D.E.Ü.İ.İ.B.F.Dergisi
Cilt:18 Sayı:1, Yıl:2003, ss:63-70

EXPLICIT DERIVATION OF THE BIAS IN THE SOLOW RESIDUAL IN THE EXISTENCE OF IMPERFECT COMPETITION AND NON-CONSTANT RETURNS TO SCALE

Sacit Hadi AKDEDE(*)

ABSTRACT

The main purpose of this paper is to explicitly derive the possible bias in the Solow residual when there is imperfect competition and non-constant returns to scale. As is known very well, the standard Solow residual assumes perfect competition and constant returns to scale. As is also known, Solow residual is one of the measures of total factor productivity. Therefore, if there is imperfect competition and non-constant returns to scale, then standard Solow measure of productivity will be biased and all the comparisons based on this residual will be unreliable. In this study, we shed some light on these issues.

Key words: *Total Factor Productivity, Bias in the Solow Residual.*

1. Introduction

Productivity varies enormously across different nations. In 1988, output per worker was 48 times higher in the most productive compared to least productive countries (Hall and Jones, 1996). In this last sentence we understand that productivity is measured by output per worker. Output per worker is not the only measure of the productivity. There are many different measures of productivity other than labor productivity. These different measures of productivity and measurement issues and procyclical nature of productivity can make the comparisons among industries and nations difficult. (Baily and Gordon, 1988: 347-420; Basu, 1993; Shapiro 1987:118-124). In this respects, in discussing statistical problems associated with measurement error, Evans (1992: 191-208) states that mismeasurement of inputs creates a serious problem. All these measurement errors and different measures of productivity and productivity growth are beyond the scope of this paper. In this paper, I assume that there is no measurement error in terms of inputs. The present paper only pays attention to the Solow residual and its standard assumptions and how

(*) Dr., Adnan Menderes Üniversitesi, İ.İ.B.F., Nazilli/Aydın.

changing the assumptions change the whole implication of the Solow residual will be the main concern of the paper.

Solow residual is one of the measures of the total factor productivity. In the standard growth accounting, Solow residual is capturing the factors that are affecting the output growth and that are not accounted by the growth of standard production factors, namely labor and capital (Hall, 1990: 71-112). The standard Solow residual is assuming perfect competition and constant returns to scale. If we relax at least one of these assumptions, then the standard Solow residual can be biased. For example, if certain industries show non-constant returns to scale and imperfect competition or either one of them, then in terms of measuring the total factor productivity, the Solow residual will give us biased measures of productivity if we don't correct for the possible biases. Therefore, these measures of productivity will not be meaningful when it comes to comparing the productivity among different countries/industries or for the same industry/country , but different time periods. For this reason, we need to be aware of the fact that in the presence of imperfect competition and non-constant returns to scale there is a possible bias in the standard measure of the Solow residual.

2. Derivation of the possible bias

In order to be able to make meaningful comparisons among different countries or different time periods, it is very important to have reliable measures of total factor productivity. That is why it is important to have unbiased measures of productivity. In this paper, we will qualitatively derive the possible bias. Empirically, this bias can be tested for different contexts and countries/industries. This, however, is a subject for another paper. Some recent studies empirically tested and measured the size of the bias such as Harrison (1994: 53-73), Kim (2000: 55-83), Krishna and Mitra(1998: 447-462). All these studies got the initial inspiration from the Hall's paper (1988). In the present paper, I will also exploit the methodology developed by Hall (1988: 921-947).

2.1. Perfect competition and constant returns to scale

As a benchmark case, I start out with the standard growth accounting assumption of constant returns and perfect competition. First, I estimate the sectoral rates of TFP growth by using the standard production function:

$$Y_{it} = A_{it} F(L_{it}, K_{it}) \tag{1}$$

Output, which is value added here, Y_{it}, is produced by industry i with inputs labor, L_{it}, and capital, K_{it}. A_{it} is an industry specific index of Hicks-neutral technical progress. Now, totally differentiating (1), and dividing through by Y, and after some manipulation, I get:

$$dY/Y_{it} = (\partial Y/\partial L)(L/Y)(dL/L)_{it} + (\partial Y/\partial K)(K/Y)(dK/K)_{it} + dA/A_{it} \qquad (2)$$

Now in a competitive economy, $\alpha_L = (\partial Y/\partial L)(L/Y)$ is the labor share in the value of output. Analogously, $\alpha_K = (\partial Y/\partial K)(K/Y)$ is the capital share in the value of output. Of course, if the production function has constant returns to scale, $\alpha_K + \alpha_L = 1$. Therefore, I have

$$\left(dY/Y\right)_{it} = \alpha_L\left(dL/L\right)_{it} + (1-\alpha_L)\left(dK/K\right)_{it} + \left(dA/A\right)_{it} \qquad (3)$$

The change in output is equal to the sum of changes in inputs and change in productivity with inputs weighted by their respective shares in the output. Now, again assuming both perfect competition and constant returns to scale, the index of the total factor productivity growth, $\left(dA/A\right)_{it}$ from eq.(3) above can be calculated easily.

2.2. Imperfect competition and non-constant returns to scale

If perfect competition and constant returns to scale assumptions are violated, total factor productivity growth estimation would yield a biased estimate of TFP growth. In particular, profit maximization by firms holding some market power would no longer imply that share of that input in total income (output) would be equal to elasticity of output with respect to that input. There will be some markup, μ_i, and this markup will be assumed to be the same all over the firms in the same industry in a given period (Levinsohn ,1993: 1-22). In addition, if the constant returns to scale assumption is violated, factor shares of production factors do not exhaust the output, but their sum is equal to a scale parameter, φ divided by the markup parameter, μ_i. In the following section, I will show this explicitly.

The share of inputs in the value of output	The elasticity of output
for labor: $(wL/pY)=\alpha_L$	$(\partial Y/\partial L)(L/Y)=\varepsilon_L$
for capital $(rK/pY)=\alpha_K$	$(\partial Y/\partial K)(K/Y)=\varepsilon_K$

When there is perfect competition, $\alpha_L=\varepsilon_L$ and $\alpha_K=\varepsilon_K$. If there is imperfect competition, it is likely that $\alpha_L\langle\varepsilon_L$ and $\alpha_K\langle\varepsilon_K$. So, there is a mark-up between factor shares and elasticity for each factor: $\alpha_L\mu=\varepsilon_L;\alpha_K\mu=\varepsilon_K$ where μ is a mark-up parameter greater than 1. Moreover, with non-constant returns to scale, $\varepsilon_L+\varepsilon_K\neq 1$. With increasing returns to scale, this sum is greater than 1 and labor and capital do not exhaust the total output created. With increasing returns to scale, perfect competition is inconsistent and there are positive economic profits. Under constant returns to scale, if F_L and F_K are partial derivatives of production function and I divide the Euler equation $F_LL+F_KK=F$ by F, I obtain $(F_LL+F_KK)/F=1$ so that the two elasticities sum to 1. Under non-constant returns to scale, the elasticities sum to a scale parameter, φ, greater than or less than 1.

I can combine imperfect competition and non-constant returns to scale:

$$\alpha_L\mu+\alpha_K\mu=\varepsilon_L+\varepsilon_K$$

$$\mu(\alpha_L+\alpha_K)=\varphi$$

$$\alpha_L+\alpha_K=(\varphi/\mu)$$

$$\alpha_K=(\varphi/\mu)-\alpha_L$$

Now, assuming Cournot behavior on the part of firms, and a mark-up that only varies across sectors, and using the first order conditions from each firm's profit maximization and eq. (2) , I get, like Harrison (1994):

$$\left(dY/Y\right)_{it} = \mu_i \alpha_L \left(dL/L\right)_{it} + \mu_i \alpha_K \left(dK/K\right)_{it} + \left(dA/A\right)_{it} \tag{4}$$

where α_K is unobservable. α_L is usually observable since we usually have the compensations to the workers in most data sets. I can divide the compensation to the workers by the value of output to get the share of labor. Note that, as shown above, the sum of factor shares can be expressed as φ/μ, where φ, scale parameter, may be greater or less than one (or equal to 1 in the constant returns case). According to Eq. (4), imperfect competition enters eq. (2) because firms with market power do not set the value of marginal product equal to factor price. The share of each input in the value of output would no longer be equal to the elasticity of output with respect to that input. In eq.(4) , the total factor productivity growth, $\left(dA/A\right)$, which is the residual in growth accounting, is incorporating imperfect competition and non-constant returns to scale. In other words, total factor productivity growth(TFPG) in eq. (4) is not the same as in eq. (2) or eq.(3). In eq.(2) –eq.(3) , TFPG is the "standard" measure of TFPG. In eq.(4), TFPG is the "true" TFPG since the standard one is assuming perfect competition and constant returns to scale, whereas "true" measure is not.

To get the unbiased estimates of total factor productivity and the unbiased coefficients of its determinants, I will offer a way of correcting these biases. To this end, I calculate the difference between the "standard" TFP growth above, $\dot{TFP}$, that does not take into account non-constant returns and imperfect competition and the "true" TFP growth denoted by $\dot{TFP}^*$. I can rewrite eq. (4) as follows since the TFPG in eq.(4) is the "true" one

$$(dY/Y)_{it} = \mu_i \alpha_L (dL/L)_{it} + \mu_i \alpha_K (dK/K)_{it} + (dA/A)^*_{it} \tag{4a}$$

The difference between the standard TFP growth in eq(3) and the true TFP growth in eq.(4a) is given by

$$(dA/A)_{it} - (dA/A)^*_{it} = \left[(dY/Y)_{it} - \alpha_L(dL/L)_{it} - (1-\alpha_L)(dK/K)_{it}\right]$$
$$-\left[(dY/Y)_{it} - \mu_i\alpha_L(dL/L)_{it} - \mu_i\alpha_K(dK/K)_{it}\right]$$

$$= \mu_i\alpha_L(dL/L)_{it} + \mu_i\alpha_K(dK/K)_{it} - \alpha_L\left[(dL/L)_{it} - (dK/K)_{it}\right] - (dK/K)_{it}$$

$$= \mu_i\alpha_L(dL/L)_{it} + \mu_i\alpha_K(dK/K)_{it} - \alpha_L dl_{it} - (dK/K)_{it}$$

where $dl_{it} = \left[(dL/L)_{it} - (dK/K)_{it}\right]$

Substituting $\alpha_K = (\varphi_i/\mu_i) - \alpha_L$, which is derived above, I find

$$(dA/A)_{it} - (dA/A)^*_{it} = \mu_i\alpha_L(dL/L)_{it} + \mu_i((\varphi_i/\mu_i) - \alpha_L)(dK/K)_{it} - \alpha_L dl_{it} - (dK/K)_{it}$$
$$= \mu_i\alpha_L(dL/L)_{it} + \varphi_i(dK/K)_{it} - \mu_i\alpha_L(dK/K)_{it} - \alpha_L dl_{it} - (dK/K)_{it}$$

$$(dA/A)_{it} - (dA/A)^*_{it} = (\mu_i - 1)\alpha_L dl_{it} + (\varphi_i - 1)(dK/K)_{it}$$

Hence, I obtain

$$(dA/A)_{it} = (\mu_i - 1)\alpha_L dl_{it} + (\varphi_i - 1)(dK/K)_{it} + (dA/A)^*_{it}$$

or

$$\dot{TFP}_{it} = (\mu_i - 1)\alpha_L dl + (\varphi_i - 1)(dK/K)_{it} + \dot{TFP}^*_{it} \qquad (5)$$

As can be seen from eq. (5), if both markup and scale parameters are 1, then standard and true measures of productivity are colliding, meaning they are same. That is, if μ=1 and φ_i=1, in eq. (5) all the right hand side terms disappear except "true" total factor productivity.

Supposing, for the moment, that φ_i=1 (ignoring non-constant returns to scale),then equation (5) shows that faster capital input growth relative to labor will lead to a negative bias in the “standard” measure when imperfect competition is present ($\mu_i \rangle 1$). When increasing returns exist, $\varphi_i \rangle 1$, the same pattern of production factor growth rates can generate either positive or negative biases.

3. Conclusion

In this paper, I explicitly derive the possible bias in the standard Solow residual when there is imperfect competition and non-constant returns to scale. As is known, standard Solow residual assumes constant returns to scale and perfect competition. If one of these assumptions is violated, then our productivity measures are unreliable and we need to take into account this bias when we do a cross-sectional analysis. In this paper, we derive the bias qualitatively and talk about the direction of the bias. In an empirical paper, this bias can be calculated/derived explicitly.

ÖZET

ÖLÇEĞE GÖRE SABİT OLMAYAN GETİRİ VE EKSİK REKABET ORTAMINDA SOLOW ARTIĞINDAKİ SAPMANIN TÜRETİLMESİ

Bu makelenin ana amacı, eksik rekabet ve ölçeğe göre sabit olmayan getiri durumunda, Solow kalıntısında doğabilecek yanlış ölçümü ayrıntılarıyla göstermektir. Bilindiği gibi, standart Solow kalıntısı tam rekabet ve ölçeğe göre sabit getiri varsayımı altında bulunur. Gene bilindiği gibi, Solow kalıntısı toplam faktör verimliliği ölçümlerinden birisidir. Dolayısıyla, eksik rekabet ve ölçeğe göre sabit olmayan getiri durumunda, verimliliğin Solow kalıntısıyla ölçülmüş değeri yanlış bir büyüklük verecek ve bu büyüklüğe göre yapılan bütün ülkeler/endüstriler arası karşılaştırmalar da güvenilir olmayacaktır. Bu çalışmada bu sorunlar üzerine ışık tutmaya çalıştık.

Anahtar Kelimeler: Toplam faktör verimliliği, Solow kalıntısında yanlış ölçüm.

REFERENCES

BAILY, Martin N. and Robert J. GORDON (1988),"The Productivity Slowdown, Measurement Issues, and the Explosion of Computer Power" *Brookings Papers Econ. Activity,*no.2 , 347-420.

BASU, Susanto.(1993), "Procylical Productivity: Overhead Inputs or Cyclical Utilization ?" *Manuscript*. Ann Arbor: Univ. Michigan, Dept. Econ.,1993.

EVANS, Charles L. (1992), "Productivity Shocks and Real Business Cycles", *Journal of Monetary Economics*. 29, 191-208.

HALL , Robert E. (1988),“ The Relation between Price and Marginal Cost in U.S. Industry”, *The Journal of Political Economy*, Volume 96, Issue 5, 921-947.

HALL, Robert E.(1990),"Invariance Properties of Solow's Productivity Residual" *in Growth/Productivity/Unemployment: Essays to Celebrate Bob's Solow's Birthday*, edited by Peter Diamond. Cambridge, Mass.: MIT press, 71-112.

HALL, Robert E. and Charles I. JONES (1996),"The Productivity of Nations" *NBER working paper series , no 5812* , NBER Cambridge.

HARRISON, Ann. (1994), “ Productivity, imperfect competition, and trade reform, theory and evidence”, *Journal of International Economics* 36, 53-73.

KIM, Euysung. (2000),“ Trade Liberalization and productivity growth in Korean manufacturing industries: price protection, market power, and scale efficiency”, *Journal of Development Economics*, vol. 62 , 55-83.

KRISHNA, Pravin and Devashish MITRA. (1998), “ Trade Liberalization, market disipline, and productivity growth: new evidence from India”, *Journal of Development Economics*,vol. 56, 447-462.

LEVINSOHN, James. (1993), “ Testing the import –as-market-discipline hypothesis”, *Journal of International Economics*, 35, 1-22.

SHAPIRO, Matthew D. (1987)," Are Cyclical Fluctuations in Productivity Due More to Supply Shocks or Demand Shocks ?" *A.E.R. papers and Proc.* 77, 118-124.