuel gases. The positive displacement compressors in use today can trace their ancestry back to the original pumping machines invented by James Watts, or the bellows and blowers of blacksmith.

1.4 Positive Displacement Compressors

Positive displacement compressors generally provide the most economic solution for systems requiring relatively high pressure. Their chief disadvantages is that the displacing mechanism provides lover mass flow rates then non-positive displacement compressors.

Positive displacement compressors are divided into those which compress air with a reciprocating motion and those which compress air with rotary motion.

1.4.1 Reciprocating Compressors

This design is widely used in commercial air compressors because of its high pressure capabilities, flexibility and ability to rapidly dissipate heat of compression.

Compression is accomplished by the reciprocating movement of a piston within a cylinder. This motion alternately fills the cylinder and then compresses the air. A connecting rod transforms the rotary motion of the crankshaft into reciprocationg piston motion in the cylinder. Depending on the application, the rotating crank is driven at constant speed by a suitable prime mover. Separate inlat and discharge valves react to variations in pressure produced by the piston movement.

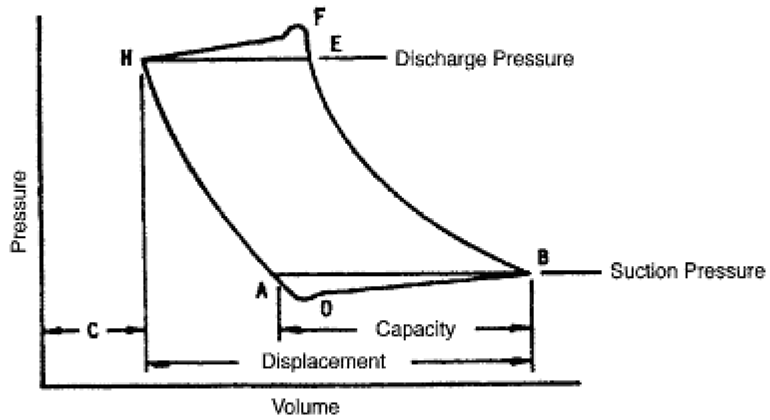


Figure 1.2 Actual Compressor indicator card

As. figure 1.2 shows, the suction stroke begins with the piston at the valve side of the cylinder, in a position providing minimum (or clearance) volume. As the piston moves to a maximum volume position, outside air flows into the cylinder through inlet valve. The discharge valve remains closed during this stroke. During the compression stroke, the piston moves in the opposite direction, decreasing the volume of air as the piston returns to the minimum position.

During these action, the spring loaded inlet and discharge valves are automatically activated by pressure differentials. That is, during the suction stroke, the piston motion reduces the pressure in the cylinder below atmospheric pressure. The inlet valve then opens against the pressures of its spring and allows air to flow into the cylinder. When the piston begins its return (compression) stroke, the inlet valve spring closes the inlet valve because there is no pressure differential to hold the valve open. As pressure increases in the cylinder, the valve is held firmly in its seat.

The discharge valve functions similarly. When pressure in the cylinder becomes greater then the combined pressures of the valve spring and the delivery pipe, the valves opens and the compressed air flows into the system.

In short, the inlet valve is opened by reduced pressure, and the discharge valve is opened by increased pressure.

Some piston compressors are double-acting. As the piston travels in a given direction, air is compressed on one side while suction is produced on the other side. On the return stroke the same thing happens with the sides reversed. In a single-acting compressor, by contrast, only one side of the piston is active.

Single-acting compressors are generally considered light-duty machines, regardless of whether they operate continuously or intermittenly. Larger double-acting compressors air considered heavy-duty machines capable of continuous operation.

Reciprocating compressors have some disadvantages. Reciprocating piston compressors inherently generate inertial forces that shake the machine. Thus, a rigid frame, fixed to a solid foundation, is often required. Also, these machines deliver a pulsating flow of air that may be objectionable under some conditions. Properly sized pulsation damping chambers or receiver tanks, however, will eliminate such problems.

In general, the reciprocating piston compressor is best suited to compression of relatively small volumes of air to high pressures.

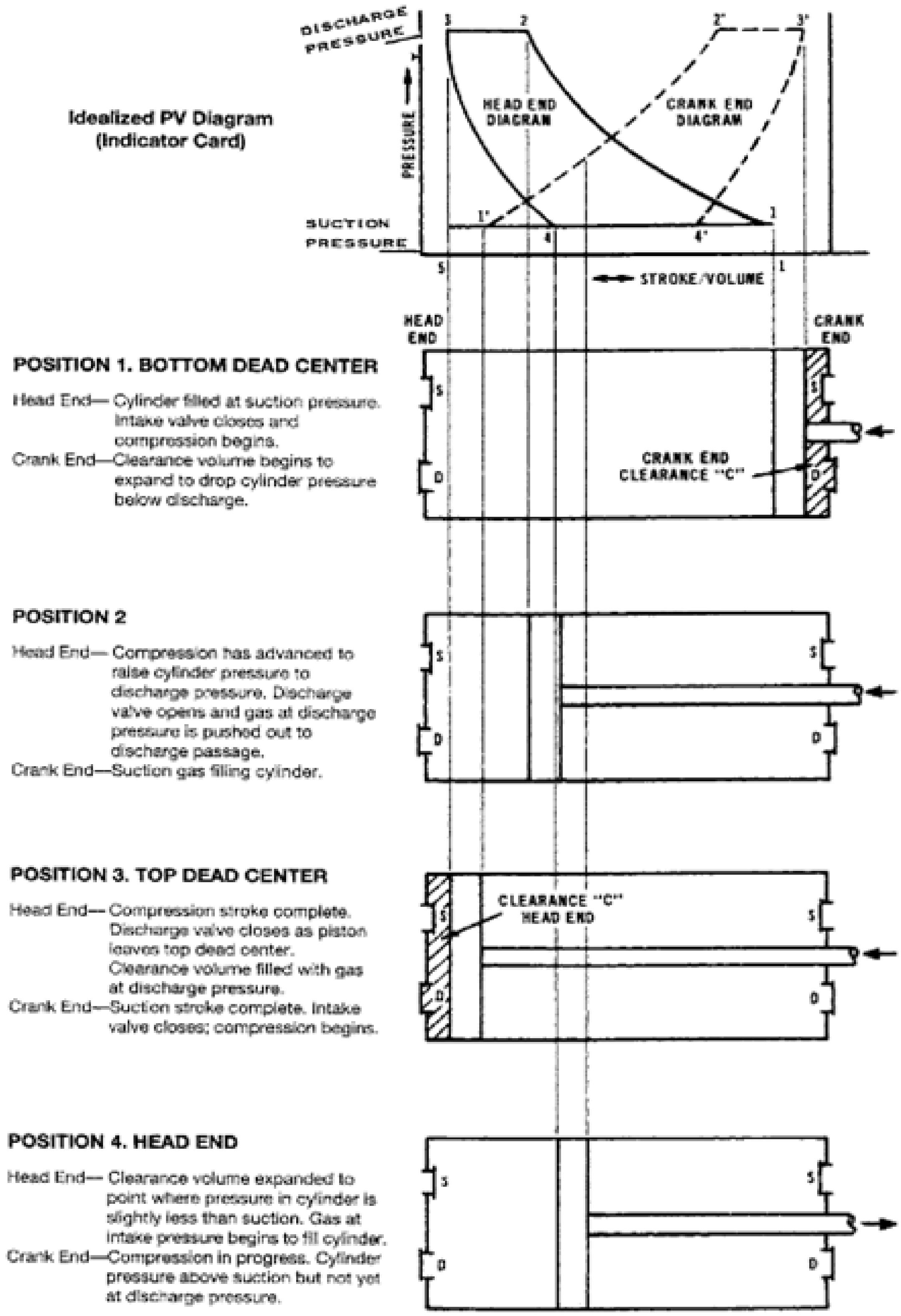


Figure 1.3 Ideal compression cycle single stage double acting compressor

CHAPTER TWO
THE FINITE ELEMENT

2.1 Introduction Of The Finite Element Method

The limitations of the human mind are such that it can not grasp the behaviour of its complex surroundings and creations in one operation. Thus the process of subdividing all systems into their individual components or elements, whose behaviour is readily understood, and then recuilding the original system from such components to study its behaviour is a natural way in which the engineer, the scientist, or even the economist proceeds.

In many situations an adequate model is obtained using a finite number of well defined components. We shall term such problems discrete. In others the subdivision is continued indefinitely and the problem can only be defined using the mathematical fiction of an infinitesimal. This leads to differential equations or equivalent statements which imply an infinite number of elements. We shall term such systems continuous.

With the advent of digital computers, discrete problems can generally be solved readily even if the number of elements is very large. As the capacity of all computers is finite, continuous problems can only be solved exactly by mathematical manipulation. Here, the available mathematical techniques usually limit the possibilities to oversimplified situations.

To overcome the intractability of realistic types of continuum problems, various methods of discretization have from time to time been proposed both by engineers and mathematicians. All involve an approximation which, hopefully, approaches in the limit the true continuum solution as the number of discrete variables increases.

The discretization of continuous problems has been approached differently by mathematicians and engineers. Mathematicians have developed general techniques applicable directly to differential equations governing the problem, such as finite difference approximations, various weighted residual procedures, or approximate techniques for determining the stationary of properly defined ‘functionals’. The engineer, on the other hand, often approaches the problem more intuitively by creating an analogy between real discrete elements and finite portions of a continuum domain. For instance, in the field of solid mechanics McHenry, Hrenikoff, Newmark, and indeed Southwell in the 1940s, showed that reasonably good solutions to an elastic continuum problem can be obtained by replacing small portions of the continuum by an arrangement of simple elastic bars. Later, in the same context, Argyris and Turner et al showed that a more direct, but no less intuitive, subsitution of properties can be made much more effectively by considering that small portions or elements in a continuum behave in a simplified manner.

It is from engineering ‘ direct analogy ‘ view that the term ‘finite element’ was born. Clough appears to be the first to use this term, which implies in it a direct use of a sandard methodology applicable to discrete systems. Both conceptually and from the computational viewpoint, this is of the utmost importance. The first allows an improved understanding to be obtained; the second offers a unified approach to the variety of problems and the development of Standard computational procedures.

Since the early 1960s much progress has been made, and today the purely mathematical and ‘analogy’ approaches are fully reconciled. It is the object of this text to present a view of the finite element method as a general discretization procedure of continuum problems posed by mathematically defined statements.

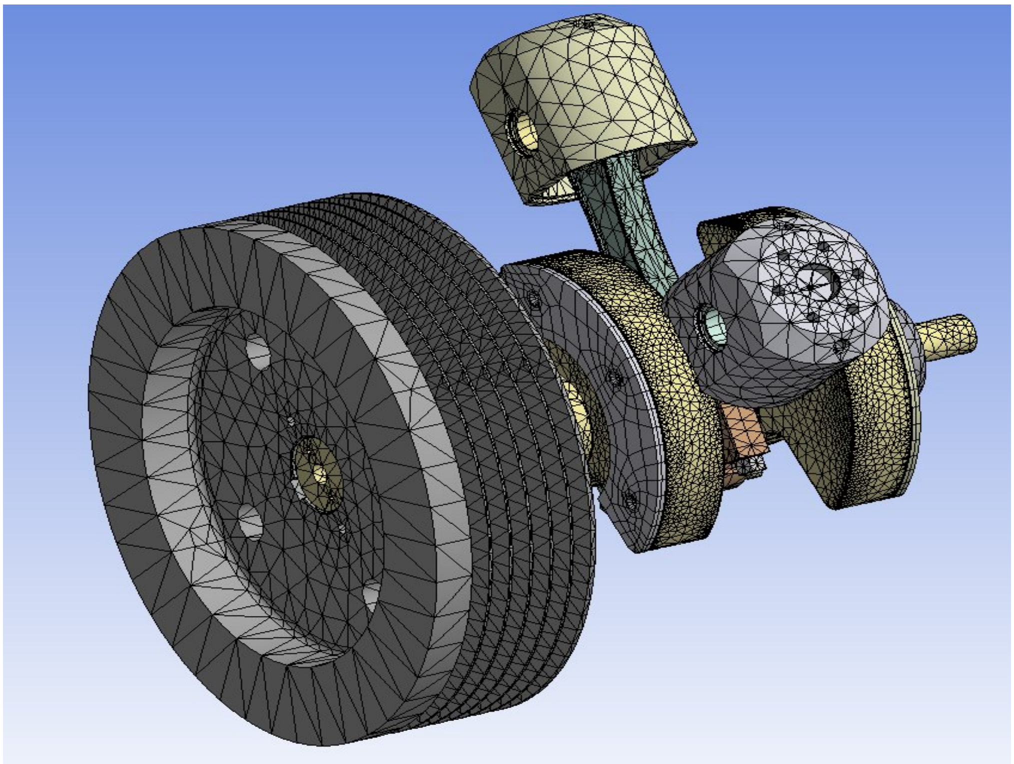


Figure 2.1 Mesh model of the system

In the analysis of the probems of a discrete nature, a Standard methodology has been developed over the years. The civil engineer, dealing with structures, first calculates force-displacement relationships for each element of the structure and then proceeds to assemble the whole by following a well-defined procedure of establishing local equilibrium at each 'node' or connecting point of the structure. The resulting equations can be solved for the unknown displacements. Similarly, the electrical or hydraulic engineer, dealing with a network of electrical components or hydraulic conduits, first establishes a relationship between currents and potentials for individual elements and then proceeds to assemble the system by ensuring continuity of flows.

All such analyses follow a Standard pattern which is universally adaptable to discrete systems. It is thus possible to define a standard discrete system, and this chapter will be primarily concerned with establishing the processes applicable to such systems. Much of what is presented here will be known to engineers, but some reiteration at this stage is advisable. As the treatment of elastic solid structures has been the most developed area of activity this will be introduced first, followed by examples from other fields, before attempting a complete generalization.

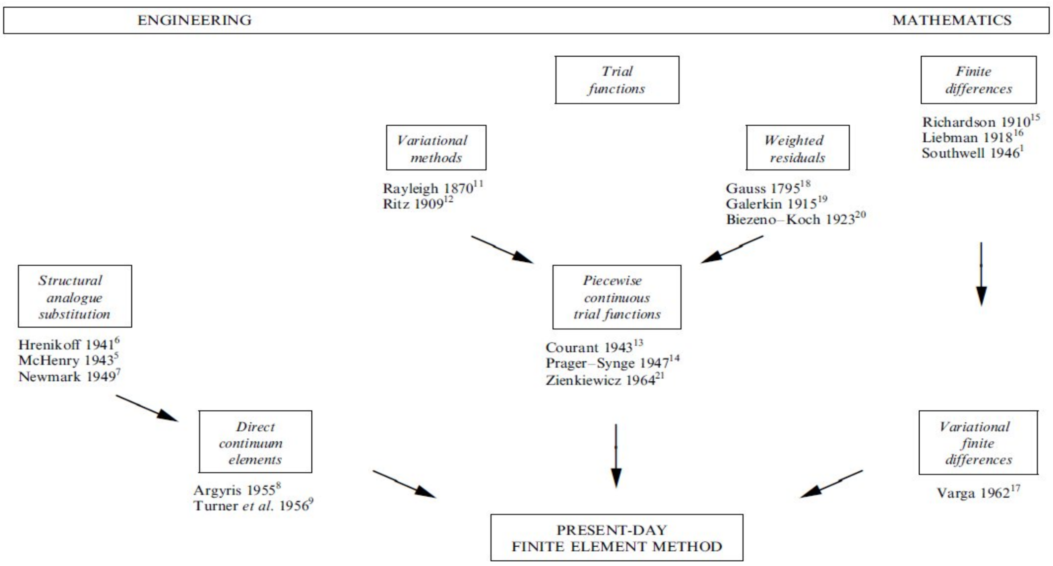
The existance of a unified treatment of 'standard discrete problems' leads us to the first definition of the finite element process as a method of approximation to continuum problems such that

- the continuum is divided into a finite number of parts (elements), the behaviour of which is specified by a finite number of parameters
- the solution of the complete system as an assembly of its elements follows precisely the same rules as those applicable to standard discrete problems.

It will be found that most classical mathematical approximation procedures as well as the various direct approximations used in engineering fall into this category. It is thus difficult to determine the origins of the finite element method and the precise moment of its invention.

Table 2.1 shows the process of evolution which led to the present day concept of finite element analysis.

Table 2.1 Finite element methods



2.2 The Structural Element And System

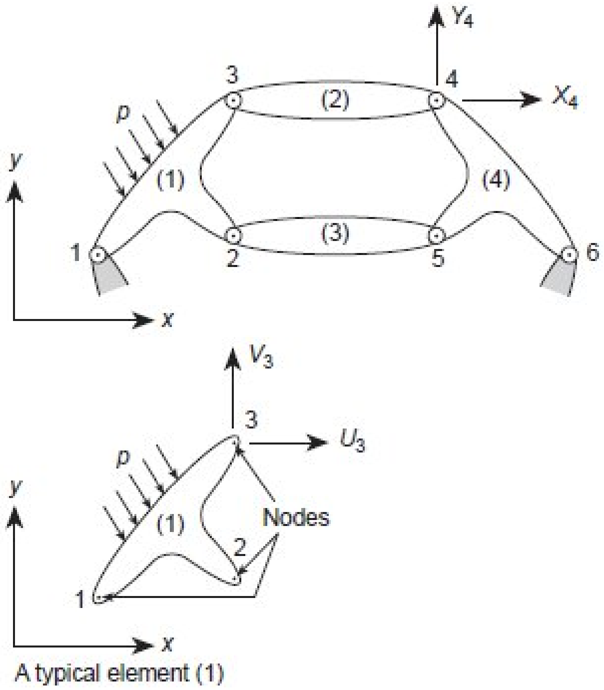


Figure 2.2 Finite element structure

Figure 2.2 represents a two dimensional structure assembled from individual components and interconnected at the nodes numbered 1 to 6. the joints at the nodes, in this case, are pinned so that moments cannot be transmitted.

As a starting point it will be assumed that by separate calculation, or for that matter from the result of an experiment, the characteristics of each element are precisely known. Thus, if a typical element labelled (1) and associated with nodes 1, 2, 3 is examined, the forces acting at the nodes are uniquely defined by the displacements of these nodes, the distributed loading acting on the element (p), and its initial strain. The last may be due to temperature, shrinkage or simply an initial 'lack of fit '. The forces and the corresponding displacements are defined by appropriate components (U, V *and* u,v) in a common coordinate system.

Listing the forces acting on all the nodes (three in the case illustrated) of the element (1) as a matrix we have,

$$\mathbf{q}^1 = \begin{Bmatrix} \mathbf{q}_1^1 \\ \mathbf{q}_2^1 \\ \mathbf{q}_3^1 \end{Bmatrix} \qquad \mathbf{q}_1^1 = \begin{Bmatrix} U_1 \\ V_1 \end{Bmatrix}, \qquad \text{etc.}$$

(2.1)

and for the corresponding nodal displacements,

$$\mathbf{a}^1 = \begin{Bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \end{Bmatrix} \qquad \mathbf{a}_1 = \begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix}, \qquad \text{etc.}$$

(2.2)

Assuming linear elastic behaviour of the element, the characteristic relationship will always be of the form,

$$\mathbf{q}^1 = \mathbf{K}^1\mathbf{a}^1 + \mathbf{f}_p^1 + \mathbf{f}_{\varepsilon_0}^1 \tag{2.3 }$$

in which f_p^1 represents the nodal forces required to balance any distributed loads acting on the element and $f_{\varepsilon o}^1$ the nodal forces required to balance any initial strains such as may be caused by temperature change if the nodes are not subject to any displacement. The first of the terms represents the forces induced by displacement of the nodes.

Similarly, a preliminary analysis or experiment will permit a unique definition of stresses or internal reactions at any specified point or points of the element in terms of the nodel displacements. Defining such stresses by a matrix $\boldsymbol{\sigma}^1$ a relationship of the form

$$\boldsymbol{\sigma}^1 = \mathbf{Q}^1\mathbf{a}^1 + \boldsymbol{\sigma}_{\varepsilon_0}^1$$

(2.4)

is obtained in which the two term gives the stresses due to the initial strains when no nodal displacement occurs.

Relationships in Eqs (2.3) and (2.4) have been illustrated by an example of an element with three nodes and with the interconnection points capable of transmitting only two components of force. Clearly, the same arguments and definitions will apply generally. An element (2) of the hypothetical structure will possess only two points of interconnection; others may have quite a large number of such points. Similarly, if the joints were considered as rigid, three components of generalized force and of generalized displacement would have to be considered, the last of these corresponding to a moment and rotation repectively. For a rigidly jointed, three dimensional structure the number of individual nodal components would be six. Quite generally, therefore,

$$\mathbf{q}^e = \begin{Bmatrix} \mathbf{q}_1^e \\ \mathbf{q}_2^e \\ \vdots \\ \mathbf{q}_m^e \end{Bmatrix} \quad \text{and} \quad \mathbf{a}^e = \begin{Bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_m \end{Bmatrix} \tag{2.5}$$

with each $\boldsymbol{q}_i^e$ and $\boldsymbol{a}_i$ possessing the same number of components or *degrees of freedom*. These quantities are conjugate to each other.

The stiffness matrices of the element will clearly always be square and of the form,

$$\mathbf{K}^e = \begin{bmatrix} \mathbf{K}_{ii}^e & \mathbf{K}_{ij}^e & \cdots & \mathbf{K}_{im}^e \\ \vdots & \vdots & & \vdots \\ \mathbf{K}_{mi}^e & \cdots & \cdots & \mathbf{K}_{mm}^e \end{bmatrix} \tag{2.6}$$

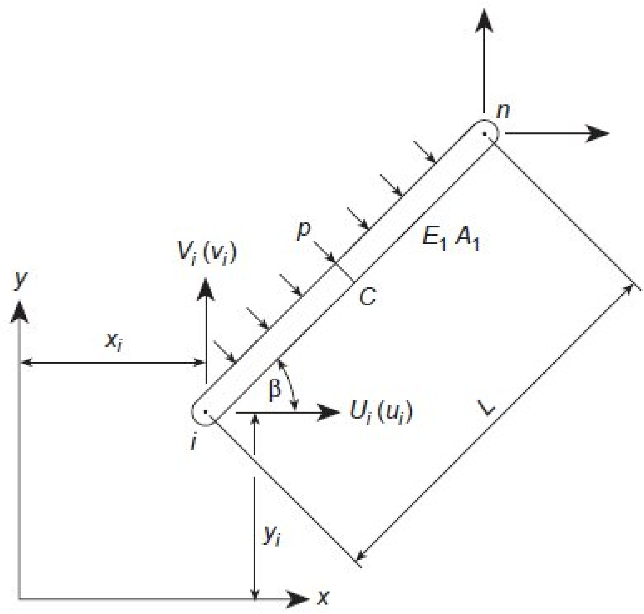


Figure 2.3 Coordinates definition

in which $\boldsymbol{K}^e_{ii}$, etc. are submatrices which are again square and of the size l x l , where l is the number of force components to be considered at each node.

If the ends of the bar are defined by the coordinates x_i , y_i, and x_n , y_n its length can be calculated as

$$L = \sqrt{[(x_n - x_i)^2 + (y_n - y_i)^2]} \tag{2.7}$$

and its inclination from the horizontal as

$$\beta = \tan^{-1}\frac{y_n - y_i}{x_n - x_i} \tag{2.8}$$

Only two components of force and displacement have to be considered at the nodes. The nodal forces due to the lateral load are clearly

$$\mathbf{f}_p^e = \begin{Bmatrix} U_i \\ V_i \\ U_n \\ V_n \end{Bmatrix}_p = -\begin{Bmatrix} -\sin\beta \\ \cos\beta \\ -\sin\beta \\ \cos\beta \end{Bmatrix} \frac{pL}{2} \tag{2.9}$$

and represent the appropriate components of simple reactions *pL/2*. similiarly, to restrain the thermal expansion ε_o an axial force ($E\alpha TA$) is needed, which gives the components

$$\mathbf{f}_{\varepsilon_0}^e = \begin{Bmatrix} U_i \\ V_i \\ U_n \\ V_n \end{Bmatrix}_{\varepsilon_0} = -\begin{Bmatrix} -\cos\beta \\ -\sin\beta \\ \cos\beta \\ \sin\beta \end{Bmatrix} (E\alpha TA) \tag{2.10}$$

Finally, the element displacements

$$\mathbf{a}^e = \begin{Bmatrix} u_i \\ v_i \\ u_n \\ v_n \end{Bmatrix} \tag{2.11}$$

will cause an elongation ($u_n - u_i$)$cos\beta$ + ($v_n - v_i$)$sin\beta$. This, when multiplied by *EA / L*, gives the axial force whose components can again be found. Rearranging these in the standart form gives,

$$\mathbf{K}^e\mathbf{a}^e = \begin{Bmatrix} U_i \\ V_i \\ U_n \\ V_n \end{Bmatrix}_\delta \tag{2.12}$$

$$= \frac{EA}{L}\left[\begin{array}{cc|cc} \cos^2\beta & \sin\beta\cos\beta & -\cos^2\beta & -\sin\beta\cos\beta \\ \sin\beta\cos\beta & \sin^2\beta & -\sin\beta\cos\beta & -\sin^2\beta \\ \hline -\cos^2\beta & -\sin\beta\cos\beta & \cos^2\beta & \sin\beta\cos\beta \\ -\sin\beta\cos\beta & -\sin^2\beta & \sin\beta\cos\beta & \sin^2\beta \end{array}\right]\begin{Bmatrix} u_i \\ v_i \\ u_n \\ v_n \end{Bmatrix}$$

(2.13)

The components of the general eq.2.3 have thus been established for the elementary case discussed. It is again quite simple to find the stresses at any section of the element in the form of relation eq.2.4. For instance, if attention is focused on the mid section *C* of the bar the average stress determined from the axial tension to the element can be shown to be,

$$\boldsymbol{\sigma}^e \approx \sigma = \frac{E}{L}[-\cos\beta, -\sin\beta, \cos\beta, \sin\beta]\mathbf{a}^e - E\alpha T \tag{2.14}$$

where all the bending effects of the lateral load *p* have been ignored.

For more complex elements more sophisticated procedures of analysis are required but the results are of the same form. The engineer will readily recognize that the socalled ‘slope-deflection’ relations used in analysis of rigid frames are only a special case of the general relations.

It may perhaps be remarked, in passing, that the complete stiffness matrix obtained for the simple element in tension turns out to be symmetric (as indeed was the case with some submatrices). This is by no means fortuitous but follows from the principle of energy conservation and form its corollary, the well-known Maxwell-Betti reciprocal theorem.

The element properties were assumed to follow a simple linear relationship. In principle, similar relationships could be established for non-linear materials, but discussion of such problems will be held over at this stage.

2.3 The Standard Discrete System

In the *standard discrete system*, whether it is structral or of any other kind, we find that;

1 – A set of discrete parameters, say $\boldsymbol{a}_i$, can be identified which describes simultaneously the behaviour of each element, *e*, and of the whole system. We shall call these the *system parameters*.

2 – For each element a set of quantities $\mathbf{q}^e_i$ can be computed in terms of the system parameters $\boldsymbol{a}_i$. The general function relationship can be non-linear

$$\mathbf{q}_i^e = \mathbf{q}_i^e(\mathbf{a}) \tag{2.15}$$

but in many cases a linear form exists giving

$$\mathbf{q}_i^e = \mathbf{K}_{i1}^e \mathbf{a}_1 + \mathbf{K}_{i2}^e \mathbf{a}_2 + \cdots + \mathbf{f}_i^e \tag{2.16}$$

3 – The *system equations* are obtained by a simple addition

$$\mathbf{r}_i = \sum_{e=1}^{m} \mathbf{q}_i^e \tag{2.17}$$

where $\mathbf{r}_i$ are system quantities. In the linear case this results in a system of equations

$$\mathbf{K}\mathbf{a} + \mathbf{f} = \mathbf{r} \tag{2.18}$$

such that

$$\mathbf{K}_{ij} = \sum_{e=1}^{m} \mathbf{K}_{ij}^e \qquad \mathbf{f}_i = \sum_{e=1}^{m} \mathbf{f}_i^e \tag{2.19}$$

from which the solution for the system variables **a** can be found after imposing necessary boundary conditions.

The reader will observe that this definiton includes the structural, hydraulic, and electrical examples already discussed. However, it is broader. In general neither linearity nor symmetry of matrices need exist – although in many problems this will arise naturally. Further, the narrowness of interconnections existing in usual elements is not essential.

2.4 Direct Formulation Of Finite Element Characteristics

The 'prescriptions' for deriving the characteristics of a 'finite element' of continuum, which were outlined in general terms will now be presented in more detailed mathematical form.

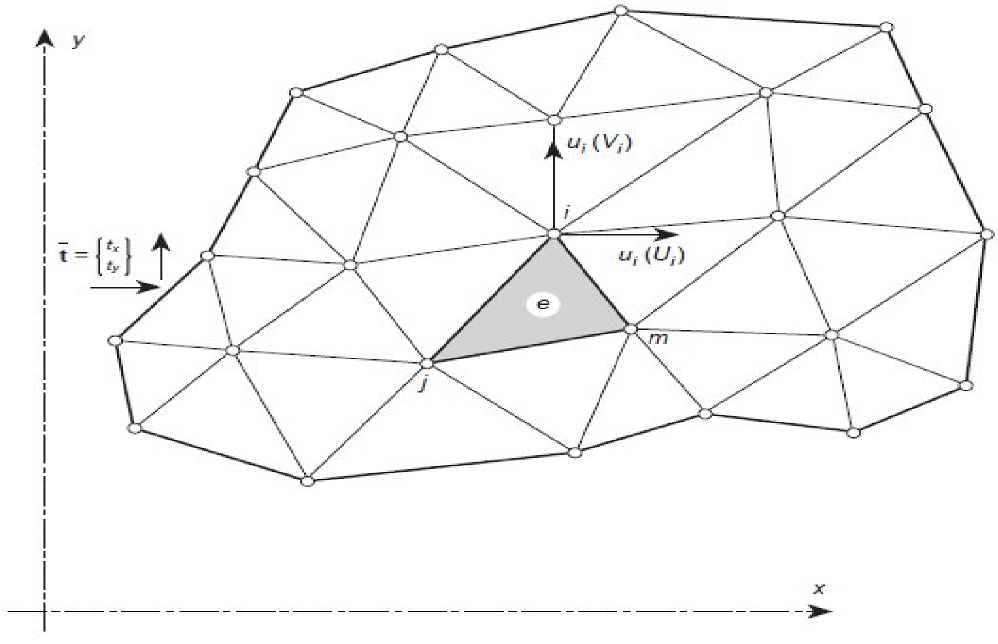


Figure 2.4 Triangular-shaped elements

It is desirable to obtain results in a general form applicable to any situation, but to avoid introducing conceptual difficulties the general relations will be illustrated with a very simple example of plane stress analysis of thin slice. In this division of the region into triangular-shaped elements is used as shown in 2.4 Relationships of general validity will be placed in a box. Again, matrix notation will be implied.

2.4.1 Displacement Function

A typical finite element, e , is defined by nodes, i,j,m, etc. and straight line boundaries. Let the displacement $\mathbf{u}$ at any point within the element be approximated as a column vector $\hat{\mathbf{u}}$

$$\mathbf{u} \approx \hat{\mathbf{u}} = \sum_k \mathbf{N}_k \mathbf{a}_k^e = [\mathbf{N}_i, \mathbf{N}_j, \ldots] \begin{Bmatrix} \mathbf{a}_i \\ \mathbf{a}_j \\ \vdots \end{Bmatrix}^e = \mathbf{N}\mathbf{a}^e \tag{2.20}$$

in which the components of $\mathbf{N}$ are prescribed functions of position and $\mathbf{a}^e$ represents a listing of nodal displacement for a particular element.

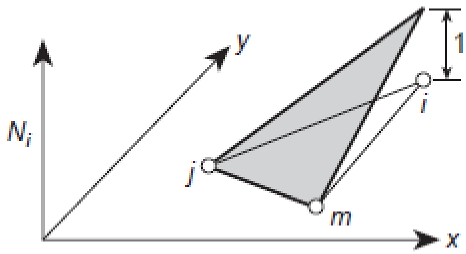


Figure 2.5 Nodal Displacement

In the case of plane stress, for instance,

$$\mathbf{u} = \begin{Bmatrix} u(x,y) \\ v(x,y) \end{Bmatrix} \tag{2.21}$$

represents horizontal and vertical movements of a typical point within the element and

$$\mathbf{a}_i = \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} \tag{2.22}$$

the corresponding displacements of a node i.

The functions N_i, N_j, N_m have to be chosen so as to give appropriate nodal displacements when the coordinates of the corresponding nodes are inserted in Eq.2.20. Clearly, in general,

$$\mathbf{N}_i(x_i, y_i) = \mathbf{I} \quad \text{(identity matrix)} \tag{2.23}$$

while

$$\mathbf{N}_i(x_j, y_j) = \mathbf{N}_i(x_m, y_m) = \mathbf{0}, \tag{2.24}$$

which simply satisfied by suitable linear functions of x and y.

If both the components of displacement are specified in an identical manner then we can write

$$\mathbf{N}_i = N_i\mathbf{I} \tag{2.25}$$

and obtain N_i from Eq. 2.20 noting that $N_i = 1$ at x_i , y_i but zero at other vertices.

The most obvious linear functions in the case of a triangle will yield the shape of N_i of the form shown in Fig.2.5. The functions **N** will be called *shape functions* and will be seen later to play a paramount role in finite element analysis.

2.4.2 Strains

With displacement known at all points within the element the 'strains' at any point can be determined. These will always result in a relationship that can be written in matrix notation as,

$$\boldsymbol{\varepsilon} \approx \hat{\boldsymbol{\varepsilon}} = \mathbf{S}\mathbf{u} \tag{2.26}$$

where **S** is a suitable linear operator. Using Eq.2.20, the above equation can be approximated as,

$$\boldsymbol{\varepsilon} \approx \hat{\boldsymbol{\varepsilon}} = \mathbf{B}\mathbf{a} \tag{2.27}$$

with

$$\mathbf{B} = \mathbf{S}\mathbf{N} \tag{2.28}$$

or the plane stress case the relevent strains of interest are those occurring in the plane or the defined in terms of the displacements by well-known relations which define the operator **S**;

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial}{\partial x}, & 0 \\ 0, & \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial y}, & \dfrac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} \tag{2.29}$$

With the shape functions N_i, N_j and N_m already determined, the matrix **B** can easily be obtained. If the linear form of these functions is adopted then, in fact, the strains will be constant throughout the element.

2.4.3 Stresses

In general, the material within the element boundaries may be subjected to initial strains such as may be due to temperature changes, shrinkage, crystal growth, and so on. If such strains are denoted by ε_0 then the stresses will be caused by the difference between the actual and intial strains.

In addition it is convenient to assume that at the outset of the analysis the body is stressed by some known system of initial residual stresses $\boldsymbol{\sigma_0}$ which, for instance, could be measured, but the prediction of which is impossible without the full knowledge of the material's history. These stresses can be simply be added on to the general definition. Thus, assuming general linear elastic behaviour, the relationship between stresses and strains will be linear and of the form

$$\boldsymbol{\sigma} = \mathbf{D}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0) + \boldsymbol{\sigma}_0 \tag{2.30}$$

where D is an elasticity matrix containing the appropriate material properties.

Again, or the particular case of plane stress three components of stress corresponding to the strains already defined have to be considered. These are, in familiar notation

$$\boldsymbol{\sigma} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \tag{2.31}$$

and the D matrix may be simply obtained from the usual isotropic stress-strain relationship

$$\begin{aligned} \varepsilon_x - (\varepsilon_x)_0 &= \frac{1}{E}\sigma_x - \frac{\nu}{E}\sigma_y \\ \varepsilon_y - (\varepsilon_y)_0 &= -\frac{\nu}{E}\sigma_x + \frac{1}{E}\sigma_y \\ \gamma_{xy} - (\gamma_{xy})_0 &= \frac{2(1+\nu)}{E}\tau_{xy} \end{aligned} \tag{2.32}$$

i.e. on solving,

$$\mathbf{D} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \tag{2.33}$$

2.5 Convergence Criteria

The assumed shape functions limit the infinite degrees of freedom of the system, and the true minimum of the energy may never be reached, irrespective of the fineness of subdivision. To ensure convergence to the correct result certain simple requirements must be satisfied. Obviously,for instance, the displacement function should be able to represent the true displacement distribution as closely as desired. It will be found that this is not so if the chosen functions are such that straining is possible when the element is subjected to rigid body displacements. Thus,the first criterion that the displacement function must obey is as follows.

> *Criterion 1*. The displacement function chosen should be such that it does not permit straining of an element to ocur when the nodal displacements are caused by a rigid body motion.

This self-evident condition can be violated easily if certain types of function are used; care must therefore be taken in the choice of displacement functions.

A second criterion stems from similar requirements. Clearly, as element get smaller nearly constant strain conditions will prevail in them. If, in fact, constant strain conditions exist, it is most desirable for good accuracy that a finite size element is able to reproduce these exactly. It is possible to formulate functions that satisfy the first criterion but at the same time require a strain variation throughout the element when the nodal displacements are compatible with a constant strain solution. Such functions will, in general, not show good convergence to an accurate solution and can not, even in the limit, represent the true strain distribution. The second criterion can therefore be formulated as follows;

> *Criterion 2*. The displacement function has to be of such a form that if nodal displacement are compatible with a constant strain condition such constant strain will in fact be obtained

It will be observed that Criterion 2 in fact incorporates the requirement of Criterion 1, as rigid body displacements are a particular case of constant strain – with a value of zero.

> *Criterion 3*. The displacement functions should be chosen such that the strains at the interface between elements are finite.

This criterion implies a certain continuity of displacements between elements. In the case of strain being defined by first derivatives, as in the plane stress example quoted here, the displacements only have to be continuous. If, however, as in the plate and shell problems, 'the strains' are defined by second derivatives of deflections, first derivatives of these have also to be continuous.

CHAPTER THREE
GAS FORCES

3.1 Thermodynamic Calculations Work Done At The Moving Boundary Of A Simple Compressible System

Work can be done by a rotating shaft, electrical work and the work done by the movement of the system boundaty, such as the work done in moving the piston in a cylinder. We will consider in some detail the work done at the moving boundary of a simple compressible system during a quasi-equilibrium process.

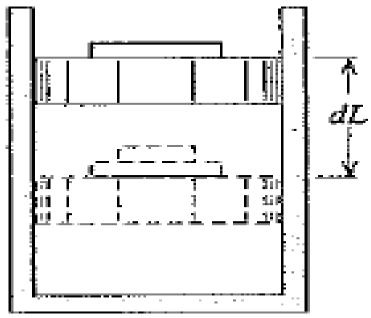


Figure 3.1 Cylinder – Piston System

Consider as a system the gas contained in a cylinder and piston as in Figure 3.1 Let one of the small weights be removed from the piston, which will cause the piston to move upward a distance *dL*. We can consider this quasi-equilibrium process and calculate the amount of work *W* done by the system during this process. The total force on the piston is *PA*, where P is the pressure of the gas and *A* is the area of the piston. Therefore the work δW is

$$\delta W = PA\ Dl \tag{3.1}$$

But $A\ dL = dV$, the change in volume of the gas. Therefore,

$$\delta W = P\ dV \tag{3.2}$$

The work done at the moving boundary during a given quasi-equilibrium process can be found by integrating Eq.3.2 However, this integration can be perfomed only if we know the relationship between *P* and *V* during this process. The relationship may be expressed in the form of an equation, or it may be shown in the form of a graph.

Let us consider a graphical solution first. We use as an example a compression process such as occurs during the compression of air in a cylinder, Fig.3.2 .At the beginning of the process the piston is at position 1, and the pressure is relatively low. This state is represented on a pressure-volume diagram. At the conclusion of the process the piston is in position 2, and the corresponding state of the gas is shown at point 2 on the P-V diagram. Let us assume that this compression was a quasi-equilibrium process and that during the process the system passed through the states shown by the line connecting states 1 and 2 on the P-V diagram. The assumption of a quasi-equilibrium process is essential here because each point on line 1-2 represents a definite state, and these states will correspond to the actual state of the system only if the deviation from equilibrium is infinitesimal. The work done on the air during this compression process can be found by integrating Eq.3.2

$$ {}_1W_2 = \int_1^2 \delta W = \int_1^2 P\,dV \tag{3.3} $$

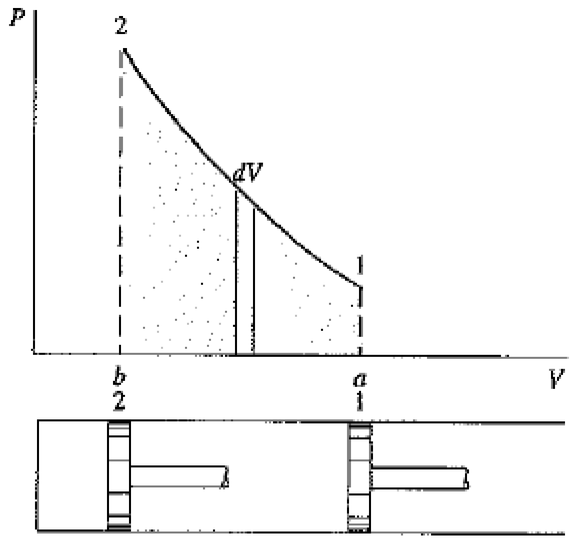


Figure 3.2 P – V relation during the compression process

The symbol $_1W_2$ is to be interpreted as the work done during the process fram state 1 to state 2. It is clear from examining the P-V diagram that the work done during this process,

$$\int_1^2 P\,dV \tag{3.4}$$

is represented by the area under the curve 1-2, area *a-1-2-b-a*. In these example the volume decreased, and the area *a-1-2-b-a* represents work done on the system. If the process had proceeded from state 2 to state 1 along the same path, the same area would represent work done by the system.

Further consideration of a *P-V* diagram, such as Fig.3.3, leads to another important conclusion. It is possible to go form state 1 to state 2 along many different quasi-equilibrium paths, such as *A,B,* or *C*. Since the area underneath each curve represents to work for each process, the amount of work done during each process not only is a function of the end states of the process but depends on the path that is followed in going from one state to another. For this reason work is called a path function or, in mathematical parlance δW is an inexact differential.

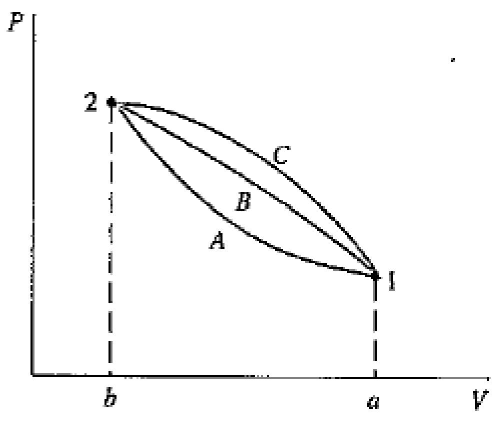


Figure 3.3 P – V (Indıcator) Diagram

This concept leads to a brief consideration of point and path functions or, to use an other term, exact and inexact differentials. Thermodynamic properties are point functions, a name that comes from the fact that for a given point on a diagram or surface, the state is fixed, and thus there is a definite value of each property corresponding to this point. The differentials of point functions are exact differentials, and the integration is simply,

$$\int_1^2 dV = V_2 - V_1 \tag{3.5}$$

Thus, we can speak of the volume in state 2 and the volume in state 1, and the change in volume depends only on the initial and final states.

Work, however, is a path function,for, as has been indicated, the work done in a quasi-equilibrium process between teo given states depends on the path followed. The differentials of path functions are inaxact differentials, and the symbol δ will be used in this text to designate inaxact differentials. Thus, for work, we write

$$\int_1^2 \delta W = {}_1W_2 \tag{3.6}$$

It would be more precise to use the notation ${}_1W_2$ which would indicate the work done during the change from state 1 to state 2 along path A. However, it is implied in the notation ${}_1W_2$ that the process between states 1 and 2 has been specified. It should be noted that we never speak about the work the system in state 1 or state 2, and thus we would never write $W_2 - W_1$.

In evaluating the integral of Eq.3.3 we should always keep in mind that we wish to determine the area under the curve in Fig.3.3 . In connection with this point, we identify the following two classes of problems.

1. The relationship between P and V is given terms of experimental data or in graphical form. Therefore, we may evaluate the integral Eq.3.3 by graphical or numerical integration.
2. The relationship between P and V makes it possible to fit an analytical relationship between them. We may then integrate directly

On common example of this second type of functional relationship is a process called a polytropic process, on in which

$$PV^n = \text{constant} \tag{3.7}$$

throughout the process. The exponent n may possible be any value from - ∞ +∞, depending on the particular process. For this type of process, we can integrate Eq.3.3 as follows

$$PV^n = \text{constant} = P_1V_1^n = P_2V_2^n$$

$$P = \frac{\text{constant}}{V^n} = \frac{P_1V_1^n}{V^n} = \frac{P_2V_2^n}{V^n}$$

$$\int_1^2 P\,dV = \text{constant}\int_1^2 \frac{dV}{V^n} = \text{constant}\left(\frac{V^{-n+1}}{-n+1}\right)\Bigg|_1^2$$

$$\int_1^2 P\,dV = \frac{\text{constant}}{1-n}\left(V_2^{1-n} - V_1^{1-n}\right) = \frac{P_2V_2^nV_2^{1-n} - P_1V_1^nV_1^{1-n}}{1-n}$$

$$= \frac{P_2V_2 - P_1V_1}{1-n} \tag{3.8}$$

Note that the resulting Eq.3.8 is valid for any exponent n except $n=1$. Where $n=1$,

$$PV = \text{constant} = P_1V_1 = P_2V_2 \tag{3.9}$$

and

$$\int_1^2 P\,dV = P_1V_1\int_1^2 \frac{dV}{V} = P_1V_1 \ln\frac{V_2}{V_1} \tag{3.10}$$

Note that in Eqs.3.8 and 3.9 we did not say that the work is equal to the expressions given in these equations. These expressions give us the value of a certain integral, that is, a mathematical results. Whether or not that integral equals the work in a particular process depends on the result of a thermodynamic analysis of that process. It is important to keep the mathematical result separate from the thermodynamic analysis, for there are many situations in which work is not given by Eq.3.3. The polytropic process as described demonstrates one special functional relationship between P and V during a process.

3.2 Calculation Of Gas Force

A relationship between the volume and the pressure is described as below,

$$P_1 V_1^{\kappa} = P_2 V_2^{\kappa} = const \tag{3.11}$$

K= polytropic index

On the basis shown in the diagram, on the Fig.3.4 XS, moving distance of the piston from the dead center to crank angle θ, is described as below,

$$x_s = r\left[1 - \cos\theta + \frac{1}{\rho}\left(1 - \sqrt{1 - \rho^2 \sin^2\theta}\right)\right] \tag{3.12}$$

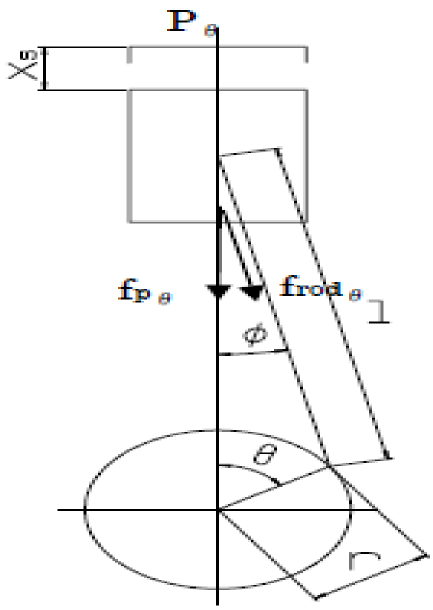


Figure 3.4 Moving distance of the piston

If inner diameter of a cylinder is D, volume of air inside of the cylinder, V_θ is the following

$$V_\theta = \frac{D^2\pi}{4} x_s = \frac{D^2\pi}{4} r\left[1 - \cos\theta + \frac{1}{\rho}\left(1 - \sqrt{1 - \rho^2 \sin^2\theta}\right)\right] \tag{3.13}$$

ρ is obtained by r/l . If bottom dead center is described by V_1,

$$V_1 = \frac{D^2\pi}{2} r \tag{3.14}$$

On the basis of a Formula above, inner pressure of the cylinder, which is a variable effected by crank angle θ, is described as below.

We describe pressure at bottom dead center (suction pressure) as P_1

The angle between Top dead center (θ = 0) and Botton dead center (θ = π) is sucking process of compressor

Therefore inner pressure of the cylinder should be suction pressure P_1

When θ = 0 ≈ π

$$f_{cy\theta}(\theta) = P_1$$

When θ = π ≈ 2π

$$f_{cy\theta}(\theta) = P_\theta = \left(\frac{V_1}{V_\theta}\right)^\kappa P_1 = \left(\frac{\frac{D^2\pi}{2} r}{\frac{D^2\pi}{4} r\left[1-\cos\theta+\frac{1}{\rho}\left(1-\sqrt{1-\rho^2\sin^2\theta}\right)\right]}\right)^\kappa P_1 = \left(\frac{2}{\left[1-\cos\theta+\frac{1}{\rho}\left(1-\sqrt{1-\rho^2\sin^2\theta}\right)\right]}\right)^\kappa P_1 \tag{3.15}$$

Load of the Piston is obtained by a formula below: multiplication of the inner pressure of the cylinder and the area of the cylinder's bore

When θ = 0 ≈ π

$$f_{p\theta}(\theta) = \frac{D^2\pi}{4} f_{cy\theta}(\theta) = \frac{D^2\pi}{4} P_1 \tag{3.16}$$

When θ = π ≈ 2π

$$f_{cy\theta}(\theta)=P_\theta=\left(\frac{V_1}{V_\theta}\right)^{\kappa}P_1=\left(\frac{\frac{D^2\pi}{2}r}{\frac{D^2\pi}{4}r\left[1-\cos\theta+\frac{1}{\rho}\left(1-\sqrt{1-\rho^2\sin^2\theta}\right)\right]}\right)^{\kappa}P_1=\left(\frac{2}{\left[1-\cos\theta+\frac{1}{\rho}\left(1-\sqrt{1-\rho^2\sin^2\theta}\right)\right]}\right)^{\kappa}P_1$$

(3.17)

For the gas force calculation we have to define necessary parameters. We are going to make a calculation for each 6 degrees of the crank angle. Necessary data are;

D_1 = 375 mm	(1. Stage Diameter)
D_2 = 260 mm	(2. Stage Diameter)
D_3 = 120 mm	(3. Stage Diameter)
r = 62.5 mm	(Crank Radius)
l = 380 mm	(Connecting Rod Length)
ρ= 0.16	(r/l value)
P_1 = 1 bar	(1. Stage Inlet Pressure)
P_2 = 4.1 bar	(2. Stage Inlet Pressure)
P_3 = 19 bar	(3. Stage Inlet Pressure)

Depends on these data, we can make a calculation by using Excel easily. In addition first stage of the compressor is double acting. So, while calculating the gas forces we have to think top side and bottom side together. In this case, we are going to think about the first stage double acting condition as 1st Group and second stage and third stage as 2nd Group.We are going to use these gas forces in Ansys dynamic load analysis and strength analysis as an input. Depends on gas forces and inertia forces we are going to obtain reaction forces in bearings and connections.

Table 3.1 First stage gas forces

		Cos(Q)	Sin(Q)	(1-RO^2*SIN^2Q)^0,5	(1-COSQ+(1/RO)*(1-(1-RO^2*SIN^2Q)^0,5	x (m)	V1 (m^3)	P2 (MPa)	F (N) Top	F (N) Bottom
0	12	1	0	1	0	0	0.001404	0.44642715	49306.36831	10623.16989
6	12.001	0.994522	0.104528	0.999852204	0.005482103	0.000342631	0.001441842	0.430190148	47513.04644	10715.27736
12	12.002	0.978148	0.207912	0.999415146	0.021868221	0.001366764	0.001554954	0.387231135	42768.36873	10871.42206
18	12.003	0.951057	0.309017	0.998707566	0.048978446	0.003061153	0.001742094	0.330537143	36506.70917	11095.68869
24	12.004	0.913545	0.406737	0.997759855	0.086515142	0.005407196	0.002001207	0.272479192	30094.40489	11394.07947
30	12.005	0.866025	0.5	0.996612814	0.134066225	0.008379139	0.002329448	0.220520839	24355.78063	11774.82491
36	12.006	0.809017	0.587785	0.995315982	0.191109716	0.011944357	0.002723214	0.177404286	19593.70314	12248.82555
42	12.007	0.743145	0.669131	0.993925568	0.257019498	0.016063719	0.003178183	0.143053241	15799.74646	12830.26284
48	12.008	0.669131	0.743145	0.992502077	0.331072224	0.020692014	0.003689363	0.096183751	10623.16989	13537.43443
54	12.009	0.587785	0.809017	0.991107715	0.412455298	0.025778456	0.004251143	0.096183751	10623.16989	14393.89314
60	12.01	0.5	0.866025	0.98980367	0.500275827	0.031267239	0.004857361	0.096183751	10623.16989	15430.00321
66	12.011	0.406737	0.913545	0.988647399	0.593570463	0.037098154	0.005501366	0.096183751	10623.16989	16685.0775
72	12.012	0.309017	0.951057	0.987690035	0.69131601	0.043207251	0.006176095	0.096183751	10623.16989	18210.3317
78	12.013	0.207912	0.978148	0.986974049	0.792440682	0.049527543	0.00687415	0.096183751	10623.16989	20072.99815
84	12.014	0.104528	0.994522	0.986531286	0.895835887	0.055989743	0.007587878	0.096183751	10623.16989	22362.09535
90	12.015	6.13E-17	1	0.986381471	1.000368403	0.062523025	0.008309457	0.096183751	10623.16989	25196.57014
96	12.016	-0.10453	0.994522	0.986531286	1.104892813	0.069055801	0.00903098	0.096183751	10623.16989	28736.83354
102	12.017	-0.20791	0.978148	0.986974049	1.208264064	0.075516504	0.009744542	0.096183751	10623.16989	33201.10133
108	12.018	-0.30902	0.951057	0.987690035	1.309349999	0.081834375	0.01044233	0.096183751	10623.16989	38888.36215
114	12.019	-0.40674	0.913545	0.988647399	1.407043749	0.087940234	0.011116701	0.096183751	10623.16989	49306.36831
120	12.02	-0.5	0.866025	0.98980367	1.500275827	0.093767239	0.011760274	0.096183751	10623.16989	49306.36831
126	12.021	-0.58779	0.809017	0.991107715	1.588025803	0.099251613	0.012366005	0.096183751	10623.16989	49306.36831
132	12.022	-0.66913	0.743145	0.992502077	1.669333437	0.10433334	0.012927264	0.096183751	10623.16989	49306.36831
138	12.023	-0.74314	0.669131	0.993925568	1.743309149	0.108956822	0.013437912	0.096183751	10623.16989	49306.36831
144	12.024	-0.80902	0.587785	0.995315982	1.809143705	0.113071482	0.013892363	0.096183751	10623.16989	49306.36831
150	12.025	-0.86603	0.5	0.996612814	1.866117033	0.116632315	0.014285645	0.096183751	10623.16989	49306.36831
156	12.026	-0.91355	0.406737	0.997759855	1.913606057	0.119600379	0.014613457	0.096183751	10623.16989	49306.36831
162	12.027	-0.95106	0.309017	0.998707566	1.951091479	0.121943217	0.014872216	0.096183751	10623.16989	49306.36831
168	12.028	-0.97815	0.207912	0.999415146	1.978163422	0.123635214	0.015059091	0.096183751	10623.16989	49306.36831
174	12.029	-0.99452	0.104528	0.999852204	1.994525893	0.124657868	0.01517204	0.096183751	10623.16989	49306.36831
180	12.03	-1	1.23E-16	1	2	0.125	0.015209827	0.096183751	10623.16989	49306.36831
186	12.031	-0.99452	-0.10453	0.999852204	1.994525893	0.124657868	0.01517204	0.096243175	10629.73304	47207.86475
192	12.032	-0.97815	-0.20791	0.999415146	1.978163422	0.123635214	0.015059091	0.097256745	10741.67843	41775.00082
198	12.033	-0.95106	-0.30902	0.998707566	1.951091479	0.121943217	0.014872216	0.098974403	10931.38795	34866.0418
204	12.034	-0.91355	-0.40674	0.997759855	1.913606057	0.119600379	0.014613457	0.10144016	11203.72249	28093.06023
210	12.035	-0.86603	-0.5	0.996612814	1.866117033	0.116632315	0.014285645	0.104718646	11565.82011	22287.20003
216	12.036	-0.80902	-0.58779	0.995315982	1.809143705	0.113071482	0.013892363	0.108898397	12027.45949	17651.14184
222	12.037	-0.74314	-0.66913	0.993925568	1.743309149	0.108956822	0.013437912	0.114096521	12601.57475	14075.45113
228	12.038	-0.66913	-0.74314	0.992502077	1.669333437	0.10433334	0.012927264	0.120465114	13304.96423	10623.16989
234	12.039	-0.58779	-0.80902	0.991107715	1.588025803	0.099251613	0.012366005	0.12819998	14159.25403	10623.16989
240	12.04	-0.5	-0.86603	0.98980367	1.500275827	0.093767239	0.011760274	0.137552448	15192.20248	10623.16989
246	12.041	-0.40674	-0.91355	0.988647399	1.407043749	0.087940234	0.011116701	0.148845369	16439.46737	10623.16989
252	12.042	-0.30902	-0.95106	0.987690035	1.309349999	0.081834375	0.01044233	0.16249486	17947.00748	10623.16989
258	12.043	-0.20791	-0.97815	0.986974049	1.208264064	0.075516504	0.009744542	0.179039988	19774.36098	10623.16989
264	12.044	-0.10453	-0.99452	0.986531286	1.104892813	0.069055801	0.00903098	0.199183481	21999.1416	10623.16989
270	12.045	-1.8E-16	-1	0.986381471	1.000368403	0.062523025	0.008309457	0.22384777	24723.22891	10623.16989
276	12.046	0.104528	-0.99452	0.986531286	0.895835887	0.055989743	0.007587878	0.254252307	28081.30705	10623.16989
282	12.047	0.207912	-0.97815	0.986974049	0.792440682	0.049527543	0.00687415	0.292019996	32252.62053	10623.16989
288	12.048	0.309017	-0.95106	0.987690035	0.69131601	0.043207251	0.006176095	0.339322489	37477.02087	10623.16989
294	12.049	0.406737	-0.91355	0.988647399	0.593570463	0.037098154	0.005501366	0.399074466	44076.42463	10623.16989
300	12.05	0.5	-0.86603	0.98980367	0.500275827	0.031267239	0.004857361	0.44642715	49306.36831	10623.16989
306	12.051	0.587785	-0.80902	0.991107715	0.412455298	0.025778456	0.004251143	0.44642715	49306.36831	10623.16989
312	12.052	0.669131	-0.74314	0.992502077	0.331072224	0.020692014	0.003689363	0.44642715	49306.36831	10623.16989
318	12.053	0.743145	-0.66913	0.993925568	0.257019498	0.016063719	0.003178183	0.44642715	49306.36831	10623.16989
324	12.054	0.809017	-0.58779	0.995315982	0.191109716	0.011944357	0.002723214	0.44642715	49306.36831	10623.16989
330	12.055	0.866025	-0.5	0.996612814	0.134066225	0.008379139	0.002329448	0.44642715	49306.36831	10623.16989
336	12.056	0.913545	-0.40674	0.997759855	0.086515142	0.005407196	0.002001207	0.44642715	49306.36831	10623.16989
342	12.057	0.951057	-0.30902	0.998707566	0.048978446	0.003061153	0.001742094	0.44642715	49306.36831	10623.16989
348	12.058	0.978148	-0.20791	0.999415146	0.021868221	0.001366764	0.001554954	0.44642715	49306.36831	10623.16989
354	12.059	0.994522	-0.10453	0.999852204	0.005482103	0.000342631	0.001441842	0.44642715	49306.36831	10623.16989
360	12.06	1	-2.5E-16	1	0	0	0.001404	0.44642715	49306.36831	10623.16989

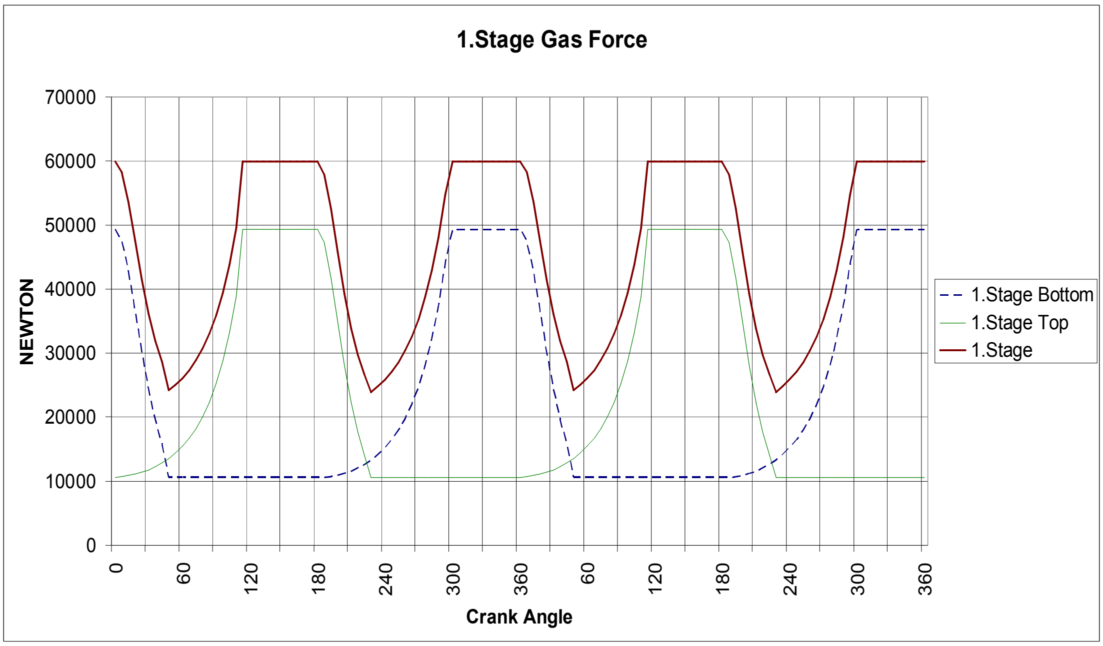


Table 3.2 Second stage gas forces

		Cos(Q)	Sin(Q)	(1-RO^2*SIN^2Q)^0,5	(1-COSQ+(1*RO)*(1-(1-RO^2*SIN^2Q)^0,5	x (m)	V1 (m^3)	P2 (Mpa)	F (N)
0	12	1	0	1	0	0	0.0009902	2.068268644	109810.4131
6	12.001	0.994522	0.104528	0.999852204	0.010941608	0.000683851	0.001026508	1.966581691	104411.5562
12	12.002	0.978148	0.207912	0.999415146	0.043472357	0.002717022	0.001134455	1.709688734	90772.3601
18	12.003	0.951057	0.309017	0.998707566	0.096720109	0.006045007	0.001311147	1.396069634	74121.40761
24	12.004	0.913545	0.406737	0.997759855	0.169264644	0.01057904	0.001551872	1.102607001	58540.62069
30	12.005	0.866025	0.5	0.996612814	0.259186654	0.016199166	0.001850261	0.861971411	45764.57557
36	12.006	0.809017	0.587785	0.995315982	0.364134279	0.022758392	0.002198509	0.677079052	35948.10111
42	12.007	0.743145	0.669131	0.993925568	0.481405065	0.030087817	0.00258765	0.538953518	28614.61377
48	12.008	0.669131	0.743145	0.992502077	0.608040614	0.038002538	0.003007866	0.436573376	23178.9535
54	12.009	0.587785	0.809017	0.991107715	0.740930513	0.046308157	0.003448835	0.402664654	21378.64059
60	12.01	0.5	0.866025	0.98980367	0.876921609	0.054807601	0.003900095	0.402664654	21378.64059
66	12.011	0.406737	0.913545	0.988647399	1.012928152	0.06330801	0.004351407	0.402664654	21378.64059
72	12.012	0.309017	0.951057	0.987690035	1.146038111	0.071627382	0.004793107	0.402664654	21378.64059
78	12.013	0.207912	0.978148	0.986974049	1.273610818	0.079600676	0.005216432	0.402664654	21378.64059
84	12.014	0.104528	0.994522	0.986531286	1.393361405	0.087085088	0.005613801	0.402664654	21378.64059
90	12.015	6.13E-17	1	0.986381471	1.503427974	0.093964248	0.005979036	0.402664654	21378.64059
96	12.016	-0.10453	0.994522	0.986531286	1.602418332	0.100151146	0.006307516	0.402664654	21378.64059
102	12.017	-0.20791	0.978148	0.986974049	1.689434199	0.105589637	0.006596262	0.402664654	21378.64059
108	12.018	-0.30902	0.951057	0.987690035	1.764072099	0.110254506	0.006843933	0.402664654	21378.64059
114	12.019	-0.40674	0.913545	0.988647399	1.826401438	0.11415009	0.007050761	0.402664654	21378.64059
120	12.02	-0.5	0.866025	0.98980367	1.876921609	0.117307601	0.007218403	0.402664654	21378.64059
126	12.021	-0.58779	0.809017	0.991107715	1.916501018	0.119781314	0.007349739	0.402664654	21378.64059
132	12.022	-0.66913	0.743145	0.992502077	1.946301826	0.121643864	0.007448627	0.402664654	21378.64059
138	12.023	-0.74314	0.669131	0.993925568	1.967694716	0.12298092	0.007519616	0.402664654	21378.64059
144	12.024	-0.80902	0.587785	0.995315982	1.982168268	0.123885517	0.007567643	0.402664654	21378.64059
150	12.025	-0.86603	0.5	0.996612814	1.991237462	0.124452341	0.007597738	0.402664654	21378.64059
156	12.026	-0.91355	0.406737	0.997759855	1.996355559	0.124772222	0.007614721	0.402664654	21378.64059
162	12.027	-0.95106	0.309017	0.998707566	1.998833142	0.124927071	0.007622942	0.402664654	21378.64059
168	12.028	-0.97815	0.207912	0.999415146	1.999767559	0.124985472	0.007626043	0.402664654	21378.64059
174	12.029	-0.99452	0.104528	0.999852204	1.999985399	0.124999087	0.007626766	0.402664654	21378.64059
180	12.03	-1	1.23E-16	1	2	0.125	0.007626814	0.402664654	21378.64059
186	12.031	-0.99452	-0.10453	0.999852204	1.999985399	0.124999087	0.007626766	0.401558846	21319.93001
192	12.032	-0.97815	-0.20791	0.999415146	1.999767559	0.124985472	0.007626043	0.401612364	21322.77143
198	12.033	-0.95106	-0.30902	0.998707566	1.998833142	0.124927071	0.007622942	0.401842065	21334.96693
204	12.034	-0.91355	-0.40674	0.997759855	1.996355559	0.124772222	0.007614721	0.402452201	21367.36082
210	12.035	-0.86603	-0.5	0.996612814	1.991237462	0.124452341	0.007597738	0.403717629	21434.54608
216	12.036	-0.80902	-0.58779	0.995315982	1.982168268	0.123885517	0.007567643	0.405976741	21554.48895
222	12.037	-0.74314	-0.66913	0.993925568	1.967694716	0.12298092	0.007519616	0.409627175	21748.30115
228	12.038	-0.66913	-0.74314	0.992502077	1.946301826	0.121643864	0.007448627	0.415126656	22040.28464
234	12.039	-0.58779	-0.80902	0.991107715	1.916501018	0.119781314	0.007349739	0.423001104	22458.36201
240	12.04	-0.5	-0.86603	0.98980367	1.876921609	0.117307601	0.007218403	0.433862034	23035.00048
246	12.041	-0.40674	-0.91355	0.988647399	1.826401438	0.11415009	0.007050761	0.448435531	23808.74991
252	12.042	-0.30902	-0.95106	0.987690035	1.764072099	0.110254506	0.006843933	0.467605844	24826.5577
258	12.043	-0.20791	-0.97815	0.986974049	1.689434199	0.105589637	0.006596262	0.492478203	26147.1038
264	12.044	-0.10453	-0.99452	0.986531286	1.602418332	0.100151146	0.006307516	0.524467938	27845.53207
270	12.045	-1.8E-16	-1	0.986381471	1.503427974	0.093964248	0.005979036	0.565426836	30020.15944
276	12.046	0.104528	-0.99452	0.986531286	1.393361405	0.087085088	0.005613801	0.617823532	32802.05279
282	12.047	0.207912	-0.97815	0.986974049	1.273610818	0.079600676	0.005216432	0.685003233	36368.819
288	12.048	0.309017	-0.95106	0.987690035	1.146038111	0.071627382	0.004793107	0.771564456	40964.60671
294	12.049	0.406737	-0.91355	0.988647399	1.012928152	0.06330801	0.004351407	0.883906833	46929.19111
300	12.05	0.5	-0.86603	0.98980367	0.876921609	0.054807601	0.003900095	1.031022329	54739.98175
306	12.051	0.587785	-0.80902	0.991107715	0.740930513	0.046308157	0.003448835	1.225610291	65071.22402
312	12.052	0.669131	-0.74314	0.992502077	0.608040614	0.038002538	0.003007866	1.485554163	78872.40215
318	12.053	0.743145	-0.66913	0.993925568	0.481405065	0.030087817	0.00258765	1.835585727	97456.59855
324	12.054	0.809017	-0.58779	0.995315982	0.364134279	0.022758392	0.002198509	2.068268644	109810.4131
330	12.055	0.866025	-0.5	0.996612814	0.259186654	0.016199166	0.001850261	2.068268644	109810.4131
336	12.056	0.913545	-0.40674	0.997759855	0.169264644	0.01057904	0.001551872	2.068268644	109810.4131
342	12.057	0.951057	-0.30902	0.998707566	0.096720109	0.006045007	0.001311147	2.068268644	109810.4131
348	12.058	0.978148	-0.20791	0.999415146	0.043472357	0.002717022	0.001134455	2.068268644	109810.4131
354	12.059	0.994522	-0.10453	0.999852204	0.010941608	0.000683851	0.001026508	2.068268644	109810.4131
360	12.06	1	-2.5E-16	1	0	0	0.0009902	2.068268644	109810.4131

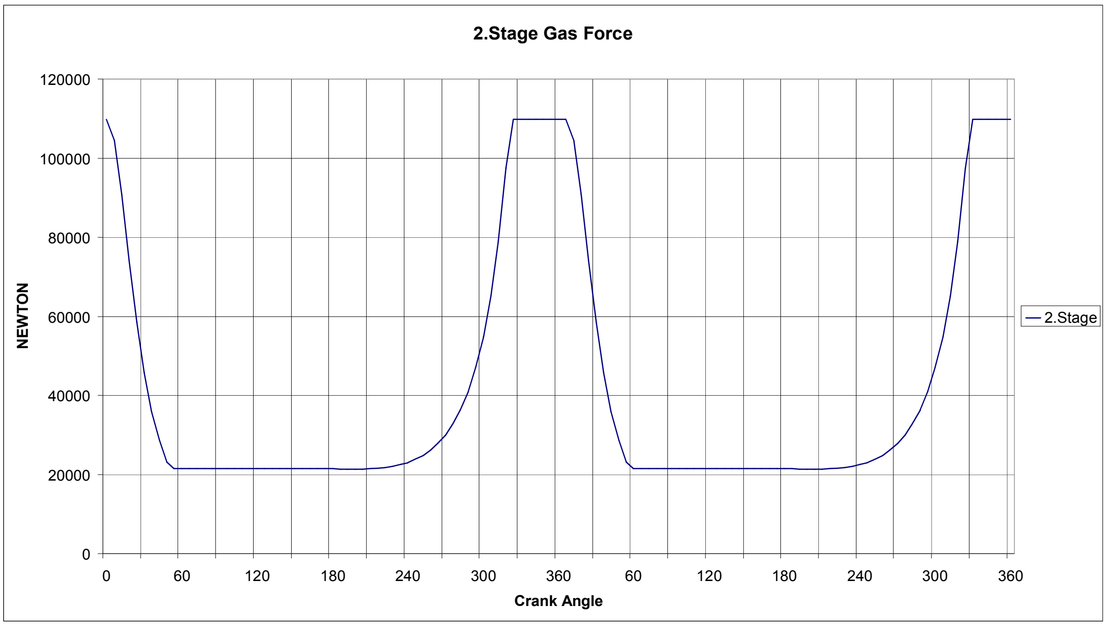


Table 3.3 Third stage gas forces

		Cos(Q)	Sin(Q)	(1-RO^2*SIN^2Q)^0,5	(1-COSQ+(1*RO)*(1-(1-RO^2*SIN^2Q)^0,5	x (m)	V1 (m^3)	P2 (Mpa)	F (N)	
0	12	1	0	1	0	0	0.0003251	4.20693015	47579.26	21524.6974
6	12.001	0.994522	0.104528	0.999852204	0.010941608	0.000683851	0.0003328	4.06822076	46010.49	21694.3291
12	12.002	0.978148	0.207912	0.999415146	0.043472357	0.002717022	0.0003558	3.69855175	41829.63	21981.3446
18	12.003	0.951057	0.309017	0.998707566	0.096720109	0.006045007	0.0003935	3.20450969	36242.15	22392.3712
24	12.004	0.913545	0.406737	0.997759855	0.169264644	0.01057904	0.0004447	2.6908751	30433.08	22937.0656
30	12.005	0.866025	0.5	0.996612814	0.259186654	0.016199166	0.0005083	2.2241505	25154.55	23628.5183
36	12.006	0.809017	0.587785	0.995315982	0.364134279	0.022758392	0.0005825	1.8982551	21468.76	24483.8145
42	12.007	0.743145	0.669131	0.993925568	0.481405065	0.030087817	0.0006654	1.8982551	21468.76	25524.7833
48	12.008	0.669131	0.743145	0.992502077	0.608040614	0.038002538	0.0007549	1.8982551	21468.76	26778.981
54	12.009	0.587785	0.809017	0.991107715	0.740930513	0.046308157	0.0008488	1.8982551	21468.76	28280.9666
60	12.01	0.5	0.866025	0.98980367	0.876921609	0.054807601	0.000945	1.8982551	21468.76	30073.9424
66	12.011	0.406737	0.913545	0.988647399	1.012928152	0.06330801	0.0010411	1.8982551	21468.76	32211.8495
72	12.012	0.309017	0.951057	0.987690035	1.146038111	0.071627382	0.0011352	1.8982551	21468.76	34762.0235
78	12.013	0.207912	0.978148	0.986974049	1.273610818	0.079600676	0.0012254	1.8982551	21468.76	37808.5147
84	12.014	0.104528	0.994522	0.986531286	1.393361405	0.087085088	0.00131	1.8982551	21468.76	41456.1639
90	12.015	6.13E-17	1	0.986381471	1.503427974	0.093964248	0.0013878	1.8982551	21468.76	45835.4413
96	12.016	-0.10453	0.994522	0.986531286	1.602418332	0.100151146	0.0014578	1.8982551	21468.76	47579.2591
102	12.017	-0.20791	0.978148	0.986974049	1.689434199	0.105589637	0.0015193	1.8982551	21468.76	47579.2591
108	12.018	-0.30902	0.951057	0.987690035	1.764072099	0.110254506	0.001572	1.8982551	21468.76	47579.2591
114	12.019	-0.40674	0.913545	0.988647399	1.826401438	0.11415009	0.0016161	1.8982551	21468.76	47579.2591
120	12.02	-0.5	0.866025	0.98980367	1.876921609	0.117307601	0.0016518	1.8982551	21468.76	47579.2591
126	12.021	-0.58779	0.809017	0.991107715	1.916501018	0.119781314	0.0016798	1.8982551	21468.76	47579.2591
132	12.022	-0.66913	0.743145	0.992502077	1.946301826	0.121643864	0.0017009	1.8982551	21468.76	47579.2591
138	12.023	-0.74314	0.669131	0.993925568	1.967694716	0.12298092	0.001716	1.8982551	21468.76	47579.2591
144	12.024	-0.80902	0.587785	0.995315982	1.982168268	0.123885517	0.0017262	1.8982551	21468.76	47579.2591
150	12.025	-0.86603	0.5	0.996612814	1.991237462	0.124452341	0.0017326	1.8982551	21468.76	47579.2591
156	12.026	-0.91355	0.406737	0.997759855	1.996355559	0.124772222	0.0017362	1.8982551	21468.76	47579.2591
162	12.027	-0.95106	0.309017	0.998707566	1.998833142	0.124927071	0.001738	1.8982551	21468.76	47579.2591
168	12.028	-0.97815	0.207912	0.999415146	1.999767559	0.124985472	0.0017387	1.8982551	21468.76	47579.2591
174	12.029	-0.99452	0.104528	0.999852204	1.999985399	0.124999087	0.0017388	1.8982551	21468.76	47579.2591
180	12.03	-1	1.23E-16	1	2	0.125	0.0017388	1.8982551	21468.76	47579.2591
186	12.031	-0.99452	-0.10453	0.999852204	1.999985399	0.124999087	0.0017388	1.8982551	21468.76	46643.6016
192	12.032	-0.97815	-0.20791	0.999415146	1.999767559	0.124985472	0.0017387	1.8984945	21471.47	44029.4205
198	12.033	-0.95106	-0.30902	0.998707566	1.998833142	0.124927071	0.001738	1.89952199	21483.09	40227.6426
204	12.034	-0.91355	-0.40674	0.997759855	1.996355559	0.124772222	0.0017362	1.90225096	21513.95	35830.5356
210	12.035	-0.86603	-0.5	0.996612814	1.991237462	0.124452341	0.0017326	1.90790953	21577.95	31352.544
216	12.036	-0.80902	-0.58779	0.995315982	1.982168268	0.123885517	0.0017262	1.91800709	21692.15	27143.6451
222	12.037	-0.74314	-0.66913	0.993925568	1.967694716	0.12298092	0.001716	1.93431142	21876.55	21524.6974
228	12.038	-0.66913	-0.74314	0.992502077	1.946301826	0.121643864	0.0017009	1.95884642	22154.03	21524.6974
234	12.039	-0.58779	-0.80902	0.991107715	1.916501018	0.119781314	0.0016798	1.99391878	22550.69	21524.6974
240	12.04	-0.5	-0.86603	0.98980367	1.876921609	0.117307601	0.0016518	2.04218114	23096.52	21524.6974
246	12.041	-0.40674	-0.91355	0.988647399	1.826401438	0.11415009	0.0016161	2.1067395	23826.66	21524.6974
252	12.042	-0.30902	-0.95106	0.987690035	1.764072099	0.110254506	0.001572	2.19131386	24783.18	21524.6974
258	12.043	-0.20791	-0.97815	0.986974049	1.689434199	0.105589637	0.0015193	2.30046547	26017.65	21524.6974
264	12.044	-0.10453	-0.99452	0.986531286	1.602418332	0.100151146	0.0014578	2.43990968	27594.73	21524.6974
270	12.045	-1.8E-16	-1	0.986381471	1.503427974	0.093964248	0.0013878	2.61694205	29596.92	21524.6974
276	12.046	0.104528	-0.99452	0.986531286	1.393361405	0.087085088	0.00131	2.84101598	32131.13	21524.6974
282	12.047	0.207912	-0.97815	0.986974049	1.273610818	0.079600676	0.0012254	4.20693015	47579.26	21524.6974
288	12.048	0.309017	-0.95106	0.987690035	1.146038111	0.071627382	0.0011352	4.20693015	47579.26	21524.6974
294	12.049	0.406737	-0.91355	0.988647399	1.012928152	0.06330801	0.0010411	4.20693015	47579.26	21524.6974
300	12.05	0.5	-0.86603	0.98980367	0.876921609	0.054807601	0.000945	4.20693015	47579.26	21524.6974
306	12.051	0.587785	-0.80902	0.991107715	0.740930513	0.046308157	0.0008488	4.20693015	47579.26	21524.6974
312	12.052	0.669131	-0.74314	0.992502077	0.608040614	0.038002538	0.0007549	4.20693015	47579.26	21524.6974
318	12.053	0.743145	-0.66913	0.993925568	0.481405065	0.030087817	0.0006654	4.20693015	47579.26	21524.6974
324	12.054	0.809017	-0.58779	0.995315982	0.364134279	0.022758392	0.0005825	4.20693015	47579.26	21524.6974
330	12.055	0.866025	-0.5	0.996612814	0.259186654	0.016199166	0.0005083	4.20693015	47579.26	21524.6974
336	12.056	0.913545	-0.40674	0.997759855	0.169264644	0.01057904	0.0004447	4.20693015	47579.26	21524.6974
342	12.057	0.951057	-0.30902	0.998707566	0.096720109	0.006045007	0.0003935	4.20693015	47579.26	21524.6974
348	12.058	0.978148	-0.20791	0.999415146	0.043472357	0.002717022	0.0003558	4.20693015	47579.26	21524.6974
354	12.059	0.994522	-0.10453	0.999852204	0.010941608	0.000683851	0.0003328	4.20693015	47579.26	21524.6974
360	12.06	1	-2.5E-16	1	0	0	0.0003251	4.20693015	47579.26	21524.6974

CHAPTER FOUR
SLIDER CRANK MECHANISM KINEMATICS

4.1 Introduction

A unique feature of all reciprocating machinery is the peculiar motion of the connecting rod linking the piston pin with the crank pin on the crankshaft. Looking at the kinematic motion of the system, the piston executes a purely translational motion, while the crank pin moves in a completely circular path. This can only be accomplished when one end of the connecting rod also moves along a linear path and the other end of the rod travels along a circular path. In other words the connecting rod has a swinging type of motion to satisfy the paths of the ends. In large compressors the forces perpendicular to the line of motion of the piston are substantial, and the piston skirt and cylinder walls are not capable of withstanding the transverse direction loads. A piston rod, crosshead and guide bar is then used to absorb these side loads. The crosshead is then connected with the connecting rod through the crosshead pin.

Since the piston moves in a straight line and the crank pin travels along a circular path, it is relatively easy to calculate the inertia forces acting on them. Once the expression for displacement is obtained, differentiating it with respect to time twice will yield acceleration, from which the inertia forces are obtained. In the connecting rod, on the other hand, besides the two end points all other points travel in an elliptical path, and this requiresa sonsiderable amount of algebraic calculations to determine the displacement, acceleration, inertia forces and subsequent integration. Fortunataly, however, this is not necessary. If the connecting rod is replaced by another structure having the same mass and center of gravity location so that the path traveled by the center of gravity is not changed, then the total inertia force of the rod is equal to that of the new structure. This follows directly from Newton's law which states that the component of the inertia force of a body in a certain direction equals the product of its mass and acceleration of the center of gravity in that direction.

With the aid of this relationship the problem can be easily solved by distributing the connecting rod mass at the reciprocating and rotating ends of the rod, so that the center of gravity location and the total weight remain uneffected. This distribution of masses of the connecting rod is the same as the one obtained by placing the two ends of the connecting rod horizontally above the center of two weight measurements scales. Note that this procedure will leave the total weight and center of gravity as in the original conencting rod, but it will not represent the inertia characteristics of the rod. The implies that the inertia forces calculated using this method will be correct, but the moments due to these forces, or the inertia couples, will not be exact. One way to obtain greater numerical accuracy is to use a finite element representation of the connecting rod.

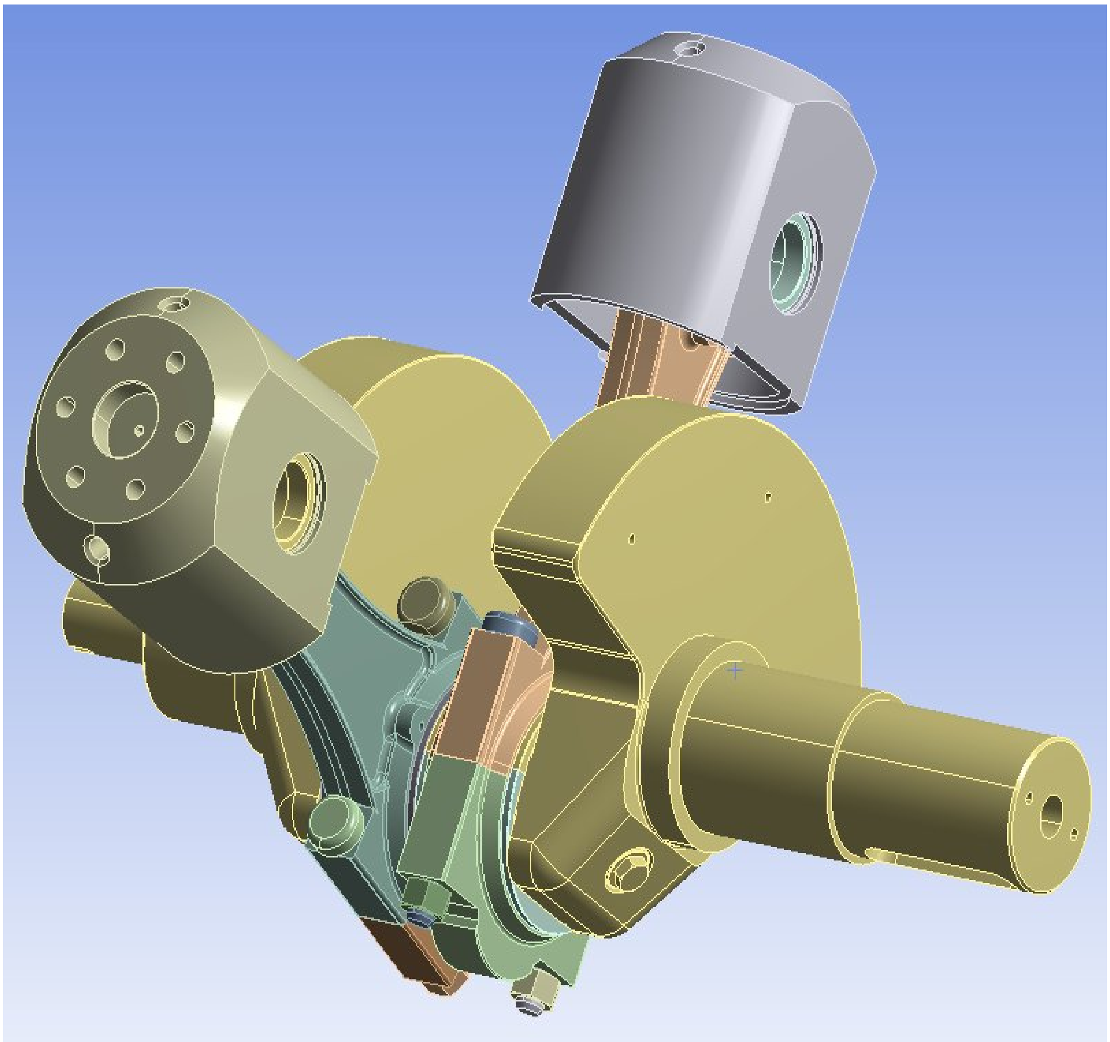


Figure 4.1 Slider crank mechanism CAD model

Mathematical expression for displacement, velocity and acceleration for both ends of the connecting rod will be developed in order to calculate inertia and gas pressure torque loads acting on it, the letter coming from the piston. The loads from the connecting rod will then traverse into the crankshaft and on to the drive train. In a single cylinder reciprocating machine only one connecting rod will be transmitting the loads to the crankshaft, as opposed to a multi-cylinder machine where more then one reciprocating mass and connecting rod act on it at a different phase angle. At first sight multi-cylinder reciprocating systems may appear very complex, but a systematic approach using phase angle relationships between individual firing systems can be developed. Once again multi-cylinder machines also come in a variety of different configurations, the most common being in line V types. In many machines, piston and connecting rod size is identical for the cylinders, which means that the reciprocating mass values for all cylinders are the same. On the case of two stage V type compressors with an intercooler, however, the cylinder sizes are different, resulting in a more elaborate mathematical model to calculate inertia and air pressure loads acting on the system components through a full cycle. A large variety of other configurations of reciprocating machines exist, each of which can be mathematically simulated and analyzed using similar guidelines that will be develped next.

4.2 Kinematic Analysis

Let Fig.4.2 represent a piston and crank, and let;

$\boldsymbol{x_p}$ = downward displacement of piston from top
$\boldsymbol{wt}$ = crank angle from top dead center
$\boldsymbol{r}$ = crank radius
$\boldsymbol{l}$ = length of connecting rod

Assume that w is constant, so the crankshaft is rotating at uniform speed. The first objective is to calculate the position of the piston in terms of the angle wt. As the crank turns through this angle, distance x_p is equal to length DB plus a term due to the fact that the connecting rod has assumed a slanting position as given by the angle ϕ. Distance *DB* is given by *r{1 – cos (wt)}*. The correction factor is given by *AC-BC*, which is equal to *l{1 – cos (ϕ)}.* Angle ϕ can be given in terms of wt by the fact that *AB=lsin(ϕ)=rsin(wt)*, or;

$$\sin(\varphi) = \frac{r}{l}\sin(\omega t) \tag{4.1}$$

and consequently,

$$\cos(\varphi) = \sqrt{1 - \frac{r^2}{l^2}\sin^2(\omega t)} \tag{4.2}$$

Thus the exact expression for piston displacement x_p in terms of crank angle *wt* is given by the expression;

$$x_p = r[1 - \cos(\omega t)] + l\left\{1 - \sqrt{1 - \frac{r^2}{l^2}\sin^2(\omega t)}\right\} \tag{4.3}$$

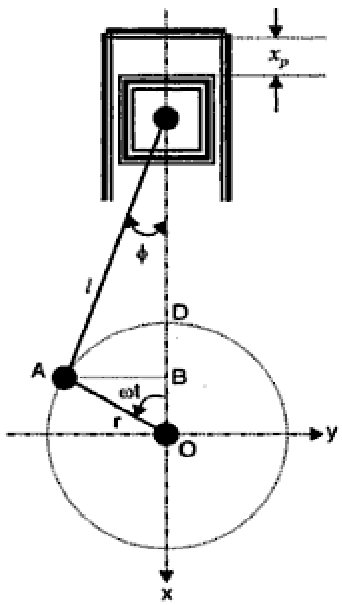


Figure 4.2 Slider crank mechanism schematic

Due to the square root this expression is not convenient for further calculation. It can be simplified by noting that for most reciprocating machines the ratio *r/l* is of the order of *1/4*,

And $(r/l)^2$ is *1/16*, so the second term under the square root sign is small in comparison to unity. Expanding into a power series and retaining only the first term,

Eq.4.3 then becomes;

$$x_p \approx r\{1 - \cos(\omega t)\} + \frac{r^2}{2l}\sin^2 \omega t \tag{4.4}$$

Further simplification is obtained through the use of trigonometric identities for $cos(2wt)$ and $sin^2(wt)$, from which the piston displacement becomes;

$$x_p = \left\{r + \frac{r^2}{4l}\right\} - r\left\{\cos(\omega t) + \frac{r}{4l}\cos(2\omega t)\right\} \tag{4.5}$$

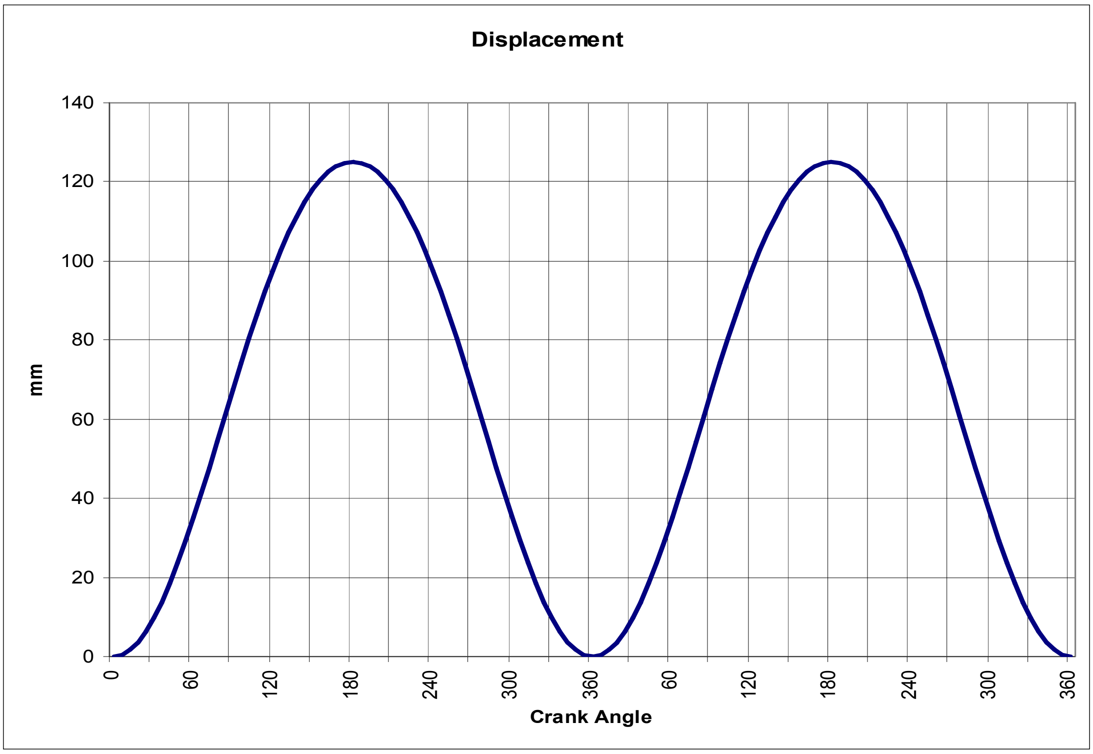
The velocity and acceleration of the piston follow from differentiation of the displacement;

$$\begin{aligned} dx_p/dt &= r\omega\,[\sin(\omega t) + (r/2l)\cos(\omega t)] \\ d^2x_p/dt^2 &= r\omega^2\,[\cos(\omega t) + (r/l)\cos(2\omega t)] \end{aligned} \tag{4.6}$$

Table 4.1 Theoretical Displacement-velocity-acceleration calculation result of the slider crank mechanism

α	Sin α	Sin2α	$sin^2α$	(r/l)*sin (α)	β	cos α	cos2α	l-cosα	x= r(l-cosα)+l(1-karekök(1-$λ^2$$sin^2$α))	v=w*r(sinα-λ/2 * sin2α)	a=w2*r(cosα-λ*cos2α)
0	0	0	0	0	0	1	1	0	0	0	572660.7086
6	0.104528	0.207912	0.010926	0.017192	0.985088	0.994522	0.978148	0.005478	0.398544199	572.2310381	571369.4627
12	0.207912	0.406737	0.043227	0.034196	1.959669	0.978148	0.913545	0.021852	1.588019587	1141.857362	567429.1999
18	0.309017	0.587785	0.095492	0.050825	2.913323	0.951057	0.809017	0.048943	3.550092582	1706.142627	560644.5988
24	0.406737	0.743145	0.165435	0.066897	3.835808	0.913545	0.669131	0.086455	6.254664055	2262.095671	550704.1052
30	0.5	0.866025	0.25	0.082237	4.717151	0.866025	0.5	0.133975	9.660542796	2806.36381	537200.211
36	0.587785	0.951057	0.345492	0.096675	5.547746	0.809017	0.309017	0.190983	13.71636459	3335.149893	519656.4882
42	0.669131	0.994522	0.447736	0.110054	6.31845	0.743145	0.104528	0.256855	18.36173265	3844.159233	497560.0979
48	0.743145	0.994522	0.552264	0.122228	7.020691	0.669131	-0.10453	0.330869	23.52854783	4328.581165	470398.2227
54	0.809017	0.951057	0.654508	0.133062	7.64657	0.587785	-0.30902	0.412215	29.14249005	4783.108259	437696.6785
60	0.866025	0.866025	0.75	0.142438	8.18897	0.5	-0.5	0.5	35.12460535	5201.994476	399058.8402
66	0.913545	0.743145	0.834565	0.150254	8.641657	0.406737	-0.66913	0.593263	41.39294824	5579.151605	354202.9803
72	0.951057	0.587785	0.904508	0.156424	8.99938	0.309017	-0.80902	0.690983	47.8642247	5908.281471	302996.1697
78	0.978148	0.406737	0.956773	0.16088	9.257952	0.207912	-0.91355	0.792088	54.45538064	6183.039604	245483.0226
84	0.994522	0.207912	0.989074	0.163573	9.414329	0.104528	-0.97815	0.895472	61.08508236	6397.22443	181907.7773
90	1	0	1	0.164474	9.466661	0	-1	1	67.67504085	6544.984695	112728.4859
96	0.994522	-0.20791	0.989074	0.163573	9.414329	-0.10453	-0.97815	1.104528	74.15114026	6621.036738	38622.41881
102	0.978148	-0.40674	0.956773	0.16088	9.257952	-0.20791	-0.91355	1.207912	80.44434199	6620.882549	-39517.83002
108	0.951057	-0.58779	0.904508	0.156424	8.99938	-0.30902	-0.80902	1.309017	86.491349	6541.019215	-120597.648
114	0.913545	-0.74314	0.834565	0.150254	8.641657	-0.40674	-0.66913	1.406737	92.23502863	6379.130472	-203342.82
120	0.866025	-0.86603	0.75	0.142438	8.18897	-0.5	-0.5	1.5	97.62460535	6134.251551	-286330.3543
126	0.809017	-0.95106	0.654508	0.133062	7.64657	-0.58779	-0.30902	1.587785	102.6156466	5806.899434	-368026.6427
132	0.743145	-0.99452	0.552264	0.122228	7.020691	-0.66913	-0.10453	1.669131	107.1698736	5399.161853	-446831.5519
138	0.669131	-0.99452	0.447736	0.110054	6.31845	-0.74314	0.104528	1.743145	111.2548358	4914.739922	-521126.7687
144	0.587785	-0.95106	0.345492	0.096675	5.547746	-0.80902	0.309017	1.809017	114.8434889	4358.941068	-589326.524
150	0.5	-0.86603	0.25	0.082237	4.717151	-0.86603	0.5	1.866025	117.9137183	3738.620885	-649928.6969
156	0.406737	-0.74314	0.165435	0.066897	3.835808	-0.91355	0.669131	1.913545	120.4478463	3062.074537	-701564.2655
162	0.309017	-0.58779	0.095492	0.050825	2.913323	-0.95106	0.809017	1.951057	122.4321571	2338.88037	-743043.1205
168	0.207912	-0.40674	0.043227	0.034196	1.959669	-0.97815	0.913545	1.978148	123.8564697	1579.700307	-773394.3925
174	0.104528	-0.20791	0.010926	0.017192	0.985088	-0.99452	0.978148	1.994522	124.7137811	796.0433464	-791899.6589
180	0	0	0	0	0	-1	1	2	125	0	-798117.6805
186	-0.10453	0.207912	0.010926	-0.01719	-0.98509	-0.99452	0.978148	1.994522	124.7137811	-796.0433464	-791899.6589
192	-0.20791	0.406737	0.043227	-0.0342	-1.95967	-0.97815	0.913545	1.978148	123.8564697	-1579.700307	-773394.3925
198	-0.30902	0.587785	0.095492	-0.05083	-2.91332	-0.95106	0.809017	1.951057	122.4321571	-2338.88037	-743043.1205
204	-0.40674	0.743145	0.165435	-0.0669	-3.83581	-0.91355	0.669131	1.913545	120.4478463	-3062.074537	-701564.2655
210	-0.5	0.866025	0.25	-0.08224	-4.71715	-0.86603	0.5	1.866025	117.9137183	-3738.620885	-649928.6969
216	-0.58779	0.951057	0.345492	-0.09668	-5.54775	-0.80902	0.309017	1.809017	114.8434889	-4358.941068	-589326.524
222	-0.66913	0.994522	0.447736	-0.11005	-6.31845	-0.74314	0.104528	1.743145	111.2548358	-4914.739922	-521126.7687
228	-0.74314	0.994522	0.552264	-0.12223	-7.02069	-0.66913	-0.10453	1.669131	107.1698736	-5399.161853	-446831.5519
234	-0.80902	0.951057	0.654508	-0.13306	-7.64657	-0.58779	-0.30902	1.587785	102.6156466	-5806.899434	-368026.6427
240	-0.86603	0.866025	0.75	-0.14244	-8.18897	-0.5	-0.5	1.5	97.62460535	-6134.251551	-286330.3543
246	-0.91355	0.743145	0.834565	-0.15025	-8.64166	-0.40674	-0.66913	1.406737	92.23502863	-6379.130472	-203342.82
252	-0.95106	0.587785	0.904508	-0.15642	-8.99938	-0.30902	-0.80902	1.309017	86.491349	-6541.019215	-120597.648
258	-0.97815	0.406737	0.956773	-0.16088	-9.25795	-0.20791	-0.91355	1.207912	80.44434199	-6620.882549	-39517.83002
264	-0.99452	0.207912	0.989074	-0.16357	-9.41433	-0.10453	-0.97815	1.104528	74.15114026	-6621.036738	38622.41881
270	-1	0	1	-0.16447	-9.46666	0	-1	1	67.67504085	-6544.984695	112728.4859
276	-0.99452	-0.20791	0.989074	-0.16357	-9.41433	0.104528	-0.97815	0.895472	61.08508236	-6397.22443	181907.7773
282	-0.97815	-0.40674	0.956773	-0.16088	-9.25795	0.207912	-0.91355	0.792088	54.45538064	-6183.039604	245483.0226
288	-0.95106	-0.58779	0.904508	-0.15642	-8.99938	0.309017	-0.80902	0.690983	47.8642247	-5908.281471	302996.1697
294	-0.91355	-0.74314	0.834565	-0.15025	-8.64166	0.406737	-0.66913	0.593263	41.39294824	-5579.151605	354202.9803
300	-0.86603	-0.86603	0.75	-0.14244	-8.18897	0.5	-0.5	0.5	35.12460535	-5201.994476	399058.8402
306	-0.80902	-0.95106	0.654508	-0.13306	-7.64657	0.587785	-0.30902	0.412215	29.14249005	-4783.108259	437696.6785
312	-0.74314	-0.99452	0.552264	-0.12223	-7.02069	0.669131	-0.10453	0.330869	23.52854783	-4328.581165	470398.2227
318	-0.66913	-0.99452	0.447736	-0.11005	-6.31845	0.743145	0.104528	0.256855	18.36173265	-3844.159233	497560.0979
324	-0.58779	-0.95106	0.345492	-0.09668	-5.54775	0.809017	0.309017	0.190983	13.71636459	-3335.149893	519656.4882
330	-0.5	-0.86603	0.25	-0.08224	-4.71715	0.866025	0.5	0.133975	9.660542796	-2806.36381	537200.211
336	-0.40674	-0.74314	0.165435	-0.0669	-3.83581	0.913545	0.669131	0.086455	6.254664055	-2262.095671	550704.1052
342	-0.30902	-0.58779	0.095492	-0.05083	-2.91332	0.951057	0.809017	0.048943	3.550092582	-1706.142627	560644.5988
348	-0.20791	-0.40674	0.043227	-0.0342	-1.95967	0.978148	0.913545	0.021852	1.588019587	-1141.857362	567429.1999
354	-0.10453	-0.20791	0.010926	-0.01719	-0.98509	0.994522	0.978148	0.005478	0.398544199	-572.2310381	571369.4627
360	-2.5E-16	-4.9E-16	6E-32	-4E-17	-2.3E-15	1	1	0	0	-1.33995E-12	572660.7086

Table 4.2 Theoretical displacement of the slider crank mechanism diagram



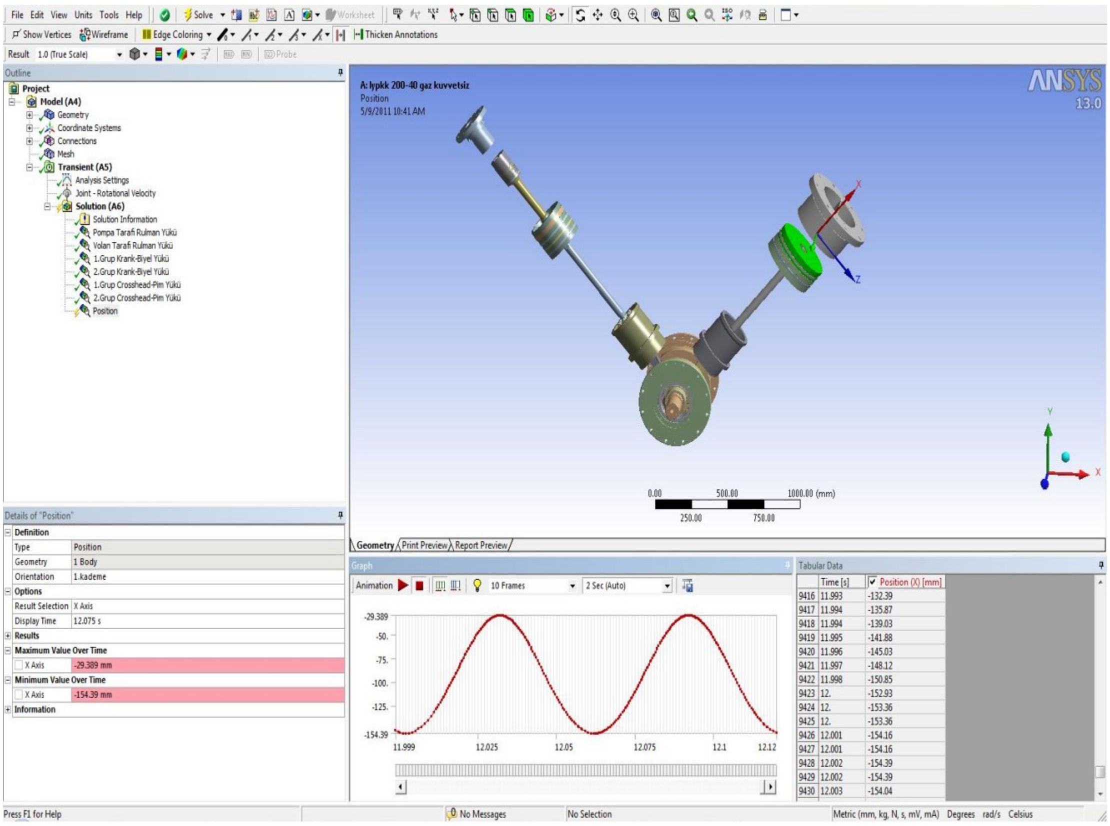
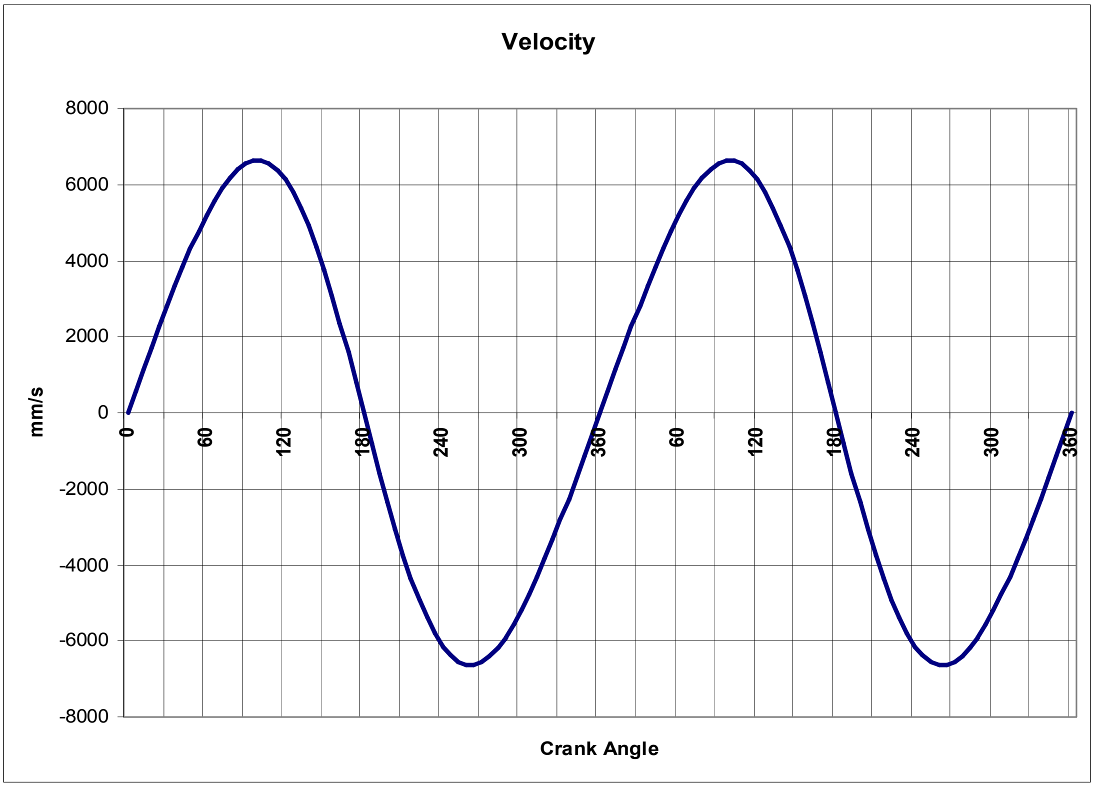


Figure 4.3 Ansys displacement result of the slider crank mechanism

Table 4.3 Theoretical velocity of the slider crank mechanism diagram



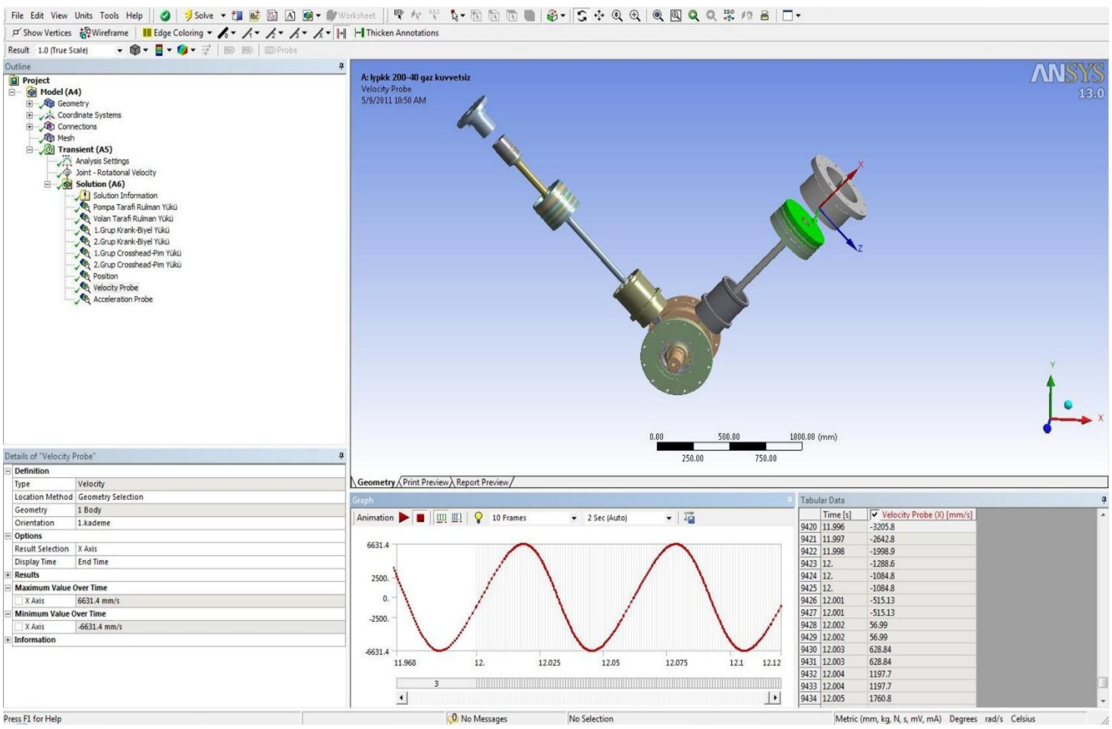
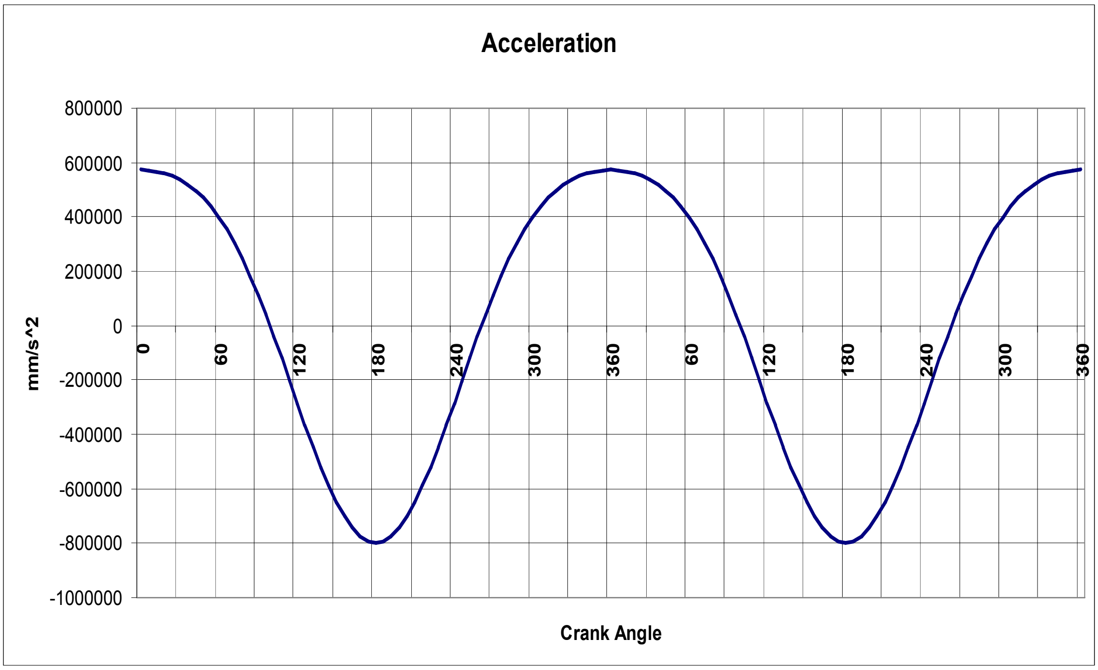


Figure 4.4 Ansys velocity result of the slider crank mechanism

Table 4.4 Theoretical acceleration of the slider crank mechanism diagram



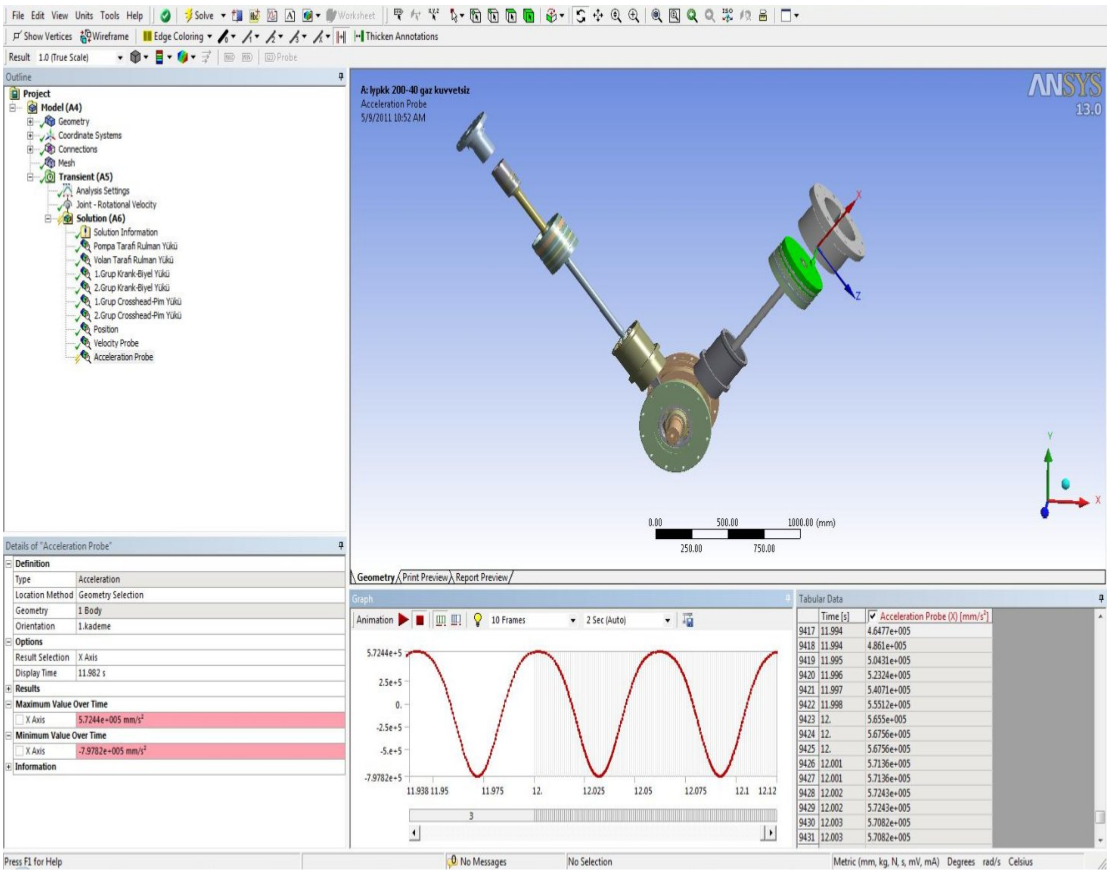


Figure 4.5 Ansys acceleration result of the slider crank mechanism

CHAPTER FIVE
DYNAMIC LOADS

5.1 Dynamic Load Analysis

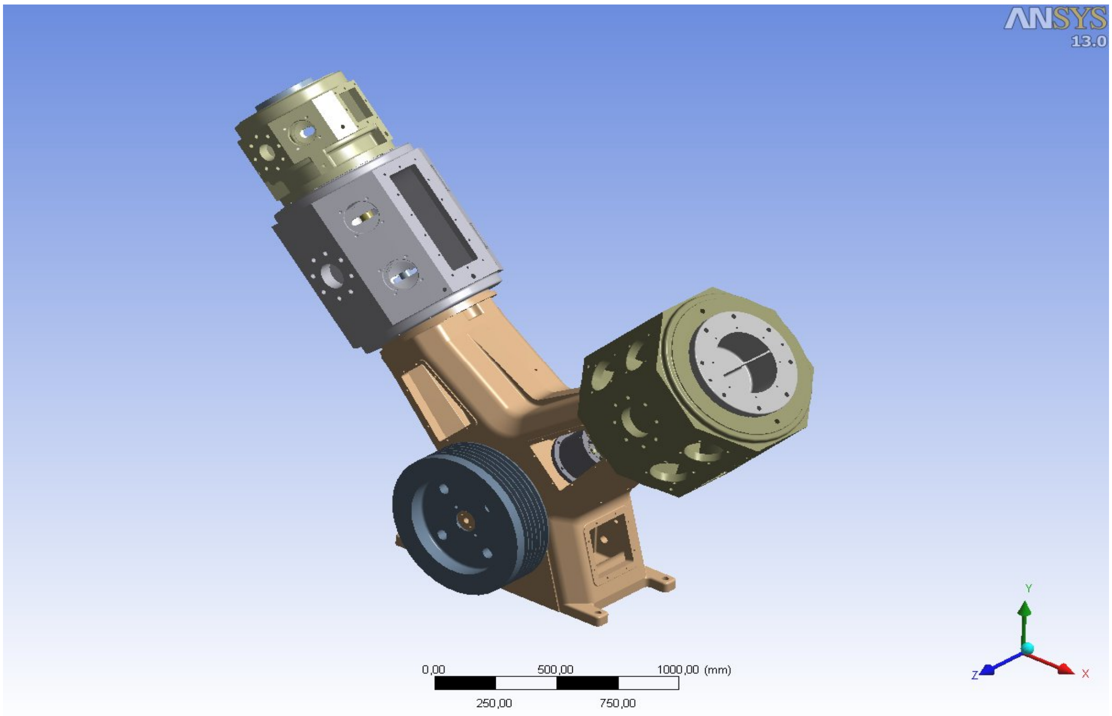


Figure 5.1 Ansys model of the oil free reciprocating compressor

Before starting the calculations and design, we have to define some criterias for the system. We can specify some of the dimensions (strok, cylinder dia, approximate thickness of the parts etc.) depends on gas calculations and main properties of the compressor. We made pre-design for each part with experience and knowladge of the Company, on the other hand examination of the same compressors of other Companies is useful way to make the pre-design.

We are going to make calculations with using this pre-design. In kinematic analysis we are going to obtain displacement, velocity, acceleration and inertia forces of the system. In dynamic analysis we are going to calculate reaction forces for each joint and connection area under the influence of gas force. In the next chapters we are going to use these forces for the strength analysis.

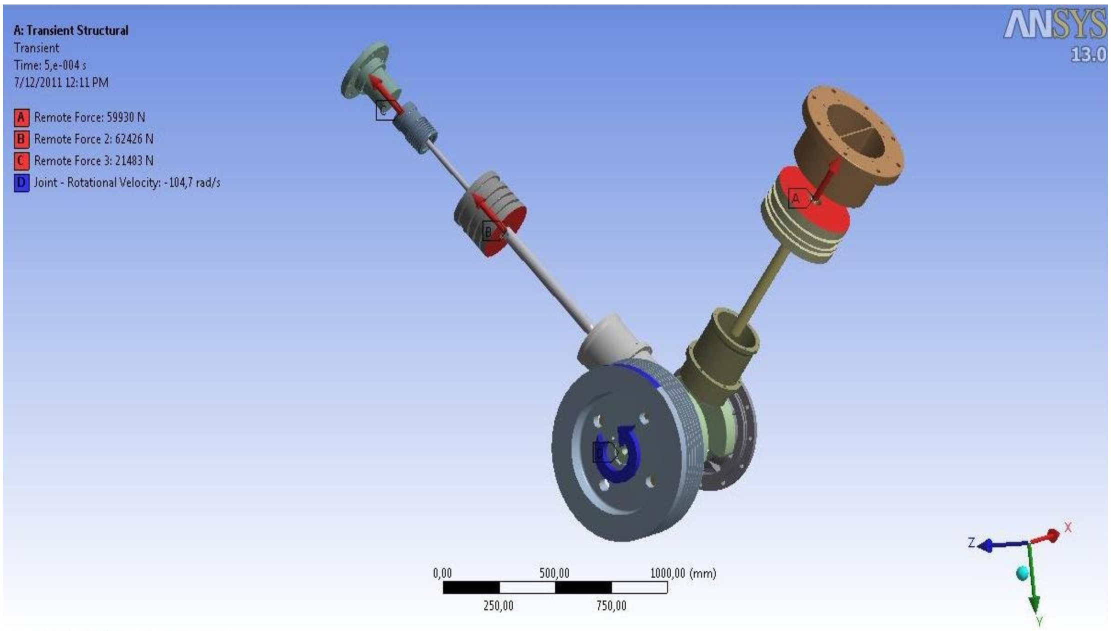
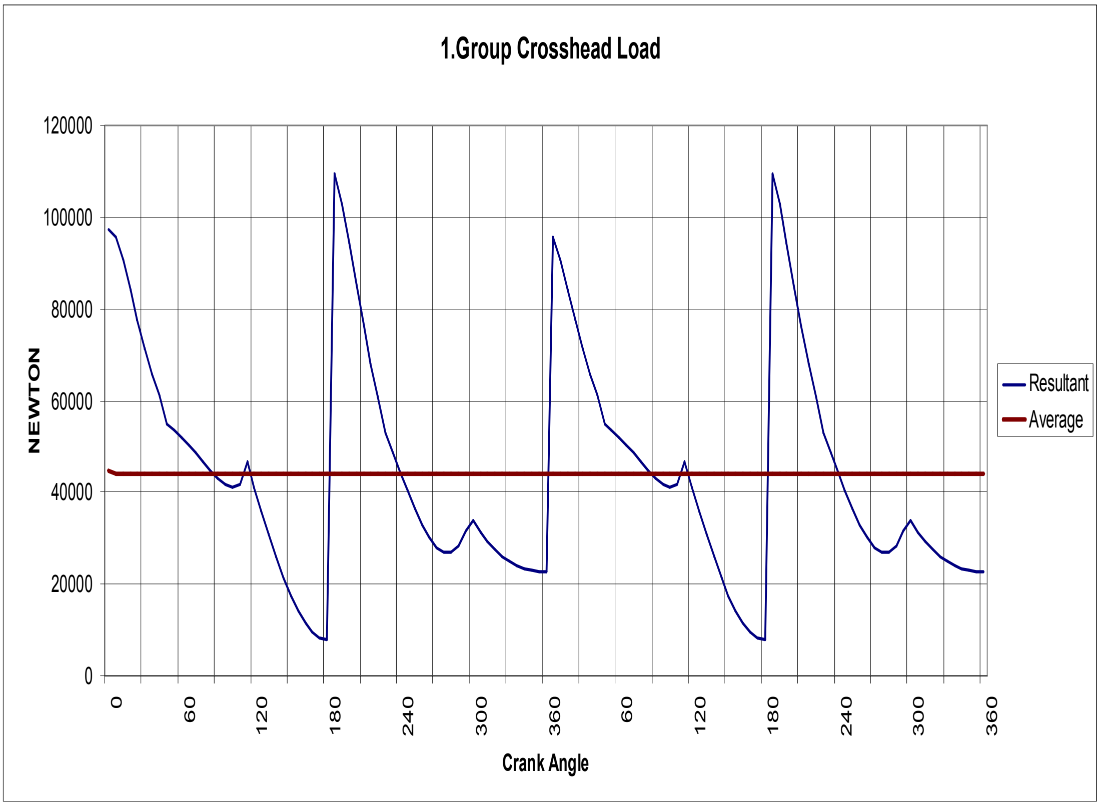


Figure 5.2 Loading conditions

In figure 5.2 we can see the inputs of the analysis. We defined rotational velocity on the flywheel as 104.7 rad/s. Also we applied gas forces which we calculated previous chapter to the pistons depends on the rotation direction and phase angle. In this analysis our system reached the max. rotational velocity in 9.6 second and than keep going working with constant rotational velocity until the end of the analysis. For the most available convergence time and value we have to specify step number and step time. For the specification of the step time and number our main factor is rotational velocity. Our compressor working speed is 1000 rpm. After making the necessary calculations, we found our rotational velocity 104.7 rad/s. We need the time which is pass during the 1 degree rotation of the crankshaft for specification of the step number and time. According to the 1000 rpm rotation speed of the compressor, 1 degree rotation of the crankshaft happens in 0.0016666 second. For suitable convergence value and time, we prepared analysis with 0.01 second step time and with this situation we examined data for each 6 degree rotation of the crankshaft. We applied gas forces for each 6 degrees of the crankshaft in 720 degree period. Also, we had results for each 6 degrees rotation of the crankshaft.

5.1.1 Crosshead Loads

Table 5.1 1st Group crosshead load Ansys result



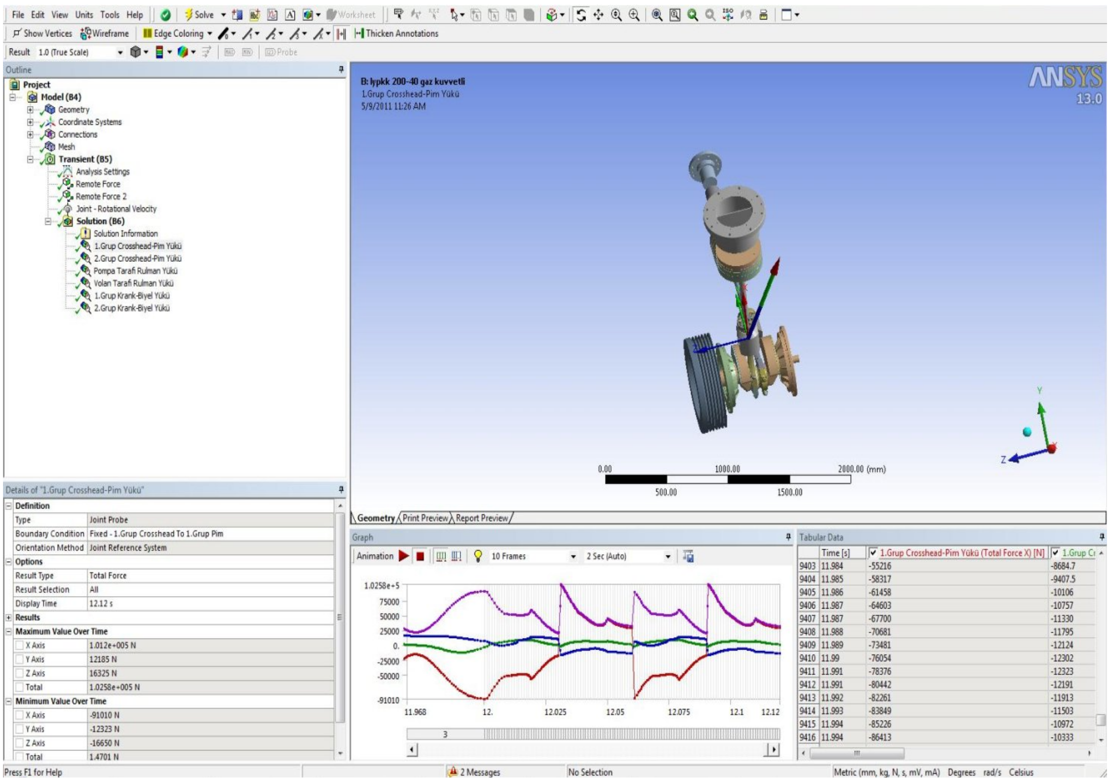
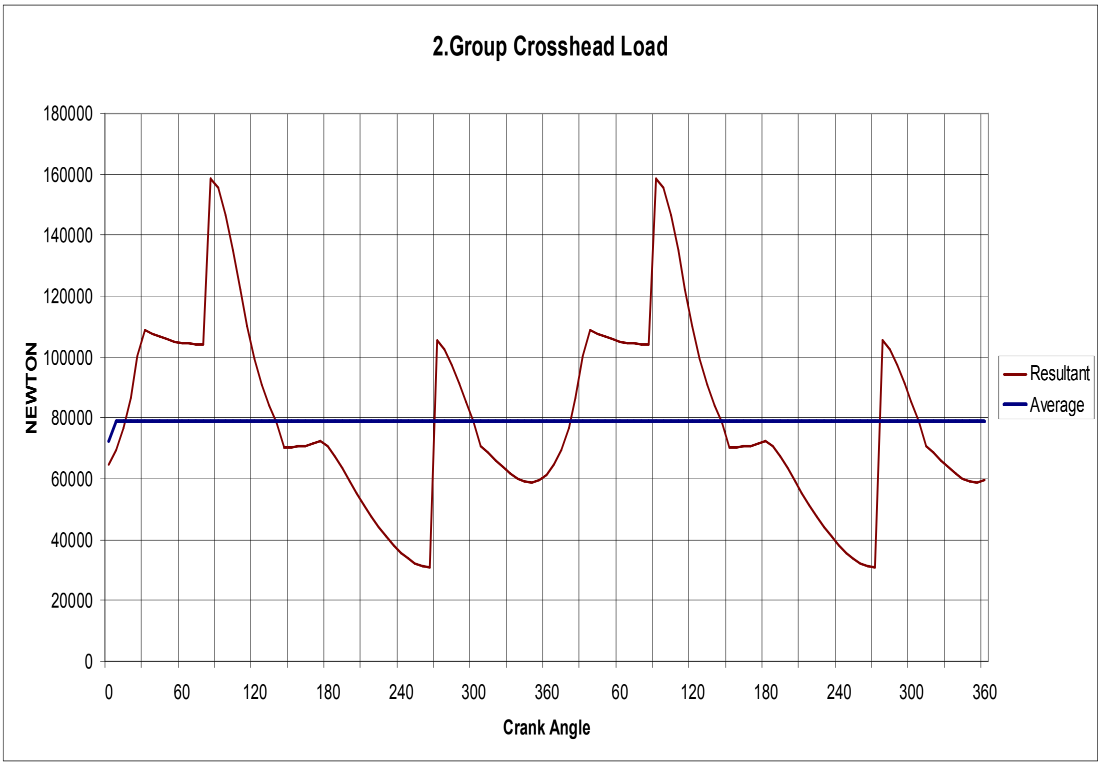


Figure 5.3 1st group crosshead load Ansys result

Table 5.2 2nd Group crosshead load Ansys result



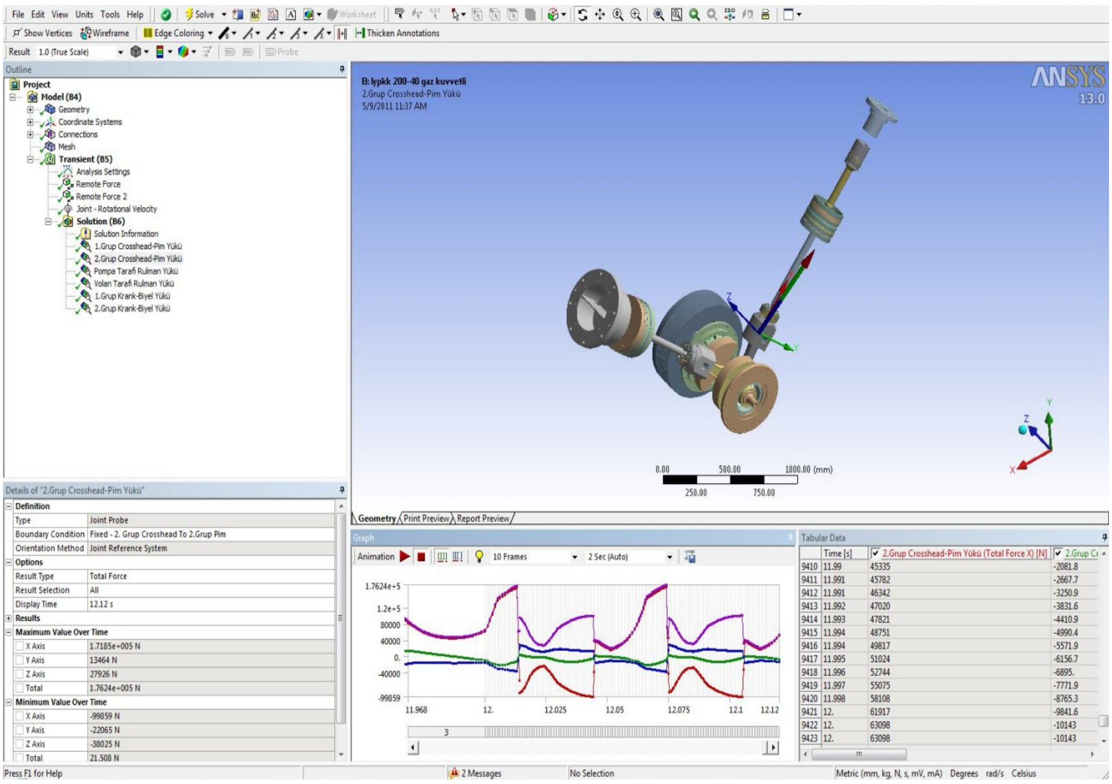
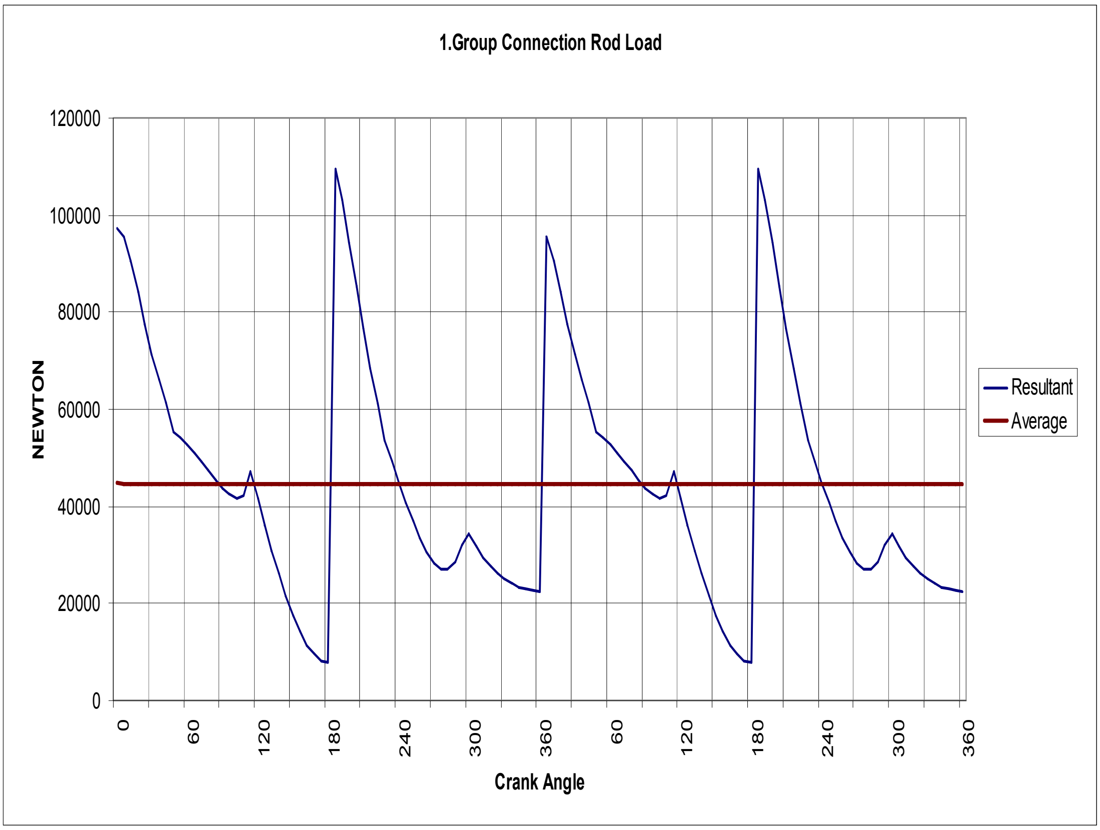


Figure 5.4 2nd group crosshead load Ansys result

5.1.2 Connecting Rod Loads

Table 5.3 1st Group connecting rod load Ansys result



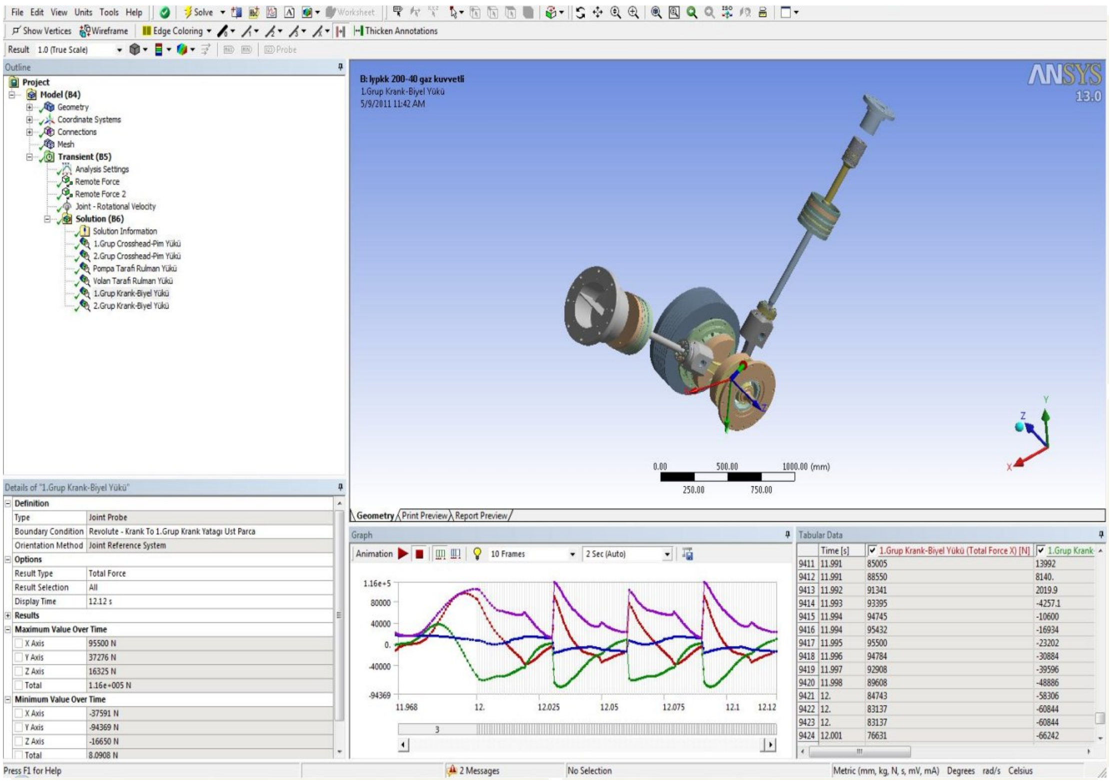
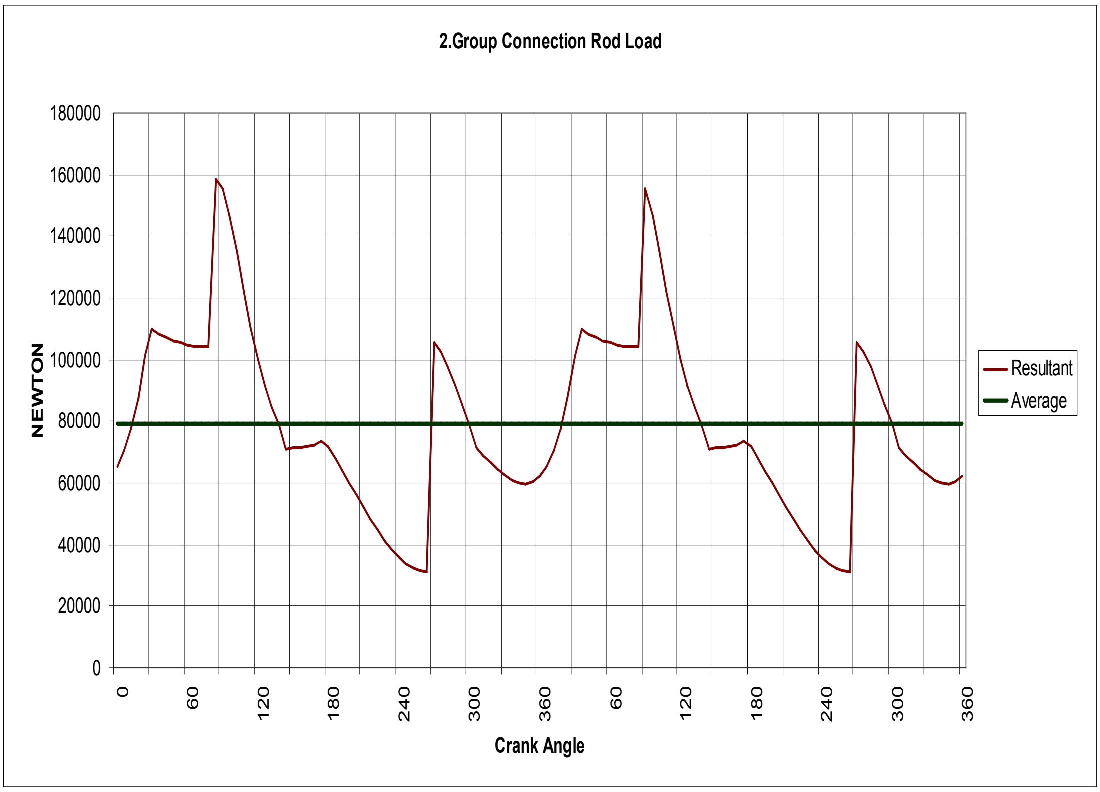


Figure 5.5 1st group connecting rod load Ansys result

Table 5.4 2nd Group connecting rod load Ansys result



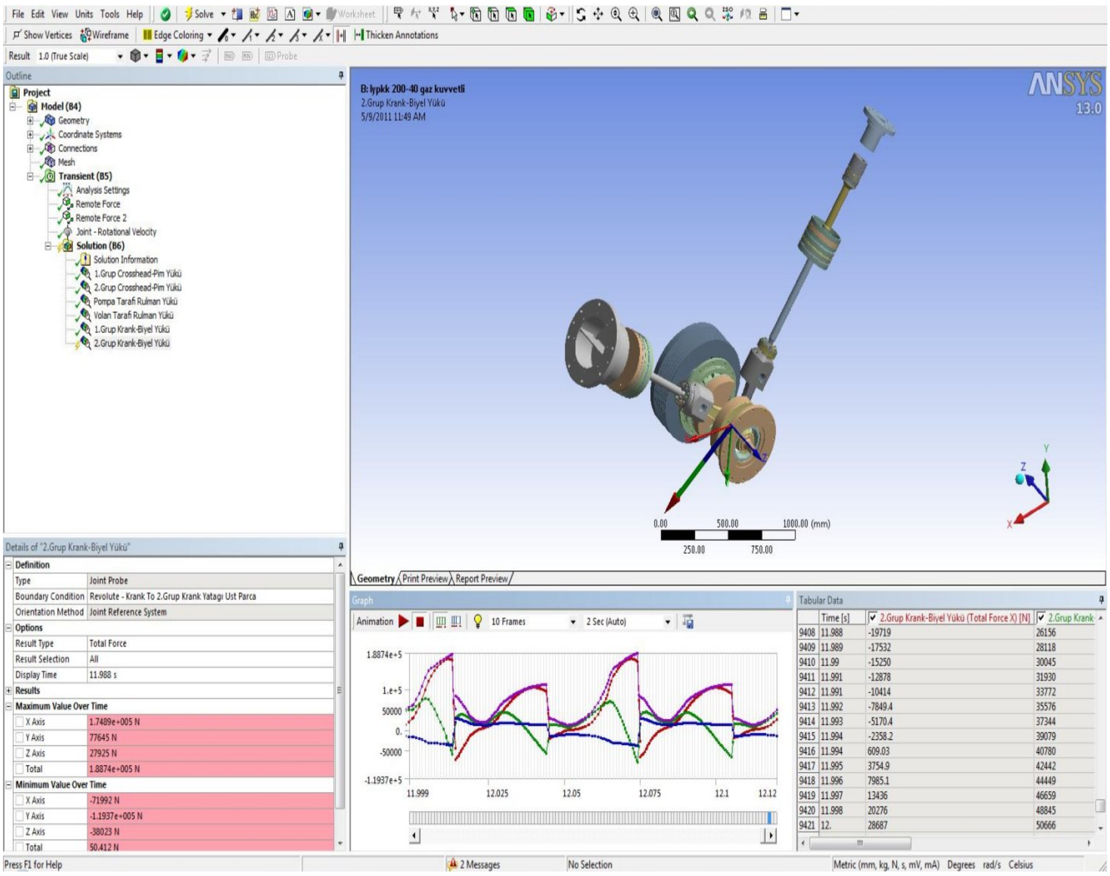
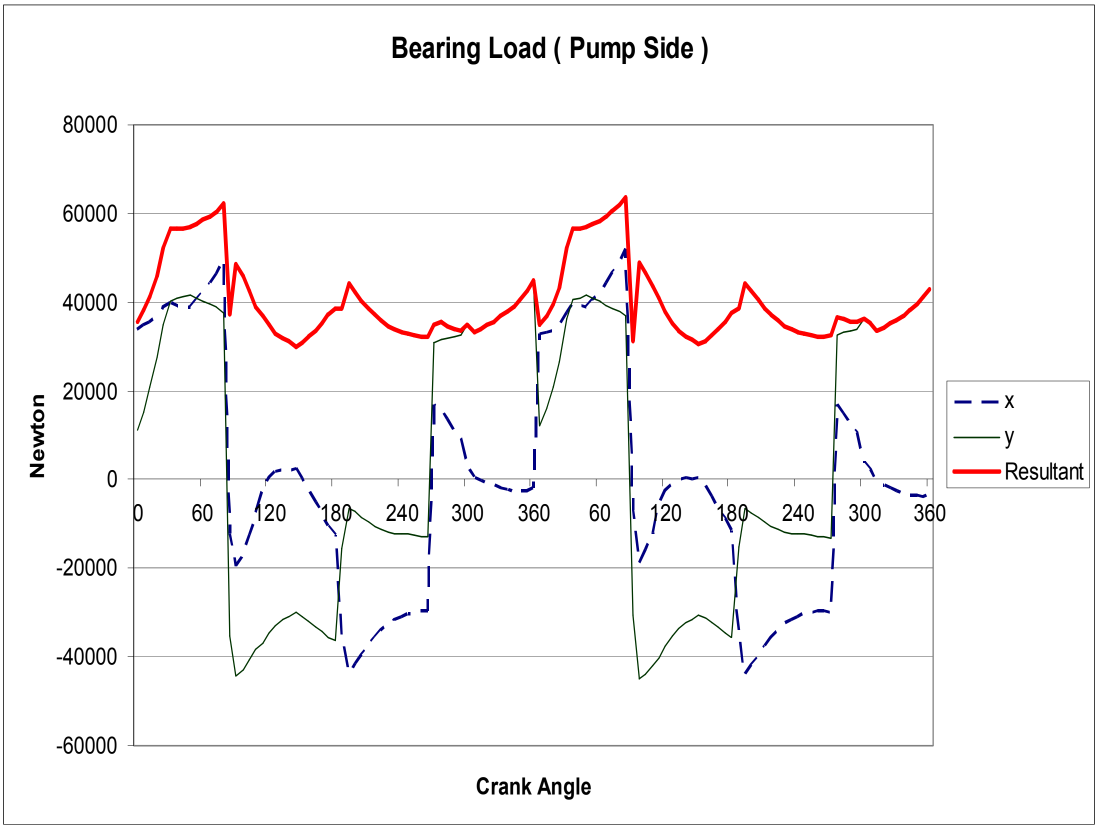


Figure 5.6 2nd group connecting rod load Ansys result

5.1.3 Bearing Loads

Table 5.5 Pump side bearing load Ansys result



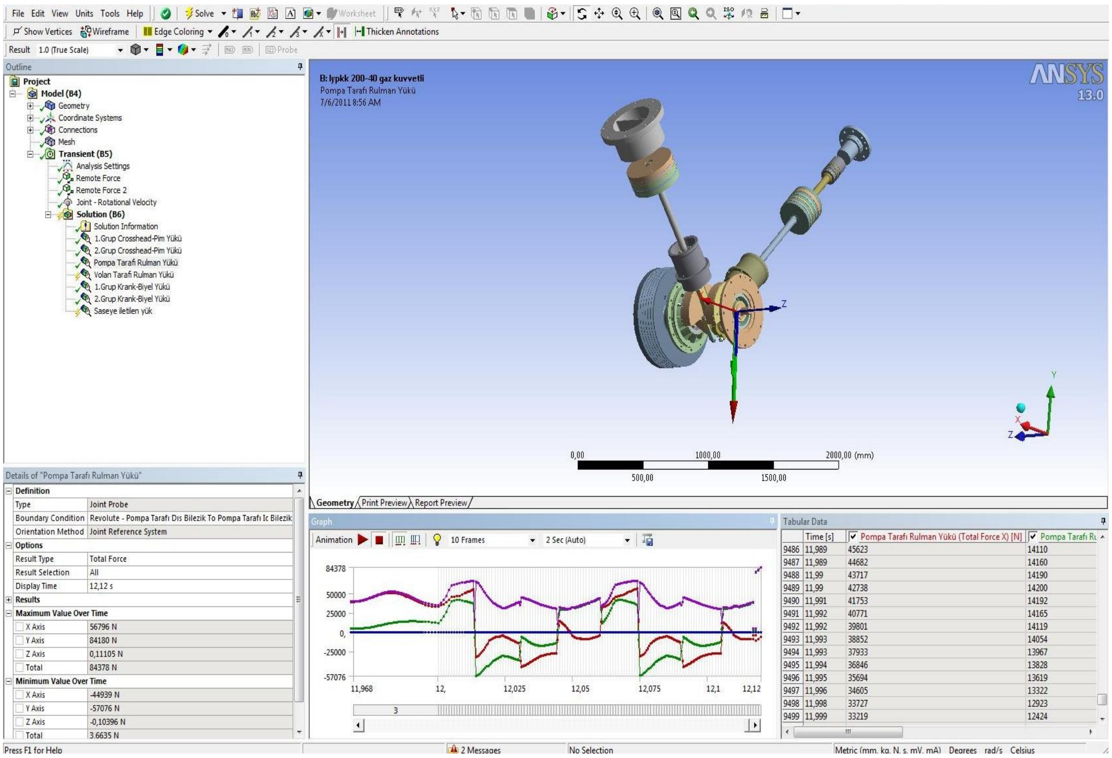
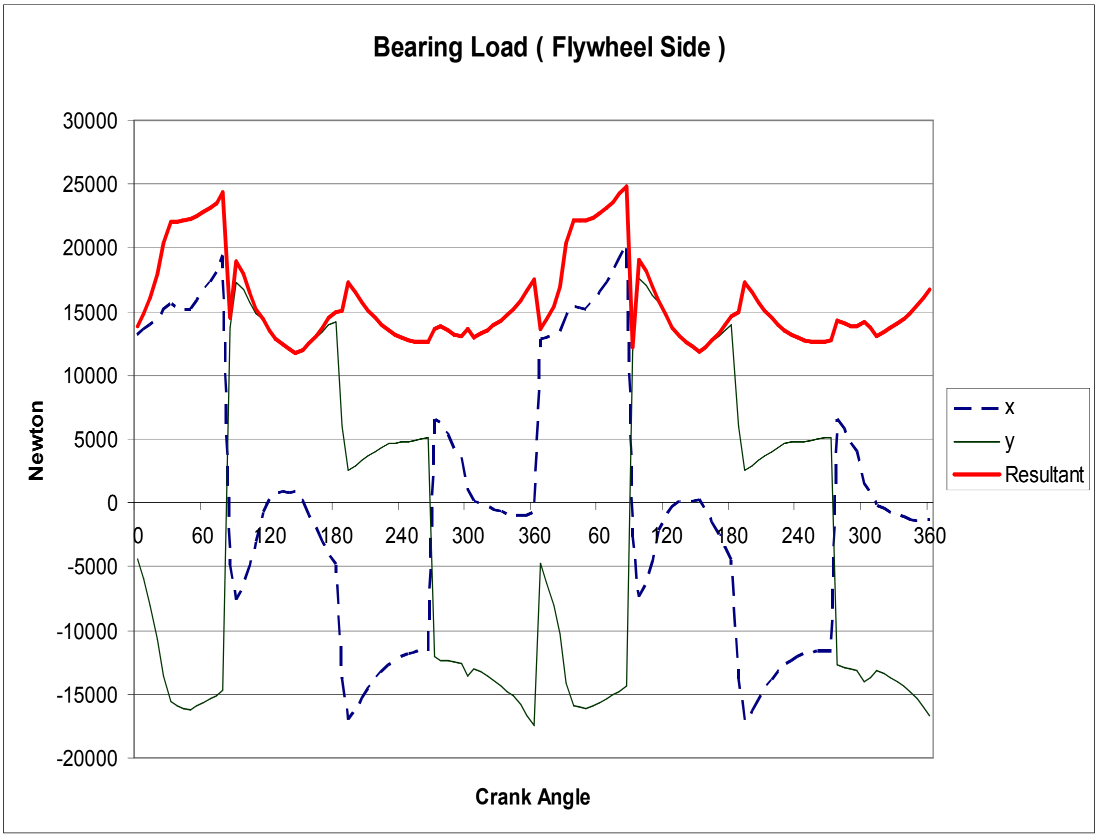


Figure 5.7 Pump side bearing load Ansys result

Table 5.6 Flywheel side bearing load Ansys result



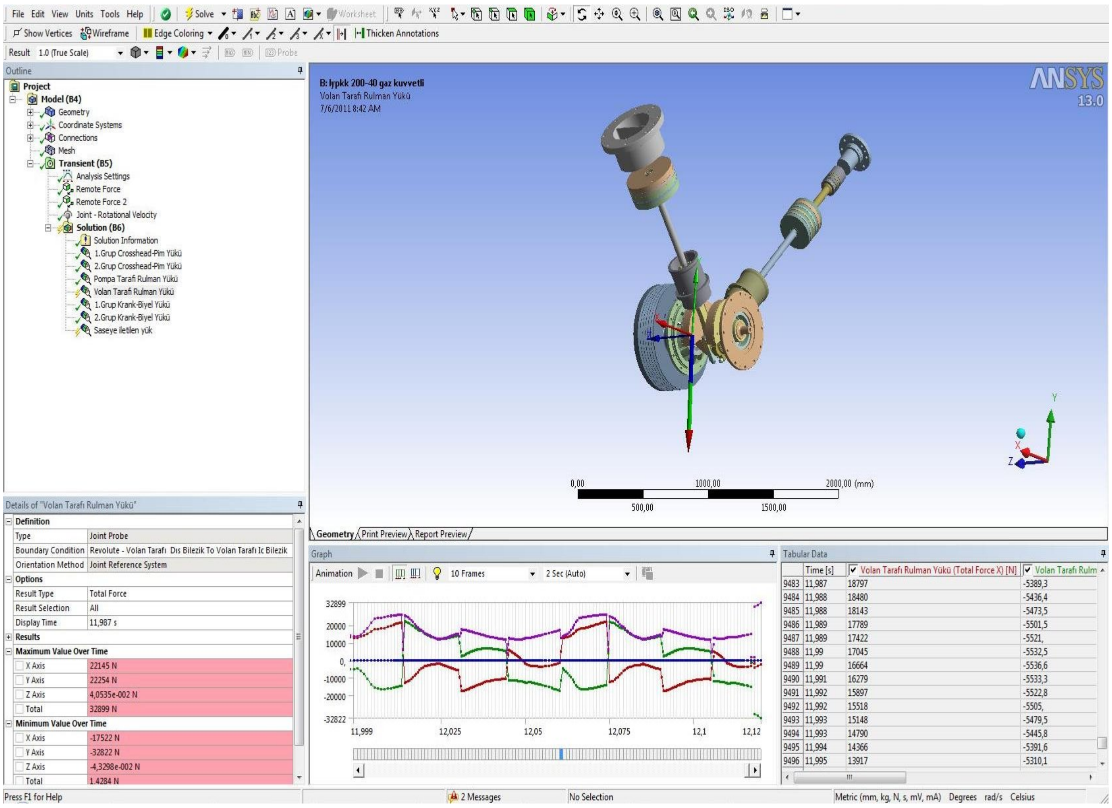


Figure 5.8 Flywheel side bearing load Ansys result

5.2 Inertia Forces

The inertia force due to the mass of the rotating crank is the same as the resultant of all the small inertia forces on the various small parts of the crank. The problem is simplified by concentrating the entire rotating crank mass at its center of gravity. Next the mass is shifted from the center of gravity to the crank pin A, but in the process it is diminished inversely proportional to the distance from the center of the shaft, so the inertia force, which in these case is centripetal in nature, remains unchanged. In addition, there is the mass of the rotating portion of the connecting rod acting at the crank pin, mentioned earlier. The whole crank is thus replaced by a single mass m_c at the crank pin, and the vertical displacement can be found directly from Fig.4.2

$$x_c = r\{1 - \cos(\omega t)\} \tag{5.1}$$

so the vertical components of velocity and acceleration become;

$$\begin{aligned} dx_c/dt &= r\omega \sin(\omega t) \\ d^2x_c/dt^2 &= r\omega^2 \cos(\omega t) \end{aligned} \tag{5.2}$$

the horizontal components are;

$$x_c = r\{1 - \cos(\omega t)\} \quad \begin{aligned} y_c &= -r \sin(\omega t) \\ dy_c/dt &= -r\omega \cos(\omega t) \\ d^2y_c/dt^2 &= -r\omega^2 \sin(\omega t) \end{aligned} \tag{5.3}$$

The momentum (or inertia force) is obtained from velocity (or acceleration) , by multiplying the expression with the rotating crank mass m_c

Next consider the conencting rod. Having divided the connecting rod mass into a part moving with the piston (reciprocating) and a part moving with the crank pin (rotating), the reciprocating and rotating masses can be represented by m_{rec} and m_{rot} Thus, m_{rec} is the total mass of the piston and a portion of the connecting rod, and m_{rot} represents the total equivalent rotating mass of the crank pin and the other part of the connecting rod. For a cylinder oriented such that the piston executes a path along the vertical direction, the vertical inertia force for all moving part is;

$$X = m_{rec}\, d^2x_p/dt^2 + m_{rot}\, d^2x_c/dt^2$$
$$X = (m_{rec} + m_{rot})\, r\omega^2 \cos(\omega t) + m_{rec}\,(r^2/l)\,\omega^2 \cos(2\omega t) \qquad (5.4)$$

and the horizontal inertia force is

$$Y = m_{rot}\, d^2y_c/dt^2 = m_{rot}\, r\omega^2 \sin(\omega t) \qquad (5.5)$$

Putting it in words, the vertical component of the inertia force consists of two parts, a 'primary part' equal to the inertia action of the combined reciprocating and the rotating masses as if they were moving up and down harmonically with crankshaft frequency and amplitude *r*, and a 'secondary part' equal to the inertia action of mass m_{rec} *(r/4l)* moving up and down with twice the frequency of the crankshaft with the same amplitude *r*. The horizontal, or lateral force has only one primary part due to the rotating mass.

The torque due to these inertia forces about the longitudinal axis *O* also needs to be calculated. Recall that to determine the vertical and horizontal forces the connecting rod when replaced by two masses at the piston and crank pin gives exact results, but for the inertia torques the results thus obtained is no longer exact. It will be correct to an acceptable degree of approximation. Thus, the piston rod and crank mechanism is replaced by a reciprocating mass m_{rec} and m_{rot} rotating uniformly about *O*, so that it has no torque about *O*.

Multiplying the reciprocating mass by the expression for piston acceleration, and ignoring all terms proportional to second or higher powers of r/l, the inertia torque becomes;

$$M = \frac{1}{2} m_{rec}\, \omega^2 r^2 \left\{ \frac{r}{2l} \sin(\omega t) - \sin(2\omega t) - \frac{3r}{2l} \sin 3(\omega t) \right\} \qquad (5.6)$$

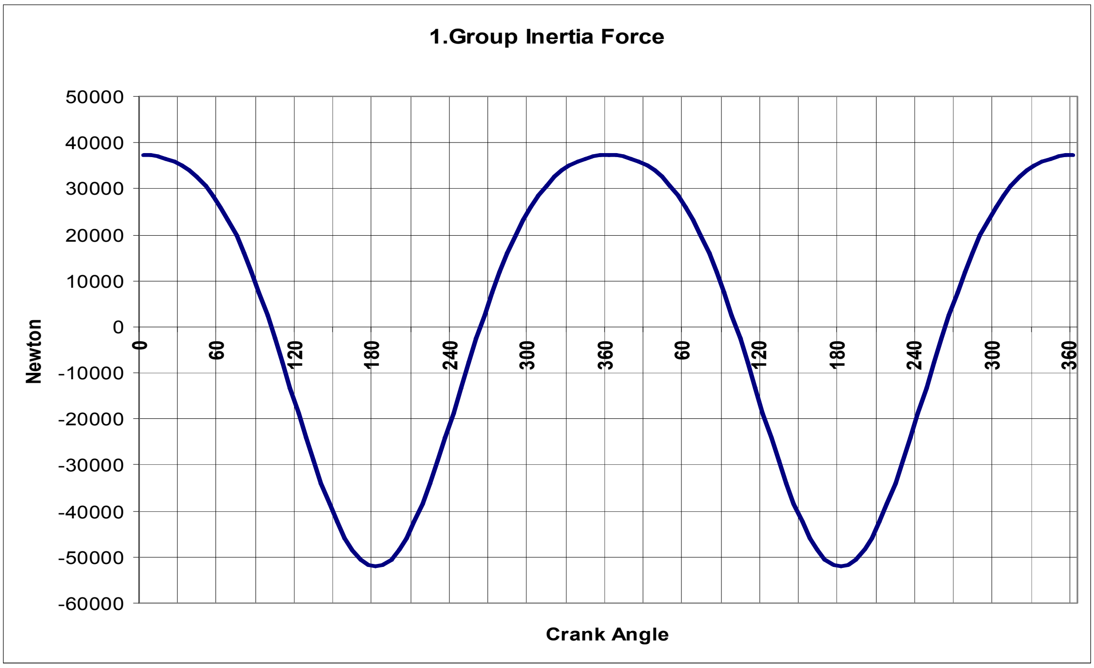
This equation fort he inertia torque, acting on the shaft in the direction of its rotation, and on the frame about *O* in opposite direction, is accurate for the usual reciprocating machine where the connecting rod design calls for two substantial bearings at its ends joined by a relatively light stem.

Table 5.7 Theoretical inertia forces of the slider crank mechanism diagram

α	Sin α	$\sin^2\alpha$	(r/l)*sin (α	β	cos α	cos2	1.Group Inertia Force $P_i=mrw^2(\cos\alpha+\lambda\cos2\alpha)$
0	0	0	0	0	1	1	37368.97454
6	0.1045	0.01	0.017192	0.99	0.995	0.98	37284.71429
12	0.2079	0.04	0.034196	1.96	0.978	0.91	37027.59244
18	0.309	0.1	0.050825	2.91	0.951	0.81	36584.86329
24	0.4067	0.17	0.066897	3.84	0.914	0.67	35936.19639
30	0.5	0.25	0.082237	4.72	0.866	0.5	35054.99977
36	0.5878	0.35	0.096675	5.55	0.809	0.31	33910.18414
42	0.6691	0.45	0.110054	6.32	0.743	0.1	32468.28419
48	0.7431	0.55	0.122228	7.02	0.669	-0.1	30695.83602
54	0.809	0.65	0.133062	7.65	0.588	-0.3	28561.89676
60	0.866	0.75	0.142438	8.19	0.5	-0.5	26040.58462
66	0.9135	0.83	0.150254	8.64	0.407	-0.7	23113.51548
72	0.9511	0.9	0.156424	9	0.309	-0.8	19772.01506
78	0.9781	0.96	0.16088	9.26	0.208	-0.9	16018.99464
84	0.9945	0.99	0.163573	9.41	0.105	-1	11870.39201
90	1	1	0.164474	9.47	6E-17	-1	7356.09735
96	0.9945	0.99	0.163573	9.41	-0.1	-1	2520.30594
102	0.9781	0.96	0.16088	9.26	-0.21	-0.9	-2578.735998
108	0.9511	0.9	0.156424	9	-0.31	-0.8	-7869.59952
114	0.9135	0.83	0.150254	8.64	-0.41	-0.7	-13269.13572
120	0.866	0.75	0.142438	8.19	-0.5	-0.5	-18684.48727
126	0.809	0.65	0.133062	7.65	-0.59	-0.3	-24015.57857
132	0.7431	0.55	0.122228	7.02	-0.67	-0.1	-29157.99292
138	0.6691	0.45	0.110054	6.32	-0.74	0.1	-34006.12729
144	0.5878	0.35	0.096675	5.55	-0.81	0.31	-38456.50233
150	0.5	0.25	0.082237	4.72	-0.87	0.5	-42411.09712
156	0.4067	0.17	0.066897	3.84	-0.91	0.67	-45780.57615
162	0.309	0.1	0.050825	2.91	-0.95	0.81	-48487.27883
168	0.2079	0.04	0.034196	1.96	-0.98	0.91	-50467.85108
174	0.1045	0.01	0.017192	0.99	-0.99	0.98	-51675.41224
180	1E-16	0	2.02E-17	0	-1	1	-52081.16924
186	-0.105	0.01	-0.017192	-1	-0.99	0.98	-51675.41224
192	-0.208	0.04	-0.034196	-2	-0.98	0.91	-50467.85108
198	-0.309	0.1	-0.050825	-2.9	-0.95	0.81	-48487.27883
204	-0.407	0.17	-0.066897	-3.8	-0.91	0.67	-45780.57615
210	-0.5	0.25	-0.082237	-4.7	-0.87	0.5	-42411.09712
216	-0.588	0.35	-0.096675	-5.5	-0.81	0.31	-38456.50233
222	-0.669	0.45	-0.110054	-6.3	-0.74	0.1	-34006.12729
228	-0.743	0.55	-0.122228	-7	-0.67	-0.1	-29157.99292
234	-0.809	0.65	-0.133062	-7.6	-0.59	-0.3	-24015.57857
240	-0.866	0.75	-0.142438	-8.2	-0.5	-0.5	-18684.48727
246	-0.914	0.83	-0.150254	-8.6	-0.41	-0.7	-13269.13572
252	-0.951	0.9	-0.156424	-9	-0.31	-0.8	-7869.59952
258	-0.978	0.96	-0.16088	-9.3	-0.21	-0.9	-2578.735998
264	-0.995	0.99	-0.163573	-9.4	-0.1	-1	2520.30594
270	-1	1	-0.164474	-9.5	-0	-1	7356.09735
276	-0.995	0.99	-0.163573	-9.4	0.105	-1	11870.39201
282	-0.978	0.96	-0.16088	-9.3	0.208	-0.9	16018.99464
288	-0.951	0.9	-0.156424	-9	0.309	-0.8	19772.01506
294	-0.914	0.83	-0.150254	-8.6	0.407	-0.7	23113.51548
300	-0.866	0.75	-0.142438	-8.2	0.5	-0.5	26040.58462
306	-0.809	0.65	-0.133062	-7.6	0.588	-0.3	28561.89676
312	-0.743	0.55	-0.122228	-7	0.669	-0.1	30695.83602
318	-0.669	0.45	-0.110054	-6.3	0.743	0.1	32468.28419
324	-0.588	0.35	-0.096675	-5.5	0.809	0.31	33910.18414
330	-0.5	0.25	-0.082237	-4.7	0.866	0.5	35054.99977
336	-0.407	0.17	-0.066897	-3.8	0.914	0.67	35936.19639
342	-0.309	0.1	-0.050825	-2.9	0.951	0.81	36584.86329
348	-0.208	0.04	-0.034196	-2	0.978	0.91	37027.59244
354	-0.105	0.01	-0.017192	-1	0.995	0.98	37284.71429
360	-2E-16	0	-4.03E-17	-0	1	1	37368.97454

α	Sin α	$\sin^2\alpha$	(r/l)*sin (α)	β	cos α	cos2α	2.Group Inertia Force $P_i=mrw^2(\cos\alpha+\lambda\cos2\alpha)$
0	0	0	0	0	1	1	37129.60236
6	0.105	0.0109	0.0171922	0.99	0.995	0.9781	37045.88186
12	0.208	0.0432	0.034196	1.96	0.978	0.9135	36790.40703
18	0.309	0.0955	0.0508252	2.91	0.951	0.809	36350.51385
24	0.407	0.1654	0.0668975	3.84	0.914	0.6691	35706.00207
30	0.5	0.25	0.0822368	4.72	0.866	0.5	34830.45008
36	0.588	0.3455	0.0966752	5.55	0.809	0.309	33692.96773
42	0.669	0.4477	0.1100544	6.32	0.743	0.1045	32260.30407
48	0.743	0.5523	0.1222278	7.02	0.669	-0.105	30499.20957
54	0.809	0.6545	0.133062	7.65	0.588	-0.309	28378.93955
60	0.866	0.75	0.1424384	8.19	0.5	-0.5	25873.77802
66	0.914	0.8346	0.1502542	8.64	0.407	-0.669	22965.45864
72	0.951	0.9045	0.1564238	9	0.309	-0.809	19645.36266
78	0.978	0.9568	0.1608795	9.26	0.208	-0.914	15916.38273
84	0.995	0.9891	0.1635727	9.41	0.105	-0.978	11794.35456
90	1	1	0.1644737	9.47	6E-17	-1	7308.976843
96	0.995	0.9891	0.1635727	9.41	-0.105	-0.978	2504.161769
102	0.978	0.9568	0.1608795	9.26	-0.208	-0.914	-2562.217545
108	0.951	0.9045	0.1564238	9	-0.309	-0.809	-7819.189703
114	0.914	0.8346	0.1502542	8.64	-0.407	-0.669	-13184.13842
120	0.866	0.75	0.1424384	8.19	-0.5	-0.5	-18564.80118
126	0.809	0.6545	0.133062	7.65	-0.588	-0.309	-23861.74343
132	0.743	0.5523	0.1222278	7.02	-0.669	-0.105	-28971.21733
138	0.669	0.4477	0.1100544	6.32	-0.743	0.1045	-33788.29631
144	0.588	0.3455	0.0966752	5.55	-0.809	0.309	-38210.16384
150	0.5	0.25	0.0822368	4.72	-0.866	0.5	-42139.42692
156	0.407	0.1654	0.0668975	3.84	-0.914	0.6691	-45487.32228
162	0.309	0.0955	0.0508252	2.91	-0.951	0.809	-48176.68681
168	0.208	0.0432	0.034196	1.96	-0.978	0.9135	-50144.57222
174	0.105	0.0109	0.0171922	0.99	-0.995	0.9781	-51344.39818
180	1E-16	2E-32	2.015E-17	0	-1	1	-51747.55605
186	-0.1	0.0109	-0.0171922	-1	-0.995	0.9781	-51344.39818
192	-0.21	0.0432	-0.034196	-2	-0.978	0.9135	-50144.57222
198	-0.31	0.0955	-0.0508252	-2.9	-0.951	0.809	-48176.68681
204	-0.41	0.1654	-0.0668975	-3.8	-0.914	0.6691	-45487.32228
210	-0.5	0.25	-0.0822368	-4.7	-0.866	0.5	-42139.42692
216	-0.59	0.3455	-0.0966752	-5.5	-0.809	0.309	-38210.16384
222	-0.67	0.4477	-0.1100544	-6.3	-0.743	0.1045	-33788.29631
228	-0.74	0.5523	-0.1222278	-7	-0.669	-0.105	-28971.21733
234	-0.81	0.6545	-0.133062	-7.6	-0.588	-0.309	-23861.74343
240	-0.87	0.75	-0.1424384	-8.2	-0.5	-0.5	-18564.80118
246	-0.91	0.8346	-0.1502542	-8.6	-0.407	-0.669	-13184.13842
252	-0.95	0.9045	-0.1564238	-9	-0.309	-0.809	-7819.189703
258	-0.98	0.9568	-0.1608795	-9.3	-0.208	-0.914	-2562.217545
264	-0.99	0.9891	-0.1635727	-9.4	-0.105	-0.978	2504.161769
270	-1	1	-0.1644737	-9.5	-2E-16	-1	7308.976843
276	-0.99	0.9891	-0.1635727	-9.4	0.105	-0.978	11794.35456
282	-0.98	0.9568	-0.1608795	-9.3	0.208	-0.914	15916.38273
288	-0.95	0.9045	-0.1564238	-9	0.309	-0.809	19645.36266
294	-0.91	0.8346	-0.1502542	-8.6	0.407	-0.669	22965.45864
300	-0.87	0.75	-0.1424384	-8.2	0.5	-0.5	25873.77802
306	-0.81	0.6545	-0.133062	-7.6	0.588	-0.309	28378.93955
312	-0.74	0.5523	-0.1222278	-7	0.669	-0.105	30499.20957
318	-0.67	0.4477	-0.1100544	-6.3	0.743	0.1045	32260.30407
324	-0.59	0.3455	-0.0966752	-5.5	0.809	0.309	33692.96773
330	-0.5	0.25	-0.0822368	-4.7	0.866	0.5	34830.45008
336	-0.41	0.1654	-0.0668975	-3.8	0.914	0.6691	35706.00207
342	-0.31	0.0955	-0.0508252	-2.9	0.951	0.809	36350.51385
348	-0.21	0.0432	-0.034196	-2	0.978	0.9135	36790.40703
354	-0.1	0.0109	-0.0171922	-1	0.995	0.9781	37045.88186
360	-0	6E-32	-4.03E-17	-0	1	1	37129.60236

Table 5.8 Theoretical 1st group inertia forces of the slider crank mechanism diagram



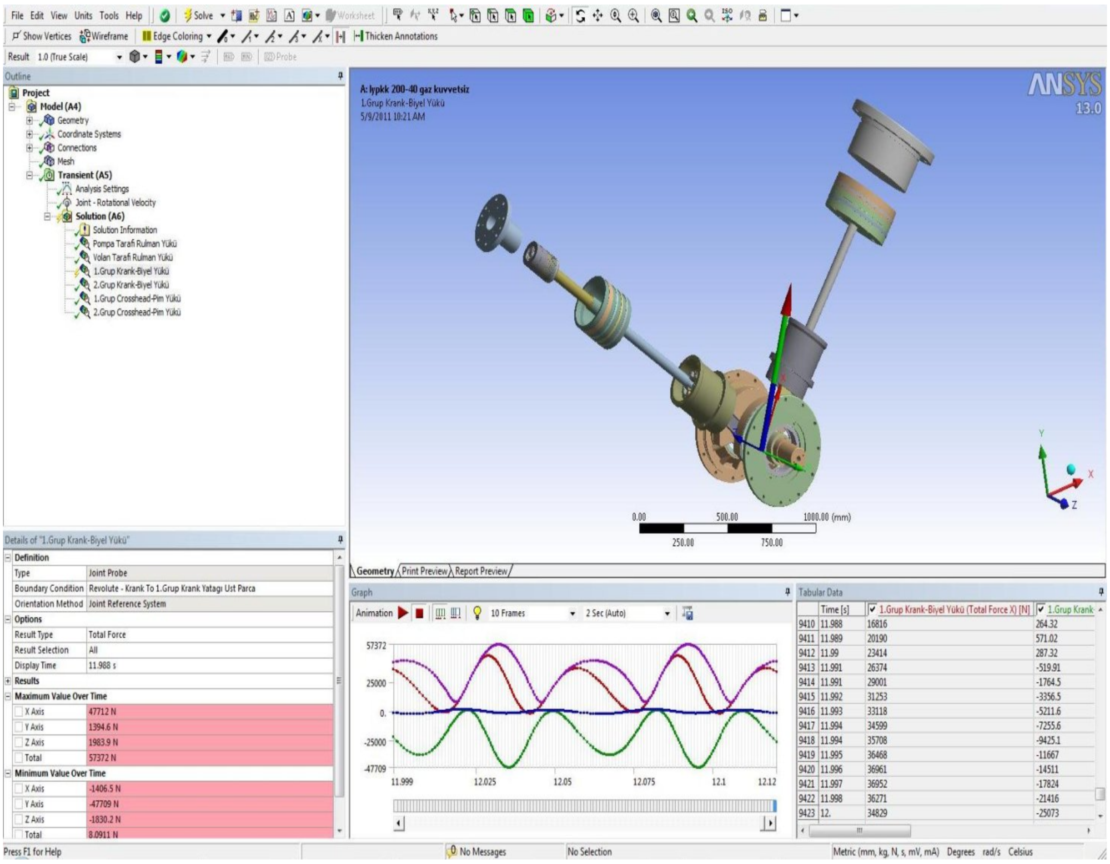
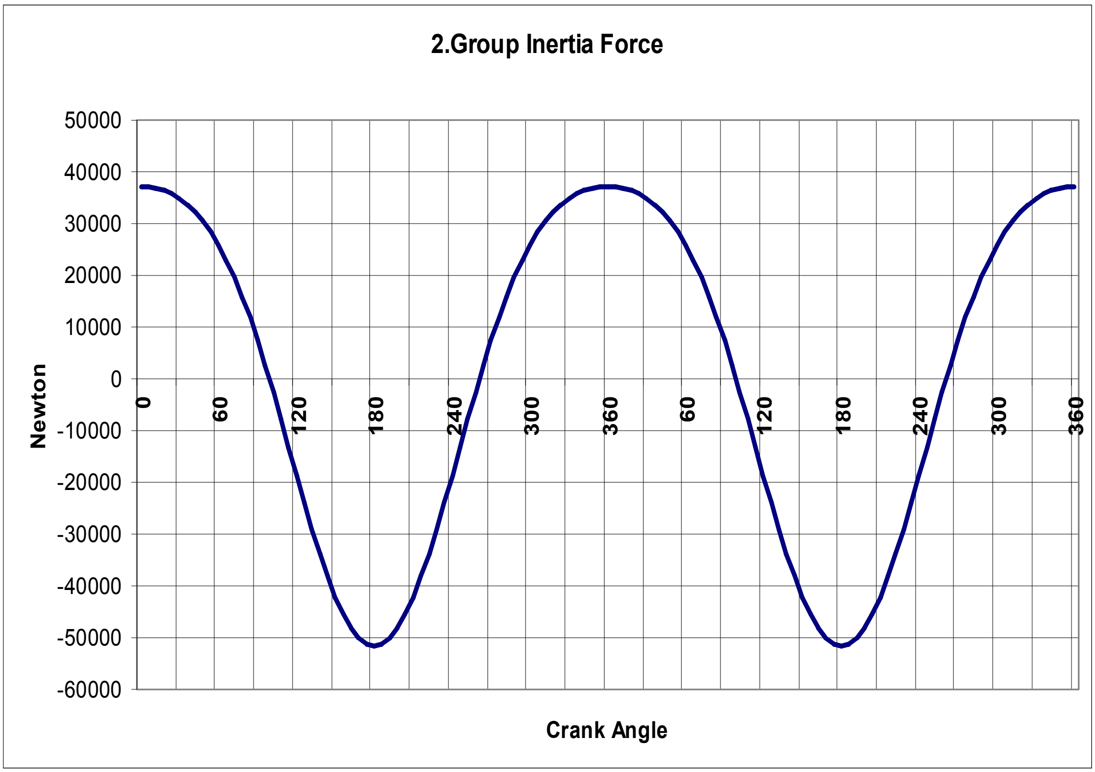


Figure 5.9 Ansys 1st group inertia force result of the slider crank mechanism

Table 5.9 Theoretical 2nd group inertia forces of the slider crank mechanism diagram



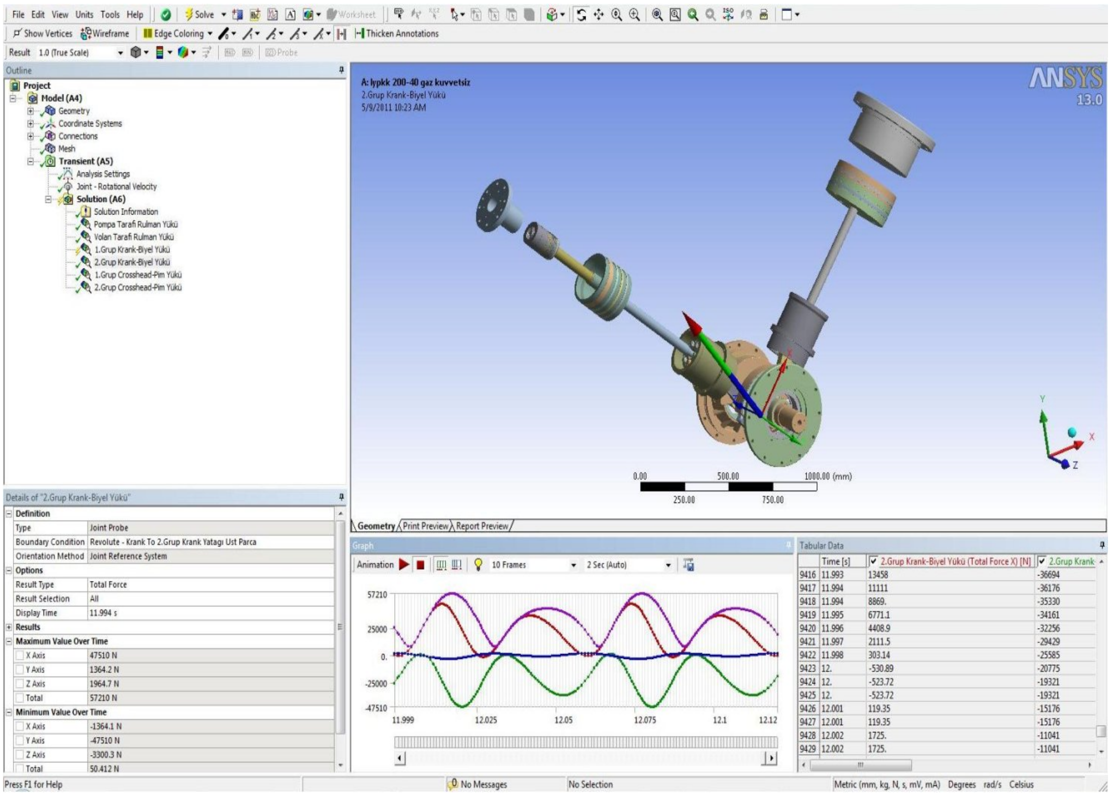
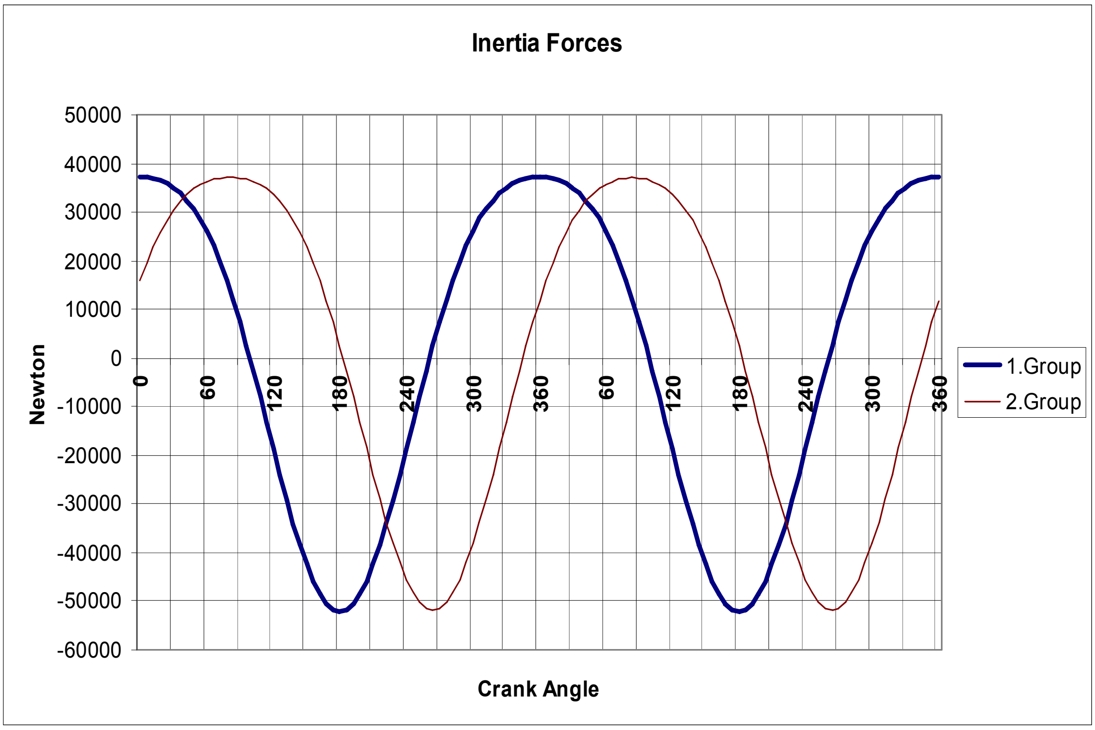


Figure 5.10 Ansys 2nd group inertia force result of the slider crank mechanism

Table 5.10 Theoretical inertia forces of the slider crank mechanism diagram



We designed the mass of moving parts for each group approximately the same becaues inertia forces for each group must be approximately the same. These 2 groups inertia forces follow each other with an specific phase angle. For V type compressors phase angle between 2 group is 90^o. In table 5.10 we can examine phase angle.

We can confirm our theoretical calculations with Ansys analysis results. If we compare our theoretical results with Ansys analysis result, we noticed that our calculation result and ansys results are alike. In these situation we can verify our design in respect of kinematic and dynamic conditions.

CHAPTER SIX
DESIGN AND CALCULATIONS

6.1 Reciprocating Compressor Components Designs And Calculations

Reciprocating piston compressor can come in two basic configurations. The simplest is a piston in a cylinder, directly driven from a crankshaft by a connecting rod attached to the piston by a wrist pin. This single acting piston can only compress gas on one face, and any leakage past the rings will go into the crankcase. Other type of reciprocating piston compressor is Double acting. In this type, the crankshaft drives a connecting rod which transmits force through a crosshead pin to a crosshead, moving in a slide. This converts the eccentric motion of the connecting rod to a pure linear force. A compressor rod connected to the crosshead transmits force to the compressor piston. So, the cylinder can be sealed on both ends, with the rod passing through a packing case to seal gas from leaking.

All of the components designed with Pro Engineer CAD program. We designed all components according to the information which we gained in previous results and results of the ANSYS calculations. We have some parameters for optimum design such as thickness, lenght, diameter etc. These parameters not only have to be optimum values for strength but also we have to think the economical conditions.

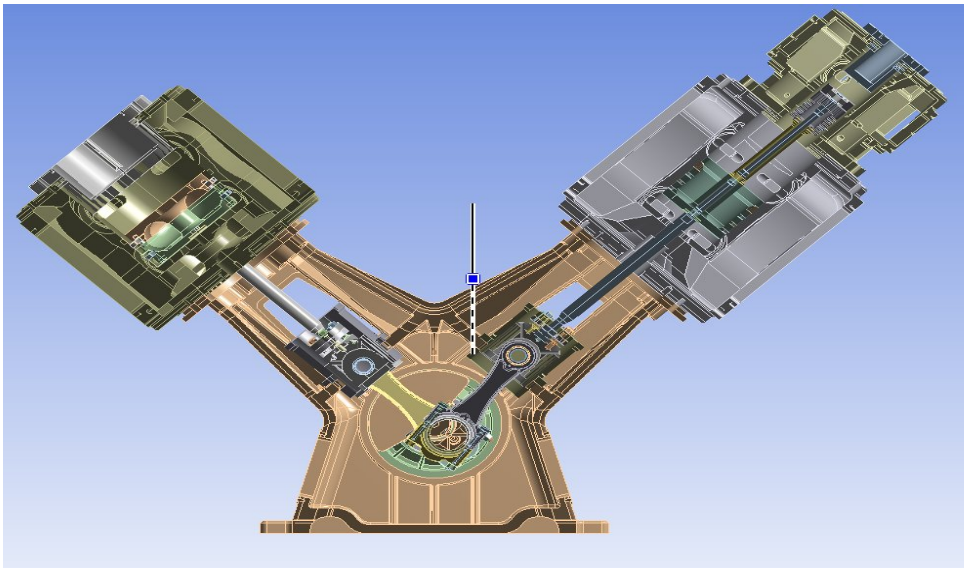


Figure 6.1 Cross-section view of the compressor

6.2 Safety Factor

Before examination of the design we have to specify the safety factor. A safety factor was originally a number which the ultimate tensile strength of a material was divided in order to obtain a value of “working stres” or “ design stress”. These design stresses, in turn, were often used in highly simplified calculations that made no allowance for such factors as stress concentration, impact, fatigue, difference between properties of the material in the standart test specimen and in the manufactured part, and so on. The part must be designed to withstand a “design overload” somewhat larger than the normally expected. Recommended values for safety factor;

1 – SF= 1.25 to 1.5 for exceptionally reliable materials used under controllable conditions and subjected to loads and stresses that can be determined with certainty used almost invariably where low weight is a particularly important consideration.
2 –SF= 1.5 to 2 for well-known materials, under reasonably constant environmental conditions, subjected to loads and stresses that can be determined readily.
3 – SF= 2 to 2.5 for average materials operated in ordinary environments and subjected to loads and stresses that can be determined.
4 – SF= 2.5 to 3 for less tried materials of for brittle materials under average conditions of environment, load and stresses.
5 – SF= 3 to 4 for untried materials used under average conditions of environment, load and stresses
6 – SF= 3 to 4 should also be used with beter-known materials that are to be used in uncertain environments or subjected to uncertain stresses.
7 – Repeated Loads = the factors established in items 1 to 6 are acceptable but must be applied to the endurance limit rather than to the yield strength of material.
8 – Impact Forces = the factors given in items 3 to 6 are acceptable, but an impact factor should be included
9 – Brittle Materials = where the ultimate strength is used as the theoretical maximum, the factors presented in items 1 to 6 should be approximately doubled

10 – Where higher factors migth appear desirable, a more thorough analysis of the problem should be undertaken before deciding on their use.

Depends on our system working conditions we have to consider the repeated loads conditions and choose safety factor between 1 to 6. We are using well-known materials with constant loading and environment conditions. Because of these situation we choose safety factor as 3.

6.3 Fixed Part Designs And Calculations

6.3.1 Cylinder And Ends

The compressor cylinder is a casting or forging designed to safely contain some maximum working pressure. It is machined to hold compressor valves and to direct gas flow to and from the cylinder cavity. In combination with the cylinder ends, it must contain the gas pressure, while having sufficiently large gas flow passages so there are minimal pressure drops due to gas flow. The cylinder and ends may also have water passages to stabilize temperature and dimensional changes. All these requirements involve compromises betweem size, strenght and efficiency. Compressor cylinders are designed for some operating range and service. If conditions change, they may not perform reliably or efficiently.

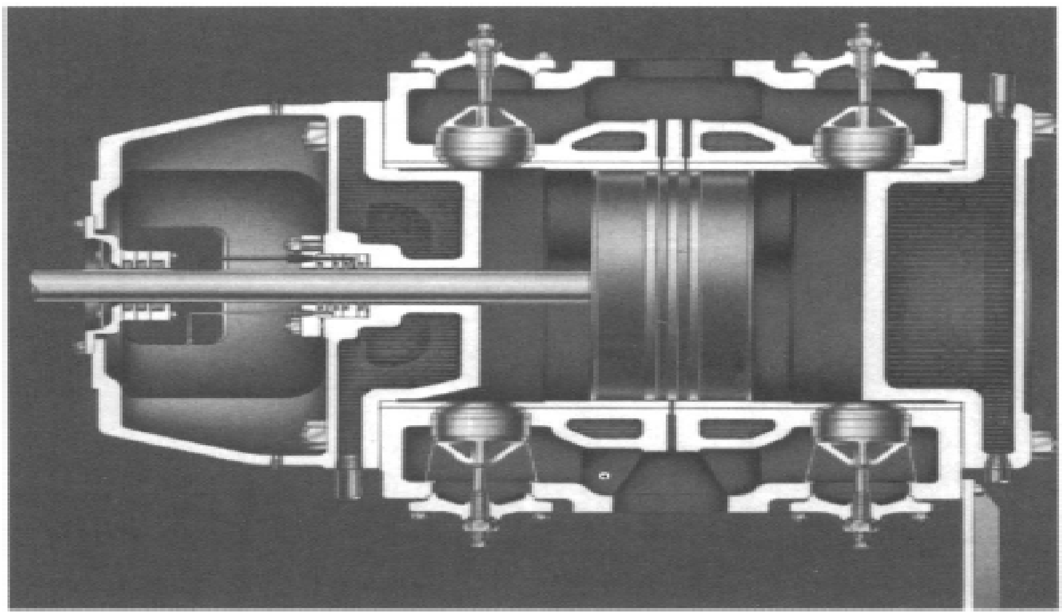


Figure 6.2 Cross-section view of the double acting cylinder and other parts

A typical double acting cylinder like that shown in figure 6.2 consist of a barrel, usually water jacketed, with a front and rear head. In some designs the rear head is an integral part of the cylinder barrel, in others it is a separate, bolted piece as shown. These heads are also water cooled to remove heat of compressor. Cylinder may be double acting, that is, compressing on both sides of the piston, or single acting, compressing at either head or crank end but not both.

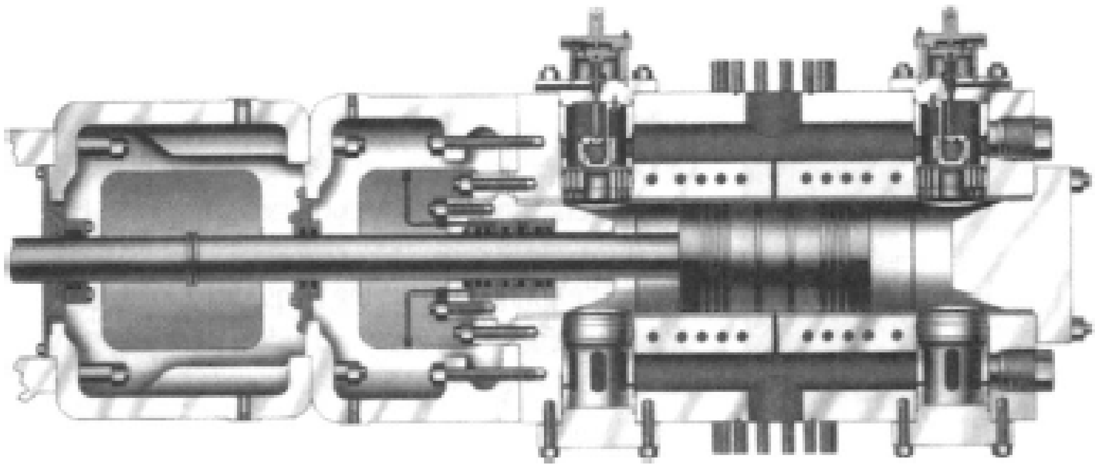


Figure 6.3 High pressure cylinder cross-section view

High pressure cylinders as shown in figure 6.3 , are made from steel forging, with only nominal water cooling as compared to the other cylinders. Because the smallest possible number of opennings in the forging is desired, capacity control mechanisms are not normally provided. Tie bolts may be installed perpendicular to the bore top re-stress the forging, decreasing the maximum tensile stress induced by the gas pressure. Valve ports are finished to a highly polished surface, which reduces the maximum tensile stresses.

Before starting the cylinder design calculation, we have to consider some parameters. Cylinder single acting or double acting? What kind of cooling system we should use? Water or oil? What is the stroke value? What is the working pressure and temperature for the compressor?

In our system, all of the cylinders has water cooling system and stroke value is 125 mm. First stage is double acting, second and third stage are single acting and our working conditions,

1st Stage : 4 bar – 200 ^{o}C

2nd Stage : 18 bar – 200 ^{o}C

3rd Stage : 40 bar – 160 ^{o}C

According to these parameters we can make calculations for our design. For water cooling system we have to design water channels around the air channels. So, this water and air channels wall thickness must have enough strength values against working pressure and working temperature. We determined our material as EN-GJL-250.

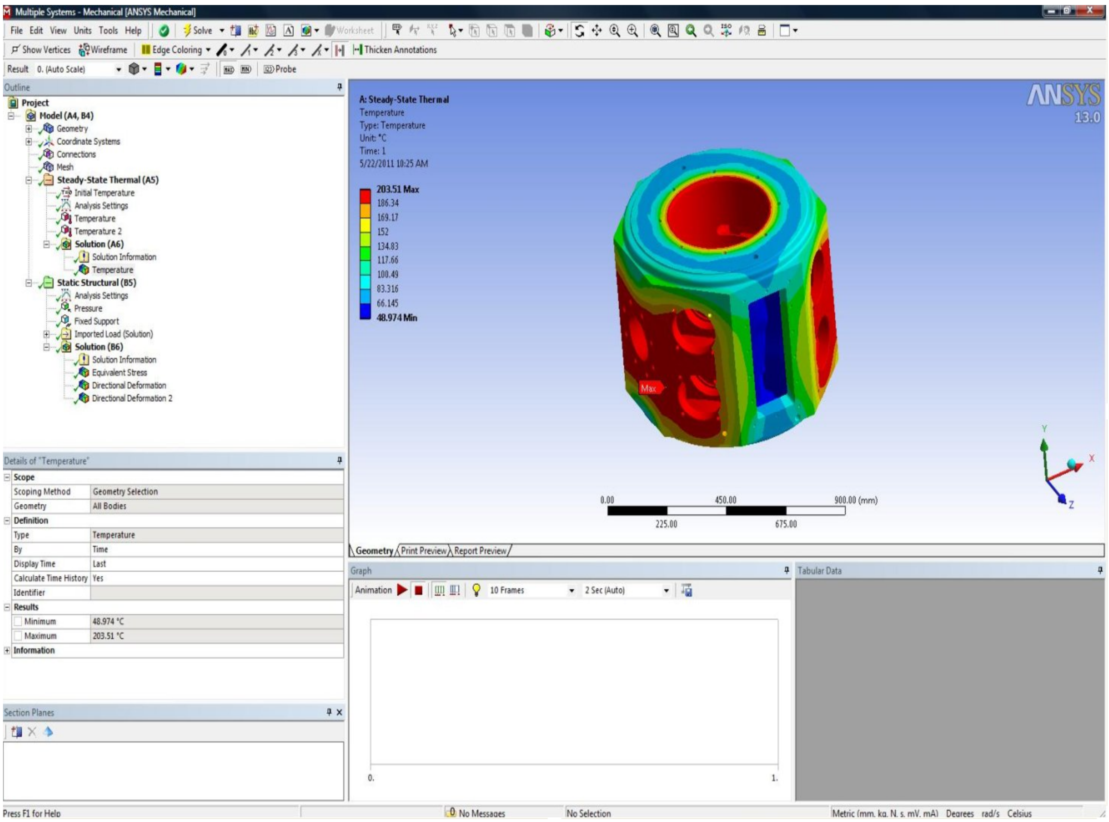


Figure 6.4 1st stage cylinder temperature distribution

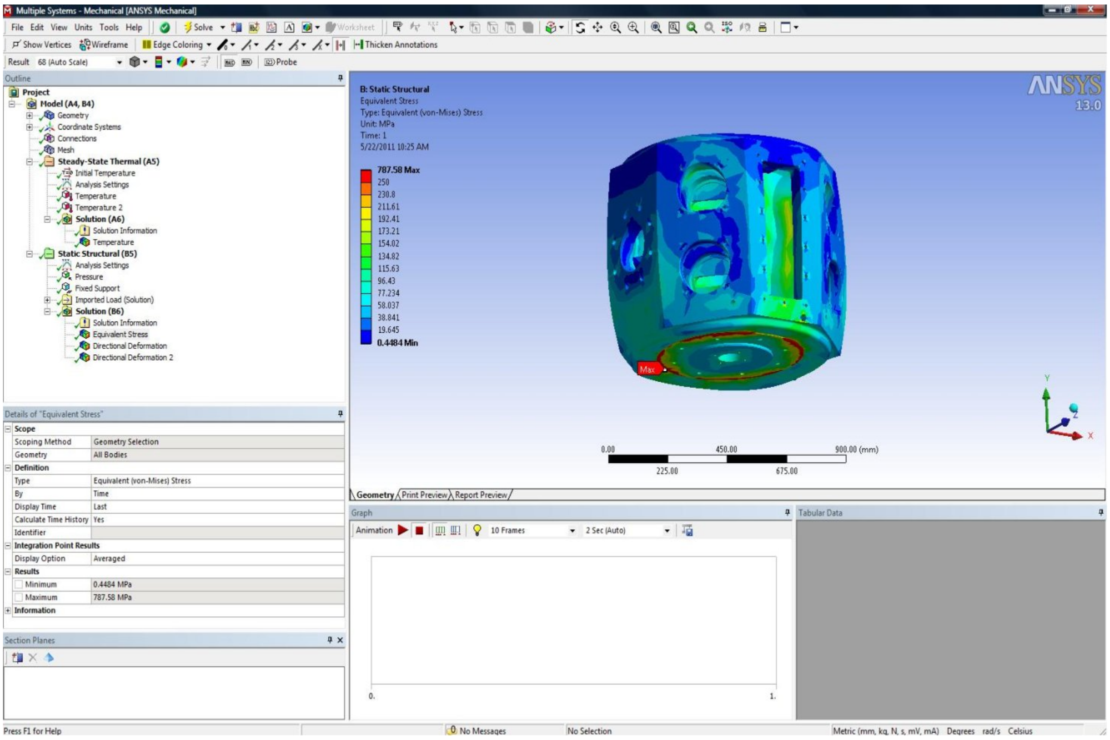


Figure 6.5 1st stage cylinder stress distribution

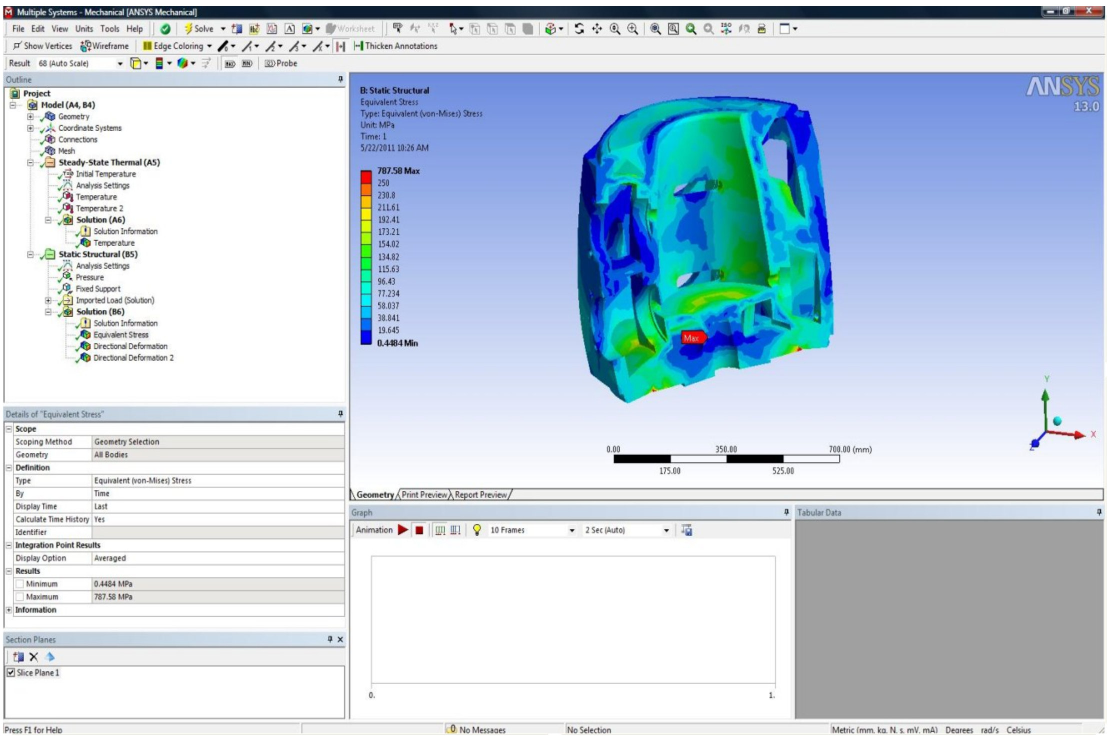


Figure 6.6 1st stage cylinder stress distribution

In Figure 6.4 we can examine the temperature distribution for the first stage cylinder. Our input values are 200 ^{o}C inside the air channels and 50 ^{o}C inside the water channels. Then we used these temperature results in strength analysis as an input. The other input value for strength analysis is 4 bar pressure inside the air channels. Figure 6.5 and Figure 6.6 are the results of the strength analysis. With these results we can examine the stress distribution with colors. We identified yielding strength value for EN-GJL-250 material as 250 MPa.

After the examinations of the results, we decided that our first stage cylinder design has optimum properties for the system. We are going to apply the same procedure to the second and third stage cylinders with different pressure values. Pressure values are 20 bar for the second stage and 40 bar for the third stage. After the examination of Figures 6.7 – 6.8 – 6.9 – 6.10 – 6.11 – 6.12 we decided that, our second and third stage cylinder design has enough strength depends on working conditions.

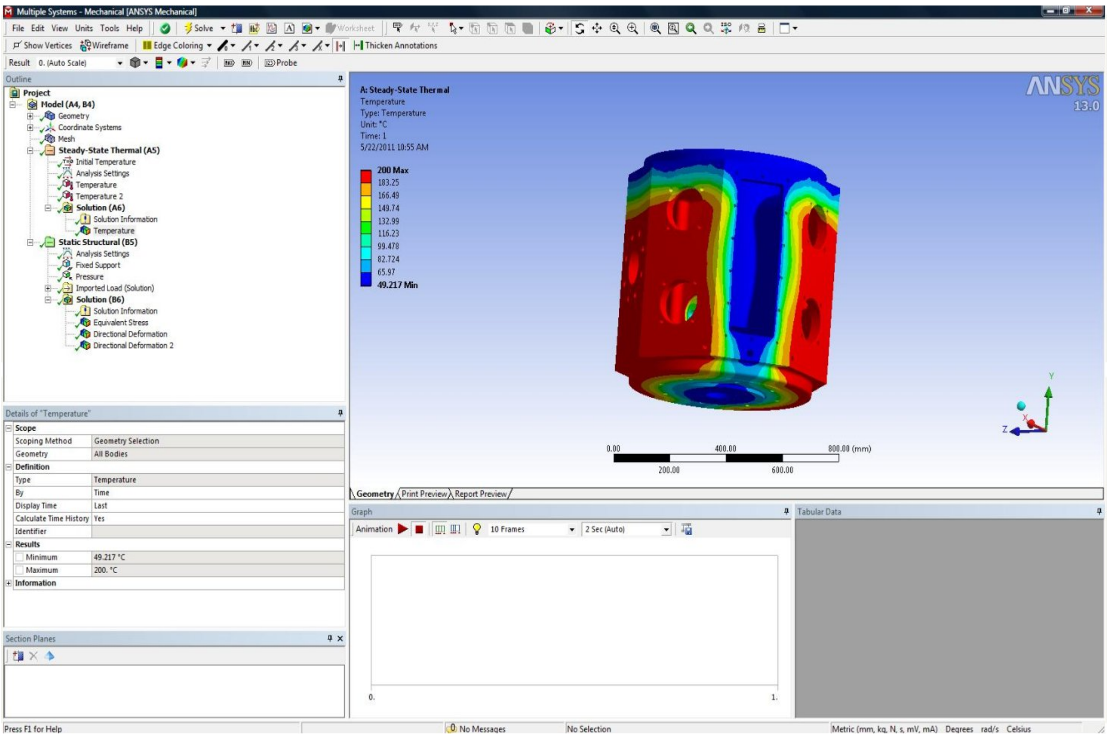


Figure 6.7 2nd stage cylinder temperature distribution

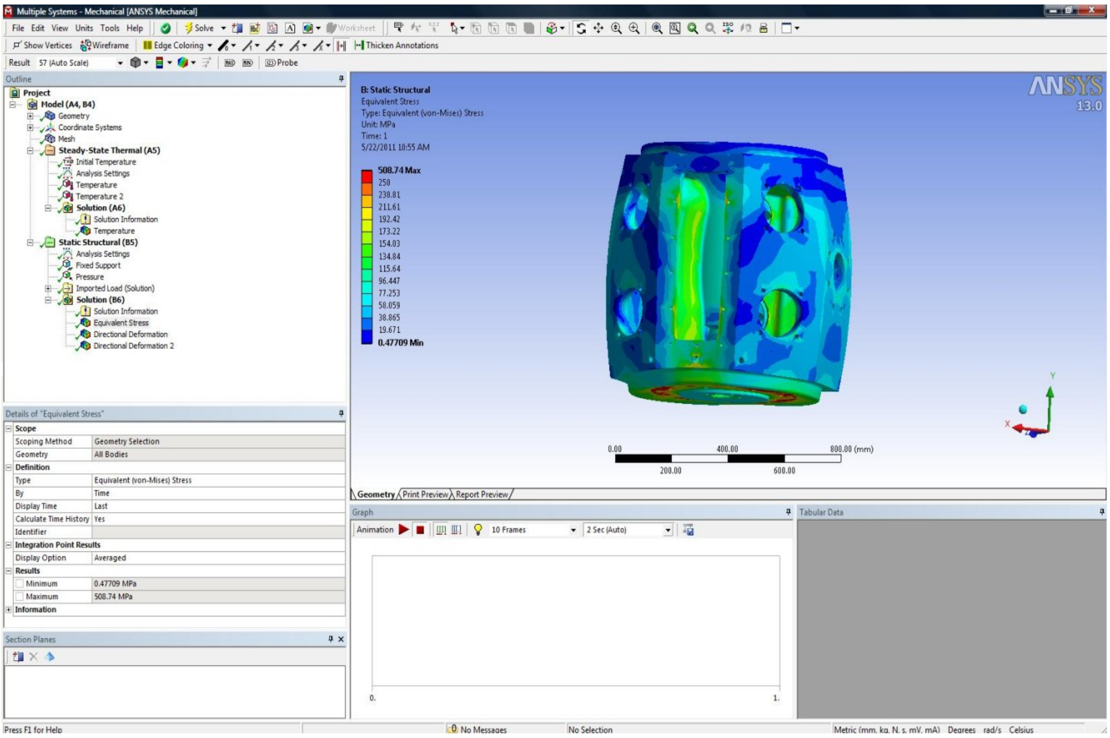


Figure 6.8 2nd stage cylinder stress distribution

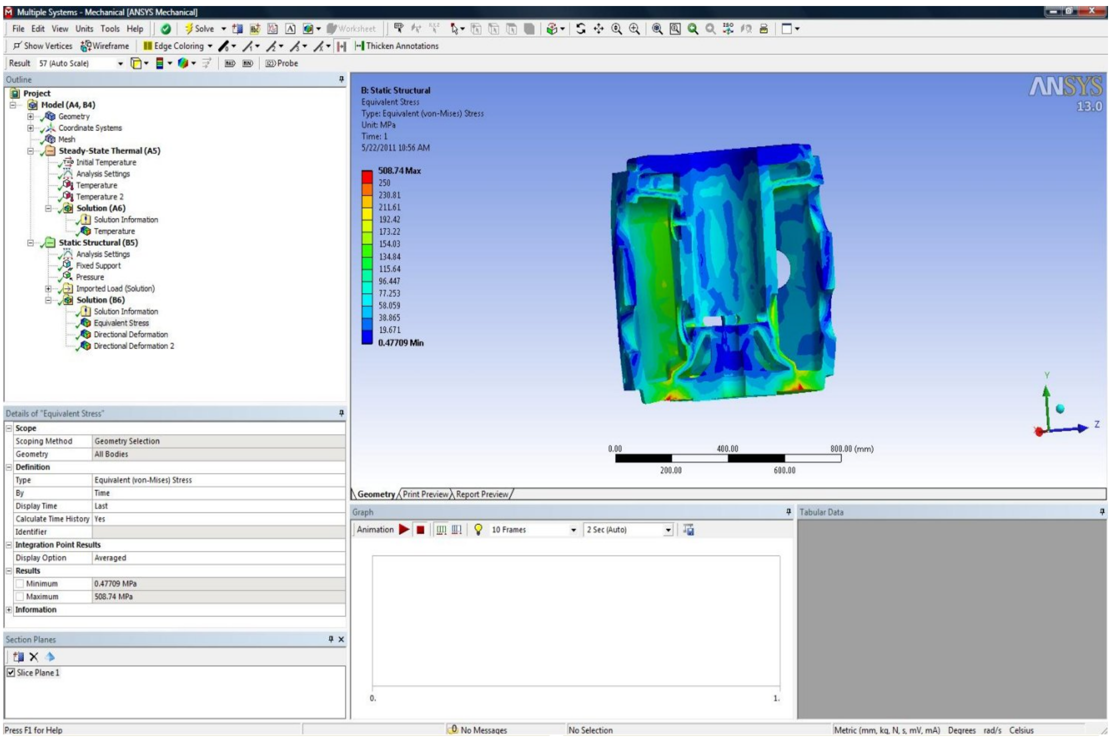


Figure 6.9 2nd stage cylinder stress distribution

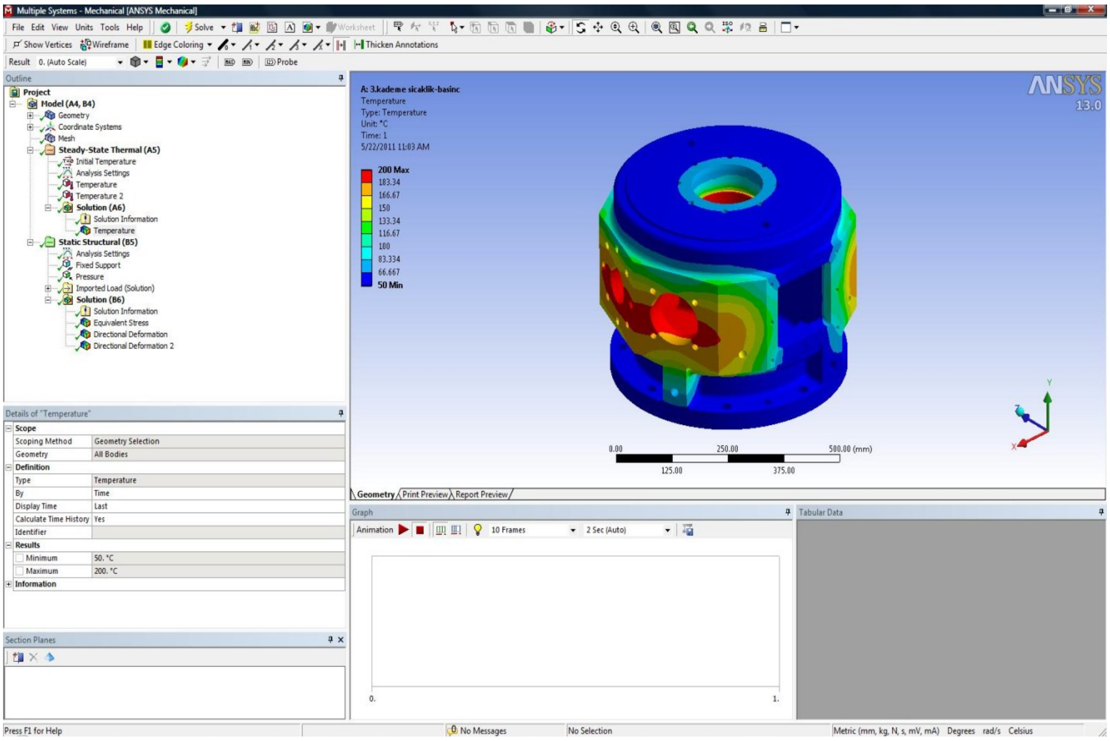


Figure 6.10 3rd stage cylinder temperature distribution

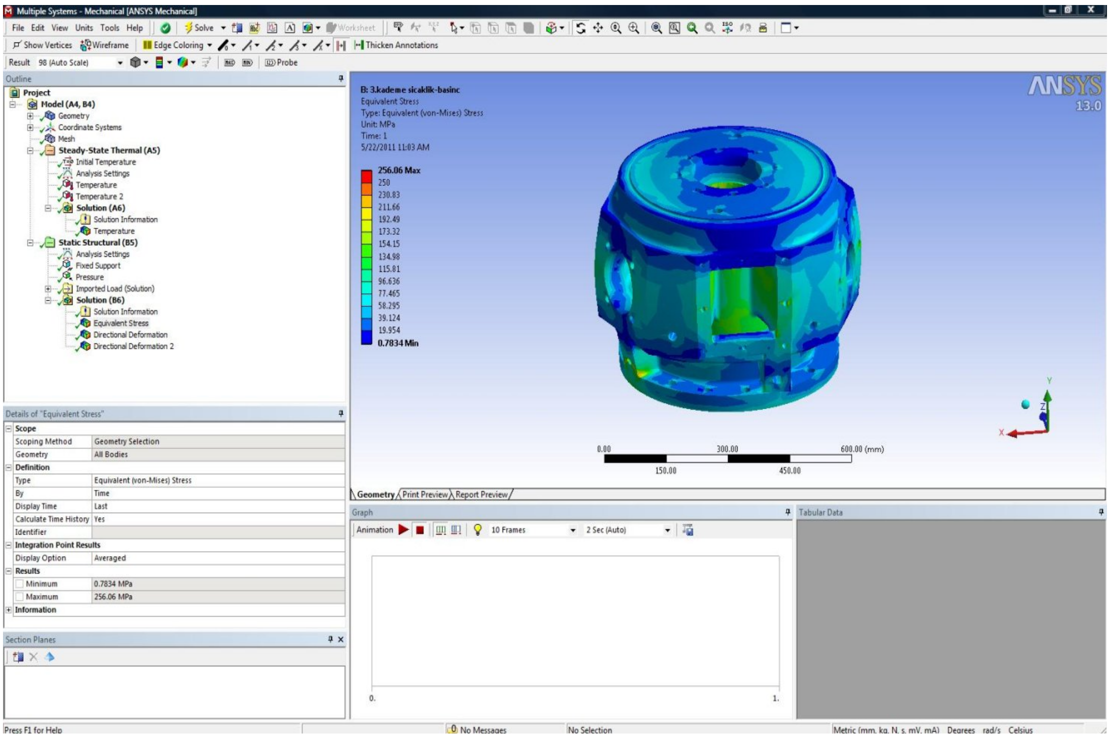


Figure 6.11 3rd stage cylinder stress distribution

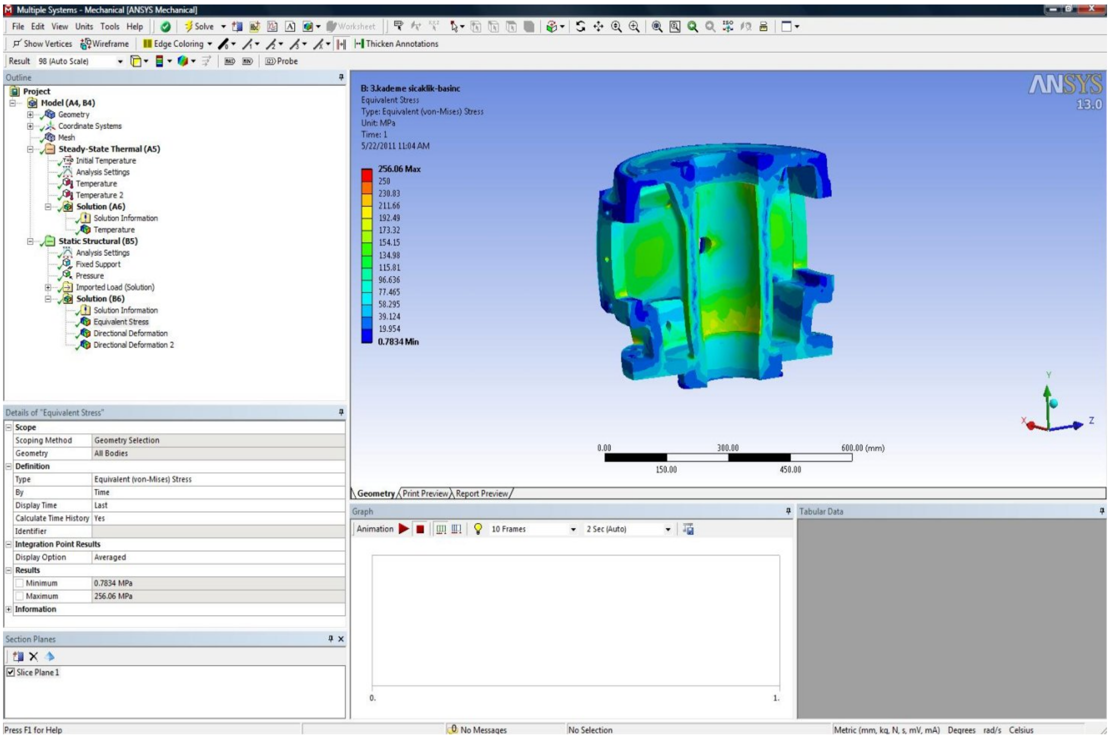


Figure 6.12 3rd stage cylinder stress distribution

6.3.2.Liners

Liners and sleeves are used in compressor cylinders either to form the cylinder wall or to be a removable part of the cylinder in case of accidental or wear over a long period of service.In general, liners are not used in low or mediu pressure cylinders where the gas being handled is non-corrosive, such as air. Liners are almost always used in medium to high pressure cylinders where the gas being handled is corrosive.

Certain industry users insist on liners in their compressors. Liners have become standart design for certain industry applications, even though the gas involved may be non-corrosive and medium pressure or with low differentials. It follows that a liner which increases the initial cost of compressor cylinder is more economical to replace then the complex cylinder casting. Another application of the liner is to reduce the cylinder bore size to meet certain capacity conditions. By installing liners with different bore sizes, the same cylinder casting can be used to accommodate a range of capacity and pressure conditions.

Figure 6.13 Cylinder liner

6.3.3 Crankcase

Design of the crankcase has more different parameters than the other components. In the other components design strength properties is more important than the geometrical properties. But during the crankcase design geometrical properties have greater importance . Because we set the distance and stroke values with the geomerty of crankcase. In addition there must be enough space inside the crankcase for the working of crank and connection rod comfortably.

On the other hand, we have to calibrate width of the crankcase sensitively because during the working, all of the components must be in the same axis. For this reason we have to identify the distance and clearance of the bearings in a sensitive way.

For the strength properties of the crankcase must carry all of the components so it must have enough strength against the bending conditions.

By using all of these parameters we can make a pre-design and calculations. For the ansys analysis we have to specify loading conditions. In this analysis we applied bending moment to the right top and the left top of the crankcase. Weight of the cylinders and other parts form the bending moments.

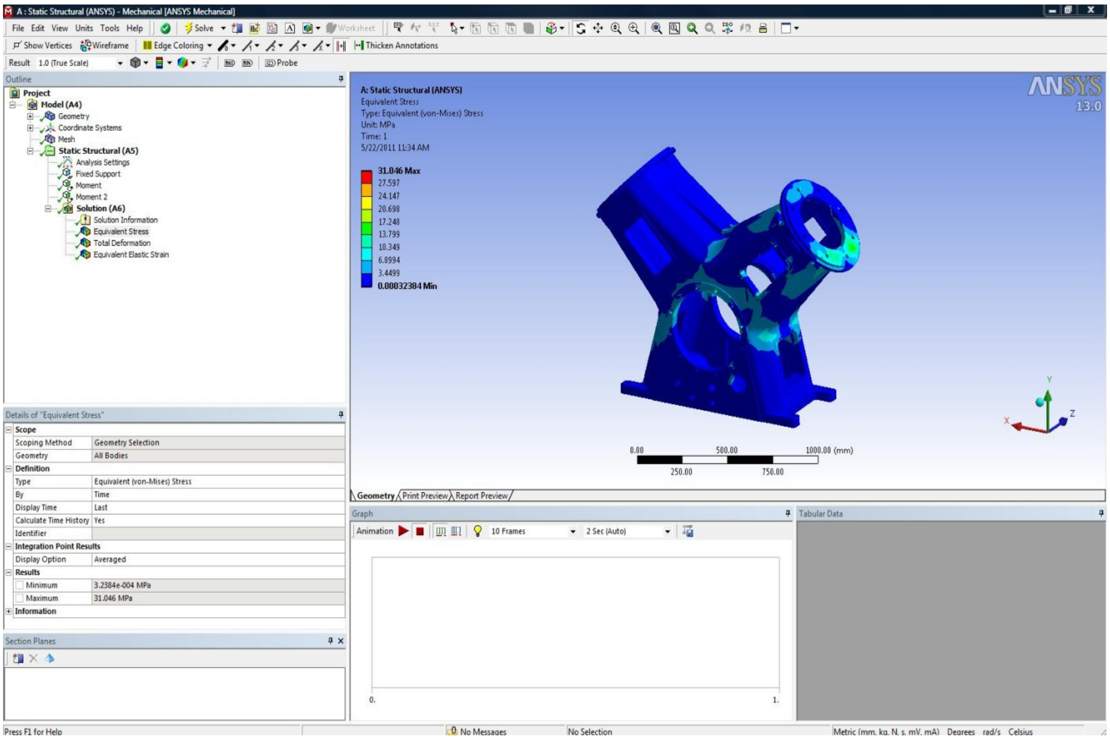


Figure 6.14 Crankcase stress distribution

We identified yielding strength value for EN-GJL-250 material as 250 MPa. In Figure 6.14 we can examine the stress distribution caused by bending moment loading. After the ANSYS stress calculation we decided that our crankcase design has enough strength values. Alongside, we made assambly with Pro-Engineer for examination of geometrical properties. Depends on these parameters we reached a conclusion that our crankcase design has acceptable properties for the system.

6.4 Moving Parts Design And Calculation

We seperated the complete moving system into three parts for the optimum calculation values and times. We applied gas forces, rotational velocity or displacement values to the these seperate systems directly as an input and inertia forces are applied automatically by ANSYS. As a result of these loading conditions we can create the system as real as possible.

We calculated gas forces in chapter 3rd and also we calculated and compared inertia forces in chapter 5th. We used these forces directly also for these time dependent analysis we have to define rotation or displacement to the systems. In figure 6.17 we can examine the crankshaft – connecting rod – crossheads group. For this group we defined 1000rpm (104.7 rad/s) rotational speed as an input. In figure 6.15 and 6.16 we can examine the piston-piston rod - crosshead connections group. For this analysis we defined 125mm. strok values as an input. We defined these loads, rotations and displacement as time variable. With this situation we can calculate exact force, stresses, deformation etc. values according to the exact positions. Now, we can examine all calculated values and use them in our design process. We are going to design moving parts 3 times bigger stress conditions because of the safety factors selection.

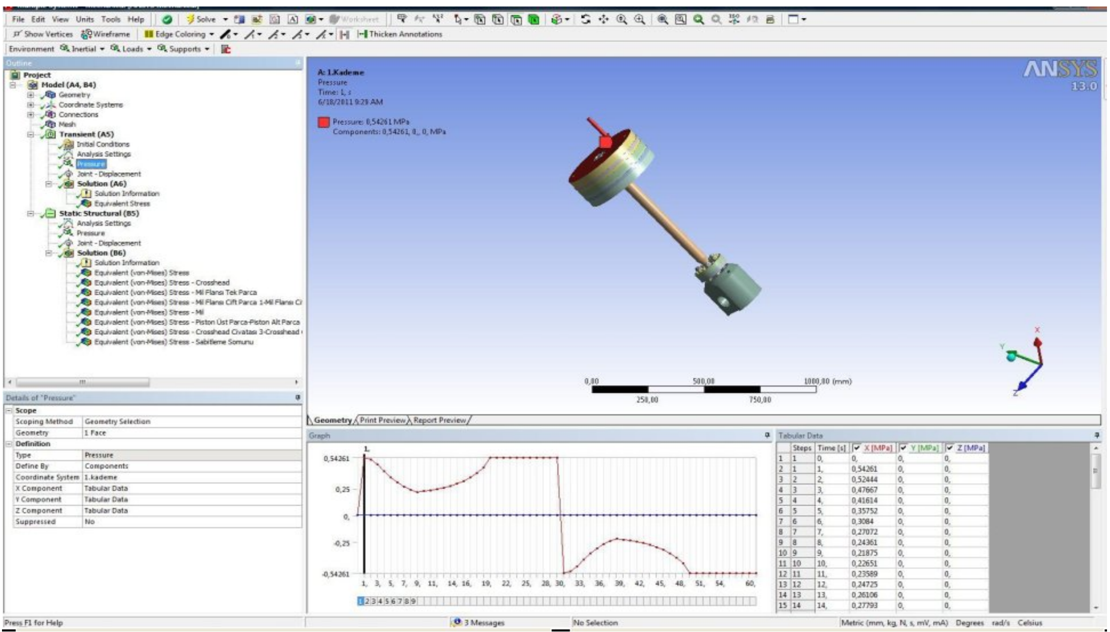


Figure 6.15 1st Group Ansys loading conditions

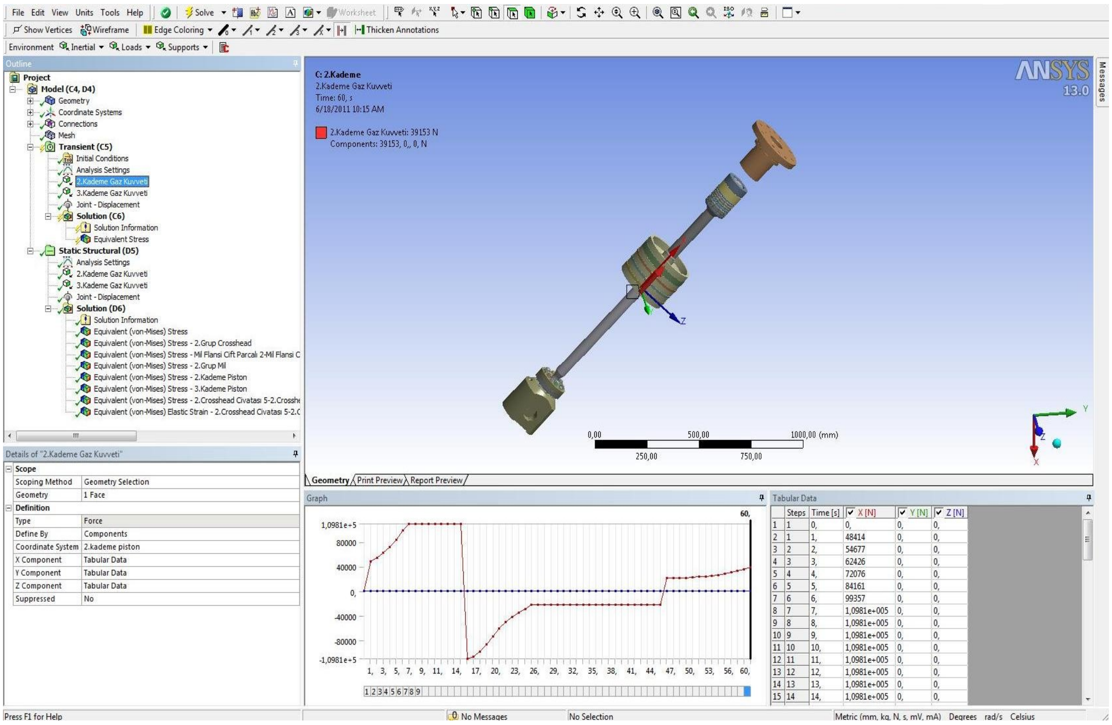


Figure 6.16 2nd Group Ansys loading conditions

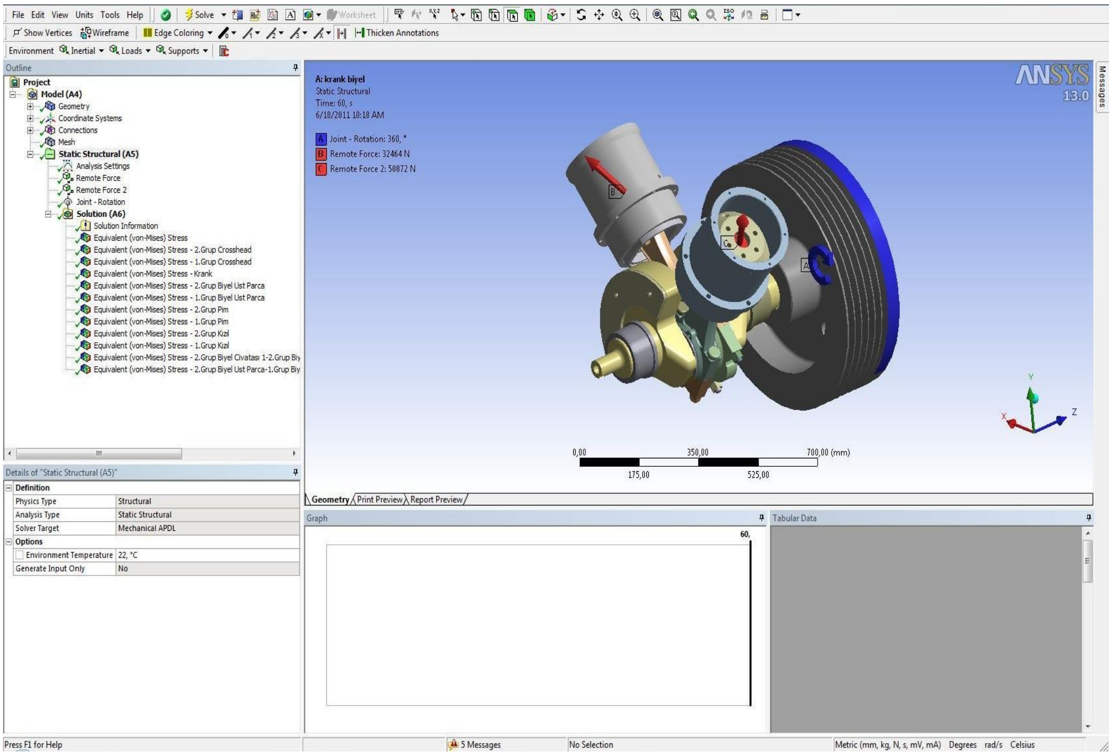


Figure 6.17 Slider – Crank system Ansys loading conditions

6.4.1 Crosshead

The crosshead guides the piston rod and transfers the side thrust to the guides. The side thrust comprises one component of the force acting along the connecting rod and in horizontal machines the weight of a part of the piston rod or of the piston. The crosshead at the same time transmits the axial force from the piston rod to the connecting rod. The force is transmitted to the pin about the axis of which the connecting rod oscillates.

There are two principal types of crossheads; crossheads supporting the pin, termed fork type (Fig.6.18 a), or pear-type (Fig6.18 b), and crossheads with a bearing.

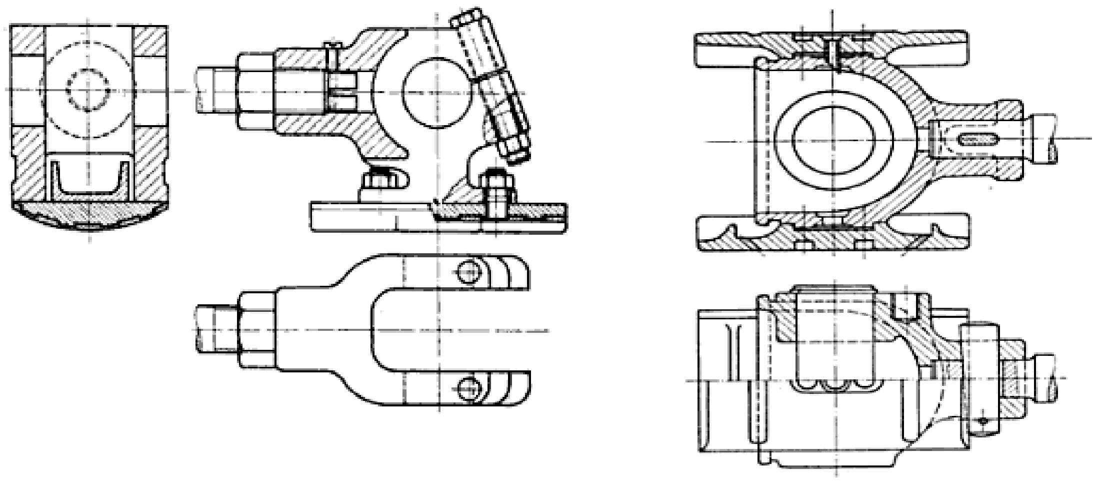


Figure 6.18 Principal types of crossheads. fork type (left), pear type (rigth)

The guading of the crossheads is either double-sided or one sided. If one sided, the smaller component of the connecting rod force, directed away from the main guide, is normally taken up either by two lateral slide bars, or one slide bar of U-profile in the centre of the crosshead. One sided slide bars are found mainly in large machines. In both cases, the guiding surfaces may be either cylindrical or plane.

All forces acting on crossheads with the connection of piston rod and connecting rod directly. Crossheads must have enough strength against this total force. Expecially, crosshead bolts are exposed to big tensile and compression stresses. On the other hand, during the working there is face to face contact between crosshead and crosshead cylinder and the gap between these two part must be as small as possible. In this situation, during the design process we have to consider these gaps for the optimum lubrication and working without impact. We have to choose crosshead material more softer than the cylinder material. In Figure 6.18 we can examine the stress distribution for the crosshead. After the examination of stress distribution, our crosshead design has enough strength properties for these working conditions. After production and assembly, we can examine the necessary design gaps for the optimum lubrication.

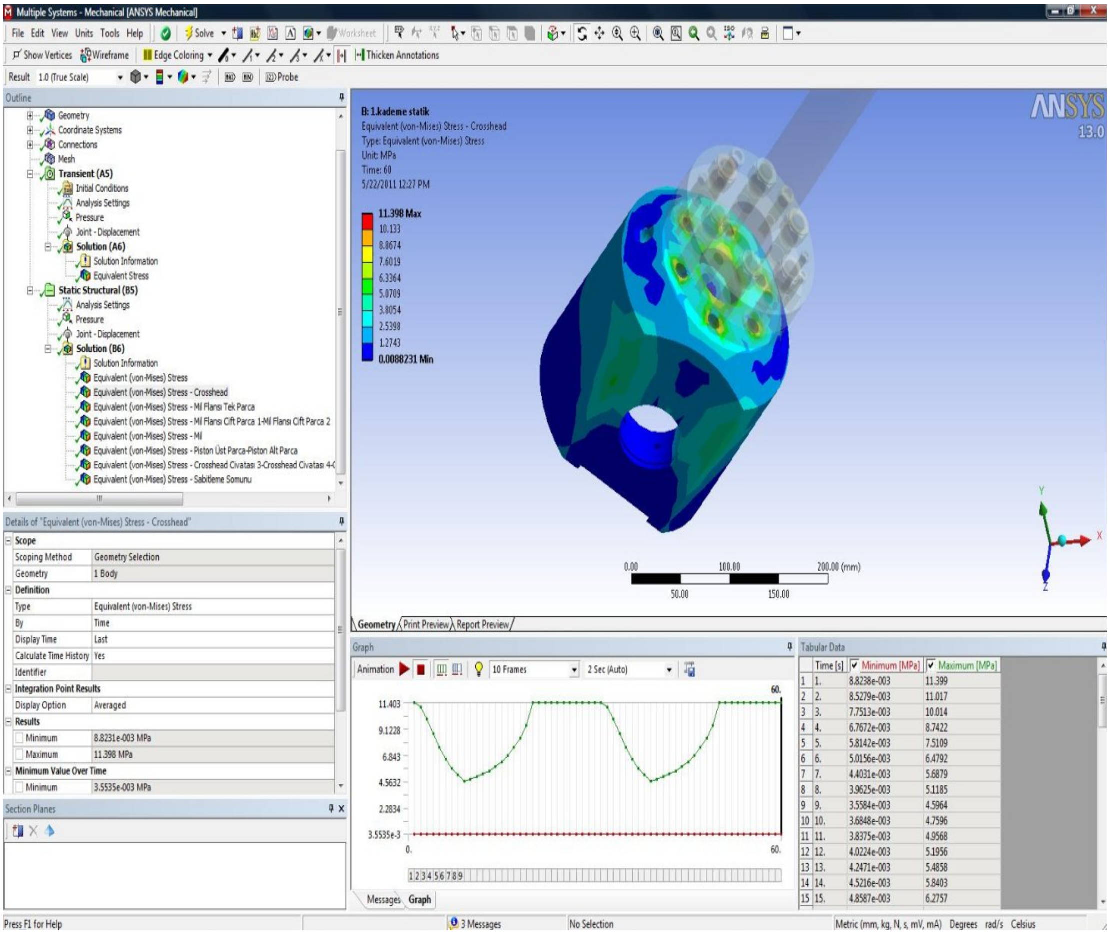


Figure 6.19 Crosshead stress distribution

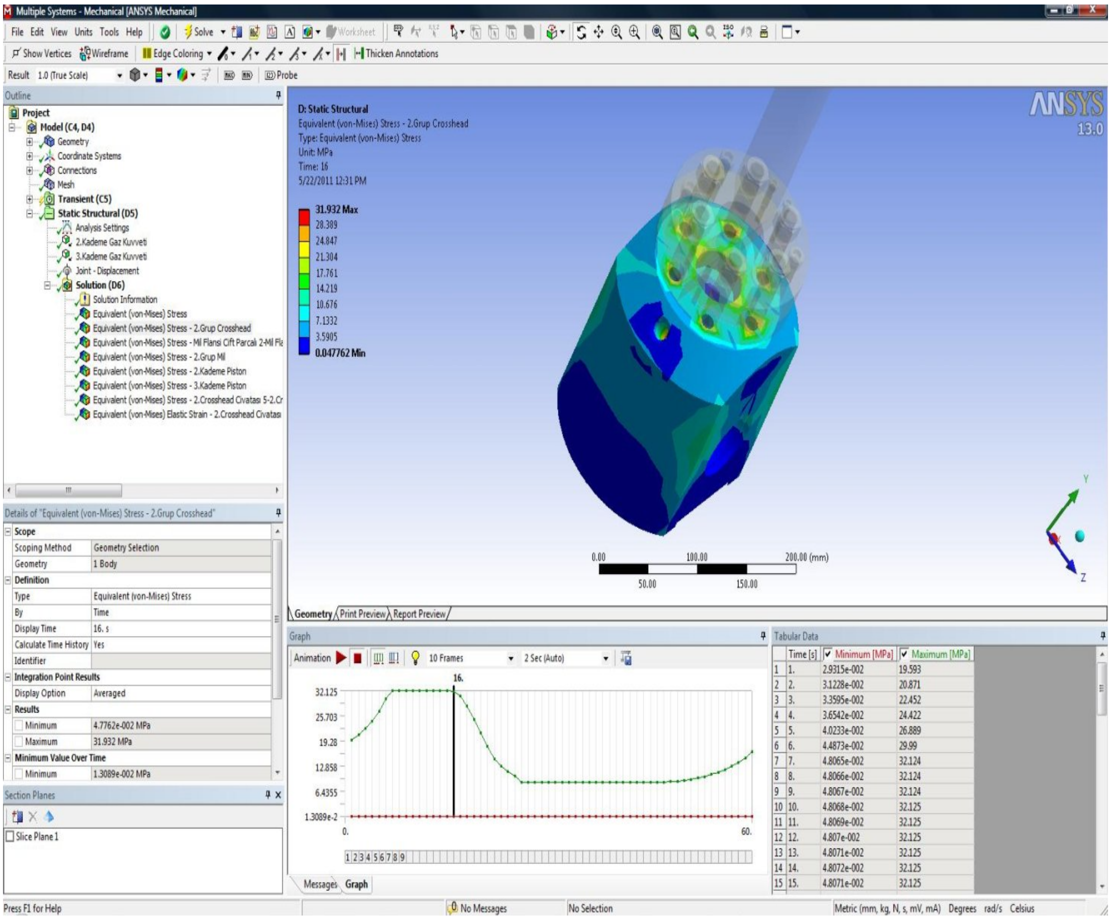


Figure 6.20 Crosshead stress distribution

6.4.2 Piston Rod

The piston rod transmits the force from the crosshead to one piston or several pistons in line, or inversely transmits the force from the piston to the crosshead.

Like other components in the modern reciprocating compressor, the piston rod is designed for specific applications such as,

- Operating pressures
- Gas composition
- Capacity
- Rotating speed

Piston rods incorporate diameter, lenght, material, composition and fastening arrangements as dictate by both the operating conditions of the compressor and the design of the piston to which they are connected.

Figure 6.21 Different types of piston rods used in reciprocating compressor.

Types of piston rods

Single Piston rods, this is the most common type. It contains a single piston.

Tandem Piston Rod, This is a piston rod on which two or more pistons are mounted in tandem. This is used where loading is low so that the combined loading does not exceed the allowable frame load.

Piston Rod for Truncated Cylinder. This is a special configuration which accomodates the truncated cylinders.

Tail Rod. In these arrangement, the piston is in the center of the pistonrod and the rod is the same diameter on both sides of piston. This prevents non-reversal loading due to equal areas and equal pressures on both sides of the piston.

Cooled Piston Rod. This rod is drilled through the center axis to allow pressure fed coolant, usually oil to circulate uo the rod core. Such circulation helps to remove the heat of compression from rod and piston.

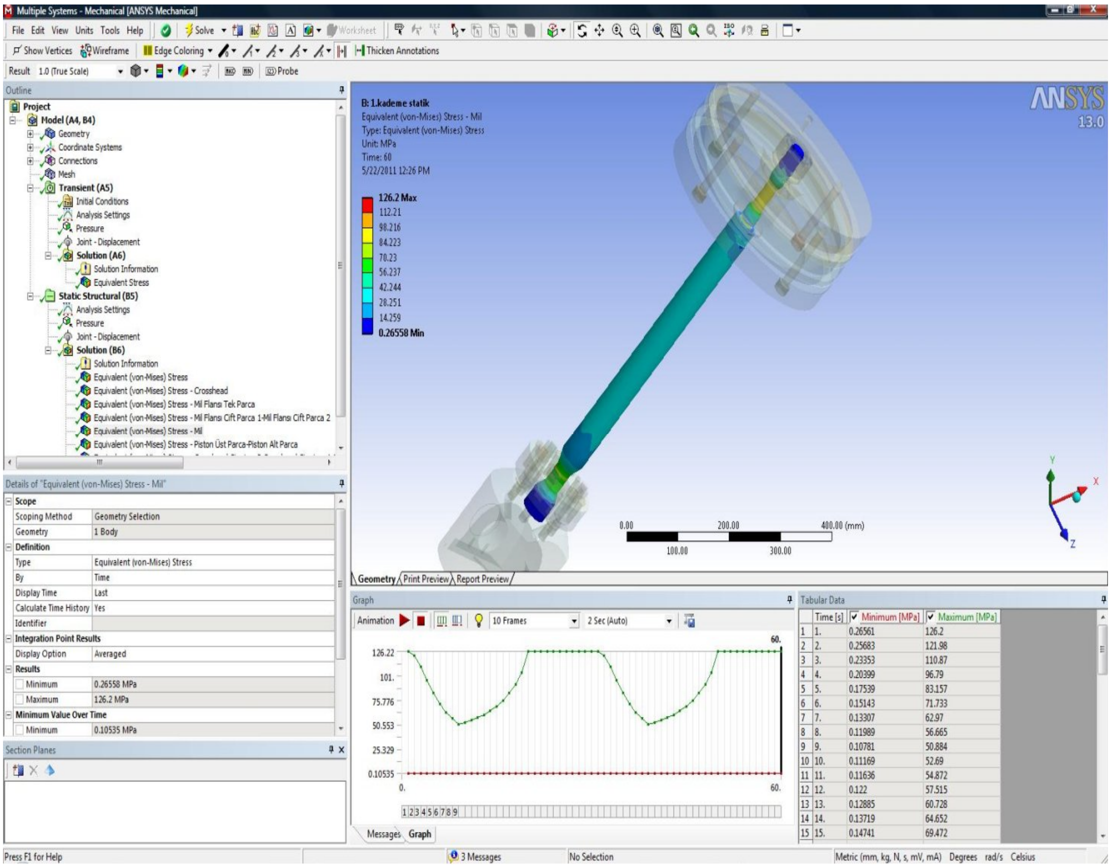


Figure 6.22 1st group piston rod stress distribution

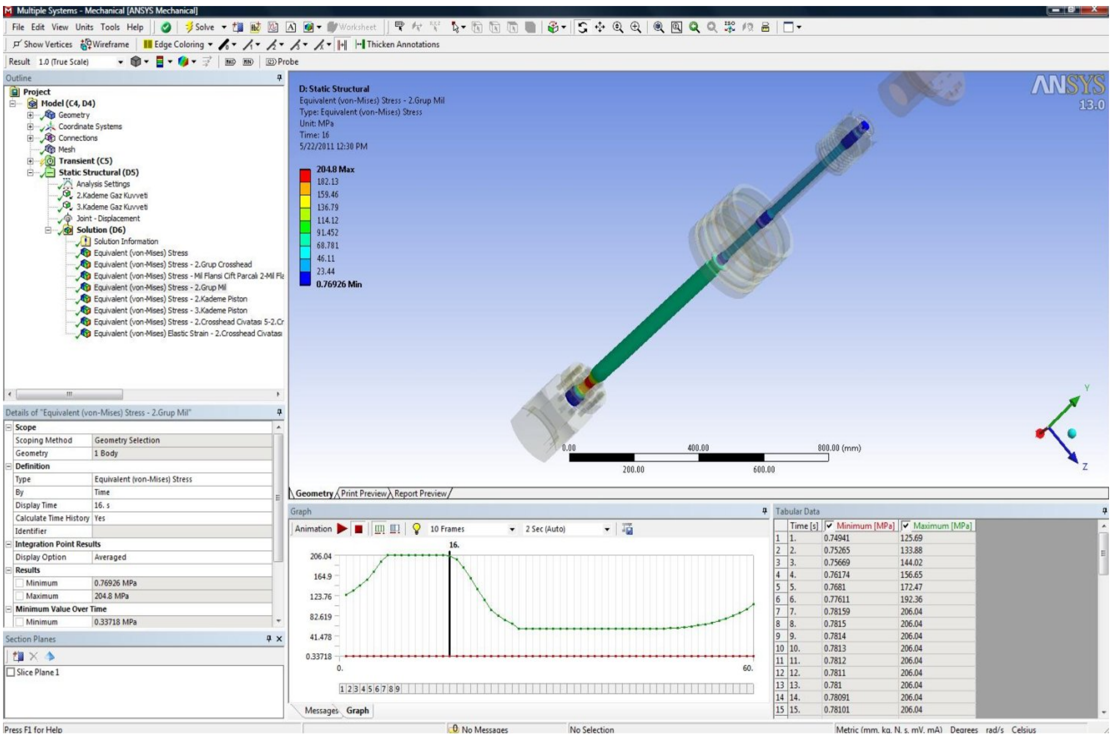


Figure 6.23 2nd group piston rod stress distribution

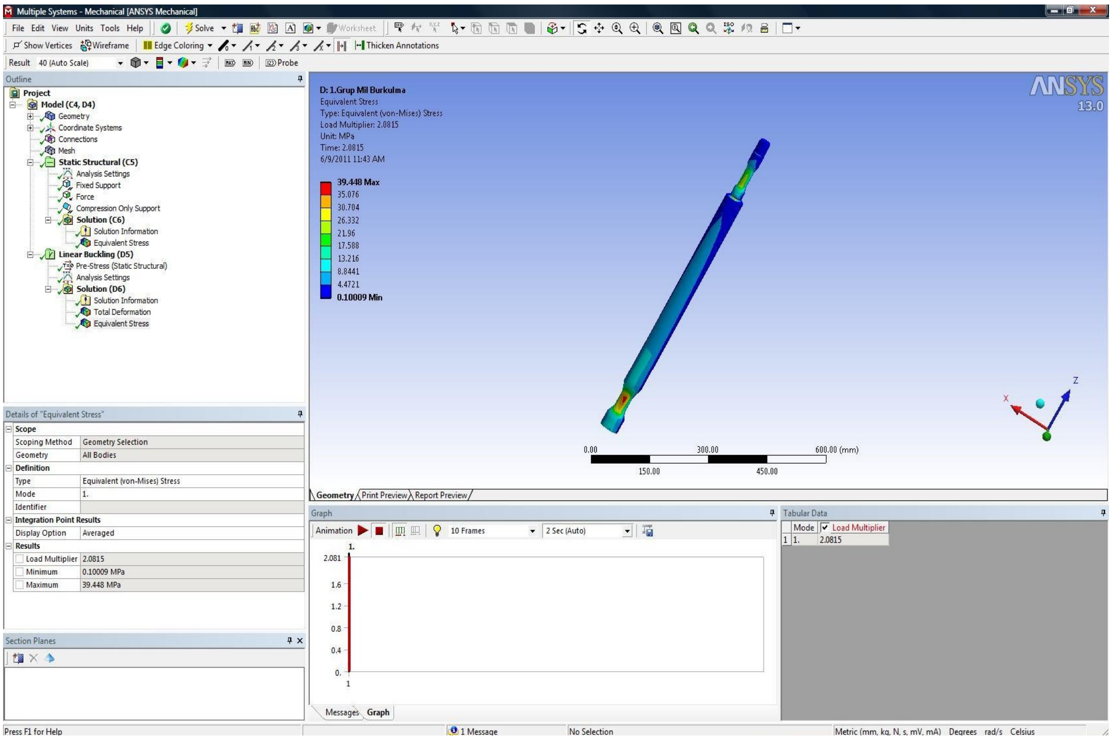


Figure 6.24 1st group piston rod buckling effect.

In Figure 6.22 and 6.23 we can understand that the stress distribution on 1st stage piston rod and 2nd-3rd stage piston rod. Because of the safety factor we have to use 3 times bigger stress values. On the other hand we have to consider endurance stress for the material during the examination. We used special Carbon alloy steel for the piston rods. Endurance stress limit of this steel is 600 MPa. Depends on material properties, safety factor and stress values of our piston rod design has enough strength. On the other hand, buckling is the another effect for the piston rod. We made buckling analysis with the Ansys buckling module. For these analysis we have to define connection and load. In our system piston rod has a fix connection with crosshead and has a translation with piston. Also we applied gas forces to the piston rod. In Figure 6.24 we can examine the buckling effect. At the end of the analysis we found load multiplier as 2,1.

6.4.3 Piston

The main requirement of a piston is good sealing of the cylinder. The second is that the weight of the piston and the entire crank mechanism is a minimum, particularly for high speed machines, in order to reduce the inertia forces and to improve the mechanical efficiency. In horizontal machines a small piston weight reduce the risk of its seizing.

The design and materials used for compressor pistons will vary with the make, type and application of the compressor. They are designed to take into account a number of conditions:

- Cylinder bore diameter
- Discharge pressure
- Compressor rotative speed
- Compressor stroke
- Required piston weight

Compressor pistons are typically designed as one of three types:

One piece, either solid cast iron or steel, for small bores and high pressure differential applications, or one piece hollow-cored cast iron or aluminum for large diameter and lower pressures.

Two piece, aluminum or cast iron, which is split for ease of hollow casting and weight control. These are generally used above 10” bore diameters. Aluminum is used when the reciprocating weight must be reduced

There piece, in which a ring carrier is added to permit band-type rider rins to be installed directly into the piston grooves. While this design adds a part, it allows thicker rings to be used since the ring does not have to be streched over the outside diameter. Is is also used as a carrier for the piston rings on large diameter pistons, where metallic rings are used which might wear into an aluminum piston.

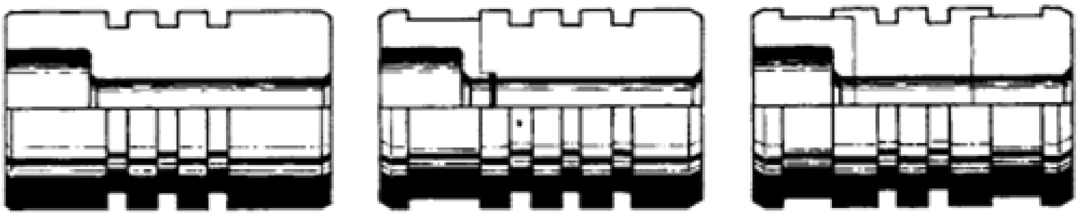


Figure 6.25 Types of pistons used in reciprocating compressors.

Piston Materials

Material selection for pistons is very important, and many factors must be considered. Some of these include,

- Weight
- Strenght, for differential pressure and inertia forces
- Corrosion resistance
- Compression and rider ring wall wear resistance
- Outside diameter wear resistance

In the first and second stage we used special aluminyum alloy as a piston material. But for the third stage we decided to use carbon steel because of the high pressure values. We added material properties for these materials in appendix. If we examine material properties, all of the piston have enough strength properties under these working conditions taking into consideration the safety factor. On the other hand, gaps and seat clereance of the rings are so important for the optimum performance. After the assembly and during the tests we are going to examine the gaps and clereance for the piston and piston rings.

In figure 6.15 and 6.16 we can see the loading condition of the system. Depends on these loading conditions we obtained stress distribution on pistons such as Figures 6.26 , 6.27 , 6.28 , 6.29 , 6.30 , 6.31. After the examination of these stress distributions on the pistons, we verified that our pistons have enough strength properties under these working conditions and safety factor.

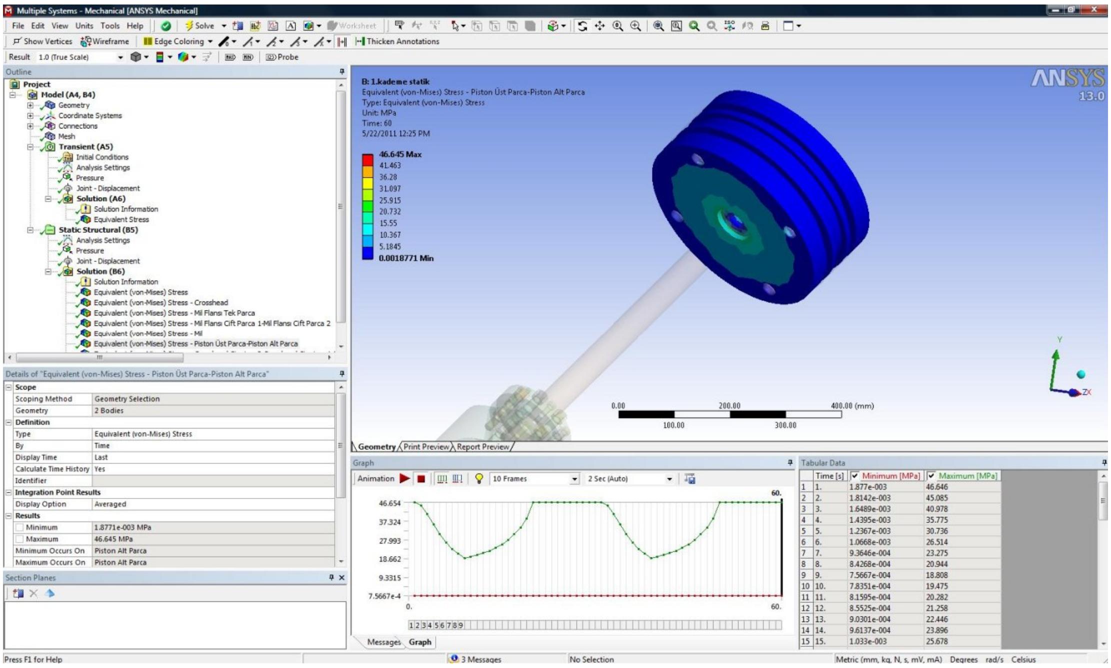


Figure 6.26 1st stage piston stress distribution.

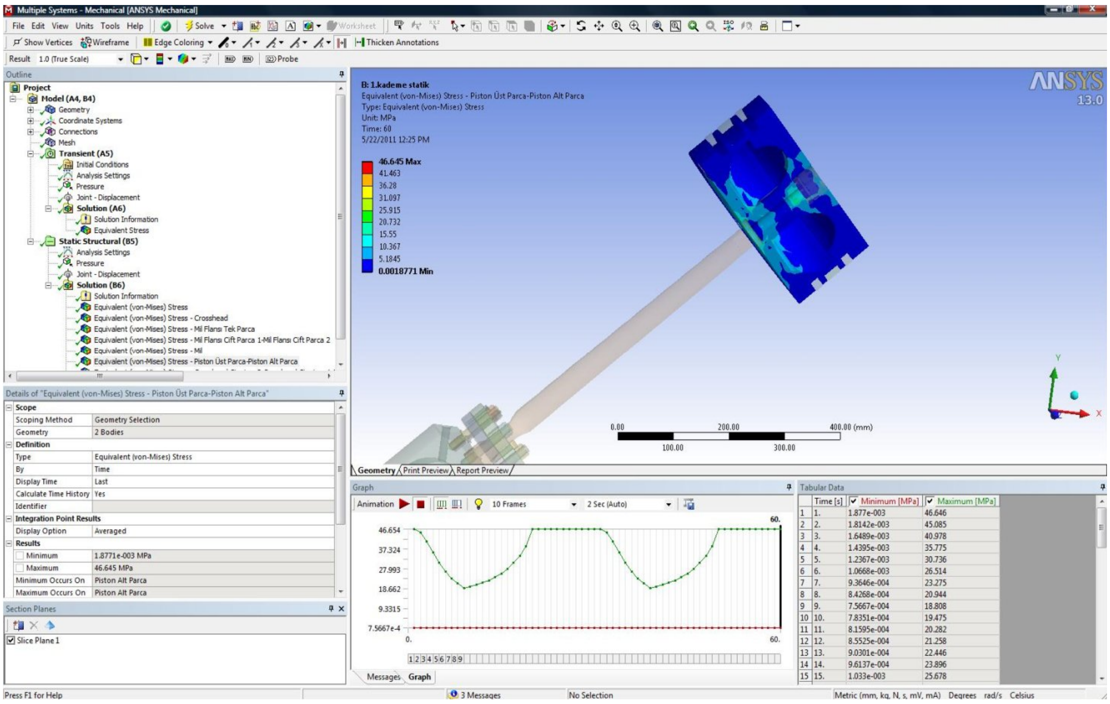


Figure 6.27 1st stage piston stress distribution.

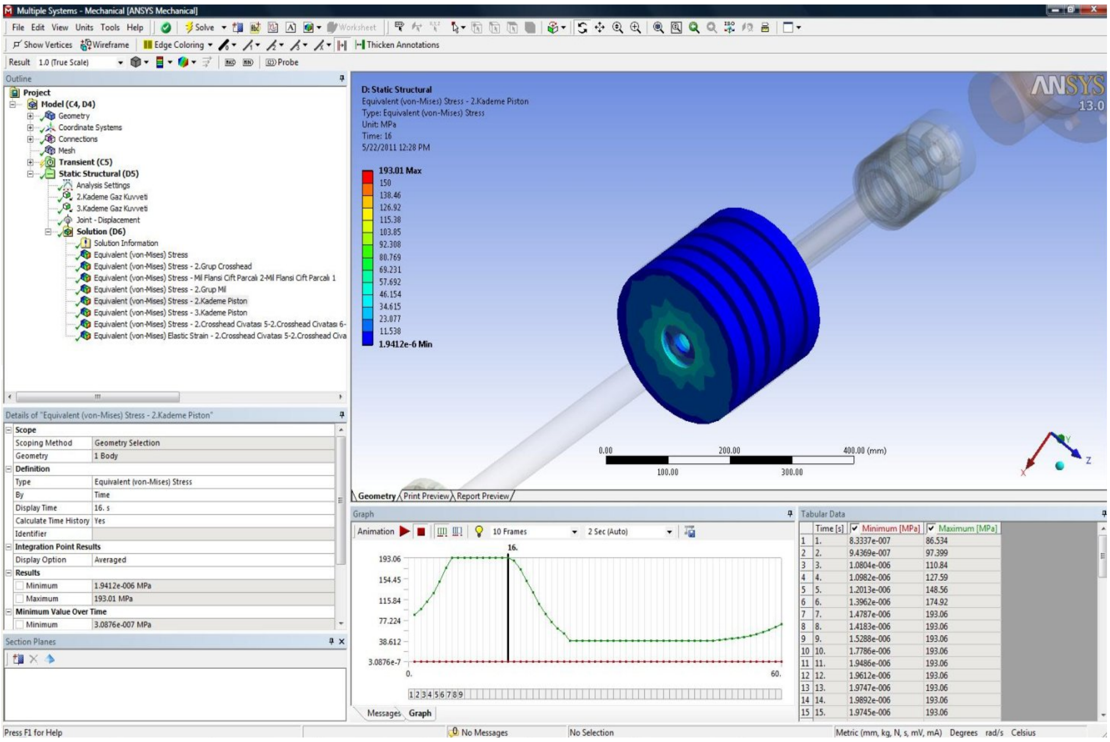


Figure 6.28 2nd stage piston stress distribution.

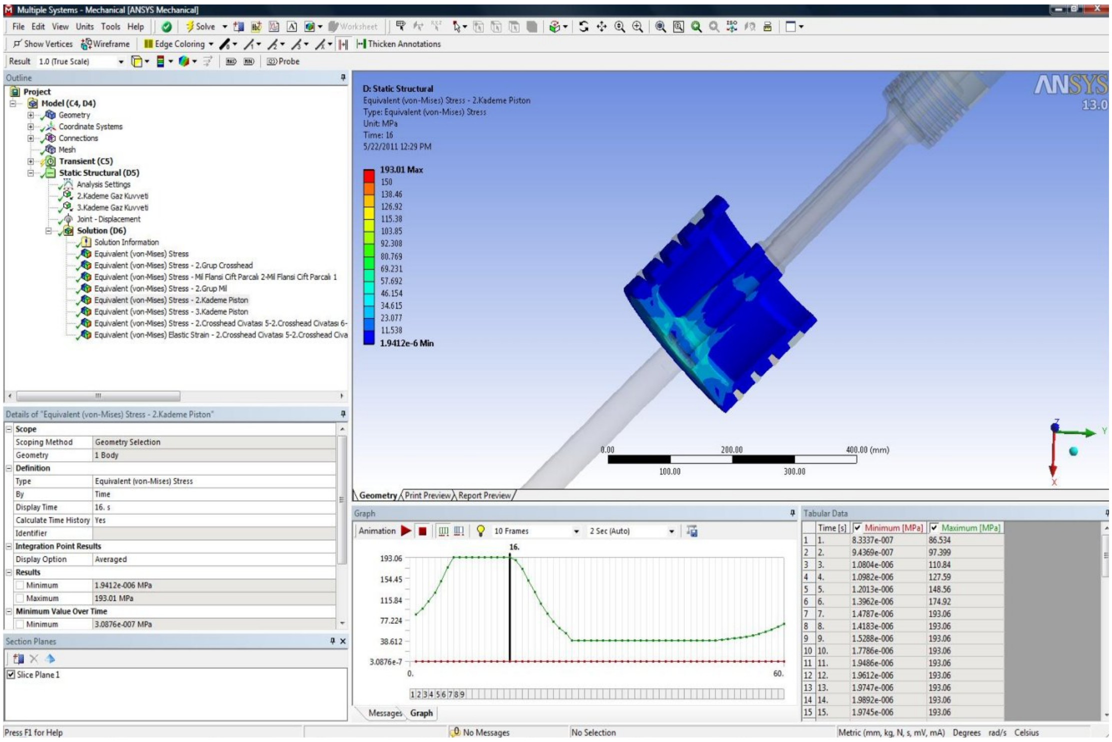


Figure 6.29 2nd stage piston stress distribution.

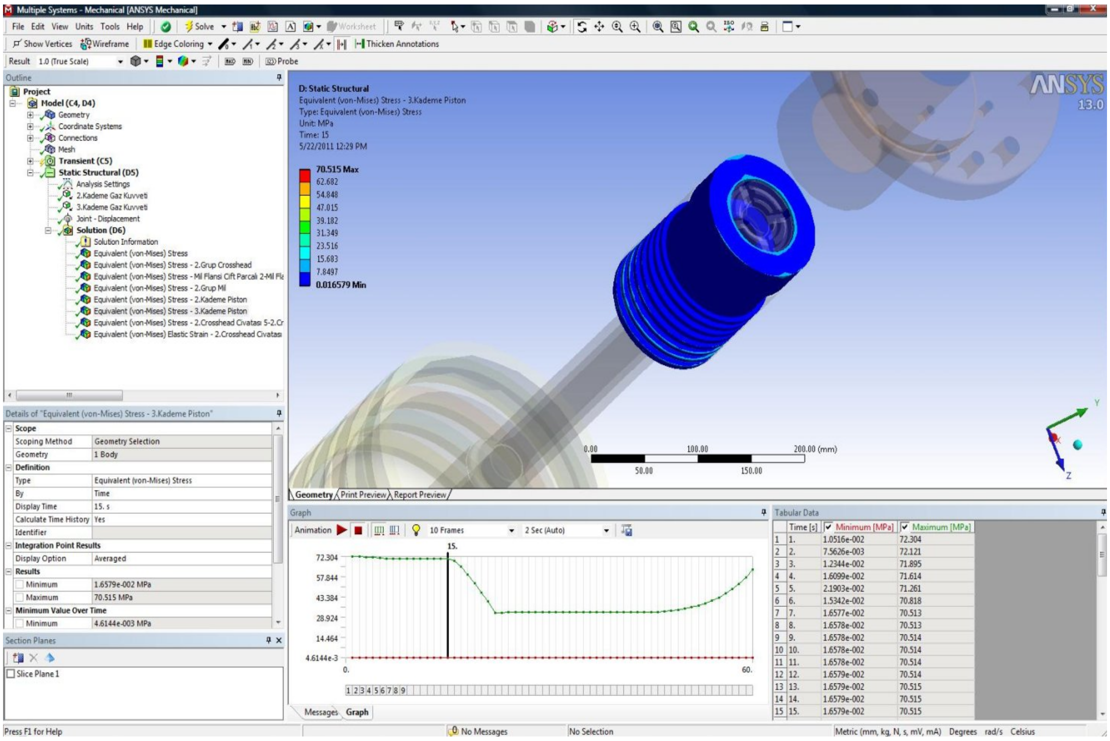


Figure 6.30 3rd stage piston stress distribution.

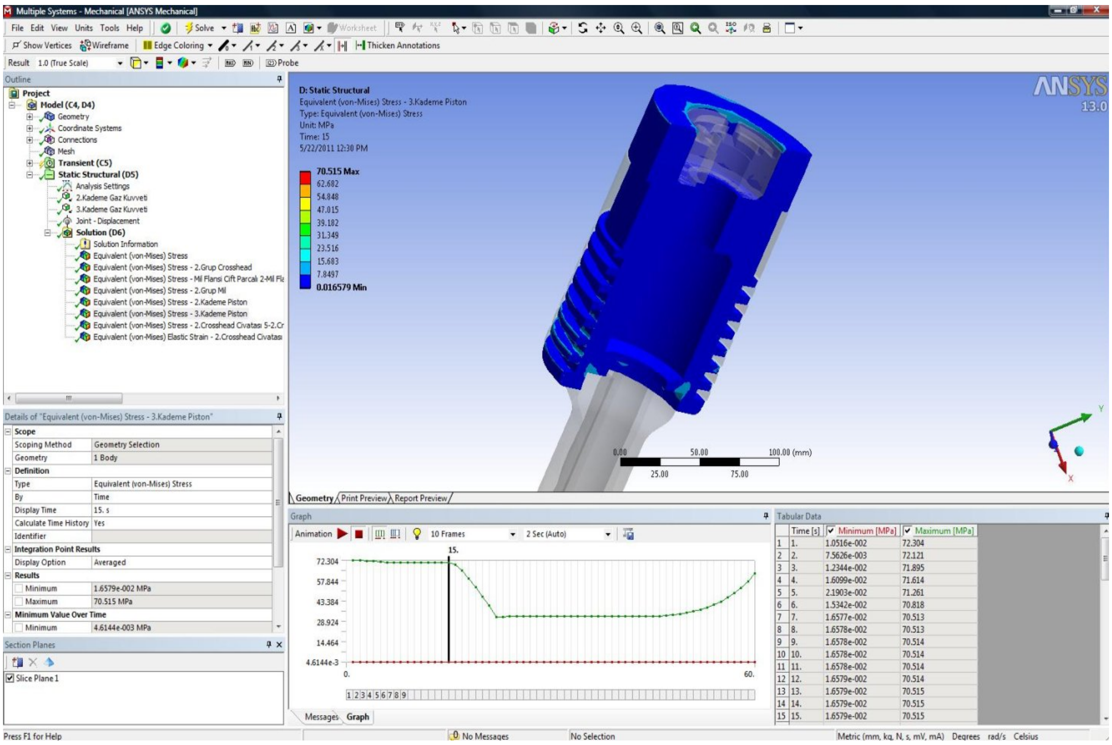


Figure 6.31 3rd stage piston stress distribution.

6.4.4 Connecting Rods

The connecting rod connects the crank pin with the crosshead pin or with the gudgeon pin. Its principal parts are; the shank and two ends, i.e. the crank end (big end) and the crosshead end or piston end (small end). The crosshead or piston end is normally closed. The crank end need not be split in machines with an overhung crank or with a composite crankshaft. Otherwise, the crank end must as a rule be split. In large motors, or in cases where variations of the length of the connecting roda re required, one or both ends are detachable from the shank. An advantage of this design is the possibility of using a direct babbitt lining of the end without using a bush, reducing the weight and at the same time improving heat dissipation. Detachable ends are often made of cast steel. Otherwise, cast steel is considered to be an unsuitable material for connecting rods due to its low strength under alternating stress conditions.

The shape of the connecting rod is influenced, on the one hand by its purpose and, on the other hand by the manufacturing procedure. The cross section of the shank is normally circular (Figure 6.32 a) if produced in small batches. In these case the connecting rod is machined from a forging blank.

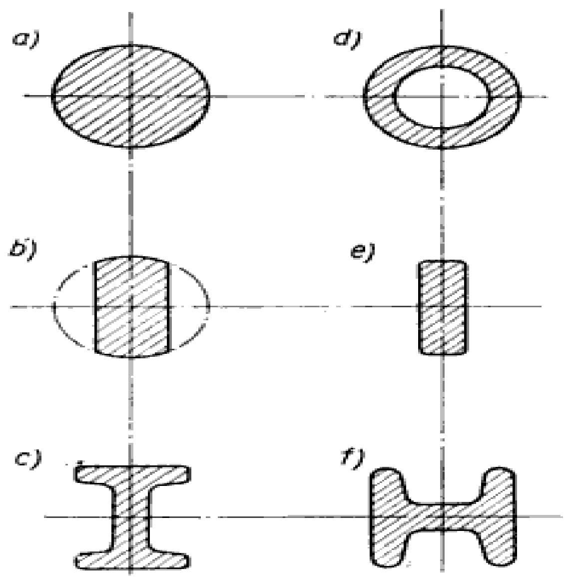


Figure 6.32 Connecting rod cross-sections

For high-speed machines the circle is either flattened at the sides (Figure 6.32 b) or a rectangular profile is selected (Figure 6.32 e) . For machines of very high speed the shank is milled into an I-section (Figure 6.32 c) or the circular shank of the connecting rod is bored (Figure 6.32 d) . In the manufacture of larger quantities, the connecting rods are die-forged an usually I- or H-shaped (Figure 6.32 f) . In these case the shank is not machined, the seam formed between the dies is removed by hand.

As a rule the length of the connecting rod of stationary machines lies between 4 or 6 r (r being the radius of the crank circle). For high speed machines a shorter connecting rod is selected. If the length of the machine is to be small, a forked connecting rod is used with the crosshead pin fixed into the eyes. The bearing may be placed in the crosshead, or two pins are forged integral with the crosshead body and both crosshead ends of the forked connecting rod are split or detachable.

We chose our connecting rod material as EN-GJS-600-3, on the other hand we chose the cross-section as H-shaped. Depend on these selections and loading conditions in fig.6.17, we got stress distribution on the connecting rod as fig.6.33. According to the material properties and safety factor our connecting rod has enough strength properties. Besides, our connecting rods under the influence of buckling conditions. We made buckling analysis with using Ansys buckling module. In fig.6.34 we can examine the buckling stress distribution and load multiplayer. Our max. stress value under the buckling conditions is 43.6 MPa and our load multiplayer is approximately 3. In this case our connecting rod material has also enough strength under the buckling conditions.

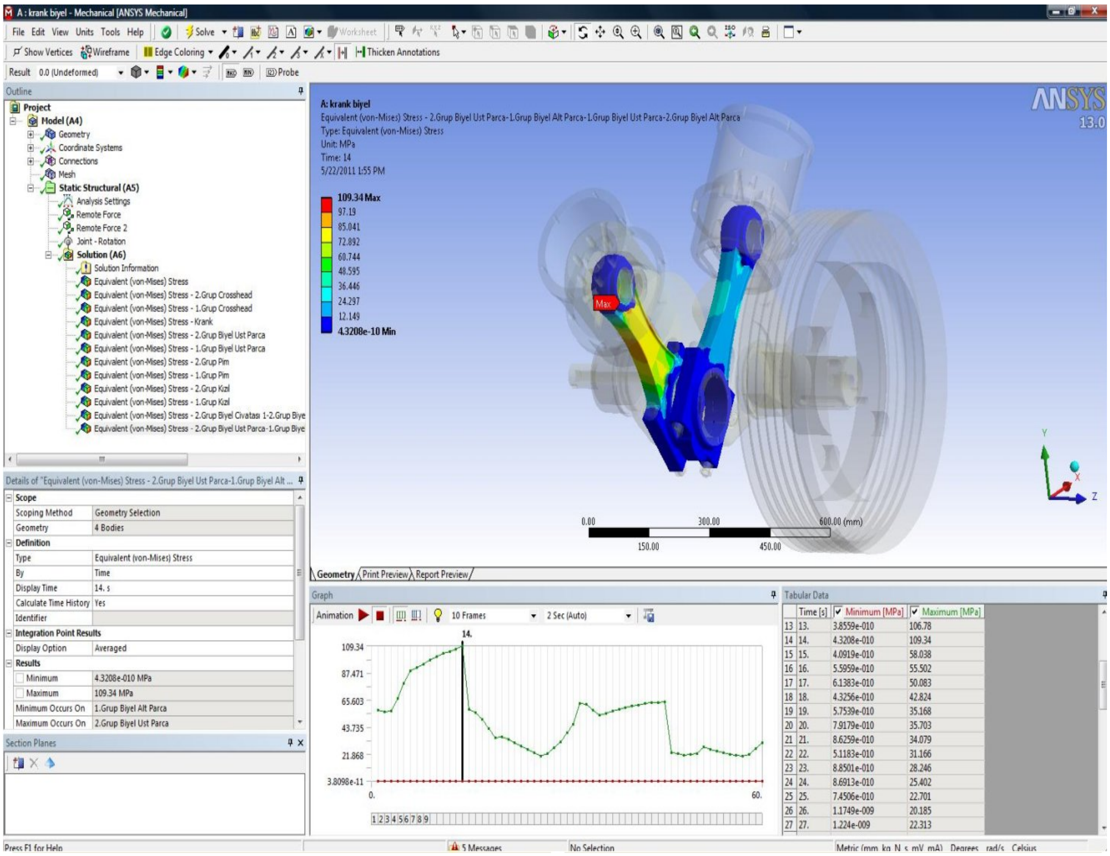


Figure 6.33 Connecting rod stress distribution

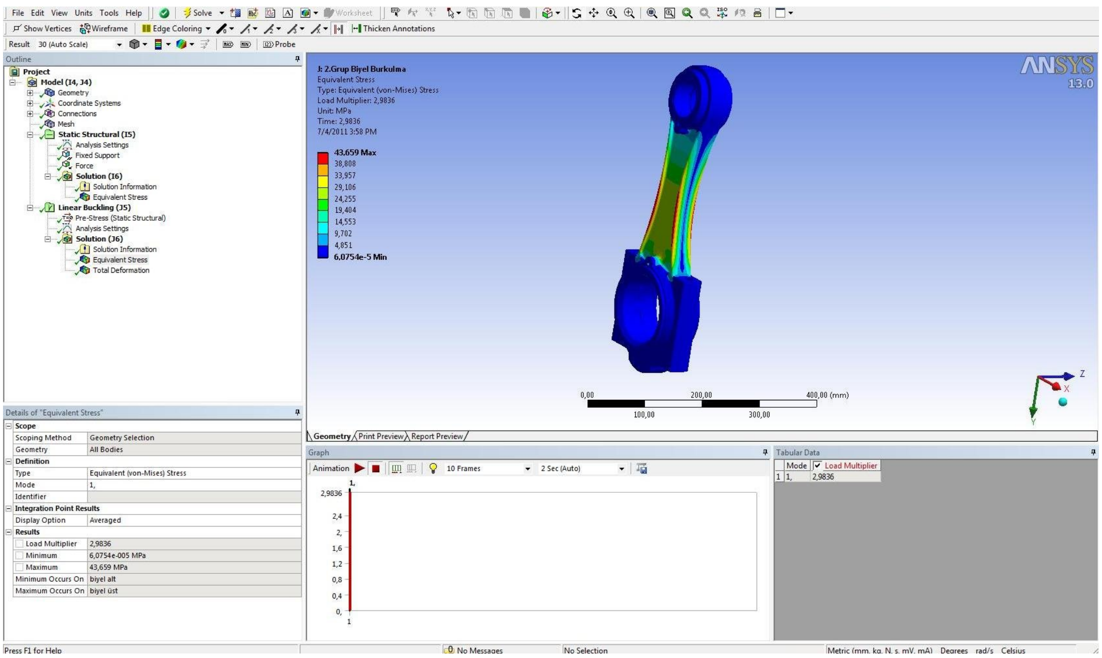


Figure 6.34 Connecting rod buckling stress distribution

6.4.5 Crankshaft

The crankshaft, sometimes casually abbreviated to crank, is the part of an engine which translates reciprocating linear piston motion into rotation. To convert the reciprocating motion into rotation, the crankshaft has "crank throws" or "crankpins", additional bearing surfaces whose axis is offset from that of the crank, to which the "big ends" of the connecting rods from each cylinder attach.

It typically connects to a flywheel, to reduce the pulsation characteristic of the four-stroke cycle, and sometimes a torsional or vibrational damper at the opposite end, to reduce the torsion vibrations often caused along the length of the crankshaft by the cylinders farthest from the output end acting on the torsional elasticity of the metal.

Crankshafts can be forged from a steel bar usually through roll forging or cast in ductile steel. Today more and more manufacturers tend to favor the use of forged crankshafts due to their lighter weight, more compact dimensions and better inherent dampening. With forged crankshafts, vanadiummicroalloyed steels are mostly used as these steels can be air cooled after reaching high strengths without additional heat treatment, with exception to the surface hardening of the bearing surfaces. Cylinder create pulsating compression forces and vibratory torque on the crankshaft with peaks that can exceed the average compressor horsepower torque by up to five times. The crankshaft design must be conservative to withstand these crank effort and vibratory stresses. For compressors over a small size of about 150 KW per crank, the crankshafts should be forged steel.

The cranks are arranged with equal angles between each crank to provide optimum unbalanced forces and the smoothest overall crank effort torque. Even-number crankthrow units are arranged with 180^{o} opposed pairs of cranks to cancel out inertia forces, odd number crankthrow units required special crank angle layout or dummy crossheads.

For some engines it is necessary to provide counterweights for the reciprocating mass of each piston and connecting rod to improve engine balance. These are typically cast as part of the crankshaft but, occasionally, are bolt-on pieces. While counter weights add a considerable amount of weight to the crankshaft, it provides a smoother running engine and allows higher RPMs to be reached. We are going to examine balancing next chapters.

We chose EN-GJS-600-3 as our crankshaft material because of the advantages of the material. After the Ansys stress calculation, we obtained stress distribution as fig.6.36 and fig.6.37. Depends on these results and safety factor our crankshaft has enough strength properties, but we have to verify our bearing dimensions, counterweigth etc. after the assembly and tests.

Figure 6.35 Crankshaft

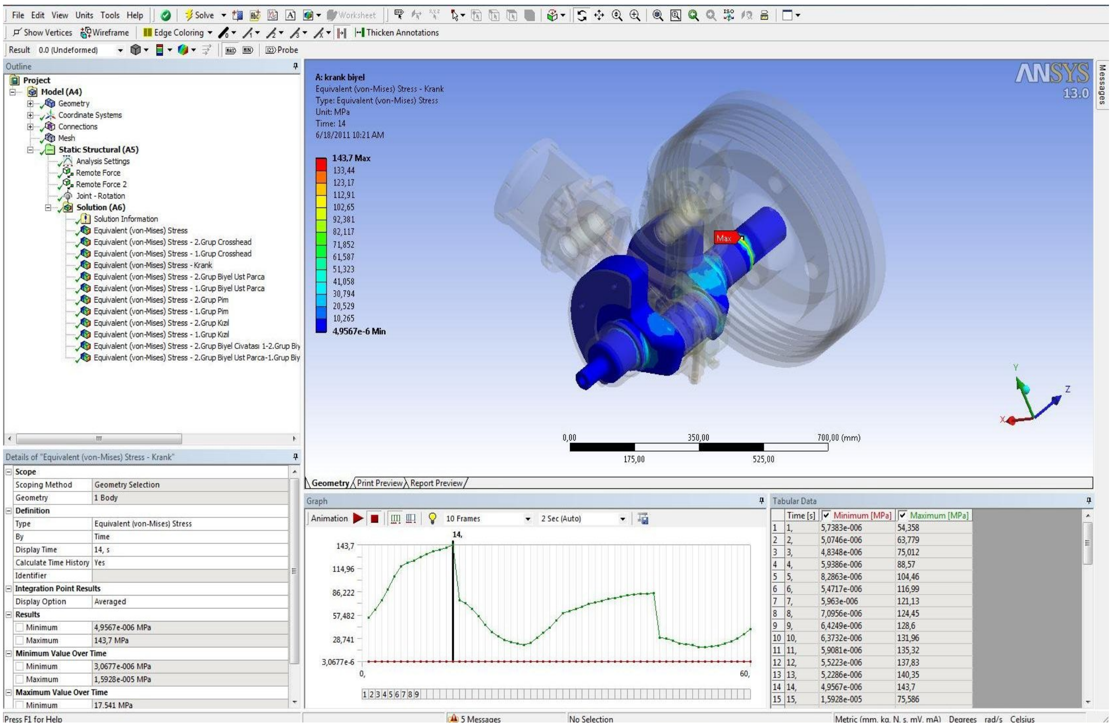


Figure 6.36 Crankshaft stress distribution

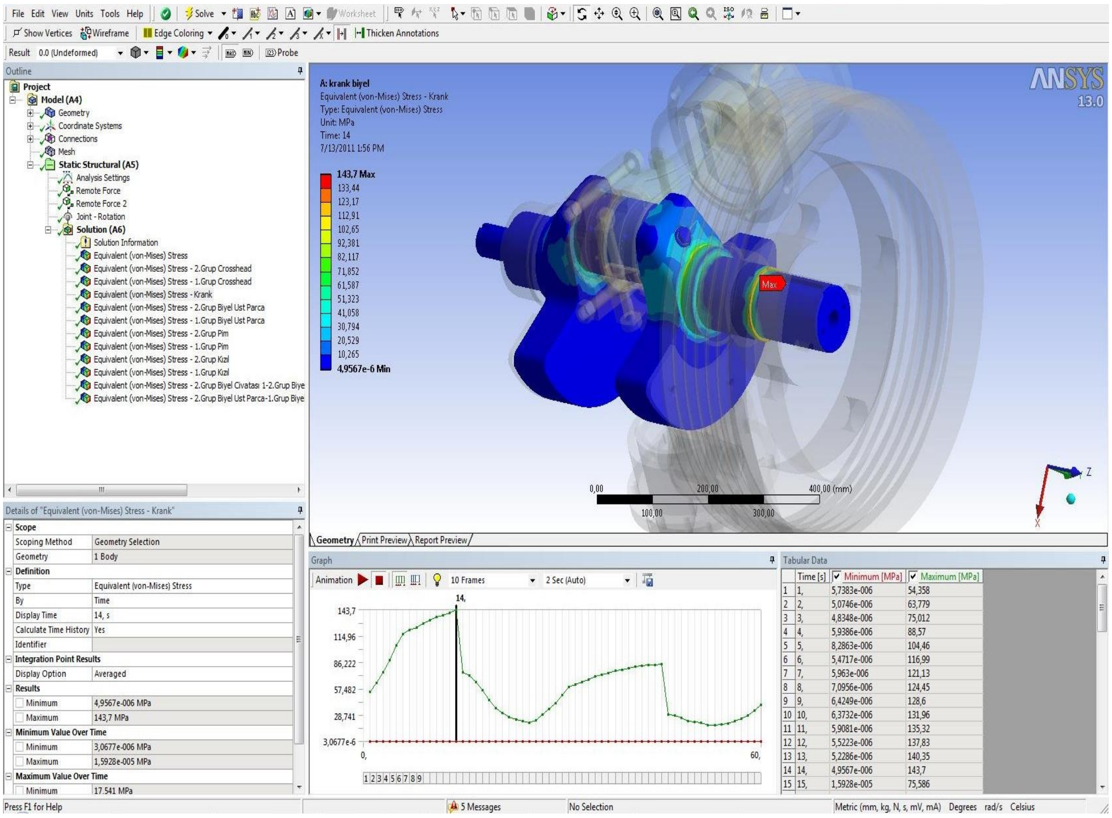


Figure 6.37 Crankshaft stress distribution

6.4.6 Valves

Compressor valves are simply fast acting check valves with a low pressure drop. They must be optimized to balance the opposing demands for long operating life and minimal pressure drop/flow loses. They may also have special features such as center ports to allow cylinder unloading.

The compressor valve is possibly the most critical component when determining the requirements for compressor service. The flow area is sensitive, as too small an area will give low efficiency, but too large area can result in valve flutter and early failure. Similarly, valve components must be designed for the expected pressure and temperature conditions.

Valves have been designed with many configurations, particularly in the sealing elements. These have progressed through steeli bakelitei glass filled Teflon or Nylon, and high strenght plastics.

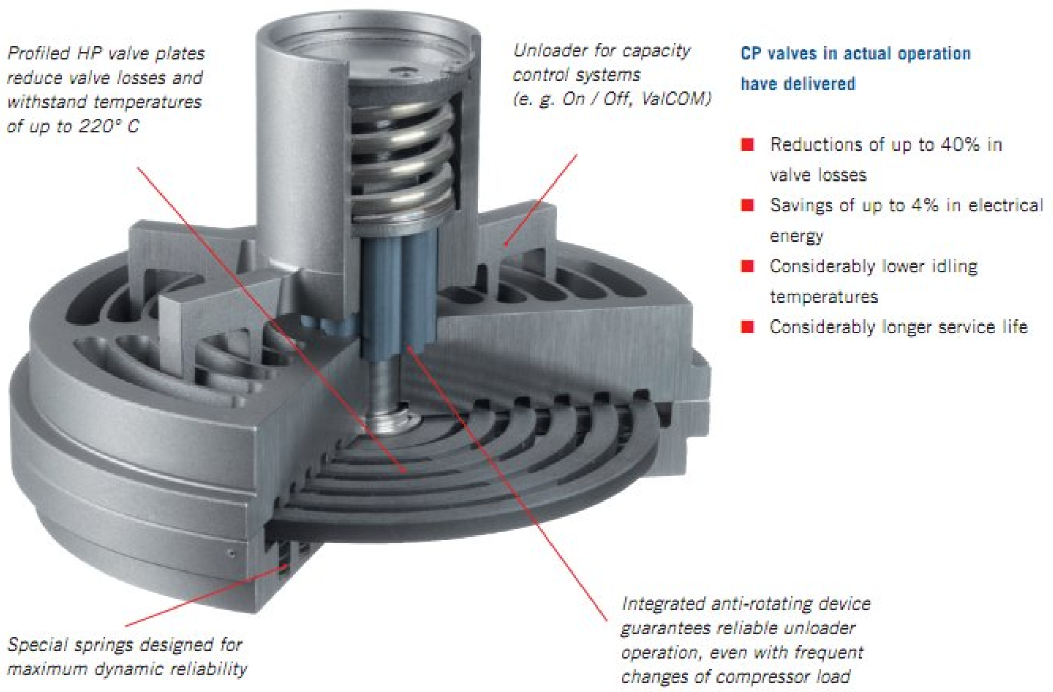


Figure 6.38 Compressor valves (Hoerbiger Company)

6.4.7 Rings And Packings

The compressor packing is a series of pressure containing rings located in the crank end of a doubleacting compressor cylinder. These seal against the piston rod and prevent leakage, so that the cylinder can compress gas on both sides of the piston. Again, as with compressor rings, the packing material is selected to provide best life and sealing with expected conditions. The packing generally pressure lubricated, and may have coolant flow to remove friction heat. There are also various specialty types to reduce gas leakage around the rod.

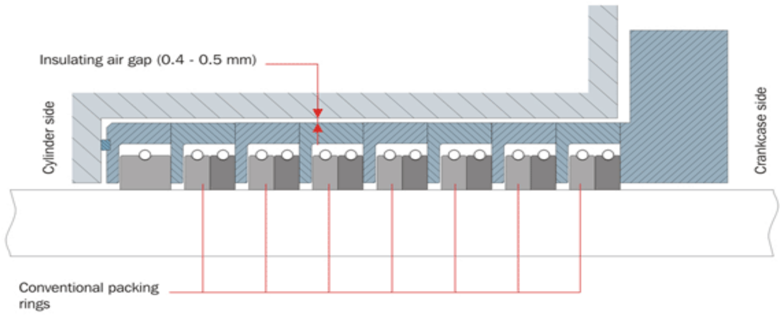


Figure 6.39 Pressure packing (Hoerbiger Company)

Most process units today are equipped with Teflon (PTFE) or other high-performance polymer piston rings. Normally, two or three single-piece diagonal-cut rings without expanders are used. For some high-pressure applications (over 300 atm absolute) three-piece bronze segmental rings are used. Also, for some nonlubricated applications other special plastics or high-performance polymers have been used. For many lubricated and all nonlubricated applications,TFE rider rings are used. The rider rings support the weight of the piston and piston rod. Rider rings may be split type, located in the center of the piston, or band type,stretched onto the piston.

Figure 6.40 Piston ring (Hoerbiger Company)

6.5 Chassis

In compressor system all of the parts such as, compressor unit, heat exchangers, seperators, motor and all of the air end water piping systems are carried by chassis. In this situation compressors chassis must have enough strength to carry of all parts weigth. Along with this, chassis is exposed dynamic loads which is the sum of the total gas force and inertia forces.

In general, chassis are designed with I or U profiles. We designed our chassis with I profile and the material of this I profile is St37-2 steel. By using Ansys, we can calculate the total forces which is acting through crankcase to chassis. Also we can obtain all of the weigths with Pro-Engineer 3D data. In fig.6.41 we can examine the loading conditions on the chassis. But we have to consider that we can not put down the chassis directly to the ground.

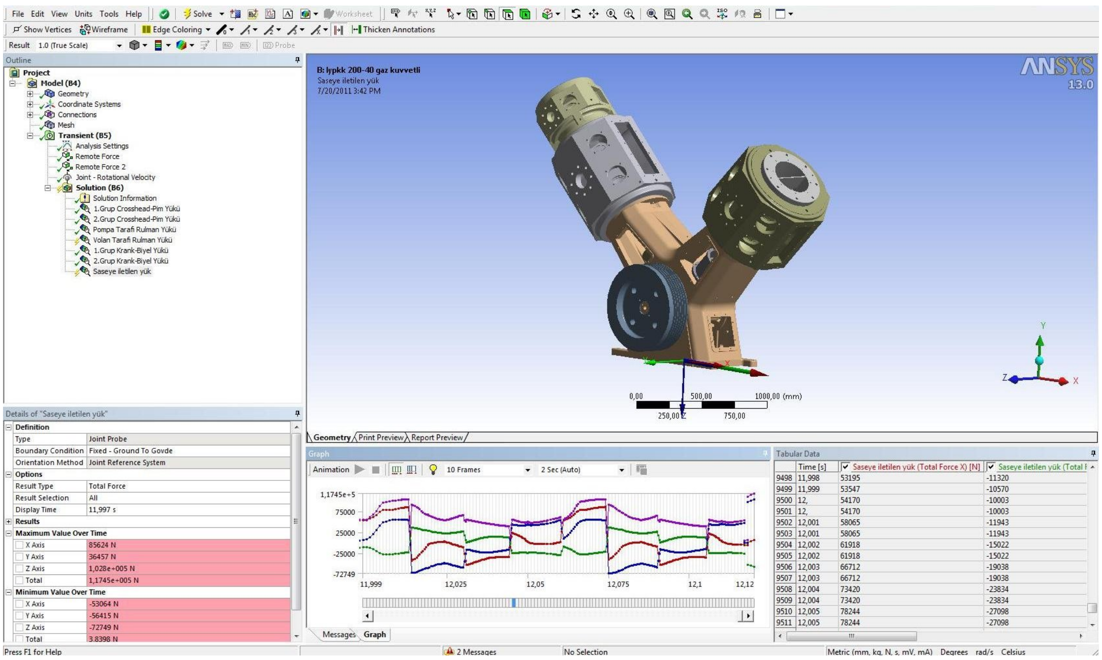
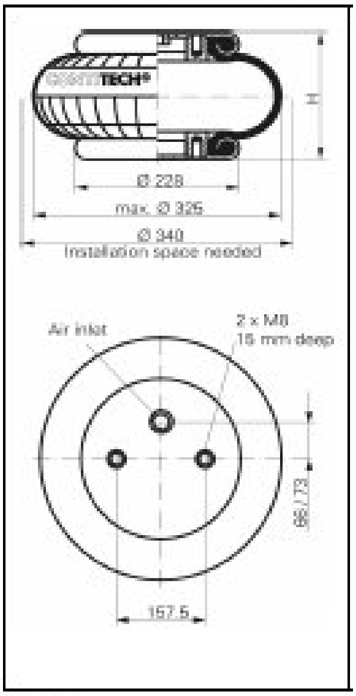
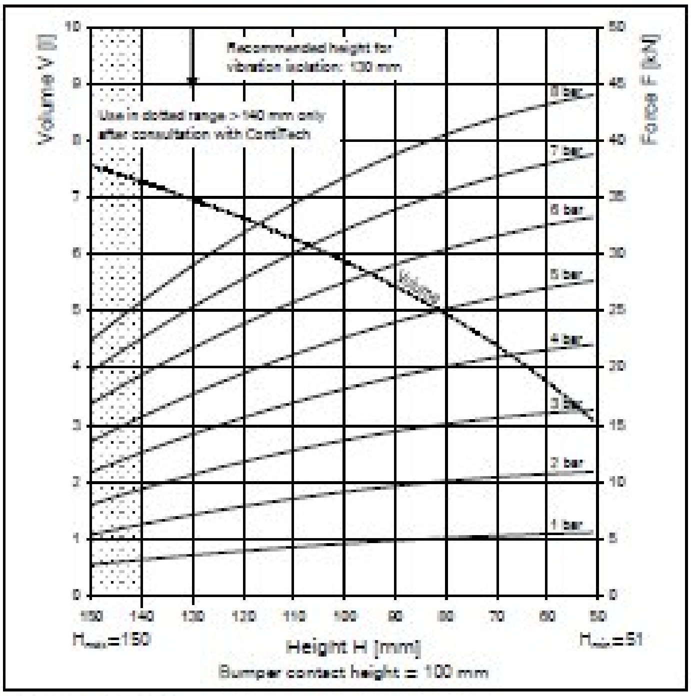


Figure 6.41 Chassis reaction forces

FS 330-11 CONTI® Single Convolution Air Spring



FS 330-11



Force-height diagram

Purchase order data

Type	Order No.
Rubber bellows only	60314
With clamped plates G 1 air inlet 66 mm excentrical	2681 060 000
With clamped plates G 1/4 air inlet 73 mm excentrical	60424
With bumper G 1/4 air inlet 73 mm excentrical	2681 062 000
With clamped plates G 3/4 air inlet 73 mm excentrical	62743

Additional types on request

Service instructions
M8 = 25 Nm
G 1 = 50 Nm
G 1/4 = 25 Nm

Technical data

Min. pressure	0 bar
Return force to min. height	≤ 300 N
Overall weight with clamped plates	4.1 kg

Vibration isolation - dynamic characteristic values

Design height H: recommended 130mm, minimum 100mm

Pressure p	[bar]	3	4	5	6	7	8	Vol. V [l]
Force (Load)	[KN]	10.7	14.2	17.8	21.8	25.4	29.0	7.0
Spring rate	[N/cm]	2645	3290	3935	4580	5225	5870	
Natural frequency	[Hz]	2.5	2.4	2.4	2.3	2.3	2.3	

Pneumatic application - static characteristic values

Force F [kN]

Pressure p	[bar]	3	4	5	6	7	8	Vol.[l]
Height H [mm]	130	10.7	14.2	17.8	21.8	25.4	29.0	7.0
	120	11.9	15.7	19.6	23.8	27.8	31.8	6.7
	110	12.7	16.9	21.1	25.8	30.1	34.4	6.3
	100	13.6	18.1	22.7	27.6	32.1	36.7	5.8
	90	14.4	19.2	24.0	29.1	34.0	38.9	5.3
	80	15.0	20.2	25.2	30.5	35.5	40.6	4.8
	70	15.5	20.9	26.2	31.6	36.9	42.1	4.3
	60	16.0	21.6	27.1	32.6	38.0	43.3	3.8

Further information available by fax: +49 (0)511-938-5162 or under http://www.contitech.de/luftfedersysteme

Continental CONTITECH

Figure 6.42 Air springs technical specifications

Actually, in our system all of the weigths and loads are carried by air springs. Air springs is a type of vehicle suspension powered by an engine driven or electric air pump or compressor. This pump pressurizes the air, using compressed air as a spring. Air suspension replaces conventional steel springs. Besides, we can use air springs as vibration dampers. In fig.6.42 we can see the technical data of the air springs. Depends on the loading we have to choose the most suitable air spring for our system. Total load is approximately 150000 N . In this situation, we decided to use 6 air springs. As a result of this decision, we have to pressurize our air spring with 4 bar. Depends on this selection our spring rate is 32900 N/mm.

For the Ansys calculation we have to define these spring rates to the system. As a mathematical model we can apply springs to the chassis and specifiy these springs with spring rate and damping ratio. After these specifications, we applied forces and weigth loads to the chassis. In fig.6.43 we can see all of the loads and springs and in fig.6.44 we can examine the stress distribution on chassis. Depends on this stress distribution our chassis design has enough strength.

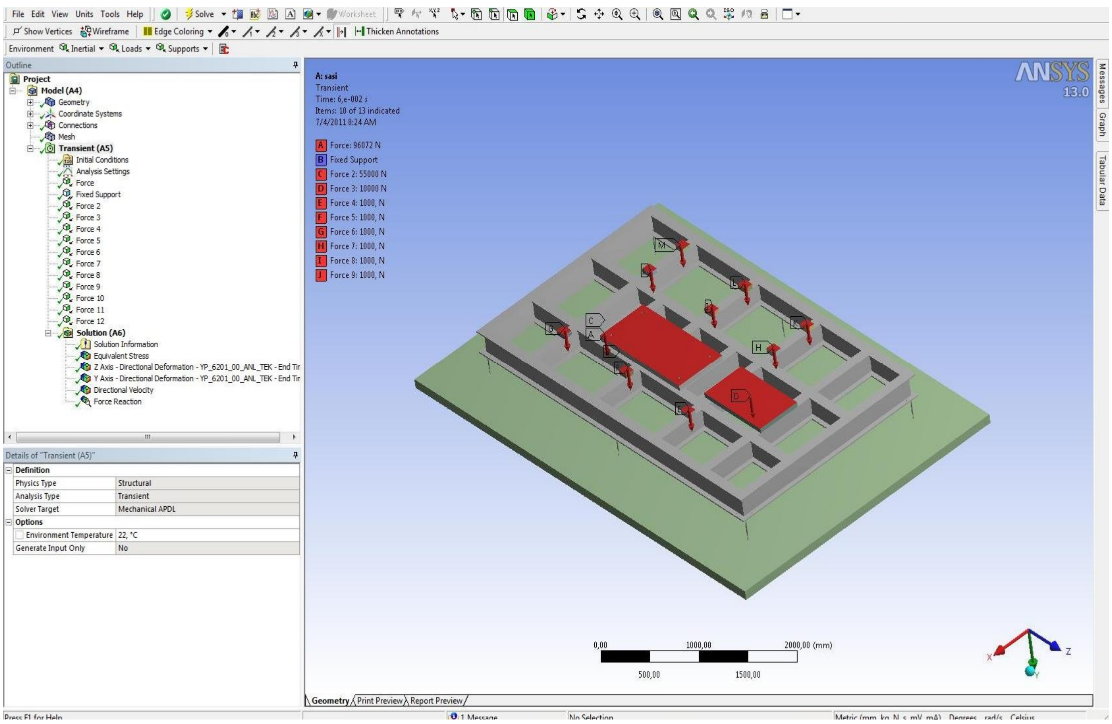


Figure 6.43 Chassis loading conditions

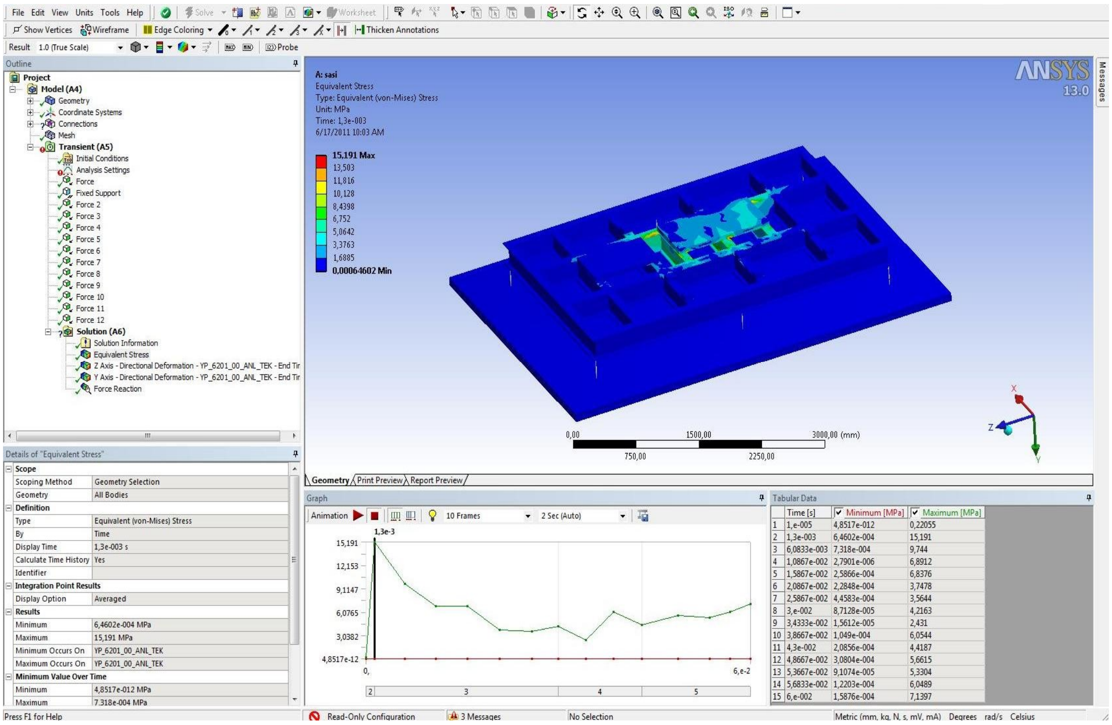
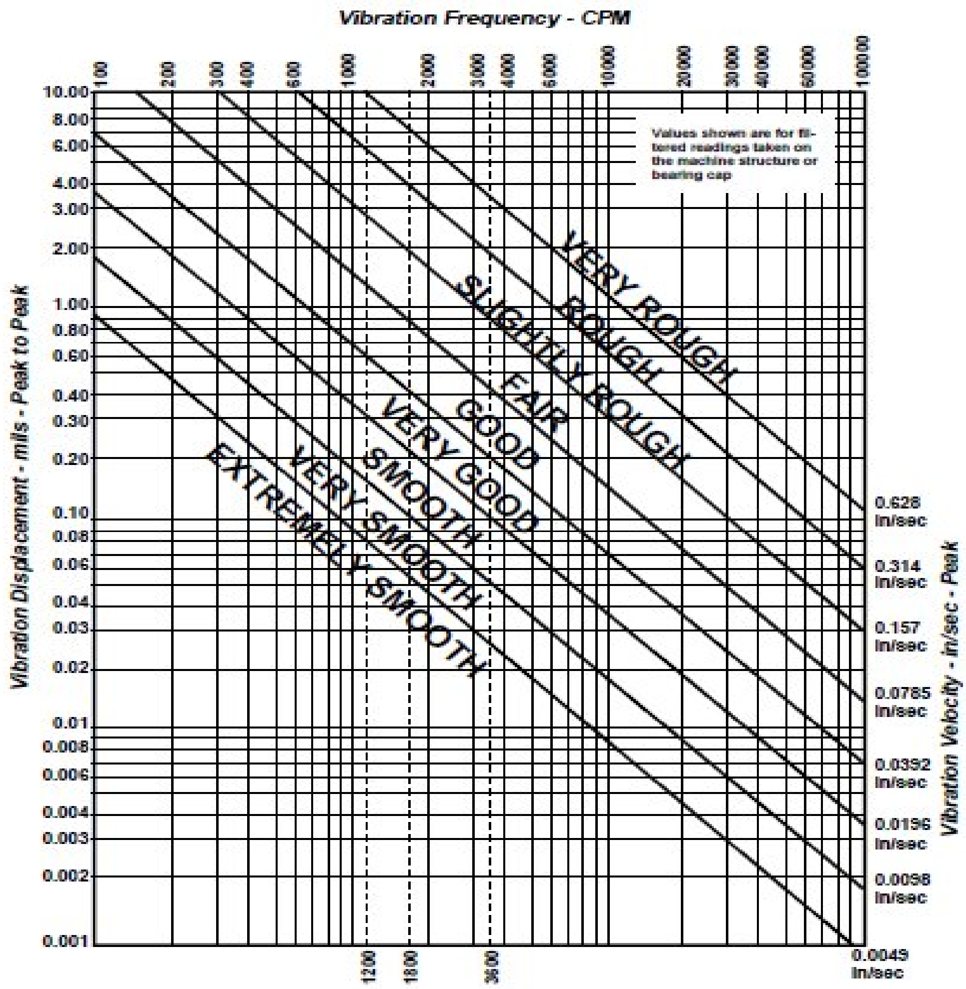


Figure 6.44 Chassis stress distribution

CHAPTER SEVEN
BALANCING OF ROTATING MACHINES
7.1 Introduction

The presence of an eccentric or unbalanced mass in rotating disc causes vibration, which may be acceptable up to a certain level. Table 7.1 is a vibration-severity chart that can be used to determine acceptable vibration levels at various frequencies or operating speeds. If the vibration caused by an unbalanced mass is not acceptable, it can be eleminated either by removing the eccentric mass or by adding an equal mass in such a position that it cancels the effect of the unbalance. In order to use this procedure, we need to determine the amount and location of the eccentric mass experimentally. The unbalance in practical machines can be attributed to such welds

Table 7.1 Vibration severity chart



The essential moving elements of a reciprocating engine are the piston, the crank and the connecting rod. Vibrations in reciprocating engines arise due to periodic variations of the gas pressure in the cylinder and inertia forces associated with moving parts. We shall now analyze a reciprocating engine and find the unbalanced forces caused by these factors.

7.2 Balancing Calculations

7.2.1 Unbalanced Forces Due to Fluctuations in Gas Pressure

Fig.7.1 is a schematic diagram of a cylinder of a reciprocating engine. The engine is driven by the expanding gas in the cylinder. The expanding gas exerts on the piston a pressure force F, which is transmitted to the crankshaft through the connecting rod. The reaction to the force F can be resolved into two components; one of magnitude F/cosφ, acting along the connecting rod, and the other magnitude F/tanφ, acting on a horizontal direction. The force F/cosφ induces a torque M_t which tends to rotate the crankshaft.

$$M_t = (F / \cos\varphi) \, r \cos\Theta \tag{7.1}$$

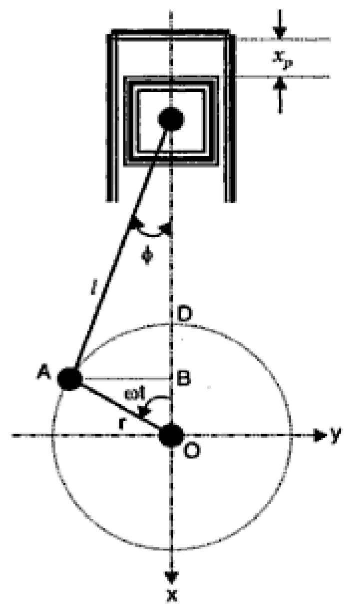


Figure 7.1 Slider crank mechanism

For force equilibrium of the overall system, the forces at the bearings of the crankshaft will be F in the vertical direction and F tanφ in the horizontal direction.

Thus the forces transmitted to the stationary parts of the engine are;

1. Force F acting upwards at the cylinder head
2. Force F tanφ acting toward the right at the cylinder head
3. Force F acting downward at the crankshaft bearing Q
4. Force F tanφ acting downward the left at the crankshaft bearing.

Although the total resultant force is zero, there is a resultant torque $M_Q = F\ h \tan\varphi$ on the body of the engine, where h can be found from the geometry of the system;

$$h = (r \cos \Theta) / (\sin \varphi) \tag{7.2}$$

thus the resultant torque is given by;

$$M_Q = (F\ r \cos\Theta) / (\cos\varphi) \tag{7.3}$$

7.2.2 Unbalanced Forces Due to Inertia of the Moving Parts

We have already calculated the displacement, velocity and acceleration of the piston and crankpin in chapter five. By using these calculation we also calculated the inertia forces.

7.3 Balancing Of V Type Compressor

For the compressor system we can neglect the fluctuation forces. In this situation we just balanced the rotating and reciprocating inertia forces. For the balancing of the system we are going to use counterweigth on the crankshaft. Calculation of the counter-weigth depends on two parameters;

1) Centrifugal force, which consist of the rotating unbalanced masses
2) Mass forces, which consist of reciprocating unbalanced masses.

7.3.1 Centrifugal forces

For the calculation of counter-weigth against the centrifugal forces, we have to consider the mass of the crankshaft eccentric part and the reduction mass of the connecting rod to the crankshaft.

Depends on these masses;

$$M_{cw,rot} = M_{rot} \, (r / r_{cw}) = (m_{ce} + (r_{ccr} / r) \, m_{ccm} + m_{cr} \, (l_2 / (l_1 + l_2))) \, (r/r_{cw}) \qquad (7.4)$$

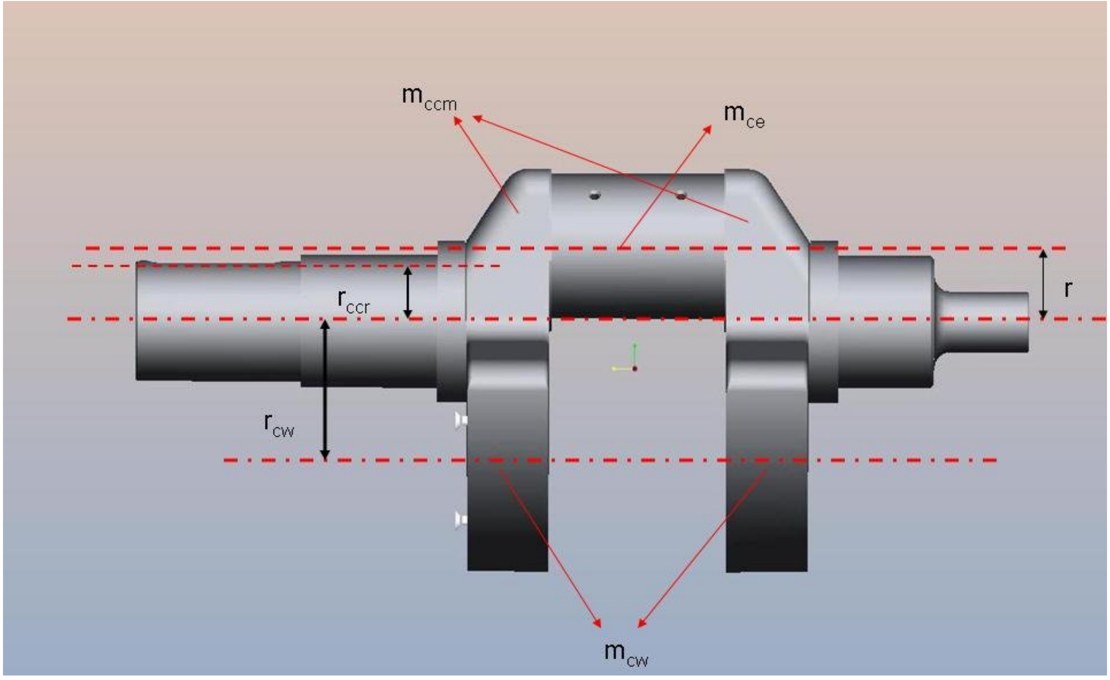


Figure 7.2 Crankshaft unbalanced masses

To obtain the necessary unbalanced mass, we used Pro-Engineer CAD program. We create every part of the crankshaft obtain the necessary masses and center of gravity of all parts.

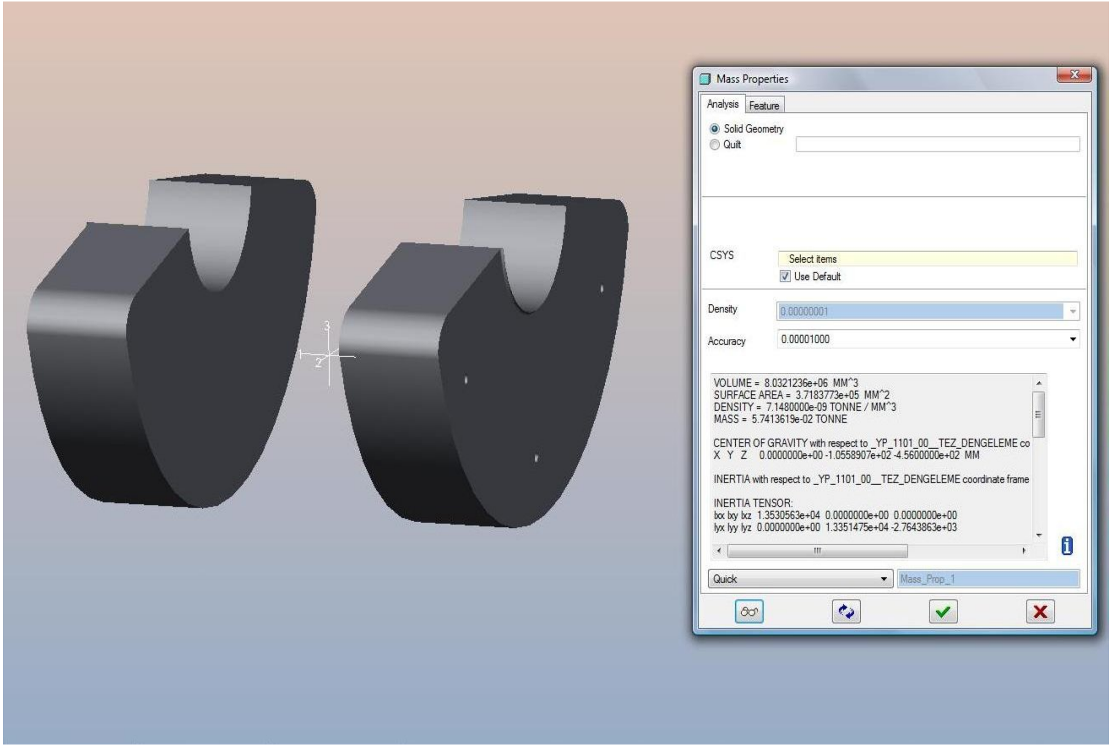


Figure 7.3 Counter weigths of the crankshaft

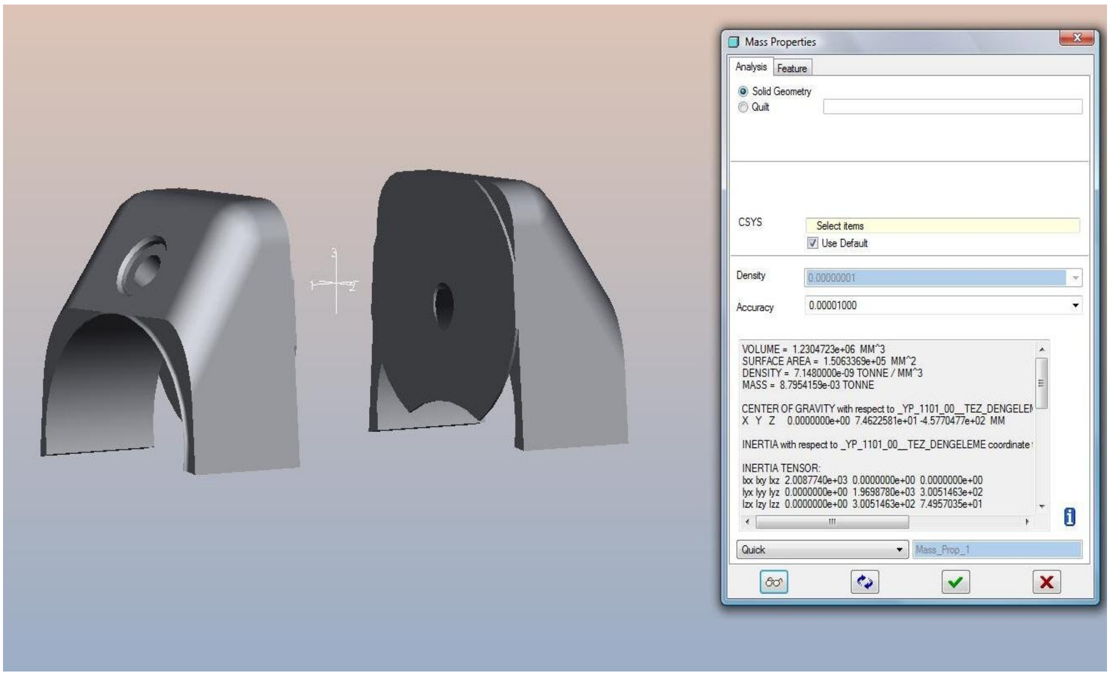


Figure 7.4 Crankshaft cheeks

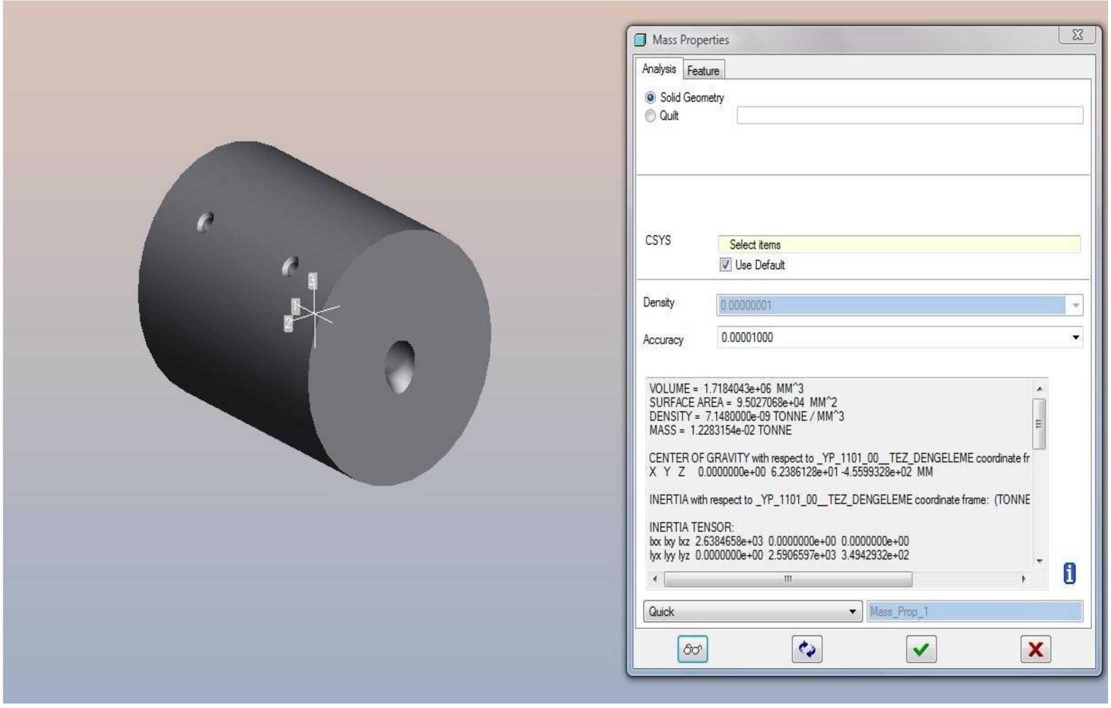


Figure 7.5 Crankshaft pin

7.3.2 Reciprocating Masses Force

During the examination of the reciprocating masses we have to consider, connecting rod, piston rod, piston and all other connection parts. Calculation of the counter weigth which balanced these masses;

$$M_{cw,rec} = ½\, m_{rec}\ (r / r_{cw}) \qquad (7.5)$$

m_{rec} = Total reciprocating mass

After the calculation of $M_{cw,rot}$ and $M_{cw,rec}$, we can reach the exact M_{cw} with the sum of the $M_{cw,rot}$ and $M_{cw,rec}$

$$M_{cw} = M_{cw,rot} + M_{cw;rec} = (M_{rot}\ (r / r_{cw})) + (½\, m_{rec}\ (r / r_{cw})) \qquad (7.6)$$

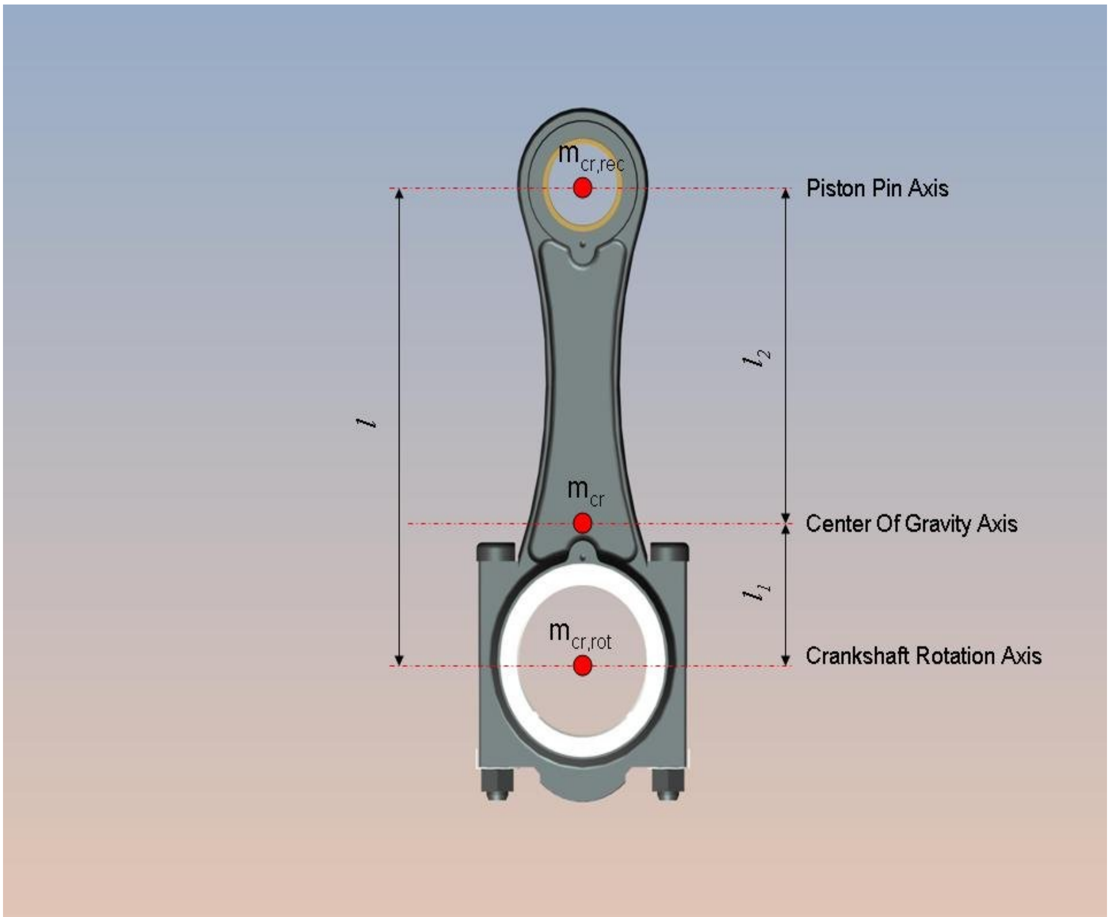


Figure 7.6 Connecting rod mass distribution

We have to calculate reduction mass of the connecting rod to the piston pin and crankshaft. We seperate total mass into two parts.

$$m_{cr,rot} = m_{cr}\ (l_2 / l) \tag{7.7}$$

$$m_{cr,rec} = m_{cr}\ (l_1 / l) \tag{7.8}$$

$l_1 = 127mm$ $l_2 = 253mm$ $l_{total} = 380$ $m_{cr} = 16.5kg$

hence;

$m_{cr,rot} =$ 11 kg

$m_{cr,rec} =$ 5.5 kg

For our system all the necessary masses and radius are;

Total Reciprocating mass (m_{rec}) = 58,128 kg
Crankshaft eccentric mass (m_{ce}) = 12,7 kg
Crankshaft cheek mass (m_{ccm}) = 8 kg
Rotation mass of the connecting rod ($m_{cr,rot}$) = 11 kg
Reciprocating mass of the Connecting rod ($m_{cr,rec}$) = 5.5 kg
Crankshaft radius (r) = 62.5 mm
Counterweigth radius (r_{cw}) = 107 mm
Crankshaft cheek radius (r_{ccr}) = 54 mm

Depends on these masses and radius,

Total rotating unbalanced mass = $m_{ce} + m_{ccm} + 2(m_{cr,rot})$ = 42.7 kg
Total Reciprocating mass = $2(m_{rec} + m_{cr,rec})$= 127.256 kg

If we use the mass and radius data in Eq.7.6, we calculate the necessary counter-weigth 62.1 kg. Our current counter-weigth is 55.5 kg. In this situation we have to add 6.6 kg to the our counter-weigth without changing the center of gravity in vertical direction.

CHAPTER EIGTH
FLYWHEEL DESIGN

8.1 Uses And Characteristics Of Flywheel

Flywheels are rotating masses installed in rotating systems of machine elements to act as a storage reservior for kinetic energy. Usually the primary task of a flywheel is to control, within an acceptable band, the angular velocity and torque *fluctuations* inherent in the power source, the load or both. Fig.8.2 illustrates the superposed torque versus angular displacement curves for a compressor.

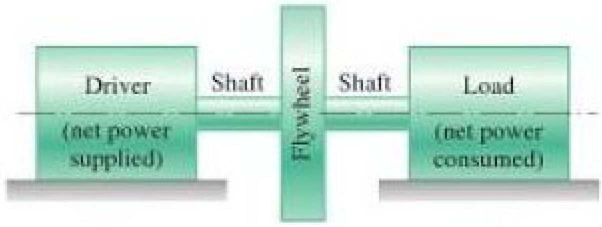


Figure 8.1 Flywheel connection

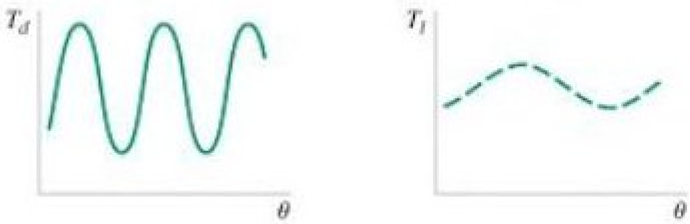


Figure 8.2 Torque – Angular velocity diagram

By definition, the *driver* torque, T_d is considered to be positive when its sense is in the direction of shaft rotation and the *driver is supplying energy* to the shaft-flywheel system. The *load* torque T_l is considered to be positive when its sense is in the direction of rotation and the *shaft-flywheel system is supplying energy* to the load. It my be noted during increments of time when the *supplied* torque *exceeds* the *required* load torque, the flywheel mass is *accelerated* and additional kinetic energy is *stored* in the flywheel. During increments of time when the *required* load torque *exceeds* the *supplied* driver torque, the flywheel mass is *decelerated* and some of the kinetic energy in the flywheel is *depleted.*

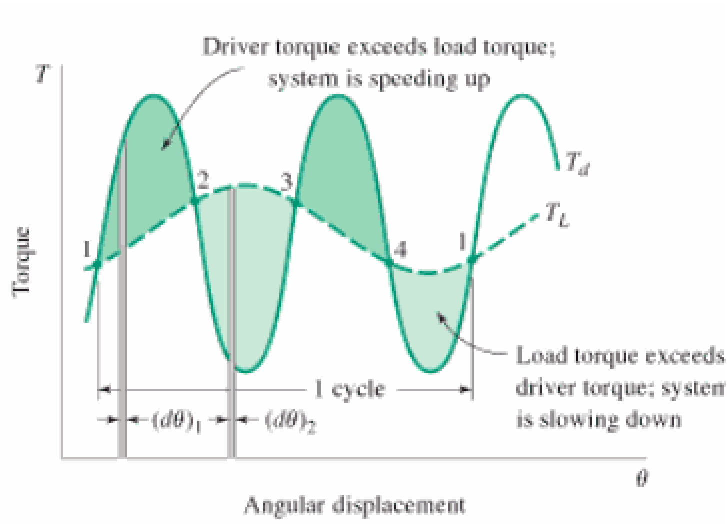


Figure 8.3 Torque – Angular displacement diagram

When the operational mission of a machine system is dependent upon restricting the speed or torque fluctuations to lie within a defined *control band*, it is possible to estimate the mass moment of inertia and rotation flywheel speed required to accomplish the task. Since all real machine elements have mass, the 'flywheel effect' from each significant rotating mass in the system must also be incorporated in the pertinent calculations.

By utiliazing a properly designed flywheel, one or more of the following potential advantages may be realized;

1 – *Reduced amplitude* of speed fluctuation

2 – *Reduced peak torque* required of the driver

3 – *Reduced stresses* in shafts, couplings and possibly other components in the system

4 – *Energy automatically stored and released* as required during the cycle.

8.1.1 Types of Flywheels

Since the kinetic energy stored in rotating mass is given by,

$$KE = \tfrac{1}{2} Jw^2 \qquad (8.1)$$

it is clear that if a large capacity for storing kinetic energy is required, the angular velocity, *w* should be as as large as possible and the mass moment of inertia af the rotating mass sould also be as large as possible

Noting that *J* is a function of the magnitudes of the mass elements comprising the flywheel and their distances from its axis of rotation, the way to achieve large values of *J* is to configure the rotating flywheel so that as much as possible lies as far as possible from the axis of rotation. This suggests a flywheel configuration in which a relatively heavy circumferential rim is connected by a lightweight structure to a hub secured to the rotating power transmission shaft of the flywheel system. Historically, in the design and construction of flywheels for *low-performance* mechanical systems,these configuration guidelines were translated into the well known spoke and rim flywheel configuration. For *high performance* rotating systems in current practice, it is more common to use uniform-strength flywheel configurations and very high strength materials.

Furthermore, the rotating flywheels mass must have an axisymmetric geometry to avoid unbalanced dynamic forces induced by eccentric mass centers. Rotating unbalanced forces typically induce excessive shaft deflection, large amplitude vibrations and large force on supporting bearings and structure. Axisymmetric geometries that have been succesfully implemented for flywheel applications.

8.2 Flywheel Selection

If the average angular velocity w_{ave} of a rotating system is to remain unchanged over time, the first law of thermodynamics requires that the *added kinetic energy* stored in the flywheel during one operational cycle must be equal to the kinetic energy *depleted* from the flywheel during the same cycle. Isolating the shaft flywheel system as a thermodynamic free body as shown in Fig.8.4 Therefore requires that the kinetic energy supplied to the flywheel system during each cycle must equal the work done by the flywheel system during each cycle or

$$d(KE) = d (W) \tag{8.2}$$

where KE= kinetic energy supplied to the rotating flywheel system

W = net work done (energy depleted) by the rotating flywheel system.

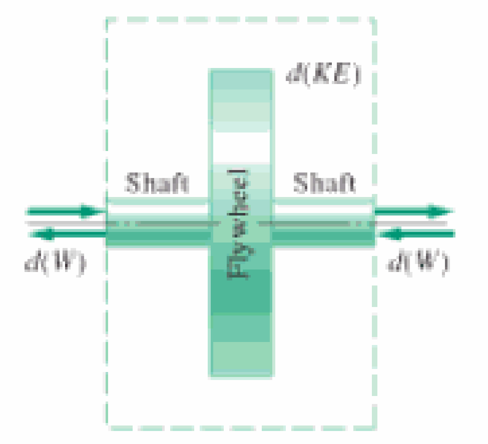


Figure 8.4 Free – body diagram of the flywheel

For a rotating mass having a mass moment of inertia *J* and angular velocity *w*, it is well known that,

$$KE= ½\, J\, w^2 \tag{8.3}$$

hence

$$d(KE) = ½\, Jd (w^2) \tag{8.4}$$

Also, presuming a rigid shaft subjected to a net torque equal to ($T_l - T_d$) ,

$$dW = (T_l - T_d)\, d\theta \tag{8.5}$$

combaining (Eq.8.4)

$$½\, Jd\, (w^2) = (T_l - T_d)\, d\theta = \mathrm{U}_{\max} \tag{8.6}$$

$\mathrm{U}_{\max}$ = by definition, the maximum change in kinetic energy of the shaft flywheel system due to speed fluctuation between minimum and maximum values.

From Eq.8.6 then,

$$½\, J\, (w^2_{max} - w^2_{min}) = U_{max} \tag{8.7}$$

Factoring the left side

$$\mathrm{J}\ \{ (w_{max} - w_{min}) / 2 \}\ \mathrm{x}\ (w_{max} - w_{min}) = U_{max} \tag{8.8}$$

but

$$\{ (w_{max} - w_{min}) / 2 \} = w_{ave} \tag{8.9}$$

so Eq.8.8 become

$$\mathrm{J}\ w_{ave}\, (w_{max} - w_{min}) = U_{max} \tag{8.10}$$

where(w_{max} - w_{min}) is the peak-to-peak fluctuation in angular velocity during each operational cycle.

$$C_f = (w_{max} - w_{min}) / w_{ave} \tag{8.11}$$

$$\mathrm{J}\ w_{ave}^2\ C_f = U_{max} \tag{8.12}$$

It is common practice to define a coefficient of speed fluctuation, C_f , as

Required Level of Speed Uniformity	C_f
Very Uniform	
Gyroscopic control systems	< 0.003
Hard disk drivers	
Uniform	
AC electric generators	0.003 - 0.012
Spinning machinery	
Some fluctuation acceptable	
Machine tools	0.012 – 0.05
Compressors, pumps	
Moderate fluctuation acceptable	
Excacators	0.05 – 0.2
Concrete mixers	
Larger fluctuation acceptable	
Crushers	>0.2
Punch presses	

In Fig.8.5 we have a tangential moments values for compressor. We can obtain U_{max} by using this graphic. If we calculate the area below the curve, we can obtain the energy value. We have three area below this curve. First of all we have to define our boundaries. We need a referans line for the calculation. Area which is below this referance line has negative effect on the system and above area has positive effect to the system. So, depends on energy equilibrium sum of these areas must be equal to zero. With this information we can determine our referance line values.

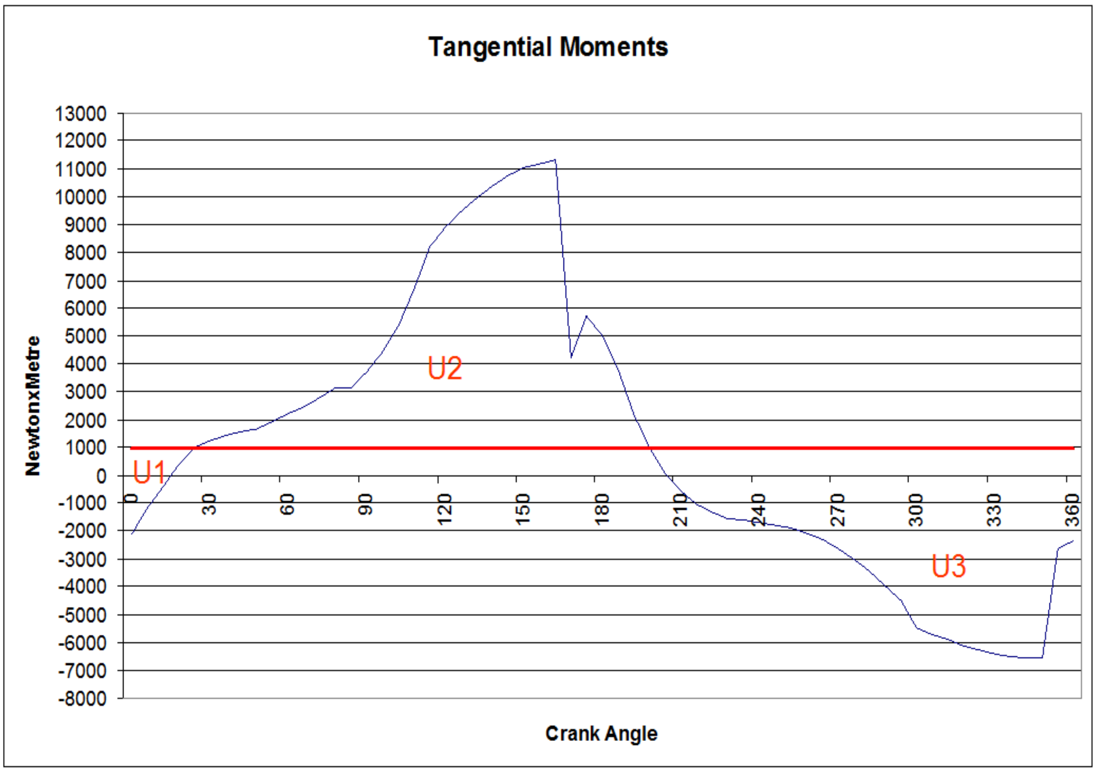


Figure 8.5 Tangential moments – Crank angle diagram

As a result of the calculation, our referance line value is approximately 1000 Newton x Metre.

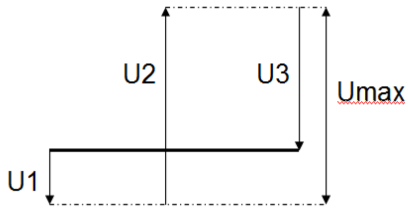


Figure 8.6 Energy vectors

U1 = 31.23 Nm^2

U2 = 515.3 Nm^2 ⟶ U2 = 8244.8 Nm

U3 = 484.065 Nm^2

If we examine Fig.8.6, we realize that, U2 value equal to the U_{max} . We can calculate necessary mass moment of inertia values by using U2 values. But, first of all we have to divide U2 value in to the crank radius to obtain the energy. If we reorginize Eg.8.12

$$J = U_{max} / (w_{ave}^2 \; C_f)$$

For the compressors C_f values must be between 0.012 – 0.05 so we choose our C_f = 0.35. Depends on this C_f value,

J = 20.46 kgm^2

8.3 Flywheel Mass Moment Of Inertia Calculation

All of the CAD programs give us the Mass moment of inertia values for each kind of geometry. But for the clarification we have to know the teorical calculation of the mass moment of inertia for complex geometry. In this system, we have to calculate the flywheels mass moment of inertia values. In this calculation we have to seperate bodies to the simple geometries.

In Fig.8.7 we can see the seperation of the simple bodies. During the selection of the flywheel we obtained the necessary mass moment of inertia values for flywheel before. Depends on these values we have to design our flywheel with the acceptable mass moment of inertia. In this case we can calculate thickness and diameter of the flywheel with;

$$\Theta = \rho \times \pi/32 \times [\, h\,(d^4 - d_1^4) + h_1\,(d_1^4 - d_2^4) + h_2\,(d_2^4 - d_3^4)\,] \qquad (8.13)$$

ρ = density

d, d_2, d_3 and h parameters depends on belt and crank dimensions so we have to calculate h_1, h_2 and d_1 dimensions. With the optimization calculations we obtained the missing dimensions.

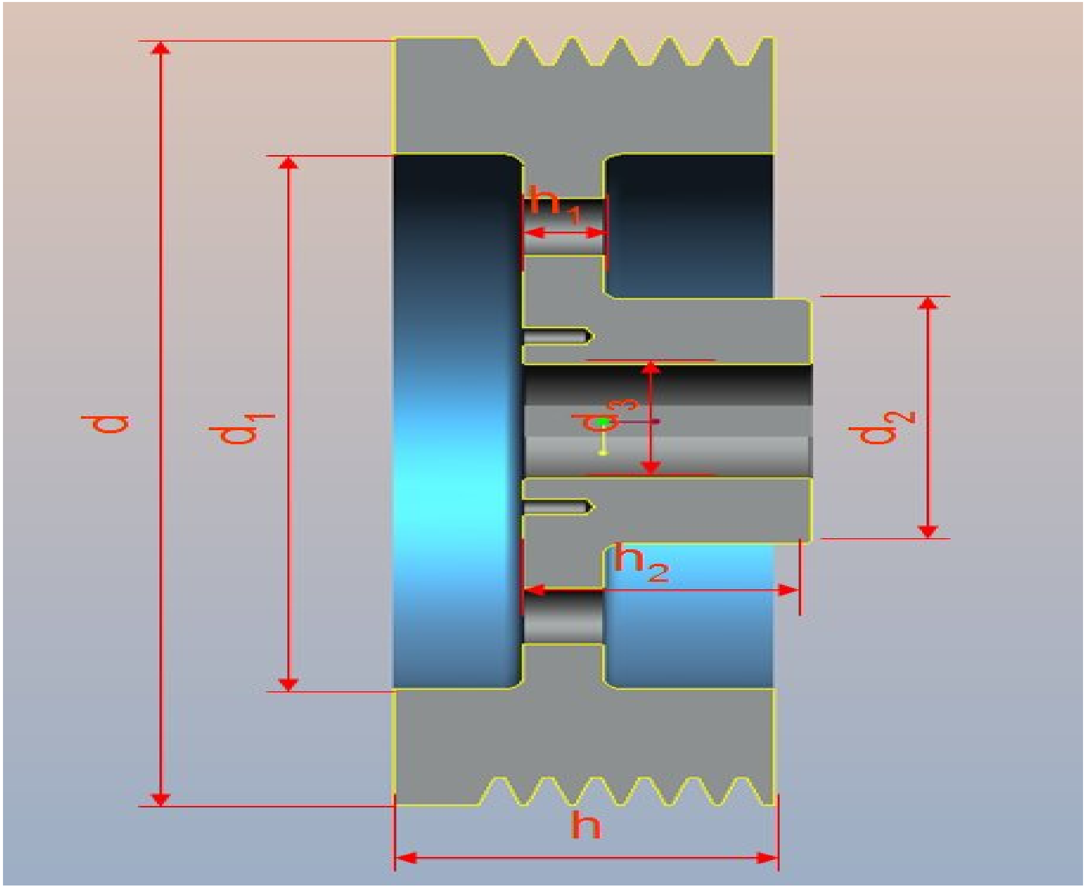


Figure 8.7 Flywheel dimensions

d = 674.6 mm	h = 200 mm
d_1 = 470 mm	h_1 = 42 mm
d_2 = 215 mm	h_2 = 152 mm
d_3 = 100 mm	

CHAPTER NINE
CONCLUSION

9.1 Conclusions

Chapter nine summarises the main findings of this study and test results about the system. This project was undertaken to design non-lubricated, multistage piston air compressor and main calculations of the system. This study has shown that all of the necessary calculations for the optimum design of the compressor system. It was also given some information of the main properties of the compressor. In this project all of the theoretical calculations were verified with Ansys analysis and test results.

The current findings add substantially to our understanding of necessary and specific calculations during the design of the compressor system. In previous chapters we examined the strength and dynamic working conditions of the system and decided our systems qualification. But some points we required test datas for the varification of the system. Especially performance and working conditions test results gives us specific datas about our system. The empirical findings in this study provide a new understanding of working conditions and available developments for the system.

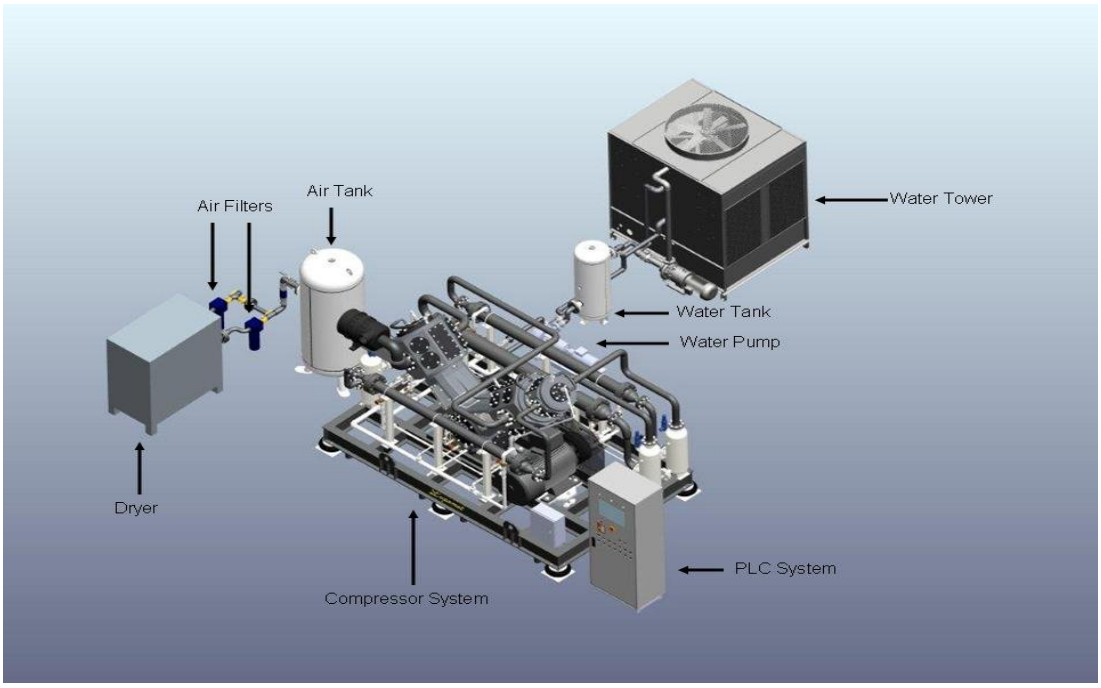


Figure 9.1 Complete system

9.1.1 Strength results

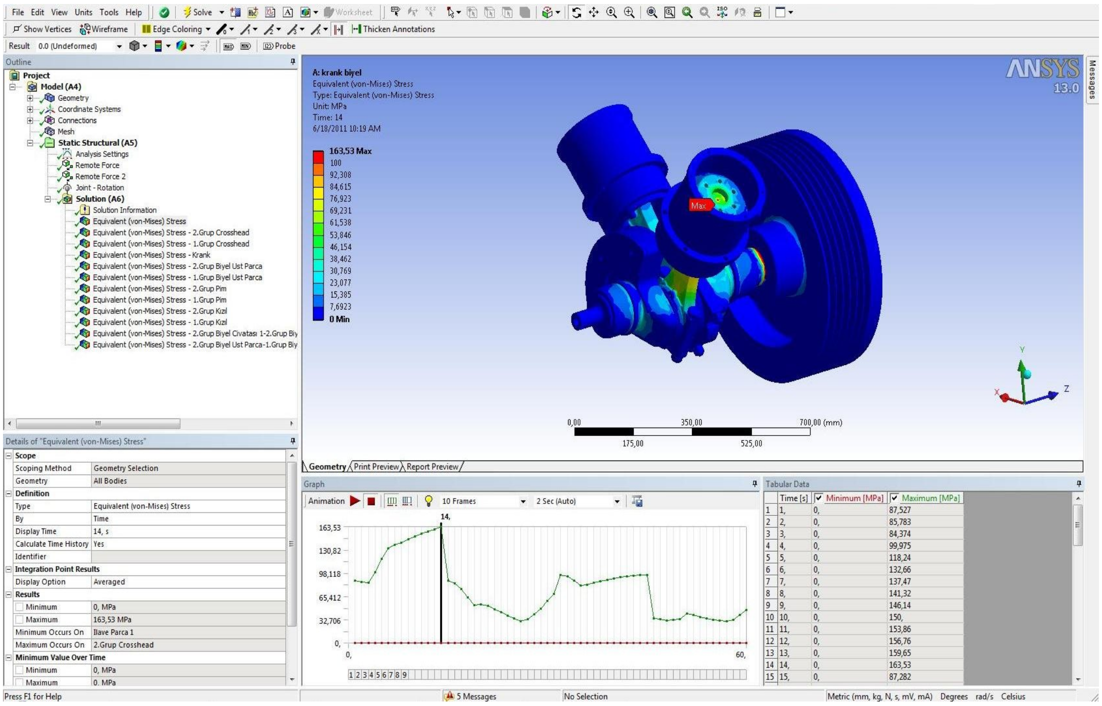


Figure 9.2 Ansys stress analysis results

We calculated, gas forces in chapter three and dynamic loads in chapter four. Depends on these calculation results we made strength analysis for each part. We examined strength analysis results in chapter six for the main parts. As a result of these analysis, pre-design of all the parts was verified under the working conditions and safety factor. But some of the parts, such as crankcase, crosshead, pistons, have high strength properties. In mass production, thickness of these kind of parts can be reduced. In this way, material costs also can be reduced. Besides, pistons and crossheads effect the reciprocating mass and inertia forces. If we decrease the mass of the these moving parts, we can decrease the inertia forces, vibration and noise.

9.1.2 Performance results

Table 9.1 Compressor performance results

TEST RESULTS

	1. Stage Air Outlet Pressure (Bar)	1. Stage Air Outlet Temperature(°C)	1. Stage Heat Exchanger Outlet Pressure (Bar)	1. Stage Heat Exchanger Outlet Temperature(°C)	2. Stage Air Outlet Pressure (Bar)	2. Stage Air Outlet Temperature (°C)	2 Stage Heat Exchanger Outlet Pressure (Bar)	2. Heat Exchanger Outlet Temperature(°C)	3. Stage Air Outlet Pressure (Bar)	3. Stage Air Outlet Temperature(°C)	3. Stage Heat Exchanger Outlet Temperature(°C)	Oil Outlet Pressure (Bar)	Oil Outlet Temperature (°C)	Tank Pressure (Bar)	Environmet Pressure (MBar)
TEST NO 124	3,62	200,36	3,29	35,41	16,84	176,48	16,46	35,44	40,70	121,86	35,49	2,97	44,78	40,16	1014,83
TEST NO 125	3,63	202,63	3,29	35,93	16,87	178,09	16,56	36,01	40,66	122,60	36,01	2,80	46,56	40,17	1014,46
TEST NO 126	3,36	193,95	3,06	33,44	16,83	163,56	16,66	33,52	37,64	116,62	34,07	2,63	48,57	37,22	1013,86
TEST NO 127	3,42	194,32	3,11	33,74	16,84	164,86	16,49	33,73	37,66	117,10	34,32	2,59	49,57	37,29	1013,66
TEST NO 128	3,43	194,48	3,11	34,07	16,78	166,18	16,44	33,98	37,65	118,05	34,60	2,63	50,06	37,26	1013,31
TEST NO 129	3,43	194,46	3,11	34,07	16,66	166,37	16,30	33,96	37,49	118,76	34,63	2,62	50,51	37,16	1013,26
TEST NO 130	3,50	196,36	3,13	34,26	16,77	167,76	16,39	34,09	38,31	119,66	34,66	2,48	50,51	37,97	1013,50
TEST NO 135	3,63	186,14	3,22	30,59	16,40	161,10	16,07	30,67	40,16	113,14	30,49	3,76	31,46	39,38	1016,61
TEST NO 136	3,59	194,68	3,28	32,58	16,50	171,30	16,19	32,69	40,06	117,68	32,76	3,28	41,46	39,40	1016,44
TEST NO 137	3,58	195,64	3,29	32,83	16,57	172,87	16,28	32,90	40,65	119,24	33,00	3,01	45,12	40,07	1016,10
TEST NO 138	3,60	196,16	3,27	32,88	16,64	173,67	16,26	32,96	40,88	120,00	33,06	2,89	46,42	40,36	1013,32
TEST NO 139	3,60	196,56	3,27	32,77	16,69	173,90	16,32	32,80	40,96	120,03	32,96	2,79	47,63	40,41	1012,71
TEST NO 140	3,62	196,69	3,30	32,78	16,64	174,00	16,39	32,81	40,92	119,97	32,96	2,73	48,63	40,41	1012,96
TEST NO 141	3,65	197,08	3,32	32,95	16,69	173,90	16,36	33,04	40,64	119,46	33,11	2,68	49,03	40,12	1012,67
TEST NO 142	3,61	197,44	3,28	33,09	16,66	174,07	16,38	33,18	40,63	119,41	33,24	2,61	49,99	40,08	1013,99

	Environmet Temperature(°C)	Environment Humidity %	Orifis Inlet Temperature(°C)	1. Stage Water Inlet Pressure(Bar)	1. Stage Water Inlet Temperature(°C)	1.Stage Water Outlet Pressure (Bar)	1.Stage Water Outlet Temperature(°C)	2-3 Stage Water Inlet Pressure (Bar)	2-3 Stage Water Inlet Temperature (°C)	2-3 Stage Water Outlet Pressure (Bar)	2-3 Stage Water Outlet Temperature(°C)	1.Stage Heat Exchanger Water Inlet Pressure (Bar)	1.Stage Heat Exchanger Water Inlet Temperature (°C)	1.Stage Heat Exchanger Water Outlet Pressure (Bar)	1.Stage Heat Exchanger Water OutletTemperature (°C)
TEST NO 124	26,53	44,61	26,88	3,52	34,92	3,36	46,68	3,45	33,16	3,63	50,96	3,57	33,50	3,44	41,46
TEST NO 125	27,60	42,66	26,72	3,52	35,44	3,34	47,67	3,46	33,70	3,62	52,00	3,56	34,04	3,44	42,02
TEST NO 126	28,42	40,77	29,10	3,49	33,07	3,30	44,10	3,40	31,45	3,59	46,99	3,53	31,56	3,41	37,89
TEST NO 127	28,71	40,53	28,95	3,48	33,35	3,30	44,55	3,43	31,73	3,58	47,83	3,52	31,92	3,40	38,19
TEST NO 128	28,99	40,03	28,90	3,48	33,67	3,31	44,89	3,41	31,97	3,59	48,91	3,53	32,23	3,40	38,47
TEST NO 129	29,02	40,14	28,90	3,48	33,65	3,30	45,00	3,42	31,97	3,59	49,30	3,53	32,20	3,40	38,45
TEST NO 130	28,67	40,23	28,51	3,48	33,82	3,30	45,02	3,43	32,11	3,59	49,34	3,52	32,39	3,40	39,13
TEST NO 135	25,54	51,56	25,77	2,12	30,08	1,87	36,92	1,95	28,44	2,19	39,26	2,12	28,87	1,93	34,88
TEST NO 136	26,87	51,29	26,66	2,14	32,08	1,89	41,21	1,99	30,47	2,22	45,36	2,13	30,74	1,96	37,21
TEST NO 137	29,56	43,48	32,05	2,14	32,24	1,89	41,61	1,98	30,70	2,22	46,30	2,13	30,90	1,96	37,40
TEST NO 138	32,23	37,64	36,42	2,14	32,31	1,88	41,80	2,01	30,71	2,22	46,60	2,11	30,96	1,95	37,42
TEST NO 139	32,66	38,07	37,75	2,13	32,21	1,88	41,80	2,00	30,61	2,21	46,69	2,11	30,82	1,95	37,33
TEST NO 140	32,70	38,24	38,05	2,13	32,20	1,88	41,80	2,00	30,65	2,21	46,63	2,11	30,87	1,95	37,36
TEST NO 141	32,79	37,34	37,19	2,13	32,40	1,88	42,00	1,94	30,83	2,21	46,83	2,13	31,06	1,95	37,49
TEST NO 142	31,47	41,12	34,94	2,13	32,53	1,89	42,20	1,97	30,93	2,20	47,00	2,12	31,14	1,95	37,61

	2.Stage Heat Exchanger Water Inlet Pressure (Bar)	2.Stage Heat Exchanger Water Inlet Temperature (°C)	2.Stage Heat Exchanger Water Outlet Pressure (Bar)	2.Stage Heat Exchanger Water OutletTemperature (°C)	3.Stage Heat Exchanger Water Inlet Pressure (Bar)	3.Stage Heat Exchanger Water Inlet Temperature(°C)	3.Stage Heat Exchanger Water Outlet Pressure (Bar)	3.Stage Heat Exchanger Water OutletTemperature (°C)	Oil Circle Water Inlet Pressure (Bar)	Oil Circle Water Inlet Temperature (°C)	Oil Circle Water Outlet Temperature (°C)	Power (System) (kW)	Power (MotorShaft) (kW)	Compressor Speed (rpm)	Flow Rate m^3/h
TEST NO 124	3,58	33,41	3,44	39,83	3,52	35,31	3,38	37,97	3,59	35,05	35,74	120,94	115,01	993,00	906,86
TEST NO 125	3,57	33,93	3,43	40,37	3,51	35,85	3,38	38,63	3,55	35,58	36,46	210,76	200,43	993,00	905,55
TEST NO 126	3,54	31,67	3,39	36,14	3,47	33,46	3,34	34,92	3,52	33,25	34,87	166,85	158,67	996,69	905,47
TEST NO 127	3,53	31,91	3,40	36,52	3,47	33,71	3,34	35,28	3,51	33,63	35,26	172,40	163,95	995,98	905,36
TEST NO 128	3,54	32,15	3,39	36,88	3,47	34,04	3,34	35,60	3,54	33,86	35,63	172,30	163,86	996,39	905,62
TEST NO 129	3,54	32,12	3,39	36,86	3,47	34,01	3,34	35,58	3,57	33,84	35,76	172,39	163,94	996,06	905,52
TEST NO 130	3,53	32,30	3,40	37,50	3,47	34,19	3,34	36,10	3,55	34,00	35,88	184,28	175,28	994,80	906,23
TEST NO 135	2,12	28,79	1,93	33,40	2,08	30,65	1,91	32,33	2,14	30,29	29,77	209,61	199,25	992,86	905,67
TEST NO 136	2,13	30,74	1,95	35,52	2,10	32,52	1,93	34,52	2,14	32,28	32,50	206,84	196,70	993,06	905,15
TEST NO 137	2,13	30,93	1,95	36,10	2,09	32,70	1,92	34,71	2,14	32,43	33,06	206,67	196,55	993,00	905,56
TEST NO 138	2,12	30,93	1,94	36,12	2,10	32,73	1,92	34,79	2,11	32,52	33,30	207,82	197,63	993,00	906,35
TEST NO 139	2,12	30,83	1,94	36,08	2,09	32,80	1,92	34,70	2,16	32,41	33,40	208,60	198,38	993,00	905,87
TEST NO 140	2,11	30,81	1,94	36,10	2,10	32,64	1,93	34,70	2,11	32,40	33,46	208,17	197,97	993,00	905,62
TEST NO 141	2,14	31,01	1,94	36,20	2,09	32,82	1,92	34,87	2,19	32,60	33,78	207,28	197,13	992,91	906,18
TEST NO 142	2,12	31,14	1,94	36,27	2,09	32,91	1,92	34,95	2,14	32,74	33,99	207,73	197,56	992,97	906,73

In table 9.1 we can examine test results about compressor performance. These performance results are average values for each test. Depends on these performance results we can verify our compressor. All of the results are within the limits.

For increasing the compressor working life and more suitable working conditions, some developments can be made. These developments are;

- 1st stage air outlet temperature is so close to the limits. For increasing the rings and valves working life, we have to decrease the air temperature. There is a direct correlation between pressure and temperature in thermodynamic laws. For decreasing the 1st stage air outlet temperature, we have to decline 1. stage pressure ratio. Therefore, we have to increase 2nd stage cylinder dia.
- For decreasing the air outlet temperatures, we can re-design the water channels inside the cylinders.
- In these kind of systems leakage in valves is the main problem. For decreasing the leakages inside the system, liquid gaskets can be used around the valves. In this way, we can increase the flow rate value of the compressor.
- For increasing the performance values of the heat exchangers, vane pipe can be used inside the heat exchangers.

9.1.3 Vibration results

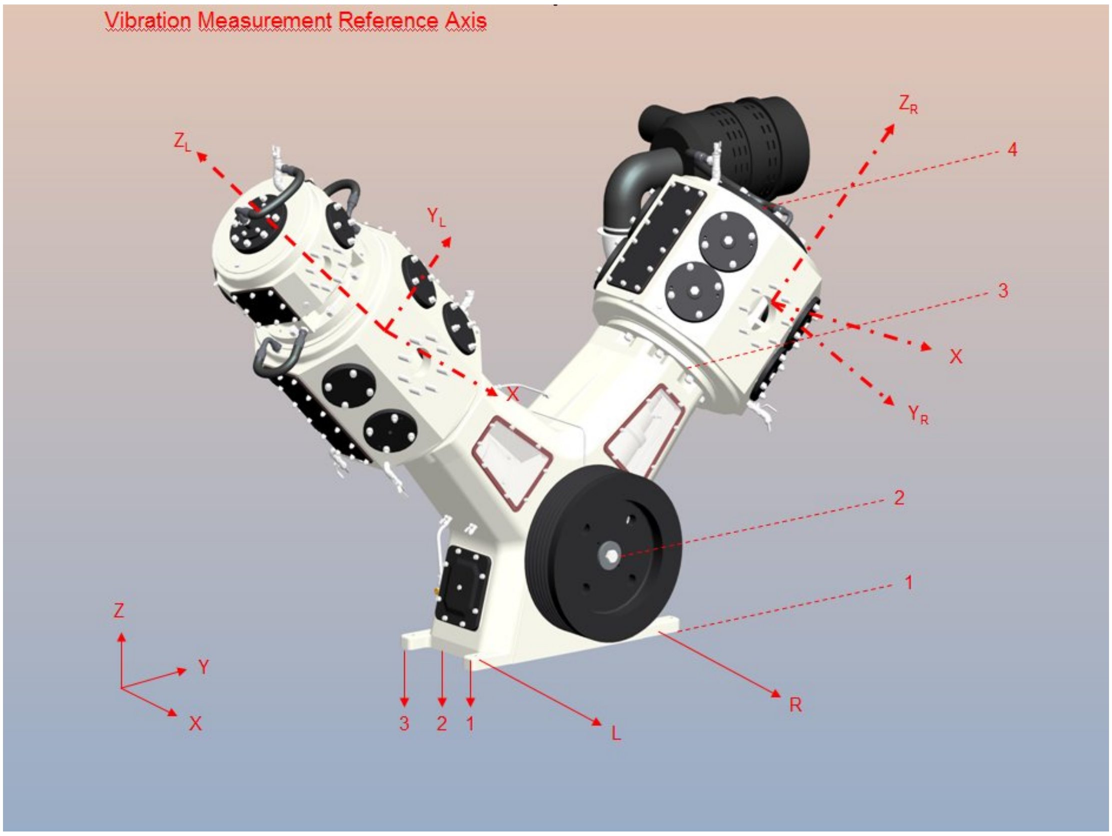


Figure 9.4 Vibration measurement reference axis

Table 9.2 Compressor vibration results

Measuring Points	X	Y	Z
	mm/s	mm/s	mm/s
L1.1	5,33	5,74	6,33
L1.2	-	5,27	6,27
L1.3	5,88	5,12	6,62
R1.1	4,68	4,03	3,25
R1.2	-	4,28	3,86
R1.3	4,3	4,16	3,22
L2.1	4,58	-	-
R2.1	4,35	-	-
L3.1	2,56	-	4,09
L3.2	-	4,61	4,52
L3.3	2,73	-	4,13
R3.1	3,37	-	5,65
R3.2	-	6,25	5,5
R3.3	3,53	-	5,24
L4.1	5,62	-	5,74
L4.2	-	10,54	7,6
L4.3	5,56	-	8,07
R4.1	4,84	-	5,5
R4.2	-	6,06	5,2
R4.3	4,27	-	5,4

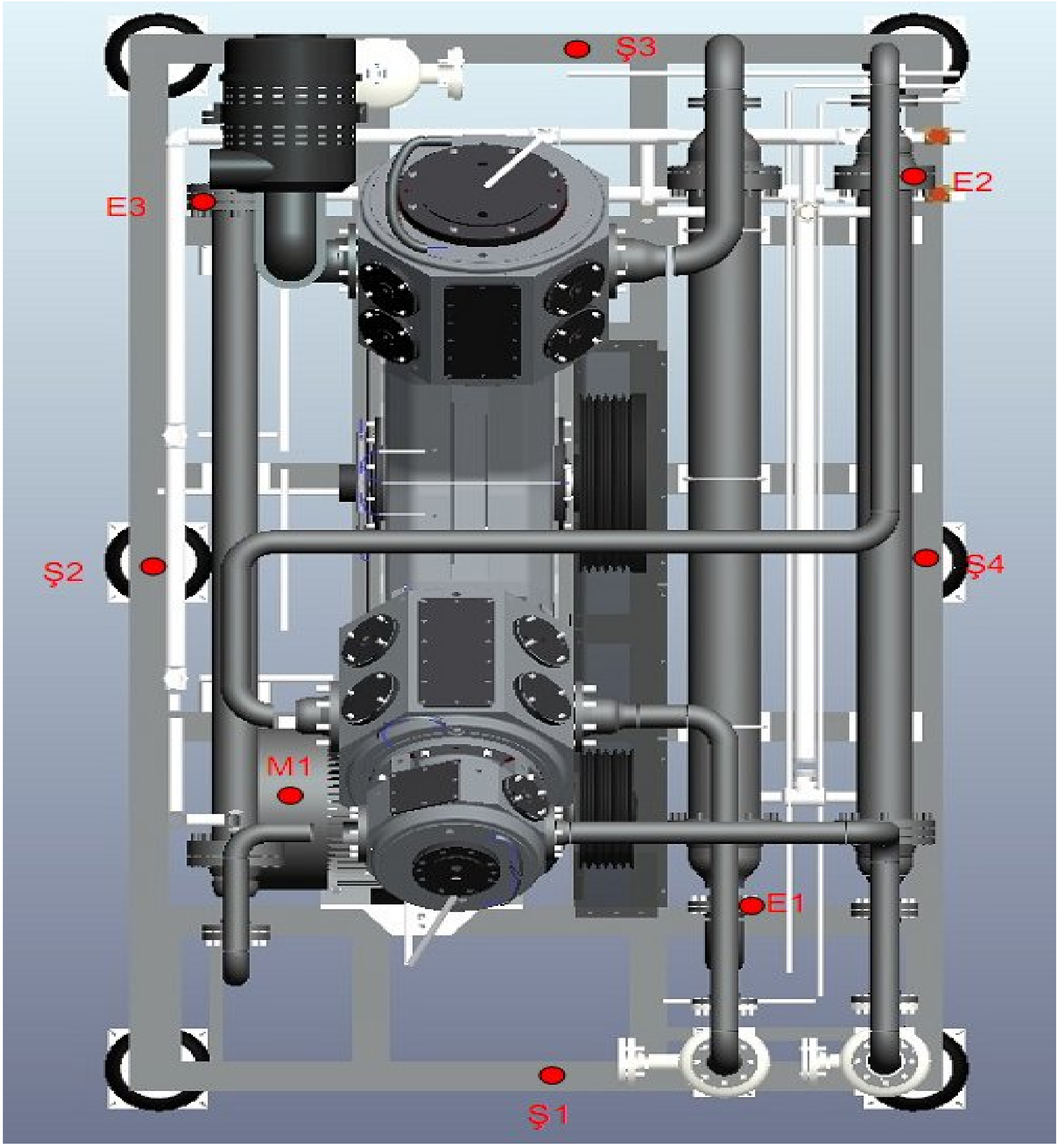


Figure 9.5 Chassis and Heat exchanger vibration measuring points

Table 9.3 Chassis and heat exchangers vibration results

Measuring Points	X	Y	Z
	mm/s	mm/s	mm/s
Ş1	9,67	-	11,41
Ş2	-	4,49	5,72
Ş3	9,56	-	11,17
Ş4	-	11,45	7,49
E1	7,39	8,13	-
E2	7,60	11,43	-
E3	9,50	9,77	-
M1	8,39	-	6,46

Vibration points for compressor, chassis and heat exchangers are shown in figure 9.4 and 9.5. Also vibration test results are shown in table 9.2 and 9.3.

According to TS EN ISO 2151/AC vibration standarts for V-Type compressors, these test results are suitable. For improving the working conditions some developments can be made,

- We can decrease total reciprocating mass with re-design of the some parts. These parts are determined depends on strength analysis. During the re-design process we should note that in V type compressors total reciprocating mass must be equal in both sides. Otherwise, because of inertia force differences vibration problem occurs.
- Some results on the heat exchangers are so close to the limits. For decreasing the vibration on heat exchangers, we can design more rigid connections between heat exchangers and chassis.
- Pulsation is the main cause the vibration on heat exchangers. For preventing the pulsation on heat exchangers, piping system can be re-designed.

Before starting the compressor, users should note that,

- Compressor must sit on a flat surface. Irregularity on the surface cause vibration.
- Air springs must inflate with certain pressure value. This pressure value is 4 bar for our system. Otherwise airsprings load capacity and damping coefficient remains inadequate. As a result, users encounter unexpected vibrations.

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SYMBOLS

A	Area
c_f	Coefficient of speed fluctuation
d	Diameter
E	Elasticity
F	Force
f	Nodal force
h	Height
J	Mass moment of inertia
K	Polytropic Index
K	Stiffness
KE	Kinetic Energy
L	Length
M	Inertia torque
m	Mass
m_{ccm}	Crankshaft cheek mass
m_{ccm}	Crankshaft cheek mass
m_{ccr}	Crankshaft cheek radius
m_{ce}	Crankshaft eccentric mass
$m_{cr,rec}$	Reciprocating mass of the connecting rod
$m_{cr,rot}$	Rotating mass of the connecting rod
m_{rec}	Total reciprocating mass
m_{rot}	Total rotating mass
N	Shape function
P	Pressure
r	Crank radius
r_{cw}	Counter weight radius
S	Suitable linear operator
T	Torque
U	Energy
W	Work

w	Angular velocity
wt	Crank angle
x_s	Moving distance of the piston
x_p	Downward displacement of piston from top
y_c	Horizontal displacement
γ	Shear strain
ε	Strain
θ	Crank angle
υ	Poisson ratio
ρ	Crankshaft radius / Connecting rod lenght
ρ	Density
σ	Stress
T	Shear stress
ϕ	Connecting rod angle
δ	Derivative index
$\int$	Integral index

APPENDICES

Quality	16MnCr5	Lucefin Group
According to standards	EN 10084: 2008	
Number	1.7131	

Chemical composition

C%	Si% max	Mn%	P% max	S% max	Cr%	
0,14-0,19	0,40	1,00-1,30	0,025	0,035	0,80-1,10	Permissible deviations
± 0.02	+ 0.03	± 0.05	+ 0.005	+ 0.005	± 0.05	on the product

16MnCrS5 n° 1.7139 S% 0.020-0.040 permissible deviation on the product ± 0.005%
By agreement this steel can be supplied with the additon of lead (Pb) 0.15-0.35%
Calcium (Ca) treatment may be supplied on request

Temperature °C

Hot forming	Normalizing	Core hardening	Carbonitriding	Carburizing	Hardening carburizing surf.	Tempering
1150-850	880 air (HB 138-187)	860-900 oil-polymer salt bath	750-930 gas	900-950	810-840 oil polymer salt bath (160-250 °C)	150 200

Soft annealing	Isothermal annealing	Spheroidizing	End quench hardenability	Preheating welding		Stress relieving after welding
750-770 slowly 15 °C/h until 680 pause after slowly to 400 pause, after air (HB max 207)	870 furnace cooling to 650 after, air (HB 156-207)	730-750 furnace cooling 50 °C/h to 680 pause, cooling to 400 after air (HB 140-187)	870 water	welding must be made on the annealing and before the carburizing		
				150-350		600 furnace cooling
				Ac1	Ac3	Ms * core ** carburizing surface
				740	840	400* 200**

Transformation annealing +FP	As-rolled	Stress relieving
950-1000 quick cooling to 630-650 holding abaut 3 h, after air (HB 140-187)	--- (HB max 230)	600-620

Mechanical and physical properties

Hot rolled properties obtained from test blanks after **core hardening** + stress relèving UNI 7846: 1978 Only reference.

size mm test banks	Testing at room temperature (longitudinal) R N/mm²	Rp 0.2 N/mm² min.	A% min.	C% min.	Kcu J min.	HB	
11	1030-1370	735	8	--	25	311-394	
30	740-1030	490	9	--	25	224-311	for information
63	640-930	440	10	--	25	198-278	for information

Hot rolled condition untreated **Lucefin** esperience

size mm	R N/mm²	Rp 0.2 N/mm² min.	A% min.	C% min.	Kcu J min.	HB max
over 10 to 100	560-720	350	15	25	--	207

Table of tempering values at room temperature for round Ø 10 mm after quenching at 870 °C in oil

HB	390	385	385	385	385	381	378	362	348	319	286	240	213	200
HRC	42	41.5	41.5	41.5	41.5	41	40.5	39	37.5	34	30	22.5	--	--
R N/mm²	1340	1335	1330	1330	1320	1300	1260	1210	1150	1050	950	800	700	650
Rp 0.2 N/mm²	1020	1060	1110	1140	1145	1140	1110	1070	1010	930	830	710	620	560
A %	12.0	12.5	12.5	12.5	12.0	12.0	12.5	13.0	14.0	15.5	17.5	20.0	23.0	25.5
C %	52.0	52.0	53.0	54.0	55.0	57.0	59.0	61.0	63.0	64.0	68.0	72.0	75.0	--
Kv J	42	46	46	45	42	40	42	62	90	124	135	155	180	194
HRC carburizing surface	64	63	62	60.5	59	57	--	--	--	--	--	--	--	--
Tempering at °C	**50**	**100**	**150**	**200**	**250**	**300**	**350**	**400**	**450**	**500**	**550**	**600**	**650**	**700**

16MnCrS5 1.7139 EN 10277-4: 2008 *Lucefin Group*

size mm		Soft annealing +A +SH Peeled Reeled, Ground +SL	Soft annealing +A +C Cold drawn	Heat treatment +FP +SH for pearlite / ferrite structure Peeled Reeled, Ground	Heat treatment +FP +C for pearlite / ferrite structure Cold drawn
over	to	HB max	HB max	HB	HB
5 [a]	10	--	260	--	--
10	16	--	250	--	--
16	40	207	245	140-187	140-240
40	63	207	240	140-187	140-235
63	100	207	240	140-187	140-235

[a] for thickness below 5 mm, hardness test, can be agreed before order placement

Forged UNI 8550: 1984 Only reference.

size mm		Testing at room temperature								
		R	$Rp_{0.2}$	A% L	A% T	A% Q	Kcu L	Kcu T	Kv L	*HB*
over	to	N/mm^2	N/mm^2 min	min	min	min	J min	J min	J min	*for inform.*
	11	1030-1375	735	8	--	--	25	--	--	*311-395*
11	25	785-1080	540	9	--	--	30	--	--	*234-327*
25	50	685-930	490	10	--	--	30	--	--	*209-278*

mechanical properties obtained from test blanks after core hardening + stress relieving
L = longitudinal T = tangential Q = radial

EN 10084: 2008 **Jominy test HRC** grain size G 5 min.

distance in mm from quenched end

	1.5	3	5	7	9	11	13	15	20	25	30	35	40	45	50	--	H
min	39	36	31	28	24	21	--	--	--	--	--	--	--	--	--	--	normal
max	47	46	44	41	39	37	35	33	31	30	29	28	27	--	--	--	

Temperature	**Mod. of elasticity** N/mm^2		**$Rp_{0.2}$** N/mm^2		**Thermal expansion**
Testing at °C	**E** long.	**G** tang.	Ø <250 mm	Ø 250-500	$[(m/m \cdot K) \cdot 10^{-6}\ °C^{-1}]$
20	210000	80000	--	--	--
100	--	--	--	--	11.1
200	--	--	--	--	12.1
300	--	--	--	--	12.9
400	--	--	--	--	13.5
500	--	--	--	--	13.9
600	--	--	--	--	--

Specific heat capacity	**Density**	**Thermal conductivity**	**Specific electric resist.**	**Conductivity**
$J/(Kg \cdot K)$	Kg/dm^3	$W/(m \cdot K)$	$Ohm \cdot mm^2/m$	$Siemens \cdot m/mm^2$
460	7.85	41	0.16	6.25

EUROPE EN	**ITALY** UNI	**CHINA**	**GERMANY** DIN	**FRANCE** AFNOR	**U.K.** B.S.	**RUSSIA**	**USA** AISI/SAE
16MnCr5	16MnCr5	15CrMn	16MnCr5	16MC	--	18HG	5115

Quality	X20Cr13 AISI 420	Martensitic	Lucefin Group
According to standards	EN 10088-3: 2005		
Number	1.4021		

Chemical composition

C%	Si% max	Mn% max	P% max	S% max	Cr%	
0,16-0,25	1,00	1,50	0,040	0,030	12,0-14,0	Permissible deviations on the product
± 0.02	+ 0.05	± 0.04	+ 0.005	± 0.005	± 0.15	

Temperature °C

Melting range	Hot forming	Normalizing	Quenching	Tempering	Welding	
1460-1490	1150-880	900-980 air	950-1050 oil or polymer	I° 700 II° 650	Preheating 300	stress relieving 700

Stress relieving	Soft annealing	Full annealing	Spheroidizing	Ac1	Ac3	Ms	Mf
150-250 air	740-820 air (HB max 230)	870-900 slow cooling (HB 170-200)	860 furnace cooling to 650 after air	790	850	240	205

Mechanical properties

Hot rolled EN 10088-3: 2005

size mm over	to	Testing at room temperature (longitudinal) R N/mm²	Rp 0.2 N/mm² min	A% min L	Kv +20 °C J min L	HB [a] max	
		760 max	–	–	–	230	+A annealed
	160	700-850	500	13	25	–	+QT quenched and tempered R 700
	160	800-950	600	12	20	–	+QT quenched and tempered R 800

[a] only for guidance

Cold processed bright bars EN 10088-3: 2005 in conditions 2H, 2B, 2G, 2P

size mm over	to	Testing at room temperature (longitudinal) R N/mm² max	HB [a] max	R N/mm²	Rp 0.2 N/mm² min	A% min L	A% min T	Kv +20 °C J min L	Kv +20 °C J min T
	10 [b]	910	290	750-1000	600	8	–	--	–
10	16	910	290	750-1000	550	8	–	--	–
16	40	850	280	700-950	500	10	–	25	–
40	63	800	250	700-900	500	12	–	25	–
63	160	760	230	700-850	500	13	–	25	–
		+A annealed		+QT 700 quenching and tempering					

[a] for information only

[b] In the range 1 mm ≤ d < 5 mm valid only for rounds – the mechanical properties of non round bars with thicknesses < 5 mm have to be agreed at the time of enquiry and order.

Work-hardness by **Cold drawing**

R	N/mm²	600	670	750	800	850	880	900	930
Rp 0.2	N/mm²	350	500	620	650	740	800	820	840
A	%	36	22	15	14	12	11	10	9
C	%	58	52	42	36	34	33	32	30
Reduction	**%**	**0**	**10**	**20**	**30**	**40**	**50**	**60**	**70**

Table of tempering values at room temperature for round Ø 10 mm after quenching at 970 °C in oil

R	N/mm²	1620	1600	1560	1540	1550	1570	1500	1230	910	780	700
Rp 0.2	N/mm²	1330	1290	1270	1260	1260	1250	1200	910	680	590	560
A	%	10.5	12.0	13.0	12.0	11.5	10.5	12.0	15.0	17.0	20.0	22.0
Kv	J	24	26	27	26	19	18	16	19	25	33	50
Tempering at °C		**200**	**250**	**300**	**350**	**400**	**450**	**500**	**550**	**600**	**650**	**700**

Min. val. for the 0.2% proof strength at high temperature EN 10088-3: 2005 EN 10250-4: 2001

Rp 0.2	N/mm²	–	460	445	430	415	395	365	330	levels +QT 700	Hot rolled & Forged
Rp 0.2	N/mm²	–	515	495	475	460	440	405	355	levels +QT 800	
Testing at °C		**50**	**100**	**150**	**200**	**250**	**300**	**350**	**400**		

X20Cr 13 AISI 420 *Lucefin Group*

Forged EN 10250-4: 2001

Testing at room temperature (longitudinal)

size d / *t*		R	Rp 0.2	A%	A%	Kv +20 °C	Kv +20 °C	
over	to	N/mm^2	N/mm^2 min	min L	min T	J min L	J min T	
	--	760 max	--	--	--	--	--	+A annealed
	250/*160*	700-850	500	13	--	25	--	+QT quenched and tempered R700
	250/*160*	800-950	600	12	--	20	--	+QT quenched and tempered R800

d = diameter *t* = *thickness*

Thermal expansion		$[m/(m \cdot K)] \cdot 10^{-6}$	--	10.5	11.0	11.5	12.0	12.0	--	--	--
Modulus of elasticity	longitudinal	N/mm^2	216000	213000	207000	200000	192000	--	--	--	--
Modulus of elasticity	tangential	N/mm^2	98000	95000	92000	89000	86000	--	--	--	--
Specific electric resistivity		$Ohm \cdot mm^2/m$	0.60	--	0.69	--	0.86	--	--	1.03	1.17
Conductivity		$Siemens \cdot m/mm^2$	1.67	--	--	--	--	--	--	--	--
Creep rupture strength 10.000 h	**R**	N/mm^2	--	--	--	--	--	147	74	33	--
Yield 10.000 h	**Rp** 0.1	N/mm^2	--	--	--	--	--	93	44	17	--
Creep rupture strength 100.000 h	**R**	N/mm^2	--	--	--	--	--	103	44	18	--
Yield 100.000 h	**Rp** 0.1	N/mm^2	--	--	--	--	--	64	25	10	--
Specific heat capacity		$J/(Kg \cdot K)$	460	--	500	--	590	--	--	720	860
Mean coefficient of linear expansion		$10^{-6}/°K$	--	10.5	10.8	11.5	11.6	12.0	--	12.2	12.7
Testing at °C			**20**	**100**	**200**	**300**	**400**	**500**	**550**	**600**	**800**

Density	**Thermal conductivity** W/(m·K)					**Magnetic permeability**	**Resistence to hot oxidation**
Kg/dm^3	**20 °C**	**200 °C**	**400 °C**	**600 °C**	**800 °C**	μ_r	
7.70	25.1	26.5	27.7	27.4	25.0	--	up to 650 °C

EUROPE EN	ITALY UNI	CHINA	GERMANY DIN	FRANCE AFNOR	U.K. B.S.	RUSSIA	USA AISI
X20Cr13	X20Cr13	2Cr13	X20Cr13	Z20C13	420S37	20H13	420

COMPOSITION Density ≈ 9,1

	Composition %										
	Cu	**Pb**	**Sn**	**Ni**	Al	Fe	Mn	P	Sb	Si	Zn
Nom	**76**	**15**	**8**	**1**							
Min	Bal	13,0	7,0	0,5							
Max	Bal	17,0	9,0	1,5	0,01	0,25	0,2	0,05	0,5	0,01	1,0

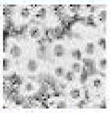
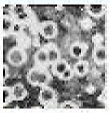
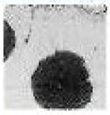
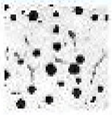
MECHANICAL PROPERTIES

			Sandcast JM4-03	**Centrifugally- & continuously cast** JM4-15
Rp0,2	Proof strength	N/mm^2	≥ 80	≥ 80
Rm	Tensile strength	N/mm^2	≥ 160	≥ 180
A5	Elongation	%	≥ 8	≥ 8
H_B	Hardness	10/1000	≥ 50	≥ 60
E	Young's modulus	N/mm^2	75 000	75 000
Coeff. of thermal expansion		$x10^{-6}$, 0-100°C	18,0	18,0
Thermal conductivity		W/m °C	45	45
Resistivity		ným, 20° C	130	130
Machinability			Excellent	Excellent
Swedish standard		SS		
Nearest equivalent foreign standards				
British standard		BS	1400, LB1	1400, LB1
German standard		DIN	1716, G-CuPb15Sn	1716, GZ/GC-CuPb15Sn
US standard		UNS	C 93800	C 93800

Values given refer to separately cast test specimen to SS 112152. Each specimen is to be cut from centrifugal- or continuous castings with a wall thickness corresponding to the test specimen diameter.

Conversion factors
1 lbf/in² = 0,07 N/mm²
1 N/mm² = 145 lbf/in²

Cast iron with nodular graphite1)

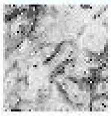
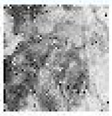
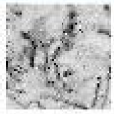
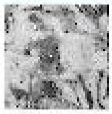
EN-GJS-500-7	EN-GJS-600-3	EN-GJS-700-2	EN-GJS-800-2	EN-GJS-1000-5[4]	EN-GJS-X SiMo4-0.5	EN-GJS-X SiMo5-1[5]	EN-GJSA-X NiSiCr35 5 2
3,50–3,70	3,50–3,70	3,50–3,70	3,50–3,70	3,50–3,70	3,00–3,40	3,00–3,40	max. 2,00
2,30–2,60	2,30–2,60	2,30–2,60	2,30–2,60	2,30–2,60	3,60–4,40	4,00–5,00	4,00–6,00
max. 0,40	max. 0,40	max. 0,40	max. 0,40	max. 0,40	max. 0,30	max. 0,30	0,50–1,50
–	–	–	–	–	0,40–0,60	0,80–1,20	–
–	–	–	–	–	–	–	34,0–36,0
 Ferrite and pearlite 1:100	 Pearlite and ferrite 1:100	 Pearlite and ferrite 1:500	 Pearlite 1:500	 Bainite and austenite 1:500	 Ferrite 1:500	 Ferrite 1:500	 Austenite 1:100
500–650	600–750	700–850	800–1000	1000–1200[6]	500–630	500–630	370–500
320–420	370–480	420–600	480–750	700–800[6]	400–520	400–520	200–290
18–7	8–3	6–2	4–2	15–5[6]	16–8	13–4	20–10
20–7	8–3	6–2	4–2	–	–	–	–
160–185	160–185	160–185	160–185	155–170[6]	160–175	160–175	110–145
–	–	–	–	–	–	–	–
–	–	–	–	–	–	–	–
–	–	–	–	–	–	–	–
170–220	200–250	235–285	270–335	300–360[6]	200–250	200–250	130–170
240	270	300	320	–	–	–	–
160	180	200	220	380	–	–	–
–	–	–	–	–	very good	very good	very good
–	–	–	–	–	good	very good	very good
good	good	moderate	moderate	moderate	moderate	moderate	good
good	good	very good	very good	very good	good	good	moderate
low	good	very good	very good	–	–	–	– –
good	good	very good	very good	–	–	–	–
		partially weldable with special electrodes					
7,10–7-30	7,10–7-30	7,10–7-30	7,10–7-30	7,10–7-30	7,10–7-30	7,10–7-30	7,45
35	33	31	31	22	28	28	13

[4] to DIN EN 1564
[5] also corresponds to quality EN-GJS-XSiMo4-1

[6] mechanical properties after annealing

special materials on request

Cast iron with lamellar graphite[1)]

Designation to DIN EN 1561				EN-GJL-150	EN-GJL-200	EN-GJL-250	EN-GJL-300
Nominal analysis[2)]	C			3,40–3,60	3,20–3,40	2,90–3,10	2,90–3,10
	Si			2,30–2,60	2,00–2,40	1,80–2,10	1,60–1,90
	Mn			0,60–0,90	0,70–1,00	0,80–1,10	0,80–1,10
Structure				 Pearlite, coarse lamellar 1:100	 Pearlite, coarse lamellar 1:100	 Pearlite, fine lamellar 1:100	 Pearlite, fine lamellar 1:100
Mechanical properties							
Tensile strength[3)]	R_m	N/mm²		150–250	200–300	250–350	300–400
			wall thickness				
Tensile strength[4)]	R_m	N/mm²	10– 20 mm	–	–	–	–
			> 20– 40 mm	120	170	210	250
			> 40– 80 mm	110	150	190	220
			> 80–150 mm	100	140	170	210
			> 150–300 mm	90	130	160	190
Reference values for other mechanical properties							
Yield point	$R_{p0,1}$	N/mm²		98–165	130–195	165–228	195–260
Breaking elongation	A	%		0,8–0,3	0,8–0,3	0,8–0,3	0,8–0,3
Compressive strength	δ_{dB}	N/mm²		600	720	840	960
Bending strength	δ_{bB}	N/mm²		250	290	340	390
Shear strength	δ_{aB}	N/mm²		170	230	290	345
Torsional strength	τ_{tB}	N/mm²		170	230	290	345
Modulus of elasticy	E	kN/mm²		78–103	88–113	103–118	108–137
Poisson's ratio	ν			0,26	0,26	0,26	0,26
Reversed bending strength	δ_{bB}	N/mm²		70	90	120	140
Alternating tensile-compr. strength	δ_{zdW}	N/mm²		40	50	60	75
Fracture toughness	K_{IC}	N/mm$^{3/2}$		320	400	480	560
Physical properties							
Density	ρ	kg/dm³		7,10	7,15	7,20	7,25
Thermical conductivity	λ to 300 °C	W/(K·m)		50	48	47	45

[1)] The mechanical properties in separately cast specimen 30 mm dia.
The mechanical properties in the casting depend on the wall thickness

[2)] Reference values, for average wall thicknesses

[3)] In separately cast specimen 30 mm dia.

[4)] In integrally cast specimen

Material Data Sheet for

Rings & Packings Materials

HOERBIGER

Material Grade: HY52

Property	Method	Value
COTE - Radial x 10^{-6}/C (20 - 200 deg C)	ASTM D696	97
COTE - Axial x 10^{-6}/C (20 - 200 deg C)	ASTM D696	135
Maximum Operating Temperature (deg C)		200
Density (g/cm^3)	ASTM D792-00	3.8 ± 0.05
Shore D Hardness	ASTM D2240-04	70 ± 2
Tensile Strength at Break (MPa)	ASTM D638-03	16 ± 1.0
Elongation at Break (%)	ASTM D638-03	101 ± 15

HY52 works well in most air compressor applications and is known for its continuous outstanding performance in non-lubricated applications. The combination of toughness, resistance to extrusion, high heat transfer coefficient, and flexibility to stretch over a piston, makes HY52 a versatile material. We recommend this grade to our customers for non-lubricated and lubricated applications where extrusion resistance is needed. HY52 is BAM certified for use in oxygen compression.