EUCLIDIAN DISTANCE FACILITY LOCATION

M. Hulûsi DEMİR (*)   Kaan YARALIOĞLU(**)

LAYOUT PROBLEMS WITH EUCLIDIAN DISTANCE

Facilities layout concerns the configuration of departments, work centers and eguipment, with particular emphasis on movement of work through the system. Layout decisions play o key in the design of productive systems, affecting operating costs and efficiency. But not all layout problems justify the use of computerized algorithms such as ALDEP, CORELAP or CRAFT. As an example, some layout and location problems involve the location of a single new facility in an existing layout or the design of a layout for a single department.

This article describes the problem of determining the localition of a single new facility with resbect to a number of existing facilities, by using Euclidean or Strainght Line Model. Here, in this model, the minimization of the defined cost function is aimed.

THE MEANING

The design of production plant or any type of physical facility requires that management consider the problem of layout. The term 'Layout' is here used to indicate the physical disposition of the facilities. The term encompasses both;

1. The location of departments upon a site

2. The location of work stations within a department.

The term indicates location strategy. Layout is one of the six strategic areas that will determine the long-run efficiency of operations. The objective of the layout strategy is to develop an economic layout that will meet

(*) Prof.Dr., D.E.Ü.İ.İ.B.F., İşletme Bölümü
(**) Araş.Gör., D.E.Ü.İ.İ.B.F., İşletme Bölümü

the requirements of:

1. product design and volume (product strategy),

2. process equipment and capacity (process strategy),

3. building and site constraints (location strategy).

Layout will affect the organization of the plant. Management's goal is to lay out the system so that it operates at peak efficiency and effectiveness. Layout decisions include the best placement of machines and equipments (in a production setting), offices and desks (in a office setting), or service centers (insettings such as department stores). To achieve these layout objectives, a multiplicity of strategies have been developed. Among them are three of very great importance.

A plant can be laid out in three ways: either to try to serve the needs of the product (product layout e.g. traditional automobile assembly line, where the product moves along an assebly or conveyor linepast a series of work stations. Workers and machines perform the required operations in processing sequence.) or to serve the needs of the process (process layout. All machines that do the same job or all processes that are the same are grouped together, such as painting, sewing, heat treating, X-rating, bookkeeping, inspecting shipping etc.) or a fixed position layout where the product remains stationary and all of the required workers and equipment come to the one work area).

Product layout results when a high production volume exists for the product, such that continuous production is justified. In a product layout, machines are not shared by different products. In a process layout a given machine can be used to process a variety of different products. Typically, a given job is produced for a period of time, and then a new job is produced. Rather than producing to inventory, the order is processed and shipped to the customer.

Of these three layout formats only the first two have undergone extensive mathematical analysis. In this conference, I shall tackle to explain Euclidian Distance Facility Location Problems. We must keep in mind, however, that unlike the straightforward mathematics of linear programming, waiting lines, forecasting etc., the layout/location and design of physical facilities is still as mucch an art as it is a science.

CRITERIA FOR A GOOD LAYOUT

Defining the boundaries on a plant layout problem is no easy undertaking. In fact, the boundaries are different from one problem to the other. There is a tremendous variety in the types of layout problems one may face. This variety is due, partly to the number of ways plant layout problems develop. e.g. A layout problem may arise because of a change in the design of a product, the addition or deletion of a product etc.

The plant layout designer might be called on to cooperate with the product designer, process designer and schedule designer. Plant layout problems can occur in a large number of ways and can have significant effects on the overall effectiveness of the production system. In searching for a plant layout design that satisfices, we must agree on the criteria for evaluating alternative designs.

It is not possible to define a good layout with any precision. However, there are certain criteria which may be satisfied by a good layout, and these are listed below.

1. Maximum flexibility

2. Maximum coordination

3. Maximum use of volume available

4. Maximum visibility

5. Maximum accessibility

6. Minimum distance

7. Minimum handling

8. Minimum discomfort

9. Inherent safety

10. Maximum security

11. Efficient material flow

12. Identification

PRODUCT AND PROCESS LAYOUT TYPES

In a product layout, the product moves down a conveyor or assembly line and through a series of work stations until completed. Thisis the way aotos are assembled, TV sets and ovens areproduced and soft drinks are bottled. Product oriented layout types uses more automated and especially designet equepment than proces layout. The main problem in product layout planning is to balance the output capacity at each work station on the production line so that it is nearly the same,while obtaining the desired amount of the product. Management's goal is to create a smooth, continuous flow along the asseembly line a minimum of idle time at each person's work station. The term most often used to describe this process is assembly line balancing.

The processlayout can simultaneously handle a wide variety of products or services. Here the operations do not justify an assembly line setup. process layout is especially good for handling the manifacture of parts in small lots and for the production of a wide variety of parts in different sizes or forms. A good example of the process layout in a service management is a hospital or clinic, where a continious inflow of patients, each with his her own problem, requires routingthourgh record areas, pharmacies, nursing stations and so on.

In process layout planning, the most common approach is to arrange departments or work centers in the most economical locations. In many facilities, optimal placement in the most economical location means minimizing material handling costs.

COMPUTERIZED LAYOUT PLANNING

The layout approaches stated above is adequate for small problems. Operations sequence analysis method does not suffice for larger problems. When 20 departments are involved in a layout problem, over 600 trillion different department configurations are possible. Fortunately, there are computer programs to handle layouts. The best known and widely discussed of these is called CRAFT (Computerized Relative Allocation of Facilities Technique) , a program that produces good but not always optimal solutions.CRAFT adopts minimization of total handling costs as the criterion. CORELAP (Computerized Releationship Layout Planning), ALDEP (Automated Layout Desing Program) and RMA Comp 1 (Richard Mather & Associates) all adopt some form of proximity criteria.

The choise of program of course depends upon the user's objektives and the particular layout problem being studied. It is important for the

user to understand the features of the program under consideration, and its limitations. This is the reason why computerized layout problems classified according to the way the final layout is generated. According to the first type classification, some algorithms constract the layout by building up a solution "from scratch". Construction Algorithms consist of successive selection and placement of activities (depaartmens) until a layout designe is achived. The second type of algorithm is of the improvement type where a complete existing layout is required initially, and locations of departments are interchanged so as to improve the layout design.

SINGLE FACILITY LOCATION PROBLEMS

As pointed above that not all layout problems justify the use of ALDEP< CORELAP or CRAFT. There are a number of layout and location problems, which involve the location of a single facility in an existing layout or the desing of a layout for a single department.

Qualitative, as well as quantative, factors should be considered in solving facility layout and location problems. However an explicit consideration of both qualitative and quantative factors in developing the optimum solution is a difficult undertaking, because it is difficult to combine them in a quantative manner in order to obtain a satisfactory evaluation of alternative solutions.

A number of layout and location problems, which are solved by using a quantative objective, are then modified based on qualitative considerations. This process is quite satisfactory, and superior to an approach using only qualitative or quantitative considerations.

Here we consider the problem of determining the location of a single new facility with respect to a number of existing facilities. The location we seek is that which minimizes and appropriately defined total cost fuction. Of cource the total cost must be proportional to distance travelled.

A number of interesting one-facility location problems exist and could be solved by the analysis represented here. Some typicle examples of one facility location problems are the location of a

- new lathe in a manifacturing jop shop,

- library in a campus area,

- new classroom building on a collage campus

- copying machine in a library

etc.

Formulation:

m existing facilities are located at known distinct points

P1,....,Pm;

a new facility is to be located at a point X;

costs of a transportation nature are incured that are directly proportional to an appropriately determined distance between the new facility and existing facility i:

Let d(X,Pi) represent the distance travelled per trip between points X and Pi.

wi, represents weigths or the product of cost per unit distance travelled and number of trips maid per year between the new facility and existing facility i,

The total anual cost due to travel between the new facility and all existing facilities thus given as follows:

$$f(X) = \sum_{i=1}^{m} w_i . d(X, P_i) \qquad (1)$$

The one-facility location problem is to determine the location of the new facility, say X*, that minimizes f(x), the total annual transportation cost.

If d(X,Pi)=10 km/trip, wi=20 DM/km x1000 trips/year, then wi.d(X,Pi)=10(20)(1000)=200.000 DM/year

In many applications the cost per unit distance is a constant. Thus, the minimization problem often reduces to a determination of the location that minimizes distance.

Here the question arises as what is meant by an appropriately determined distance. The distance that is the straight line, or euclidian distance.

If the coordinates for the new facility are x and y and for the existing facility i are ai and bi, so that

$$X = (x,y) \text{ and } Pi = (ai,bi)$$

the Euclidian distance between X and Pi is defined as

$$d (X, Pi) = \left((x-ai)^2 + (y-bi)^2 \right)^{1/2} \qquad (2)$$

Euclidian distance applies for some network location problems as well as some instances involving conveyors and air travel. Some electrical-wiring problems and pipeline desing problems are also examples of Euclidian distance problems.

In most machine location problems, travel occurs along a set of aisles arranged in a rectangular pattern parallel to the walls of the building. In such a sutiation, the appropriate distances is referred as rectangular, rectilinear, metropolitan or Manhattan distance. The rectilinear distance between X and Pi is

$$d(X,Pi) = Ix-aiI+Iy-biI \qquad (3)$$

Rectilenear distance is appropriate in some urban location analyses where travel occurs along an orthogonal set of streets.

In some facility location problems, cost is not a simple linear function of distances.It can be a gravity problem type.

THE EUCLIDIAN-DISTANCE LOCATION PROBLEMS

The euclidian problem is also refered as The Steiner-Weber Problem or The General Fermat proble.In fact, a version of the problem for the case m=3 and wi=1, for i=1,2,3, was posed, purely as a problem in geometry, by Fermat early in the 17th C., was solved by Toricelli prior to 1640.The problem was studied by a Swiss Mathematician called Steiner in the 19th C., and by a german economist Weber early in the 20th C.. A duality result was obtained by Fasbender in 1846. However the problem completely solved only in 1963 by H.W.Kuhn.

The euclidian problem may be stated as:

$$\text{minimize } f(x,y) = \sum_{i=1}^{m} wi \left((x - ai)^2 + (y - bi)^2 \right)^{1/2} \qquad (4)$$

$$x,y$$

In order to solve the problem, we have to compute the partial derivatives of (4) and set them to zero. Assuming

$(x,y) \neq (ai,bi), i=1,...,m$, the partial derivatives are

$$\frac{\partial f(x,y)}{\partial x} = \sum_{i=1}^{m} \frac{wi(x-ai)}{\left((x-ai)^2 + (y-bi)^2 \right)^{1/2}} \qquad (5)$$

$$\frac{\partial f(x,y)}{\partial y} = \sum_{i=1}^{m} \frac{wi(y-bi)}{\left((y-ai)^2 + (y-bi)^2 \right)^{1/2}} \qquad (6)$$

Equation (5) and (6) are undefined. Difficulties may arise when the location for the new facility coincides mathematically with the location of some existing facility. If it does not coincide, then the above equations would still give the necessary and sufficient conditions for a minimum location of the new proposed facility. However, there is no such quarantee and a modification of the partial derivative approach is required. This modification is based on the two-tuple R(x,y). If $(x,y) \neq (ai,bi)$, $i=1,...,m$; then the definition is as follows:

$$R(x,y) = \left(\frac{\partial f(x,y)}{\partial x}, \frac{\partial f(x,y)}{\partial y} \right)$$

And if $(x,y) = (ak,bk)$, $k=1,...,m$,

$$R(x,y) = R(ak, bk) = \begin{cases} \left(\dfrac{(uk - wk)}{uk} sk, \dfrac{(uk - wk)}{uk} tk \right), & \text{if } uk > wk \\ (0,0) & \text{if } uk \leq wk \end{cases}$$

where

$$s_k = \sum_{\substack{i=1 \\ \neq k}}^{m} \frac{w_i(a_k - a_i)}{\left((a_i - a_k)^2 + (b_i - b_k)^2\right)^{1/2}}$$

$$t_k = \sum_{\substack{i=1 \\ \neq k}}^{m} \frac{w_i(b_k - b_i)}{\left((a_i - a_k)^2 + (b_i - b_k)^2\right)^{1/2}}$$

$$u_k = \left(s_k^2 + t_k^2\right)^{1/2}$$

The two-tuple $R(x,y)$ is defined for all points in the plane. Kuhn installs that a necessary and adequate condition for (x^*,y^*) to be a least cost new facility location is that $R(x^*,y^*) = (0,0)$. The location of some existing facility, (a_k, b_k) will be optimum, if and only if $w_k \geq u_k$. Whereas we have available necessary and adequate conditions for an optimum solution to the Euclidian problem, we still do not have a way of determining (x^*,y^*).

From setting the expression (5) equal to zero, we obtain

$$x \sum_{i=1}^{m} \frac{w_i}{\left[(x-a_i)^2(y-b_i)^2\right]^{1/2}} = \sum_{i=1}^{m} \frac{w_i a_i}{\left[(x-a_i)^2 + (y-b_i)^2\right]^{1/2}}$$

and letting

$$g_i(x,y) = \frac{w_i}{\left[(x-a_i)^2 + (y-b_i)^2\right]^{1/2}}, \quad i=1,\ldots,m$$

Equation (7) can be given as

$$x = \frac{\sum\limits_{i=1}^{m} a_i g_i(x,y)}{\sum\limits_{i=1}^{m} g_i(x,y)} \qquad \text{Sim. from (6) we obtain} \qquad y = \frac{\sum\limits_{i=1}^{m} b_i g_i(x,y)}{\sum\limits_{i=1}^{m} g_i(x,y)}$$

So long as $g_i(x,y)$ is defined, we can employ the following iterative procedure:

$$x^{(k)} = \frac{\sum\limits_{i=1}^{m} a_i g_i(x^{(k-1)}, y^{(k-1)})}{\sum\limits_{i=1}^{m} g_i(x^{(k-1)}, y^{(k-1)})} \qquad (8)$$

$$y^{(k)} = \frac{\sum\limits_{i=1}^{m} b_i g_i(x^{(k-1)}, y^{(k-1)})}{\sum\limits_{i=1}^{m} g_i(x^{(k-1)}, y^{(k-1)})} \qquad (9)$$

Note that the superscripts denote the iteration number. Thus, a starting value $(x^{(0)}, y^{(0)})$ is required to define $(x^{(1)}, y^{(1)})$

The iterative procedure continous until no improvement is possible or until a location is found which satisfies Kuhn's modified gradient con-

dition. The iterative procedure is guarented to focus to the optimum location and gravity solution is used as the starting value for this procedure.

As an illustirative examaple for the iterative procedure, let

P1=(0,0), P2(0,10), P3(5,10), P4(12,6) and all wi values be equal.

The starting values of $x^{(0)}$ and $y^{(0)}$ is found as follows:

$$x^{(0)} = \frac{0+0+5+12}{4} = 4.25, \quad y^{(0)} = \frac{0+10+0+6}{4} = 4.00$$

Therefore

$$P(x^{(0)}, y^{(0)}) = P(4.25, 4.00)$$

By using the formula gi(x,y) weobtain the following values;

$$g1(x^{(0)}, y^{(0)}) = \frac{1}{[(4.25-0)^2 + (4.00-0)^2]^{1/2}} = 0.71$$

$$g2(x^{(0)}, y^{(0)}) = \frac{1}{[(4.25-0)^2 + (4.00-10)^2]^{1/2}} = 0.136$$

$$g3(x^{(0)}, y^{(0)}) = \frac{1}{[(4.25-5)^2 + (4.00-0)^2]^{1/2}} = 0.246$$

$$g4(x^{(0)}, y^{(0)}) = \frac{1}{[(4.25-12)^2 + (4.00-0)^2]^{1/2}} = 0.125$$

$$\therefore x^{(1)} = \frac{0.71(0) + 0.136(10)+0.246(5)+0.125(12)}{0.71+0.136+0.246+0.125} = 4.023$$

$$\therefore y^{(1)} = \frac{0.71(0) + 0.136(0)+0.2460)+0.125(6)}{0.71+0.136+0.246+0.125} = 3.11$$

Similarly we obtain

$$(x^{(1)},y^{(1)}) = (4.023 , 3.111) , \quad (x^{(6)}, y^{(6)}) = (3.971 , 2.074)$$

$$(x^{(2)},y^{(2)}) = (3.949 , 2.627) , \quad (x^{(7)}, y^{(7)}) = (3.981 , 2.045)$$

$$(x^{(3)},y^{(3)}) = (3.935 , 2.358) , \quad (x^{(8)}, y^{(8)}) = (3.987 , 2.028)$$

$$(x^{(4)},y^{(4)}) = (3.944 , 2.209) , \quad (x^{(9)}, y^{(9)}) = (3.992 , 2.011)$$

$$(x^{(5)},y^{(5)}) = (3.958 , 2.124) , \quad (x^{(10)}, y^{(10)}) = (3.995, 2.011)$$

By examining the values one can easily say that the least -- cost new facilty location is the point $(x^*,y^*) = (4.0,2.0)$.

Before giving a detailed example of the above stated, we must assert that if you place the new facilty close to the optimum, a very good solution is normally realized. When using the iterative solution procedure, we should strive to obtain a very good solution by choosing an appropriate stopping criterion for the procedure.

Example

Dokuz Eylül University has employed a campus plan layout for its two faculties, Faculty of Economics and Administrative Sciences and Faculty of Law. A district heating system is to be installed, which will heat each of four building. Considering the cost of installation and the heat

losses, it is agreed that the cost for the system is proportional to the square of the euclidean distance between the heating facilty and each building. The constant of proportionality is the number of avarage student and /or administrative and academic personnel at each building. The buildings to be served by the heating system are located as follows; P1(12.9 , 23.6), P2 (18.4 , 23.6), P3(22.8 , 23.6), P4 (22.8 , 9.2). The location points are extracted from 1:500 scale campus, taking 2cm =1 unit. The weights taken are the avarage number of students / faculty staff. wa=200, wb=200, wc=2000, wd=1000.

By using the squared Euclidian distance formulas, the least cost location for the central heating facilty is found as follows;

In order to use the iterative algorithm we have to find the starting coordinates.

$$x^{(0)} = \frac{\sum\limits_{i=1}^{4} w_i(a_i)}{\sum\limits_{i=1}^{4} w_i} = \frac{200(12.9)+200+(18.4)+2000(22.8)+1000(22.8)}{200+200+2000+1000}$$

$$x^{(0)} = 21.96$$

$$y^{(0)} = \frac{\sum\limits_{i=1}^{4} w_i(b_i)}{\sum\limits_{i=1}^{4} w_i} = \frac{200(23.6)+200+(23.6)+2000(23.6)+1000(8.2)}{200+200+2000+1000}$$

$$y^{(0)} = 19.07$$

$$\therefore \left(x^{(0)}, y^{(0)}\right) = (21.96 , 19.07)$$

In the first iteration, gi values are calculated first, then by using the formulas given by (8) and (9) $(x^{(1)}, y^{(1)})$ values are found.

$$g_1\left(x^{(0)}, y^{(0)}\right) = \frac{200}{\left[(21.96 - 12.9)^2 + (19.07 - 23.6)^2\right]^{1/2}} = 19.745$$

$$g_2\left(x^{(0)}, y^{(0)}\right) = \frac{200}{\left[(21.96 - 18.4)^2 + (19.07 - 23.6)^2\right]^{1/2}} = 34.713$$

$$g_3\left(x^{(0)}, y^{(0)}\right) = \frac{2000}{\left[(21.96 - 22.8)^2 + (19.07 - 23.6)^2\right]^{1/2}} = 434.101$$

$$g_4\left(x^{(0)}, y^{(0)}\right) = \frac{1000}{\left[(21.96 - 22.8)^2 + (19.07 - 8.2)^2\right]^{1/2}} = 91.723$$

$$\therefore x^{(1)} = \frac{12.9\,(19.745) + 18.4\,(34.713) + 22.8\,(434.101) + 22.8\,(91.723)}{19.745 + 34.713 + 434.101 + 91.723} = 22.19987$$

$$\therefore y^{(1)} = \frac{23.6\,(19.745) + 23.6\,(34.713) + 23.6\,(434.101) + 8.2\,(91.723)}{19.745 + 34.713 + 434.101 + 91.723} = 21.1661$$

Twenty iterative calculations are made by means of a computer program developt by us. The results are :

$$\left(x^{(1)}, y^{(1)}\right) = (22.19987, 21.1661)$$

$$\left(x^{(2)}, y^{(2)}\right) = (22.37342, 22.33783)$$

$$\left(x^{(3)}, y^{(3)}\right) = (22.54508, 22.93641)$$

$$\left(x^{(4)}, y^{(4)}\right) = (22.6595, 23.24577)$$

$$\left(x^{(5)}, y^{(5)}\right) = (22.7241, 23.40980)$$

.

.

.

$$\left(x^{(19)}, y^{(19)}\right) = (22.7999, 23.59997)$$

$$\left(x^{(20)}, y^{(20)}\right) = (22.8, 23.59998)$$

Therefore;

$(x^*, y^*) = (22.8, 23.6)$, which coincides with point C.

Using Kuhn's modified gradient, it can be verified that the point $(x^*, y^*) = (22.8, 23.6)$ is a least-cost new facility location.

$$s3 = \frac{200\,(22.8 - 12.9)}{\left[(12.9 - 22.8)^2 + (23.6 - 23.6)^2\right]^{1/2}} \quad \frac{200\,(22.8 - 18.4)}{\left[(18.4 - 22.8)^2 + (23.6 - 23.6)^2\right]^{1/2}} +$$

$$\frac{1000\,(22.8 - 22.8)}{\left[(22.8 - 22.8)^2 + (8.2 - 23.6)^2\right]^{1/2}} = 400$$

$$t3 = \frac{200\,(23.6 - 23.6)}{\left[(12.9 - 22.8)^2 + (23.6 - 23.6)^2\right]^{1/2}} + \frac{200\,(23.6 - 18.4)}{\left[(18.4 - 22.8)^2 + (23.6 - 23.6)^2\right]^{1/2}} +$$

$$\frac{1000\,(23.6 - 8.2)}{\left[(22.8 - 22.8)^2 + (8.2 - 23.6)^2\right]^{1/2}} = 1000$$

$$u3 = \left[(400)^2 + (1000)^2\right]^{1/2} = 1077$$

. .

. $w3 = 2000$ and $w3 > u3$

The point $(x^*, y^*) = (22.8, 23.6)$ is a least-cost location point.

EUCLİD UZAKLIKLI YERLEŞİM

EUCLID sorunu 17. yüzyılda Fermat tarafından geliştirilmiş ve çözülmüş bir geometri problemidir. Problem daha sonraları Steine-Weber ikilisi tarafından tekrar ele alınmış ve maliyet minimizasyonu açısından mevcut tesislere en iyi uyumu sağlayacak yeni tesisin kuruluş yerinin belirlenmesi sorununa adapte edilmiştir. Bu çalışmada model, tekrar ele alınmış, fakülte binalarına hizmet verecek bir kazan dairesi sorununu çözecek şekilde kurulmuş ve hazırlanan bir bilgisayar programıyla 20 iterasyona kadar çözülmüştür.

KAYNAKÇA

DEMİR, M. Hulûsi: "**Üretim Yönetimi**", c.1, Aydın Yayınevi. İzmir, 1984.

FRANCİS, Richard L., John A. White: "**Facility Layout and Location; An Analytical Approach**", Prentice-Hall Inc., Englevood Cliffs, New Jersey 1974.

KUHN, H. W: "**On a pair of Dual Nonlinear Programms**", Nonlinear Programming (ed. J. Abadie), John Wiley and Sons Inc. New York 1967.

LAUNHARDT, W: Die Bestimmung des zweckmaessigster Standorts einer gewerblichen Anlage, Ztf. VDI, Vol. 26, no. 3, 1982.

NOBLE, B: "**Optimum Localition Problems**", Applications of Undergraduate Matematics in Engineering, Bölüm 2, The Macmillan Co., New York 1967,

SİVAZLİAN, B. D., L. E. Stanfel: "**Analysis of Systems in-Operations Research**", Prentice Hall, New York 1975.