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THE STRUCTURAL PROPERTIES OF THE INTEGER-LINEAR PARAMETRIC PROGRAMMING PROBLEMS WHICH HAVE RIM-PARAMETRIC CASE.

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ÖZET

Genelde doğrusal parametrik proğramlama problemlerinin parametreler uzayına bağlı olarak beş farklı çeşidi vardır. Çalışmada amaç fonksiyon ve sağ-tarf sabitlerinde parametreler (yapısal almayan) ve tam sayı koşullar mevcut doğrusal parametrik proğramlama problemlerin bir kısım cebirsel ve topolojik özellikleri incelendi.

I. INTRODUCTION

The parametric mathematical programming problems are generally denoted over the metric (or compact Hausdorff) spaces whit the point-to-set mappings (set-valued mappings). However, for explaining their properties, we will need some algebraical and topological structures that defined over these spaces. So we will first explain some basic mathematical conceps briefly .

Let F be a non-empty set. İf in F, there are two operations (+) and (.) such that

i-) a,b Ɛ F implies that a+b Ɛ F

ii-) a+b = b+a for a,b Ɛ F

iii-) (a+b)+c = a+(b+c) for a,b,c Ɛ F

iv-) There exists an element o Ɛ F such that a+o = o+a = a for every a Ɛ F

v-) Given a Ɛ F, there exist a,b Ɛ F such that a+b = o.(We will write b as -a)

vi-) a,b Ɛ F implies that a.b Ɛ F

vii-) a.(b.c) = (a.b).c for a,b,c Ɛ F

viii-) a. (b+c) a.b + a.c and (b+c).c = b.a + c.a for a,b,c Ɛ F then the set F is said to be a ring.

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Let F be a ring. If for every a Ɛ F there exists an element ı Ɛ F such that a.I = I.a = a then F is callad a ring with unit and if for every a,b Ɛ F a.b = b.a then F is called a comutavite ring. However if for a.b Ɛ F a.b = o implies that a = o or b = o then F is said to be a domain and if F is acommutative ring and for a,b Ɛ F a.b = o in F implies that a = o or b = o then F is said to be an integral domain. A ring whit unit F is said to be division ring if for $a \neq o$ in F there exists s an element b Ɛ F (usually written as a^{-1}) such that $a.a^{-1} = a^{-1}.a = ı$ and so if F is a commutative division ring then F is said to be a field (HERSTEİN, 1988,p.146-147) , (LANG, 1968,p.43-46) and (JACOBSON,1967, p.53-54).

Let F be a field. If V is an abelian (commutative) group under " + " such that for every $\alpha \varepsilon$ V there exit an element $\propto v$ e V such that

i-) $\propto (v_1 + v_2) = \propto v_1 + \propto v_2$, for $\propto \varepsilon F$, $v_1, v_2 \varepsilon V$

ii-) $(\propto+\beta)v = \propto v + \beta v$ for $\propto , \beta \varepsilon F , v \varepsilon F$

iii-) $\propto (\beta v) = (\propto \beta v)$, for $\propto , \beta \varepsilon F , v \varepsilon V$

iv-) ı.v = v for all v ε V where ı is the unit element of F

then V is called a vector space over a field F (HERSTEİN, 1988, p.210), (LANG,1968, p.86), (JACOBSON,1967,p.3-9) and (JACOBSON, 1967,p.3-4).

Let V be a vector space over F and the mapping II.II be denoted by d : V x V--->R^+ If the following axioms are satisfied then V is called a norm over the space V and (V,II.II) or only V is called a normed space.

NI II v II = o for every v ε V

N2 II v II = o <==> v = o for v ε V

N3 II ∝ v II = I∝I II v II , for v e V , ∝ ε F

N4 II v+w II ≤ II v II + II w II , for every v, w ε V

If the second axiom is not satisfied then II . II is called a seminorm over V and in this case V is called a semi-normed space. However, if a mapping d is denoted by d(v,w) = II v-w II for every v,w ε V where V is a normed space then mapping d is satisfies the following metric axioms and it is called a metric and (V,d) or only V is called a metric space (DİXMİER, 1984, p.90-93).

MI $d(v,w) = o$ for every $v,w \varepsilon V$

M2 $d(v,w) = o \Leftrightarrow v=w$ for every $v, w \varepsilon V$

M3 $d(v,w) = d(w,v)$ for every $v,w \varepsilon V$

M4 $d(v,w) \leq d(v,q) + d(q,w)$ for every $p.v.w \varepsilon V$

Suppose that the mapping d is given as above and the second axiom (M2) is realized as only " $d(v,w) = o \Rightarrow v=w$, for every $v,w \varepsilon V$ ".Then d is called a pseudometric (KELLY, 1967, p.II8-119). Let V be a vector space and W a subset of V. If the following axioms are satisfied then W is called a subvector aspace of V.

SVI If $v,w \varepsilon W$ then $v+w \varepsilon W$

SV2 If $c \varepsilon V$ then $o \varepsilon W$

SV3 If $v \varepsilon W$ and $o \varepsilon F$ then $cv \varepsilon W$

Let V be a vector space, $w_1,....,w_n \varepsilon V$ and W be the set of oll elements $w = x_1 w_1 +...+ x_n w_n$ with $x_i \varepsilon F$. Then W is a subspace of V and W is called The subspace generated by $w_1,.....,w_n$ and $w_1,.....,w_n$ is also called generators for this subspace (LANG, 1968, p.87) and (LANG, 1972, p.43).

Let X be a set and τ be a collection of subsets of X. If the following axioms are satisfied then τ is a topology for X. (HOCKİNG-YOUNG,1988,p.5), (HALL-SPENCER II,1965,p.53), (KELLY, 1967, p.37) and (MOORE, 1964, p.21).

TOPI Both the empty set and X are members of τ

TOP2 The union of each subcollection of is a member of . τ

TOP3 The intersection of each subcellection of is a member of τ

However let $\mathfrak{B}$ be a family of subsets of a set X. Then the family of all sets, aech of which is the union of a subfamily of $\mathfrak{B}$, is called the family generated by $\mathfrak{B}$ and is denetod by $\mathfrak{B}$. It is generally said that $\mathfrak{B}$ generates $\mathfrak{B}$ (MOORE,1964,p.33). Let (x,d) be $\mathfrak{B}$ a metric space and β be the famly of r-spheres in X(for all p in X and all pozitive real numbers r]. Then $\mathfrak{B}$ is base for a topology generated by the

family $\mathfrak{B}$ is called the metric topology for X induced by d (KELLY, 1967,p.II9) and (MOORE, 1964, p.46-47).

Let τ be a topology over X. If for any $x,y \varepsilon X$ $(x \neq y)$, there are two neighbourhoods U_x and V_y such that $x \varepsilon U_x, y \varepsilon V_y$ and $U_x \cap V_y = \varnothing$ then (X,τ) is called Hausdorff or T_2 space (HALL-SPENCER II,1965,p.61),(HOCKİNG-YOUNG, 1988,p.38) and (KELLY, 1967,p.67).Suppose that A is a subset of the topological space X. Then A is compact iff each open cover for A has a finited subcover for A (MOORE, 1964,p.67) and if A is compact then A is closed and bounded and also every sequence in A has a subsequence which converges to a point of A (MARSDEN, 1974,p.62). Thus the topological space X is compact iff each familly of closed sets which has the finite ihtersection property has a non-void intersection or each net (or sequence) in X has a clus er point (KELLY,1967,p.135). However any topological space X is sequentially compact iff every sequence in X has a convergent subsequence and X iscountable compact iff every countable open cover has a finite subcover or each sequence in it has a cluster point (KELLY, 1967,p.162) and (WİLLARD,1970,p.125)and also a topological space X is local compact iff each point of X has at least one compact neighbourhood (KELLY, 1967,p.146) and (MOORE, 1964,p.92). Now let X be any topological space and E be a subset of X. İf there exist a sequence $\{C_n\}$ of compact sets such that $E = \bigcup_{n=1}^{\infty} C_n$ then E is said to be σ - compact (KELLY,1967,p.172) and (HALMOS,1988,p.4).

Let X be any topological space and A be a subset of X and also $x \varepsilon X$. If every open set U containing x contains some points of A other than x then the point $x \varepsilon X$ is called an occumulation point of A. Sometimes it is called cluster point or limit point (KELLY, 1967,P.41) and (MARSDEN.1974,p.39). If $A^C = X - A = X \setminus A$ is open or there exits an open neighbourhood U_x of $x \varepsilon X$ such that $x \varepsilon U_x \subseteq A$ then A is the complement of A (KELLY, 1967,p.23) and (MARSDEN,1974,p.37). If for each $\varepsilon > o$ there exitsts $M > o$ $(M \varepsilon R)$ such that for every $x \varepsilon A$, $\| x \| \leq M$ then A is called bounded (MARSDEN,1974,p.86). If the topological space X is not union of two disjoint, non-empty open sets then it is called a seperated space (HOCKİNG-YOUNG,1988,p.14) and it is connected iff it is not the union of two non-void seperated subsetsand also a subset Y of $(X\tau)$ is connected iff the topological space Y with the relative topology is connected (KELLY,1967,p.53). Let $Y \subseteq X$ where (X, τ) is a topological space and $\mathfrak{F}$ be the

family of sets such that a set belongs to $\mathcal{F}$ (iff there is a member U in τ such that $W = Y \cap U$). Then the family $\mathcal{F}$ is called the relativization of τ to Y and therefore the family $\mathcal{F}$ is sometimes called the relative topolgy for Y and also $(Y, \mathcal{F})$ is a subspace of (x,τ) (KELLY, 1967,p.50-51) and (MOORE, 1964,p.38-39).A topological space X is called a complate space iff every Cauchy seguence has a limit in X and if the space is a complate normed space then it is called a Banah space (DİXMİER,1984, p.104).

Suppose that x and Y are metric (or compact T_2) space and $\wp$ (Y) is a collection of the proper subsets of Y. Then the mapping F which is denoted by F : X ---> $\wp$ (Y) or F : XxY ---> $\wp$ (Y) = 2^Y is called a point-to-set mapping or set-valued mapping (DELAHEYE-DENEL, 1979,p.I-3).We can say that various proterties of the point-to set mappings had been investigated in many different ways by the several suthority people (DENEL, 1979, p.48-68), (POLAK-KLESSİG, 1979,p.179-190), (POLOVİNKİN, 1975,p.405-410), (CONTESE),1968, p.51-68), (SAVARAGI-NAKAYAMA-TANINO, 1986, p.21-24) and (DEMYANOV-VASİLEV, 1985, p.II-20). Thus we will be content with point out that the point-to-set mappings are continuous (BANK-GUDDAT-KLATıE-KUMMER-'ı AMMEK, 1983, p.25).

Suppose that X is a normed topological space and A is any subset of X.If for every $x,y \varepsilon A$ and $\alpha \varepsilon (0,I)$, $\alpha x + (I-\alpha) y \varepsilon A$ then A is said to be a convex subset of X(KELLY, 1967, p.103),(JAMESON, 1974,p.203) and (HOCKİNG-YOUNG,1988,p.199). Suppose that X and Y are normed topological spaces, $A_1,.....,A_n$ are convex subsets of X and f : X---->Y is any mapping. Then the following properties are satisfied over the space X (JAMESON, 1974,p.203-210), (PRESSİNİ-SULLİVAN-UHL, Jr, 1988, p. 37-42) and (JOHN, 1986, p.3-18).

i-) $A_i + A_j = \{x+y \mid x \varepsilon A_i, y \varepsilon A_j\}$; i, j = 1,..., n ($i \neq j$) is convex.

ii-) $A = \bigcap_{i=1}^{n} A_i$ is convex.

iii-) If f : X -->Y. is a linear mapping then for every i (i=1,...,n) $f(A_i)$ is convex.

iv-) If $X_j \varepsilon A_i$, i=1,....,n, j=1,....,k and aj $\alpha_j \varepsilon R^+$ such that

$$\sum_{i=1}^{n} \alpha_i = I \text{ then } x = \sum_{j=1}^{n} \alpha_j x_j \varepsilon A_i$$

Suppose that X be a normed topological space, A is a subset of X and $\mathfrak{B}$ = { B | B⊆ X, B is convex}. Then B ε $\mathfrak{B}$ is called a convex hull if $\bigcap_{B\varepsilon\mathfrak{B}} B$ is the smallest convex set that contains A. Then we can clearly say that the convex hull of the set A⊆X is the intersection of all convexsets that contains A and the convex hull of the set A is generally shown by Co (A) (JAMESON, 1974, p.204) and (PRESSİNİ-SULLİVAN-UHL, Jr, 1988, p. 42). If X and Y are any topological al spaces and M = {XεX | Ax ≤ b, b ε Y, A is mxn-matrix } then M is compact, connected and convex subset (clearly half-space) of X (PRESSİNİ-SULLİVAN-UHL, Jr, 1988, p.40)

Suppose that X is a normed space, f: X --> **R** ∪ {- ∞, + ∞ } is any mapping* and α ε (0,1). For every x, y ε X f [αx + (1-α)y] ≤ αf (x) + (1-α) f (y) then fis called a convex mapping, if f [αx + (1-α) y < αf (x) + (1-α) f (y) then f is called a strictly, convex mapping, if f [αx + (1-α)y] ≥ αf (x) + (I-α) f(y) then f is called a concave mapping and f [αx + (1-α)y] ≥ αf (x) + (I-α) f(y) then f is called a strictly concave mapping (BANK-GUDDAT-KLATTE-KUMMER-TAMMER, 1993, p.22), (PRESSİNİ-SULLİVAN-UHL, Jr, 1988, p. 45-46), (SAATY-BRAM, 1964, 1964, p. 100), (BEALE, 1986, p.4-5), (MİNOUX, 1988, p. 14-15) and (EVTUSHENKO, 1985, p. 7). However, if "for every x,y ε X, ∇ f (x)^T (y-x) > 0 => f(x) ≤ f (y)where ∇ f (x) is the gradient of f (x)" then f is called a pseudo-convex mapping and if for every α ε **R**, , { x | f (x) ≤ α} is a convex set then f is called a quasi-convex mapping and also if -f (x) is aquasi convex mapping then f (x) is a quasi-concave mapping. So a mapping f is aquasi-convex mapping iff for every x, y ε X and α ε (0.1) max { f (x), f (y)} ≤ f [αx + (1+α)y] and if f is a quasiconvex mapping then f is a convex mapping (ZOUTENDJİCK, 1975, p.267-269).

Suppose that X and Y are normed topological spaces and f : X--> Y is any mapping. If for any given ε>o there exists a > o (depending on f, x and ε) such that for all xεX, x, ≠ x_o || x-x_o|| < implies that || f(x)-b || < ε then b is the limit of f at the point x ε X and if for all ε > o there exists a > o such that for all x ε X, || x-x_o || < imples || f (x) - f(x_o)|| < ε then f is called a continuouus mapping at the point x_o ε X (MARSDEN, 1974, p. 79-80). However if for each y ε Y, f^{-1} (y, +∞) is open in x then of is lowersemicontinuous and if for each y ε Y, f^{-1} (-∞,y) is open in X then f is uppersemicontinuous (JAMESON, 1974, p.49). The mapping f is said to be closed iff the image of every closed subset of A is closed subset of Yand f is said to be open iff the image of every open subset of is open sub-

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$\overline{R}$* = R ∪ {-∞, +∞} is the extended set of all real numbers.

implies $f(x_1) \neq f(x_2)$ then f is said to be one-to-one mapping and f is a one-to-one mapping then f is continuous iff the inverse of the mapping f^{-1}; Y-->X is open. Thus the mapping f is said to be a homeomorfizm iff f is one-to-one, continuous and f^{-1} is continuous. In this case, the spaces X and Y are called the homeomorfic spaces (clearly the space X is homeomorfic to Y) (DİXMİER, 1984, p. 20-22), (MORE, 1964, p.69), (HOCKİNG-YOUNG, 1988, p.4) and (HALL-SPENCER II, 1965, p.74).

Let x and Y be normed topological spaces and A be a subset of X.If there is a linear mapping that is denoted by $Df(x_o)$: X --> Y such that

$$\lim_{X \to x_o} \frac{\| f(x) - f(x_o) - Df(x_o)(x-x_o) \|}{\| x-x_o \|} \qquad \text{.....(I.I)}$$

then the mapping f is said to be differentiable at $x \varepsilon X$ (MARSDEN, 1974, p. 154) and D is called the gradient of the mapping f at the point $x_o \varepsilon X$ and also it is generally denoted by $\nabla f(x_o)$. If $f : \mathbf{R}^n \to \mathbf{R}^m$ then

$$\nabla f(xo) = \left[\frac{\partial f(x_o)}{\partial x_1}, \ldots\ldots, \frac{\partial f(x_o)}{\partial x_n} \right] \qquad \text{.....(1.2)}$$

and $\dfrac{\partial f(x_o)}{\partial x_i}$ is the partial derivative of the mapping f with respect to the ith component of $x \varepsilon X$. If f is a real-valued mapping that is defined in a neighbourhood of $x_o \varepsilon X$ and $e \varepsilon X$ is aunit vector then

$$\frac{d}{dt} f(x_o + te) \Big|_{t=o} = \lim_{t \to 0} \frac{f(x_o + te) - f(x_o)}{t} \qquad \text{.....(1.3)}$$

is called the directional derivative of f at $x_o \varepsilon X$ then the directional derivatives of f ara generally given by

$$\nabla f(x_o)\, e = \lim_{t \to 0} \frac{f(x_o + te) - f(x_o)}{t} \qquad \text{.....(1.3)}$$

Let X be a normed topological space, $A \subseteq X$ be open set and $f : A \to \mathbf{R}$. If there is a neighbourhood U_x of $x_o \varepsilon A$ such that for every $x \varepsilon U_x$, $f(x) \geq f(x_o)$ then $f(x_o)$ local minimum for f and similarly if $f(x) \leq f(x_o)$ then $f(x_o)$ is alocal maximum for f and also if f is differentiable at $x_0 \varepsilon U \subseteq A$ and $\nabla f(xo) = o$ then x_o is a critical point for f (MARSDEN, 1974, p.183).

critical point for f (MARSDEN, 1974, p.183).

2. GENERAL FORMULATION AND CLASSIFICATION

It is seen that the parametric mathematical programming problems had been generalized with any maximizing or minimizing problem in many further studies. The main criteria in classifying them is their functional structures and therefore we will try to classify tthem according to this main criteria by denoting over the metric (or compact T_2) spaces.

We can say that the parametric mathemetical programming problems were denoted in warious ways. But we will accept the following definition for giving the general formulation of them (GAL, 1984, p.44) and (BANK-GUDDAT-KLATTE-KUMMER-TAMMER, 1983, P. II-12).

DEFİNİTİON 2.I : Let X and Y be metric spaces and the mappings f and M be given as $f: X \times \Lambda \to \mathbf{R} \cup \{-\infty, +\infty\}$ and $M : \lambda \to M(\lambda)$, $M(\lambda) \subseteq X$, $\forall \lambda \in \Lambda$. Then the parametric mathematical programming problem can be deno ted as follows; ..... (2.1)

$$P_\lambda : \inf \{ f(x,\lambda) \mid x \in M(\lambda) \} ; \lambda \in \Lambda \quad \text{..... (2.2)}$$

where the restrictions set mapping is

$$M : \lambda \to M(\lambda); \lambda \in \Lambda \quad \text{..... (2.3)}$$

the optimal value mapping is

$$\varphi : \lambda \to \varphi(\lambda) = \inf_{x \in M(\lambda)} f(x,\lambda); \lambda \in \Lambda \quad \text{..... (2.4)}$$

and the optimal set mapping is

$$\Psi : \lambda \to \Psi(\lambda) = \{x \in M(\lambda) \mid f(x,\lambda) = \varphi(\lambda)\}$$

It is clear that P_λ defines a minimizing problem if the expressions (2.1) and (2.3) are only defined by the greates lower bounds (infimums) and P_λ defines a maximizing problem if these expressions are only defined by the least upper bounds (supremums or sup). We can say that the parametric mathematical programming problems are the most general forms of the mathematical pogramming

Because it is clearly seen from definition 2.1 that a non-parametric mathematical programming problem can be obtained for every value of the parameter $\lambda \varepsilon \Lambda$. Therefore we will accept that the non-parametric problems are the special manners of the parametric problems. So we can deno te the non-parametric mathematical programming problem asfollows (MİNOUX, 1988, p.I) and (FİACCO-MC CORMİCK, 1968, p.I).

DEFİNİTİON 2.2: Let X be a metric space and the mappings f, $\{g_i\}$ and $\{h_j\}$ be real-valued mappings that are denoted over X where i=1,..., m and j=1,..., n. Then the nonparametric mathematical programming problem can be denoted as follows,

$$\begin{aligned} &\text{Max (min) } f(x) \\ &\text{Subject to : } g_i(x) \geq 0;\ i=1,\ldots, m \\ &\qquad\qquad h_j(x) = 0;\ j=1,\ldots, n \end{aligned} \qquad \ldots\ldots(2.5)$$

We can say that a non-parametric mathematical programming problem can be explained as determining a vector $x = (x_1,\ldots x_n) \varepsilon X$ that solves the problem in (2.5) and it is sometimes denoted by

$$P : \underset{(\max)}{\text{Min}} \{f(x) \mid g_i(x) \geq 0, h_j(x) = o;\ i=1,\ldots, m, j=1,\ldots, n\} \qquad \ldots\ldots(2.6)$$

If the mappings f; $\{g_i\}$ and $\{h_j\}$ are the linear then the problem in (2.5) is called a non-parametric linear programming problem and it is generally denoted as follows (HADLEY,1963, p.71), (COOPER-STEİNBERG, 1974, p.I), (TAHA, 1967, p.30) and (MİNOUX, 1988, p.27).

DEFİNİTİON 2.3 : Let X and Y be normed topological spaces (clearly $X = \mathbf{R}^n$ and $Y = R^m$) and $f : X \to \mathbf{R}^+$, $g_i : X \to Y$, i=1,..., m be linear mappings such that $f(x) = c^T x$, $[g_1(x),\ldots, g(x)]^T = Ax$ where c, $x \varepsilon X$, $b \varepsilon Y$ and A is mxn matrix. Then the most general form of the linear non-parametric programming problem can be expressed as follows,

$$\begin{aligned} &\text{Max (min) } f(x) = c^T x \\ &\text{Subject to} \quad : Ax \{\leq, =, \geq\} b \\ &\qquad\qquad x \geq 0 \end{aligned} \qquad \ldots\ldots(2.7)$$

The mapping f (x) = cTx is generally called the objective function the set Ax {≤, =, ≥} b is called the restrictions and x≥0 is called the potitiveness conditions of the problem. However the problem can sometimes expressed by

$$P : \underset{(\min)}{\max} \{c^T x \mid Ax \{\leq, =, \geq, \} b, x \geq 0\} \qquad \ldots\ldots(2.8)$$

As it is known that if any one of the mappings f, $\{g_i\}$ and $\{h_j\}$ is not linear then the problem in (2.5) is a non-parametric non-linear programming problem and the general form of it is expressed as in (2.5) or (2.6).

Now, we can denote the linear parametric and non-linear parametric programming problems by using the definition 2.I. If the restrictions set mapping and the optimal set mapping in definition 2.I are linear then P_λ is a linear parametric programming problem and any one of these mappings is not linear then P_λ is a non-linear parametric programming problem. So we will first explain the non-linear parametric programming problems.

Let X and Λ be metric (or compact T_2) spaces and I={1,...,m}, J={1,...,k}. Suppose that for every $i \varepsilon I$, $j \varepsilon J$ the mappings f, g_i and h_j are denoted form $X \times \Lambda$ into **R** where atleast one of these mapping is non-linear. If the mapping φ and Ψ are denoted as in definition 2.I where

$$M(\lambda) = \{ x \varepsilon X \mid g_i(x,\lambda) \geq o,\ h_j(x,\lambda) = o,\ i\varepsilon I,\ j\varepsilon J \} \qquad ...(2.9)$$

then

$$P_\lambda : \underset{(min)}{max} \{ f(x,\lambda) \mid x \varepsilon M(\lambda) \}\ \lambda\varepsilon\Lambda\} \qquad ...(2.10)$$

defines a non-linear parametric programming problem. The set

$$U = \{ \lambda \varepsilon \Lambda \mid \varphi(\lambda) \neq \varnothing \} \qquad ...(2.11)$$

is said to be a solvability set where $\Lambda \subseteq U$ (BANK-GUDDAT-KLATTE-KUMMER-TAMMER, 1983, p.I5) If for every $\lambda \varepsilon \Lambda$, $M(\lambda) \neq \varnothing$ then P_λ is a constrained non-linear parametric programming problem and if for every $\lambda\varepsilon\Lambda$, $M(\lambda) = 0$ then P_λ is a uncostrained non-linear parametric programming problem and it is denoted by

$$P_\lambda : \underset{(min)}{max} \{ f(x,\lambda) \mid x \varepsilon X \}; \lambda \varepsilon \Lambda \qquad ...(2.12)$$

However constrained non-linear parametric programming problem is denoted as in (2.10) where $M(\lambda)$ is given as in (2.9) (PİERRE-LOEWE, 1975, p. 3-5), (KOWALIK-OSBERNE, 1968, p. I-4), (BEİGHTLER-PHİLLİPS-VİLDE, 1979, p. 288-292) and (LOOTSMA, 1975, p. 3I3-3I5). So we will give the definition of non-linear parametric programming problem more specifically that make use of in application frequently(FİACCO-1973, p.3), (JANIN, 1984, p.III), (KOJİMA-HİRABAYASI, 1984, p.I50-I5I) and (ROBİNSON, 1987

p.3).

DEFİNİTİON 2.4 : Let X and Y be metric(or compact T_2) spaces and I={1,...,m}, J={1,...,k}. Suppose that the mapping f,g_i and h_j are denoted from $X \times \Lambda$ into $\mathbf{R}$ and continuous over $X \times \Lambda$, $\forall\, i \in I, j \in J$. Then a non-linear parametric programming problem can be denoted as follows,

$$P\lambda : \max(\min)\ Z(\lambda) = f(x,\lambda)$$
$$\text{Subjecj to} \quad : g_i(x,\lambda) \geq 0 \qquad i \in I, j \in J, \lambda \in \Lambda \qquad \ldots(2.13)$$
$$h_j(x,\lambda) = 0$$

İn this case, the set (restrictions set mapping)

$$M(\lambda) = \{x \in X \mid g_i(x,\lambda) \geq o, h_j(x,\lambda) = o, i \in I, j \in J\}, \lambda \in \Lambda \quad \ldots(2.14)$$

is called the set of the feasible solutions, the set

$$G(\lambda) = \{ (\lambda,x) \mid x \in X \}; \quad \lambda \in \Lambda \qquad \ldots(2.15)$$

is called the graph of the point-to-set mapping M(λ),

$$P_\lambda : \inf_{(\sup)} \{ f(x,\lambda) \mid x \in M(\lambda) \} \qquad \ldots(2.16)$$

is called the marginal or perturbation mapping, the set

$$S(\lambda) = \{ x_o \in M(\lambda) \mid f(x,\lambda) = P(\lambda) \}; \quad \lambda \in \Lambda \qquad \ldots(2.17)$$

is called the set of the optimal solutions, the set

$$S = \{ \lambda \in \Lambda \mid M(\lambda) \neq \varnothing \} \qquad \ldots(2.18)$$

is called the set of feasible perturbations (JANIN, 1984, p.II0-I26) and (GAUVİN-DEBAU, 1984, p.I0I-I02).

Let $X = R^n \times R^k$ and $\Lambda = \mathbf{R}^s$. Suppose that $f(x,\lambda) = f_{(x)} + \lambda_o a_o(x)$; $\forall \lambda \in \Lambda, \exists \lambda = \lambda_o$ and $g_i(x,\lambda) = g_i(x) + \lambda_i b_i(x)$, $h_j(x,\lambda) = h_j(x) + \lambda_{j+m} q_j(x)$; $i \in I = \{1,...,m\}$, $j \in J = \{1,...,p\}$, $\{\lambda_o, \lambda_1, ..., \lambda_m, \lambda_{m+1}, ..., \lambda_{m+p}\} \subseteq \Lambda$ and a_o

, b_i and c_j are the scalar-valued (real-valued) mappings. Then we can denote the non-linear parametric programming problem which is used in practice as follows (FİACCO-MC CORMİCK, 1968, p.34),

$$P_\lambda : \max(\min)\ f(x)+\lambda_o a_o(x)$$

$$\text{Subject to : } g_i(x) + \lambda_i b_i(x) \geq o\ ;\ i=1,\ldots,m \qquad \ldots(2.19)$$

$$h_j(x) + \lambda_{j+m}\ c_j(x) = o\ ;\ j=1,\ldots,p$$

At present we will try to give the definitions of the various kinds of the linear parametric programming problems briefly. Let X and Λ be metric spaces $I=\{1,\ldots,m\}$, $J=\{1,\ldots,n\}$ be index sets and the linear mappings f and g_i. $i \in I$ be denoted by $f: X\times\Lambda \rightarrow \mathbf{R}+$ and $g_i : X\times\Lambda \rightarrow \mathbf{R}$ such that $f(x,\lambda)=[c(\lambda)]^T_x$ and $g(x,\lambda)=[a_i(\lambda)]^T_x$ where a_i, $x \in X$ and $\lambda\in\Lambda$. Then the restrictions set mapping of a linear parametric programming problem can be denoted as follows (BANK-GUDDAT-KLATTE-KUMMER-TAMMER, 1983, p.43),

$$M(\lambda) = \{x \in X \mid g_i : X\times\Lambda \rightarrow \mathbf{R}, i\in I\}\ ;\ \lambda\in\Lambda \qquad \ldots(2.20)$$

It is clear that $g_i(.\ ,\lambda) = a_i(\lambda)$ and $a_i(\lambda)$ is the i^{th} row-vector of the matrix $A(\lambda)$. Therefore we can write $M(\lambda) = \{\ x\in X \mid A(\lambda)\ x = b(\lambda)\ \}$, $\lambda\in\Lambda$ where $b\in Y$ ($X=\mathbf{R}^n$, $Y=\mathbf{R}^m$) and thus the general form of the linear parametric programming problem is denoted by

$$P_\lambda : \max_{(\min)}\{\ [c(\lambda)]^T x \mid A(\lambda)x = b(\lambda),\ x \geq o\ \}\ ;\ \lambda\in\Lambda \qquad \ldots(2.21)$$

In this case the vectors b and c and the matrix A are not the mapping (function)s of the parameter $\lambda\in\Lambda$. The parameter λ shows the changes in the original parameters A,b and c of the problem. Therefore we will denote the linear parametric programming problem more specifically such that the PARAMETERS SPACE Λ of the problem is a product space of the three subspaces T,U and V

DEFİNİTİON 2.5 : Let $X=\mathbf{R}^n$ and $\Lambda = T\times U\times V$ where $T,U \subseteq \mathbf{R}^s$ and $V \subseteq \mathbf{R}^r$. Then the most general form of the linear parametric programming problem is denoted as follows where A is mxn matrix, $c,x \in \mathbf{R}^n$, $b \in \mathbf{R}^m$ and $\lambda=(t,u,v) \in T\times U\times V=\Lambda$.

$$P_\lambda : \max(\min)\ Z(t,u,v) = [c(t)]^T x$$

Subject to : $A(u)x = b(v)$

$x \geq o$ ...(2.22)

If $T,U,V \subseteq \mathbf{R}$ (clearly T,U,V are intervals) then P_λ is called a linear scalar parametric programming problem and $T,U \subseteq R^s$ and $V \subseteq \mathbf{R}^r$ $(s, r > 1)$ then P_λ is called a linear multi parametric programming problem. If $\Lambda = TxUxV = \{o\}$ then P_λ is a linear non-parametric problems can be investigated in four different forms as depend on the changes in original parameters A,b and c of the problem and therefore we will formulate them in four different ways.

I-) $\Lambda = T\,x\{o\}x\{o\}$ where $U=V=\{o\}$ and $\lambda=(t,o,o)$

$P_\lambda : \max(\min)\, Z(t) = [c(t)]^T x$

Subject to : $Ax = b$

$x \geq o$ ...(2.23)

II-) $\Lambda = \{o\}x\{o\}x\,V$ where $T = U = \{o\}$ and $\lambda=(o,o,v)$

$P_\lambda : \max(\min)\, Z(v) = c^T x$

Subject to : $Ax = b(v)$

$x \geq o$ ...(2.24)

III-) $\Lambda = \{o\}xU\;\; x\{o\}$ where $T=V=\{o\}$ and $\lambda=(o,u,o)$

$P_\lambda : \max(\min)\, Z(u) = c^T x$

Subject to : $A(u)x = b$

$x \geq o$

IV-) $\Lambda = T\,x\{o\}x\,V$ where $U=\{o\}$ $\lambda=(t,o,v)$ ...(2.25)

$P_\lambda : \max(\min)\, Z(t,v) = [c(t)]^T x$

Subject to : $Ax = b(v)$

$x \geq o$ ...(2.26)

The linear parametric programming problem in (2.26) is generally known as RIM-parametric case(GAL,1984, p.47) and (ORCHARD-HAYS, 1966, p.I7I) and for the pronlem which has the RIM-parametric case, one of the following conditions is satisfied,

i-) t and v are mutually independent.

ii-) t and v are dependent.

If the case (ii) is valid then there exists an implicit mapping $g(t,v)=o$ such that

$t=g(v)$ or $v=\tilde{g}(t)$. It is clear that the restrictions set mappings $M(\lambda)$ of the promlem in (2.24), (2.25) and (2.26) are the function of $\lambda \varepsilon \Lambda$ but this is not valid for the problem in (2.23).

As it is known that the above classification has only the finite linear parametric programming problems. The second kind is the semi-infinite linear parametric programming problems and the other (third) kind is infinite linear parametric programming problems. We will only be content with defining the second kind of them.

Let T be a compact Hausdorff (T_2) space, C(T) be the space of all continuous mappings b: T---> $\mathbf{R}$ normed by $\| b \|_\infty = \sup | b(t) |$ and $C(T,R^n)$ be the space of all continuous mappings $B : T \dashrightarrow \mathbf{R}^n$ normed by $\| B \|_\infty = \sup_{t \varepsilon T} \|B(T)\|_2$ where $\| . \|_2$ denotes the Euclidean norm in $\mathbf{R}^n$. If the space $\mathfrak{B}$ is denoted by

$$= C(T, \mathbf{R}^n) \times C(T) \times \mathbf{R}^n \qquad ..(2.27)$$

where $\sigma = (B,b,p) \varepsilon \mathfrak{B}$ such that $B \varepsilon C(T, \mathbf{R}^n)$, $b \varepsilon C(T)$ and $p \varepsilon \mathbf{R}^n$ then we can denote the semi-infinite linear parametric $\mathfrak{B}$ programming problem as fallows (BROSOWSKİ, 1984, p.I8-I9).

DEFİNİTİON 2.6 : Let the space be given as in (2.27). Then the parametric mathematical programming problem

$$LM(\sigma) : \min \langle p,x \rangle$$
$$\text{subject to} : \langle B(t), x \rangle = b(t) \; ; \quad \forall t \varepsilon T \qquad ...(2.28)$$

is called the semi-infinite linear parametric programming problem.

The space is said to be the parameter space of the problem and for every $\sigma \varepsilon \mathfrak{B}$, $\| \sigma \|_\infty = \| B \|_\infty + \| b \|_\infty + \| p \|_\infty$. Howewer the set of all feasible points is

$$Z_\sigma = \{ x \varepsilon \mathbf{R}^n \mid \langle B(t), x \rangle = B(t), t \varepsilon T \} \qquad ...(2.29)$$

The minimum value is

$$E = \inf \{ p(x) \varepsilon \mathbf{R} \mid x \varepsilon Z_\sigma \} \qquad ...(2.30)$$

and the set of minimal points is

$$P_\sigma = \{ v \varepsilon Z_\sigma \mid \langle p,v \rangle = E_\sigma \} \qquad ...(2.3I)$$

3. THE BASIC CONCEPTIONAL PROPERTIES

The integer linear programming problems are generally subdivided into two groups such as the mix-integer linear programming problems and the pure-integer linear programming problems. Therefore it is possible that every one on the linear parametric programming problems which are formulated in four different ways in (2.23), (2.24), (2.25) and (2.26) can be subdivided into two problems.

For investigating the properties of the integer linear parametric programming problems which have the RIM-parametric case we will now try to give the definition of them by using the formulation in (2.26) as the pure-integer and the mixed-integer problem (BANK- GUDDAT- KLATTE - KUMMER - TAMMER, 1983, p.54).

DEFİNİTİON 3.I : Let $X=R^n$, $\Lambda=R^m$, A be a (m,n)-matrix, Z be the set of all integers and $o \leq s \leq n$. Suppose that the linear parametric programming problem is given as follows,

$$(P_s(p,\lambda)) : \underset{(\sup)}{\inf} \{ p^T x \mid x \varepsilon G(\lambda) \} ; (p,\lambda) \varepsilon \mathbf{R}^{n+m} \qquad ...(3.I)$$

i -) If s=n and the constraint set of the problem is

$$G(\lambda) = \{ x \varepsilon \mathbf{R}^n \mid Ax \leq \lambda, x_1, ..., x_n \varepsilon Z^s \} ; \lambda \varepsilon \mathbf{R}^m \qquad ...(3.2)$$

then it is said to be a pure-integer linear parametric programming problem.

ii -) If $s < n$ and the constraint set of the problem is

$$G(\lambda) = \{ x \varepsilon \mathbf{R}^n \mid x = (z,y),\ Bz + Cy \leq \lambda,\ z \varepsilon Z^s \};\ \lambda \varepsilon \mathbf{R}^m \qquad \ldots(3.3)$$

where the subvectors z and y consist of the variables $x_1, \ldots, x_s$ and $x_{s+1}, x_{s+2}, \ldots, x_n$ and the (m,s)-matrix B, (m,n-s)-matrix C are the submatrices of the m,n matrix A then it is said to be a mixed-integer linear parametric programming problem.

Let the integer linear parametric programming problem be given as in definition (3.I)

Then the set

$$\mathbf{B}_s = \{ \lambda \varepsilon \mathbf{R}^m \mid G(\lambda) \neq \varnothing \} \qquad \ldots(3.4)$$

where the restricted constraint set mapping $G : \mathbf{R}^m \dashrightarrow \mathbf{R}^n$, $\lambda \dashrightarrow G(\lambda)$ is continuous is said to be feasible parameter set and if the set $M(\lambda)$ is defined by

$$M(\lambda) = \{ x \varepsilon \mathbf{R}^n \mid Ax \leq \lambda \};\ \lambda \varepsilon R^m \qquad \ldots(3.5)$$

and the set $\mathbf{B}_o$ is defined by

$$\mathbf{B}_o = \{ Az \mid z \varepsilon \mathbf{R}^n \} \qquad \ldots(3.6)$$

then $\mathbf{B}_s \subset \mathbf{B}_o$ and $G(\lambda) \subset M(\lambda)$. Howewer the set

$$U_s = \{ (p,\lambda) \mid \Psi_s (p,\lambda) \neq \varnothing \};\ \lambda \varepsilon \mathbf{R}^m,\ o \leq s \leq n \qquad \ldots(3.7)$$

İs said to the solubility set where $\Psi_s(p, \lambda)$ is restricted optimal set mapping and $\varphi_s(p, \lambda)$ is the resricted extreme (optimal) value mapping and the set

$$Q(\lambda o) = \{ \lambda \varepsilon \ \mathbb{B}_s \mid T(\lambda) = T(\lambda_o) \} \qquad \ldots(3.8)$$

is said to be the local stability set where $\lambda_o \varepsilon \ \mathbb{B}_s$ is a fixed parameter point and $T(\lambda) = P_z G(\lambda) \subset Z^s$.

Thus we can investigate some properties of the parametric problem which is given in definition 3.I with the theorems that have some topological and algebraical structure such as convexity, connectedness, continuity, et.c.

THEOREM 3.I: The feasible parameter set $\mathbb{B}_s$ is for all s, $o \leq s \leq n$, a connected set equal to the union of a countable family of convex polyhedral cones each of which is obtained by a translation of the m-dimensional cone

$$K = \{v \varepsilon R^m \mid Cy \leq v\} \qquad \ldots(3.9)$$

Proof: For the definition of the set $G(\lambda)$, we can write the set $G(z,\lambda)$ as follows

$$G(z,\lambda) = \{x \varepsilon \mathbf{R}^n \mid x=(z,y)\ ,\ Cy=\lambda - Bz\} \qquad \ldots(3.10)$$

where $z \varepsilon Z^s$ and $\lambda \varepsilon \mathbf{R}^m$ are fixed. In this case, it is clearly seen that there is a vector $v \varepsilon \mathbf{R}^m$ such that $v=\lambda - Bz \varepsilon K$. Then we have "$G(z,\lambda) \neq \varnothing$ $(\lambda - Bz) = v \varepsilon K$". $G(\lambda) \neq \varnothing$, $G(z,\lambda) \neq \varnothing$ and $G(\lambda)$ is the union of poly hedra $G(z,\lambda)$. So we can write

$$s = \bigcup_{z \varepsilon Z^s} (\mathbb{B}z+K) \qquad \ldots(3.11)$$

At present we will show that $\mathbb{B}$ is connected. Let z and $\bar{z}$ be any two points of Z^s. From the definition of $\mathbb{B}s$ in (3.11), there exist the two non-void sets (bz+K) and ($B\bar{z}$+K) such that

$$(Bz+K) \cap (B\bar{z}+K) \neq \varnothing \qquad \ldots(3.12)$$

each of them is closed (half space) and the subset (subspace) of their union. If this union is denoted by $\mathfrak{B}$ then $\mathfrak{B} \subseteq \mathfrak{B}_s$ and the sets (Bz+K) and (Bz̄+K) are closed in $\mathfrak{B}$. So they are seperated (MOORE,1964,P.40), (KELLY,1967,p.52-54), (HOCKİG-YOUNG,1988,P.14) and (HALL-SPECER,II,1964,P.80 AND 151). Then $\mathfrak{B}$ is not the union of the two seperated sets and therefore $\mathfrak{B}$ is connected

HİNT: The feasible parameter set is generally nonconvex.

THEOREM 3.2: If the matrix A=(B,C) has only rational elemets than the feasible parameter set $\mathfrak{B}_s$ is closed for $o < s \leq n$.

Proof: To show closedness of $_s$,we will prove that the property" $\mathfrak{B}_s$ " is closed iff for every sequence $\lambda_t \varepsilon \mathfrak{B}_s$ which converges to the limit (λ_o) lies in $\mathfrak{B}_s$ is satisfied (MARSDEN,1974,P.45).Let $\{\lambda_t\} \subset \mathfrak{B}_s$ be arbitrary sequence with the property $\lambda_t \to \lambda_o$. If $G(\lambda) \neq \varnothing$ then $\lambda_t \varepsilon \mathfrak{B}_s$. Now we must show that $G(\lambda_o) \neq \varnothing$. For each t=1,2,....

$$G(\lambda_t) = K_G(\lambda_t) + U_G \qquad (3.13)$$

where $K_G(\lambda_t)$ are compact sets which are denoted by $K_G : \mathbf{R}^m \to 2^{R^n}$,

$$K_G(\lambda_t) = \{x \varepsilon \mathbf{R}^n \mid x = v + \sum_{i=1}^{\tau} \alpha_i u_i,\ v \varepsilon K(\lambda_t),\ \alpha_i \varepsilon [0,1], i=1,\ldots,\tau\ x_1,\ldots x_s \varepsilon Z\} \qquad (3.14)$$

and U_G is the set of all lattice points of the recession cone $M(\lambda_t)$ wich is denoted by

$$U_G = \{u \varepsilon \mathbf{R}^n \mid Au \leq o,\ u_1, \ldots, u_s \varepsilon Z\} \qquad \ldots(3.15)$$

and so for λ_o, the equality in (3.2) is satisfied. $G(\lambda_o) \neq \varnothing$ and $_s$ is closed.

COROLLARY 3.1: The feasible parameter set $\mathfrak{B}_s$ is compact.

Proof:From the above theorem 3.2, $\mathfrak{B}_s$ is closed.We know that $\mathfrak{B}_s \subset \mathfrak{B}_o \subset \mathbf{R}^n$. Then for every $z \varepsilon \mathfrak{B}_s$, there exists a $M \varepsilon \mathbf{R}$ (M>o) such that $\| z \| = | z | < M$ and thus $\mathfrak{B}_s$ is bounded. If a set is closed and bounded then it is compact (MARSDEN,1974, p.62). Therefore $\mathfrak{B}_s$ is compact.

We have investigated the properties (such as closedness, boundedness and compactness) of the problem in definition 3.1 by proving the theorem 3.1, theorem 3.2 and corollary 3.1. Now we will try to explain the o ther properties under the stability conditions by proving the following theorems (BANK-GUDDAT-KLATTE-KUMMER-TAMMER,1983,p.145-153).

LEMMA 3.1: Let the matrix A=(B,C) be rational, $\lambda_o \varepsilon$ be an arbitrary fixed parameter point and s satisfying o< s ≤ n and be o therwise arbitrary. The set

$$D(\lambda_o) = \bigcap_{z \varepsilon T(\lambda_o)} (Bz+K) \qquad \ldots(3.16)$$

is then an m-dimensional convex polyhedral cone. Furthermore, there then exists a $v_o \varepsilon \mathbb{B}_s$ such that

$$D(\lambda_o)\ v_o + K \subset_s \qquad \ldots(3.17)$$

holds, K is the polyhedral cone given by (3.9).

Proof: Suppose that K has a face of the smallest dimension. If the mapping $T(\lambda_o)$ is finite then $D(\lambda_o)$ is a m-dimensional convex polyhedral set (cone) because of (3.12). Let $T(\lambda_o)$ be not finite. Then for each subset $\Omega \subset T(\lambda_o)$, the following relation is satisfied

$$D(\lambda_o) \subset \bigcap_{z \varepsilon \Omega} (Bz+K) \qquad \ldots(3.18)$$

We will now show that a finite subset $\Omega' \subset T(\lambda_o)$ exists sich that the inclusion may be reversed as

$$\bigcap_{z \varepsilon \Omega} (Bz+K) \subset D(\lambda_o) \qquad \ldots(3.19)$$

Because of the rationality of the matrix A, the set $G(\lambda_o)$ can be shown by

$$G(\lambda_o) = K_G(\lambda_o) + U_G \qquad \ldots(3.20)$$

where $K_G(\lambda_o)$ and U_G are respectively given in (3.14) and (3.15). If the mapping Ω' is choosen as the projection $\Omega' = P_\lambda K_G(\lambda_o)$ of the compact set $K_G(\lambda_o)$ onto the subspace spanned by the integer variables then the following properties are satisfied.

$$\Omega' \neq \varnothing \ , \ \Omega' \subset T(\lambda_o) \ , \Omega' \text{ finite}$$

We will show that the reverse of (3.18) is satisfied for $\Omega = \Omega'$. Suppose that $\bar{\lambda} \in \bigcap_{z \in \Omega'} (Bz+K)$ is an arbitrary point such that for every $z \in \Omega'$,

$G(z, \bar{\lambda}) \neq \varnothing \iff (\bar{\lambda}-Bz) \in K$ implies

$$G(z, \bar{\lambda}) \neq \varnothing \qquad \ldots(3.21)$$

Because of (3.15) and (3.21) we must demonstrate that $\bar{\lambda}$ is contained in $D(\lambda_o)$ which is equivalent to

$$G(z, \lambda) \neq \varnothing \ , \ \forall z \in T(\lambda_o) \qquad \ldots(3.22)$$

Suppose that $\bar{z} \in T(\lambda_o)$ is an arbitrary point and $\bar{z} \notin \Omega'$. Then because of $\Omega' \subset T(\lambda_o)$, $\bar{z} \in T(\lambda_o) \setminus \Omega'$. If $T(\lambda_o)$ is defined by

$$T(\lambda_o) = P_z G(\lambda_o) \subset Z^s \qquad \ldots(3.23)$$

then $G(z,\lambda_o) \neq \varnothing$. Let $\bar{x} \in G(z,\lambda_o)$ be choosen by $x = (\bar{z}, \bar{y})$. From $G(z,\lambda_o) \subset G(\lambda_o)$ and (3.20) we can find the two points $\hat{x} = (\hat{z}, \hat{y}) \in K_G(\lambda_o)$ and $\hat{u} = (v,w) \in U_G$ such that

$$\begin{aligned} \bar{x} &= \hat{x} + \hat{u} \\ \bar{z} &= \hat{z} + v \\ \hat{z} &\in \Omega' \end{aligned} \qquad \ldots(3.24)$$

(*) For more knowledge, see (LANG,1972,p.109-114) and (HALMOS,1967,p.16-18)

By (3.21),G $(\hat{z},\hat{\lambda}) \neq \varnothing$ and we may choose a point $\hat{x}=(\hat{y},\hat{w}) \varepsilon G(\hat{z},\hat{\lambda})$. So we can obtain the point x as follows

$$x=(z,y)\ ,\ \hat{x}=\hat{x}+\hat{u}\ ,\ z=\hat{z}+\hat{v}\ ,\ y=\hat{y}+\hat{w} \qquad ...(3.25)$$

By the definition of the polyhedron G (z,λ) in (3.10) and U_G in (3.15), we have $Cy \leq \lambda - Bz, Cw \leq -Bv$ and

$$C(\hat{y}+\hat{w}) = \bar{\lambda} - B(z+v) \qquad ...(3.26)$$

However, to gether with (3.23), this implies $z=\bar{z}$, thus $x \varepsilon G(\bar{z},\bar{\lambda})$ and consequently $G(\bar{z},\bar{\lambda})$ is non-empty. Since $\bar{z} \varepsilon T(\lambda_o)$ was arbitrary we have demonstrated the validity of (3.22). Hence

$$D(\lambda_o) = \bigcap_{z \varepsilon \Omega'} (Bz+K) \qquad ...(3.27)$$

Ω' is finite and so $D(\lambda_o)$ is an m-dimensional convex polyhedral cone. Now we will show that (3.17) holds. K has a face of the smallest dimension and thus each cone (Bz+K) has a vertex set which also applies to $D(\lambda_o)$.For each $v_o \varepsilon \mathbb{B}_s$ (3.17) holds.

LEMMA 3.2 : Let the matrix A = (B,C) be arbitrary, s satisfy $o<s \leq n$ and otherwise be arbitrary and $Q(\lambda_o)$ be the local stability set corresponding to an arbitrary fixed parameter point $\lambda_o \varepsilon \mathbb{B}_s$. Furthermore, assume $T(\lambda_o) \neq Z^s$. There exists a finite set $\Omega \subset Z^s \setminus T(\lambda_o)$ such that

$$Q(\lambda_o) = D(\lambda_o) \setminus \bigcap_{z \varepsilon \Omega'} D^z \qquad ...(3.28)$$

holds wher $D(\lambda_o)$ is the m-dimensional convex polyhedral cone in (3.15) and

$$D^z = Bz + K\ ,\ z \varepsilon Z^z \qquad ...(3.29)$$

Proof : From the definition of the local stability set in (3.8), we can write the fallowing equality

$$Q(\lambda_o) = \bigcap_{z \varepsilon T(\lambda_o)} (Bz+K) \setminus \bigcap_{z \varepsilon Z^s \setminus T(\lambda_o)} (Bz+K) \qquad ...(3.30)$$

and because of (3.15) and (3.29), $Q(\lambda_o)$ can be olso write as follows

$$Q(\lambda_o) = \bigcap_{z \varepsilon Z^s(\lambda_o) \setminus T(\lambda_o)} [D(\lambda_o) \setminus D^s] \qquad ...(3.31)$$

$Q(\lambda_o) \neq \varnothing$ and thus for every $z \varepsilon Z^s(\lambda_o) \setminus T(\lambda_o)$

$$D(\lambda_o) \setminus D^z \neq \varnothing \qquad ...(3.32)$$

holds which implies $(\lambda_o) \subset D^z$. Let $v_o \varepsilon$ **B** be a point satisfying (3.19). If we define

$$\tilde{D}(\lambda_o) = v_o - K \qquad ...(3.33)$$

Then we can prove the relation

$$Bz \varepsilon D(\lambda_o') \Leftrightarrow D(\lambda_o) \; D^z \qquad ...(3.34)$$

If $D(\lambda_o) \subset D^z$ then there exists a $v \varepsilon K$ such that

$$v = Bz + v \Rightarrow Bz = v_o - v \varepsilon D(\lambda_o) \qquad ...(3.35)$$

On the o ther hand $Bz \varepsilon D(\lambda_o)$ implies the existence of a $v \varepsilon K$ with the property

$$Bz = v_o - v \Rightarrow v_o \varepsilon D^z \Rightarrow D(\lambda_o) \subset D^z \qquad ...(3.36)$$

Thus (3.34) is proved. Baceuse of (3.32) and (3.34) the following inclusion must be hold

$$Z^s \setminus T(\lambda_o) \subset Z^s \setminus \{ z \varepsilon Z^s \mid Bz \varepsilon D(\lambda_o) \} \qquad ...(3.37)$$

For arbitrary $\Omega'' \subset Z^s \setminus T(\lambda_o)$, the following inclusion is valid.

$$\bigcap_{z \varepsilon \Omega'} [D(\lambda_o) \setminus D^z] \subset \bigcap_{z \varepsilon Z^s \setminus T(\lambda_o)} [D(\lambda_o) \setminus D^z] \qquad ...(3.38)$$

Now we need only demons trate the existence of a finite set Ω'' contained in $Z^s \setminus T(\lambda_o)$ and for which the reverse of (3.28) holds. Because of the rationality of the matrix C there

$$K = \{ \lambda \varepsilon \mathbf{R}^m \mid F \; \lambda \geq 0 \} \qquad ...(3.39)$$

which gives the representation

$$D(\lambda_o)\} = \{\lambda \varepsilon R^m \mid F\ \lambda \geq f_o\} \qquad \ldots(3.40)$$

for the cone $D(\lambda_o)$. Let F be a (p,m)-matrix such that (3.39) and (3.40) do not contain redundant inequalities. If the rows of the matrix F are defined by f_i, i=1,...,p then at the same time the hyperplanes

$$H_i = \{\lambda \varepsilon R^m \mid f_i^T = f_{io}\} \qquad i=1,\ldots,p \qquad \ldots(3.41)$$

are boundary hyperplanes of $D(\lambda_o)$ and $D(\lambda_o)$. Let the sets $\mathcal{S}$ and H_i^+ are denoted by

$$\mathcal{S} = \{\lambda \varepsilon R^m \mid Bz = \lambda, z \varepsilon Z^s\} \setminus D(\lambda_o) \qquad \ldots(3.42)$$

$$H_i^+ = \{\lambda \varepsilon R^m \mid f_i^T \lambda > f_{io}\} \qquad \ldots(3.43)$$

then $\cap H_i$ are the non-empty sets that we interest where $i \varepsilon I$ is an arbitrary index.

$$I = \{i \varepsilon \{1,\ldots,p\} \mid H_i^+ \cap \mathcal{S} \neq \varnothing\} \qquad \ldots(3.44)$$

Let $i \varepsilon I$ be an arbitrary index. Then there exists a $\lambda_i \varepsilon H_i^+ \cap \mathcal{S}$ with the property

$$d(\lambda_i, H_i) = \min \{d(\lambda, H_i) \mid \lambda \varepsilon H_i^+ \cap \mathcal{S}\} > o \qquad \ldots(3.45)$$

We willnow prove this indirectly. Suppose that no $\lambda_i \varepsilon H_i^+ \cap \mathcal{S}$ exists with the property in (3.45). Then there is a sequence $\{\lambda_t\} \varepsilon H_i^+ \cap \mathcal{S}$ such that

$d(\lambda_t, Hi) \to \mathcal{S} \geq o$ which is equivalent to

$$|f_i^T \lambda_t - f_i| \to o \qquad \ldots(3.46)$$

Because of (3.42) and the definition of $H_i^+ \cap \mathcal{S}$, each λ_t has the form $Bz_t = \lambda_t$ where $z_t \varepsilon Z^s$. If the elements of the matrix B and the companent of f_i are respectively denoted by

$$b_{ik} = \sum_{j=1}^{m} f_{ij}\, b_{jk} \qquad i=1,\ldots,p \quad ,k=1,\ldots,s \qquad \ldots(3.47)$$

and f_{ij} then the relation (3.46) is equivalent to

$$\text{"}\ \forall \varepsilon > o\ ,\ \exists\, t'\ \ni \forall\, t \geq t' \Rightarrow |\sum_{k=1}^{s}{}' \ b_{ik}\, z_{kt}| < \varepsilon \text{"} \qquad \ldots(3.48)$$

where $z_{kt} \varepsilon Z$, k=1,..., s , t=1,2,.. This contradicts Kronecker's approximation theorem (MARSDEN,1974, p.465) and (HEWITT-STROMBERG,1965,p.293) and there exists a $\lambda_i \varepsilon H_i^+ \cap \mathcal{S}$ satisfying (3.46).

Let the set $_i$ be defined as follows for any iεı.

$$\mathcal{S}\ i = \{\ \lambda_i \varepsilon\ H^t_i \cap \mid d\ (\lambda, H_i) = D\ (\lambda_i, H_i)\} \qquad \ldots.(3.49)$$

If there exists a $\lambda \varepsilon\ \mathcal{S}$ i such that $\lambda \varepsilon\ D\ (\lambda)$ then we choose a $z \varepsilon\ Z^s$ satisfying $Bz = \lambda\ z$ and denote it by z_i. On the other hand i f $\mathcal{S}$ i c $D\ (\lambda_0)$ then we determine a $\lambda \varepsilon\ \mathcal{S}_i$ with the property

$$d\ (\hat{\lambda}, v_o) = \min\ \{\ d\ (\lambda, v_o) \mid \lambda \varepsilon\ _i\ \} \qquad \ldots.(3.50)$$

Now we can choose a $z\ \varepsilon\ Z^s$ satisfying $\hat{\lambda} = Bz$ which we call z_i. Then from the choice of z_i we can write the following properties,

$$D\ (\lambda\ o) \setminus D^{zi} \subset D^{Z}, \forall\ Z \varepsilon\ Z^s,\ \lambda = Bz \varepsilon\ H^{+}i\ \cap \mathcal{S} \qquad \ldots.(3.51)$$

If we proceed in this fashion for all i ε I and define

$$\Omega'' = \{z_i \mid i \varepsilon\ I\} \qquad \ldots.(3.51)$$

then Ω is obviously finite and because of (3.51), (3.26) holds.

COROLLAYRY 3.2 : If in lemma 3.2 the condition that A is rational is lelated then in general a finite set Ω" with the property in (3.26) does not exist.

Proof : This follows directly from the proof of lemma 3.2 since kronecker's approximation theorem implies that parameter points λ_i contained in $H^+_i \cap \mathcal{S}$ i ε I arbitrary, may lie arbirtrarily close to the hyper plane H_i.

THEOREM 3.3 : If the matrix A = (B,C) is rational and s is arbitrary apart from $o < s \leq n$ then the following statements are true.

I-) If $\lambda_o\ \varepsilon$ $\mathbb{B}_s$ then the local stability set $Q\ (\lambda o)$ in (3.8) has the properties.

i-) $\lambda \varepsilon\ Q\ (\lambda_o) ==> Q\ (\lambda_o)$

ii-) $\lambda \varepsilon\ \mathbb{B}_s \setminus Q\ (\lambda_o) ==> Q\ (\lambda) \cap Q\ (\lambda_o) = \varnothing$

iii) Q (λ_o) is star-shaped and in general neither open nor closed.

2-) The feasible parameter set s may be decomposed in to a countable system of disjoint local stability sets Q (λ_o).

Prof : I-) The property (f) f2ollows from the definition of Q (λ_o). If (ii) were false then there would exist a $\lambda \varepsilon Q(\lambda) \cap Q(\lambda_0)$ which according to (3.8) implies $\hat{T}(\lambda) = T(\lambda)$. Since then $\lambda \varepsilon Q(\lambda o)$ follows we have a contradıcation. In order to prove (iii) we will take lemma 3. I and lemma 3.2 into account and write Q (λ_o) in the following form,

$$Q(\lambda_o) = (v_o + K) \setminus \cup_{z \varepsilon \Omega} D^z \qquad \ldots(3.53)$$

Then we will show that the line segment S (λ, v_o) is contained in Q (λ_o) is contined in Q (λ_o) for arbitrary $\lambda \varepsilon Q(\lambda_o)$. Let K be a cone with a face of the smallest dimension. Now we will first demonstrate that for all $\lambda \varepsilon Q(\lambda_o)$, and

$$V_o \varepsilon \lambda = K \qquad \ldots(3.54)$$

$$(\lambda\text{-}K) \cap \cup_{z \varepsilon \Omega''} D^z \neq \varnothing \qquad \ldots(3.55)$$

This mean that each cone (λ-K) seperates the point v of the vertex se of D (λ_o) in (3.8), from the set $\cup D^z$. Each $\lambda \varepsilon$ Q (λ) is contained in D (λ_o), hence there exis ts a $v \varepsilon K$ with the property $\lambda = v_o + v$ and (3.54) follows. If (3.55) were false then we could find a $z\ \Omega'' \varepsilon$ and $v \varepsilon K$ such that $Bz + v \varepsilon \lambda\text{-}K$. Then this would imply the existence of a $v \varepsilon k$ which gives $\lambda = Bz + v = v+v$ and thus $\lambda \varepsilon \cup_{z \varepsilon \Omega''}$ Dz. This last contradicts $\lambda \varepsilon Q(\lambda_0)$ and therefore (3.55) is correct. We next show that

$$Q(\lambda_o) \cap (\lambda\text{-}K) = D(\lambda_o) \cap (\lambda\text{-}K) \qquad \ldots(3.57)$$

holds for arbitrary $\lambda \varepsilon Q(\lambda_o)$. Then from (3.26) we have

$$Q(\lambda_o) \cap \lambda\text{-}K) = [\, D(\lambda_o) \cap (\lambda\text{-}K)\,] \setminus (\cup_{z \varepsilon \Omega''} D^z \cap (\lambda\text{-}K)) \cap D(\lambda_o)]$$

(*) A star-shaped subset of R^n is defined as "A subset SCR^n is said table star - Shaped if there exists a point $z \varepsilon S$ Such that $S(x,Z) = \{\, y \varepsilon R^m \mid y = \alpha x + (1\text{-}\alpha) z,\ \alpha \varepsilon (0.1)\,\}$ $\subset$, $\forall\ x \varepsilon S$ holds"

since λ is contained in Q (λ_o) in (3.55) and thus (3.56) follows. Because of (3.54) we have

$$v_o \varepsilon (\lambda\text{-}K) \cap D(\lambda_o) \qquad .....(3.58)$$

Since K contains the zero point of $\mathbf{R}^n$ we can alsı find $\lambda \varepsilon (\lambda\text{-}K) \cap D(\lambda_o)$. The convexity of ($\lambda$-K) D ($\lambda$)o) implies that the line segments S (λ, v_o)) is contained in the same.. Because of (3.56), s (λ, v_o) lies entirely in Q (λ). The second part of (ii) follows immediately fro the representation in (3.26) of Q (λ_o) and the fact that λ_o is always contained in Q (λ).

2-) This follows from (ii) of I, lemma 3.2 and the fact that the system of finite subsets of Z^s is countable.

COROLLARY 3.3 : If the matrix A = (B,C) is rational and s, $o < s \leq n$, is arbitrary then the restriction $G_{Q(\lambda_o)}$ of the point-to-set mapping defined by

$$G(\lambda) = \{ X \varepsilon \mathbf{R}^n \mid Ax = \lambda, x_1, ..., x_s \varepsilon Z \} \qquad .....(3.59)$$

is continuous.

Prof : We know that the restriction of any point-to-set mapping is a set-valued mapping. Thus from the continuity properties of the set-valued mappings (BANK-GUDDAT-KLATTE-KUMMER-TAMMER, 1983, p. 25-28) and the poperties of the restriction of any mapping (HALL-SPENCER II, 1965, p.46 and 71-72). (HEWİTT-STROMBERG, 1965, p: 7-12), (HOCKİNG-YOUNG, 1988, p.4), (KELLY, 1967, p. 10-11) and 18), (LANG, 1968, p. 17-21 and 144-148) and(MARSDEN, 1974, p.5), the proof is clearly seen as the result of preceding theorems.

TEOREM 3.4 : Let the matrix A = (B,C) be rational and s, $0 < s \leq n$, be arbitrary. The stability set U_s of (P_s (p, λ)) is then non-empty, closed and connected and

$$U_s = -U^P \times \mathbf{B}_s \qquad .....(3.60)$$

holds, where U^P is the polar cone of the recession cone U and $\mathbf{u}_s$ is the feasible parameter set.

Prof : Since (P_s (p,λ)) is soluble, $U_s \neq \varnothing$ follows. We will first show that

if A is rational then

$$U^P_G = U^P \qquad \text{....(3.61)}$$

is satisfied for U_G. For all s satisfying $o<s \leq n$, $U_G \subset U$ and therefore we will need only de monstrate $U^P_G \subset Up$. If $u \varepsilon U$ is choosen arbitrarily then we can write

$$u = \sum_{i=o}^{n} \alpha_i u_i \qquad \alpha_i \geq 0,\ u_i \varepsilon U_G,\ i=1,\ldots,n, \qquad \sum_{i=1}^{n} \alpha_i = 1 \qquad \text{....(3.62)}$$

Because of rationality of A, for an arbitrary $v \varepsilon U^P_G$ then we have

$$V^T_u = V^T(\sum_{i=1}^{n} \alpha_i u_i) = \sum_{i=0}^{n} \alpha_i (v^T_{u_i}) \leq 0 \qquad \text{....(3.63)}$$

Which verifies (3.61). Now we will consider (3.60). If $(p, \lambda) \varepsilon U_s$ then $G(\lambda)$ is non-empty because this is the case for $\Psi_s(p,\lambda)$ which implies $\lambda \varepsilon$ $_s$. For arbitrary $x \varepsilon \Psi s(p,\lambda)$

$$P^T x \leq P^T x, \ \forall\ x \varepsilon G(\lambda)$$

holds. Since the point $x + \alpha u$ is always CONTAİNED in $G(\lambda)$, if $x \varepsilon G(\lambda)$ $\alpha \varepsilon Z_\tau$ and $u \varepsilon U_G$ are

arbitrary, it follows that $p^T x \geq o$ must hold for all $u \varepsilon U_G$ and this implies $p \varepsilon - U_G^p$. We have $\mathbf{u}_s \subset - U_G^p x_s$ from (3.6I). Assume now $(p,\lambda) \varepsilon U^p x \mathbf{u}_s$. An arbitrary $x \varepsilon G(\lambda)$ can be represented as

$$x=x+u \ , x \varepsilon K_G(\lambda) \ , \ u \varepsilon U_G \qquad ...(3.64)$$

and then we have

$$p^T u = p^T x + p^T u = p^T x = \inf_{x \varepsilon K_G(\lambda)} p^T x \qquad ...(3.65)$$

Since $K_G(\lambda)$ is non-empty and compact and is a subset of $G(\lambda)$ it follows that $\Psi_s(p,,\lambda) \neq \varnothing$ and thus $- U^p x \mathbf{B}_s \subset \mathbf{u}_s$. Since U^p is a convex polyhedral cone, from theorem 3.I and theorem 3.2, the remaininding assertions follow.

THEOREM 3.5 : Let the matrix A=(B,C) be rational and s, $o<s \leq n$, be arbitrary. The following statements are then true.

i- The restriction of the extreme value function $\varphi_s \setminus_{\mathbf{u}_s}$ is continuous on the solubility set $\mathbf{u}_s$.

ii-) If $Q(\lambda_o)$ is the local stability set corresponding to an arbitrary fixed parameter point $\lambda_o \varepsilon \mathbf{B}_s$ then the restriction of the extreme value function $\varphi_s \setminus - U^p x Q(\lambda_o)$ is continuous on the set $- U^p x Q(\lambda_o)$, and likewise the restriction of the optimal set mapping $\Psi_s \setminus_{U^p x Q(\lambda_o)}$ is uppersemicontinuous (H-u.s.c) on $-U^p x Q(\lambda_o)$.

Proof : (i) follows immediately from theorem 3.2 and (ii) holds since the conditions of corollary 3.I.

4. CONCLUSION

In this study, first, some general topological and algebraical concepts have been explained briefly and then the relations which are between the original (strucrural) parameters and the other parameters of the integer-linear parametric programming problems which have RIM-parametric case were stated under the stability conditions by expressing as the theorems.

RIM-PARAMETRİK DURUMA SAHİP TAMSAYILI DOĞRUSAL PARAMETRİC PROGRAMLAMA PROBLEMLERİNİN YAPISAL ÖZELLİKLERİ

Genelde doğrusal parametrik programlama problemlerinin parametreler uzayına bağlı olarak beş farklı çeşidi vardır. Çalışmada amaç fonksiyon ve sağ-taraf sabitlerinde parametreler (yapısal almayan) ve tam sayı koşullar mevcut doğrusal parametrik programlama problemlerinin bir kısım cebirsel ve topolojik özellikleri incelendi.

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